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The Distributive Implications of Patents on Indivisible Goods

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The Distributive Implication of Indivisible Goods[1]

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Abstract: *An indivisible good is an ideal type with interesting properties and strong implications about public policy. It is a good - such as a heart transplant or a treatment for AIDS - that must be consumed in a fixed amount or not at all. The community's demand curve for an indivisible good is a rotation of the distribution of income. Monopolization of ordinary goods can be expected to reduce everybody's consumption; monopolization of indivisible goods knocks out low income consumers. Deadweight loss from monopolization has a distinct distributional aspect best captured in a utility-weighted measure. Indivisible goods are strong candidates for public provision and for the expropriation of patents.*

Introduction

A good is defined here as "indivisible" when a fixed amount is useful, less is useless and more is superfluous, so that each person wants to buy either that fixed amount (typically one unit) or none at all. You cannot have half an appendectomy and wouldn't want a second. Nor can you benefit from half an AIDS treatment or half a course of antibiotics; you must take the entire treatment if it is to do you any good. The indivisible good is, of course, an ideal type to which actual goods conform to a greater or lesser extent, but the study of the indivisible goods may cast light on economic phenomena and suggest economic arguments that might otherwise be overlooked.

This article establishes several propositions about indivisible goods.:

- A person's demand curve for an indivisible good is a pair of vertical lines.

- A community's demand curve for an indivisible good is a rotation of the distribution of income.

- The community's elasticity of demand for an indivisible good is the measure of equality in the distribution of income.

- Everybody deterred by monopolization from consuming an indivisible good is poorer than anybody not deterred.

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[1]With thanks for helpful suggestions by my colleagues Sumon Majumdar, Marvin McInnis and Klaus Stegemann and two anonymous reviewers.

- The deadweight loss from the monopolization of an indivisible good arises entirely from its impact on poor people.

These propositions will be "proved" conclusively. They are proved in the sense that they are implied by certain axioms. In high school geometry, we looked upon axioms as self-evident truths. In the social sciences we have no such luxury. These propositions will be proved on the strength of whatever assumptions about the economy are necessary to make them true, in the hope that there is enough space between assumptions and propositions to render the propositions interesting and useful.

Three large implications will be drawn from these propositions, one analytical and the other two about public policy. The analytical implication is that a measure of deadweight loss might be constructed in utils rather than in dollars. The political implications are that

- A strong objection to the socialization of commodities - that a given expenditure on redistribution is always more beneficial to the recipients when provided in cash rather than in kind - is much weaker for indivisible goods than for ordinary goods, which is why medical care is socialized in many countries, but never hats or sweaters.

- The case for expropriation of patents is much stronger for indivisible goods than for ordinary goods.

Both of these political implication are best thought of as factors in the choice of public policy, rather than as decisive all by itself.

The plan of the article is this: The first section is about the derivation of the demand curves for indivisible goods, with emphasis on the contrast between the shape of the demand curves for the individual and for society as a whole. The demand curve for society is nothing more than a rotation of the distribution of income. When the distribution of income conforms to a Pareto function, the elasticity of demand and the measure of equality are one and the same. The next section is about he consequence of the monopolization of indivisible goods. Typically, a rise in the price of an ordinary good induces everybody, rich or poor, to consume less. By contrast, a rise in the price of an indivisible good leaves richer people's consumption unaffected and stops poorer people from consuming altogether, so that the deadweight loss from monopolization corresponds exactly to the harm inflicted upon people who are too poor to consume the good at the higher price but not too poor to consume it at the original price. In these circumstances, it may be instructive to construct measures of surplus and deadweight loss in utils rather than in dollars. The article concludes with a discussion of the relevance of the distinction between ordinary and indivisible goods in weighing the pros and cons of the socialization of commodities and of the choice between prizes and patents as ways of encouraging invention

The Demand for Indivisible Goods

Indivisibility, as defined here, is a property of taste rather than technology. There need be nothing unusual about the supply curve of an indivisible good, and, for convenience, it is assumed to be flat so that market price is invariant. Indivisibility is defined in the first instance with reference to the utility function. A distinction is drawn between *indivisible* goods and *ordinary* goods. For ordinary goods, utility increases steadily with the amount consumed. For an indivisible good, utility is higher if one consumes the appropriate amount than if one does not, but utility is not augmented by consuming less than the appropriate amount or by additional consumption over and above the appropriate amount. Throughout this paper, units of the indivisible good are graduated so that the appropriate amount is just equal to 1.

The distinction between ordinary goods and indivisible goods is reflected in their demand prices. A person's demand price for an ordinary good is a continuously decreasing function of the amount consumed. A person's demand price for an indivisible good is a reservation price. A unit of the indivisible good is purchased if one' reservation price exceeds the market price, but not otherwise.

A person's utility function covering an indivisible good as well as a vector of ordinary goods must be of the form

$$u = u(\delta, x^*) \qquad (1)$$

where u is utility, x* is a vector of quantities of ordinary goods consumed and δ signifies the presence or absence of the indivisible good; $\delta = 1$ when the indivisible good is consumed and $\delta = 0$ when it is not.

Indivisible goods are endowed with two associated properties. First, they must not be indispensable. The concept of indivisible goods is uninteresting unless a person has a real option to consume or not to consume. The reservation price would otherwise be meaningless. Second, and more problematically, indivisible goods are assumed to be separable from ordinary goods within the utility function. Separability means that, at any given set of market prices, the amount consumed of each and every ordinary good is dependent on total expenditure on ordinary goods alone, regardless of whether or not the indivisible good is consumed as well. Expenditure on each and every ordinary good is the same for a person with $90 to spend and who desists from buying the indivisible good as it would be if that person had $100 to spend of which $10 was spent on the indivisible good. Separability ensures that a person's reservation price for the indivisible is an increasing function of his income. The higher a person's income, the more he would be prepared to pay for the indivisible good rather than do without it altogether.

Separability restricts the form of the utility function. With separability, the utility function in equation (1) reduces to

$$u = u\{\delta, v(x^*)\} \qquad (2)$$

where $v(x^*)$ is a subordinate utility function of the vector of ordinary goods alone, where $u\{1, v(x^*)\}$ is necessarily greater than $u\{0, v(x^*)\}$ and where $u\{0, v(x^*)\}$ must be finite as long as the indivisible good is not indispensable.

In an atemporal context, indivisible goods might be exemplified by medical insurance on the understanding that care is the same for every insured person and that, without insurance, one receives no medical care at all. Newly-invented goods may approximate pure indivisibility when the difference between having the good and not having it dwarfs the difference in usefulness between better or worse variations of the good. Newly-invented goods are obviously not indispensable because there was a time when people had to make do without them. Local telephone service is indivisible in so far as one pays a fixed fee regardless of how much one's phone is used.

