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Entrepreneurship and Asymmetric Information in Input Markets

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ENTREPRENEURSHIP AND ASYMMETRIC INFORMATION IN INPUT MARKETS

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ABSTRACT

Entrepreneurs starting new firms face two sorts of asymmetric information problems. Information about the quality of new investments may be private, leading to adverse selection in credit markets. And, entrepreneurs may not observe the quality of workers applying for jobs, resulting in adverse selection in labor markets. We construct a simple model to illustrate some consequences of new firms facing both sorts of asymmetric information. Multiple equilibria can occur. Stable equilibria can be in the interior, or at a corner in which no entrepreneurs enter. Stable interior equilibria can involve involuntary unemployment, as well as credit rationing. Equilibrium outcomes mismatch workers to firms, and will generally result in an inefficient number of new firms. With involuntary unemployment, there will be too few new firms, but with full employment, there may be too many or too few. Taxes or subsidies on new firms and employment can be used to achieve a second-best optimum. Alternative information assumptions are explored.

Key Words: entrepreneurship, asymmetric information, adverse selection

JEL Classification: D82, G14, H25

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1. Introduction

New firms and the entrepreneurs that initiate them are beset by problems of asymmetric information with respect to their prospects for success, as well as with respect to the quality of labor they are able to hire and their ability to obtain credit on good terms. The fact that new entrepreneurs are to a large extent indistinguishable from one another means that creditors are unable to tailor financial terms to entrepreneurs' project qualities. As well, since they are hiring workers for the first time, they do not have the experience to discern the quality of potential workers and whether they will be a good match for the particular projects being initiated. This puts new firms at a significant disadvantage with respect to existing firms whose track records have been proven, who may have internal finance, and who have had a chance to sort out good or suitable workers from bad or unsuitable ones. These problems naturally lead to the question of whether public policies should actively encourage the entry of new entrepreneurial firms. That is the focus of this paper.

The literature has recognized in a piecemeal way some of the problems that new firms face due to asymmetric information, and has come to some surprising results. The seminal paper of Stiglitz and Weiss (1981) studied the adverse selection problems that arise when banks are unable to distinguish high- from low-quality projects and must offer the same financial terms to all. In their case, the expected return could be observed, but not the riskiness of individual projects of a given expected return. In this setting, too few projects would be financed—those with the highest risk—suggesting a subsidy on the financing of new firms. Moreover, the possibility of credit rationing existed which exacerbated the underinvestment. Subsequently, de Meza and Webb (1987) considered the case in which banks could observe ex post project returns but not the probability of success. In this setting, the findings of Stiglitz and Weiss were reversed: there would be overinvestment in low-probability projects and no possibility of credit rationing, leading to a presumption of taxing new firms. These results have been generalized by Boadway and Keen (2005) to allow for more general patterns of project characteristics in the pool, and to allow for alternative forms of finance. What emerges is a general presumption of overinvestment as low-risk investments opt for debt finance and high-risk ones for equity finance.[1]

Asymmetric information has also been the focus of the venture capital (VC) literature where the financing of new entrepreneurs is combined with managerial advice. Here, the emphasis has been more on moral hazard problems associated with the effort of both the VC and the entrepreneur. Keuschnigg and Neilsen (2003) have argued that these moral hazard problems can be addressed by a tax on new firms combined with a reduced capital gains tax. Dietz (2002) has added adverse selection to the VC problem and allowed entrepreneurs to choose between VC financing (with managerial advice) and bank financing. He finds that high-risk projects choose the former and low-risk projects the latter, but that too many low-risk projects end up being financed by VCs.

There has been a limited amount of attention paid to the consequences for new firms of imperfect information on other markets. Weiss (1980) considers the case of adverse selection in labor markets when firms cannot observe the quality of workers they hire. He shows that workers will tend to be drawn from the bottom of the skill distribution—since a common wage is paid regardless of quality—and too few will be hired. Moreover, there is a possibility of excess labor supply, or involuntary unemployment, in equilibrium. A subsidy on employment would be welfare-improving in this context. The Weiss model focuses entirely on adverse selection in labor markets: there is no heterogeneity of entrepreneurs and there is no uncertainty of project success. Presumably other forms of asymmetric information in labor markets would particularly affect new firms as well, such as unobservable effort (Shapiro and Stiglitz, 1984) or search problems (Diamond, 1982).

There are potentially many other ways in which the entry of new firms is rendered inefficient because of asymmetric information or externalities. Some of these include signaling problems, knowledge externalities, and strategic barriers to entry. The potential consequences of various sources of inefficiency for tax policy toward new firms are surveyed in broad terms in Boadway and Tremblay (2005). Rosen (2005) reviews the empirical effects of existing policies on entrepreneurship in the United States.

[1] This overinvestment result is in sharp contrast to the consequences of asymmetric information for already existing firms. Myers and Majluf (1984) argue that managers of existing firms will pass up good projects when new equity finance is used if insiders have more information about the value of a firm's projects that outside investors.

Our purpose is to study how adverse selection in both credit markets and labor markets affects equilibrium and efficiency in the formation of new firms, and to consider the consequences for policy. To do so, we develop a simple but rather specific model designed to capture the main features of information asymmetry facing new firms, while at the same time avoiding needless complications. This asymmetry is two-sided: potential entrepreneurs do not know quality of individual workers, and workers do not know quality, or ability, of new entrepreneurs. And, banks and governments know neither.

The model we use builds on the one used by de Meza and Webb (1987) to study adverse selection in credit markets by adding an employment dimension. Entrepreneurs of varying ability each hire a fixed number of workers who are of varying quality. If successful, an entrepreneur's firm produces a fixed output, where the possibility of success depends jointly on the ability of entrepreneurs and the quality of workers. Entrepreneurs have no initial wealth, so must rely on credit to finance their operations. Equilibrium will involve the best entrepreneurs hiring workers randomly from the set of lowest-quality workers. There will be several sorts of inefficiencies in this context. Workers of different qualities will be mismatched with entrepreneurs of different ability. Neither labor markets nor credit markets may clear: there may be involuntary unemployment or credit rationing. And, there will be an inefficient number of entrepreneurs, either too many or too few depending on the nature of the equilibrium outcome. This will lead to the possibility of efficiency-enhancing policy intervention.

The basic model is outlined in the following section, and the full-information equilibria in Section 3. This is followed by the analysis of equilibrium when the quality of workers and the ability of entrepreneurs are both private. In Section 5, we investigate the efficiency of markets outcomes under asymmetric information and the implications for policy. The following section briefly discusses alternative information assumptions in which only of one worker quality or entrepreneurial ability is private knowledge. A final section concludes.

2. Elements of the Model

The model we use has several specific features. They are chosen to highlight the kinds of issues that can arise when there is asymmetric information in labor and credit markets, while at the same time avoiding complications that can obscure the phenomena we are trying to illustrate and can lead to excessively complex analysis. Many of our simplifications parallel those found in the literature on adverse selection in credit markets, such as the simple structure of project returns and the limitation on the number of dimensions of decision-making. Naturally, this leads to results that are model-specific, but hopefully are still suggestive.

The model is partial equilibrium in the sense that it focuses on the entrepreneurial sector of the economy, that is, the sector consisting of new entrepreneurial firms. There is a continuum of potential entrepreneurs, as well as a (separate) continuum of potential workers.[2] Entrepreneurs differ in a single dimension called ability, denoted a, while workers differ by quality, denoted q, and both a and q are distributed uniformly over $[0, 1]$. The total population of entrepreneurs is normalized to unity, while that of workers is normalized to n, so there are n workers per entrepreneur. These assumptions about the distributions of entrepreneurs and workers are important because, as we shall see, they lead to perfect matching of workers and entrepreneurs in the full-information outcome, thereby avoiding the complications that arise when matching is imperfect. In fact, the supports of the two distributions need not be the same, and are assumed to be so only for simplicity. Only a portion of both potential entrepreneurs and workers end up choosing the entrepreneurial sector, and those who do not have a fallback option as discussed below.

Every potential entrepreneur has a project that may either be successful or unsuccessful. Success occurs with probability p and yields a return R, where R is fixed exogenously and is the same for all projects. If the project fails, zero revenue is obtained ($R = 0$).[3] To

[2] An alternative approach would be to assume that entrepreneurs and workers come from the same population, as in Kanbur (1981) for example, and discussed in de Meza (2002). In our approach, differences in attributes of entrepreneurs and workers play a key role, and it simplifies matters to abstract from the possibility that individuals may possess various amounts of each attribute. This would add an occupational choice dimension to our analysis that would complicate things considerably.

[3] These assumptions are equivalent to the de Meza and Webb (1987) case, as opposed to the

undertake a project, each entrepreneur hires n workers, taken to be fixed for simplicity. The probability of success of a project p depends upon both the ability of the entrepreneur a and the qualities of the n workers hired, $\underline{q} \equiv (q_1, \cdots, q_n)$, according to:[4]

$$p(a, \underline{q}) = \beta a \overline{q^\alpha}, \qquad 0 < \beta, \alpha < 1 \tag{1}$$

where $\overline{q^\alpha}$ is the average value of q^α. Both a and q may be private information to the entrepreneur or the worker respectively. Moreover, neither can be inferred ex post since all that might be observed is whether the project has succeeded or failed and not the probability of success.

When a project is undertaken, each worker is paid a wage up front, and for simplicity wage costs are the only costs to the entrepreneurs. Let c denote total wage costs. Entrepreneurs are assumed to have no wealth so the amount c must be obtained from the credit market.[5] We assume credit takes the form of a loan extended by a bank at a gross interest rate of r (i.e., one plus the market interest rate). If the project is successful, the entrepreneur repays rc to the bank. Otherwise, the firm goes bankrupt and the bank receives no payment. We assume that banks can costlessly observe whether the project succeeds or fails. Since it would be in the interest of entrepreneurs to declare bankruptcy even if the project is successful, it may be more realistic to require that banks monitor project returns ex post in the event of such a declaration. Adding an ex post monitoring cost would not change the results, so we leave it out for simplicity. Some consequences of ex post monitoring costs when new firms face adverse selection in credit markets are

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Stiglitz and Weiss (1981) case where expected returns on projects are the same, or Boadway and Keen (2005) where projects are distributed over both R and the probability of success p. Allowing returns to be greater than zero in the bad outcome would be inconsequential, provided the bad return leads to bankruptcy.

[4] Alternatively, (1) could be written $p(a, \underline{q}) = \beta a \overline{q}^\alpha$, where $\overline{q}$ is the average value of q in the firm. The two forms are equivalent for large n, but (1) leads to simpler analytics without affecting the results. It could also be generalized so that $p(a, \underline{q}) = \beta a^\gamma \overline{q^\alpha}$, but this would not have a qualitative effect on the results. An alternative approach would be to allow the quality of workers to affect the return R rather than the probability of success, as in Weiss (1980). This also leads to qualitatively similar results.

[5] Adding a fixed capital cost that must be financed by credit, as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), adds nothing of substance in our context since credit is already required to finance wage costs.

discussed in Boadway and Keen (2005).

All agents—entrepreneurs, workers and banks—are assumed to be risk-neutral. Potential entrepreneurs choose whether or not to undertake a project. Those who do not undertake their project have a perfectly certain alternative income, denoted π_0. The expected profit of the project to an active entrepreneur is then:

$$\pi = p(a, \underline{q})(R - rc) \geqslant \pi_0 \tag{2}$$

where $p(a, \underline{q})$ is determined by (1) and π_0 is assumed to be the same for all entrepreneurs. The marginal entrepreneur will be the one with ability a such that (2) holds with equality.

In the credit market, banks are perfectly competitive. Let ρ be the risk-free gross rate of return that banks must pay to their depositors. If banks know the probability of success p of a given project—that is, they know the ability of the entrepreneur and the quality of the worker as in the full-information case—they will in equilibrium charge a gross interest rate r that, by their zero-expected profit condition, satisfies:[6]

$$r(p) = \frac{\rho}{p} \tag{3}$$

However, if p for individual entrepreneurs is not known, banks will charge a common gross interest rate $\overline{r}$ that satisfies:

$$\overline{r} = \frac{\rho}{\overline{p}} \tag{4}$$

where $\overline{p}$ is the expected probability of success of entrepreneurs who obtain bank finance.

Workers can seek employment either in the entrepreneurial sector or in a 'traditional' sector, where there is full information and no uncertainty. To allow for the possibility of unemployment, we assume that there is ex post immobility between the two sectors: a worker who chooses to seek employment in the entrepreneurial sector cannot move to the traditional sector in the same period if a job is not obtained. Those who opt for the traditional sector produce an output equal to their quality q for certain and earn a wage

[6] This assumes that there are no operating costs for banks and that banks need not monitor projects ex post to verify that bankruptcy has in fact occurred. As mentioned, monitoring costs would have no qualitative effect on the results.

equal to q. There is no need for loan intermediation in the traditional sector since wage payments are perfectly certain and there is no bankruptcy. However, in the entrepreneurial sector, there may be asymmetric information in the sense that entrepreneurs cannot observe the quality of workers they hire. In this case, following Weiss (1980), all those who are employed in the entrepreneurial sector earn the same wage w despite their quality, and the cost of workers per firm is $c = nw$. Note that if q is private information, workers cannot observe the quality of other employees of the same entrepreneur. We assume that n is large enough that each worker in a firm takes as given $\overline{q^\alpha}$, and therefore the expected probability of success of the firm.

