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Clarifying Poverty Decomposition

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Clarifying Poverty Decomposition

Adrian Muller*†

Abstract: I discuss how poverty decomposition methods relate to integral approximation, which ultimately is the foundation of every decomposition of the temporal change of a quantity into key drivers. This offers a common framework for the different decomposition methods used in the literature, clarifies their often somewhat unclear theoretical underpinning and identifies the methods' shortcomings. In light of integral approximation, many methods actually lack a sound theoretical basis and they usually have an ad-hoc character in assigning the residual terms to the different key effects. I illustrate these claims for the Shapley-value decomposition and methods related to the Datt-Ravaillon approach and point out difficulties in axiomatic approaches to poverty decomposition. Recent developments in energy and pollutant decomposition offer some improved methods, but ultimately, a further development of poverty decomposition should account for the basis in integral approximation.

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NOTE: this is a working paper; for a the next version and the presentation, I will in particular give additional illustration and motivation for equation (1) and its interpretation as the fundamental equation for decomposition.

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1 Introduction

Decomposing some key variable in components assigned to different driving forces is a common exercise in many areas. Classical areas are the evolution of energy use and pollutant emissions (e.g. Ang 1995, 2004; Bruvoll and Larsen 2004) or poverty and inequality measures (e.g. Shorrocks 1999; Kakwani 2000; for a recent review, see Heshmati 2004).

Such decomposition is related to the general index number theory as developed for price and quantity indices. The calculation of such indices is based on integral approximations and the price and quantity index literature is aware of this (Trivedi 1981). The awareness of this basis in integral approximation has, however, been lost in most of the literature on energy and pollutant decomposition (Muller 2006), and in poverty decomposition in particular. The lack of this connection to the underlying basic formalism makes current efforts to develop optimal decomposition approaches somewhat arbitrary and often difficult to understand. This is the case for the recently developed Logarithmic Mean Divisia Index (LMDI) approach in the energy and pollutant context (Ang 2004, Muller 2006), for example, and for the discussion of the residual and zero or negative values in this same context.

Decomposition of poverty measures is even more arbitrary in that the motivations for the choice of a certain method usually lie even further from the underlying integral approximations. This is especially confusing in the context of the Shapley-value decomposition (Shorrocks 1999; Baye 2005) where reference is made to game theoretic concepts. In addition, critique emerges regarding the performance of these methods and claims arise that they are only suitable in special cases (Sastre and Trannoy 2000; Fiorio 2006).

Explicit reference to integral approximation as the underlying formalism of decomposition would not only clarify issues and make the methods easier to access, it would also add to the understanding of the problem of zero and negative values virulent in energy and pollution decomposition and it would shed a new light on the discussion of the residual. This has been done for energy and pollutant decomposition in Muller (2006). Zero and negative values are no topic in poverty decomposition because they either do not occur or they do not pose any problem in the methods currently applied. The residual, on the other hand, is present in some classical approaches to poverty decomposition and usually given the somewhat vague interpretation of interaction effects (e.g. Datt and Ravaillon 1992). This interpretation is often criticized and the absence of a residual term in the newer approaches related to the Shapley-value is seen as an advantage (Baye 2005).

As in energy decomposition, though, the zero residual is not a good criterion to identify optimal decomposition methods. The methods are much better understood if tied to the underlying integral approximations, where the presence of some residual due to approximation errors is natural. Referring to this basis offers a common framework for the decomposition approaches mainly applied in the poverty and inequality context, i.e. the Shapley-value based decomposition and the decomposition methods similar to the one presented in Datt and Ravaillon (1992). I will show that these methods are special and not entirely consistent approaches to approximate the underlying integrals. In general, decomposition would gain, irrespective of where it is applied, if this common ground in integral approximation would be appreciated. Ultimately, it could be promising to develop improved methods on this basis.

Section 2 introduces the general formalism of decomposition, illustrates how it is linked to integral approximation and how decomposition is discussed in the energy and pollution context. Section 3 presents some of the main methods of poverty decomposition currently applied and illustrates how they relate to each other and to the general formalism based on integral approximation. Conclusions are drawn in section 4.

2 A General Formalism for Decomposition

The aim of dynamic decomposition analysis[1] is to identify and assess the (relative) magnitude of different variables driving the time development of a key quantity. One example is the total industrial energy use and how much the changes in sector-wise energy efficiency, in the relative size of the different

[1] In a static decomposition analysis, a key variable is decomposed for one period to investigate the differences between several groups, such as states or castes. As often done in energy analysis, a dynamic approach can also be differentiated to account for the effects of group structures. A static decomposition is usually of only restricted interest, as the information on time development is missing. I therefore focus on the dynamic approach, but add some more remarks on the static approach in section 3.

sectors, and in the size of the total industry contribute to its development. Another example is how much changes in mean income and inequality contribute to changes in total poverty within a country.

