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**ABSTRACT**

Recent evidence on the importance of cross-border equity flows calls for a rethinking of the standard theory of external adjustment. We introduce equity holdings and portfolio choice into an otherwise conventional open-economy dynamic equilibrium model. Our model is simple and admits a closed-form solution regardless of whether financial markets are complete or incomplete. We find that the excessive emphasis put in the literature on solving models with incomplete markets for the sole purpose of obtaining nontrivial implications for the current account is misplaced. We revisit the current debate on the relative importance of the standard vs. the capital-gains-based (or "valuation") channels of the external adjustment and establish that in our framework they are congruent. Our model's implications are consistent with a number of intriguing stylized facts documented in the recent empirical literature.

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# 1. Introduction

An unprecedented rise in cross-border equity holdings over the past two decades generated a source of income previously disregarded in the national accounts: capital gains on equity holdings. The current practice incorporates capital gains only after they are redeemed, and this lack of marking to market may result in a potentially dangerous misrepresentation of the extent of external imbalances worldwide—especially in the US, most of Europe, and Japan. This is because if one were to use the standard measure of external sustainability, based on the official figures, one could predict a global economic disaster. Summarizing the view expressed by many influential economists, Martin Wolf (2004) proclaims that “The US is now on the comfortable path to ruin.”

Burgeoning empirical literature has emphasized that capital gains on the industrialized countries’ net foreign asset (NFA) positions—or “valuation changes”—have become sizable.<sup>1</sup> These exciting new empirical developments call for a modification of the standard external adjustment theory that includes valuation changes. This task is not an easy one. A formal study of the dynamic properties of the NFA position requires not only the analysis of how exchange rates and asset prices change in the presence of a multitude of shocks, but also the knowledge of countries’ portfolio decisions and their interaction with the real side of the economy. Furthermore, this has to be studied in an economic environment where the current account implications are not trivial.

Recent literature offers several ambitious attempts to incorporate portfolio choice and asset pricing in models of international macroeconomics. The complexity of the proposed models, however, rules out analytical solutions and hence the literature has concentrated on developing sophisticated approximate or numerical methods to be able to analyze the dynamic properties of such economies.<sup>2</sup> Moreover, it has been suggested that to get nontrivial implications for the current account dynamics, the literature needs to move away from the complete markets paradigm, which complicated the matters further.

In this paper, we intend to take a step back and simplify the economic setting so that it

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<sup>1</sup>Prominent examples of papers belonging to this strand of literature include Gourinchas and Rey (2007a), Gourinchas and Rey (2007b), Lane and Milesi-Ferretti (2001), Lane and Milesi-Ferretti (2007), Tille (2003), and Tille (2005).

<sup>2</sup>See, e.g., Ghironi, Lee, and Rebucci (2006) and Kollmann (2006) for the first-generation analyses that employ standard first-order approximation techniques. The second-generation methodologies were developed by Devereux and Sutherland (2006), Evans and Hnatkowska (2007), and Tille and van Wincoop (2007) who solve stochastic portfolio models with incomplete markets using more complex higher-order approximations. Cavallo and Tille (2006) propose a shortcut solution method in which optimal portfolios are specified to be an (exogenous) fraction of trade flows.

becomes analytically tractable and allows us to disentangle the different channels through which capital markets and external adjustment are interrelated. In particular, we develop a two-country pure-exchange economy with multiple risky assets and incomplete markets in which equilibrium can be characterized in closed form. To our knowledge, ours is the first fully-fledged dynamic general equilibrium model of this kind that admits an analytical solution.

An advantage of our approach is that it provides a theoretical framework in which we can examine (and clarify) some of the conjectures that have been made in the literature. First, we reassess the role that incomplete markets play in generating nontrivial current account dynamics. We argue that to produce fluctuations in the (conventional) current account it is neither necessary nor sufficient that markets are incomplete, and provide two simple examples to support this argument. Instead, the necessary and sufficient condition for generating current account imbalances is that the countries' optimal portfolios include holdings of bonds—which depends on hedging demands of agents and not on market incompleteness *per se*.

Second, we revisit the traditional intertemporal approach to the current account that says that, for the budget constraint to be satisfied, a country's current negative NFA position must be compensated by future trade surpluses (Obstfeld and Rogoff (1995)). A new view that has recently emerged in the literature criticizes the traditional approach for neglecting the possibility that changes in asset returns may lead to changes in the discount factor that could raise the *present value* of future trade surpluses without the need to actually adjust the trade balance.<sup>3</sup> Surprisingly, we find that in our model, once the endogenous responses of asset prices to underlying shocks are taken into account, the NFA position ends up being tightly linked to the current trade balance—thus making the traditional and the new views of the external adjustment congruent.

Third, we find that our theoretical implications are consistent with many stylized facts obtained in the literature. We adjust the traditional measure of the current account for capital gains, and analyze separately the two elements that are missing from that measure: the *expected* and the *unexpected* capital gains. We show that the former have a stabilizing property, offsetting the fluctuations in the trade balance and the traditional current account. Gourinchas and Rey document

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<sup>3</sup>As pointed out by Gourinchas and Rey (2007b). See also Hausmann and Sturzenegger (2006) and Tille (2003) for related arguments. This new view sheds a fresh light on the question of the sustainability of the US external imbalances, suggesting that the widening current account deficit in the US could be part of the normal adjustment process and does not necessarily spell any economic disaster. This conclusion is contested by the proponents of the traditional view who believe that a significant adjustment of the trade balance and in particular a large US dollar depreciation needs to take place (see e.g., Edwards (2005), Frankel (2006), Obstfeld (2004), Obstfeld and Rogoff (2007), Roubini and Setser (2004)).

a similar effect occurring in their dataset. It is the *unexpected* part of the capital gains, however, that is key to the dramatically different dynamic properties of the traditional and the capital-gains adjusted current account in our model. The traditional current account follows a persistent process, while the capital-gains adjusted current account is highly volatile and serially uncorrelated. This is consistent with the evidence presented in Kollmann (2006) and Lane and Shambaugh (2007). In other words, the capital-gains adjusted current account behaves much like asset returns, whose short-term dynamics are also dominated by unexpected capital gains.

Finally, because we are able to fully characterize the equilibrium in our economy, we describe the adjustment process, and what role portfolio reallocations, changes in expected returns, unexpected capital gains, and the trade balance play. We study the behavior of these and other variables in response to supply and demand shocks and explain the economic mechanisms behind the patterns that we find.

We have already cited the papers that are conceptually related to this work. Methodologically, the most closely related works are He and Pearson (1991), Cuoco and He (1994), and Basak and Cuoco (1998). At a partial equilibrium level, He and Pearson derive a solution to a consumption-portfolio problem under incomplete markets. Cuoco and He develop a method for solving for equilibrium under incomplete markets via a “planner” with stochastic weights. Basak and Cuoco were the first to apply this method to study financial markets with frictions (restricted participation, in their case). None of these papers, however, offers a model with multiple risky assets and incomplete markets that can be analyzed analytically. The model that we develop builds on Cole and Obstfeld (1991), Helpman and Razin (1978), Pavlova and Rigobon (2007), and Zapatero (1995). All these are tractable multi-asset multi-good models like ours, but in contrast to our work, in each of these papers markets are complete or effectively complete.

The rest of the paper is organized as follows. In Section 2, we describe the model and characterize its equilibrium. In Section 3, we derive a number of implications of our model for the current account and its dynamics. In Section 4, we present several special cases of our economy in which the dynamics of portfolios and the current account simplify significantly, and in Section 5, simulate the model in order to analyze its properties in the general case. Section 6 discusses several caveats and desirable extensions, and Section 7 concludes.

## 2. The Model

### 2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy with a finite horizon,  $[0, T]$  along the lines of Pavlova and Rigobon (2007). Uncertainty is represented by a filtered probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ , on which is defined a standard four-dimensional Brownian motion  $\vec{w}(t) = (w(t), w^*(t), w^\alpha, w^\beta)^\top$ ,  $t \in [0, T]$ . All stochastic processes are assumed adapted to  $\{\mathcal{F}_t; t \in [0, T]\}$ , the augmented filtration generated by  $\vec{w}$ . All stated (in)equalities involving random variables hold  $P$ -almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are two countries in the world economy: Home and Foreign. The Home country represents a large industrialized country, while Foreign stands for the rest of the world. Each country is endowed with a Lucas tree producing a strictly positive amount of a country-specific perishable good:

$$dY(t) = \mu_Y(t)Y(t)dt + \sigma_Y(t)Y(t)dw(t) \quad (\text{Home}), \quad (1)$$

$$dY^*(t) = \mu_{Y^*}(t)Y^*(t)dt + \sigma_{Y^*}(t)Y^*(t)dw^*(t) \quad (\text{Foreign}), \quad (2)$$

where  $\mu_Y, \mu_{Y^*}, \sigma_Y > 0$ , and  $\sigma_{Y^*} > 0$  are arbitrary adapted processes. The claims to the trees, Home and Foreign stocks  $S$  and  $S^*$ , respectively, are available for trade by all investors and are in fixed supply of one share each. The prices of the Home and Foreign goods are denoted by  $p$  and  $p^*$ , respectively. We fix the world numeraire basket to contain  $a \in (0, 1)$  units of the Home good and  $(1-a)$  units of the Foreign good, and normalize the price of this basket to be equal to unity. The terms of trade,  $q$ , are defined as the price of the Home good relative to that of the Foreign good:  $q \equiv p/p^*$ .

In addition to the stocks  $S$  and  $S^*$ , there is also the “world” bond  $B$  available for investment, which is a money market account locally riskless in units of the numeraire. The bond is in zero net supply. Since there are four independent Brownian motions driving the economy and only three investment opportunities in place, financial markets are incomplete. To fix notation, the posited

dynamics of the investment opportunity set of the agents is given by

$$dB(t) = B(t)r(t)dt, \quad (3)$$

$$dS(t) + p(t)Y(t)dt = S(t)[\mu_S(t)dt + \sigma_S(t)d\vec{w}(t)], \quad (4)$$

$$dS^*(t) + p^*(t)Y^*(t)dt = S^*(t)[\mu_{S^*}(t)dt + \sigma_{S^*}(t)d\vec{w}(t)], \quad (5)$$

where the interest rate  $r$ , the stocks expected returns  $\mu_S$  and  $\mu_{S^*}$  and their volatilities  $\sigma_S$  and  $\sigma_{S^*}$  are to be determined in equilibrium. The *volatility matrix* of the stock returns is then defined as

$$\sigma = \begin{bmatrix} \sigma_S \\ \sigma_{S^*} \end{bmatrix}.$$

The initial shareholdings of a representative consumer-investor of each country consist of no shares of the bond and a total supply of the stock market of his country. Thus, the initial wealth of the Home resident is  $W_H(0) = S(0)$  and that of the Foreign resident is  $W_F(0) = S^*(0)$ . Each consumer  $i$ ,  $i \in \{H, F\}$ , chooses nonnegative consumption of each good  $(C_i(t), C_i^*(t))$  and a portfolio of the available securities  $x_i(t) \equiv (x_i^S(t), x_i^{S^*}(t))$ , where  $x_i^j$  denotes the fraction of wealth of consumer  $i$  invested in asset  $j$ . The dynamic budget constraint of each consumer has the standard form

$$\begin{aligned} dW_i(t) = & \left[ W_i(t)r(t) + x_i^S(t)W_i(t)(\mu_S(t) - r(t)) + x_i^{S^*}(t)W_i(t)(\mu_{S^*}(t) - r(t)) \right] dt \\ & + \left[ x_i^S(t)W_i(t)\sigma_S(t) + x_i^{S^*}(t)W_i(t)\sigma_{S^*}(t) \right] d\vec{w}(t) - \left[ p(t)C_i(t) + p^*(t)C_i^*(t) \right] dt, \end{aligned} \quad (6)$$

where  $W_i(T) \geq 0$ ,  $i \in \{H, F\}$ . Preferences of consumer  $i$ , are represented by a time-additive utility function defined over consumption of both goods:

$$E \left[ \int_0^T e^{-\rho t} u_i(C_i(t), C_i^*(t)) dt \right], \quad \rho > 0, \quad i \in \{H, F\}, \quad (7)$$

where

$$u_H(C_H(t), C_H^*(t)) = \alpha_H(t) \log C_H(t) + \beta_H(t) \log C_H^*(t),$$

$$u_F(C_F(t), C_F^*(t)) = \beta_F \log C_F(t) + \alpha_F \log C_F^*(t).$$

In our specification of the countries' utilities, we allow for the possibility of preference shifts towards the home or the foreign good (or "demand shocks"), modeled along the lines of Dornbusch, Fischer, and Samuelson (1977). The role of this assumption is twofold. First, in the absence of the demand shocks, free trade in goods makes stock prices perfectly correlated and financial markets irrelevant (Helpman and Razin (1978), Cole and Obstfeld (1991), Zapatero (1995)). Second,

empirical evidence indicates that demand shocks are important drivers of the real-world dynamics. For example, Stockman and Tesar (1995) calibrate preference shocks to be roughly 85% of the size of supply shocks, while of Pavlova and Rigobon (2007) estimate a similar model and conclude that they have about the same volatility as supply shocks. Formally, we assume that  $\alpha_H$  and  $\beta_H$  are positive adapted stochastic processes, martingales, and have dynamics

$$d\alpha_H(t) = \sigma_{\alpha_H}(t)d\vec{w}(t), \quad d\beta_H(t) = \sigma_{\beta_H}(t)d\vec{w}(t).$$

In the analysis that follows, we consider primarily two types of demand shocks: (i) demand shocks that are completely independent of the supply (output) shocks  $w$  and  $w^*$  and (ii) demand shocks that are allowed to be correlated with the supply shocks. For simplicity, we assume that there are no demand shocks at Foreign, but our model can be easily extended to accommodate these.<sup>4</sup>

## 2.2. Countries' Portfolio Choice

The first step in our solution procedure is to derive the countries' optimal portfolios at a partial equilibrium level. To do so, we are going to employ techniques developed in the portfolio choice literature. However, relative to that literature, there are two non-standard ingredients in the optimization problem that the countries are facing: multiple consumption goods and incomplete markets. We address them in turn.

For concreteness, we focus our exposition on the Home consumer. The portfolio of the Foreign consumer is derived analogously. Following the early literature in finance (Breedon (1979), Adler and Dumas (1983)), we decompose the problem of maximizing his utility (7) subject to the budget constraint (6) into two parts. First, at each  $t$ , we derive the consumer's demands for the Home and the Foreign goods, keeping the overall consumption expenditure fixed. Second, we derive his optimal consumption expenditure process and the optimal portfolio.