As we shall see, when worker quality q cannot be observed there may be involuntary unemployment, in which case jobs are filled randomly from those who have opted for that sector. Let e be the proportion of workers in the entrepreneurial sector who become employed. Then, given risk neutrality, workers will seek work in the entrepreneurial sector only if the following reservation constraint is satisfied:

$$ew \geqslant q \tag{5}$$

where $e \leqslant 1$. Let $\tilde{q}$ be the quality of workers who are just indifferent between the traditional and the entrepreneurial sectors, so $\tilde{q} = ew$. All workers with $q \leqslant \tilde{q}$ enter the entrepreneurial sector, so the number of workers in the entrepreneurial sector, given the uniform distribution assumed, is $n\tilde{q}$. This result that the lowest quality workers enter the entrepreneurial sector only applies when worker quality cannot be observed. As we shall see, higher-quality workers will generally be attracted to the entrepreneurial sector when q is observable by entrepreneurs. This constitutes one important source of inefficiency induced by asymmetric information.

We have assumed that worker remuneration takes the form of a wage paid up front. However, in principle, payments to workers could include an ex post bonus paid in the event of success. If worker quality cannot be observed, the use of a two-part wage consisting of an up-front wage and an ex post bonus might potentially be used to separate workers by quality. However, an ex post bonus will not be useful under the informational assumptions we are making. To see this, suppose b is an ex post bonus paid by an entrepreneur in the second period. Then, the cost to the entrepreneur of hiring a worker measured in terms of second period income is $c = \overline{r}w + b = \overline{r}(w + b/\overline{r})$, where $\overline{r} = \rho/\overline{p}$ by (4). Workers cannot be separated according to their qualities with the use of an ex post bonus since each of them takes $\overline{p}$ to be given.[7] Each of them will discount the second-period bonus payment at the rate $\overline{r}' = \rho'/\overline{p}$, where $\rho' \geqslant \rho$, with the inequality applying if workers are liquidity-constrained. Then, for the marginal worker, we have $\tilde{q} = e \cdot (w + b/\overline{r}')$. Using this, the cost per worker c to the entrepreneur can be written:

$$c = \overline{r}\left(\frac{\tilde{q}}{e} + \frac{b}{\overline{r}} - \frac{b}{\overline{r}'}\right)$$

Given that $\overline{r}' \geqslant \overline{r}$, the firm would never gain by using a bonus.

In what follows, our analysis focuses mainly on two cases, the benchmark full-information case in which both a and q are public information, and the asymmetric-information case in which both are private information. As we shall see, while the former is fully efficient, the latter is generally not even constrained efficient, thereby motivating policy intervention. In a later section, we briefly discuss the intermediate cases where either a or q is public information.

3. Equilibrium and Optimality with Full Information

With full information, both the ability of each entrepreneur and the quality of each worker is known to all agents, including the government. Equilibrium is characterized first by the set of potential entrepreneurs and workers who choose to enter the entrepreneurial sector, and second by the assignment of workers to entrepreneurs. The nature of the equilibrium outcome depends on the parameters of the problem as well as the return R. For concreteness, we shall focus on a particular case: that in which the highest-quality workers opt for the entrepreneurial sector, although it is possible that the set of workers choosing the entrepreneurial sector may be in the interior of the quality distribution. The lowest-quality workers will go to the traditional sector since the probability of success

[7] If there were few enough workers at a firm such that each one recognized the effect of their quality on p, it might be possible to use a bonus to separate workers by quality. In a background paper, we studied this possibility for the extreme case in which each firm hires only one worker. See Boadway and Sato (2005).

will be too low in the entrepreneurial sector. For example, no entrepreneur would hire a worker with $q = 0$, since $p = 0$ in that case. (The highest-ability entrepreneurs will always enter the sector.) In the case where the highest-quality workers enter, the nature of the equilibrium outcome is intuitive.

Consider first the optimal matching of workers by quality with entrepreneurs by ability. The probability of success for a type-a entrepreneur employing a set of workers $\underline{q}$ is $p(a, \underline{q}) = \beta a \overline{q^\alpha}$, where a and each element of $\underline{q}$ are public information, and therefore $\overline{q^\alpha}$ and p are known to the workers and the banks. For a given distribution of worker qualities and entrepreneurial abilities in the entrepreneurial sector, aggregate expected output will be highest if higher-quality workers are matched with higher-ability entrepreneurs. To see this, consider two entrepreneurs of ability $a_2 > a_1$ and two sets of workers, $\underline{q}_1$ and $\underline{q}_2$, whose qualities are such that $\overline{q^\alpha}_2 > \overline{q^\alpha}_1$. Thus, workers $\underline{q}_2$ are of higher quality than $\underline{q}_1$ in the sense that their average value of q^α is higher. Expected output will be highest if $\overline{q^\alpha}_2$ is matched with a_2, and $\overline{q^\alpha}_1$ with a_1, since:

$$\beta a_1 \overline{q^\alpha}_1 R + \beta a_2 \overline{q^\alpha}_2 R > \beta a_1 \overline{q^\alpha}_2 R + \beta a_2 \overline{q^\alpha}_1 R, \quad \text{or} \quad a_2(\overline{q^\alpha}_2 - \overline{q^\alpha}_1) > a_1(\overline{q^\alpha}_2 - \overline{q^\alpha}_1)$$

Extending this logic to many types of entrepreneurs, output will be maximized by matching the highest-quality workers with the highest-ability entrepreneurs. In our context where the distributions of entrepreneurs and workers are uniform, and there are n workers for each entrepreneur, matching will be perfect. Each entrepreneur will hire n workers of identical quality, and workers of higher quality will be matched with entrepreneurs of higher ability.

Next, consider how the market might generate such an outcome. With full information, wage rates will be specific to workers' abilities, so there will be no adverse selection and no involuntary unemployment (unlike in the asymmetric-information case as we shall see in the next section). Therefore, there will be an equal number of workers and jobs in the entrepreneurial sector (n per entrepreneur). Given that the densities of q and a are identical by assumption, we might expect that the full-information equilibrium will entail perfect matching of workers to entrepreneurs with q increasing in a. Moreover, if the highest-ability entrepreneurs and the highest-quality workers are the ones that opt for the entrepreneurial sector, which is the case we shall assume, the matching outcome will imply

$q = a$ since the upper support of both distributions is the same. We proceed by showing that $q = a$ is in fact an equilibrium in the full-information case when both entrepreneurs and workers are drawn from the top of their respective distributions. The cutoff ability of active entrepreneurs will then be determined by a zero-net-expected-profit condition, and since each entrepreneur hires n workers, that will determine the cutoff quality of workers in the entrepreneurial sector.

To see that perfect matching will be an equilibrium, suppose the wage function is $w(q)$, where $w(q) \geqslant q$ to ensure participation. Consider an entrepreneur of type a, and suppose that entrepreneur hires $n(q)$ workers of type q, where $\int_0^1 n(q)dq = n$. Given that a and q are public information, the banks charge an entrepreneur-specific gross interest rate of $r(p) = \rho/p$, where by (1):

$$p(a, \underline{q}) = \beta a \frac{\int_0^1 n(q)q^\alpha dq}{n}$$

Then given the wage function $w(q)$, the entrepreneur's expected profits can be written, using (2) and (3), as:

$$\text{(6)} \qquad \pi = p(a, \underline{q}) \left(R - r \int_0^1 w(q)n(q)dq \right) = \beta a R \frac{\int_0^1 n(q)q^\alpha dq}{n} - \rho \int_0^1 w(q)n(q)dq$$

Entrepreneurs choose their mix of workers to maximize expected profits, given the wage function $w(q)$.

Suppose the wage function $w(q)$ determined by the market for workers is convex, so $w''(q) \geqslant 0$. We shall confirm below that this will be the case in equilibrium. Then, the following lemma applies:[8]

Lemma 1: *If $w''(q) \geqslant 0$, then entrepreneur a prefers to hire all n workers of the same quality q^*:*

$n^*(q) = n \qquad \text{for} \qquad q = q^*$

[8] Proof: Expected profits in (6) may be written $\pi = \beta a R E[q^\alpha] - \rho n E[w(q)]$. Then since $E[q^\alpha] < E[q]^\alpha$ for $\alpha < 1$ and $E[w(q)] \geqslant w(E[q])$ for $w''(q) \geqslant 0$, we have $\pi = \beta a R E[q^\alpha] - \rho n E[w(q)] < \beta a R E[q]^\alpha - \rho n w(E[q])$. Therefore, starting in a situation in which workers of different qualities are hired, the entrepreneur can increase profits by replacing them with n workers of a type equal to the average quality of existing workers, $\overline{q} = E[q]$, since then expected profits will be $\beta a R E[q]^\alpha - \rho n w(E[q])$.

$n^*(q) = 0 \qquad \text{for} \qquad q \neq q^*$

Given that entrepreneurs all hire workers of a single quality, expected profits (6) can be rewritten

$$\pi(a, q) = \beta a R q^\alpha - \rho n w(q) \tag{6'}$$

It must also be the case that, since there are n workers of each quality, all entrepreneurs hire a different quality of worker. With workers being drawn from the top of the quality distribution, we expect that $q = a$ will be an equilibrium. This will be so if profits $\pi(a, q)$ are maximized where $q = a$. The first-order condition for an entrepreneur a's choice of q is:

$$\frac{\partial \pi(a, q)}{\partial q} = \beta a R \alpha q^{\alpha - 1} - \rho n w'(q) = 0$$

This will be satisfied at $q = a$ if:

$$w'(q) = \frac{\alpha \beta R}{\rho n} q^\alpha \tag{7}$$

The second-order condition for the entrepreneur's problem, evaluated at $q = a$, is:

$$\frac{\partial^2 \pi(a, a)}{\partial q^2} = \beta a R(\alpha - 1)\alpha q^{\alpha - 2} - \rho n w''(q) < 0$$

Since the first term is negative, this will be satisfied if $w'' \geqslant 0$, which by (7) is the case. Thus, $q = a$ is clearly a candidate for equilibrium since it can be supported by a wage function satisfying (7).

To verify that $q = a$ is an equilibrium, we can obtain an expression for the wage function $w(q)$ by integrating (7) to obtain:

$$w(q) = \frac{\alpha \beta R}{\rho n (1 + \alpha)} q^{\alpha + 1} + F \tag{8}$$

where F is a constant of integration, whose value will be determined below. We assume that $w(q)$ is defined over $[0, 1]$ and is a continuous function. Moreover, we assume that R is sufficiently large that in (7), $w'(q) > 1$ for all workers in the entrepreneurial sector. In fact, this will be the case if $w'(q) > 1$ for the marginal worker, since $w'(q)$ is increasing in

q along the wage profile defined by (8). This is sufficient to ensure that the highest-quality workers are the ones that enter the entrepreneurial sector.

Let $\tilde{q}$ be the ability of the marginal worker. Then, competition for workers will ensure that $w(\tilde{q}) = \tilde{q}$, where $\tilde{q}$ is the wage rate that can be obtained in the traditional sector.[9] Since $w'(q) > 1$, $w(q) > q$ for all workers with $q > \tilde{q}$, implying that the highest-quality workers enter the entrepreneurial sector. (Of course, had it been the case that $w'(q) < 1$ for the marginal worker, a segment of workers from the interior of the wage distribution would enter.) Since, as we shall confirm below, the highest-ability entrepreneurs are the active ones, the marginal entrepreneur with lowest ability $\tilde{a}$ will hire the marginal workers $\tilde{q}$. And, since the densities of the two distributions are the same, this implies that $q(a) = a$ so there is perfect matching. Therefore, from $w(\tilde{q}) = \tilde{q} = \tilde{a}$, we can infer by applying (8) for the marginal workers that F satisfies:

$$F = \tilde{a} - \frac{\alpha\beta R}{\rho n(1+\alpha)}\tilde{a}^{\alpha+1} \tag{9}$$

The wage profile will adjust so that (9) is satisfied. All workers with $q > \tilde{a}$ will therefore earn $w(q) > q$ by (8) and the fact that $w'(q) > 1$, implying that they will earn a surplus.