Indivisibility lies at an extreme of a continuum; it is a property that goods possess to a greater or lesser extent. You either have a kidney transplant or you don't, but a kidney transplant is not perfectly indivisible in so far as there is a choice in the quality of the surgeon or the standard of hospital care. In practice, most goods differ in quality as well as in quantity. Purely and completely indivisible good would differ in neither dimension. While it is hard to imagine goods with no variation in quality whatsoever, some goods are close enough to complete indivisibility, and the practical implications of indivisibility are striking enough, to make the analysis of indivisible goods interesting and useful.[2]

To establish the strong propositions about indivisible goods at the beginning of the paper, the utility function in equations (1) or (2) will, from now on, be confined to a very simple form. Assume there are only two goods, one ordinary good with a price per unit of 1 and one indivisible good with a price per unit of p, so that the budget constraint of a person with an income of Y becomes

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[2]The concept of indivisible goods in this paper is similar but not quite identical to a concept employed by Nichols and Zeckhauser (1982), Blackorby and Donaldson(1988) and Besley and Coate (1991). Their "indivisible good" is introduced as an instrument for redistribution when the government cannot see who is poor . The good must be indivisible in the sense that a person can only make use of one unit at a time, but it must also be amenable to variation in quality, so that, for example, a poor person would accept the offer of a low quality house but a rich person would not. Houses in their context would be indivisible in the sense that a rich person would be worse off well off with two low quality houses than with one high quality house. By contrast, an indivisible good in this paper must be of uniform quality so that a person acquires a full dose or none at all. For houses to be indivisible in this sense, there must be only one quality of house. Such goods are sometimes called "preclusive"; ignoring the condo in Florida, consumption of one unit, large or small, precludes consumption of another.

$$x + \delta p = Y \tag{3}$$

where x is the quantity of the ordinary good, where, once again, $\delta = 1$ if the indivisible good is consumed and $\delta = 0$ otherwise. The postulated utility function is

$$u = A^{\delta}x \tag{3}$$

where A is a constant greater than 1. In effect, $u = Ax$ if the person consumes the indivisible good as well as x units of the indivisible good, and $u = x$ if the person consumes x units of the ordinary good but not the indivisible good. It must be the case that $A > 1$ to ensure that a person is better off consuming than not consuming the indivisible good for any given consumption of ordinary goods is. When the indivisible good is medical, the ordinary good x can be thought of as available together with a certain survival probability and risk of discomfort from illness, and the parameter A can be thought of as the multiple by which survival probability or comfort increases as a consequence of the consumption of the indivisible good.

On these assumptions, the person's utility becomes

$$u = A^{\delta}(Y - \delta p) \tag{5}$$

and the person can be thought of as maximizing utility by his choice of δ. This formulation is somewhat less restrictive than might at first appears because Y, x and p can be interpreted as total real, real income devoted to ordinary goods and the price of the indivisible goods in terms of ordinary goods rather than dollars.[3]

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[3]Replace the scalar x in the utility function of equation (3) with utility of ordinary goods, v(x), where x is a vector of quantities of ordinary goods consumed and where v(x) is Cobb-Douglas. Specifically, the person's utility function becomes

$$u = A^{\delta}v(x)$$

where $$v(x) = \Pi x_j^{\alpha j}$$

where x_j is his consumption of the good j, where each α_j is a fixed coefficient and where the sum of all such coefficients is equal to 1. The person maximizes utility in equation (1*) subject to his budget constraint

$$\Sigma p_j x_j + \delta p = Y$$

where p_j is the price of the j^{th} good and p is, once again, the price of the indivisible good.

A distinction must now be drawn between a person's total income, Y, and his expenditure on ordinary goods, M, where, necessarily, As a function of income, Y, a person's reservation price for the indivisible good is an immediate derivable from the utility function in equation (5). Without the indivisible good, utility becomes

$$u_0 = Y \qquad (6)$$

With the indivisible good, utility becomes

$$M = Y - \delta p$$

Regardless of whether or nor a person purchases the indivisible good, he can be thought of as choosing a vector of ordinary goods to maximize v(x) subject to his budget constraint

$$\Sigma p_j \, x_j = M$$

where p_j is the market price of the ordinary good j. The process gives rise to an indirect utility function of ordinary goods

$$v = KM/I$$

where K is a constant term (equal to $\Pi\alpha_j^{\alpha j}$) and I is a geometric price index (equal to $\Pi p_j^{\alpha j}$).

Replacing the direct utility of ordinary goods in equation (3) with the indirect utility in equation (13), we arrive at the utility function

$$u = A^{\delta}KM/I = A^{\delta}K[(Y/I) - \delta(p/I)]$$

where Y/I can be interpreted as real income, p/I can be interpreted as the real price of the indivisible good and the constant term K will cancels out in the determination of a person's reservation price for the indivisible good. The expression Y/I plays the role of x in equation (4), and the expression p/I plays the role of p in equation (3). The preceding equation is virtually the same as equation (5) in the text. All of the propositions to be derived about the one can be derived about the other as well.Nor would it matter if v were concave in the limited sense that the sum of the exponents in the Cobb-douglas utility function is less than 1. But the restriction of v(x*) to the Cobb-Douglas form is important. Without it results to be derived below must be seen as approximations.

$$u_1 = A(Y - p) \tag{7}$$

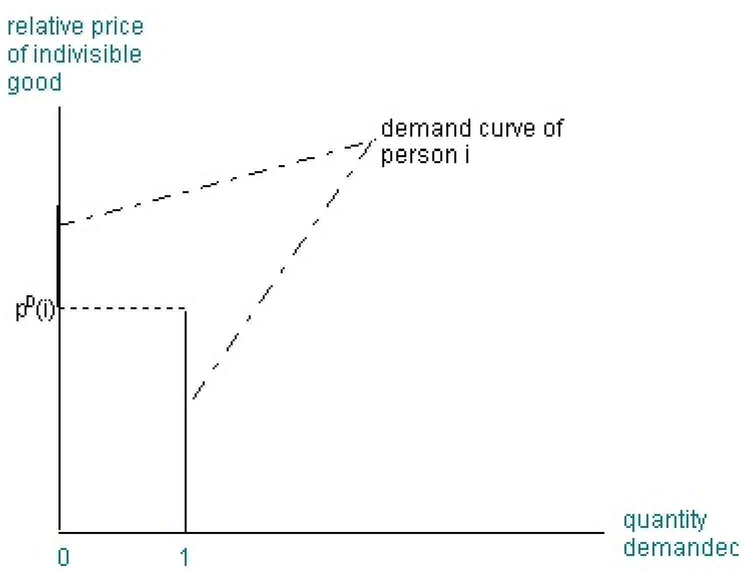
One's reservation price for the indivisible good is the price just high enough to make one indifferent between consuming and not consuming the indivisible good. It is the price for which $u_1 = u_0$, so that

$$Y = A(Y - p) \tag{8}$$

Thus $$p^D(Y) = Y(A - 1)/A \tag{9}$$

where $p^D(Y)$ is the reservation price when one's income is Y.[4]