The first step is the standard static consumer problem under certainty:

$$\begin{aligned} & \max_{\{C_H(t), C_H^*(t)\}} \alpha_H(t) \log C_H(t) + \beta_H(t) \log C_H^*(t) \\ \text{s.t.} \quad & p(t)C_H(t) + p^*(t)C_H^*(t) \leq \mathbb{C}_H(t), \end{aligned}$$

where  $\mathbb{C}_H(t)$  denotes overall consumption expenditure at time  $t$ . Solving this problem, we obtain

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<sup>4</sup>At this point, for generality, we are not requiring that each country has a stronger preference for the home good ( $\alpha_i > \beta_i$ ,  $i \in \{H, F\}$ ). However, a realistic calibration of a model like ours would typically incorporate such a preference bias. We delay calibrating our model until Section 5.



the following demands for the individual goods as fractions of the overall expenditure:

$$\bar{C}_H(t) = \frac{\alpha_H(t)}{\alpha_H(t) + \beta_H(t)} \frac{\mathbb{C}_H(t)}{p(t)}, \quad \bar{C}_H^*(t) = \frac{\beta_H(t)}{\alpha_H(t) + \beta_H(t)} \frac{\mathbb{C}_H(t)}{p^*(t)}. \quad (8)$$

The indirect utility function defined as  $U_H(\mathbb{C}_H(t); p(t), p^*(t)) \equiv u_H(\bar{C}_H(t), \bar{C}_H^*(t))$  is then given by

$$U_H(\mathbb{C}_H(t); p(t), p^*(t)) = (\alpha_H(t) + \beta_H(t)) \log \mathbb{C}_H(t) + F(\alpha_H(t), \beta_H(t), p(t), p^*(t)),$$

where  $F(\cdot)$  is a function the form of which does not affect our analysis. This function  $F$  depends only on the variables that are exogenous from the viewpoint of the consumer and therefore, because of the separability of the indirect utility, it drops out of his portfolio choice.

The second step is to reformulate the portfolio choice problem of the consumer in terms of his indirect utility:

$$\max_{x_H^S, x_H^{S^*}, \mathbb{C}_H} E \left[ \int_0^T e^{-\rho t} (\alpha_H(t) + \beta_H(t)) \log \mathbb{C}_H(t) dt \right] \quad (9)$$

$$\begin{aligned} \text{s.t.} \quad dW_H(t) &= \left[ W_H(t) r(t) + x_H^S(t) W_H(t) (\mu_S(t) - r(t)) + x_H^{S^*}(t) W_H(t) (\mu_{S^*}(t) - r(t)) \right] dt \\ &+ \left[ x_H^S(t) W_H(t) \sigma_S(t) + x_H^{S^*}(t) W_H(t) \sigma_{S^*}(t) \right] d\vec{w}(t) - \mathbb{C}_H(t) dt. \end{aligned} \quad (10)$$

The optimization problem is thus formally equivalent to a familiar single-good consumption-investment problem, with consumption expenditure  $\mathbb{C}_H$  replacing consumption. Consumption of individual goods can then be recovered from (8). It is important to note that the prices of the individual goods,  $p$  and  $p^*$ , and hence the terms of trade have dropped out of the optimization problem. This implies that fluctuations in the terms of trade do not pose a risk that the consumer desires to hedge. In contrast, one would generally expect him to hedge against the preference shifts  $\alpha_H$  and  $\beta_H$ , which enter as state variables in his optimization problem.

The next issue we need to address is market incompleteness. A technique for solving such problems in a single-good framework via martingale methods has been developed in a seminal contribution of He and Pearson (1991). These authors show that, just like for the case of complete markets, one can replace the dynamic optimization problem (9)–(10) by the following static variational problem:

$$\max_{\mathbb{C}_H} E \left[ \int_0^T e^{-\rho t} (\alpha_H(t) + \beta_H(t)) \log \mathbb{C}_H(t) dt \right] \quad (11)$$

$$\text{s.t.} \quad E \left[ \int_0^T \xi_\nu(t) \mathbb{C}_H(t) dt \right] \leq W_H(0), \quad (12)$$

where  $\xi_\nu$  denotes an appropriate state price density—i.e., an Arrow-Debreu state price per unit of probability  $P$ . The difficulty arises from the fact that in incomplete markets, there is an infinite number of such state price densities consistent with no arbitrage and hence potentially an infinite number of static budget constraints (12). However, this set of budget constraints is known to possess some special structure. Let  $m$  denote the market price of risk process

$$m(t) \equiv \sigma(t)^\top (\sigma(t)\sigma(t)^\top)^{-1} (\mu(t) - r(t)\mathbf{1}), \quad (13)$$

where  $\mu \equiv (\mu_S, \mu_{S^*})^\top$  and  $\mathbf{1}$  is a two-dimensional vector of ones. Then the set of state price densities can be represented as (He and Pearson, Proposition 1):

$$d\xi_\nu(t) = -r(t)\xi_\nu(t)dt - (m(t) + \nu(t))^\top \xi_\nu(t)d\vec{w}(t), \quad (14)$$

with  $\nu(t) \in \mathbb{R}^4$  satisfying  $\sigma(t)\nu(t) = 0$ ,  $\forall t \in [0, T]$  and  $\int_0^T \|\nu(t)\|^2 dt < \infty$ .<sup>5</sup> It is easy to see that if the volatility matrix sigma is a nondegenerate square matrix, the condition  $\sigma(t)\nu(t) = 0$  can be satisfied only for  $\nu(t) = \mathbf{0}$ , where  $\mathbf{0}$  is a four-dimensional vector of zeros. This is precisely the case when markets are complete: the state price density is unique and  $\nu(t) = \mathbf{0}$  at all  $t$ . If, however, the volatility matrix has fewer rows than there are Brownian motions (and hence columns), many possible  $\nu(t)$ 's can satisfy the restriction  $\sigma(t)\nu(t) = 0$ . This is the case when markets are intrinsically incomplete.

He and Pearson go on to prove that there exists a unique individual-specific  $\nu$ , which we denote by  $\nu_H$ , that minimizes the maximum expected utility in (11). We derive the expression for it in the proposition below. The only relevant budget constraint in (12) is then the one corresponding to  $\nu_H$ . Establishing the portfolio that solves the optimization problem (11)–(12) is then straightforward. We report this portfolio, as well as the portfolio of Foreign, in the following proposition.

**Proposition 1.** *(i) The fractions of wealth  $x_H$  and  $x_F$  invested in the risky stocks by the Home and the Foreign country, respectively, are given by*

$$x_H(t) = \underbrace{(\sigma(t)\sigma(t)^\top)^{-1}\sigma(t)m(t)}_{\text{mean-variance portfolio}} + \underbrace{(\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) \frac{\sigma_{\alpha_H}(t)^\top + \sigma_{\beta_H}(t)^\top}{\alpha_H(t) + \beta_H(t)}}_{\text{hedging portfolio}} \quad (15)$$

$$x_F(t) = (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t)m(t). \quad (16)$$

The fractions of wealth invested in the bond by Home and Foreign are given by  $1 - \mathbf{1}^\top x_H(t)$  and  $1 - \mathbf{1}^\top x_F(t)$ , respectively.

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<sup>5</sup>The notation  $\|z\|^2$  stands for the dot product  $z \cdot z$ .

(ii) The processes  $\nu_H$  and  $\nu_F$ , entering the specification of the personalized state price densities of Home and Foreign, respectively, are given by

$$\nu_H(t)^\top = -\frac{\sigma_{\alpha_H}(t) + \sigma_{\beta_H}(t)}{\alpha_H(t) + \beta_H(t)}(I_{4 \times 4} - \sigma(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\sigma(t)) \quad \text{and} \quad \nu_F(t) = \mathbf{0}, \quad (17)$$

where  $I_{4 \times 4}$  is a  $4 \times 4$  identity matrix.

Consider first the portfolio of the Home consumer. It consists of two parts: the mean-variance efficient portfolio and the hedging portfolio. This decomposition is standard in the portfolio choice literature. The optimal mean-variance portfolio was first derived by Markowitz (1952) in a one-period setting and later generalized by Merton (1971) to a continuous-time stochastic environment. Furthermore, Merton shows that in addition to the mean-variance portfolio an investor optimally selects a hedging portfolio whose role is to offset fluctuations in the state variables in his optimization problem. As is well-known, investors with logarithmic preferences do not wish to hedge against changes in their investment opportunity set (stock and bond price dynamics)—in that sense they behave myopically. However, they do wish to hedge against fluctuations in the state variables entering their preferences, namely the preference shifts. When markets are complete (or effectively complete), the gains made by the hedging portfolio are perfectly positively correlated with the fluctuations in state variable Home desires to hedge:  $\alpha_H + \beta_H$ . (This is the state variable entering Home's objective function (9)). When markets are incomplete, not every payoff can be replicated and so it is typically not possible to construct a portfolio whose gains are perfectly correlated with a state variable. In that case, the Home investor chooses the portfolio most highly correlated with  $\alpha_H + \beta_H$ .

In contrast, the Foreign investor demands no hedging portfolio. This is because the term  $\alpha_F + \beta_F$  entering his objective function is non-stochastic. Consequently, the inability to hedge perfectly under incomplete markets does not hurt the Foreign investor: in contrast to that of the Home investor, his personalized  $\nu_F$  remains the same as it would be under complete markets.

As we elaborate later, (heterogeneous) hedging demands are key vehicles for generating trade between consumers in equilibrium. For example, in the absence of preference shifts, agents have no hedging demands and hence they have no reason to trade assets.

### 2.3. Characterization of Equilibrium

An equilibrium in our economy is defined in a standard way: it is a collection of goods and asset prices  $(p, p^*, S, S^*, B)$  and consumption-investment policies  $(C_i(t), C_i^*(t), x_i^S(t), x_i^{S^*}(t))$ ,  $i \in \{H, F\}$  such that (i) each consumer-investor maximizes his utility (7) subject to the budget constraint (6) and (ii) goods, stock, and bond markets clear.

In the economy with incomplete markets the equilibrium allocation would not be Pareto optimal. Hence, the usual construction of a representative agent's (planner's) utility as a weighted sum, with constant weights, of individual utility functions is not possible. Instead, we are going to employ a fictitious representative agent with stochastic weights (introduced in an important contribution by Cuoco and He (1994)), with these stochastic weights reflecting the effects of market incompleteness.<sup>6</sup> This fictitious representative agent maximizes his utility subject to the resource constraints:

$$\begin{aligned} \max_{\{C_H, C_H^*, C_F, C_F^*\}} \quad & E \left[ \int_0^T e^{-\rho t} (u_H(C_H(t), C_H^*(t)) + \lambda(t) u_F(C_F(t), C_F^*(t))) dt \right] \\ \text{s. t.} \quad & C_H(t) + C_F(t) = Y(t), \\ & C_H^*(t) + C_F^*(t) = Y^*(t), \end{aligned}$$

where we have normalized the weight on the Home consumer to be equal to one and assigned the weight  $\lambda$  to the Foreign consumer. The possibly stochastic weighting process  $\lambda$  will be linked to the wealth distribution in the economy and will be determined as part of the equilibrium. In the event that in an equilibrium  $\lambda$  ends up being a constant (we encounter this situation in some of the special cases we consider later), the allocation is Pareto optimal. This situation corresponds to the case of complete or so-called effectively complete financial markets.

Solving the representative agent's optimization problem, we obtain the sharing rules

$$C_H(t) = \frac{\alpha_H(t)}{\alpha_H(t) + \lambda(t)\beta_F} Y(t), \quad C_H^*(t) = \frac{\beta_H(t)}{\beta_H(t) + \lambda(t)\alpha_F} Y^*(t), \quad (18)$$

$$C_F(t) = \frac{\lambda(t)\beta_F}{\alpha_H(t) + \lambda(t)\beta_F} Y(t), \quad C_F^*(t) = \frac{\lambda(t)\alpha_F}{\beta_H(t) + \lambda(t)\alpha_F} Y^*(t). \quad (19)$$

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<sup>6</sup>Alternatively, we could have solved for equilibrium directly from the system of equilibrium equations. We prefer the method we are presenting because of the clarity of the ensuing intuitions. The construction of a representative agent with stochastic weights has been employed extensively in dynamic asset pricing models with financial market frictions. See, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), and Detemple and Serrat (2003). A related approach is the extra-state-variable methodology of Kehoe and Perri (2002). For the original solution method utilizing weights in the representative agent, see Negishi (1960).

We can now derive the terms of trade that prevail in a competitive equilibrium. They are identified with the ratio of either country's marginal utilities of the Home and Foreign goods:

$$q(t) = \frac{\alpha_H(t) + \lambda(t)\beta_F}{\beta_H(t) + \lambda(t)\alpha_F} \frac{Y^*(t)}{Y(t)}. \quad (20)$$

We next use the no-arbitrage valuation principle to obtain stock prices and equilibrium wealth of the countries.

**Lemma 1.** *Equilibrium stock prices in our economy are given by*

$$S(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{q(t)}{aq(t) + 1 - a} Y(t), \quad (21)$$

$$S^*(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{1}{aq(t) + 1 - a} Y^*(t) \quad (22)$$

and the wealth of the countries by

$$W_H(t) = \frac{\alpha_H(t) + \beta_H(t)}{\alpha_H(t) + \lambda(t)\beta_F} S(t), \quad W_F(t) = \frac{\lambda(t)(\alpha_F + \beta_F)}{\beta_H(t) + \lambda(t)\alpha_F} S^*(t). \quad (23)$$

Lemma 1 yields a simple interpretation of the weight  $\lambda$ . One can see that

$$\lambda(t) = \frac{W_F(t)(\alpha_H(t) + \beta_H(t))}{W_H(t)(\alpha_F + \beta_F)}. \quad (24)$$

That is, incomplete markets enrich the dynamics of the economy with an additional state variable  $\lambda$ , which is related to the wealth distribution, but not given exactly by the wealth distribution unless  $\alpha_H(t) + \beta_H(t)$  is constant. We have already encountered the expression  $\alpha_H(t) + \beta_H(t)$  earlier in our analysis: it was the state variable giving rise to the hedging portfolio held by Home.

Lemma 1 allows us to characterize the dynamics of stock returns and the market price of risk in equilibrium, which are tedious but straightforward to compute. Equation (24) lets us pin down the weight  $\lambda$ . We relegate the details of the necessary calculations to the Appendix, and report the resulting dynamics of  $\lambda$  below.

**Proposition 2.** *(i) In an equilibrium, the weight of the Foreign country in the fictitious representative agent follows*

$$d\lambda(t) = -\lambda(t) \nu_H(t) d\vec{w}(t), \quad \text{with } \lambda(0) = \beta_H(0)/\beta_F.$$

*(ii) When such equilibrium exists, the volatility matrix  $\sigma$  and the market price of risk  $m$  can be computed as functions of exogenous state variables. They are reported in the Appendix.*

Note that our characterizations the terms of trade, consumption, and stock prices presented in this section all involve the exogenous state variables of the model and one endogenous quantity: the weight  $\lambda$ . With this weight  $\lambda$  now characterized in Proposition 2, we can then pin down these equilibrium quantities and their dynamics. Moreover, the countries' portfolios held in equilibrium are also fully determined now, with the volatility matrix of stock returns and the market price of risk characterized fully in terms of exogenous state variables (see the Appendix). Admittedly, the equilibrium characterizations of the portfolios are not particularly transparent. To develop intuitions, in Section 4 we are going to consider several special cases in which the expressions for the portfolios are simple. The analysis of these special cases relies in part on the result of the following lemma.