Consider now the entrepreneurs. The expected profit of an entrepreneur of ability a is given by: $\pi = \beta a q^{\alpha} R - \rho n w(q)$, where $w(q)$ satisfies (8) and (9). The solution to the entrepreneur's first-order condition will be $q = a$, and it will be unique since the second-order condition for the entrepreneur's problem is satisfied. Therefore, expected profits may be written:

$$\pi(a) = \beta a^{1+\alpha} R - \rho n \left(\frac{\alpha\beta R}{\rho n(1+\alpha)} a^{\alpha+1} + F \right) = \frac{\beta R}{(1+\alpha)} a^{\alpha+1} - \rho n F$$

where F is given by (9). Since $\pi(a)$ is increasing in a, that implies that the marginal entrepreneur will have the lowest ability among active entrepreneurs. The ability of the marginal entrepreneur $\tilde{a}$ is determined, using (9), by:

$$\pi_0 = \beta R \tilde{a}^{\alpha+1} - \rho n \tilde{a} \tag{10}$$

[9] To see this, note that if $w(\tilde{q}) > \tilde{q}$, a worker of slightly lower quality, say $\tilde{q} - \varepsilon$, can offer to work for a slightly lower wage. The marginal entrepreneur would then prefer to employ this worker than one with quality $\tilde{q}$.

All entrepreneurs with $a \geqslant \tilde{a}$ will enter, and the number of active entrepreneurs, denoted m, will be given by $m = 1 - \tilde{a}$.

It is apparent that this full-information equilibrium is efficient. To see that, we only have to show that $\tilde{a}$ is optimal. This determines the number of active entrepreneurs and workers, and we already know that output is maximized when matching is perfect, which will be the case when $q = a$. Social surplus is given by the following, where $q = a$ in equilibrium:

$$S = \int_{\tilde{a}}^{1} [p(a,q)R - \rho n w(q) - \pi_0] da + \rho \int_{\tilde{q}}^{1} n[w(q) - q] dq$$

$$= \int_{\tilde{a}}^{1} \left[\beta a^{\alpha+1} R - \rho n w(a) - \pi_0\right] da + \rho \int_{\tilde{a}}^{1} n[w(a) - a] da$$

where the first term is the surplus obtained by entrepreneurs and the second term the surplus of workers, both measured in terms of second-period income.[10] Differentiating S by $\tilde{a}$, we obtain:

$$\frac{dS}{d\tilde{a}} = -\left[\beta \tilde{a}^{\alpha+1} R - \rho n w(\tilde{a}) - \pi_0\right] - \rho n[w(\tilde{a}) - \tilde{a}] = -\left[\beta \tilde{a}^{\alpha+1} R - \rho n \tilde{a} - \pi_0\right] = 0$$

where the last equality follows from (10). Therefore, the number of entrepreneurs is optimal, and the full-information equilibrium is efficient.

The equilibrium outcome we have described in this section is only one of many that can occur. We have chosen it partly for simplicity, but partly because of the stark differences that will exist between it and equilibria under asymmetric information discussed below. As mentioned, if we had assumed that $w'(q) < 1$ in (7), the set of workers who opt for the entrepreneurial sector would fall along an interval of dimension $1 - \tilde{a}$ in the interior of the quality distribution. Of course, higher-quality workers would be matched with higher-entrepreneurs, and the equilibrium outcome under full information would still be socially optimal.

[10] Alternatively, dividing through by ρ would yield the surplus in present value terms. Since workers get paid in the first period, their surplus occurs then, while entrepreneurs' surplus occurs in the second period.

4. Equilibrium with Asymmetric Information

In this case, both a and q are private information. All workers who are employed in the entrepreneurial sector obtain a common wage rate w, while all active entrepreneurs face a common interest rate $\overline{r}$. We begin with a general overview of the relationships that must hold in equilibrium before turning to the qualitative features of equilibria.

Equilibrium Relationships

Consider first the decision of workers to seek employment in the entrepreneurial versus the traditional sector. Suppose workers believe—correctly in equilibrium—that the probability of being employed in the entrepreneurial sector is e. All entrepreneurs will offer the same wage rate w in equilibrium since all workers have the same expected quality from the point of view of entrepreneurs. Given w, the expected income of workers in the entrepreneurial sector is ew. Since a worker of quality q can receive a wage of q in the traditional sector, the cutoff quality of workers by (5) is $\tilde{q} = ew$. All workers with $q < \tilde{q}$—those with the lowest quality—choose the entrepreneurial sector regardless of the parameters of the problem. This is in contrast with the social optimum achieved with full information where a segment of higher-quality workers will generally be attracted to the entrepreneurial sector.

Each entrepreneur also takes e as given. Recall from (1) that the probability of success for an entrepreneur depends on $\overline{q^\alpha}$, which cannot be observed here. When an entrepreneur hires workers, those workers are selected randomly from the pool of available workers with $q \in [0, \tilde{q}]$, so $E[q^a]$ is the same for all entrepreneurs. Given e and the uniform distribution of workers, $E[q^a]$ is given by:

$$E[q^a] = E\left[q^\alpha | q \leqslant \tilde{q}\right] = \frac{1}{\tilde{q}} \int_0^{\tilde{q}} q^\alpha dq = \frac{\tilde{q}^\alpha}{1+\alpha} = \frac{(ew)^\alpha}{1+\alpha} \tag{11}$$

Then, for an entrepreneur of ability a, the expected probability of success will be given by

$$E[p] = \beta a E[q^a] = \frac{\beta a}{1+\alpha}(ew)^\alpha \tag{12}$$

Expected profits for this entrepreneur can therefore be written, using (2), as:

$$\pi = E[p](R - \overline{r}nw) = \frac{\beta a}{1+\alpha}(ew)^\alpha(R - \overline{r}nw) \tag{13}$$

This assumes that all entrepreneurs who choose to become active can receive a loan at the rate $\overline{r}$. We return later to the issue of whether there can be credit rationing in this context, which would complicate matters considerably.

Each active entrepreneur can be thought of as choosing a wage rate to maximize profits, given e. All entrepreneurs who become active offer the same wage rate w in equilibrium, as we shall confirm. Moreover, workers will be indifferent among active entrepreneurs since they are paid the wage w in advance, so are not affected by bankruptcy. Let m be the number of active entrepreneurs. Since there are $n\tilde{q}$ workers in the entrepreneurial sector and since each entrepreneur hires n workers, the employment rate e is given by:

$$e = \min\left\{\frac{nm}{n\tilde{q}}, 1\right\} = \min\left\{\frac{m}{\tilde{q}}, 1\right\} \tag{14}$$

For the case in which $m < \tilde{q}$, there is unemployment ($e < 1$) and we have by (5) and (14), $\tilde{q} = ew = mw/\tilde{q}$. Therefore, the labor force in the entrepreneurial sector and the rate of employment can be expressed respectively as $\tilde{q} = \sqrt{mw}$ and $e = \sqrt{m/w}$. This implies that the quality of the marginal worker with full employment and unemployment may be written:

$$\tilde{q} = ew = \begin{cases} w & \text{if} \quad m = w \\ \sqrt{mw} & \text{if} \quad m < w \end{cases} \tag{15}$$

Banks can observe neither a nor q. Assuming that projects are allocated randomly among banks, the interest rate they offer will be $\overline{r} = \rho/\overline{p}$ by (4), where $\overline{p}$ is the expected probability of success of any given project. Using (12) and (15), $\overline{p}$ is given by:

$$\overline{p} = \begin{cases} \frac{\beta\overline{a}}{1+\alpha} w^{\alpha} & \text{if} \quad m = w \\ \frac{\beta\overline{a}}{1+\alpha} (mw)^{\frac{\alpha}{2}} & \text{if} \quad m < w \end{cases} \tag{16}$$

where $\overline{a}$ is the average quality of active entrepreneurs, discussed below. Expected profits of an entrepreneur of ability a in equilibrium can then be written, using (13) and (15), as:

$$\pi = \begin{cases} \frac{\beta a}{1+\alpha} w^{\alpha} (R - \overline{r}nw) & \text{if} \quad m = w \\ \frac{\beta a}{1+\alpha} (mw)^{\frac{\alpha}{2}} (R - \overline{r}nw) & \text{if} \quad m < w \end{cases} \tag{17}$$

Finally, we can use these expressions for expected profits to determine the surplus accruing to society from a given allocation of resources. Note that inactive entrepreneurs (who obtain reservation profits π_0) and workers who remain in the traditional sector earn no surplus. Moreover, workers who enter the entrepreneurial sector but are unemployed produce nothing. (If there were credit rationing, active entrepreneurs who are unable to obtain a loan would earn no surplus if there is ex post immobility between the two sectors.) Let A and Q be the respective sets of entrepreneurs and workers in the entrepreneurial sector, where $Q = [nq \mid q \leqslant \tilde{q}]$ and A is discussed below. Social surplus S is given by:

$$S = \int_A [p \cdot (R - \overline{r}nw) - \pi_0] da + \rho \int_Q n[ew - q] dq = \overline{p}m[R - \overline{r}nw] - m\pi_0 + \rho \left[new\tilde{q} - \frac{n\tilde{q}^2}{2} \right]$$

where, recall, m is the number of entrepreneurs and $\overline{p}$ is the expected probability of success of all active entrepreneurs. Since, $e\tilde{q} = m$ and $\overline{r} = \rho/\overline{p}$, social surplus can be written, using (15) and (16), as:

$$(18) \qquad S = m[\overline{p}R - \pi_0] - \frac{\rho n\tilde{q}^2}{2} = \begin{cases} m\left[\frac{\beta\overline{a}R}{1+\alpha} m^\alpha - \pi_0 - \frac{\rho nm}{2}\right] & \text{if} \quad m = w \\ m\left[\frac{\beta\overline{a}R}{1+\alpha} (mw)^{\frac{\alpha}{2}} - \pi_0 - \frac{\rho nw}{2}\right] & \text{if} \quad m < w \end{cases}$$

We turn now to the determination of the two key endogenous variables in the model, the wage rate (which determines the number of workers who opt for the entrepreneurial sector) and the number of entrepreneurs.

Determination of the Wage Rate

Consider first the wage rate preferred by any given entrepreneur. Using (13), the value of w that maximizes the profits of an entrepreneur of any ability, given the employment rate e, is the solution to the following problem:

$$\max_{\{w\}} \; w^\alpha \cdot (R - \overline{r}nw)$$

Using the first-order conditions and (4), the solution, denoted $\widehat{w}$, is given by:

$$(19) \qquad \widehat{w} = \frac{\alpha R}{(1+\alpha)\overline{r}n} = \frac{\alpha R\overline{p}}{(1+\alpha)\rho n}$$

which is independent of the ability level of the entrepreneur. The second-order conditions are satisfied given our assumption that $\alpha < 1$. Starting at $w = 0$, profits will initially rise with w and eventually reach a peak at $\widehat{w}$, assumed to be at $\widehat{w} < 1$ so that we have an interior solution. The intuition here is that the rise in w attracts better quality workers, which increases the probability of success, but also increases labor costs. Given that p is concave in q, the latter eventually offsets the former.

Denote by w^e the market clearing, or equilibrium wage rate, that is, the wage rate such that $e = 1$. Given m, the market clearing wage will be $w^e = m$, where the number of workers just equals the number of entrepreneurs, $\tilde{q} = m$. Whether involuntary unemployment exists depends upon the relative size of w^e and $\widehat{w}$. If $\widehat{w} > w^e = m$, entrepreneurs will bid up the wage rate above the market clearing level, attracting excess workers into the entrepreneurial sector and generating involuntary unemployment. On the other hand, if $\widehat{w} \leqslant w^e = m$, the wage rate will be bid up only to w^e by competition for workers, so there will be full employment. Consider the consequences for entrepreneurial profits of each outcome in turn.

Unemployment Case: $\widehat{w} > w^e = m$

In this case, the market wage is $\widehat{w}$ given by (19) and $\overline{p}$ is given by the second row of (16). These consist of two equations in $\widehat{w}$ and $\overline{p}$ whose solutions are:

$$\overline{p}(\overline{a}, m) = \left(\frac{\beta\overline{a}}{1+\alpha}\right)^{\frac{2}{2-\alpha}} \left(\frac{\alpha}{1+\alpha}\frac{mR}{\rho n}\right)^{\frac{\alpha}{2-\alpha}} \tag{20}$$

$$\widehat{w}(\overline{a}, m) = \left[\frac{\beta\overline{a}}{1+\alpha}\frac{\alpha}{1+\alpha}\frac{R}{\rho n}\right]^{\frac{2}{2-\alpha}} m^{\frac{\alpha}{2-\alpha}} \tag{21}$$

where both $\overline{p}(\overline{a}, m)$ and $\widehat{w}(\overline{a}, m)$ are increasing in $\overline{a}$ and m.

The expected profits of an entrepreneur of ability a can be written as follows, using the second row of (17) with $w = \widehat{w}$:

$$\pi = \frac{\beta a(m\widehat{w})^{\frac{\alpha}{2}}}{1+\alpha}(R - \overline{r}n\widehat{w})$$

Using (21), this yields:

$$\widehat{\pi}(a, \overline{a}, m) = a\left(\frac{\beta}{1+\alpha}\frac{R}{1+\alpha}\right)^{\frac{2}{2-\alpha}} \left(\frac{\alpha\overline{a}m}{\rho n}\right)^{\frac{\alpha}{2-\alpha}} \tag{22}$$

where $\widehat{\pi}(\cdot)$ refers to expected profits when the wage rate is $\widehat{w}$. This function for expected profits is increasing in all three arguments, a, $\overline{a}$ and m.