I develop the following general formalism. I will show in section 3 how the methods commonly used for poverty decomposition can be seen as special cases of this general formalism. The key quantity of interest shall be $P(t)$ $=P(x_1(t), ..., x_m(t))$, depending on m time-dependent drivers $x_i(t), i = 1, ...m, t \in [T_0, T_n]$.[2] The change in P is given by its total derivative $\frac{\mathrm{d}P}{\mathrm{d}t}$ and the change from T_0 to T_n can be written as

$$\begin{aligned}
\Delta P_{T_0,T_n} &:= P(T_n) - P(T_0) = \int_{T_0}^{T_n} \frac{\mathrm{d}P}{\mathrm{d}t}\mathrm{d}t = \\
&= \int_{T_0}^{T_n} \Big(\frac{\partial P}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial P}{\partial x_2}\frac{\partial x_2}{\partial t} + ... + \frac{\partial P}{\partial x_m}\frac{\partial x_m}{\partial t}\Big)\mathrm{d}t = \qquad (1) \\
&= \int_{T_0}^{T_n} \frac{\partial P}{\partial x_1}\frac{\partial x_1}{\partial t}\mathrm{d}t + \int_{T_0}^{T_n} \frac{\partial P}{\partial x_2}\frac{\partial x_2}{\partial t}\mathrm{d}t + ... + \int_{T_0}^{T_n} \frac{\partial P}{\partial x_m}\frac{\partial x_m}{\partial t}\mathrm{d}t.
\end{aligned}$$

The part containing the derivative with respect to x_i is then interpreted as the contribution of changes in x_i to the total change in P. I denote this by $\Delta P^{x_i}_{T_0,T_n}$. Usually, the functions involved are not known for all points $t \in [T_0, T_n]$, but only for some discrete points of time, most often equally spaced (e.g. annually): $T_0, T_1, T_2, ..., T_{n-1}, T_n$. The integrals then have the following structure and the integrands are basically only known at the endpoints ($i = 1, ..., m$):

[2] P can further be differentiated according to some group-structure of interest, i.e. $P = \sum_{g=1}^{G} P_g$, where P^g is the value for P referring to group g, but this does not change the general argument and I use the simpler notation without this additional structure.

$$\Delta P^{x_i}_{T,T+1} = \int_T^{T+1} \frac{\partial P(x_1, ..., x_m)}{\partial x_i} \frac{\partial x_i}{\partial t} \mathrm{d}t. \tag{2}$$

Decomposing P thus boils down to solving such integrals. Because of the lack of information, though, i.e. the lack of knowledge of the underlying functions besides for the boundary values T and $T+1$, this is essentially an approximation problem. The integral has to be approximated by the values of the integrand at the endpoints of the integration range. In addition, the presence of derivatives may cause a problem as usually only the functions but not their derivatives are known for the endpoints. In this case, some approximation of the derivatives is necessary as well. The integral can thus be written as a function J or $\tilde{J}$ of the values at the end-points:[3]

$$\begin{aligned}
& \Delta P^{x_i}_{T,T+1} \approx \\
\approx \; & J\Big(P(T), x_i(T), \frac{\partial P}{\partial x_i}(T), \frac{\partial x_i}{\partial t}(T), \\
& P(T+1), x_i(T+1), \frac{\partial P}{\partial x_i}(T+1), \frac{\partial x_i}{\partial t}(T+1)\Big) \approx \\
\approx \; & \tilde{J}\Big(P(T), x_i(T), P(T+1), x_i(T+1), \\
& P(T-1), x_i(T-1), P(T+2), x_i(T+2)\Big),
\end{aligned} \tag{3}$$

As the decomposition problem in energy and pollutant analysis is framed, $\frac{\partial P}{\partial x_i}$ is usually known due to the particular structure of P being a product of

[3] J includes the derivatives directly, while they are approximated in $\tilde{J}$. For $\tilde{J}$, I chose the general formulation including $P(T-1), x_i(T-1), P(T+2)$ and $x_i(T+2)$, as they may enter the formula dependent on how the derivatives are approximated. An approximation of the derivative at $T+1$ from the right side, for example, usually depends on the value at $T+2$.

the various x_i (Muller 2006). Besides the integral, only the derivative of x_i remains then to be approximated.