**Lemma 2.** *The countries hold no bond in their portfolios if and only if the value of the hedging portfolio demanded by Home is equal to zero.*

**Proof.** Suppose that bondholdings of the countries are zero. This is equivalent to saying that the fraction of wealth each country invests in the stocks is equal to one:

$$\begin{aligned}\mathbf{1}^\top x_H(t) &= \mathbf{1}^\top \left[ (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) m(t) + (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) \frac{\sigma_{\alpha_H}(t) + \sigma_{\beta_H}(t)}{\alpha_H(t) + \beta_H(t)} \right] = 1 \\ \mathbf{1}^\top x_F(t) &= \mathbf{1}^\top (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) m(t) = 1,\end{aligned}$$

where we have substituted the formulas for the portfolios derived in Proposition 1. This can happen only if  $\mathbf{1}^\top (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) \frac{\sigma_{\alpha_H}(t) + \sigma_{\beta_H}(t)}{\alpha_H(t) + \beta_H(t)} = 0$ —i.e., the fraction of wealth invested in the hedging portfolio is zero.

Conversely, if the value of the hedging portfolio is zero, then  $\mathbf{1}^\top x_H(t) = \mathbf{1}^\top x_F(t) = \mathbf{1}^\top (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) m(t)$ . Bond market clearing then implies that  $\mathbf{1}^\top (\sigma(t)\sigma(t)^\top)^{-1}\sigma(t) m(t) = 1$ .

■

### 3. The Current Account

Before we turn to investigating equilibrium behavior in the general case of our model and make a link between portfolio rebalancing and external adjustment mechanisms, we explore our model's implications for the current account and its dynamics. We first start with the textbook definition of the current account and then consider the definition based on changes in a country's net foreign asset position.

In principle, the two definitions should yield similar measures. However, once expected and realized capital gains are accounted for, the differences between the two measures, and especially their dynamic properties, can be striking, as we demonstrate below.

### 3.1. The Textbook Current Account

Let us concentrate on the Home country. The traditional measure of the current account, commonly employed in international finance textbooks, is

$$CA = \text{Trade Balance} + \text{Net Dividend Payments} + \text{Net Interest Payments}.$$

In our model, the trade balance is given by

$$TB_H(t) = p(t)(Y(t) - C_H(t)) - p^*(t)C_H^*(t),$$

and the current account is

$$CA_H(t) = \left[ TB_H(t) + s_H^{S^*}(t)p^*(t)Y^*(t) - s_F^S(t)p(t)Y(t) + s_H^B(t)B(t)r(t) \right] dt, \quad (25)$$

where  $s_i^j$  denotes the number of shares of asset  $j$  held by country  $i$ . The second and the third terms in (25) are dividend receipts from foreign assets minus dividend payments to Foreign, and the last term is net interest payments. Recall that each of the above quantities in our model is defined as a rate (e.g., the export rate, the dividend rate, etc.) and hence need to be scaled by a time increment. This is the reason behind the term “dt” appearing in (25).

An often cited shortcoming of pure-exchange models with log-linear preferences is that they are unable to generate a nontrivial current account. Having a current account equal to zero at all times would clearly hinder any quantitative analysis of current account deficits that we intend to undertake in this paper. It is therefore worth highlighting the situations under which the current account is zero in our model.

**Lemma 3.** *The current account of the Home country can be represented as follows:*

$$CA_H(t) = s_H^B(t)B(t) \left( r(t) - \frac{\rho}{1 - e^{-\rho(T-t)}} \right) dt. \quad (26)$$

**Proof.** Note that

$$\begin{aligned}
s_H^B(t)B(t) &= W_H(t) - s_H^S(t)S(t) - s_H^{S^*}(t)S^*(t) \\
&= \frac{\alpha_H(t) + \beta_H(t)}{\alpha_H(t) + \lambda(t)\beta_F}S(t) - (1 - s_F^S(t))S(t) - s_H^{S^*}(t)S^*(t) \\
&= \frac{1 - e^{-\rho(T-t)}}{\rho} \left[ p(t)Y(t) \frac{\beta_H(t) - \lambda(t)\beta_F}{\alpha_H(t) + \lambda(t)\beta_F} - s_H^{S^*}(t)p^*(t)Y^*(t) + s_F^S(t)p(t)Y(t) \right],
\end{aligned}$$

where the second equality follows from Lemma 1 and stock market clearing ( $s_H^S(t) = 1 - s_F^S(t)$ ), and the last one, again, from Lemma 1. On the other hand, by substituting (18) into (25) and simplifying, one can show that

$$\begin{aligned}
CA_H(t) &= \left[ -p(t)Y(t) \frac{\beta_H(t) - \lambda(t)\beta_F}{\alpha_H(t) + \lambda(t)\beta_F} + s_H^{S^*}(t)p^*(t)Y^*(t) - s_F^S(t)p(t)Y(t) + s_H^B(t)B(t)r(t) \right] dt \\
&= s_H^B(t)B(t) \left( -\frac{\rho}{1 - e^{-\rho(T-t)}} + r(t) \right) dt.
\end{aligned}$$

This proves the statement in (26). ■

This lemma reveals that the first sufficient condition for the current account to be equal to zero is that the Home country (and hence the Foreign) holds no bonds. This is indeed a common implication of models with log-linear preferences (e.g., Pavlova and Rigobon (2005), Pavlova and Rigobon (2007)), however, nothing in our model prevents the bond holdings from being different from zero. That is, preference shifts may potentially introduce enough heterogeneity among the countries so that they are willing to trade in all available financial assets for risk-sharing purposes. The second condition under which the current account is zero is when the interest rate  $r(t)$  is equal to  $\rho/(1 - e^{-\rho(T-t)})$ . The latter quantity is deterministic, while the interest rate is a stochastic process. Hence, it is true only on the measure zero set of parameter values.

### 3.2. The Capital-Gains Adjusted Current Account

Defining the current account as the change in the net foreign asset position of a country, we have

$$CGCA_H(t) = d \left[ s_H^{S^*}(t)S^*(t) - s_F^S(t)S(t) + s_H^B(t)B(t) \right], \quad (27)$$

where the first two terms in the square brackets are Home's investment in the Foreign stock minus Foreign' investment in the Home stock, and the last term is Home's balance on the bond account. The label "CGCA" stands for "capital-gains adjusted current account," the rationale for which will



become clear shortly. Note that, by market clearing,  $s_F^S(t) = 1 - s_H^S(t)$  and that, by definition, Home's financial wealth equals its portfolio value,  $W_H(t) = s_H^S(t)S(t) + s_H^{S^*}(t)S^*(t) + s_H^B(t)B(t)$ . Hence, we can rewrite (27) as

$$CGCA_H(t) = dW_H(t) - dS(t) \quad (28)$$

Note further that the budget constraint of Home (6) can be equivalently represented as

$$\begin{aligned} dW_H(t) = & \left[ s_H^B(t)B(t)r(t) + s_H^S(t)S(t)\mu_S(t) + s_H^{S^*}(t)S^*(t)\mu_{S^*}(t) \right] dt \\ & + \left[ s_H^S(t)S(t)\sigma_S(t) + s_H^{S^*}(t)S^*(t)\sigma_{S^*}(t) \right] d\vec{w}(t) + \left[ TB_H(t) - p(t)Y(t) \right] dt. \end{aligned}$$

Substituting it into (28) and then using (4) and the stock market clearing, we arrive at the following proposition.

**Proposition 3.** *The capital-gains adjusted current account is given by*

$$\begin{aligned} CGCA_H(t) = & \left[ TB_H(t) + s_H^{S^*}(t)S^*(t)\mu_{S^*}(t) - s_H^S(t)S(t)\mu_S(t) + s_H^B(t)B(t)r(t) \right] dt \\ & + \left[ s_H^{S^*}(t)S^*(t)\sigma_{S^*}(t) - s_H^S(t)S(t)\sigma_S(t) \right] d\vec{w}(t), \end{aligned} \quad (29)$$

where  $\sigma_S$  and  $\sigma_{S^*}$  are derived in Proposition 2 and  $r$ ,  $\mu_S$ , and  $\mu_{S^*}$  are reported in the Appendix.

The first difference between the traditional and the capital-gains adjusted current account revealed by Proposition 3 is the presence of the diffusion ( $d\vec{w}$ ) component in (29) and its absence in (25). This component is the *unexpected* part of the realized capital gains on Home's net foreign assets. A shock  $d\vec{w}$  typically has a differential impact on the stock portfolios of Home and Foreign. The capital-gains adjusted current account of Home improves if its return on foreign asset holdings exceeds the return the Foreign country makes on its holdings of Home's assets. Equation (29) assumes, however, that all capital gains are marked to market. This aspect makes *CGCA* different from book-value based measures of current account such as the one in (25), commonly employed in practice.

It is important to note that the two measures of the current account have fundamentally different dynamic properties. The traditional current account is a persistent process, in line with the results in the existing literature. In contrast, the capital-gains adjusted current account features additionally an increment of a random walk ( $d\vec{w}$ ) process, and therefore it bears a closer resemblance to the dynamics of a stock market rather than a persistent macroeconomic series such as

dividends or the traditional current account. It is amply documented by the proponents of the efficient markets hypothesis in finance that stocks' capital gains are large, volatile, and serially uncorrelated. And so should be the fluctuations in a NFA position of a country, captured here by *CGCA*!

The second difference between the two measures of the current account stems from the differences in the *expected* ( $dt$ ) component. Comparing equations (25) and (29), we find two elements that are common to the traditional and the capital-gains adjusted current account measures: the trade balance and net interest payments on the (locally) riskless bond. The remaining terms, due to holdings of the risky stocks, are different. To better highlight the differences, note from (4)–(5) that

$$S(t) \mu_S(t) dt = E_t[dS(t)] + p(t)Y(t)dt \quad \text{and} \quad S^*(t) \mu_{S^*}(t) dt = E_t[dS^*(t)] + p^*(t)Y^*(t)dt,$$

where  $E_t[\cdot]$  is a shortcut for  $E[\cdot | \mathcal{F}_t]$ . The first term on the right-hand side of these expressions is the expected capital gains and the second one is the dividends per share. Denoting the expected capital gains on holding a share of stocks  $S$  and  $S^*$  by  $ECG_S$  and  $ECG_{S^*}$ , respectively, we can express the difference between the expected capital-gains adjusted current account and the traditional one as

$$E_t[CGCA_H(t)] - CA_H(t) = -s_F^S(t) ECG_S(t) + s_H^{S^*}(t) ECG_{S^*}(t). \quad (30)$$

This simple formula describes what Hausmann and Sturzenegger (2006) label “the missing dark matter” in the measurement the current account. This “dark matter” accumulates due to the fact that the countries hold risky assets which are issued at home as well as abroad, and these risky assets have different expected capital gains. The expected capital gain on an asset, of course, reflects its risk and return tradeoff. If domestic assets are safer and have lower expected returns than foreign, a country earns a higher expected return on its assets than it owes on its liabilities. In this scenario, the difference between the capital-gains adjusted and the traditional current account is expected to be positive. We expect such pattern to be occurring in the US whose gains on domestic assets have been lower than gains on assets abroad (Gourinchas and Rey (2007b)).

The sum of net expected capital gains and net unexpected capital gains is what has been typically labeled as “valuation effects” in the current account adjustment literature. Empirically, it is very hard to disentangle the two individual components. However, as we will point out below, their stochastic properties are very distinct.

One may wonder why the terms of trade were mentioned nowhere in this discussion. After all, the original arguments highlighting valuation effects stress primarily the fact that the values of a country’s asset and liabilities change in response to fluctuations in the exchange rate (or the terms of trade), and these fluctuations need to be taken into account when evaluating the NFA position of a country. This argument definitely applies in our model—in fact, as will become clear below, the terms of trade play a key role in our intuitions. The terms of trade are embedded in the prices of stocks and their dynamics.

It is worth mentioning that in our derivations of the capital-gains adjusted current account and the relationship between CA and CGCA we did not need to specialize consumer preferences to be log-linear. The expressions for both the current account in equation (25) and the capital-gains adjusted current account in equation (29) are valid for a general utility function. The log-linear form of the preferences is needed only for obtaining a closed-form characterization of the inputs into the formulas, namely the stock prices and the countries’ portfolios.

### 3.3. Congruence between NFA and Trade Balance

To conclude this section, we derive the expression for NFA in our model and draw a connection between the NFA position and the trade balance. Note that

$$\begin{aligned} NFA_H(t) &= W_H(t) - S(t) \\ &= \frac{1}{\xi_{\nu_H}(t)} E_t \left[ \int_t^T \xi_{\nu_H}(s) (p(s) C(s) + p^*(s) C^*(s)) ds \right] - \frac{1}{\xi_{\nu_H}(t)} E_t \left[ \int_t^T \xi_{\nu_H}(s) p(s) Y(s) ds \right]. \end{aligned}$$

Hence, by definition of a trade balance,

$$NFA_H(t) = -\frac{1}{\xi_{\nu_H}(t)} E_t \left[ \int_t^T \xi_{\nu_H}(s) TB_H(s) ds \right]. \quad (31)$$

Equation (31) is nothing else but the familiar statement that the NFA position is given by the present value of the future trade deficits. The traditional intertemporal approach to the external adjustment—that ignores changes in the state price density (or the stochastic discount factor)  $\xi_{\nu_H}$ —says that, for example, for a country with a negative NFA position, adjustment must come through future trade surpluses. Recent literature challenges this conclusion and draws attention to the “valuation channel” of the external adjustment that operates precisely through changes in the stochastic discount factor. It is argued that such changes are large and volatile, and hence the valuation channel should have a substantial contribution to the NFA dynamics. Surprisingly, it

turns out that in our model, after the endogenous responses of asset prices and hence the stochastic discount factor to underlying shocks are taken into account, the NFA adjustment takes place instantaneously and entirely through the trade balance. In that sense, the traditional and the new views are not at all inconsistent: much can be learned about the NFA adjustment by simply looking at the most conventional measure of external imbalances—the trade balance.<sup>7</sup>

**Lemma 4 (Congruence between NFA and trade balance).** *The relationship between the net foreign assets and the trade balance is given by*

$$NFA_H(t) = -\frac{1 - e^{-\rho(T-t)}}{\rho} TB_H(t). \quad (32)$$

*The net foreign asset position of Home is*

$$NFA_H(t) = \frac{\beta_H(t) - \lambda(t) \beta_F}{\alpha_H(t) + \lambda(t) \beta_F} S(t). \quad (33)$$

We remark that the perfect negative correlation between the current trade balance and the NFA position is due primarily to our assumption that the agents have log-linear preferences that rule out intertemporal hedging motives. It is important to evaluate the robustness of this result under alternative preferences that give rise to intertemporal hedging.

## 4. Equilibrium Portfolios in Several Special Cases

To better understand the composition of the equilibrium portfolios, we consider several special cases of our model before turning to the general case (analyzed in Section 5).

### 4.1. Example 1: The Irrelevance Result

The first example we study is one in which the financial markets’ structure is irrelevant, there are no net portfolio flows, and therefore, capital gains on financial assets play no role in the international adjustment process. This is the case considered in Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995). The three examples that follow relax some of the assumptions we make here in order to clarify the role that capital gains play.

