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The Discount Rate and the Value of Remaining Years of Life

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Abstract

We back out an estimate of a personal discount rate of between 3 and 4 percent for a person with a life expectancy of 74 years who dies at age 30 (or 40) and has a value of statistical life of $6.3 million. Central to these calculations is the series generated by Murphy and Topel of value of life years (the dollar value of consumption plus the dollar value of leisure, with some smoothing for income in retirement). We employ the Makeham "model" of life expectancy in our calculations.

- key words: discount rate, value of life
- JEL classification: J170, I120

1. Introduction

We back out an estimate for a representative individual's discount rate, given a value of statistical life of $6.3 million, the Makeham "model" of life expectancy, and the schedule of "value of life years" reported in Murphy and Topel [2006;

*Morten Nielsen provided helpful guidance as I commenced calculation of life expectancy.

Figure 2b].[1] For a person dying suddenly at age 30, we find that a discount rate of just under 4% yields a present value of future expected years of life of \$6.3 and for a person dying suddenly at age 40, our backed-out discount rate is somewhat lower, but still between 3% and 4%. We link values of life years to values of remaining years of life via the Makeham "model" and various trials for a discount rate. Our "best" trial discount rate yields an expected value of remaining years of life at \$6.3 (the value of a statistical life favored by Murphy and Topel).

The Makeham function for arriving at an expected age at death derives directly from the well-known Gompertz function and includes an additional parameter.[2] Since the probability of a person of age k dying over the current period is easily obtained from survey data, one can estimate the parameters for the "force of mortality" function (a hazard rate function) quite easily. These parameters also parameterize the survival function in turn and one thus has a convenient, empirically based life expectancy "model" to work with.

2. Makeham's Formulation of Life Expectancy

The Makeham hazard function is $h(t) = B + \alpha e^{\beta t}$ and the survival function is $S(t) = e^{(-Bt - \frac{\alpha}{\beta}(e^{\beta t}-1))}$. Then minus the derivative of the survival function is the density function $f(t) = [B + \alpha e^{\beta t}]e^{(-Bt - \frac{\alpha}{\beta}(e^{\beta t}-1))}$. This function[3] is of course a product of the hazard rate and the survival function. Expected age at death, for

[1] We will report on what a "value of life year" is below and what estimates Murphy and Topel came up with for this central series.

[2] For the very special case of a constant "death rate" (hazard rate) D, the expected age at death, starting from age 0 is $\int_0^\infty tDe^{-Dt}dt$ which equals $1/D$. Here e^{-Dt} is the survival function and $\int_0^\infty De^{-Dt}dt = 1$. The variance for this case is $\int_0^\infty (t - \frac{1}{D})^2 De^{-Dt}dt$ which equals $\frac{1}{D^2}$.

[3] Bebbington, Lai and Zitikis [2007] reports on the Gompertz and Makeham density functions for life expectancy madeling. They also report on an approach to refining functions of the Gompertz sort in order to capture better real world data on life expectancy.

a person of age Z, is now

$$A(Z) = \int_{Z}^{\infty} t * h(t) * \frac{S(t)}{S(Z)} dt.$$

and the variance is $V(Z) = \int_{Z}^{\infty} (t - A(Z))^2 * h(t) * \frac{S(t)}{S(Z)} dt$. One needs appropriate parameter values for B, α and β. The issue now is to fit observed data on age-specific death rates to the function, $B + \alpha e^{\beta t}$ with three parameters. There is a sizable literature on this issue. Gavrilov and Gavrilova [1991; p.76] report values based on Swedish men from 1926-1930 of $B = 0.00376$, $\alpha = 0.0000274$ and $\beta = 0.104$. These parameter values yield an expected age at death of $A = 64.17$ years and a corresponding variance of 495.72. The plot of the death rate (hazard function) stays flat along the time axis up to about age 45 and then rises fairly rapidly up past 80 years. Hence we have a theoretical model of expected life span that works and is ultimately derived from data on observed death rates. Of interest is that the standard deviation of 22.25 years is quite large, a datum that empirically-minded researchers do not appear to have focused their attention on. The coefficient of variation is $\frac{22.25}{64.17} = 0.347$.

Gavrilov and Gavrilova (p. 77) emphasize that the data on the increase in life expectancy of the Swedish men satisfy best a Makeham equation with a simple shift down in the value of parameter B[4]. Hence we proceeded to "create" a base equation with $B = 0.0000000006$, $\alpha = 0.0000274$ and $\beta = 0.104$. This yields a life expectancy of 73.72 years and a variance of 150.27 years.[5] We have plotted the mortality schedule in Figure 1 and the density function in Figure 2.