How to best approximate $\Delta P^{x_i}_{T,T+1}$, i.e. how to optimally choose J or $\tilde{J}$ is implicitly driving all the different approaches to decompose energy use and pollutant emissions. "Implicitly" only, though, as awareness of the basis in integral approximation is largely missing in the literature. The problem is basically seen as one of choosing the correct weights for the known values at the two end-points to best calculate the change in P over the whole range in between. Choosing weights actually has its roots in integral approximation, as the simplest method to approximate an expression such as equation (2) consists in replacing the integral with the product of the value of the integrand at the upper or lower end-point times the distance on the ordinate ΔT, in this case equaling one: $J = \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}|_{T+1 \text{ resp. } T}$. This gives a weight of one to the upper or lower boundary and a weight zero to the other. Weighting both boundaries equally results in the average of the two values times ΔT, equaling one again: $J = [\frac{\partial P}{\partial x}\frac{\partial x}{\partial t}(T+1) + \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}(T)]/2$. These are three approximations replacing the true function by different types of step-functions. They are analogous to classical indices in the price/quantity context (the Laspeyres, Paasche and Marshall-Edgeworth index) and especially the first two were also applied in (early) energy decomposition, but they have several disadvantages, such as a usually rather large residual term or the asymmetry regarding the boundaries (Ang 2004).

The energy decomposition literature has then developed further approaches based on more flexible weights (e.g. the Divisia approach $J = \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}(T) + \alpha[\frac{\partial P}{\partial x}\frac{\partial x}{\partial t}(T+1) - \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}(T)]$ with $\alpha \in [0,1]$; $\alpha = 0, 1$ or $\frac{1}{2}$ replicate the three methods mentioned above).[4] This strategy separated decomposition even further from its base in integral approximation, as the single most important criterion for good weights became associated with a vanishing residual, meaning that J or $\tilde{J}$ not only approximate, but exactly replicate the left-hand side in equation (3).[5] This however is a misleading criterion, as the chance to exactly approximate the unknown integral based on the boundary values only is very small. A zero residual thus bears the danger of having been forced to be zero by just randomly or without strong reasons apportioning it to the different parts of a decomposition. A decomposition with zero residual thus needs not at all be superior to one with some residual - which, if it is too large, however clearly also spoils the explanatory power of the result.[6]

[4] $\frac{\partial x}{\partial t}(T+1)$ and $\frac{\partial x}{\partial t}(T)$ are usually approximated by the slope of the straight line joining the endpoints, i.e. $\frac{\partial x}{\partial t}(T+1) \approx x(T+1) - x(T) \approx \frac{\partial x}{\partial t}(T)$, thus giving the same value and simplifying formulae such as the Divisia Index above. This strategy could be criticized because of its inconsistency by taking the approximation from the right for the value at the left boundary T and the value from the left at $T+1$. This leads to potentially different results for $\frac{\partial x}{\partial t}(T)$ depending on whether it is part of a term between $T-1$ and T or between T and $T+1$. However, the strategy makes sense if seen in the context of replacing the whole unknown function with straight lines joining the known values (as for the Divisia with $\alpha = \frac{1}{2}$, i.e. the Marshall-Edgworth Index), for example.

[5] This criticism applies for example to the LMDI currently advocated as the most adequate index for energy and pollutant decomposition (Ang 2004).

[6] See Muller (2006) for an illustrative simulation of these issues in energy/pollution decomposition.

3 Poverty Decomposition

As for energy decomposition, there is a range of methods for poverty decomposition. The choice of a certain method is sometimes based on some formal symmetry arguments or axioms (Shorrocks 1982; Tsui 1996; Kakwani 2000), but in most cases rather ad-hoc. There is no awareness of the underlying approximation problem, although the decomposition methods proposed can be understood in this frame (see section 3.2 below). Poverty decomposition usually refers to decomposing some kind of poverty measure P, often the classical measure introduced in Foster et al. (1984), into parts corresponding to the effects of temporal changes in the mean income μ, the income distribution L and the poverty line z: $P = P(\mu, L, z)$. This can be normalized by z, i.e. the function to be investigated afterward depends on only two instead of three variables: $\bar{P}(\frac{\mu}{z}, \frac{L}{z})$. It follows from the discussion above that the general decomposition of the poverty measure reads

$$\Delta P_{T,T+1} = \int_T^{T+1} \frac{\partial P}{\partial \mu}\frac{\partial \mu}{\partial t}\mathrm{d}t + \int_T^{T+1} \frac{\partial P}{\partial L}\frac{\partial L}{\partial t}\mathrm{d}t + \int_T^{T+1} \frac{\partial P}{\partial z}\frac{\partial z}{\partial t}\mathrm{d}t, \tag{4}$$

where the integrals involved have the same structure as discussed above and similar problems related to their approximation are encountered. This also illustrates the formal equivalence of poverty and energy decomposition.