In our model, we obtain our irrelevance result by specializing the Home consumer’s preferences so that  $\alpha_H$  and  $\beta_H$  are constant ( $\sigma_{\alpha_H}(t) = \sigma_{\beta_H}(t) = \mathbf{0}$ ). As is well-known, under this specification

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<sup>7</sup>It is important to note that the “connection” is between the NFA and the trade balance of goods and services, and not between NFA and the conventional current account.

the returns on the two stocks are perfectly correlated in equilibrium. Hence, portfolio allocations into these stocks are indeterminate. Only positions in the composite stock market,  $S(t) + S^*(t)$ , can be uniquely determined. Using an argument analogous to that we employed in Section 2.2, we can derive the investors' optimal fractions of wealth invested in the composite stock market. One can easily see that the portfolios demanded by the two agents are going to be identical: in the absence of preference shifts, they both demand the mean-variance but no hedging portfolios.

It is also well-known that financial markets are effectively complete in this special case. This is equivalent to saying that  $\nu_H(t) = \nu_F(t) = \mathbf{0}$  at all times and hence, from Proposition 2, the weight  $\lambda$  is constant. Thus, Pareto optimality obtains despite market incompleteness—or, in other words, markets are effectively complete. Investors are not adversely affected by market incompleteness because they do not make use of financial markets to construct portfolios hedging against fluctuations in any state variables: there is no state variable either agent desires to hold a hedge against. The intuition for why the financial markets are not needed in this case comes from the fact that movements in the terms of trade exactly offset output shocks and hence the values of the dividends on the Home and the Foreign stock markets,  $p(t)Y(t)$  and  $p^*(t)Y^*(t)$ , respectively, are always the same (up to a multiplicative constant). Fluctuations in the terms of trade therefore fully offset the supply shocks; i.e., with no demand uncertainty the capital gains on the two stocks are always perfectly correlated. This feature of our model is due to the way we specified preferences (log-linear) and endowments (shares of trees), and represents a simple benchmark for comparison.

Finally, unlike holdings of individual stocks, the bond holdings of the countries are uniquely determined: there are equal to zero at all times.<sup>8</sup> This is because, the two countries demand the same portfolio and in particular, wish to invest the same fraction of wealth in the bond. For the bond market to clear, this fraction has to be zero. Consequently,  $CA(t) = 0$  at all times. Moreover, the capital-gains adjusted current account  $CGCA$  is also zero. Each country is holding the same portfolio and stock markets are perfectly correlated. Therefore, the net dividend payments and the net capital gains account have to be zero as well.

## 4.2. Example 2: No Asset Cross-Holdings

The purpose of this example is to illustrate the role of international asset cross-holdings and portfolio rebalancing. We consider a version of our model in which portfolio holdings are unique (as opposed

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<sup>8</sup>This result is not new. See Cass and Pavlova (2004).

to indeterminate as in the previous case), but portfolio holdings are fixed (hence there is never a rebalancing after any of the shocks). We show that in this case the current account and the capital-gains current account are zero at all times. Hence, movements in asset prices are unrelated to the external adjustment.

To highlight these dynamics, consider the special case of the model in which  $\beta_H$  remains constant ( $\sigma_{\beta_H}(t) = \mathbf{0}$ ), and  $\alpha_H$  is stochastic. In the presence of preference shifts—even one possible shift, as we specify here—the two stocks are no longer perfectly correlated, the volatility matrix is invertible, and hence the expressions in Propositions 1 and 2 readily apply. It turns out that the equity portfolios of the countries, expressed as numbers of shares, take a particularly simple form

$$s_H(t) = \left(1, \frac{\beta_H - \lambda(t)\beta_F}{\lambda(t)\alpha_F + \beta_H}\right) \quad \text{and} \quad s_F(t) = \left(0, \frac{\lambda(t)(\alpha_F + \beta_F)}{\lambda(t)\alpha_F + \beta_H}\right),$$

where  $s_i \equiv (s_i^S, s_i^{S^*})$  are obtained from  $x_i$  using Lemma 1.

The hedging portfolio held by Home, in numbers of shares, is

$$h(t) = \left(1, -\frac{\alpha_H(t) + \lambda(t)\beta_F}{\lambda(t)\alpha_F + \beta_H}\right).$$

The (instantaneous) gain on this hedging portfolio is given by

$$dh(t) = [\dots] dt + \frac{W_H(t)}{\alpha_H(t) + \beta_H} \sigma_{\alpha_H}(t) d\vec{w}(t),$$

where the drift term need not concern us here. Note that the gain on the hedging portfolio is *perfectly instantaneously correlated* with fluctuations in the preference shifts  $\alpha_H$ . (Recall that  $d\alpha_H(t) = \sigma_{\alpha_H}(t) d\vec{w}(t)$ .) Therefore, despite market incompleteness, the Home investor is able to construct a portfolio perfectly correlated with its preference shock. It is of no surprise then that it turns out that  $\nu_H(t) = \mathbf{0}$ : the investor is able to achieve the same efficiency of hedging as under complete markets. Consequently (Proposition 2), again, the weight  $\lambda$  is constant and markets are effectively complete. In contrast to the no preference shifts case, however, one can see that effective market completeness does not lead to the indeterminacy of equilibrium portfolios.

Having established that  $\lambda$  is a constant, we can further simplify the expressions for the countries' portfolios. From Proposition 2,  $\lambda = \lambda(0) = \beta_H/\beta_F$ , and hence the portfolios of the countries, in numbers of shares, are simply  $s_H(t) = (1, 0)$  and  $s_F(t) = (0, 1)$ . Note that Home ends up holding the entire supply of the Home stock, and Foreign holds the entire supply of its stock as well.

Note that in this example we have an extreme portfolio home bias (an apparent home bias, because in this case the optimal portfolio is to hold all of the home stock and none of the foreign).

It is important to point out that the home bias is coming because the demand shock is affecting home demand for home goods (this is equivalent to explicitly modeling shocks to the non-tradable demand which has been already highlighted in the literature). It is equally important to highlight that the home bias in consumption has *nothing* to do with the home bias in portfolios in this case. The home bias in consumption in country  $i$  occurs when  $\alpha_i$  is larger than  $\beta_i$ . However, for this result  $\alpha_i$  can be larger or smaller than  $\beta_i$  in either country, and the home bias in portfolios will remain as long as the demand shocks affect the preference for the home good (i.e.,  $\alpha_H$  is stochastic) as opposed to that for the foreign good.

Regarding bond holdings, just like in the previous special case, the countries invest nothing in the bond. To see this, we compute the value of the hedging portfolio  $h$  and conclude (from Lemma 1) that it is equal to zero at all times. The result then follows from Lemma 2. Intuitively, the hedging portfolio held by Home is a costless long-short portfolio of the two available stocks. If it were not costless, the investor would need to borrow or lend on his bond account in order to finance it.

Finally, note that

$$CA_i(t) = CGCA_i(t) = 0, \quad i \in \{H, F\}.$$

The textbook current account is zero because none of the countries invests in the bond (Lemma 3). The capital-gains adjusted current account is also zero simply because the countries' end up owning no foreign assets and no bonds. Hence, by definition (equation (27)), both the net foreign asset positions of the countries and their capital-gains adjusted current accounts are zero. In this case, the net capital gains and net expected return accounts are zero because the countries exhibit a 100 percent home bias, and not because the stock markets are perfectly correlated (as they were in the previous case).

### 4.3. Example 3: Valuation Effects

We now consider a more general case of our model in which the current account is still identically equal to zero but the capital-gains adjusted current account is now different from zero. The difference between the two definitions is, of course, due to the expected and unexpected capital gains on NFA. The external adjustment process in this example is therefore driven entirely by the valuation effects and has nothing to do with traditional channels.

We consider a special case of the model in which the preference shifter  $\alpha_H$  is driven only by

the Brownian motion  $w^\alpha$  and  $\beta_H$  by the Brownian motion  $w^\beta$ . Formally,  $\sigma_{\alpha_H}(t) = (0, 0, \sigma_{\alpha_1}(t), 0)$  and  $\sigma_{\beta_H}(t) = (0, 0, 0, \sigma_{\beta_2}(t))$ , with  $\sigma_{\alpha_1} > 0$  and  $\sigma_{\beta_2} > 0$ . The stockholdings of Home and Foreign, respectively, are as follows:

$$s_H = \frac{1}{G} \left( \frac{\beta_H}{\sigma_{\beta_2}^2} (\lambda \alpha_F + \beta_H) + \frac{\alpha_H}{\sigma_{\alpha_1}^2} (\alpha_H - \lambda \alpha_F), \quad \frac{\beta_H}{\sigma_{\beta_2}^2} (\beta_H - \lambda \beta_F) + \frac{\alpha_H}{\sigma_{\alpha_1}^2} (\alpha_H + \lambda \beta_F) \right), \quad (34)$$

$$s_F = \frac{\lambda (\alpha_F + \beta_F)}{G} \left( \frac{\alpha_H}{\sigma_{\alpha_1}^2}, \frac{\beta_H}{\sigma_{\beta_2}^2} \right), \quad (35)$$

where  $G \equiv \frac{\beta_H}{\sigma_{\beta_2}^2} (\lambda \alpha_F + \beta_H) + \frac{\alpha_H}{\sigma_{\alpha_1}^2} (\alpha_H + \lambda \beta_F)$ . In the expressions above and for the remainder of this section, we suppress the argument  $t$ . It already becomes clear at this point that depending on parameter values, our model can produce large gross portfolios.<sup>9</sup>

The hedging portfolio,  $h$ , held by Home, in numbers of shares, is

$$h = \frac{\frac{\beta_H}{\sigma_{\beta_2}^2} - \frac{\alpha_H}{\sigma_{\alpha_1}^2}}{G} (\lambda \alpha_F + \beta_H, \quad -(\alpha_H + \lambda \beta_F)). \quad (36)$$

Consider again the gain on the hedging portfolio:

$$dh = [\dots] dt + \frac{W_H \beta_H \left( \frac{\beta_H}{\sigma_{\beta_2}^2} - \frac{\alpha_H}{\sigma_{\alpha_1}^2} \right)}{(\alpha_H + \beta_H) \left( \frac{\beta_H^2}{\sigma_{\beta_2}^2} + \frac{\alpha_H^2}{\sigma_{\alpha_1}^2} \right)} (0, 0, \sigma_{\alpha_1}, \sigma_{\beta_2}) d\vec{w},$$

and compare it to the fluctuations in the state variable,  $\alpha_H + \beta_H$ , that Home desires to hedge against:

$$d(\alpha_H + \beta_H) = (0, 0, \sigma_{\alpha_1}, \sigma_{\beta_2}) d\vec{w}.$$

(Recall that  $\alpha_H + \beta_H$  is the state variable entering Home's objective function (9).) Unlike in the previous special case, the hedge is no longer perfect. If  $\beta_H/\sigma_{\beta_2}^2 > \alpha_H/\sigma_{\alpha_1}^2$ , the hedging portfolio gains in response to a positive shock in  $\alpha_H$  (an innovation to  $w^\alpha$ ). But it loses value if the economy is hit by a positive  $\beta_H$  shock. The opposite is true for  $\beta_H/\sigma_{\beta_2}^2 < \alpha_H/\sigma_{\alpha_1}^2$ . In any event, Home is able to perfectly hedge against an  $\alpha_H$  or a  $\beta_H$  shock but not both. The condition determining which shock to focus on reflects the relative importance of a shock. Ceteris paribus, if the volatility

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<sup>9</sup>The shareholdings simplify considerably in the case of full symmetry ( $\alpha_H = \alpha_F = \alpha$ ,  $\beta_H = \beta_F = \beta$ ,  $\lambda = 1$ , and  $\sigma_{\alpha_1} = \sigma_{\beta_2} = \sigma$ ).

$$s_H = \frac{1}{\alpha + \beta} (\beta, \alpha), \quad s_F = \frac{1}{\alpha + \beta} (\alpha, \beta),$$

In this case, the portfolio holdings of the two countries are mirror images of each other and the extent of the portfolio home bias is directly related to the degree of consumption home bias.



of, say, the  $\alpha_H$  shock,  $\sigma_{\alpha_1}$ , is high, Home holds a hedging portfolio that is positively correlated with  $\alpha_H$ ; otherwise, it prefers instead a portfolio positively correlated with the  $\beta_H$  shock. Note that the holdings of the two stocks in the hedging portfolio have the opposite sign, and this sign depends on the sign of  $\beta_H/\sigma_{\beta_2}^2 - \alpha_H/\sigma_{\alpha_1}^2$ . This implies that, depending on the relative importance of the two demand shocks, our model can produce a home bias or a reverse home bias in portfolios. Note that the condition determining the direction of the bias is not same one as that for the home bias in consumption ( $\alpha_H > \beta_H$ ), as is often thought.

The inability to hedge perfectly is indicative of the fact that market incompleteness matters. Indeed, in equilibrium,

$$\nu_H = \left( 0, \quad 0, \quad -\frac{\alpha_H \sigma_{\alpha_1} \sigma_{\beta_2}^2}{\beta_H^2 \sigma_{\alpha_1}^2 + \alpha_H^2 \sigma_{\beta_2}^2}, \quad -\frac{\beta_H \sigma_{\alpha_1}^2 \sigma_{\beta_2}}{\beta_H^2 \sigma_{\alpha_1}^2 + \alpha_H^2 \sigma_{\beta_2}^2} \right), \quad (37)$$

and hence the weight  $\lambda$  follows a stochastic process. The zeros in the first two positions of  $\nu_H$  are not accidental. Since the preference shifts that the Home country faces are uncorrelated with the output shocks, it demands a hedge correlated with the Brownian motions  $w^\alpha$  and  $w^\beta$  but not  $w$  and  $w^*$ . Constructing such a hedging portfolio is possible: one can easily show that any zero-cost portfolio of the two stocks is going to be uncorrelated with the output shocks. The hedging portfolio  $h$  must then have a value of zero, and one can easily verify from (36) and Lemma 1 that this is indeed the case. As a corollary, none of the countries holds any bond (Lemma 2) and hence their current accounts are zero (Lemma 3).

Now let us examine the capital-gains adjusted current account of, say, Home. Recall from Lemma 4 that  $NFA_H = \frac{\beta_H - \lambda \beta_F}{\alpha_H + \lambda \beta_F} S$ . Unlike in the two previous special cases, where it turned out that  $\beta_H - \lambda \beta_F = 0$ , Home's net foreign asset position is no longer zero at all times. We find that in our model, the sign of the responses of the capital-gains adjusted current account to the underlying innovations depends on whether the country is a net debtor or a net creditor. We thus need the following condition:

**Condition A1: Home is Net Creditor.**  $\beta_H - \lambda \beta_F > 0$ .

We can then sign (or characterize) the direction the valuation effects. Table 1 presents the effects.

For the signs of the unexpected gain/loss on the Home's net foreign asset position in response to the output shocks, Condition A1 is necessary and sufficient. This observation is similar to the one made in Kraay and Ventura (2000). However, the condition is only a sufficient one for the signs

	$dw$	$dw^*$	$dw^\alpha$	$dw^\beta$
Unexpected change in $S$	+	+	+	-
Unexpected change in $S^*$	+	+	-	+
$CGCA_H$	$+^{A1}$	$+^{A1}$	$-^{A1}$	$+^{A1}$

Table 1: The valuation effects: Unexpected gains on stocks and Home's the net foreign assets ( $CGCA_H$ ) in response to the underlying shocks. The superscript  $A1$  indicates that a sign is valid under Condition A1.

of the responses to the demand shocks, which give rise to more complex dynamics.