Figure 1: A Person's Demand Curve for an Indivisible Good



Regardless of the form of utility function, a person's

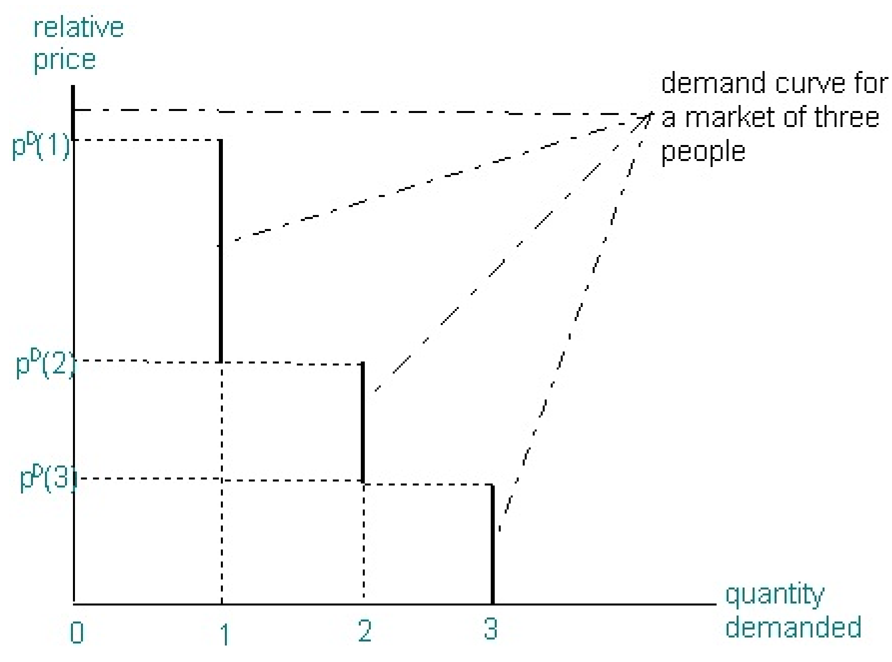
[4]Could an indivisible good ever be inferior in the sense that its reservation price is higher for the poor than for the rich? Not as the indivisible good is defined in equation (3) where, analytically, it acts as a magnifier of the utility of all ordinary goods together. An indivisible good might be inferior within the more general utility function of equation (1) where there is nothing to forbid the indivisible good from magnifying the utility of some ordinary goods more than others. If the rich consume relatively more meat and the poor consume relatively more potatoes, an indivisible good that magnifies the nutritional impact of potatoes but not meat might be worth more to the poor than to the rich.

reservation price for an indivisible good must be as shown in figure 1. The reservation price of person i is designated as $p^D(i)$, and the corresponding demand curve must consist of two vertical lines, one above $p^D(i)$ at a quantity of 0 and the other below $p^D(i)$ at a quantity of 1. This is in sharp contrast with the demand curve for an ordinary good in which the demand price is a continuously decreasing function of the person's quantity consumed. If person i's utility function is as indicated in equation (4) and if his income is Y, his reservation price, $p^D(i)$, must equal $p^D(Y)$ in equation (9).

The next step in the argument is to connect the demand curve of a person to the demand curve of a community of people with identical utility functions corresponding to equation (4) but different incomes. It turns out that the community's demand curve is downward-sloping, just like a person's demand curve for an ordinary good. More interestingly, the community's demand curve for an indivisible good is a simple inversion of the distribution of income.

We shall illustrate this in two stages: first for a group of three people and then for a large economy. Designate the three people as 1, 2, and 3, and let their demand prices be $p^D(1)$, $p^D(2)$, $p^D(3)$ such that $p^D(1) > p^D(2) > p^D(3)$. As shown in figure 2, the demand curve for these three people together is obtained by adding quantities horizontally. Nobody buys the indivisible good when the price exceeds $p^D(1)$. Only person 1 buys the indivisible good - so that only 1 unit is purchased -when the market price is between $p^D(1)$ and $p^D(2)$. Persons 1 and 2 buy the indivisible good - so that 2 units are purchased - when the market price is between $p^D(1)$ and $p^D(2)$. Finally, all three people buy the indivisible good - so that 3 units are purchased - when the market price is below $p^D(3)$ That being so, the demand curve consists of four vertical segments as shown by the four bold lined in figure 2.

Figure 2: The Demand Curve for a Group of Three People

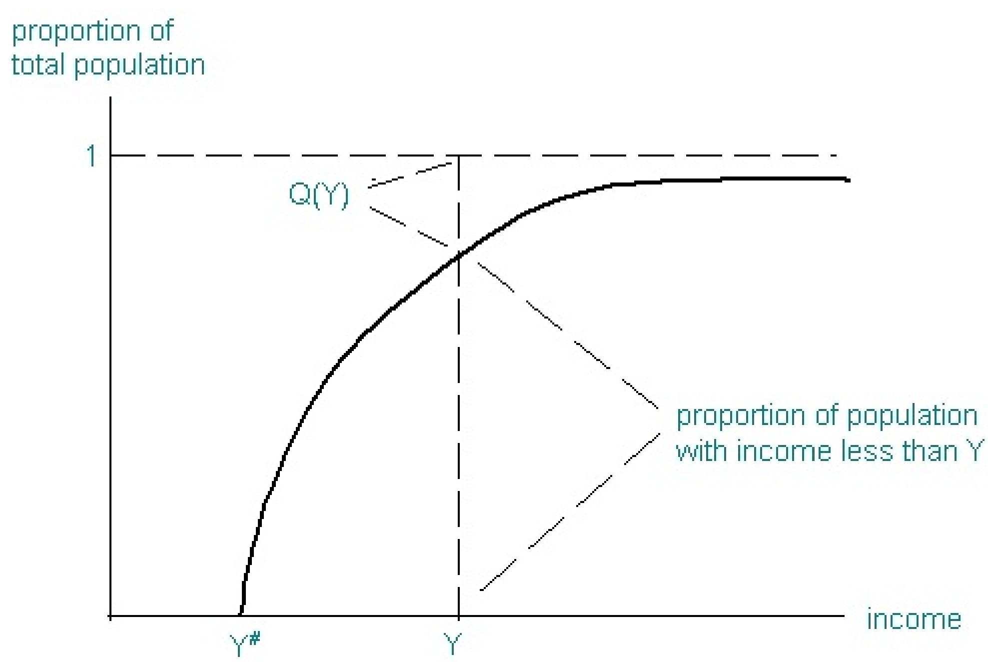


A smooth downward-sloping demand curve for an indivisible good emerges when the market is composed of a large number of potential buyers ordered in accordance with their demand prices, and with very small differences between contiguous people in that ordering. Information about the distribution of income can then be employed to convert a function $p^D(Y)$ connecting people's demand prices to their incomes into a proper demand function $p^D(Q)$ connecting demand prices to the quantity, Q, consumed. Since each person consumes either 1 unit or no units of the indivisible good, the quantity consumed in the population as a whole can be represented equally well by the number of consumers or by as the proportion of the population that chooses to consume it. We adopt the second interpretation so that Q is at once a a variable in the demand function and an aspect of the distribution of income. The distribution of income can be represented by a function Q(Y) where Q is the proportion of the population with incomes in excess of Y. Corresponding to any distribution function Q(Y), there must be an inverse function Y(Q). Plugging function Y(Q) into the demand function $p^D(Y)$ connecting price and income, we arrive at the demand curve