In the following, I will introduce the most common methods for decomposition of changes in poverty or inequality measures (the decompositions in the spirit of Datt and Ravaillon (1992), the Shapley-value decomposition and some further related approaches) and show how they relate to the general framework presented above.

3.1 Common Approaches to Poverty Decomposition

Most poverty measure decomposition approaches assume that the contribution of one variable to total change in poverty can be separated if all other variables are kept constant, i.e. if an unobserved "counterfactual situation" is correctly constructed. In particular, the choice of the time period, in which to keep the other variables constant, is crucial and various possibilities for this differentiate the methods. This approach leads to decompositions such as (taking the normalized form with $\bar{\mu} := \frac{\mu}{z}$ and $\bar{L} := \frac{L}{z}$)

$$\begin{aligned} \Delta\bar{P}_{T,T+1} &= \bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T)) = \\ &= \Big[\bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T+1))\Big] + \\ &\quad + \Big[\bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T+1), \bar{L}(T))\Big] + \bar{R} = \\ &= \bar{\mu}(\text{i.e. growth})\text{-effect} + \bar{L}(\text{i.e. inequality})\text{-effect} + \bar{R}, \end{aligned} \tag{5}$$

where $\bar{R}$ is the residual - also referred to as the interaction effect between growth and changes in inequality, given by $\bar{R} = \bar{P}(\bar{\mu}(T), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) + \bar{P}(\bar{\mu}(T+1), \bar{L}(T)) - \bar{P}(\bar{\mu}(T), \bar{L}(T))$ (Datt and Ravallion 1992; Baye 2004).

The residual thus has a similar structure as the decomposition itself and the whole formula has a rather ad-hoc character by adding and subtracting terms to get the effects of interest and then correcting for it by collecting their corresponding negatives in the residual, thus guaranteeing the validity of the formula. Datt and Ravallion (1992) also observe that this residual can be quite large, thus invalidating the whole approach. Equation (5) also depends on the period chosen as base period, as it is not symmetrical in T and

$T+1$. This method nevertheless is applied without discussion of potential problems, e.g. in Grootaert (1995) or Kraay (2006).

A similar approach is proposed by Jain and Tendulkar (1990),

$$\begin{aligned} \Delta \bar{P}_{T,T+1} &= \Big[\bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T+1))\Big] + \\ &\quad + \Big[\bar{P}(\bar{\mu}(T), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T))\Big], \end{aligned} \quad (6)$$

where the residual is zero, but the two effects are calculated with reference to different base periods and the decomposition is again not symmetric in T and $T+1$.

This situation led other authors (e.g. Kakwani 2000; Mazumdar and Son 2001; Bhanumurthy and Mitra 2003; Son 2003) to suggest a symmetric alternative of this decomposition by averaging the formulae with base periods T and $T+1$. Kakwani (2000) in particular motivates this by proposing a set of axioms any poverty decomposition should fulfill (cf. footnote 12 below). This leads to a symmetric decomposition without residual and the growth and inequality effects have the same combination of mixed base periods:

$$\begin{aligned} \Delta \bar{P}_{T,T+1} &= \frac{1}{2}\Big[\bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T+1)) + \\ &\quad + \bar{P}(\bar{\mu}(T+1), \bar{L}(T)) - \bar{P}(\bar{\mu}(T), \bar{L}(T)\Big] + \\ &+ \frac{1}{2}\Big[\bar{P}(\bar{\mu}(T+1), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T+1), \bar{L}(T)) + \\ &\quad + \bar{P}(\bar{\mu}(T), \bar{L}(T+1)) - \bar{P}(\bar{\mu}(T), \bar{L}(T))\Big]. \end{aligned} \quad (7)$$

This decomposition can be applied to any numbers of variables. Given a poverty measure P depending on m variables $x_1, ..., x_m$, the contribution of x_i to changes in P can be defined to be a combination of all terms

$$\Delta P^{x_i}_{T,T+1}(\pi_{s-1,m-s}) = [P(..., x_i(T+1), ...) - P(..., x_i(T), ...)], \qquad (8)$$

where $\pi_{s-1,m-s}$ is any $m-1$-vector with $s-1$ entries $T+1$ and $m-s$ entries T. The elements of this vector indicate at which time the variables other than x_i, i.e. $x_1, ..., x_{i-1}, x_{i+1}, ..., x_m$, are taken in both the terms on the right hand side in equation (8).[7] For m variables, a certain combination of s variables taken at $T+1$ and $m-s$ at T thus shows up in the final expression s times with a positive sign, stemming from the positive part of equation (8), for each variable at $T+1$. And correspondingly, it shows up $m-s$ times in the final expression with a negative sign, stemming from the negative part, but referring to the corresponding expression for $s+1$.[8] The condition that in the end only the original terms remain, i.e. $\Delta\bar{P}_{T,T+1} = P(x_1(T+1), ..., x_i(T+1), ..., x_m(T+1)) - P(x_1(T), ..., x_i(T), ..., x_m(T))$, requires coefficients unequal 1 for the various terms. In the simplest case, the coefficients of the positive terms can be chosen to be $\frac{1}{s}$ and for the negative ones $\frac{1}{m-s}$, for $s \neq 0$ and $s \neq m$, and $\frac{1}{m-s} = \frac{1}{m}$ for $s = 0$ while the positive part is absent, and $\frac{1}{s} = \frac{1}{m}$ for $s = m$, where the negative part is absent.