[4] As B gets smaller, remaining positive, the mortality schedule in Figure 1 shifts to the right. The Gompertz case of $B = 0$ corresponds to the mortality schedule implausibly "far" to the right.

[5] There is a problem here. Gavrilov and Gavrilova require us to change parameter B in order

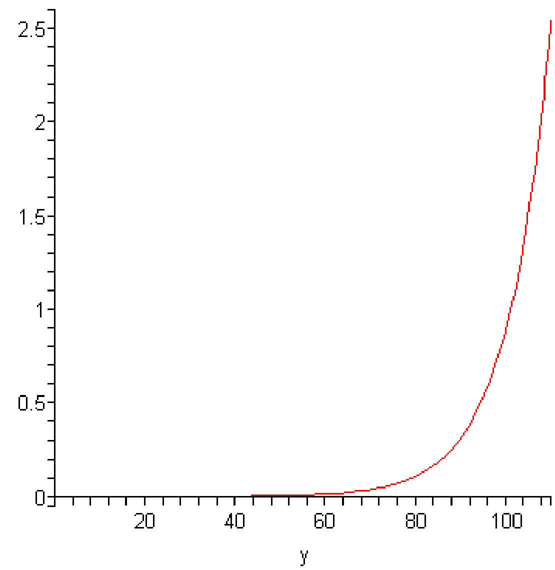

Figure 1: Hazard Rate

Recall that 73.72 was the expected age at death for this case, with age zero the "base year".

Figure 2: Density Function

With age 30 the base year (our person has attained age 30) for our choice of parameters above, we observe that life expectancy is 74.015 and the variance is

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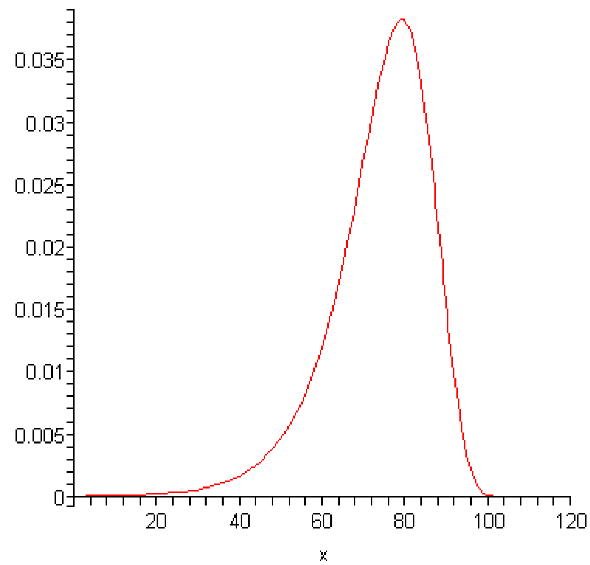
to track an increase in life expectancy. For B very small we are stuck with a life expectancy of about 73.7 years. In the limit of $B = 0$, we retrieve of classic Gompertz equation which does not work well empirically. For life expectancies above 74 years, we must either change an additional parameter, which we have done as exercises, or appeal to another model. This Makeham-Gompertz model responds fairly sluggishly to parameter changes. For β reduced from 0.104 to 0.076 ($B = 0.00376$, $\alpha = 0.0000274$), we observe that the expected age at death rises to 80.74 and the variance rises to 1261.43. The coefficient of variation rises to 0.44, a sizable jump from 0.35.



135.27. With age 40 the base year, we observe life expectancy at 74.434 and the variance is 120.55.

3. Value of Remaining Life

We proceed now to estimate the value-loss to a person dying at age 30, say in an auto accident, using the "value per life-year" estimates from Figure 2b in Murphy and Topel [2006]. Murphy and Topel define the "value of life year" (p. 881), $m(t)$ as the representative agent's value of her current "full consumption" weighted by "surplus per dollar of full consumption" plus "full income". On page 880 "full income and consumption" are defined by "adding the shadow value of non-market time to each". In addition lifetime income is smoothed so that the is a positive flow of income to a person in retirement (post age 65). The value of a life year is then current consumption in dollars plus current value of leisure in dollars. The series is constructed from actual life-cycle wage and consumption data and the series is calibrated to fit with a value of a statistical life of \$6.3 million. The ultimate series (Figure 2b) starts at \$200,000 at age 20, rises to \$360,000 at age 50, and declines to \$200,000 at about age 75. It then declines slowly to about \$80,000 at age 110. For our calculations we obtained values for life-years by simply reading them off the schedule in Figure 2b of Murphy and Topel.