Table 1 reveals that on impact, both stocks yield unexpected capital gains in response to a positive output shock in either country ( $dw$  or  $dw^*$ ). This is because a positive output shock in say, Home, raises the dividend on the Home tree. At the same time, it causes a deterioration of Home's terms of trade because the Home good becomes less scarce. This in turn improves Foreign's terms of trade and hence raises the value of the output of the Foreign tree. Hence, both stock markets go up. The remaining signs of the unexpected gains on the stocks are summarized in the last two columns. The reaction of stock prices to preference shifts has a distinctly different pattern: the preference shifts make the stock prices always move in opposite directions. As Home shifts its preference towards the Home good, there is an excess demand for the Home good in the world. This pushes the price of the Home good up, or equivalently, causes an appreciation of the terms of trade,  $q$ . This raises the value of the Home output relative to Foreign. Consequently, the price of the Home stock increases, while that of the Foreign stock falls.

To understand the intuition behind the impact of the valuation effects on the current account, note that Foreign always holds a positive position in Home stock and, under Condition A1, Home has a positive position in the Foreign stock (see equations (34)–(35)). It is then immediate from (29) that the unexpected component term has to be negative for the  $dw^\alpha$  and positive for the  $dw^\beta$  shock. In the case of a preference shift towards the Home good, a positive  $dw^\alpha$ , Home loses on its investment in the Foreign stock while Foreign gains on its investment in the Home stock—hence the fall in the current account surplus of Home. The opposite is true for a preference shift towards the Foreign good, a positive  $dw^\beta$ .

Finally, it turns out that in this example the directions of the agents' portfolio reallocations in response to the underlying shocks are unambiguous. In particular, portfolios respond to demand shocks, but not to the supply shocks, as reported in Table 2.

Variable/ Effects of	$dw(t)$	$dw^*(t)$	$dw^\alpha(t)$	$dw^\beta(t)$
$ds_H^S$	0	0	-	+
$ds_H^{S^*}$	0	0	+	-

Table 2: Impact responses of Home’s portfolio holdings to the underlying shocks.

#### 4.4. Example 4: Bondholdings and the Current Account

This final example illustrates what is required for the current account to have a nontrivial dynamics. As we can anticipate from the previous examples, the key implication that produces such dynamics is that the countries have nontrivial bondholdings.

To induce the countries to trade in bonds, we now allow for the correlation between the preference shifts and the output shocks. To keep the model tractable, however, we reduce the number of Brownian motions driving the economy from four to two. In particular, we shut down Brownian motions  $w^\alpha$  and  $w^\beta$  and require that all processes are adapted to the filtration generated by the output shocks  $w$  and  $w^*$ . Under this modification, all four-dimensional vectors in our analysis in Section 2 and the Appendix become two-dimensional. This implies further that the volatility matrix of stock returns  $\sigma$  is a  $2 \times 2$  square matrix. If this matrix is nondegenerate—which is always the case in the presence of stochastic preference shifts—financial markets are complete. Equilibrium allocation is then Pareto optimal and the weight  $\lambda$  is constant.

In the interest of space, we do not report the countries’ portfolios in this case. It suffices to say that now portfolios depend on all of the parameters of the model except for the drifts of outputs. As to be expected, the gain on the hedging portfolio in this case is perfectly correlated with the fluctuations in  $\alpha_H + \beta_H$ :

$$dh = [\dots] dt + \frac{W_H}{\alpha_H + \beta_H} (\sigma_{\alpha_H} + \sigma_{\beta_H}) d\vec{w}.$$

In contrast to all the special cases we have considered so far however, the value of the hedging portfolio is not equal to zero. Lemma 2 then implies that now the countries engage in borrowing and lending. Furthermore, for some special cases of this economy, the bondholdings always have a unique sign. For example, if we set  $\sigma_{\alpha_H} = (\sigma_{\alpha_1}, 0)$  and  $\sigma_{\beta_H} = (0, \sigma_{\beta_2})$ , with  $\sigma_{\alpha_1} > 0$  and  $\sigma_{\beta_2} > 0$ , the value of the bondholdings of the Home country becomes

$$-\frac{(1 - e^{-\rho(T-t)})}{\rho} \frac{Y Y^* \lambda (\alpha_F + \beta_F) \sigma_{\alpha_1} \sigma_{\beta_2}}{(1 - a)Y (\lambda \alpha_F + \beta_H)^2 \sigma_{Y^*} \sigma_{\alpha_1} + aY^* (\alpha_H + \lambda \beta_F)^2 \sigma_Y \sigma_{\beta_2}}.$$

Home borrows from Foreign to finance its hedging portfolio, whose value is always greater than

zero in this case. This example demonstrates that in our model it is possible to have a negative bond position forever. This does not in any way contradict sustainability of a country’s external position: if an equilibrium exists, the budget constraints of both countries are always satisfied, and so a negative position in the bond account is offset by positive positions in the stocks.

It follows from Lemma 3 that the countries current accounts are nonzero. This is the first time we encounter a nonzero current account in this section. As the case we are considering here demonstrates, enough heterogeneity in hedging demands that is sufficient to give rise to trade in bonds for risk sharing purposes guarantees that the current account deviates from zero.

The analysis of the capital-gains adjusted current account is less transparent when the preference shifts depend on the output shocks. This is because the capital gains on the stocks in response to all shocks no longer have unique signs. Recall from our earlier discussions (Section 4.3) that the Home stock responds positively to an output shock in either country, positively to the preference shift towards the Home good ( $d\alpha_H > 0$ ), but negatively to the preference shift towards the Foreign good. The demand and supply effect reinforce each other for the case of the Home output shock (because  $\alpha_H$  loads positively on  $dw$ ), but they go in the opposite direction for the case of the Foreign output shock (because  $\beta_H$  loads positively on the Foreign output shock). The analogous argument holds for the Foreign stock.

## 5. Simulations

Section 4 studies particular examples that highlight the different channels through which asset prices affect the adjustment process. In those examples we have either incomplete markets with independent demand shocks or complete markets with correlated demand and supply shocks. In this section, we study a general case of our economy. Although our characterizations are in closed form, it is easier to present the results in terms of tables and plots.

The parameter values chosen are not meant to represent any particular country—they are just for expositional purposes—and so should not be viewed as a formal calibration of the model. Nevertheless, we chose them to be such that countries have positive cross-holdings, the portfolios exhibit a home bias, and one of the countries is running a current account deficit (conventionally defined). Table 3 lists the parameter values employed in this section.

Parameter	Value	Parameter	Value	Parameter	Value
$Y(0)$	1	$\alpha_H(0)$	0.6	$\sigma_Y$	0.01
$Y^*(0)$	1	$\alpha_F$	0.6	$\sigma_{Y^*}$	0.01
$\mu_Y$	0.02	$\beta_H(0)$	0.4	$\sigma_{\alpha_H}$	$(z_1, 0, z_2, 0)$
$\mu_{Y^*}$	0.02	$\beta_F$	0.4	$\sigma_{\beta_H}$	$(0, z_1, 0, y_2)$
$\rho$	0.05	$a$	0.5		

Table 3: Parameter choices.

where  $z_1 \in [0, 0.006]$ ,  $z_2 \in [0.002, 0.014]$ .<sup>10</sup> According to this parametrization, the countries' output processes are geometric Brownian motions (since both the drift and diffusion are constant). This is a specification most commonly assumed in the literature. The literature provides little guidance for the form of preference shifts, and so we simply assume that the parameters  $\sigma_{\alpha_H}$  and  $\sigma_{\beta_H}$  are constant. This creates a potential problem that  $\alpha_H(t)$  and  $\beta_H(t)$  may become negative, but we guard against this possibility in our simulations. The results are qualitatively the same if  $\alpha_H(t)$  and  $\beta_H(t)$  follow geometric Brownian motions with no drift. Furthermore, motivated by the estimated dynamics of demand shocks in Pavlova and Rigobon (2007), we posit that  $\alpha_H$  loads positively on the Home output shock  $w$  and  $\beta_H$  loads positively on the Foreign output shock  $w^*$ . We vary the relative sizes of the demand and supply shocks by varying the volatility of the demand shocks and keeping that of the supply shocks fixed.

We define the steady state as the path in which all shocks are equal to zero at all times. Figure 2 reports the steady-state values of the key variables in our model. Unless stated otherwise, all variables are for the Home country. We plot the number of shares of the Home stock held by the Home residents (panel (a)), the number of shares of the Foreign stock held by Home residents (panel (b)), the value of their bondholdings (panel (c)), the annualized trade balance and (conventional) current account measured as fractions of GDP (panels (d) and (e)), the capital-gains adjusted current account as a fraction of GDP (panel (f)), the interest rate (panel (g)), and the expected returns on Home and Foreign stocks (panels (h) and (i)). In all figures, the x-axis measures the instantaneous correlation between the demand and supply shocks (we vary it from 0 to 45 percent), and the y-axis the relative size of the demand and supply shocks, as captured by the ratio of their volatilities (we vary it from 0.05 to 1.85).<sup>11</sup> The points where the correlation is zero— $z_1 = 0$ —

<sup>10</sup>The ranges of  $\sigma_{\alpha_H}$  and  $\sigma_{\beta_H}$  are determined by choosing the relative variance of the supply and demand shocks to vary from 0.05 to 1.85, and the correlation between supply and demand shocks from 0 to 45 percent.

<sup>11</sup>The former is formally defined as  $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$ , where  $i_1 = (1, 0, 0, 0)$ , and the latter as  $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ . That is, the supply shock is represented by the Home output shock and the demand shock by a preference shift towards the Home good. We could have defined these quantities using the output shock at Foreign and the demand shift towards the Foreign good—the results below would be essentially the same.

correspond to our Example 3 (Section 4). Finally, the value of the relative variance of 0.05 implies that demand shocks are almost negligible and the model approaches Example 1 (although we cannot simulate Example 1 directly because portfolio holdings become indeterminate).

Our choice of parameters implies a home bias in portfolios, whereby Home holds more than 70 percent of the supply of Home shares, and between 25 to 50 percent of that of Foreign shares. When the correlations between demand and supply shocks are zero and their relative variance is small, this ratio is close to 70/30.<sup>12</sup> As in our Example 3, when demand shocks are uncorrelated, bond holdings are exactly zero. When the correlation increases, Home's bondholdings become negative.<sup>13</sup> Home demands more of both the Home and Foreign stock, ending up owning more shares of the Foreign stock than foreigners own abroad, and finances these stock purchases not by selling domestic shares (rebalancing), but by borrowing. The selling of the bond by Home implies a current account deficit, as shown in panel (e) and implied by Lemma 3. The current account deficit varies from zero up to 12 percent of GDP, while the trade balance and CGCA are exactly zero in steady state (panels (d) and (f)).<sup>14</sup> Finally, the last row of Figure 2 plots the interest rate and the expected return on Home and Foreign stocks. The interest rate and the expected returns are increasing with the degree of correlation and the size of the demand shocks. The equity premia on both stocks are positive but small.<sup>15</sup> Finally, note that our parametrization implies that the expected return on the Foreign stock is larger than the expected return on the Home stock.

## 5.1. Unconditional responses

In the simulation we generate 500 histories, each of 95 periods (we set the shocks to be zero in the first five periods), by randomly drawing all four shocks. Each period corresponds to one hundredth of a year.

We start by examining the serial correlation of the textbook current account and the serial correlation of the capital-gains adjusted current account. To provide an illustration, Figure 1 depicts

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<sup>12</sup>A home bias in portfolios is achieved by selecting appropriate variances of the demand shocks.

<sup>13</sup>It is important to highlight that whether bondholdings are positive or negative depends on the sign of the correlation between the demand shocks and the supply shocks. In our case, home supply shocks are positively correlated with the agents' demand for the Home good, and therefore, the Home stock is a good hedge against that correlation.

<sup>14</sup>The trade balance and the CGCA are zero because of the symmetry of the agents' preferences and their identically valued endowments.

<sup>15</sup>This should have been expected, given our assumptions on the agents' preferences. The purpose of this paper is not to match the equity premium, and we make a number of simplifying assumptions in order to achieve tractability. We discuss extensions of our model that incorporate alternative preferences in Section 6.

a variety of sample paths of the current accounts emerging from the Monte-Carlo simulations.

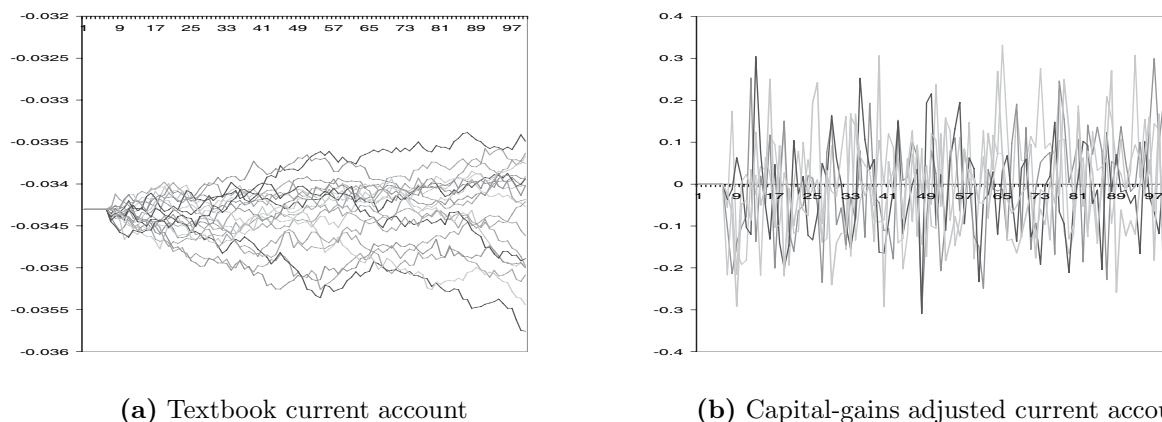


Figure 1: Simulated sample paths of the textbook current account ( $CA_H$ ) and the capital-gains adjusted current account ( $CGCA_H$ ) (in fractions of GDP). Each sample path corresponds to one simulated history.

The difference in the dynamic behavior of the two series is striking. The textbook current account is clearly highly serially correlated, while the capital-gains adjusted current account is not. The main reason why the latter does not exhibit significant serial correlation is the fact that much of its variation is explained by the *unexpected* capital gains, which are not serially correlated. The series in panel (b) looks much like a return on a financial asset: very volatile and largely unpredictable. Table 4 validates these observations and reports the serial correlations of each of the two variables. We estimate a simple AR(1) and average the coefficients across all simulations, for the entire parameter space.

	Serial Correlation			Standard Deviation
	Average	Max	Min	
$CA_H$	95.91%	97.09%	94.96%	0.53%
$CGCA_H$	-0.95%	-0.55%	-1.25%	0.19%

Table 4: The variability and the serial correlation of the textbook current account ( $CA_H$ ) and the capital-gains adjusted current account ( $CGCA_H$ ) (in fractions of GDP). The Average is computed as the simple average of the estimates across our parameter space. The variation of the estimates across the different parameters is measured as the standard deviation in column 4, and as the maximum and minimum in the second and third columns. All of the point estimates for the serial correlation of  $CA_H$  are significant at the 1% level, while all those of  $CGCA_H$  are not significant even at the 10% level (the average t-statistics is 0.01).