A more general choice of the coefficients is then $\gamma(m,s)\frac{1}{s}$ and $\gamma(m,s)\frac{1}{m-s}$ with $\gamma(m,0) = 1 = \gamma(m,m)$. This decomposition is symmetric and residual-free. Choosing $\gamma(m,s) = \frac{s!(m-s)!}{m!}$ then gives the Shapley-value coefficients (see e.g. Baye (2005), taking $s+1$ instead of s for the negative terms)

[7] This is a type of *ceteris paribus* reasoning employing all combinations of how the other variables can stay constant: each at T or at $T+1$

[8] $\pi_{s-1,m-s}$ gives s variables at $T+1$ in the positive parts of $\Delta P^{x_i}_{T,T+1}(\pi_{s-1,m-s})$ and $s-1$ at $T+1$ in the negative ones. Correspondingly, $s+1$ gives s variables at $T+1$ in the negative parts that combine with the corresponding terms referring to s.

and the decomposition coincides with the Shapley-value based poverty decomposition as introduced in Shorrocks (1999), which is seen as one of the best methods currently available (Baye 2004, 2005; Kolenikov and Shorrocks 2005). For two variables, this is equivalent to equation (7). It is not clear, though, how this or any other specific choice of weights might be motivated. The derivation of this decomposition as just described is transparent but it lacks a sound motivation. However, in my opinion, the game-theoretic background of the Shapley-value neither offers additional relevant motivation (i.e. motivation related to the problem of poverty decomposition, which has no tie to game theory) on how the decomposition should best be done, respectively on why to choose $\gamma(m, s)$ in this particular way. Admittedly, the Shapley-value has some distinct axiomatic background (symmetry, no essential player, additivity), but I will show in the next subsection that the Shapley-value is not optimal in the light of decomposition as integral approximation, and that thus these axioms cannot be employed as a motivation for the method's optimality.

There are other approaches aiming at improving poverty decomposition. Dercon (2006) bases decomposition on a micro-level assessment of single households and their status as poor or non-poor and how this changes between periods. Another different approach is based on the linkages captured in the Social Accounting Matrix (e.g. Thorbecke and Jung (1996) and references therein). Thirdly, Fournier (2001) discusses an approach explicitly taking into account changes in the different underlying variables and their correlations separately. This is, in fact, similar to taking some terms of the Shapley-value approach into account and explaining part of the remaining residual by building counterfactuals based on the rank-correlation structure. Usually, some residual remains. Fourthly, there are regression-based approaches to decomposition (see e.g. Juhn et al. (1993), Borooah (2005) or Wan and Zhou (2005) and references therein). The regressions, however, refer to the definition, choice, or identification of the variables the decomposition is based on or the construction of the counterfactual case, while the decomposition itself (i.e. the combination of the terms where only one variable changes) is again made according to the common approaches as described in this subsection. Similarly, Di Nardo et al. (1996) discuss a kernel estimation approach to construct counterfactuals needed for the decomposition into changes attributable to single variables, while the decomposition ultimately is again a variant of the approaches discussed above. This should not be seen as an encompassing list and I do not discuss these alternative approaches in more detail (for a recent review of methods, see also Heshmati (2004)).

3.2 Poverty Decomposition and Integral Approximation

In this subsection, I discuss the poverty decomposition approaches introduced above in the light of general decomposition as integral approximation as presented in section 2. This establishes a common basis for and a new understanding of poverty decomposition methods.