Full Employment Case: $\widehat{w} \leqslant w^e = m$

In this case, by the first row of (16), we have

$$\overline{p} = \frac{\beta \overline{a} m^{\alpha}}{1+\alpha}$$

Expected profits of an entrepreneur of ability a can immediately be written, using $\overline{r} = \rho/\overline{p}$ and the above expression for $\overline{p}$, as:

$$\pi^e(a, \overline{a}, m) = \frac{\beta a m^{\alpha}}{1+\alpha}(R - \overline{r}nm) = a\left(\frac{\beta R m^{\alpha}}{1+\alpha} - \frac{\rho n m}{\overline{a}}\right) \tag{23}$$

In this case, $\pi^e(\cdot)$, expected profits when $w = w^e$, is increasing in a and $\overline{a}$, but the effect of m is ambiguous.

The Number of Active Entrepreneurs

Given that expected profits are increasing in ability a with or without unemployment, there is a cutoff ability level $\tilde{a}$ such that all entrepreneurs with $a \geqslant \tilde{a}$ become active and the remainder obtain their reservation profits π_0. Then, the number of active entrepreneurs is $m = 1 - \tilde{a}$, and given the uniform distribution that we have assumed, their average (expected) ability is $\overline{a} = (1 + \tilde{a})/2$, so

$$m = 2 - 2\overline{a} \tag{24}$$

Recall from (21) that the profit-maximizing wage rate $\widehat{w}$ depends on $\overline{a}$ and m. Therefore, whether $\widehat{w} \gtreqless w^e = m$ depends on $\overline{a}$. In particular, using (21) and (24), we obtain that:

$$\widehat{w} > w^e \quad \Longleftrightarrow \quad \frac{\beta \overline{a}}{1+\alpha} \frac{\alpha}{1+\alpha} \frac{R}{\rho n} \; > \; (2 - 2\overline{a})^{1-\alpha} \tag{25}$$

The left-hand side of (25) is increasing in $\overline{a}$, while the right-hand side is decreasing. Moreover, at $\overline{a} = 0$, the right-hand side exceeds the left-hand side. Therefore, there will be a value of $\overline{a}$, denoted $\overline{a}'$ such that

$$\frac{\beta \overline{a}'}{1+\alpha} \frac{\alpha}{1+\alpha} \frac{R}{\rho n} = (2 - 2\overline{a}')^{1-\alpha} \tag{26}$$

For $\overline{a} \leqslant \overline{a}'$, $\widehat{w} \leqslant w^e$ (there is full employment), and vice versa. It must be the case that $0 < \overline{a}' < 1$ (since the right-hand side is less that the left-hand side at $\overline{a} = 1$). Note that $\overline{a} \geqslant 1/2$ since if $m = 0$, $\overline{a} = 1/2$. In what follows, we shall assume that $\overline{a}' > 1/2$ to allow for the possibility that there is full employment. If $\overline{a}' < 1/2$, $\overline{a}$ would always exceed $\overline{a}'$ so there would always be involuntary unemployment.

Whether the equilibrium involves full employment or unemployment depends on the relationship between $\overline{a}$ and $\overline{a}'$. In turn, the market wage w as well as the expected probability of success $\overline{p}$ and expected profits π depend on this relationship. We can summarize these results for future reference as follows:

$$w(\overline{a}) = \begin{cases} m & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ \widehat{w}(\overline{a}, m) & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{27.1}$$

$$\overline{p}(\overline{a}) = \begin{cases} \frac{\beta \overline{a} m^{\alpha}}{(1+\alpha)} & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ \left(\frac{\beta \overline{a}}{1+\alpha}\right)^{\frac{2}{2-\alpha}} \left(\frac{\alpha}{1+\alpha} \frac{mR}{\rho n}\right)^{\frac{\alpha}{2-\alpha}} & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{27.2}$$

$$\pi(a, \overline{a}) = \begin{cases} a\left(\frac{\beta R m^{\alpha}}{1+\alpha} - \frac{\rho n m}{\overline{a}}\right) & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ a\left(\frac{\beta}{1+\alpha} \frac{R}{1+\alpha}\right)^{\frac{2}{2-\alpha}} \left(\frac{\alpha \overline{a} m}{\rho n}\right)^{\frac{\alpha}{2-\alpha}} & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{27.3}$$

where $\widehat{w}(\overline{a}, m)$ is given by (21), $\overline{a}'$ is given by (26) and $m = 2 - 2\overline{a}$ by (24).

It remains to determine $\overline{a}$, the average quality of entrepreneurs, which depends upon how many entrepreneurs become active. For the marginal entrepreneur, $\pi(\tilde{a}, \overline{a}) = \pi_0$. By (27.3), for $a > \tilde{a}$, we have $\pi(a, \overline{a}) > \pi_0$ since $\pi(a, \overline{a})$ is increasing in a, confirming our presumption that active entrepreneurs are those such that $a \geqslant \tilde{a}$. In characterizing the number of active entrepreneurs, we can focus on the expected profit function for the marginal entrepreneur, defined as $\tilde{\pi}(\overline{a}) \equiv \pi(\tilde{a}, \overline{a})$.

Using (27.3), we obtain

$$\tilde{\pi}(\overline{a}) = \pi(\tilde{a}, \overline{a}) = \begin{cases} \tilde{a}\left[\frac{\beta R (2-2\overline{a})^{\alpha}}{1+\alpha} - \frac{\rho n (2-2\overline{a})}{\overline{a}}\right] & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ \tilde{a}\left[\frac{\beta}{1+\alpha} \frac{R}{1+\alpha}\right]^{\frac{2}{2-\alpha}} \left[\frac{\alpha \overline{a} (2-2\overline{a})}{\rho n}\right]^{\frac{\alpha}{2-\alpha}} & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{28}$$

where $\tilde{a} = 2\overline{a} - 1$. Differentiating $\tilde{\pi}(\overline{a})$ with respect to $\overline{a}$ gives:

$$\frac{d\tilde{\pi}(\overline{a})}{d\overline{a}} = \frac{\partial\pi(\tilde{a},\overline{a})}{\partial\tilde{a}}\frac{\partial\tilde{a}}{\partial\overline{a}} + \frac{\partial\pi(\tilde{a},\overline{a})}{\partial\overline{a}} = 2\frac{\partial\pi(\tilde{a},\overline{a})}{\partial\tilde{a}} + \frac{\partial\pi(\tilde{a},\overline{a})}{\partial\overline{a}} \tag{29}$$

The first term on the right-hand side of (29) is positive since $\partial\pi/\partial\tilde{a} > 0$. The second term, $\partial\pi/\partial\overline{a}$, is initially positive and then becomes negative, as differentiation will confirm. Moreover, at $\overline{a} = 1/2$ and $\overline{a} = 1$, $\tilde{\pi}(\overline{a}) = 0$. A typical shape for the $\tilde{\pi}(\overline{a})$ function might be single-peaked as shown in Figure 1.[11]

Interior equilibrium values for $\overline{a}$ will be those such that $\tilde{\pi}(\overline{a}) = \pi_0$. Figure 1 depicts possible equilibria. For given values of π_0, there are generally two interior equilibria, one stable and the other unstable. The stable one is denoted $\overline{a}^*$, and is to the left of the peak of the $\tilde{\pi}(\overline{a})$ curve. The other equilibrium $\overline{a}^u$ is unstable: for $\overline{a} > \overline{a}^u$, entrepreneurs will exit causing $\overline{a}$ to rise, and vice versa. That implies that the two stable equilibria will be the interior one at $\overline{a} = \overline{a}^*$ and the corner equilibrium $\overline{a} = 1$ (where there are no active entrepreneurs).[12] Depending on the size of $\overline{a}$ relative to $\overline{a}'$, the stable equilibrium may involve unemployment. The higher the value of π_0, the higher will be $\overline{a}^*$, and the more likely will there be unemployment in equilibrium.

The Possibility of Credit Rationing

Stiglitz and Weiss (1981) found that credit rationing could arise when projects were pooled by their expected return (pR), which was exogenously given. In the de Meza and Webb (1987) case where projects were pooled by their return R and the distribution of the probability of success p across entrepreneurs was given, credit rationing could not arise. Our model is an extension of the de Meza-Webb model to allow for p for a given entrepreneur to be endogenously determined by the quality of workers hired. It turns out that in this case, credit rationing might arise. We simply show that possibility here without exploiting its consequences for the form of equilibrium achieved and its efficiency properties.

[11] Twice differentiating (28) with respect to $\overline{a}$, we obtain that for $\overline{a} > \overline{a}'$, $d^2\tilde{\pi}/d\overline{a}^2 < 0$. However, for $\overline{a} < \overline{a}'$, the sign of $d^2\tilde{\pi}/d\overline{a}^2$ is ambiguous. Differentiating (28) with respect to $\overline{a}$ also reveals that $\tilde{\pi}(\overline{a})$ can be increasing at $\overline{a} = \overline{a}'$ as shown in Figure 1. In fact, the slope of $\tilde{\pi}(\overline{a})$ will generally be discontinuous at the point $\overline{a} = \overline{a}'$, but it can either rise of fall discontinuously.

[12] Of course, if π_0 is very high, the only equilibrium will be one in which there are no entrepreneurs. In Figure 2, the π_0 curve lies above the peak of the $\tilde{\pi}(\overline{a})$ curve. We are ruling this out as being not interesting for our purposes.

Consider first the marginal entrepreneur. Using $\rho = \overline{r}\ \overline{p}$ by (4) and the expressions for $\overline{p}$ in (27.2), the expected profits for the marginal entrepreneur in (28) can be rewritten as follows:

$$\text{(30)} \qquad \tilde{\pi}(\overline{a}, \overline{r}) = \begin{cases} \frac{(2\overline{a}-1)\beta}{1+\alpha}(2-2\overline{a})^{\alpha}(R-(2-2\overline{a})\overline{r}n) & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ \frac{(2\overline{a}-1)\beta R}{(1+\alpha)^2}\left[\frac{\alpha R(2-2\overline{a})}{(1+\alpha)\overline{r}n}\right]^{\frac{\alpha}{2}} & \text{if} \quad \overline{a} > \overline{a}' \end{cases}$$

where, in equilibrium, $\tilde{\pi}(\overline{a}, \overline{r}) = \pi_0$. Suppose we focus on a stable interior equilibrium, which requires that $\partial\tilde{\pi}(\overline{a}, \overline{r})/\partial\overline{a} > 0$. Differentiating condition $\tilde{\pi}(\overline{a}, \overline{r}) = \pi_0$, we obtain:

$$\left.\frac{d\overline{a}}{d\overline{r}}\right|_{\tilde{\pi}=\pi_0} = -\left.\frac{\partial\tilde{\pi}/\partial\overline{r}}{\partial\tilde{\pi}/\partial\overline{a}}\right|_{\tilde{\pi}=\pi_0} > 0$$

where the sign follows from the stability condition and the fact that $\tilde{\pi}(\overline{a}, \overline{r})$ in (30) is decreasing in $\overline{r}$. Intuitively, an increase in the interest rate pushes the lowest-ability entrepreneurs out of the sector and increases the average quality of those remaining, $\overline{a}$.

Next, turn to the banks. The expected profit per unit of lending is $\Pi_B = \overline{p}\ \overline{r} - \rho$. Using (27.2) and the fact that $m = 2-2\overline{a}$ by (24), expected profits per unit can be written:

$$\text{(31)} \qquad \Pi_B = \begin{cases} \frac{\beta 2^{\alpha}}{(1+\alpha)}\overline{a}(1-\overline{a})^{\alpha}\overline{r} - \rho & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ \frac{\beta}{1+\alpha}\left(\frac{2\alpha}{1+\alpha}\frac{R}{n}\right)^{\frac{\alpha}{2}}\overline{a}(1-\overline{a})^{\frac{\alpha}{2}}\overline{r}^{1-\frac{\alpha}{2}} - \rho & \text{if} \quad \overline{a} > \overline{a}' \end{cases}$$

Credit rationing can only occur if an increase in the interest rate $\overline{r}$ causes bank expected profits to fall. Given that $\overline{a}$ is increasing in $\overline{r}$ as shown above, a necessary condition for this is that $\partial\Pi_B/\partial\overline{a} < 0$. From (31), we find by differentiation that for the full employment case where $\overline{a} \leqslant \overline{a}'$, $\partial\Pi_B/\partial\overline{a} < 0$ if $\overline{a} > 1/(1+\alpha)$, which is clearly possible. Similarly, in the unemployment case, we obtain that $\partial\Pi_B/\partial\overline{a} < 0$ if $\overline{a} > 2/(2+\alpha)$, which is again possible. Thus, unlike in the de Meza-Webb case, credit rationing could occur in our model. Intuitively, $\overline{p}$ can fall in $\overline{a}$ since lower-quality workers are left in the entrepreneurial sector when the number of entrepreneurs decreases ($\overline{a}$ increases). Exploring the consequences of that would be rather complicated and would take us too far afield from our present purpose, so in what follows we rule out credit rationing.