One can see that irrespectively of the parameters chosen for a simulation, the current account is highly serially correlated, while the CGCA is not. In terms of their volatilities, the CGCA has

a standard deviation of 1.16% while the current account of only 0.27% (these standard deviations have been annualized).<sup>16</sup> Finally, the average current account deficit is around 3% of GDP, while that of the capital-gains adjusted current account is about zero.

Recent empirical literature has produced an intriguing stylized fact: valuation changes (capital gains) have a stabilizing effect on the current account (Gourinchas and Rey (2007a)). We investigate whether our model generates similar implications. We have already noted that the unexpected capital gains drastically increase the variability of the current account, and so our focus here is primarily on the *expected* current account and the effects of net expected capital gains.

Recall our decomposition of the current account:

$$CA_H(t) = \left[ TB_H(t) + \underbrace{s_H^{S^*}(t)p^*(t)Y^*(t) - s_F^S(t)p(t)Y(t)}_{\text{Net Dividend Payments}} + \underbrace{s_H^B(t)B(t)r(t)}_{\text{Net Interest Payments}} \right] dt$$

and

$$CGCA_H(t) = \left[ CA_H(t) + \underbrace{s_H^{S^*}(t)ECG_{S^*}(t) - s_F^S(t)ECG_S(t)}_{\text{Net Expected Capital Gains}} \right] dt + \left[ \underbrace{s_H^{S^*}(t)S^*(t)\sigma_{S^*}(t) - s_F^S(t)S(t)\sigma_S(t)}_{\text{Net Unexpected Capital Gains}} \right] d\vec{w}(t)$$

In the current simulation we computed a variance decomposition of both measures of the current account by looking at the net contribution of their components. This is not a standard variance decomposition because we are not studying the contribution of independent shocks (as is typical of VAR variance decompositions). Instead, we are interested in highlighting the negative correlation of the variables comprising the current account. Therefore, we first compare the variance of the trade balance with the variance of the traditional the current account ( $CA_H$ ). If the net dividend and interest payments play a stabilizing role, then the variance of the current account should be smaller than the variance of the trade balance. We compute the ratio of the standard deviations to capture the extent of this stabilizing role. We then contrast the traditional and the *expected* capital-gains adjusted current account and examine the variance reduction that the net expected capital gains produce. Finally, we add *unexpected* capital gains. Table 5 presents the results of the variance decomposition.

Table 5 reveals that there is a sizeable stabilizing effect that the net dividend and interest payments and the expected capital gains provide. Together, they generate a reduction of 60 percent of the standard deviation of the trade balance. Especially notable is the contribution of the net

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<sup>16</sup>The orders of magnitude and their relative importance are consistent with the evidence presented in Kollmann (2006) and Lane and Shambaugh (2007).



Variable	Std. Dev.			Ratio
	Average	Max	Min	Average
$TB_H$	0.41%	0.54%	0.37%	1
$CA_H$	0.27%	0.42%	0.07%	0.69
$CA_H + \text{Net Expected Capital Gains}$	0.16%	0.21%	0.15%	0.40
$CGCA_H$	1.16%	1.55%	1.04%	2.83

Table 5: A variance decomposition.

expected capital gains, suggesting that there is a negative correlation between the traditional current account and the net expected capital gains. However, the unexpected capital gains, although slightly negatively correlated with the trade balance in our simulations (about -14%), increase the overall variance. This is because the unexpected capital gains themselves are extremely volatile.

## 5.2. Impulse responses

We now turn to investigating impulse responses of the economy to supply and demand shocks. More specifically, we examine the following shocks: a permanent shift in preferences toward the home good (a demand shock) and a permanent increase in productivity at Home (a supply shock). In our model we can also investigate the effects of a preference shift towards the Foreign good and a productivity shock at Foreign, but for brevity we omit that discussion. We emphasize that the shocks that we study are permanent.<sup>17</sup>

For each shock we present a sequence of figures that capture the impact responses of the real side of the economy, of the financial side of the economy and then highlight the resulting response of the current account of the Home country. To compute the impact responses, we simulate the model without shocks ( $d\vec{w}(t) = 0$ , all  $t$ )—the steady state—and then introduce one shock occurring at an arbitrarily chosen time  $t = 5$ . Then, for each pertinent variable, we subtract the steady-state series for this variable from the one with the shock, and report the resulting change at the time of the shock.

Figure 3 presents impact responses to a shock  $dw^\alpha$ . By construction, such a shock has no effect on the Home or Foreign output, and the only change in the the primitives of our model that takes place in response to the stock is a preference shift at Home towards the domestic good

<sup>17</sup>Some of the existing literature has focused on the effects of transitory shocks. Our focus on permanent shocks is motivated largely by the limitations of the model: while productivity shocks can be modeled as transitory, preference shifts have to be permanent (recall that, for tractability, we have assumed that  $\alpha_H$  and  $\beta_H$  are martingales). The analysis of transitory shocks is thus beyond the scope of this paper, and we leave it for future research.

( $\alpha_H$  increases). As we have discussed in Section 4, the resulting excess demand for the Home good pushes its price up and therefore improves Home’s terms of trade. This is why panel (a) of Figure 3 reports an increase in  $q$  for all parameter values. Output in both countries stays the same, but because of the movement in the terms of trade, the value of the output increases at Home and decreases at Foreign. Hence the increase in the stock market values at Home (panel (b)) and the decrease at Foreign (panel (c)). It is now easy to understand why the Home country suffers an net unexpected capital loss (panel (d)). Remember that our parameters imply positive steady state holdings of both stocks by both countries. The increase in the stock price at Home thus implies a capital gain for foreigners, while the decline in foreign stock prices implies a loss for Home agents. Both effects act in the same direction, against the Home residents. Notice that this is exactly the same intuition as we have gained from Example 3 (Section 4), except here it generalizes to the case where the supply and demand shocks are correlated and the countries trade in the bond market. The capital loss at Home induces the “planner” to reassign weights to the countries in its objective function—in favor of Foreign (panel (e)). This could be thought of as a wealth transfer from Home to Foreign, but this is not entirely accurate, as can be seen from equation (24):  $\lambda(t) = \frac{W_F(t)}{W_H(t)} \times \frac{\alpha_H(t)+\beta_H(t)}{\alpha_F+\beta_F}$ . Indeed, a part of a change in the relative weight  $\lambda$  coincides with a change in the wealth distribution (a pure wealth transfer), but the remaining part is due to imperfect demand risk sharing: under complete markets,  $\lambda$  does not move in response to any shock, but it does respond to shocks under incomplete markets because the shocks cannot be perfectly hedged. For our parametrization, the wealth transfer effect dominates, but this need not be the case in general.

As shown in panel (f), the trade balance improves for all parameter values. This improvement of the trade balance is simply the flip side of the deterioration of the NFA position due to the unexpected net capital loss (Lemma 4). The current NFA position is no longer sufficient to finance the existing trade imbalance, and so Home has to cut it down. For all parameters, except the ones that give rise to our Example 3 (i.e., no correlation between the demand and supply shocks), Home purchases the bond. The purchases decrease the deficit in the current account (Lemma 3)—an improvement in the current account is evident in panel (h). Finally, note that the movements in the net unexpected capital gains dominate the dynamics of the NFA position. The response of the capital-gains adjusted current account mimics that of the net unexpected capital gains—CGCA is negative for all parameter values (panel (i)). Note that the direction of the response of the conventional current account is the opposite (panel (h)).

In Figure 4 we provide the impulse responses to a shock to the Home output  $dw$ . Remember that the case in which the instantaneous correlation between the demand and supply shocks is zero corresponds to our Example 3. However, when this correlation is positive, as we assume through our choice of parameters here, a positive Home output shock also causes an increase in  $\alpha_H$ . As mentioned earlier, a positive shock to the Home output  $Y_H$  increases the value of its stock market (panel (b)) but at the same time deteriorates its terms of trade because the Home good becomes less scarce (panel (b)). The resulting improvement in Foreign’s terms of trade benefits its stock market (panel (c)). These responses are somewhat mitigated by the increase in  $\alpha_H$  that occurs contemporaneously. As we have seen in the previous figure, a positive shock to  $\alpha_H$  improves Home’s terms of trade, boosts the stock market at Home, while decreasing the stock market abroad. We can see that this confounding effect becomes more pronounced when the demand shocks have a high loading on the supply shock  $dw$ —i.e., when both the correlation between the demand and supply shocks and the relative variance of the demand and supply shocks is high.

Furthermore, we find that Home enjoys a net unexpected capital gain in response to the supply shock (panel (d)). The intuition of this result is complicated because it depends on how prices and portfolio choices interact. A “partial” intuition is as follows. When the correlation between the demand and supply shocks is zero (as in Example 3) the net unexpected capital gain have to be zero as well. The main reason for this result is that for our parameter values  $\beta_H(t) = \lambda(t)\beta_F$  in the steady state, and hence our condition A1 holds with equality. The net unexpected capital gains then have to be zero for all remaining parameter values, as becomes evident from Table 1. When we move away from the case of zero correlation, home agents demand more of both Home and Foreign stocks and purchase them from the foreigners (see Figure 2). For the exact same movement in the stock prices, this implies a net unexpected capital gain. A capital gain at Home causes a wealth transfer from Foreign to Home, or, more accurately, the weight  $\lambda$  of Foreign in the “planner’s” problem falls, increasing the allocation to the Home country (panel (e)). Since the NFA position of Home has improved, it can afford to deteriorate its trade balance, which is what we see in panel (f). Part of the financing of the increased purchases of the two stocks comes from selling the bond (panel (g)), which according to our Lemma 3 deteriorates the conventional current account (panel (h)). The capital-gains adjusted current account, however, moves in the opposite direction—the same direction as the net unexpected capital gains, which are positive here (panel (i)).

## 6. Discussion

Some of the recent literature has drawn attention to the relevance of the quality of international assets for the discussion of global imbalances (see Caballero, Farhi, and Gourinchas (2006), Dooley, Folkerts-Landau, and Garber (2004), and Blanchard, Giavazzi, and Sa (2005) for the link to global imbalances, and Kouri (1982) for an earlier discussion). Because in our model the entire output of each country is capitalizable and there are no restrictions on capital flows, financial assets do not vary in their quality. But we believe that differences in asset quality is an important feature of international capital markets, and therefore it would be interesting to extend our framework to include this element into the analysis.

Extending the framework beyond log-linear preferences may also prove fruitful. This would introduce some of the intertemporal hedging motives that have been shut down in our model. Moving away from the log-linear specification, however, has the drawback that the model loses its tractability. For instance, for the case of CES preferences, it is not possible to obtain closed-form characterizations for portfolios and asset prices.<sup>18</sup> There are three ways in which one can tackle such a model. First, one can attempt to solve the model numerically. To our knowledge, this has been done only for the complete-markets case (Gourinchas and Rey (2006))—an extension to the incomplete-markets case is a daunting task. Second, one can follow, for example, Devereux and Sutherland (2006) and Tille and van Wincoop (2007) and approximate around a deterministic steady state. Finally, one can recognize that log-linear preferences are a special case of CES preferences and build on our model to find an approximate solution for the CES case. To do so, one can perturb the equilibrium in our economy by expanding around the unitary elasticity of substitution, for which the solutions are analytical.<sup>19</sup> The advantage of this approach is that the approximation is done around a stochastic equilibrium as opposed to a deterministic steady state.

## 7. Conclusion

In his Harms Lecture at the Kiel Institute, Obstfeld (2004) stresses that “recent changes in the functioning of international capital markets require a new view of external adjustment” and moreover, that any notion of “external balance adjustment cannot be defined without reference to the

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<sup>18</sup>The analysis of the NFA position is analytically tractable only for the case of complete markets (see ).

<sup>19</sup>This idea is closely related to the works of Judd (1998) and Kogan and Uppal (2003) who develop applications of perturbation methods to solving problems in economics and finance.

structure of national portfolios.” In this paper, we take a step in that direction. We develop an open economy model with endogenous portfolio decisions, in which we investigate the interaction between capital markets and the external adjustment process.

From the methodological point of view, our contribution is to construct a framework that is rich enough to include multiple risky assets, incomplete markets, and supply- and demand-side uncertainty, while at the same time simple enough to allow for closed-form characterizations of asset prices, net foreign asset positions, and equity portfolios. It is within this framework that we are able to establish the interconnections between the real side of the economy represented by the trade balance, current account, and consumption allocations and the financial side such as portfolio holdings, stock prices, and valuation changes.

From the policy point of view, one surprising result in our paper is that even though valuation effects play an important role in the adjustment process, there is a tight link between the trade balance of good and services—the traditional and the preferred policy target—and the new measure of external sustainability based on the market value of net foreign assets—that is extremely difficult to measure and target. On the other hand, this relationship does not exist between the current account and the valuation-effects adjusted measures. Hence, the discussion regarding the disconnect between the new measures of sustainability and the classical ones is far from over.

Of course, the implications that we highlight have been derived in the context of our model, and as any model, it is a highly simplified depiction of reality. Future research must go beyond our stylized framework and establish tighter links with the data.

## Appendix

**Proof of Proposition 1.** In this proof, we closely follow He and Pearson (1991). Their analysis is presented in the context of a single-good economy, but this does not present a difficulty for us because (in the main text) we have reduced our problem to a representation that is equivalent to a familiar single-good one. In particular, the first-order conditions for the consumer problem (11)–(12) have the familiar form

$$e^{-\rho t} \frac{\alpha_i(t) + \beta_i(t)}{\bar{C}_i(t)} = y_i \xi_{\nu_i}(t), \quad i \in \{H, F\}, \quad (\text{A.1})$$

where the Lagrange multiplier  $y_i$  is such that the budget constraint evaluated at the optimal consumption expenditure,  $\bar{C}$ , is satisfied with equality:

$$E \left[ \int_0^T \xi_{\nu}(t) \bar{C}_H(t) dt \right] = W_H(0), \quad i \in \{H, F\}.$$

It follows that, by no-arbitrage, the time- $t$  wealth of a consumer is given by

$$W_i(t) = E_t \left[ \int_t^T \frac{1}{\xi_{\nu_i}(t)} \xi_{\nu_i}(s) \bar{C}_i(s) ds \right], \quad i \in \{H, F\},$$

and hence, making use of (A.1) and the assumption that  $\alpha_i$  and  $\beta_i$  are martingales, we have

$$W_i(t) = \frac{\alpha_i(t) + \beta_i(t)}{y_i \xi_{\nu_i}(t)} \frac{e^{-\rho t} - e^{-\rho T}}{\rho}, \quad i \in \{H, F\}. \quad (\text{A.2})$$

Of course, for the case of the Foreign country, the arguments  $\alpha_F$  and  $\beta_F$  are constant over time.