3.2.1 Most Common Approaches and the Shapley-Value

Approximating the terms in equation (4) by their values at the upper boundary leads to expressions such as $J \approx \frac{\partial P}{\partial \mu}\frac{\partial \mu}{\partial t}|_{T+1}\Delta T$, and approximating the derivatives by the slope of the straight line joining the end-points as discussed in footnote 4 gives

$$J \approx \frac{P(\bar{\mu}(T+1), \bar{L}(T+1)) - P(\bar{\mu}(T), \bar{L}(T+1))}{\bar{\mu}(T+1) - \bar{\mu}(T)} \frac{\bar{\mu}(T+1) - \bar{\mu}(T)}{\Delta T} \Delta T, \quad (9)$$

which is the Laspeyres index. The corresponding expression can be calculated for the variable $\bar{L}$ and both can also be evaluated at time T, thus giving the Paasche index. The combination of the Laspeyres for both $\bar{\mu}$ and $\bar{L}$ gives the Datt-Ravaillon decomposition equation (5), and the combination of Laspeyres for $\bar{\mu}$ and Paasche for $\bar{L}$ gives the Jain-Tendulkar formula (6). Taking the average of the Laspeyres and Paasche indices gives the Marshall-Edgeworth index (equivalent to the Divisia index with $\alpha = \frac{1}{2}$). This, finally, is the same as the Shapley-value decomposition for two variables, equation (7).

So far, I have shown how the basic poverty decomposition methods can be seen as special cases of integral approximation. This is however not true any longer for the generalised formulae used in the literature and presented above, i.e. for the Shapley-value with more than two variables. One criticism is that in the light of the equivalence of the Shapley-value decomposition and the decomposition method introduced in Sun (1998) (Ang et al. 2003), the various terms in the Shapley-value can be understood as an assignment of the residual to the various effects based on some symmetry arguments but without further basis in the properties of the underlying functions or integral approximations. Thus, all variables are treated equally, irrespective of their properties. I illustrate this for three variables and a total which is their multiplication:

$$\begin{aligned} \Delta P = P(T) - P(0) &= x_1(T)x_2(T)x_3(T) - x_1(0)x_2(0)x_3(0) \\ &= \Delta P_1 + \Delta P_2 + \Delta P_3, \end{aligned} \tag{10}$$

where ΔP_i is the contribution of the variable x_i to the decomposition of P. Replacing x_i^T with $x_i^0 + \Delta x_i$, seeing Δx_i as the incremental change in x_i from period 0 to T, and symmetrically rearranging terms, we thus have

$$\begin{aligned} \Delta P &= (x_1^0 + \Delta x_1)(x_2^0 + \Delta x_2)(x_3^0 + \Delta x_3) - x_1^0 x_2^0 x_3^0 = \\ &= \Delta x_1 x_2^0 x_3^0 + \Delta x_2 x_1^0 x_3^0 + \Delta x_3 x_1^0 x_2^0 + \\ &\quad + \Delta x_1 \Delta x_2 x_3^0 + \Delta x_1 \Delta x_3 x_2^0 + \Delta x_2 \Delta x_3 x_1^0 + \Delta x_1 \Delta x_2 \Delta x_3 = \\ &= \Delta x_1 x_2^0 x_3^0 + \frac{1}{2}[\Delta x_1 \Delta x_2 x_3^0 + \Delta x_1 \Delta x_3^0 x_2^0] + \frac{1}{3}\Delta x_1 \Delta x_2 \Delta x_3 + \\ &\quad + \Delta x_2 x_1^0 x_3^0 + \frac{1}{2}[\Delta x_1 \Delta x_2 x_3^0 + \Delta x_2 \Delta x_3^0 x_1^0] + \frac{1}{3}\Delta x_1 \Delta x_2 \Delta x_3 + \\ &\quad + \Delta x_3 x_1^0 x_2^0 + \frac{1}{2}[\Delta x_1 \Delta x_3 x_2^0 + \Delta x_2 \Delta x_3^0 x_1^0] + \frac{1}{3}\Delta x_1 \Delta x_2 \Delta x_3. \end{aligned} \tag{11}$$

The three last lines are ΔP_1, ΔP_2 and ΔP_3, respectively, and equal the contributions of the three variables as identified in Sun (1998). As shown in Ang et al. (2003), they are equal to the Shapley-value decomposition, as can also be seen by further rearranging terms and comparing to the formulae for the Shapley-value given above. As already indicated, the logic behind this formula is to equally assign all the difference-terms involving Δx_i's to the contributions of the variables x_i, i.e. a term involving s Δ-factors is divided by s. A pictorial illustration for this simple example are the volumes of two cubes with edges x_i^0 and $x_i^0 + \Delta x_i$, respectively, and how to assign the difference in volume between the two to each of the differences in the single edges.

A decomposition rule is based on the goal to decompose a general function of some m variables into m additive parts, each one corresponding to the contribution of one of these variables. This is thus some type of linearisation, and basing a decomposition procedure on some symmetries on the level of these linearised summands, as it is done in the Shapley-value approach, treating all variables symmetrically, need not be correct. This is so as we are not primarily interested in Δx_i itself, but rather in $\Delta x_i = x_i(t + \Delta t) - x_i(t)$ *as a function of* Δt, which, in general, will *not* be linear.