To summarize the results of this section, equilibrium in the asymmetric-information case will have the following features. The highest-ability entrepreneurs and the lowest-quality workers will enter the entrepreneurial sector, in contrast with the full-information case. As well, workers will be randomly assigned to entrepreneurs contrary to the efficient matching of the full-information case. All entrepreneurs will pay a common wage rate, which will be paid up-front, and a single interest rate facing all firms. There will generally be multiple equilibria, unless π_0 is high enough to rule out an entrepreneurial sector entirely. Two equilibria will be stable and one unstable. The stable equilibria will include one interior one and one corner solution in which there are no entrepreneurs. The interior stable equilibrium may or may not involve involuntary unemployment.

5. Efficiency and Policy with Asymmetric Information

In this section, we study the optimality properties of equilibrium outcomes with asymmetric information in credit and labor markets. Our main interest will be in stable interior equilibria with and without unemployment. We begin by investigating the efficiency of market equilibria, and then look at the consequences for government policy.

Local Efficiency Properties of Equilibria

Recall the expressions for social surplus S in (18). Rewriting these using the fact that $m = 2 - 2\overline{a}$, we obtain:

$$S = \begin{cases} (2-2\overline{a})\left[\frac{\beta\overline{a}R}{1+\alpha}(2-2\overline{a})^{\alpha} - \pi_0 - (1-\overline{a})\rho n\right] & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ (2-2\overline{a})\left[\frac{\beta\overline{a}R}{1+\alpha}(2-2\overline{a})^{\frac{\alpha}{2}}\widehat{w}^{\frac{\alpha}{2}} - \pi_0 - \frac{\rho n\widehat{w}}{2}\right] & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{32}$$

where $\widehat{w}$ in the case of unemployment is given by (21), with $m = 2 - 2\overline{a}$. The efficiency of the equilibrium outcomes can be investigated by considering the effects on social surplus of incremental changes in $\overline{a}$ and, in the case of an unemployment equilibrium, in $\widehat{w}$. Consider the unemployment case first, concentrating on the interior equilibrium ($\overline{a} < 1$).

Unemployment Equilibrium: $1 > \overline{a} > \overline{a}'$

In this case, S depends on $\overline{a}$ directly and also indirectly via $\widehat{w}(\overline{a})$. Consider the two effect in turn. Differentiating the second row in (32) partially with respect to $\overline{a}$, we obtain, after straightforward manipulation and using the expressions for $\overline{p}$ and $\widehat{w}$ in (27.2) and (19):[13]

$$\left.\frac{\partial S}{\partial \overline{a}}\right|_{\widehat{w}} = \left[\frac{2}{\overline{a}} - 4 - \alpha\right] \rho n \widehat{w} < 0 \tag{33}$$

where the sign follows from the fact that $\overline{a} > 1/2$. Then, differentiating S with respect to $\widehat{w}$ and using (27.2), (24) and (19), we obtain:

$$\left.\frac{\partial S}{\partial \widehat{w}}\right|_{\overline{a}} = \frac{\alpha \rho n m}{2} > 0 \tag{34}$$

Thus, (33) and (34) indicate that an unemployment equilibrium is inefficient. If $\overline{a}$ and $\widehat{w}$ could be manipulated separately, $\overline{a}$ should be reduced (the number of entrepreneurs increased) and $\widehat{w}$ should be increased (more high-ability workers should be attracted into the entrepreneurial sector).

However, $\widehat{w}$ depends on $\overline{a}$ through (21). Substituting $m = 2 - 2\overline{a}$ in (21) and differentiating with respect to $\overline{a}$, we obtain:

$$\frac{\partial \widehat{w}}{\partial \overline{a}} \gtreqless 0 \quad \text{as} \quad \frac{2}{2+\alpha} \gtreqless \overline{a}$$

This relationship between w and $\overline{a}$ applies only for $\overline{a} > \overline{a}'$. For $\overline{a} \leqslant \overline{a}'$, full employment exists, so $w = m = 2 - 2\overline{a}$ implying that w is declining in $\overline{a}$. The two panels of Figure 2 depict possible cases for the relationship between the w and $\overline{a}$, depending on whether $\overline{a}' \gtreqless 2/(2+\alpha)$. If $\overline{a}' \geqslant 2/(2+\alpha)$, the market wage is monotonically decreasing in $\overline{a}$, while if $\overline{a}' < 2/(2+\alpha)$, the wage rate is hump-shaped in the range where there is unemployment. These figures will be useful again below.

[13] Specifically, differentiating (32) and using (27.2), we obtain:

$$\frac{1}{2}\left.\frac{\partial S}{\partial \overline{a}}\right|_{\widehat{w}} = \overline{p}R\left(\frac{1-2\overline{a}}{\overline{a}}\right) + \pi_0 + \frac{\rho n \widehat{w}}{2} - \frac{\alpha \overline{p} R}{2}$$

Since $\tilde{a} = 2\overline{a} - 1$, $\overline{p}\tilde{a}/\overline{a} = \tilde{p}$ (by (1)), and $\tilde{p}(R - \rho n \widehat{w}/\overline{p}) = \pi_0$ for the marginal entrepreneur, this becomes:

$$\frac{1}{2}\left.\frac{\partial S}{\partial \overline{a}}\right|_{\widehat{w}} = \rho n \widehat{w}\left(\frac{1}{\overline{a}} - \frac{3}{2}\right) - \frac{\alpha \overline{p} R}{2}$$

which reduces to the expression in the text using (19).

The total effect of a change in $\overline{a}$ on surplus can be expressed as follows:

$$\frac{dS}{d\overline{a}} = \frac{\partial S}{\partial \overline{a}} + \frac{\partial S}{\partial \widehat{w}}\frac{\partial \widehat{w}}{\partial \overline{a}} \tag{35}$$

Given (33) and (34), this will be unambiguously negative if $\partial\widehat{w}/\partial\overline{a} < 0$, which will be the case if $\overline{a} > 2/(2+\alpha)$. That is, an increase in the number of entrepreneurs would increase efficiency. Otherwise, it will be ambiguous. We return below to how policy might be used to enhance efficiency.[14]

Full Employment Equilibrium: $1/2 < \overline{a} \leqslant \overline{a}'$

With full employment, $\overline{a}$ has only a direct effect on S in the first row of (32). Differentiating with respect to $\overline{a}$, we obtain after similar manipulation:

$$\frac{1}{2}\frac{dS}{d\overline{a}} = \frac{2\rho n(1-\overline{a})^2}{\overline{a}} - \alpha\overline{p}R \tag{36}$$

To interpret this, use $w^e = m = 2 - 2\overline{a}$ and $\alpha\overline{p}R = \rho n(1+\alpha)\widehat{w}$ by (19) to find:

$$\frac{1}{2}\frac{dS}{d\overline{a}} = \rho n\left(\frac{1-\overline{a}}{\overline{a}}w^e - (1+\alpha)\widehat{w}\right) \gtreqless 0$$

since $w^e > \widehat{w}$. Thus, there may be too few or too many entrepreneurs in the full-employment equilibrium.

These efficiency results contrast with those of de Meza and Webb (1987) who show that in the absence of heterogeneous worker quality, there are unambiguously too many entrepreneurs in equilibrium ($\overline{a}$ is too low in our notation). In the de Meza-Webb model, there is an adverse selection effect that allows low-quality entrepreneurs to take advantage of a common interest rate, and too many do so. That effect is present in our model as well, but in addition the quality of workers hired in the entrepreneurial sector tends to

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[14] Note that an increase in the number of entrepreneurs will increase employment, even if it also increases $\widehat{w}$. To see this, use $m = 2 - 2\overline{a}$ and (21) to give:

$$e = \sqrt{m/\widehat{w}} = \left[(2-2\overline{a})^{1-\frac{\alpha}{2-\alpha}}\left(\frac{\beta\overline{a}}{1+\alpha}\frac{\alpha}{1+\alpha}\frac{R}{\rho}\right)^{\frac{-2}{2-\alpha}}\right]^{\frac{1}{2}}$$

Differentiating this with respect to $\overline{a}$, we obtain $de/d\overline{a} < 0$.

be too low. Increasing the number of entrepreneurs is a way of attracting higher-quality workers, but at the expense of taking lower-quality entrepreneurs. Either of those effects can dominate.

The above welfare effects are local ones. Unfortunately in our model, social surplus in (32) is not globally concave. Differentiation with respect to $\overline{a}$ indicates that $d^2S/d\overline{a}^2$ is generally of ambiguous sign. Therefore, there are various possibilities for global optima, as our discussion next illustrates.

Policy Implications

In the above analysis, we considered a hypothetical perturbation of $\overline{a}$, and thus $\widehat{w}$ in the unemployment case, around the equilibrium. The government cannot control $\overline{a}$ directly since that is determined by the decision of entrepreneurs to become active. Instead, in a decentralized market economy, the government can influence equilibrium outcomes by intervening with taxes or subsidies. Two kinds of policy instruments might be used to influence $\overline{a}$ and $\widehat{w}$: a tax or subsidy on entrepreneurs who become active and a tax or subsidy on wages. We begin by analyzing how these policy instruments affect equilibrium values of $\overline{a}$ and $\widehat{w}$. Then, we turn to the effect of policies on the social surplus, S. In evaluating the potential for policy intervention, it is useful to note that since the government can observe neither the quality of workers nor the ability of entrepreneurs, the full-information optimum cannot be achieved. In particular, workers cannot be optimally matched to firms, and nothing can be done to avoid the fact that it will be the lowest quality of workers that will be attracted to the entrepreneurial sector. A more far-reaching analysis might consider ways in which information about q and a could be elicited, such as by signaling or ex ante monitoring.

Effect of Policies on Equilibrium Outcomes

Let τ be a subsidy on entrepreneurs, and σ a wage subsidy. Then, (2) can be revised to give the after-subsidy expected profit of an active entrepreneur:

$$\pi = E[p](R - (1-\sigma)nw\overline{r}) + \tau \geqslant \pi_0$$

where $E[p] = \beta a(ew)^\alpha/(1+\alpha)$ by (12) and $\overline{r} = \rho/\overline{p}$ by (4). The equilibrium value of $\overline{a}$

is determined by the zero-net-profit condition of the marginal entrepreneur. Revising (28) to incorporate the subsidies and using $\tilde{a} = 2\overline{a} - 1$, we obtain:

$$\tilde{\pi}(\overline{a}, \sigma, \tau) = \begin{cases} (2\overline{a}-1)\left[\frac{\beta R(2-2\overline{a})^{\alpha}}{1+\alpha} - \frac{(1-\sigma)\rho n(2-2\overline{a})}{\overline{a}}\right] + \tau & \text{if} \quad \overline{a} \leqslant \overline{a}' \\ (2\overline{a}-1)\left[\frac{\beta}{1+\alpha}\frac{R}{1+\alpha}\right]^{\frac{2}{2-\alpha}} \left[\frac{\alpha\overline{a}(2-2\overline{a})}{(1-\sigma)\rho n}\right]^{\frac{\alpha}{2-\alpha}} + \tau & \text{if} \quad \overline{a} > \overline{a}' \end{cases} \tag{37}$$

where $\tilde{\pi}(\overline{a}, \sigma, \tau) = \pi_0$ in equilibrium. Equation (37) determines how $\overline{a}$, and therefore $\tilde{a}$ responds to changes in σ and τ. Let us focus on the stable interior solution in Figure 1. Stability requires that $\partial\tilde{\pi}/\partial\overline{a} > 0$ for both the full employment and unemployment cases. Since $\partial\tilde{\pi}/\partial\sigma > 0$ and $\partial\tilde{\pi}/\partial\tau > 0$ in (37), we have that:

$$\frac{\partial\overline{a}}{\partial\sigma} = -\frac{\partial\tilde{\pi}/\partial\sigma}{\partial\tilde{\pi}/\partial\overline{a}} < 0, \qquad \frac{\partial\overline{a}}{\partial\tau} = -\frac{\partial\tilde{\pi}/\partial\tau}{\partial\tilde{\pi}/\partial\overline{a}} < 0 \tag{38}$$

which imply that $\partial\tilde{a}/\partial\sigma < 0$ and $\partial\tilde{a}/\partial\tau < 0$ as well. An increase in either subsidy increases the number of active entrepreneurs by attracting more low-ability ones into the entrepreneurial sector.