We proceeded to calculate wealth (value of remaining uncertain years) $W(30; r, 74) = \int_{30}^{\infty} L(t) * h(t) * \frac{S(t)}{S(30)} dt$ for a person with an expected life of 74 years at birth, using the Makeham function for $S(t)$ (and $h(t)$) and Murphy-Topel annual values of $m(t)$ in $L(t) = \int_{30}^{t} m(z)e^{-rz}dz$. We made exploratory runs with different values of the discount rate. See the detail in Table 1. (Parameter values for the Makeham function: $B = 0.0000000006$, $\alpha = 0.0000274$, and $\beta = 0.104$.)

Table 1

r	0.01	0.02	0.03	0.04
W(30;r,74)	$10,790,648	$8,857,493	$7,383,426	$6,243,112.3
SD	$2,068,605	$1,492,942	$1,113,090	$840,037

Observe first that a discount rate just under 0.04 yields an expected value of future life equal to $6.3. Standard deviations are listed below in the SD row. We repeated our calculations for the case of a person dying accidentally at age 40 rather than age 30. The results in Table 2 are for the same parameter values.

Table 2

r	0.01	0.02	0.03	0.04
W(40;r,74)	$8,660,002	$7,598,688	$6,547,183	$5,759,806
SD	$2,039,565	$1,582,626	$1,252,408	$1,003,362

A discount rate slightly above 0.03 yields a value of a statistical life at $6.3 million. This suggests the inference (a) that a discount rate of between 0.03 and 0.04 is quite robust in the sense that the "appropriate" value of a statistical life emerges and (b) that a personal discount rate declining slowly in middle age gets support from this analysis.[6]

A typical series of terms, $L(t) * h(t) * \frac{S(t)}{S(30)}$ in our sum rose to about age 80 and then slowly declined to age 110. Of interest is that $L(t)$ is always rising over time while $h(t) * \frac{S(t)}{S(30)}$ rises to about 80 and then falls off rapidly. For an discount rate of 3.0% (favored by Maurphy and Topel p. 898) our values of remaining years are $7.383 and $6.547 million above for base age 30 and 40 respectively. These compare with $7 and $6.2 million from Figure 3 ("Value of remaining life") of Murphy and Topel. We are unable to say more in this comparison since there

[6] Azfar [1999] provides an analysis justifying a personal discount rate declining over a lifetime.

is no detail in Murphy and Topel about their construction of the series in their Figure 3.[7]

Our two inferences (discount rate between 0.03 and 0.04, and discount rate declining slowly between age 30 and 40) received support when we repeated our analysis above for two very different selections of parameters for the Makeham function. Each of these two parameter selections ($B = 0.00376$, $\beta = 0.104$ and $\alpha = 0.0000034$; and $B = 0.00376$, $\beta = 0.076$ and $\alpha = 0.0000274$) yielded an expected age at death of 80 years and each yielded expected values for remaining life of \$6.3 million from ages 30 and 40 with a discount rate between 0.03 and 0.04. In addition between base years 30 and 40 the appropriate discount rate declined slightly for each case. These parameter values have the merit of yielding reasonable life expectancies but are quite far off those endorsed by Gavrilov and Gavrilova. Hence one should interpret the outputs from these "extra" analyses as weak support for the our two main inferences.

References

[1] Azfar, O. [1999] "Rationalizing Hyperbolic Discounting" *Journal of Economic Behavior and Organization*, 38, pp. 245-52.

[2] Bebbington, Mark, Chin-Diew Lai and Ricardas Zitikis [2007] "Modeling Human Mortality using Mixtures of Bathtub-shaped Failure Distributions", *Journal of Theoretical Biology*, 245, pp. 528-538.

[7] With regard to Firgure 3 (Value of remaining life) we have the comment: "The effects of discounting and future mortality are apparent: the value of remaining life reaches \$7 million near age 30 and then falls, but Figure 2b showed that the value of a life-year rises until age 50." Since Murphy and Topel are explicit that the "value of life years" series is calibrated to a value of statistical life of \$6.3 million, it seems reasonable to assume that the two series (Figures 2 and 3) were generated simultaneously.

[3] Gavrilov, L.A. and N.S. Gavrilova [1991] *The Biology of Life Span: A Quantitative Approach*, Chur: Harwood.

[4] Murphy, Kevin M., Robert H. Topel, [2006] "The Value of Health and Longevity", *Journal of Political Economy*, vol. 114, no. 5, October, pp. 871-904.