$$p^D(Q) = p^D(Y(Q)) \tag{10}$$

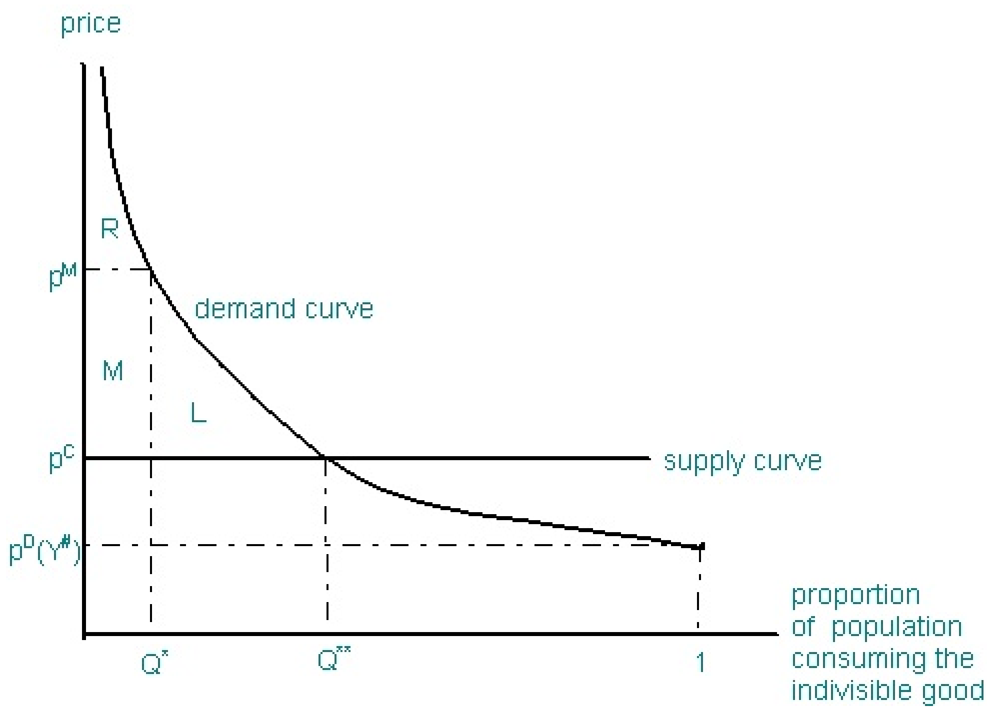
which is the demand curve we seek. The demand curve is downward-sloping because p^D is an increasing function of Y, and Y is a decreasing function of Q.

Figure 3: The Distribution of Income



The process is simplified enormously when the demand price is a multiple of income as exemplified by equation (9) because, in that case, the demand function for the indivisible good and the distribution of income are virtually the same. The distribution of income is commonly represented on a graph like that in figure 3 with income on the horizontal axis and the proportion of the population with less than that income on the vertical axis.[5] But, for constructing a demand curve, we are interested in the proportion of people with incomes above, rather than below, any given income. This is represented by the distance Q(Y) between the curve in figure 3 and a flat line at a height of 1 above the horizontal axis. For reasons that will soon become evident, we are supposing there to be a minimal income, $Y^{\#}$.

Figure 4: The Demand and Supply Curves for the Indivisible Good



[5]It is customary to describe the distribution of income as a distorted S with two long tails, one, as shown in figure 3, for high incomes and the other for low incomes, but with no minimum income such as $Y^{\#}$ in figure 3. The shape in figure 3 is postulated to conform to a Pareto distribution because the Pareto distribution has nice properties that simplify the exposition and facilitate the development of a numerical example. A normal or lognormal distribution might be more realistic, but far less tractable. A case can also be made for the shape in figure 3 as appropriate for the study of indivisible goods, where the relevant Y is some notion of purchasing power rather than actual disposable income. For example, a person who earns a great deal of money every three years and loses money in between would be shown in the statistics as having negative a income every two out of three years, but his purchasing power - which is what matters in his demand for the indivisible good - would always be positive. There must be some minimum income in the sense of purchasing power as long as nobody starves to death.

The function Y(Q) (which is only an inch away from the demand curve for the indivisible good when p^D is a fixed multiple of Y) is obtained by a 90^0 anti-clockwise rotation distribution of income function in figure 3 and then by measuring income on what was the flat line a distance of 1 above the horizontal axis. The demand curve itself is obtained by replacing Y with the corresponding value of p^D. The resulting demand curve, together with the supply curve for the indivisible good, is shown in figure 4. In the figure, the supply curve is assumed to be flat, and the demand price is assumed to be connected to income by equation (9) so that, except for the rotation and the uniform contraction of the vertical axis, the curves in figures 3 and 4 are identical.

The construction of the demand curve for the indivisible good is especially simple when the distribution of income conforms to a Pareto function

$$Q(Y) = (Y^{\#}/Y)^{\mu} \qquad (11)$$

where Y# is the income of the poorest person in the economy and where μ is an equality parameter; the larger μ, the more equal the distribution of income must be[6]. Converting income into the demand price in accordance with equation (9), the demand curve becomes

$$Q = (Y^{\#}/Y)^{\mu} = [Y^{\#} (A - 1)/A](p^D)^{-\mu} \qquad (12)$$

where the expression in square brackets is a constant and there the parameter μ which began life as a measure of the degree of equality in the income distribution is now reborn as the elasticity of demand for an indivisible good. In general, one would expect that, the more equal the

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[6]The density function of the Pareto distribution is

$$f(Y) = \mu(Y^{\#})^{\mu} (Y)^{-\mu -1}$$

where $Y^{\#}$ is lowest income, there is no upper limit on y, and μ is the equality parameter. The parameter μ has been observed to be about 3 in many countries, and it is assumed here to be 3 exactly. It follows that the proportion of the population with incomes between $Y^{\#}$ and Y is

$$\int_{Y^{\#}}^{Y} \mu(Y^{\#})^{\mu} Y^{-\mu-1} dY = 1 - (Y^{\#}/Y)^{\mu}$$

which must equal 1 when y rises to infinity. Since the proportion of the population with incomes between $Y^{\#}$ and y is $1 - (Y^{\#}/Y)^{\mu}$, the proportion, Q(Y), of the population with incomes equal to or greater than y must be $(Y^{\#}/Y)^{\mu}$ as indicated in equation (11). Estimates of μ for many times and places vary from a low of about 1.25 to a high of about 2.50. See Bronfenbrenner (1971), page 46. For Rome at the time of Cicero, the value of μ was estimated to be 1.5.

distribution of income, the greater the elasticity of demand for an indivisible good, but the clean equality in equation (12) is dependent on our simplifying assumptions.[7]

When the supply curve of the indivisible good is flat and at a height, p^C, (the cost of production) above the horizontal axis, the quantity demanded per person is Q^* at the intersection of the supply and demand curves. The quantity demanded person is less than 1 because the price, p^C , exceeds the demand price of the poorest person in the community. At a higher price, p^M , a larger proportion, Q^{**} , of the population would prefer not to purchase the indivisible good. Areas of surplus, R, M and L, will be discussed below.