Next, consider the wage rate. In the full employment case, the market-clearing wage is $w = m = 2 - 2\overline{a}$ as before. An increase in the number of entrepreneurs reduces $\overline{a}$ and therefore w. Lower-ability entrepreneurs and lower-quality workers enter the entrepreneurial sector.

With unemployment, the preferred wage rate in (19) becomes:

$$\widehat{w} = \frac{\alpha R\overline{p}}{(1-\sigma)(1+\alpha)\rho n} \tag{19$'$}$$

which, combined the expression for $\overline{p}$ in (16), then leads to a revised version of (21):

$$\widehat{w}(\overline{a}) = \left[\frac{\beta}{1+\alpha}\frac{\alpha}{1+\alpha}\frac{R}{(1-\sigma)\rho n}\right]^{\frac{2}{2-\alpha}} \left[\overline{a}^2(2-2\overline{a})^{\alpha}\right]^{\frac{1}{2-\alpha}} \tag{21$'$}$$

In equilibrium, $\overline{a}$ will be determined by $\tilde{\pi}(\overline{a}, \sigma, \tau) = \pi_0$ in (37). Combining the second row in (37) with (21$'$) yields $\widehat{w}$ as a function of the equilibrium value of $\overline{a}$ and the subsidies:

$$\widehat{w}(\overline{a}, \tau, \sigma) = \frac{(\pi_0 - \tau)\alpha}{(1-\sigma)\rho n}\frac{\overline{a}}{2\overline{a}-1} \qquad \text{if} \qquad \overline{a}^* > \overline{a}' \tag{39}$$

where

$$(40) \qquad \frac{\partial \widehat{w}}{\partial \overline{a}} < 0, \qquad \frac{\partial \widehat{w}}{\partial \tau} < 0, \qquad \frac{\partial \widehat{w}}{\partial \sigma} > 0$$

We are now in a position to investigate the effect of subsidy policy on social surplus. We begin with local welfare analysis, evaluating the effect of introducing small subsidies starting at laissez-faire equilibria. Then, optimal policies are considered.

The Efficiency Effect of Incremental Policies

Social surplus is again given by (32), where now $\overline{a}$ and $\widehat{w}$ depend on τ and σ. We consider the effect of changes in τ and σ on S in the unemployment and the full-employment equilibria in sequence.

Unemployment Case

Differentiating the second row of (32) with respect to $\overline{a}$ and $\widehat{w}$ and using (27.2) and (19′), we obtain the analogs of (33) and (34):[15]

$$(41) \qquad \frac{\partial S}{\partial \overline{a}} = \left[\frac{2}{\overline{a}} - 4 - \alpha + \frac{\sigma}{1-\sigma}\right](1-\sigma)\rho n \widehat{w} + 2\tau$$

$$(42) \qquad \left.\frac{\partial S}{\partial \widehat{w}}\right|_{\overline{a}} = [\alpha - (1+\alpha)\sigma]\,\frac{m\rho n}{2}$$

At the no-subsidy equilibrium with $\tau = \sigma = 0$, these reduce to (33) and (34) with $\partial S/\partial \overline{a} < 0$ and $\partial S/\partial \widehat{w} > 0$.

Consider now the effect of small changes in subsidies on social surplus. Differentiating the second row in (32) with respect to τ and σ yields:

$$(43) \qquad \frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial \overline{a}}\frac{\partial \overline{a}}{\partial \tau} + \frac{\partial S}{\partial \widehat{w}}\frac{d\widehat{w}}{d\tau}, \qquad \frac{\partial S}{\partial \sigma} = \frac{\partial S}{\partial \overline{a}}\frac{\partial \overline{a}}{\partial \sigma} + \frac{\partial S}{\partial \widehat{w}}\frac{d\widehat{w}}{d\sigma}$$

where $d\widehat{w}/d\tau$ and $d\widehat{w}/d\sigma$ include both the direct effects of these policies on $\widehat{w}$ by (40) and the indirect effect through changes in $\overline{a}$ using (38). These imply $d\widehat{w}/d\tau \gtreqless 0$ and

[15] We are assuming an interior solution with $\widehat{w} < 1$, although technically a corner solution with all workers choosing the entrepreneurial sector is possible.

$d\widehat{w}/d\sigma > 0$. Suppose we evaluate this starting at the no-intervention equilibrium. Then, using the signs obtained from (41) and (42) when $\tau = \sigma = 0$, we obtain:

$$\left.\frac{\partial S}{\partial \tau}\right|_{\tau=\sigma=0} \gtreqless 0, \qquad \left.\frac{\partial S}{\partial \sigma}\right|_{\tau=\sigma=0} > 0$$

Thus, starting from the laissez-faire unemployment equilibrium, welfare will be unambiguously increased if we impose a small subsidy on wages.

Full Employment Case

In this case, S depends only on $\overline{a}$. Differentiating the first row of (32) with respect to $\overline{a}$ and using (27.2) yields the analog of (36) with subsidies incorporated:

$$\frac{1}{2}\frac{dS}{d\overline{a}} = \frac{2\rho n(1-\overline{a})^2}{\overline{a}} - \alpha\overline{p}R + \sigma\rho n\frac{(2\overline{a}-1)(2-2\overline{a})}{\overline{a}} + \tau \tag{44}$$

This again reduces to (36) and has an ambiguous sign in the no-intervention case.

The effects of small changes in τ and σ on social surplus are now:

$$\frac{\partial S}{\partial \tau} = \frac{dS}{d\overline{a}}\frac{\partial \overline{a}}{\partial \tau}, \qquad \frac{\partial S}{\partial \sigma} = \frac{dS}{d\overline{a}}\frac{\partial \overline{a}}{\partial \sigma} \tag{45}$$

Both of these have ambiguous signs at the no-intervention full-employment equilibrium.

Optimal Policies

Suppose now that the government can choose subsidies σ and τ to maximize social surplus. It turns out that the social optimum may involve either full employment or unemployment depending on the parameters of the problem.

If the optimum involves unemployment, the government will choose τ and σ such that in (43), $\partial S/\partial\tau = \partial S/\partial\sigma = 0$. This will be the case if $\partial S/\partial\overline{a} = 0$ and $\partial S/\partial\widehat{w} = 0$, where these are given by (41) and (42). This leads to a straightforward characterization of optimal policies. Setting (42) to zero, we immediately obtain the optimal wage subsidy:

$$\sigma = \frac{\alpha}{1+\alpha} > 0$$

Then, setting (41) to zero and using this expression for σ (which implies that $\sigma/(1-\sigma) = \alpha$), the optimal subsidy on entrepreneurs satisfies:

$$\frac{\partial S}{\partial \overline{a}} = \left[\frac{2}{\overline{a}} - 4\right](1-\sigma)\rho n\widehat{w} + 2\tau = 0 \quad \Longrightarrow \quad \tau > 0$$

where the sign of τ follows from the fact that $\overline{a} > 1/2$. Thus, if unemployment exists in the optimum, both the wage subsidy and the subsidy on entrepreneurs should be positive.

On the other hand, if the optimum involves full employment, S depends only on $\overline{a}$. Optimal subsidy policies require setting (44) to zero, and this requires only one policy instrument. It is apparent that either τ or σ can be used. Here, the sign of the optimal subsidy is ambiguous. Suppose, for example, that τ is used. Then, its sign depends on the sign of the first two terms on the right-hand side of (44), or equivalently the right-hand side of (36), which is ambiguous. This parallels the result found above for incremental policy changes.

Whether there is full employment or unemployment in the optimum depends upon the parameters of the problem. To see this, consider how S varies with $\overline{a}$. In the case of unemployment, differentiating the second row of (32) with respect to $\overline{a}$ and using (27.2), we obtain:[16]

(46) $$\left.\frac{\partial S}{\partial \overline{a}}\right|_{\overline{a}>\overline{a}'} = \overline{p}R\left[\frac{2}{\overline{a}} - \alpha - 4\right] + \rho n\widehat{w} + 2\pi_0$$

For the full employment case, we found earlier that differentiating the first row of (32) gives:

(47) $$\left.\frac{dS}{d\overline{a}}\right|_{\overline{a}<\overline{a}'} = \overline{p}R\left[\frac{2}{\overline{a}} - 2\alpha - 4\right] + (2-2\overline{a})\rho n + 2\pi_0$$

As $\overline{a}$ approaches $\overline{a}'$, we move from one case to the other. Let $\overline{a} \rightarrow \overline{a}'_+$ denote $\overline{a}$ approaching $\overline{a}'$ from above (in the unemployment region), and vice versa for $\overline{a} \rightarrow \overline{a}'_-$. Then we obtain from (46) and (47), and using the fact that $\widehat{w} = 2 - 2\overline{a}$ at $\overline{a} = \overline{a}'$:

$$\lim_{\overline{a}\rightarrow\overline{a}'_+}\frac{dS}{d\overline{a}} - \lim_{\overline{a}\rightarrow\overline{a}'_-}\frac{dS}{d\overline{a}} = \alpha\rho n\widehat{w} > 0$$

Therefore, the slope of $S(\overline{a})$ increases discontinuously at $\overline{a} = \overline{a}'$, implying that $S(\overline{a})$ cannot be concave. Moreover, whether the slope of $S(\overline{a})$ at $\overline{a} = \overline{a}'$ is positive or negative depends upon the parameters of the problem, as inspection of (46) and (47) indicates.

[16] This is just the first equation in footnote 11. Note that we are assuming that σ is chosen optimally so that $\partial S/\partial\widehat{w} = 0$. Therefore, we need not take account of changes in $\widehat{w}$ as $\overline{a}$ changes.

Suppose first that $S(\overline{a})$ is single-peaked in $\overline{a}$. Then, if $\partial S/\partial \overline{a} < 0$ at $\overline{a} = \overline{a}'$, it will be optimal to induce a reduction in $\overline{a}$ thereby moving into the range of full employment. By the same token, if $\partial S/\partial \overline{a} > 0$, there will be unemployment in the optimum. From (46) and (47), we can see that $\partial S/\partial \overline{a}$ will be positive at $\overline{a} = \overline{a}'$ if π_0 is large enough. A large value of π_0 will make it more difficult to attract entrepreneurs into the sector, thereby increasing the chances of an unemployment equilibrium.

However, it is quite possible that $S(\overline{a})$ is not single-peaked. That is, $\lim_{\overline{a} \to \overline{a}'_+} dS/d\overline{a}$ may be positive, while $\lim_{\overline{a} \to \overline{a}'_-} dS/d\overline{a}$ is negative. This can occur if the wage function is as depicted in Panel B of Figure 2. Then, a reduction in $\overline{a}$ will cause S to rise. However, starting at $\overline{a} = \overline{a}'$, increases in $\overline{a}$ will also cause $\widehat{w}$ to rise in this case. This increase in $\widehat{w}$ together with the increase in $\overline{a}$ could cause the right-hand side of (46) to rise. There would be local optima in both the full-employment and unemployment ranges of $\overline{a}$, and either one could be the global optimum.

Thus, policy prescriptions depend on the parameters of the problem: with unemployment both σ and τ should be positive, while with full employment only one of σ or τ is needed and it could be positive or negative. Indeed, optimal policies are even more ambiguous when we recall that the laissez-faire equilibrium could be a corner solution in which no entrepreneurs are active. In this case, it will be necessary to impose sufficiently large subsidies to move the initial equilibrium to a stable interior one. To study this case properly would involve an explicitly dynamic analysis.

6. Alternative Information Assumptions

In the previous analysis, we assumed that both worker qualities and entrepreneur abilities were private information. This results in adverse selection in two markets, which leads to various sorts of ambiguity: ambiguity about the possibility of unemployment, ambiguity about policy prescriptions, and multiple equilibria. Moreover, equilibrium outcomes vary considerably from the full-information case in terms of the quality of workers that opt for the entrepreneurial sector, the number of entrepreneurs, and the mismatch between worker quality and entrepreneurial ability. In this section, we relax the information assumptions by allowing either a or q to be public information. In each case, we shall simply sketch the outlines of the analysis and summarize the results rather than providing a full-fledged treatment, which would be too space-consuming. The intuition will apparent given what we have learned in the case already considered.

Entrepreneurial Ability Known

Suppose first that a is public information, but q remains private. Since workers cannot be distinguished, a common wage w will be offered, and the expected quality of workers employed by all entrepreneurs will be the same. Banks can offer ability-specific gross interest rates $r(a) = \rho/p(a)$, where $p(a)$ is given by the expression for $E[p]$ in (12). The expected profit of a type-a entrepreneur then becomes:

$$\pi = p(R - r(a)nw) = \frac{\beta a R(ew)^{\alpha}}{1+\alpha} - \rho nw \tag{48}$$

where $e \leqslant 1$ is the employment rate for workers who opt for the entrepreneurial sector.