To find optimal portfolios, we apply Itô's lemma to (A.2) and match the corresponding diffusion term to that in the dynamic budget constraint (10). This operation yields

$$x_i^\top(t) \sigma(t) = \frac{\sigma_{\alpha_i}(t) + \sigma_{\beta_i}(t)}{\alpha_i(t) + \beta_i(t)} + (m(t) + \nu_i(t))^\top, \quad (\text{A.3})$$

where we have used equation (14). Recall that in incomplete markets the matrix  $\sigma$  is not a square matrix, and hence the above system of equations contains 4 equations (dimensionality of the vector of Brownian motions) in 2 unknowns (the number of stocks). It has a solution if and only if its right-hand side lies in  $\text{Span}(\sigma)$ . This entails a restriction

$$(I_{4 \times 4} - \sigma(t)^\top (\sigma(t) \sigma(t)^\top)^{-1} \sigma(t)) \frac{\sigma_{\alpha_i}(t)^\top + \sigma_{\beta_i}(t)^\top}{\alpha_i(t) + \beta_i(t)} + \nu_i(t) = 0, \quad (\text{A.4})$$

where we have applied the projection operator  $I_{4 \times 4} - \sigma(t)^\top (\sigma(t) \sigma(t)^\top)^{-1} \sigma(t)$ . Equation (17) then follows immediately. Note that, for the case of Foreign, equation (A.4) simplifies to yield  $\nu_F(t) = \mathbf{0}$  because  $\sigma_{\alpha_F}(t)$  and  $\sigma_{\beta_F}(t)$  are both equal to zero.

The optimal portfolios are obtained from (A.3) via simple algebraic manipulations that, in particular, make use of the property that  $\sigma(t) \nu_i(t) = 0$ . ■

**Proof of Lemma 1.** We use the construct of the representative agent to value stocks in the economy. The representative agent's utility evaluated at the aggregate output is given by

$$u(Y(t), Y^*(t); \lambda(t)) = \max_{\substack{C_H(t) + C_F(t) = Y(t), \\ C_H^*(t) + C_F^*(t) = Y^*(t)}} u_H(C_H(t), C_H^*(t)) + \lambda(t)u_F(C_F(t), C_F^*(t)).$$

It follows from this definition that the marginal utilities of the representative agent and the individual agents, evaluated at the optimum, are related as

$$\nabla u(Y(t), Y^*(t); \lambda(t)) = \nabla u_H(\bar{C}_H(t), \bar{C}_H^*(t)) = \lambda(t)\nabla u_F(\bar{C}_F(t), \bar{C}_F^*(t)),$$

where the symbol  $\nabla$  is used to denote the gradient. From the first-order conditions of the Home consumer,

$$\nabla u_H(\bar{C}_H(t), \bar{C}_H^*(t)) = \left( y_H p(t) \xi_{\nu_H}(t), y_H p^*(t) \xi_{\nu_H}(t) \right).$$

To derive this we used the fact that  $\nabla u_H(\bar{C}_H(t), \bar{C}_H^*(t)) = (\alpha_H(t)/\bar{C}_H(t), \beta_H(t)/\bar{C}_H^*(t))$  combined with (8) and (A.1). Substituting the sharing rules of the representative agent (18), we can then derive the personalized state price density of the Home consumer and hence that of the representative agent:

$$\xi_{\nu_H}(t) = e^{-\rho t} \frac{p(0)}{p(t)} \frac{\bar{C}_H(0)}{\bar{C}_H(t)} = e^{-\rho t} \frac{p(0)Y(0)}{p(t)Y(t)} \frac{\alpha_H(t) + \lambda(t)\beta_F}{\alpha_H(0) + \lambda(0)\beta_F}. \quad (\text{A.5})$$

This state price density can be used to price assets by no-arbitrage:

$$S(t) = \frac{1}{\xi_{\nu_H}(t)} E_t \left[ \int_t^T \xi_{\nu_H}(s) p(s) Y(s) ds \right], \quad S^*(t) = \frac{1}{\xi_{\nu_H}(t)} E_t \left[ \int_t^T \xi_{\nu_H}(s) p^*(s) Y^*(s) ds \right].$$

Hence, the price of the Home stock is

$$\begin{aligned} S(t) &= \frac{e^{\rho t} p(t) Y(t)}{\alpha_H(t) + \lambda(t)\beta_F} E_t \left[ \int_t^T e^{-\rho s} (\alpha_H(s) + \lambda(s)\beta_F) ds \right] \\ &= \frac{1 - e^{-\rho(T-t)}}{\rho} p(t) Y(t) + \frac{e^{\rho t} \beta_F p(t) Y(t)}{\alpha_H(t) + \lambda(t)\beta_F} E_t \left[ \int_t^T e^{-\rho s} (\lambda(s) - \lambda(t)) ds \right], \end{aligned} \quad (\text{A.6})$$

where we used the fact that  $\alpha_H$  is a martingale (i.e.,  $E_t[\alpha_H(s)] = \alpha_H(t)$ ). Analogously, using the fact that  $\beta_H$  is a martingale, we find the price of the Foreign stock to be

$$\begin{aligned} S^*(t) &= \frac{e^{\rho t} p^*(t) Y^*(t)}{\beta_H(t) + \lambda(t)\alpha_F} E_t \left[ \int_t^T e^{-\rho s} (\beta_H(s) + \lambda(s)\alpha_F) ds \right] \\ &= \frac{1 - e^{-\rho(T-t)}}{\rho} p^*(t) Y^*(t) + \frac{e^{\rho t} \alpha_F p^*(t) Y^*(t)}{\beta_H(t) + \lambda(t)\alpha_F} E_t \left[ \int_t^T e^{-\rho s} (\lambda(s) - \lambda(t)) ds \right]. \end{aligned} \quad (\text{A.7})$$

There are two ways to proceed in evaluating the above conditional expectations. The first is to assume that  $\lambda$  is a martingale (and hence  $E_t[\lambda(s) - \lambda(t)] = 0$ ) and then verify that it is indeed the case in equilibrium. From Proposition 2, however, we can only conclude that  $\lambda$  is a local martingale. In all special cases that we consider in Section 4, it is easy to verify that  $\lambda$  is also a true martingale

under some additional mild regularity conditions imposed on the preference shifts.<sup>20</sup> However, for the general case it is not immediate how to show it.

An alternative approach is to use the following, less direct, argument based on market clearing. In particular, from (A.1)–(A.2), we have

$$W_H(t) + W_F(t) = (\bar{C}_H(t) + \bar{C}_F(t)) \frac{1 - e^{-\rho(T-t)}}{\rho} = p(t) Y(t) \frac{1 - e^{-\rho(T-t)}}{\rho} + p^*(t) Y^*(t) \frac{1 - e^{-\rho(T-t)}}{\rho},$$

where in the last equality we used the fact that the total consumption expenditure at time  $t$  equals  $p(t)Y(t) + p^*(t)Y^*(t)$ . On the other hand, from stock market clearing, we have

$$W_H(t) + W_F(t) = S(t) + S^*(t).$$

Combining the resulting restriction that

$$S(t) + S^*(t) = p(t) Y(t) \frac{1 - e^{-\rho(T-t)}}{\rho} + p^*(t) Y^*(t) \frac{1 - e^{-\rho(T-t)}}{\rho}$$

with (A.6)–(A.7), we conclude that

$$S(t) = p(t) Y(t) \frac{1 - e^{-\rho(T-t)}}{\rho} \quad \text{and} \quad S^*(t) = p^*(t) Y^*(t) \frac{1 - e^{-\rho(T-t)}}{\rho}.$$

This establishes (21)–(22).

To derive (23), we combine (A.1)–(A.2) with (18)–(19) and use the representation of the stock prices  $S$  and  $S^*$  derived in this lemma. ■

**Proof of Lemma 4.** Equation (33) follows from  $NFA_H(t) = W_H(t) - S(t)$  and Lemma 1. To derive (32), we use the definition of the trade balance,  $TB_H(t) = p(t)(Y(t) - C_H(t)) - p^*(t)Y^*(t)$ , substitute the equilibrium expressions for consumption and the terms of trade, (18) and (20), and simplify. ■

Before we proceed to the rest of the proofs, we need to define several auxiliary vectors to be used throughout the remainder of this appendix. Let

$$i_1 \equiv (1, 0, 0, 0), \quad i_2 \equiv (0, 1, 0, 0), \quad \text{and} \tag{A.8}$$

$$A(t) \equiv \frac{\sigma_{\alpha_H}(t) - \beta_F \lambda(t) \nu(t)^\top}{\alpha_H(t) + \lambda(t) \beta_F} - \frac{\sigma_{\beta_H}(t) - \alpha_F \lambda(t) \nu(t)^\top}{\beta_H(t) + \lambda(t) \alpha_F} - \sigma_Y(t) i_1 + \sigma_{Y^*}(t) i_2. \tag{A.9}$$

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<sup>20</sup>The only special case that requires these additional assumptions is that presented in Section 3.3. In particular, one needs to bound the preference shifts in such a way that the expression in (37) satisfies  $E \left[ e^{\frac{1}{2} \int_0^T \nu(t)^\top \nu(t) dt} \right] < \infty$ . This condition is known as the Novikov condition.



**Proof of Proposition 2.** By substituting (A.2) into (24) we derive

$$\lambda(t) = \frac{y_H \xi_{\nu_H}(t)}{y_F \xi_{\nu_F}(t)}.$$

Applying Itô's lemma and using the representation of the countries' state price densities from (14), we have

$$d\lambda(t) = -\lambda(t)m(t)^\top \nu_H(t)dt - \lambda(t)\nu_H(t)d\vec{w}(t), \quad (\text{A.10})$$

where we have substituted the finding that  $\nu_F(t) = \mathbf{0}$  established in Proposition 1. To show that the drift term in (A.10) is equal to zero, we use the definition of  $m$  from (13) and the restriction that  $\sigma(t)\nu_H(t) = 0$ .

To determine  $\lambda(0)$ , note from Lemma 1 that the initial financial wealth of, say, the Home country is given by (23) evaluated at  $t = 0$ . On the other hand,  $W_H(0) = S(0)$  because the initial portfolio of Home consists of one share of the Home stock. This allows us to pin down  $\lambda(0)$ . It is easy to show that  $\lambda(0) = \beta_H(0)/\beta_F$ .

We now report the volatility matrix of stock returns.

$$\sigma(t) = \begin{bmatrix} \frac{1-a}{aq(t)+1-a} A(t) + \sigma_Y(t) i_1 \\ -\frac{aq(t)}{aq(t)+1-a} A(t) + \sigma_{Y^*}(t) i_2 \end{bmatrix},$$

where  $A(t)$ ,  $i_1$ , and  $i_2$  are defined in (A.8)–(A.9). This volatility matrix is obtained by applying Itô's lemma to the closed-form expressions for the stock prices (21)–(22).

The market price of risk process  $m$  can be derived from the dynamics of  $\xi_H$  in (14). Using the identity  $\xi_H(t) = ap(t)\xi_H(t) + (1-a)p^*(t)\xi_H(t)$  and equations (20) and (A.5), we derive

$$\xi_H(t) = ae^{-\rho t}p(0)\frac{\alpha_H(t) + \lambda(t)\beta_F}{\alpha_H(0) + \lambda(0)\beta_F} \left( \frac{Y(0)}{Y(t)} \right) + (1-a)e^{-\rho t}p(0)\frac{\beta_H(t) + \lambda(t)\alpha_F}{\alpha_H(0) + \lambda(0)\beta_F} \left( \frac{Y(0)}{Y^*(t)} \right). \quad (\text{A.11})$$

Applying Itô's lemma and identifying the diffusion term with that in the representation of  $\xi_H$  in (14), we obtain

$$\begin{aligned} m(t) = & -\frac{p(0)e^{-\rho t}}{\xi_H(t)} \frac{Y(0)}{\alpha_H(0) + \lambda(0)\beta_F} \left[ a \frac{\sigma_{\alpha_H}(t) - \beta_F\lambda(t)\nu_H(t)}{Y(t)} - a \frac{\alpha_H(t) + \beta_F\lambda(t)}{Y(t)} \sigma_Y(t) i_1 \right. \\ & \left. + (1-a) \frac{\sigma_{\beta_H}(t) - \alpha_F\lambda(t)\nu_H(t)}{Y^*(t)} - (1-a) \frac{\beta_H(t) + \alpha_F\lambda(t)}{Y^*(t)} \sigma_{Y^*}(t) i_2 \right] - \nu_H(t). \end{aligned}$$

This completes the proof of the proposition. ■

**Proof of Proposition 3.** We first report the interest rate  $r$  and the stocks' expected returns  $\mu_S$  and  $\mu_{S^*}$  and then explain how we derived these expressions.

$$r(t) = \rho + \frac{aq(t)}{aq(t) + 1 - a} \left( \mu_Y(t) - \sigma_Y(t)^2 + \frac{\sigma_Y(t)i_1(\sigma_{\alpha_H}(t)^\top - \lambda(t)\nu(t)\beta_F)}{\alpha_H(t) + \lambda\beta_F(t)} \right) \\ + \frac{1 - a}{aq(t) + 1 - a} \left( \mu_{Y^*}(t) - \sigma_{Y^*}(t)^2 + \frac{\sigma_{Y^*}(t)i_2(\sigma_{\beta_H}(t)^\top - \lambda(t)\nu(t)\alpha_F)}{\beta_H(t) + \lambda\alpha_F} \right) \quad (\text{A.12})$$

$$\mu_S(t) = \rho + \mu_Y(t) + \frac{1 - a}{aq(t) + 1 - a} \left( \mu_q(t) - \frac{aq(t)}{aq(t) + 1 - a} \|A(t)\|^2 + \sigma_Y(t)A(t)i_1^\top \right), \quad (\text{A.13})$$

$$\mu_{S^*}(t) = \rho + \mu_{Y^*}(t) + \frac{aq(t)}{aq(t) + 1 - a} \left( -\mu_q(t) + \frac{aq(t)}{aq(t) + 1 - a} \|A(t)\|^2 - \sigma_{Y^*}(t)A(t)i_2^\top \right), \quad (\text{A.14})$$

where  $\mu_q$  is the expected improvement in the terms of trade, given by

$$\mu_q(t) = \mu_{Y^*}(t) - \mu_Y(t) + \frac{1}{2} \|A(t)\|^2 - \frac{1}{2} \frac{\|\sigma_{\alpha_H}(t)^\top - \lambda(t)\nu(t)\beta_F\|^2}{(\alpha_H(t) + \lambda\beta_F(t))^2} \\ + \frac{1}{2} \frac{\|\sigma_{\beta_H}(t)^\top - \lambda(t)\nu(t)\alpha_F\|^2}{(\beta_H(t) + \lambda\alpha_F(t))^2} + \frac{1}{2} \sigma_Y(t)^2 - \frac{1}{2} \sigma_{Y^*}(t)^2,$$

and where  $A(t)$ ,  $i_1$ , and  $i_2$  are defined in (A.8)–(A.9).