I illustrate this criticism of the Shapley-value with a simulation based on some concrete choice of the variables x_i as functions of t: let's choose $x_1 = t, x_2 = t^2, x_3 = \frac{t}{4}$. Inserting this in equation (1), where again $P = x_1 x_2 x_3$, and solving the integrals gives $\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 = \frac{T^4}{4}$ and the following (exact) decomposition

$$\Delta P_1 = \int_0^T \frac{\partial x_1}{\partial t} x_2 x_3 \mathrm{d}t = \int_0^T \frac{t^3}{4} \mathrm{d}t = \frac{T^4}{16},\ \Delta P_2 = \frac{T^4}{8}, \Delta P_3 = \frac{T^4}{16}. \tag{12}$$

Using the Shapley-value equation (11), the result is different (but also exact), which shows that the Shapley-value does not necessarily lead to the correct decomposition[9]:

[9]Most terms are equal zero in this simple example, as $x_i^0 = 0$ for $i = 1, 2, 3$, but this special property is not crucial for the general argument.

$$\Delta P_1 = \Delta P_2 = \Delta P_3 = \frac{T^4}{12}. \qquad (13)$$

For further illustration, I also state the condition for the Shapley-value for three variables to be exact. It is, for the contribution of the first variable ($x_i^t = x_i(t)$), the requirement that

$$\begin{aligned} & \int_0^T \frac{\partial x_1(t)}{\partial t} x_2(t) x_3(t) \mathrm{d}t \stackrel{!}{=} \\ \stackrel{!}{=} \;& (x_1^T - x_1^0) x_2^0 x_3^0 + \frac{1}{2}[(x_1^T - x_1^0)(x_2^T - x_2^0) x_3^0 + (x_1^T - x_1^0)(x_3^T - x_3^0) x_2^0] + \\ & + \frac{1}{3}(x_1^T - x_1^0)(x_2^T - x_2^0)(x_3^T - x_3^0). \end{aligned} \qquad (14)$$

Comparing this to integral approximation as discussed above shows also that the Shapley-value contains too many terms mixing values referring to the two different boundaries. In correct integral approximation, such mixture only occurs via the derivative-term, i.e. for one variable only, while all the others are evaluated either at the upper or lower boundary only.

3.2.2 Static and Axiomatic Decomposition

A somewhat different approach to decomposition is taken by authors that address the static decomposition of differences between various groups in the society such as spatial groups, e.g. states in a nation (Dhongde 2003; Kolenikov and Shorrocks 2005), or different castes (Borooah 2005) rather than changes between time periods. Formally this could be seen as the same problem as temporal decomposition, and the same methods could be applied. This however would assume some continuous range of parameters between spatial groups, states, castes etc., which clearly is not the case for most group-variables in reality - although approximation formulae based on the values at the endpoints (i.e. for two groups, for example) can be applied due to formal equivalence. Thus, the framework of integral approximation is not adequate for such static analysis as the notion of a path connecting the groups generally does not make sense. Postulating such a path makes the formulae from integral approximation applicable but it will likely lack a sound interpretation. The case is different as soon as some temporal information is available. Then, the decomposition can be undertaken as discussed above, employing separate group-wise analysis (i.e. separately for each state, caste, etc.), or it could be done by directly incorporating the different group effects as they are usually incorporated in energy decomposition (for groupings such as by fuel type or industry sector, see e.g. Muller 2006).[10]

Finally, I link the poverty decomposition method based on integral approximation as described above to some axiomatic approaches in the literature. Most recent is Kakwani (2000), who sets up a system of 5 simple rather intuitive axioms any poverty decomposition should fulfil (mainly symmetry and consistency properties, see the formalism below and footnote 12), discusses and criticises existing decomposition methods in the light of these axioms and proposes a new method that fulfils all 5 axioms. His discussion is framed in a two-variable setting and the method he finally recommends is just the Shapley-value for two variables.[11] Adopting his notation, we consider

[10] A separate analysis is udnertaken for each group and then aggregated for all groups. This leads to results quantifying the relative effects of changes in fuel-composition or sectoral structure without further specification of how changes in the single fuels or sectors contribute.

[11] He does not mention this, though - but the Shapley-value decomposition was also introduced after this paper was originally written in 1997.

a poverty measure in period i depending on a measure for inequality (like the Lorenz curve L) and a measure of the average income level μ: $\Theta(\mu_i, L_i)$. Employing the integral approximation approach to decomposition, the change in this measure between two periods can then be written as

$$\Theta_{ij} = \int_i^j \frac{\partial\Theta}{\partial\mu_t}\frac{\partial\mu_t}{\partial t}\mathrm{d}t + \int_i^j \frac{\partial\Theta}{\partial L_t}\frac{\partial L_t}{\partial t}\mathrm{d}t =: G_{ij} + I_{ij}, \qquad (15)$$

where G is the growth and I the inequality component. Due to the properties of integration, the decomposition based on integral approximation thus fulfills these 5 axioms set up by Kakwani (2000)[12].