As before, entrepreneurs take e as given and choose a wage to offer. As we shall see, each entrepreneur will have a different preferred wage rate, but competition among entrepreneurs will cause a common wage rate to emerge (which may or may not clear the labor market). To see this, consider the population of active entrepreneurs. Suppose that all entrepreneurs with $a \geqslant \tilde{a}$ will become active, and let $\widehat{w}(a)$ be the wage offered by a type-a entrepreneur. Workers who seek a job in the entrepreneurial sector will apply to the entrepreneurs offering the highest $\widehat{w}(a)$. We assume, critically, that there is ex post immobility not only between sectors as above, but also from one entrepreneur to another once a job application is made. This implies that competition for workers will equalize $\widehat{w}(a)$ among entrepreneurs, so we can simply write $\widehat{w}$ in what follows. Given the probability of employment e, workers with $q \leqslant e\widehat{w}$ are attracted to the entrepreneurial sector, and all are indifferent among the active entrepreneurs.

Suppose as before that there are m active entrepreneurs. In equilibrium, all entrepreneurs will perceive the same e (although out of equilibrium they may well perceive different ones), and that will be given by (14) as before. Moreover, given that $\widehat{w}$ is the same for all entrepreneurs—the highest one preferred by any active entrepreneur—the equilibrium wage rate will again be given by $\widehat{w}$ with $\tilde{q} = \sqrt{m\widehat{w}}$ if there is unemployment by (15), and $w^e = m$ if there is full employment. Unemployment will occur if $\widehat{w} > w^e$ as before.

Consider now what determines the common value of $\widehat{w}$ that is offered by all entrepreneurs in the unemployment case. Given e, an entrepreneur of ability a prefers the wage rate w that maximizes expected profits π given by (48). The solution to this problem yields:

$$\widehat{w}(a) = \left(\frac{\beta a R \alpha}{(1+\alpha)\rho n}\right)^{1/(1-\alpha)} e^{\alpha/(1-\alpha)}$$

which is increasing in a. The entrepreneur with $a = 1$ prefers to offer the highest wage rate, and all other entrepreneurs will be obliged to follow. Therefore, $\widehat{w} = \widehat{w}(1)$. As long as $\widehat{w} > m$, that will be the prevailing wage rate. In effect, perfect information in capital markets intensifies competition for workers thereby pushing up $\widehat{w}$. Using $e = \sqrt{m/\widehat{w}}$, the above equation for entrepreneur $a = 1$ becomes:

$$\widehat{w} = \left(\frac{\beta R \alpha}{(1+\alpha)\rho n}\right)^{2/(2-\alpha)} m^{\alpha/(2-\alpha)} \tag{49}$$

The characterization of equilibrium is similar to that for the asymmetric-information case considered earlier. The market clearing wage rate will be $w^e = m$, where $m = 2 - 2\overline{a}$. There will be a value of $\overline{a}$, say $\overline{a}'$, such that $\widehat{w} = w^e$. The equilibrium wage rate will be $w = w^e$ if $\overline{a} \leqslant \overline{a}'$, and $w = \widehat{w}$ if $\overline{a} > \overline{a}'$, where $\widehat{w}$ is given by (49). We can proceed as above to derive expected profits for the marginal entrepreneur, $\tilde{\pi}(\overline{a})$, the analog of (28). The analog of Figure 1 applies here as well. There will generally be two interior equilibria, only one of which is stable. The other stable equilibrium is that at which $\overline{a} = 1$ where there are no active entrepreneurs. In the stable equilibrium, there may be either full employment or unemployment.

It is apparent that this case has similar qualitative features to the asymmetric-information case analyzed earlier. In both cases, there may be full employment or involuntary unemployment. Workers are paid a common wage and are drawn from the bottom of the quality distribution. There is, however, no adverse selection problem in credit markets in the sense that the interest rate can be conditioned on the ability of entrepreneurs. This implies some differences in the equilibrium possibilities under the two regimes. It will be the case that $\overline{a}'$ will be lower and $\widehat{w}$ higher in this case than in the case where both a and q are private information. This is a consequence of the fact that the wage rate is bid up to that preferred by the highest-ability entrepreneur. If there is full employment, wage rates will be the same for a given number of entrepreneurs. Unfortunately, although one might expect there to be fewer entrepreneurs financed in this case given the absence of adverse selection, that turns out not to be unambiguously the case. Nonetheless, from a social point of view, the number of entrepreneurs is now unambiguously too low ($\overline{a}$ is too high) if there is full employment: the tendency for over-entry due to adverse selection in credit markets (the De Meza-Webb effect) no longer applies.

Worker Quality Known

In this case, worker quality can be observed by all agents, but each entrepreneur's ability is private knowledge. This information structure leads to a somewhat more complicated outcome. Since the qualities of the workers hired by a given entrepreneur are known to the bank, the interest rate charged can reflect those qualities. Consider an entrepreneur that hires $n(q)$ workers of quality q, where $\int n(q)dq = n$ as before. The gross interest rate charged to this entrepreneur is $\overline{r} = \rho/\overline{p}(q)$ with $\overline{p}(q) = \beta\overline{a}\overline{q^\alpha}$, where $\overline{q^\alpha} = \int n(q)q^\alpha dq/n$. Since worker quality is known, a wage function $w(q)$ can be offered. The expected profit of a type-a entrepreneur is then given by:

$$\pi = \beta a\overline{q^\alpha}\left(R - \overline{r}\int n(q)w(q)dq\right) = a\int_0^1 \frac{n(q)}{n}\left(\beta Rq^\alpha - \frac{\rho n}{\overline{a}}w(q)\right)dq = a\overline{\pi} \tag{50}$$

where $\overline{\pi}$ is common to all entrepreneurs.

With worker quality observable to entrepreneurs, there will be full employment. But, wage setting is more complicated than before since wages can be quality-specific. First note that offering wages equal to each worker's quality cannot be an equilibrium in the entrepreneurial sector. That is because for all entrepreneurs, the same uniform quality of workers would maximize profits $a\overline{\pi}$ when $w = q$, so wages of this worker would be bid up. Competition among entrepreneurs implies that $w(q)$ must be such that all entrepreneurs will be indifferent about the quality of workers that they hire. To determine the equilibrium pattern of wages that will ensure that, consider a type-a entrepreneur. Given the wage function $w(q)$, that entrepreneur would choose $n(q)$ to maximize π in (50) subject to $\int n(q)dq/n = 1$. If $w(q)$ were such that the entrepreneur were indifferent about the quality of workers hired, the first-order conditions for all q would be satisfied with equality, yielding:

$$\lambda = \beta R q^{\alpha} - \frac{\rho n}{\overline{a}} w(q), \qquad \text{or} \qquad w(q) = \frac{\overline{a}}{\rho n}(\beta R q^{\alpha} - \lambda) \qquad \forall\, q \tag{51}$$

where the value of λ is the same for all entrepreneurs. Since $\alpha < 1$, $w(q)$ is increasing and strictly concave: $w'(q) > 0 > w''(q)$. Assuming that workers are drawn from the interior of the quality distribution, there will be two values of q, denoted $\underline{q}$ and $\overline{q}$ with $\overline{q} > \underline{q}$, such that $w(\underline{q}) = \underline{q}$, and $w(\overline{q}) = \overline{q}$.[17] Figure 3 illustrates.

For worker qualities q such that $\underline{q} < q < \overline{q}$, the wage payment $w(q) > q$, so these workers are attracted into the entrepreneurial sector. Workers with $q < \underline{q}$ or $q > \overline{q}$ (if there are any of the latter) will choose the traditional sector. Then the supply of labor in the entrepreneurial sector, denoted s, will be given by $s(\overline{a}, \pi_a) = n(\overline{q} - \underline{q})$. where $\underline{q}$ and $\overline{q} \leqslant 1$ are defined as above. Note that by (51), $s(\overline{a}, \pi_a)$ is increasing in $\overline{a}$ and decreasing in π_a. In terms of Figure 3, an increase in $\overline{a}$ shifts the curve $w(q)$ up, while an increase in π_a shifts it down.

Since the number of entrepreneurs is m and each entrepreneur hires n workers, equilibrium in the labor market requires $s(\overline{a}, \pi_a) = nm$. Given m, π_a adjusts to satisfy this equilibrium condition. Since, $s(\cdot)$ is monotonic in m, we can solve the equilibrium condition for the value of π_a that ensures labor market clearing, $\pi_a(\overline{a}, m)$, where $\pi_a(\cdot)$ is increasing in $\overline{a}$ and decreasing in m.

Since entrepreneurs that are not active obtain π_0, the profit of the marginal entrepreneur will satisfy $\pi_0 = \tilde{a}\pi_a(\overline{a}, m)$. Entrepreneurial expected profits are increasing in a, so it will be the case as before that entrepreneurs with ability $a \geqslant \tilde{a}$ will become active, while the remainder will choose the alternative option. Therefore, $m = 1 - \tilde{a}$ and $\overline{a} = (1 + \tilde{a})/2$. Using these relationships, the condition determining the quality of the marginal entrepreneur may be written:

$$\tilde{a}\pi_a\left(\frac{1+\tilde{a}}{2}, 1 - \tilde{a}\right) = \pi_0$$

The value of $\tilde{a}$, or equivalently $\overline{a} = (1 + \tilde{a})/2$, that satisfies this equation will be uniquely determined.

[17] If $w(1) \geqslant 1$, it will be the case that $\overline{q} = 1$, since the upper bound of q is unity. Conversely, if $w(0) > 0$, we have $\underline{q} = 0$, which is the lower bound of q.

Given that entrepreneurs are indifferent about the quality of workers they hire, and workers are indifferent to whom they work for, there will not be perfect matching of a and q. Indeed, we might expect that matching is random. That being the case, there will be three sources of inefficiency. There will be a tendency for too many entrepreneurs to enter due to the standard adverse selection effect on credit markets. There will be a mismatch of workers with entrepreneurs. And, the set of workers by quality may not be correct. Unlike in the case where q is private, here higher-quality workers will generally be attracted into the entrepreneurial sector. However, it is not clear whether the average quality of workers is too high or too low.

7. Concluding Comments

The results in this paper are obviously model-specific. Nonetheless, they are suggestive and do indicate that once one combines adverse selection in labor markets with those in credit markets, matters become much more complicated and policy prescriptions less clearcut. Multiple stable equilibria exist, one of which can be a corner solution in which no entrepreneurs are active (and therefore no surplus is generated). Even if the market equilibrium is interior, it may involve involuntary unemployment, or even credit rationing. Depending on the equilibrium, efficiency consequences, and therefore policy prescriptions, may differ. If there is involuntary unemployment, a presumption exists that there will be too few entrepreneurs and therefore too much unemployment, although even that depends on parameter values. In a full-employment equilibrium, no such presumption exists. Unlike the case with adverse selection applying only in credit markets, there may be too few or too many entrepreneurs. Adverse selection in credit markets tends to induce too many low-ability entrepreneurs to enter since the interest rate they face is too generous given their ability. At the same time, the entry of more entrepreneurs mitigates the adverse selection problem in labor markets which results in too-few high-quality workers.

There are a number of ways in which the model could be fruitfully enriched, albeit at the further expense of simplicity A straightforward extension would be to allow firms to vary the number of workers they employ, as in Weiss (1980). As he shows, firms will tend to hire too few workers because of adverse selection, leading to an argument for subsidizing employment in the entrepreneurial sector. A more ambitious extension would be to have both old firms and new firms competing with one another both for workers and in output markets. Assuming that established firms have informational advantages over new ones, one would expect to obtain a case for differential tax treatment of the two sorts of firms, although to which type's advantage may not be obvious. Finally, we have assumed in our basic model, following the literature as well as our own assumptions, pooling on both labor and credit markets. One can imagine extending the model to allow for the possibility of separating firms either on the basis of information acquired from ex ante monitoring by banks or signaling by entrepreneurs, or on the basis of other firm characteristics or behavior, such as firm size or the ability to provide collateral.[18] These extensions would complicate the analysis considerably.

References

Boadway, R. and M. Keen (2005), 'Financing and Taxing New Firms under Asymmetric Information,' Queen's University, mimeo.

Boadway, R. and M. Sato (2005), 'Entrepreneurship and Asymmetric Information in Input Markets,' Queen's University, mimeo.

Boadway, R. and M. Sato (1999), 'Information Acquisition and Government Intervention in Credit Markets,' *Journal of Public Economic Theory* **1**, 283–308.

Boadway, R. and J-F Tremblay (2005), 'Public Economics and Start-up Entrepreneurs,' in Kanniainen and Keuschnigg (2005), 181–219.

de Meza, D. (2002), 'Overlending,' *Economic Journal* **112**, F17–F31.

de Meza, D. and D.C. Webb (1987), 'Too Much Investment: A Problem of Asymmetric Information,' *Quarterly Journal of Economics* **102**, 281–92.

[18] Elsewhere, we have analyzed the efficiency and policy consequences of ex ante monitoring (Boadway and Sato, 1999). For a consideration of the consequences of banks separating firms using collateral and variable loan size, see Boadway and Keen (2005). And, as mentioned, we have studied the possibility of separating workers using ex post bonuses when workers have a perceptible influence on the firm in which they are employed.