The difference in the forms of the demand curves for an indivisible good, for a person and for the community, has its counterpart in a difference in the measures of surplus, defined here in the usual way as the dollar value of having the good available at the market price as compared with not having it at all. For an ordinary good, the surplus to the entire population is the surplus to an individual writ large. If the demand curve in figure 4 were for an ordinary good and if peoples utility function were Cobb-Douglas as in equation (2), the total surplus would be the area between the demand curve and the supply curve, and that surplus would be apportioned among people in accordance with their incomes

For an indivisible good, a person's surplus is the difference between *his* demand price and the market price. When the cost of production is p^C , the surplus, S(Y), of a person with an income Y is

$$S(Y) = p^D(Y) - p^C \qquad (13)$$

as long as his demand price exceeds the market price, and is 0 otherwise because none of the indivisible good would be consumed. If everybody's utility function was as specified in equation (5) ands if it just so happens that the poorest person is just indifferent between buying and not buying the indivisible good, then $p^D(Y^\#) = p^C$ and $p^D(Y) = [(A - 1)/A]Y$ so that

$$S(Y) = p^D(Y) - p^D(Y^\#) = [(A - 1)/A][Y - Y^\#] \qquad (14)$$

For both types of goods, the community's surplus per head, S, is the area between the demand curve and the supply curve as shown in figure 4, but, for an indivisible good, the surplus becomes

$$\int_{Y^\#}^{\infty} S(Y) f(Y) dY = \int_{Y^\#}^{\infty} [p^D(Y) - p^D(Y^\#)] f(Y) dY \qquad (15)$$

[7]With a normal or lognormal distribution of income, there must still be a function Q(Y) for each distribution of income, but there would be no simple counterpart to the identification of μ with the elasticity of demand for the indivisible good.

where f(Y) is the distribution function of income and as long as the demand price of the poorest person and the cost of production are assumed to be the same. If there is a Pareto distribution of income and if everybody's utility is in accordance with equation (3), the surplus in equation (15) reduces to

$$S = [(A - 1)/A][1/(\mu - 1)]Y^{\#} \qquad (16)$$

The surplus is directly proportional to the income of the poorest person in the community and inversely proportional to the degree of equality in the distribution of income. The less equal the distribution of income, the larger the surplus turns out to be.[8]

The Social Cost of the Monopolization of an Indivisible Goods

For all goods, whether indivisible or ordinary, monopolization breaks the surplus, S, as it would be if the good were not monopolized into three parts: the monopoly revenue, M, the residual surplus, R, associated with consumption not deterred by monopolization, and the loss of surplus or deadweight loss, L, from the reduction in consumption induced by the rise in price, as shown in figure 4.

$$S = M + R + L \qquad (17)$$

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[8]Replacing S(Y)by [(A - 1)/A][Y - $Y^{\#}$] from equation (14) and replacing f(Y) by its value in the Pareto distribution, equation (15) becomes

$$S = \int_{Y^{\#}}^{\infty} S(Y)f(Y)dY = [(A - 1) \int_{Y^{\#}}^{\infty} [Y - Y^{\#}]\mu(Y^{\#})^{\mu} (Y)^{-\mu-1}dY$$

$$= [(A - 1)/A]\mu(Y^{\#})^{\mu} \int_{Y^{\#}}^{\infty} [Y - Y^{\#}](Y)^{-\mu-1}dY$$

$$=[(A - 1)/A]\mu(Y^{\#})^{\mu} \int_{Y^{\#}}^{\infty} (Y)^{-\mu} - Y^{\#}(Y)^{-\mu-1}]dY$$

where the upper limit of the integral is 0, so that

$$S = - [(A - 1)/A]\mu(Y^{\#})^{\mu}\{[(1/(-\mu + 1)](Y^{\#})^{-\mu+1} - [1/(-\mu)]Y^{\#}(Y^{\#})^{-\mu}\}$$

$$= - [(A - 1)/A]\mu(Y^{\#})^{\mu}\{[(1/(-\mu + 1)](Y^{\#})^{-\mu+1} - [1/(-\mu)]Y^{\#}(Y^{\#})^{-\mu}\}$$

$$= - [(A - 1)/A]\mu(Y^{\#})\{[(1/(-\mu + 1)] - [1/(-\mu)]\} = [(A - 1)/A](Y^{\#})[(1/(\mu - 1)]$$

For ordinary goods, the cost of the monopolization, the residual surplus and the loss of surplus are spread throughout the income distribution, more or less in proportion to income, exactly so if everybody's income elasticity of demand is equal to 1. For indivisible goods, monopolization has a marked distributional bias, funnelling the cost of monopolization onto some people more than others. It is the purpose of this section and the next to show the nature of bias, to explain how M, R and L come to have different impacts on rich and poor, to estimate the magnitudes of M, R and L from parameters of the utility function and the distribution of income, and to suggest an alternative measure of deadweight loss taking account of the distributional impact of monopolization.

The monopolist sets a price, p^M, to maximize his profit, M, where

$$M = Q(p^M - p^C) \qquad (18)$$

For an indivisible good where everybody's preference is represented by the utility function in equation (3) and where the demand curve is a reflection of the distribution of income in accordance with equation (12), the monopolist's revenue-maximizing price, turns out to be[9]

$$p^M = p^C\mu/(\mu - 1) \qquad (19)$$

With in p^M figure 4 interpreted as the monopoly price, the corresponding quantity Q^* can be computed from equation (12). It follows at once from equation (19) that the minimum income, Y^M, at which one buys a unit of the indivisible good when it is monopolized is

$$Y^M = p^M A/(A - 1) = Y^\# \mu/(\mu - 1) \qquad (20)$$

Our object in what follows is to show how the original surplus, S, when an indivisible good is available at its cost of production, p^C, is apportioned between the revenue of the monopolist, M, the residual surplus, R, to people who continue to consume the indivisible good even at the monopoly price, and the loss of surplus, L, to people deterred by the higher from consuming the good at all. Specifically, we wish to quantify the proportions, M/S, R/S and L/S. On the strength of the special assumptions we have made about the common utility function and the distribution of income, it turns out that these proportions will depend on only one parameter,

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[9]Equation (19) is derived bymaximizing M with respect to p^M. From equation (18),

$$M = Q(p^M - p^c) = [N(p^C)^\mu(p^M)^{-\mu}][p^M - p^C] = [N(p^C)^\mu][(p^M)^{-\mu+1} - p^C(p^M)^{-\mu}]$$

Therefore $\qquad \delta M/\delta p^M = [N(p^C)^\mu][(-\mu + 1)(p^M)^{-\mu} - p^C(-\mu)(-p^M)^{-\mu-1}] = 0$

On factoring out $[N(p^C)^\mu](p^M)^{-\mu}$, we see that $(-\mu + 1) = (-\mu)(p^C)(p^M)^{-1}$

from which equation (19) follows immediately.

the measure, μ, of the degree of equality in the distribution of income. Our task is simplified if it is supposed that Q^{**} in figure 4 is equal to 1, in other words that

$$p^C = [(A - 1)/A]Y^{\#} \qquad (21)$$

signifying that the poorest person is indifferent between buying and not buying the indivisible good.

Determination of the ratios M/S, R/S and L/S, requires assumptions about the parameters in the formulae we have derived. Let $\mu = 2$, $Y^{\#} = \$30,000$, $A = 1.2$, and $p^C = \$5,000$, which is consistent with equation (21) above. Then,

- from equation (19), it follows that $p^M = p^C\mu/(\mu - 1) = \$10,000$

- from equation (20), it follows that $Y^M = p^M A/(A - 1) = Y^{\#}\mu/(\mu - 1) = \$60,000$

- from equation (11), it follows that $Q^* = Q(Y^M) = (Y^{\#}/Y^M)^{\mu} = .25$
  meaning that monopolization reduces consumption of the indivisible good by 75%.