Other axiomatic systems are presented in Shorrocks (1982) and Tsui (1996), for example. The axiomatisation in Tsui, however, mainly refers to the poverty measure itself and less to its decomposition, which is basically the same as finally derived in Kakwani (2000).

Decomposition based on integral approximation does however not fulfil the axioms of Shorrocks (1982). The assumption on symmetric treatment of factors (his assumption 2b) is not fulfilled in his sense, where it refers to the functional dependence of the contribution to inequality of one factor being the same for all factors. Using his notation, in my approach, the contribution of a factor Y^k to the general inequality measure $I(Y)$, where $Y = \sum_{k=0}^{K} Y^k$ is total income built of several types of income Y^k, is $S_k(Y^1, ..., Y^K; K) := \int_0^T \frac{\partial I(Y)}{\partial Y^k}\frac{\partial Y^k}{\partial t}\mathrm{d}t$. As a functional description, this is symmetric in the different factors, but not necessarily on the level of S as a function of Y^k. Furthermore, Shorrocks' approach is also criticised by several authors, e.g. by Paul (2004)

[12]The axioms are 1) If $I_{ij} = 0$ then $\Theta_{ij} = G_{ij}$ and if $G_{ij} = 0$ then $\Theta_{ij} = I_{ij}$; 2) if $G_{ij} \leq 0$ and $I_{ij} \leq 0$ then $\Theta_{ij} \leq 0$ and if $G_{ij} \geq 0$ and $I_{ij} \geq 0$ then $\Theta_{ij} \geq 0$; 3) $G_{ij} = -G_{ji}$ and $I_{ij} = -I_{ji}$; 4) $G_{ij} = G_{ik} + G_{kj}$; 5) $I_{ij} = I_{ik} + I_{kj}$ for all periods i, j, k;

for the lack of motivation for some of his conditions[13] and by Fournier (2001) as being too restrictive and, being static, as not being of primary interest — although static decomposition is applied frequently.

It may be concluded from this discussion that generally, as in energy and pollutant decomposition, formulating axioms for the decomposition of changes should not be given too much weight to, especially if decomposition is seen in the light of integral approximation. Furthermore, although the general formulation of decomposition based on integrals may fulfil the axioms, due to the unavoidable errors, they may not be fulfilled when it comes to concrete approximations (cf. also Muller 2006). Investigating axioms for poverty measures themselves, however, may clearly make sense, but this is not the topic of this paper.

4 Conclusions

A wide range of methods for poverty or general inequality measure decomposition is currently being applied. None of these methods, however, has a sound basis, as none refers to integral approximation, which is the ultimate starting point of any dynamic decomposition analysis. Muller (2006) recently analysed these issues in the context of energy and pollutant decomposition, where similar problems are encountered. The methods used in energy decomposition perform somewhat more satisfactorily from a theoretical point of view than the common poverty decomposition methods. To assess the adequacy of the methods most often applied in poverty decomposition, such as the Shapley-value, comparison with methods more directly related to integral

[13] Paul (2004) however retains the symmetry axiom I criticise here.

approximation is necessary. For energy decomposition, such an assessment has been done for the LMDI (Ang 2004; Muller 2006), which is seen as one of the best methods in this context. Although lacking a sound theoretical basis, this method performs reasonably well also in relation to integral approximation, and the LMDI may be used as a reasonably reliable option for most cases.

Thus I suggest to apply the LMDI also for the decomposition of changes in poverty or general inequality measures. This method is more appropriate than the Shapley-value, which has desirable features, but assigns the residual term in an inadequate manner to the different drivers behind changes in poverty. This does not mean that results based on the Shapley-value necessarily are wrong - but it is difficult to assess when it is adequate and how large potential errors may be. Admittedly, an assessment of the performance of the LMDI may not be easy and the best practice would be to solely rely on integral approximation. This would work best if it is possible to collect or access additional data for the case at hand, thus gaining additional information on the functions to be approximated and improving the reliability of the result.

Finally, I emphasize that, as in energy and pollutant decomposition, increased reliance on axiomatic approaches is no solution to identify optimal methods. In the light of integral approximation, desirable properties only need to be fulfilled approximately, thus spoiling assessments of methods based on a system of axioms. The prime example for this may be the desirability of a zero residual, i.e. of a complete decomposition, which does not need to hold for an approach based on approximations. It is only natural to encounter some errors when approximating - which simply lies in the nature of an approximation in comparison to an exact solution.

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