Diamond, P. A. (1982), 'Wage Determination and Efficiency in Search Equilibrium,' *Review of Economic Studies* **49**, 271–7.

Dietz, M.D. (2002), 'Risk, Self Selection and Advice: Banks versus Venture Capitalists,' University of St. Gallen, Institut für Finanzwirtschaft und Finanzrecht (IFF-HSG), mimeo.

Kanbur, R.M. (1981), 'Risk Taking and Taxation: An Alternative Perspective,' *Journal of Public Economics* **15**, 163–84.

Kanniainen, V. and C. Keuschnigg (2005), *Venture Capital, Entrepreneurship, and Public Policy* (Cambridge, Mass.: MIT Press).

Keuschnigg, C. and S.B. Neilsen (2003), 'Tax Policy, Venture Capital, and Entrepreneurship,' *Journal of Public Economics* **87**, 175–203.

Keuschnigg, C. and S.B. Neilsen (2004), 'Start-ups, Venture Capitalists, and the Capital Gains Tax,' *Journal of Public Economics* **88**, 1011–42.

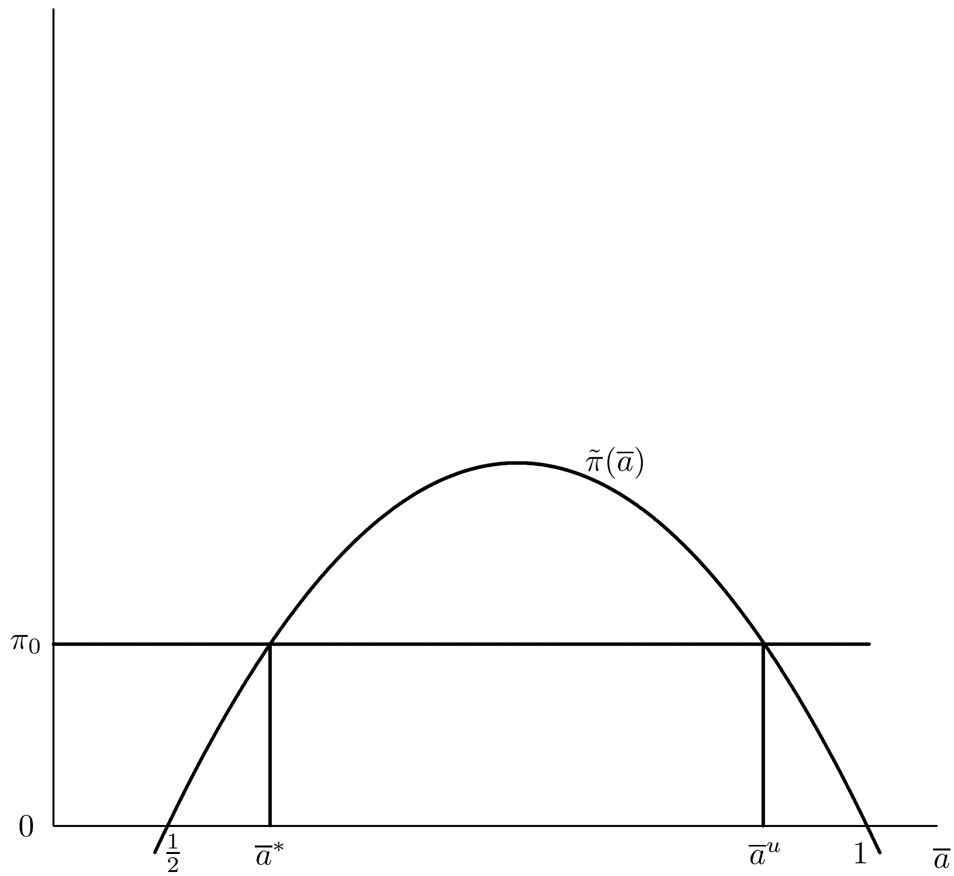
Myers, S.C. and N.S. Majluf (1984), 'Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have,' *Journal of Financial Economics* **13**, 187–221.

Rosen, H.S. (2005), 'Entrepreneurship and Taxation,' in Kanniainen and Keuschnigg (2005), 181–219.251–79.

Shapiro, C. and J.E. Stiglitz (1984), 'Equilibrium Unemployment as a Worker Discipline Device,' *American Economic Review* **74**, 433–444.

Stiglitz, J.E. and A. Weiss (1981), 'Credit Rationing in Markets with Imperfect Information,' *American Economic Review* **71**, 393–410.

Weiss, A. (1980), 'Job Queues and Layoffs in Labor Markets with Flexible Wages,' *Journal of Political Economy* **88**, 526–38.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π_0
0
$\frac{1}{2}$
$\overline{a}^*$
$\overline{a}^u$
1
$\overline{a}$

Figure 1

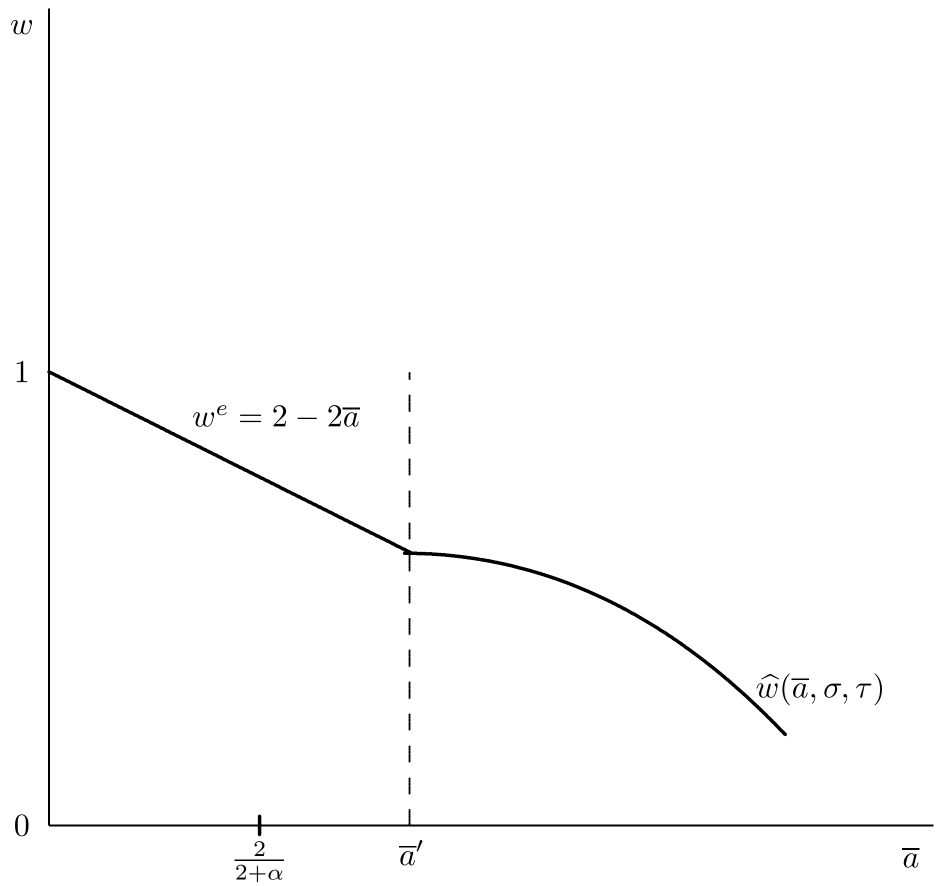

Figure 2, Panel A

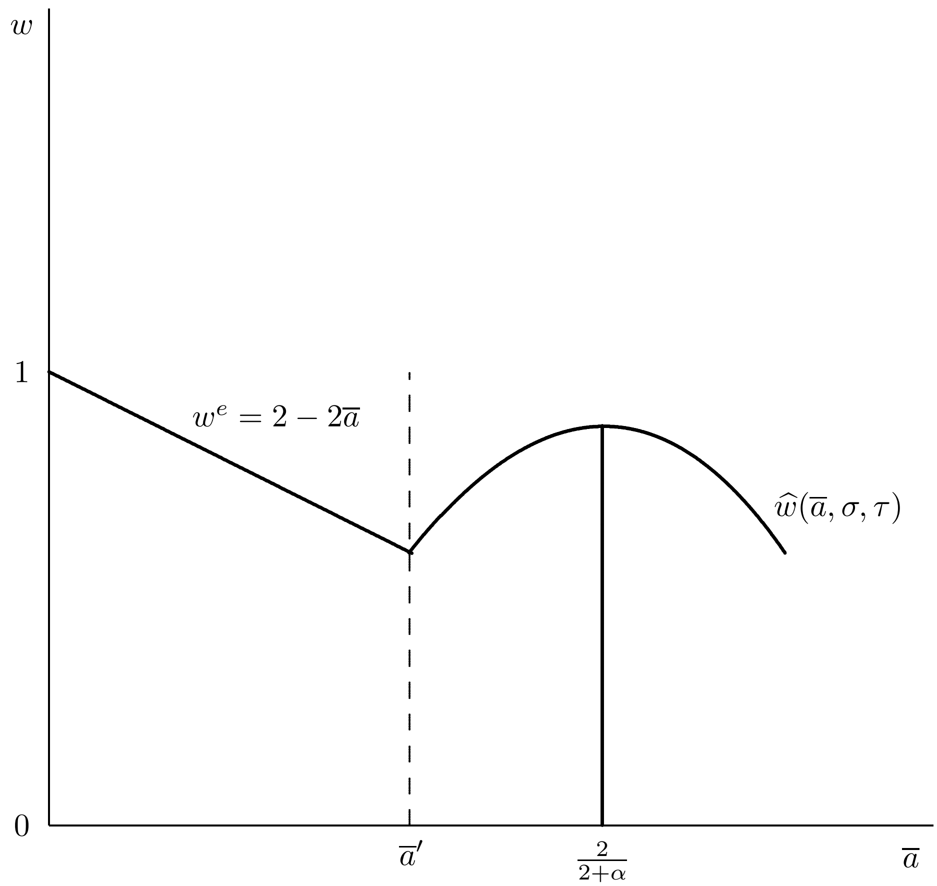

Figure 2, Panel B

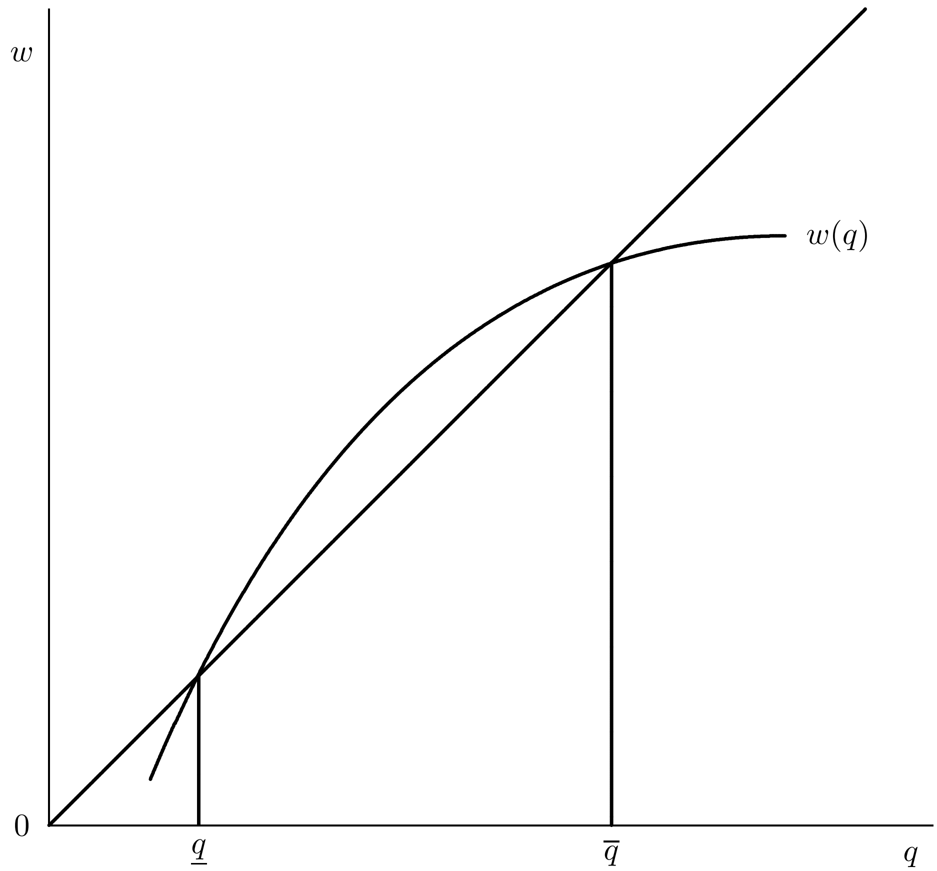

Figure 3