- from equation (16), it follows that the total surplus per person is

$$S = [(A - 1)/A][1/(\mu - 1)]Y^{\#} = \$5,000$$

- the revenue of the monopolist per person is

$$M = (p^M - p^C)Q(Y^M) = ([\mu/(\mu - 1)] - 1)\, p^C\, (Y^{\#}/Y^M)^{\mu}$$

$$= S\, [(\mu - 1)/\mu]^{\mu} = \$1,250$$

The residual surplus per person once the indivisible good is monopolized becomes

$$R = \int_{Y^{\#}}^{Y^{\#}} (p^D(Y) - p^C) f(Y)\, dY \qquad (22)$$

which is the product of the surplus as it would be if the minimal income were Y^M rather than $Y^{\#}$, weighted by the proportion, $Q(Y^M)$ of the population with incomes in excess of Y^M. Using equation (16),

$$R = S\{Y^M/Y^*\}\{Y^{\#}/Y^M\}^{\mu} = \$2,500 \qquad (23)$$

Finally, the loss of surplus per person becomes

$$L = \int_{Y^{\#}}^{Y^{d\#}} (p^D(Y) - p^C) f(Y) dY \tag{24}$$

which can be obtained as a residual,

L = S - M - L = $1,250

Thus, as ratios of total surplus, the three components become

M/S = 1,250/5,000 = .25

R/S = 2,500/5,000 = .50

and L/S = 1,250/5000 = .25

Of the total surplus from the indivisible good if it were not monopolized, 75% (.25 + .50) would accrue to the richer 25% of the population with incomes over $60,000, and the remaining 25% would accrue to the poorer 75% of the population with incomes between $30,000 and $60,000. These estimates of M/S, R/S and L/S are cooked in that they depend entirely upon the parameters we have chosen, but the story they tell might have a more general application. Monopolization takes away more than half of what would otherwise be the surplus to the rich, but it takes away the whole of what would otherwise be the surplus of the poor. In fact, if the monopolist were among the rich, then the rich, as a class, would lose nothing from monopolization, but the poor would remain no better off than if the monopolized good did not exist at all.

When the monopolized good is indivisible, the area M is at once a profit of the monopolist and a cost of monopolization to the rich, while the area L is at once a loss to the poor and the deadweight loss to the economy as a whole. This is fundamentally different from the monopolization of ordinary goods where the burden of monopoly is shared by everyone more or less in proportion to income.

The cost of production of the indivisible good is assumed to be just high enough to make the poorest person indifferent between consuming a unit or not. Dropping that assumption does not change the story very much. If the cost of production were higher than the demand price of the poorest person - that is, if $p^C > p^D(Y^{\#})$ - then a group of very poor people would desist from consuming the indivisible good even if it were not monopolized, and the story we have told would be confined to segment of the population that does consume the indivisible good at the competitive price. If the cost of production were lower - that is, if $p^C < p^D(Y^{\#})$ - the monopolist's price would at, a minimum, be set equal to $p^D(Y^{\#})$ because the demand curve is vertical up to that price. One cannot say a priori whether a higher price would be advantageous.

Measuring Deadweight Loss in Utils

The measure, L/S, of deadweight loss as a proportion of total potential surplus is the usual indicator of the harm from monopoly. The measure is typically employed as part of the case against monopolization or as an indicator of cost in a comparison of costs and benefits to determine whether monopolization is justified in some circumstances, notably as a goad to invention. For ordinary goods, the ratio L/S seems a reasonable indicator of harm because the harm is widespread throughout the income distribution. For indivisible goods, the ratio may be an understatement because it fails to take account of monopoly's distributive consequences.

In any society where income is redistributed - through progressive taxation, unemployment insurance, welfare, food stamps, socialized medical care, and so on - the choice of redistributive programs may be thought of as the maximization of a utilitarian measure of social welfare where the interpersonal trade-off is in utils rather than in dollars and where redistribution is always expensive in the sense that a dollar's worth of benefit to the poor can only be attained at the cost of something more than one dollar to the rich. It may be appropriate in this context to measure the ratio of deadweight loss to total original surplus is utils as well.

This measure is easily constructed within the assumptions in this article. It follows from the special utility function in equation (5) that the marginal utility of a person with an income of Y is 1/Y. Measured in utils, a person's surplus from the an indivisible good purchased at a price p^C becomes

$$S^*(Y) = u'(Y)(p^D(Y) - p^C) = (1/Y)(p^D(Y) - p^C) \tag{25}$$

and the surplus to the community, S^*, as a whole becomes

$$S^* \int_{Y^{\#}}^{\infty} = (1/Y)(p^D(Y) - p^C)f(Y)dy \tag{26}$$

the loss of surplus in utils, L^*, becomes

$$L^* = \int_{Y^{\#}}^{Y^{\#\#}} (1/Y)(p^D(Y) -p^C)f(Y)dY \tag{27}$$

and, as shown in the appendix, the utility-weighted ratio of loss of surplus to total surplus becomes

$$L^*/S^* = 1 - 2[\mu/(\mu - 1)]^{-\mu} = 1 - 2(1/2)^2 = ..5$$

The the ratio of deadweight loss to total potential surplus from monopolization of indivisible goods rises from 25% to 50% when measurement is converted from dollars to utils. Monopolization may sometimes appear to be justified by one measure but not the other.

The Politics of Indivisible Goods

Indivisibility has two large political implications. It avoids a principal defect in the public provision of certain goods and services, and it strengthens the case for alternatives to patents as a way of encouraging innovation. We consider these in turn.

<u>Public Provision of Private Goods</u>: The main task of the government is to provide public goods such as the army and the police, but governments also provide private goods such as education, health care and public housing. Among the arguments for the public provision of certain private goods are these:

- Assistance to the Poor: Universal public provision of private goods is redistributive when financed by progressive taxation. It is even more redistributive if the good provided is something the poor will accept but the rich will not. Public housing is the prime example. Even if public housing were offered to everyone, the rich would refuse the gift because publically-provided houses would be too small and incommodious.

- Altruism: No matter what my own income, I may prefer to live in a society with a relatively narrow distribution than to live in a society with great discrepancies between the incomes of the rich and incomes of the poor. I may care not only about my children, but about the entire next generation.

- Paternalistic Altruism: One's concern for other people may carry more weight for some commodities than for others. I may not care whether you ride a car or a bicycle, or whether you live in a big house or a little house, but it may care if you are unable to afford medical treatment when you are sick or if your children are denied a proper education...

- Externalities: I want your children to be well educated so that my children will live in a country of well-educated people. Externalities may flow from a good itself, regardless of how it is financed, or it may flow from public, but not private, provision. All schooling may create more productive citizens, but it could be the case that only public schooling creates civic virtues.

- Insurance: Public provision, especially of medical care, avoids some of the disadvantages of private insurance. Insurance companies have a strong incentive to let chronically ill people die Insurance companies are understandably reluctant to insure people known to be in ill-health.. Ideally, we should insure ourselves while still in the womb, but there are certain difficulties in doing so. Public provision does so automatically.

Against these arguments for public provision, it may be objected that each person should stand on his own feet and should consume no more than can be purchased with his earnings, or that government activities are invariably inefficient and corrupt. These objections are ignored in this paper to focus on another: that redistribution should always be in cash rather than in kind. Better to provide an old age pension, unemployment insurance or a negative income tax than to provide health care, education or public housing. The rationale of this objection is that, any given amount of public expenditure for the benefit of the poor leaves recipients are worse off when provided with goods than when provided with enough money to buy those goods, or something else, as they please.

Consider a program to supply every person with some commodity. If the commodity can be bought or sold at the going market price, then the only difference between redistribution in cash or in kind is the government's waste of time and money in procuring and distributing the commodity. A person who wants more than the government supplies, buys an additional amount. A person who wants less sells the excess. Better for the government to supply each person with an equivalent amount of money to be spend on whatever that person values most. It would make no sense whatever for the government to supply everybody with a head of lettuce per week. The case against public provision in these circumstances is that it is entirely ineffectual.

Public provision can only influence consumption when, for some reason, people have to be content with the amount that the government supplies. Nobody can sell part of his right to public medical care or public education. But the influence of public provision may be perverse. Suppose that the government supplies x units, and suppose, as would often be the case, that some people prefer to buy more than x while other people would prefer to buy less than x if the good were available on the market instead. Everybody would become better off with a cash grant of enough money to buy x units because consumption of the good would no longer be tied to what the government, in its wisdom, chooses to supply. If the good in question is housing and if the government supplies everybody with a certain size of house, then people who would prefer a smaller house and a bigger car, or a bigger house and a smaller car, are both disappointed.

The difficulty is compounded when goods come in many shapes and sizes. Some people own one necktie, others own a great many, others own none at all. Some neckties are cheap; other neckties are expensive. Everybody has his own preferences for colour and design. Public provision of a single uniform neck tie would be as ridiculous as a law requiring everybody to buy identical neckties. Almost everybody, rich or poor, would prefer to devote the cost to something else. Nobody would be better off than if he were given the money instead.

The "cash rather than kind" objection is powerful but not decisive. Other things being equal, the less people's diversity of preference for a good, the weaker the "cash rather than kind" argument turns out to be.[10] The objection is at its weakest for the socialization of indivisible

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[10]Financed by progressive taxation, public provision of private goods may be evaluated by voters as a trade off between an implicit redistribution of income and the possibility of a mismatch between the amount of the good provided by the government and the amount each

goods. If everybody would be prepared to buy the indivisible good voluntarily - as would be the case when $p^D(Y^\#) > p^C$ - public provision would be superfluous. Otherwise, it would have no effect upon the purchase by the rich, would supply the good to poor people who might be unwilling to buy it for themselves, and would be somewhat redistributive when financed by proportional or progressive income taxation. In such circumstances, arguments for public provision based upon paternalistic altruism, externalities or insurance might easily prevail.

Though relatively free of the major defect of the socialization of commodities - that redistribution in kind is, cet par, inferior to redistribution in cash - the socialization of indivisible goods has a special problem of its own. Ordinary goods can always be supplied by the government equally. Whatever the total amount of a commodity to be distributed, each person's portion can be the same. In a population of - say - a million people, a total supply of 5 million loaves of bread would provide 5 loaves per person, while a total supply of 3 million loaves would provide 3 loaves per person. That does not work for indivisible goods. If there are 5 thousand kidneys available and 20 thousand people who need them, then 15 thousand people must do without, for 1/4 kidney per person would be of no use to anybody. If indivisible goods are to be socialized, governments should make every effort, and bear whatever cost is necessary, to be sure there is enough to go round. Sometimes that is impossible, and a system of rationing must be devised, notwithstanding the risk of corruption, rent seeking and favouritism. But socialized medicine on the cheap may be worse for most people than either private medical care or socialized medicine properly financed.[11]

Expropriation of Patents: A second political implication of indivisible goods is about the relative merits of patents and prizes as inducements to invention. To induce invention, the inventor must be rewarded, by a salary as university professors or scientists in government labs are rewarded, by prize money bearing some relation to what an agency of government deems the invention to be worth or by a patent granting the inventor a monopoly on his invention for a number of years.[12] . Our concern here is with the relative merits of patents and prizes[13], and with

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person would prefer the government to supply. There may or may not be a majority in favour of public provision depending on the relative strengths of these two considerations. The latter has little or no importance for indivisible goods. See, Usher, D., "The Welfare Economics of the Socialization of Commodities", *Journal of Public Economics*, 1977, 151-68.

[11]On this topic, see Usher, D., "Public Provision of Indivisible Private Goods in Short Supply", *Public Finance Review*, 2002, 385-415.

[12]Other aspects of patents - the boundary between patentable inventions and unpatentable science, the complex interplay between public and private funding of research, the arcane economics of patent scope, legal ploys to extend patent life, international conflict over patent protection, the benefits and costs of the race among would-be patent holders to invent first and the occasional failure of the patent system to draw forth socially advantageous research on new products - are all ignored in this paper. There is a vast literature on each of these topics. See, for

the pros and cons expropriation. Governments might expropriate patents to some inventions, just as governments expropriate land in the path of a new highway, on the understanding that expropriation would be matched by just compensation.[14] All methods of rewarding invention have serious defects, though it is better to reward inventions defectively then to forgo discoveries by not rewarding then at all.

The principal defect in the rewarding invention by prizes or in expropriating patents with just compensation is the absence of a universally-recognized rule for deciding what an invention is worth. Even under a patent regime, there is some requirement for legal or administrative discretion in identifying patent-worthy inventions and resolving disputes about priority, but, once an invention is patented, the income to the patent-holder is determined by the market. Governmental discretion would be much greater with prizes or expropriation. Some public agency would have to decide on the value of an invention and on the appropriate fraction of the full surplus from invention to offer as a reward or as compensation for expropriation. The agency charged with the responsibility of rewarding inventors directly would surely make mistakes, over-valuing some inventions and undervaluing others. Absence of a clear standard, would open the rewarding process to rent seeking and corruption.

The corresponding advantage of prizes or expropriation of patents is that the full surplus from invention is preserved. With reference to figure 4 above, the market price of the invented product would be the cost of production, p^C , rather than the monopoly price, p^M , and the entire surplus from the availability of the invented good would be become available. If the direct reward to the inventor were set equal to the monopoly profit, M, the surplus to consumers would increase from R to R + L. The deadweight loss would be regained. These advantages and disadvantages pertain to ordinary goods and to indivisible goods as well.

What differentiates indivisible goods from ordinary goods in this context is the nature of the deadweight loss. A central theme in this paper is the difference between ordinary goods and indivisible goods in their response to monopolization. When an ordinary good is patented, the burden of the rise in price is borne by everybody more or less in proportion to their incomes, exactly so for the particular function we have employed as an example. When an indivisible good is patented, the effect of the rise in price is borne disproportionately by the poor. Among the richer people not deterred from consumption, the cost is uniform rather than in proportion to their incomes. Poorer people are deterred altogether, left in exactly as they would be if the newly-invented good had not been invented at all. The case for rewarding invention with prizes and for

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example, Arrow (1963), Heller and Eisenberg (1998), Merges and Nelson (1990), Nordhaus (1969), Tandon (1983), Stegemann and Pazderka (2003), and Usher (believe it or not, 1964).

[13]On prizes and payments for research, see Wright (1983).

[14]On expropriation of patents, see Kremer (1989). On compulsory licencing, see Hollis and Flynn (forthcoming).

expropriation with just compensation is much stronger for indivisible goods than for ordinary goods.

Bibliography

Arrow, K., “Economic Welfare and the Allocation of Resources for Invention”, in R. R. Nelson ed. *The Rate and Direction of Inventive Activity: Economic and Social Factors*, 1963.

Besley, T. and Coate, S., “Public Provision of Private Goods and the Redistribution of Income”, *American Economic Review*, 1991, 979-84.

Blackerby, C. and Donaldson, D., “Cash versus Kind, Self-selection and Efficient Transfers”, *American Economic Review*, 1988, 691-700.

Bronfenbrenner, M, *Income Distribution Theory*, 1971.

Heller, M. and Eisenberg, R., “Can Patents Deter Innovation? The Anticommons in Biomedical Research”, *Science*, 1998, 188-91.

Hollis, A. and Flynn, S., “An Economic Analysis of Compulsory Licences for Needed Medicines”, *International Review of Law and Economics*, forthcoming.

Kremer, M., “Patent Buyouts: A Mechanism for Encouraging Innovation”, *Quarterly Journal of Economics*, 1998, 1137-67.

Merges, R. and Nelson, R., “On the Complex Economics of Patent Scope”, *Columbia Law Review*, 1990, 839-916.

Nichols, A.L. and Zeckhauser, R.J., “Targeting Transfers through Restrictions on Recipients”, *American Economic Review*, 1982, 372-77.

Tandon, P., “Rivalry and Excessive Allocation of Resources to Research” *Bell Journal of Economics*, 1983, 152-65.

Stegemann K. and Pazderka, B., “The TRIPS Agreement as an Alliance for Knowledge Production: The Funding of Pharmaceutical Innovation” *The Journal of World Intellectual Property*, 2003, 529-70.

Usher, D, “The Welfare Economics of Invention” *Economica*, 1964, 279-87.

Usher, D., “The Welfare economics of the Socialization of Commodities”, *Journal of Public Economics*, 1977, 151-168.

Usher, D., “Public Provision of Indivisible Goods in Short Supply”, *Public Finance Review*, 2002, 385-415.

Wright, B., "The Economics of Invention Incentives: Patents Prizes and Research Contracts", *American Economic Review*, 1983, 691-707.

Appendix: Measuring Deadweight Loss from Monopolization in Utils rather than in Dollars from Monopolization

Measured in dollars, a person's surplus from the purchase of a unit of the indivisible good is $[p^D(Y) - p^C]$ where $p^D(Y)$ is the demand price of a person with an income of Y and p^C is the cost of production. Aperson's surplus in utils is his surplus in dollars weighted by his marginal utility of income, dU/dy.

$$(dU/dY)\,[p^D(Y) - p^C].$$

With the logarithmic utility function in equation (5), the marginal utility of income is equal to 1/Y and the utility-weighted loss of surplus as a proportion of the utility-weighted total surplus becomes

$$\frac{\text{dead weight loss}}{\text{total potential surplus}} = \frac{\int_{Y^{\#}}^{Y^{M}} \mu'(Y)\left[p^D(Y) - p^C\right] f(Y)\,dY}{\int_{Y^{\#}}^{\infty} \mu'(Y)\left[p^D(Y) - p^C\right] f(Y)\,dY}$$

where $y^{\#}$, y^M and the distribution function f(y) are as specified in the text. Cancelling out constants in the numerator and the denominator, the ratio becomes

$$\frac{\int_{Y^{\#}}^{Y^{M}} (Y - Y^{\#})Y^{-\mu-2}\,dY}{\int_{Y^{\#}}^{\infty} (Y - Y^{\#})y^{-\mu-2}\,dY}$$

$$\frac{\int_{Y^{\#}}^{Y^{\#}\mu/(\mu-1)} Y^{-\mu-1}\,dY^{-\mu-1} - Y^{\#}\int_{Y^{\#}}^{Y^{\#}\mu/(\mu-1)} Y^{-\mu-2}\,dY}{\int_{Y^{\#}}^{\infty} Y^{-\mu-1}\,dY - Y^{\#}\int_{Y^{\#}}^{\infty} Y^{-\mu-2}\,dY}$$

$$= \frac{\left.\frac{Y^{-\mu}}{-\mu}\right|_{Y\#}^{Y\#\mu/(\mu-1)} - Y\#\left.\frac{Y^{-\mu-1}}{-\mu-1}\right|_{Y\#}^{Y\#\mu/(\mu-1)}}{\left.\frac{Y^{-\mu}}{-\mu}\right|_{Y^{\#}}^{\infty} - Y\#\left.\frac{Y^{-\mu-1}}{-\mu-1}\right|_{Y^{\#}}^{\infty}}$$

$$= \frac{\dfrac{\left(Y^{\#}(\mu/\mu-1)\right)^{-\mu}}{-\mu} - \dfrac{\left(Y^{\#}\right)^{-\mu}}{-\mu} - \dfrac{Y^{\#}\left((\mu/\mu-1)\right)^{-\mu-1}}{-\mu-1} + \dfrac{Y^{\#}\left(Y^{\#}\right)^{-\mu-1}}{-\mu-1}}{-\dfrac{\left(Y^{\#}\right)^{-\mu}}{-\mu} + \dfrac{Y^{\#}\left(Y^{\#}\right)^{-\mu-1}}{-\mu-1}}$$

When $(y^{\#})^{-\mu}$ is cancelled out of the numerator and the denominator, the ratio becomes

$$\frac{\left[\dfrac{1}{\mu} - \dfrac{1}{\mu+1} - \left(\dfrac{\mu}{\mu-1}\right)^{-\mu}\dfrac{1}{\mu} + \left(\dfrac{\mu}{\mu-1}\right)^{-\mu-1}\dfrac{1}{\mu+1}\right]}{\left[\dfrac{1}{\mu} - \dfrac{1}{\mu+1}\right]}$$

$$= 1 - \left(\frac{\mu}{\mu-1}\right)^{-\mu}\mu(\mu+1)\frac{1}{\mu}\left(\frac{2}{\mu+1}\right) = 1 - 2\left(\frac{\mu}{\mu-1}\right)^{-\mu}$$

Setting $\mu = 2$,

$$\frac{\text{deadweight loss}}{\text{total potential surplus}} = 1 - 2(1/2)^2 = 50\%$$

If $\mu = 1.25$, signifying a much less equal distribution of income,

$$\frac{\text{deadweight loss}}{\text{total potential surplus}} = 1 - 2\left(\frac{.25}{1.25}\right)^{1.25} = .7325\%$$

The less equal the distribution of income, the larger the loss of utility-weighted surplus from monopolization.