The interest rate  $r$  in (A.12) is equal to the drift term from the Itô expansion of the equilibrium state price density reported in (A.11). The formulas in (A.13)–(A.14) are obtained by applying Itô's lemma to the closed-form expressions for the stock prices (21)–(22) and the terms of trade (20), and then using the definitions of  $\mu_S$  and  $\mu_{S^*}$  from (4)–(5). ■

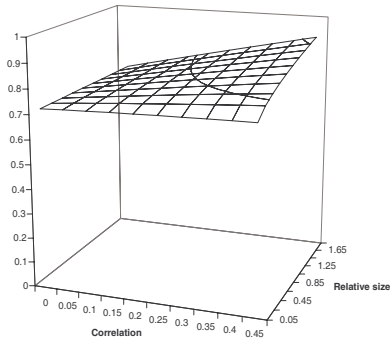
**Derivations for Section 4.** All derivations for the special cases examined in Section 4 are tedious but straightforward. Perhaps the easiest way to obtain the formulas and signs reported in that section is to use *Mathematica* to simplify the expressions in Propositions 1–3 and manipulate them in *Mathematica* to verify the desired properties. Our programs are available upon request.

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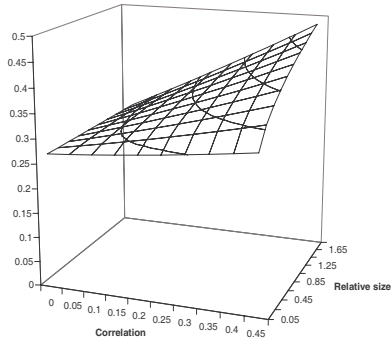
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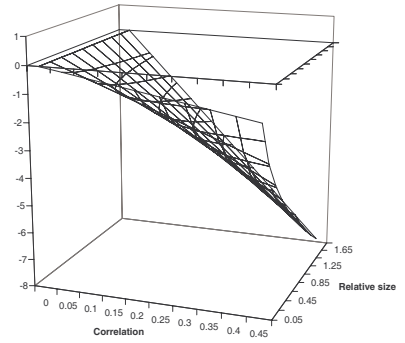
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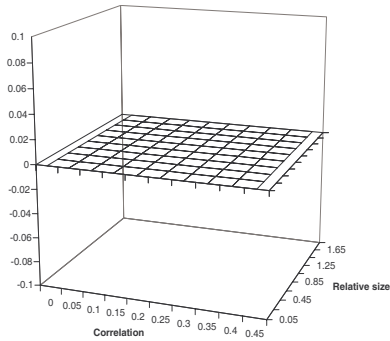
(a) Home's holdings of Home stock  $s_H^H$



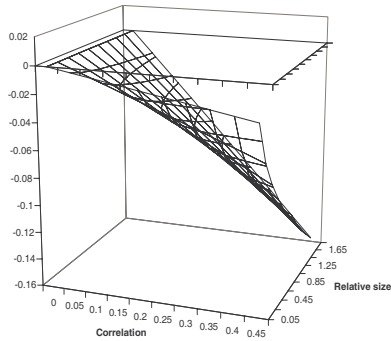
(b) Home's holdings of Foreign stock  $s_H^F$



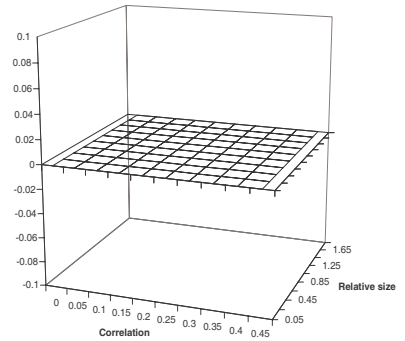
(c) Home's bondholdings (value)  $s_H^B B$



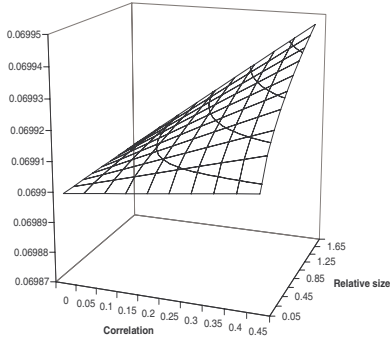
(d) Trade balance  $TB_H$



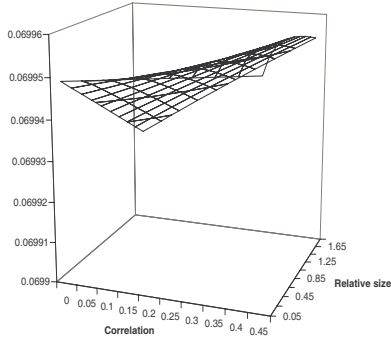
(e) Current account  $CA_H$



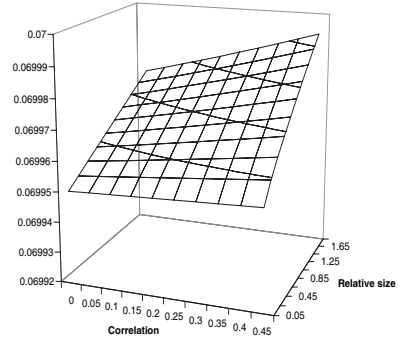
(f) Capital-gains adjusted current account  $CGCA_H$



(g) Interest rate  $r$

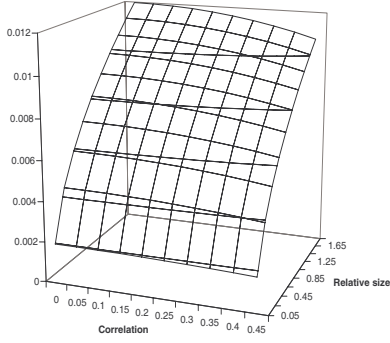


(h) Expected return on the Home stock  $\mu_S$

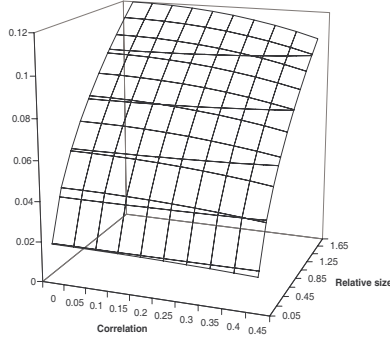


(i) Expected return on the Home stock  $\mu_{S^*}$

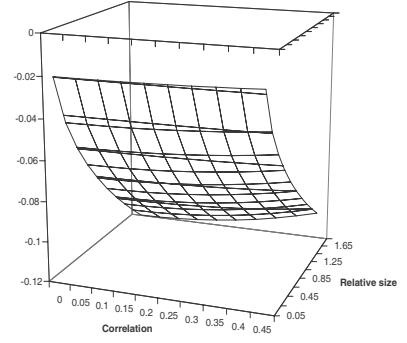
Figure 2: Steady state values. All underlying shocks,  $dw$ ,  $dw^*$ ,  $dw^\alpha$ ,  $dw^\beta$ , are set to be zero at all times. The horizontal axes measure the instantaneous correlation between the demand and supply shocks  $\sigma_Y i_1 \sigma_{\alpha H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha H}\|^2})$ , where  $i_1 = (1, 0, 0, 0)$ , and the ratio of the volatilities of the demand and supply shocks  $\sqrt{\|\sigma_{\alpha H}\|^2} / \sigma_Y$ .



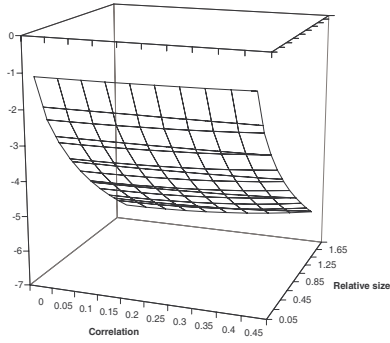
(a) Terms of trade  $q$



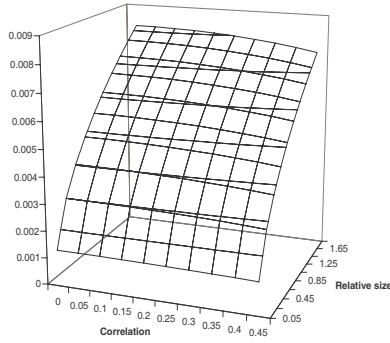
(b) Home stock  $S$



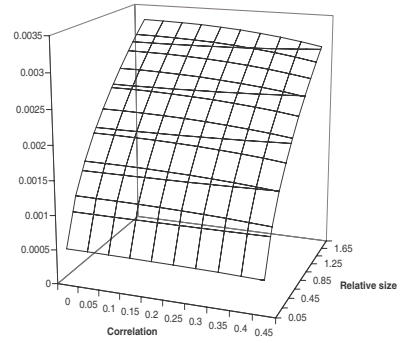
(c) Foreign stock  $S^*$



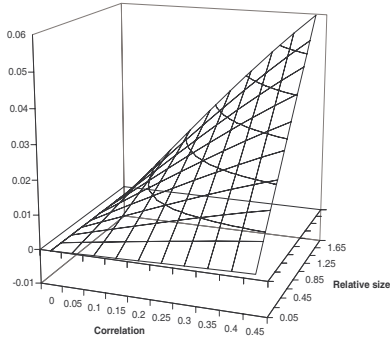
(d) Net unexpected capital gains of Home



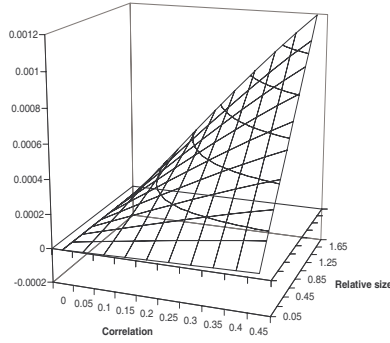
(e) Weight of Foreign  $\lambda$



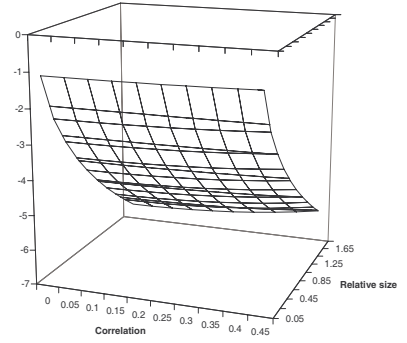
(f) Trade balance  $TB_H$



(g) Home's bondholdings (value)  
 $s_H^B B$

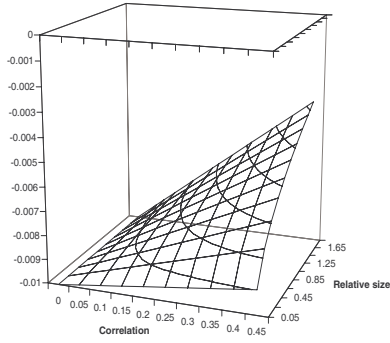


(h) Current account  $CA_H$

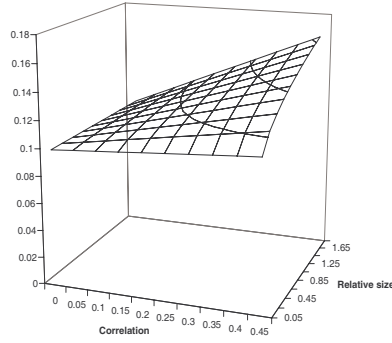


(i) Capital-gains adjusted current account  $CGCA_H$

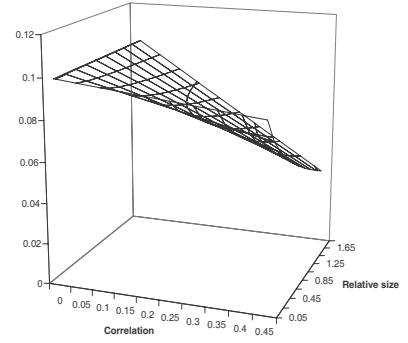
Figure 3: Impact responses to a shock  $dw^\alpha$  (a shock to the preference-shifter  $\alpha_H$ ). The horizontal axes measure the instantaneous correlation between the demand and supply shocks  $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$ , where  $i_1 = (1, 0, 0, 0)$ , and the ratio of the volatilities of the demand and supply shocks  $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ .



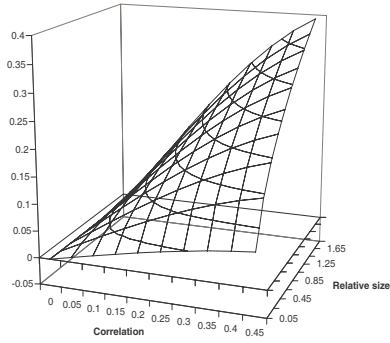
(a) Terms of trade  $q$



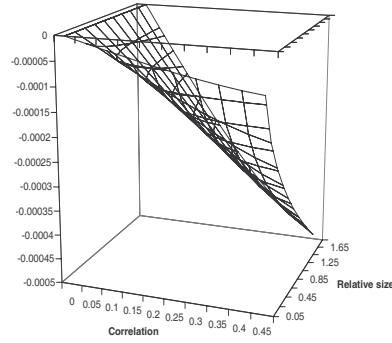
(b) Home stock  $S$



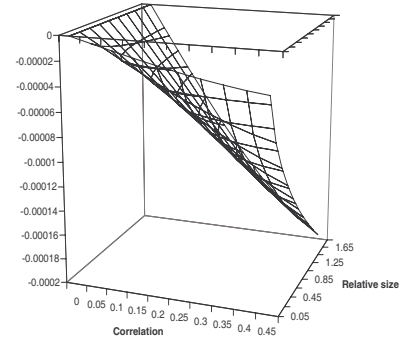
(c) Foreign stock  $S^*$



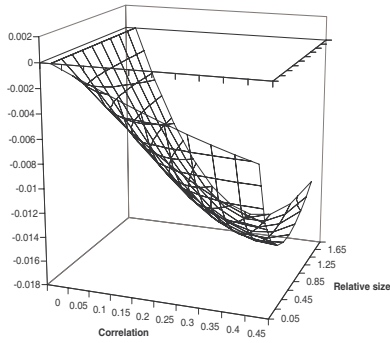
(d) Net unexpected capital gains of Home



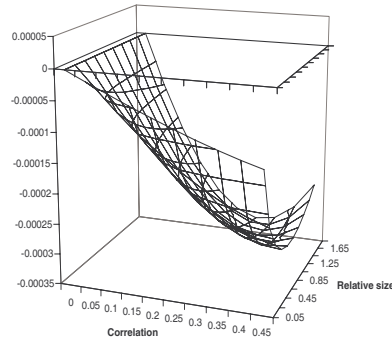
(e) Weight of Foreign  $\lambda$



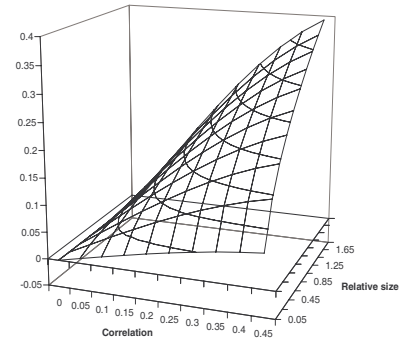
(f) Trade balance  $TB_H$



(g) Home's bondholdings (value)  
 $s_H^B B$



(h) Current account  $CA_H$



(i) Capital-gains adjusted current account  $CGCA_H$

Figure 4: Impact responses to an output shock at Home  $dw$ . The horizontal axes measure the instantaneous correlation between the demand and supply shocks  $\sigma_Y i_1 \sigma_{\alpha_H}^T / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$ , where  $i_1 = (1, 0, 0, 0)$ , and the ratio of the volatilities of the demand and supply shocks  $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ .