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TACIT COLLUSION IN THE PRESENCE OF CYCLICAL DEMAND AND ENDOGENOUS CAPACITY LEVELS

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ABSTRACT

We analyze tacit collusion in an industry characterized by cyclical demand and long-run scale decisions; firms face deterministic demand cycles and choose capacity levels prior to competing in prices. Our focus is on the nature of prices. We find that two types of price wars may exist. In one, collusion can involve periods of mixed strategy price wars. In the other, consistent with the Rotemberg and Saloner (1986) definition of price wars, we show that collusive prices can also become countercyclical. We also establish pricing patterns with respect to the relative prices in booms and recessions. If the marginal cost of capacity is high enough, holding current demand constant, prices in the boom will be generally lower than the prices in the recession; this reverses the results of Haltiwanger and Harrington (1991). In contrast, if the marginal cost of capacity is low enough, then prices in the boom will be generally higher than the prices in the recession. For costs in an intermediate range, numerical examples are calculated to show specific pricing patterns.

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1 Introduction

Inferring collusion is difficult because there rarely is a "smoking gun." Instead, empirical research has focused on dynamic pricing patterns predicted by theories of collusion that are inconsistent with static models of competition. One such strand of literature tests whether observed equilibrium prices are consistent with the predicted collusive prices when firms face cyclical demand. This literature leverages the result that when firms face demand cycles, conditional on current demand, prices will be higher if demand is expected to rise in the future, compared to if it is expected to fall.[1] We show that when firms face endogenous capacity constraints, these predicted pricing patterns can change; therefore, ignoring capacity constraints may lead us to conclude collusion does not exist when, in fact, it exists.

Collusion when firms face capacity constraints, has recently been the focus of a number of antitrust cases in both the US and Europe. The US Department of Justice recently launched an investigation into capacity collusion in the DRAM market. Mergers increasing the ability of firms to coordinate have also concerned the European Commission. The Commission blocked the Airtours and First Choice Holidays merger in the package holiday travel market partly because of concerns about an increased coordination in capacities; in this industry capacity levels are chosen well in advance of consumer bookings. Coordinating on capacity was also the Commission's initial objection to the UPM-Kymmene/Haindl newsprint merger.[2]

Antitrust policy-makers have noted the difficulty of uncovering collusion in markets with strict capacity constraints. Coordinating on capacities can lead to lower capacity levels and outputs closer to these capacity levels. Therefore, if one observed the market, *taking capacity levels as given*, they may conclude that collusion does not exist, presuming instead that firms are simply capacity constrained. This added difficulty increases the importance of understanding how prices behave when firms collude on both capacities and prices.

In this paper, we analyze the collusive behavior of firms where changing the scale of operation takes a significant period of time and market demand fluctuates cyclically. Our goal is to establish testable implications with respect to pricing behavior along the demand cycle. Firms face

[1] See, for example, Borenstein and Shepard (1996) and Rosenbaum and Sukharomana (2001).

[2] While the Commission ultimately concluded that coordinating on capacities was too difficult, our analysis suggests that this conclusion may have been unwarranted.

deterministic demand cycles and choose capacity levels before competing in prices. This represents industries where the cycles are frequent, or capacity is difficult to alter; for example, the holiday travel market cited above, electricity markets where demand cycles each day and capacity changes can take over 18 months, or the gasoline refining market where demand exhibits annual fluctuations while refining capacity has remained fairly constant.[3]

The inclusion of the scale decision as a formal choice variable in the dynamic game can drastically change the collusive pricing patterns and the effectiveness of collusion. Capacity constraints have two countervailing effects on firms' ability to collude. First, low capacity levels may reduce the incentive for a firm to deviate from collusion by limiting the immediate gain from defection. That is, if a firm's capacity is less than the market demand at the collusive price, the firm cannot supply the entire market after a low price defection. On the other hand, low capacity levels can decrease the severity of the credible punishment after a deviation, decreasing the incentive to collude. We find that whether low or high capacities best facilitate collusion depends on how expensive it is to install; hence, the price of capacity is a key determinant of collusive behavior. Furthermore, we find that endogenizing capacity constraints can have a significant impact on the effectiveness of collusion, since firms are able to choose capacity levels to increase the collusive profits.

Our primary analysis is concerned with collusive pricing patterns. Consistent with the existing literature, we identify two types of price wars. Rotemberg and Saloner (1986) show that when each period's demand depends on an independent identically distributed (iid) shock and there are no capacity constraints, prices may be inversely correlated with the level of demand. Because the gain from cheating is greater when demand is higher (and the punishment is independent of the current level of demand), prices may fall when demand increases in order to counteract the incentive to cheat; we refer to these counter-cyclical prices as *mild* price wars.[4]

A second type of price war also exists. Consistent with Staiger and Wolak (1992), we find under certain capital prices and discount levels, firms will switch between cooperative and non-cooperative mixed-pricing behavior; we refer to these periods as *severe* price wars. As the marginal cost of capacity increases, severe price wars are only possible in periods of higher and higher demand. Unlike the mild price wars, severe price wars correspond to periods where one firm actually undercuts the other in equilibrium.[5]

[3]Other industries such as cement, railroad, steel, heavy electrical equipment and petroleum also loosely fit this abstract description. Scherer and Ross (1990) describe these industries as all having relatively high concentration, high fixed costs, relatively low marginal costs and non-stationary demand patterns. The high fixed costs come from the requirement of long-term pre-commitment to production technologies and/or resource investment.

[4]While this does not represent a price war in the sense that firms revert to non-cooperative pricing, we keep Rotemberg and Saloner's nomenclature.

[5]Green and Porter (1984) also find reversion to non-cooperative behavior in a model of collusion.; however, this

We also establish testable implications with respect to the relative prices during booms and recessions. The first of these states, if the marginal cost of capacity is *high* enough, then for equal current demand levels, prices in booms will generally be *lower* than prices in recessions. In this case, capacity is too costly to hold as extra punishment.[6] Instead, firms choose low capacity levels to limit the gain from deviations in high demand periods. Since low capacity levels lead non-cooperative prices to be highest in these periods, the near-term punishment after a defection is smallest when demand is growing. For most model specifications, the converse is also true: If the cost of capacity is *low* enough, then prices in the booms will be generally *higher* than prices in the recessions when firms collude. In this case, capacity is cheap enough that holding large amounts, to increase the credible level of punishment, is most helpful to maximize collusive profits. Therefore, the near-term loss after a deviation is largest when demand is growing. If the cost is in an intermediate range, no such blanket pricing patterns can be established; the relationship can change along the demand cycle.

To further examine pricing patterns, we calculate numerical examples for different costs at varying discount factors. Numerical examples with extremely low costs show very similar pricing patterns to the limitless capacity model of Haltiwanger and Harrington (1991). As in Haltiwanger and Harrington (1991), we also find that prices are pro-cyclical for high discount factors, but can become extremely counter-cyclical if firms are impatient enough. With high capacity costs, at equal current demand levels, collusive prices are lower in the boom than in the recession and never become counter-cyclical. The prices always remain high in the largest demand periods, while *severe* price wars can occur in the lowest demand periods. An intermediate cost specification shows patterns of both the low and high cost cases. For most discounts, prices are higher in the low demand periods of the boom than the low demand periods in the recession, while lower in the high demand periods of the boom than in the high demand periods of the recession.

Our results are not limited to a model with perfectly inflexible capacities. In Section 7 we show that as long as there is sufficient time lag to adjust capacity and a positive fixed cost of adjustment, the pricing implications of the model are identical to the case of perfectly inflexible capacities. This is not surprising given that our model can be construed as one with infinite adjustment costs.

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is driven by information asymmetries rather than capacity constraints.

[6] We follow the existing literature and refer to booms as periods where demand is growing and recessions as periods where demand is contracting.

1.1 Relation to existing literature

This paper builds on a literature that began with Rotemberg and Saloner (1986). Rotemberg and Saloner analyze an industry where firms compete over an infinite horizon, face iid demand shocks in each period, and have infinite capacities; firms observe these demand shocks prior to choosing prices. In this setting, as the discount factor falls away from unity, collusive prices will be lower when demand is high. This is because the gains from cheating are highest during high demand periods and the punishment is independent of the current demand state (because of the iid demand shocks). As such, prices may be lower the greater is demand. Rotemberg and Saloner refer to this lowering of prices as a price war.

Kandori (1991) generalizes the model of Rotemberg and Saloner to the setting of Markov uncertainty in demand levels. For a class of demand shocks, the collusive prices are shown to exhibit the same price wars during booms—prices lowered from monopoly levels in high demand states—as in the iid case. The key to this result is that the Markov distribution approaches a stationary distribution over time.

Haltiwanger and Harrington (1991) extend the model of Rotemberg and Saloner in a different direction by assuming firms face deterministic demand cycles. Their results demonstrate that stationary future demand is far from an innocuous assumption and, in fact, drives the price war results of Rotemberg and Saloner. Once demand movements are no longer independent, the loss from punishment in the future depends on the state of demand today. With deterministic demand cycles, the losses from punishment differ for all periods of the cycle; the losses will be greater when demand is going to grow in the near future. Based on this key insight, Haltiwanger and Harrington show that, holding current demand fixed, firms sustain weakly higher collusive prices in a boom period than a recession period. When the firms are sufficiently impatient, prices can become counter-cyclical just as in the Rotemberg and Saloner model.

Bagwell and Staiger (1997) study a model of repeated price competition with Markov uncertainty in demand growth rates; the growth rate switches randomly between a high and a low rate. Under this specification the concept of recessions and booms can be defined analogously to Haltiwanger and Harrington, if shocks are positively correlated. With positive correlation, if the growth rate is high today, it is more likely to be high tomorrow, hence this is a boom. Conversely, when the growth rate is low, it is more likely to be low tomorrow, this is a recession. As in the Haltiwanger and Harrington model, holding the current level of demand constant, the expected future loss after a deviation is greater during a boom than in the recession, since demand levels are expected to remain high.[7] Therefore, because the immediate gain from a deviation is the same, firms sustain higher collusive price in the boom.

Staiger and Wolak (1992) extend the model of Rotemberg and Saloner in another direction by keeping iid demand shocks, but including endogenous capacity constraints. The supergame is the infinite repetition of a game that involves first, a choice of capacities by both firms when the demand for the period is uncertain. Then, after observing the other firm's capacity and the realized demand state, the firms choose their prices. The single-period subgame is an extension of the Kreps and Scheinkman (1983) model to include iid demand uncertainty.[8] The introduction of capacity constraints changes one primary analytical feature of the Rotemberg and Saloner model: The capacity constraints limit the gains from cheating in high demand states by limiting the quantity a firm can sell when they cheat. Clearly, if the demand realization is high enough such that the sum of the capacities of the two firms is less than the demand at the monopoly price, then no firm gains from deviating in that state. Staiger and Wolak show that the nature of price wars depends on the degree of excess capacity in an industry. *Mild* price wars, occur in demand states with little excess capacity, while *severe*, mixed-strategy, price wars occur in states with more excess capacity.

Fabra (2006) also studies collusion under capacity constraints and cyclical demand. Our analysis differs in two important respects. First, Fabra assumes exogenous capacity levels; we find endogenizing capacity can have large effects. More importantly, Fabra limits the analysis to the *critical point* for mild price wars—the point on the cycle where prices first deviate from monopoly levels; we derive more general results with respect to prices. She finds that if the exogenous capacity constraints are large enough, the critical point for price wars is during the recession. While if the capacity constraints are low enough, the critical point is in the boom of the cycle.

2 Preliminaries

Consider an industry with two firms and a market for a single homogeneous product. The index i is used to identify an arbitrary firm, where $i = 1, 2$. The two firms are infinitely lived and modeled as initially choosing capacities, then interacting in a Bertrand-Edgeworth price game in each period

[7] One difference being that in the Haltiwanger and Harrington model there is no uncertainty. Another distinction between the two models is that Bagwell and Staiger's model has only multiplicative demand shifts. This restriction leads to a single-collusive price for all boom states and a single collusive price for all recession states both independent of the demand level.

[8] There is a substantial literature on dynamic oligopoly games of capacity and price competition with stationary demand, including Brock and Scheinkman (1985), Benoit and Krishna (1987), Davidson and Deneckere (1990), Compte, Frédéric and Rey (2002) and Besanko and Dorazelski (2004).

that follows. The parameter θ is a finite real number that represents the state of demand. The market demand function at any time t, given the state $\theta_t \in \mathbb{R}_+$, is $D(\cdot;\theta_t) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. The inverse demand for any time t, given the state $\theta_t \in \mathbb{R}_+$, is $P(\cdot;\theta_t) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. Demand is assumed to follow deterministic cyclical fluctuations over time based on the parameter θ. The structure of demand movements is given by cycles that repeat every τ (finite) periods.[9] The market demand function follows the deterministic cyclical time path:

$$\theta_t = \begin{cases} \theta_1 & \text{if} \quad t \in \{1, \tau+1, 2\tau+1, ...\}, \\ \theta_2 & \text{if} \quad t \in \{2, \tau+2, 2\tau+2, ...\}, \\ \vdots & \\ \theta_\tau & \text{if} \qquad t \in \{\tau, 2\tau, 3\tau, ...\}. \end{cases} \tag{1}$$

Each firm has a capacity $x_i \in \mathbb{R}_+$, the absolute limit on the number of units it can produce. The marginal cost of production is zero up to the firm's capacity and infinite for any quantity beyond. The two firms have a common discount factor $\delta \in (0,1)$. We label the generic τ-period cycle of demand parameters as $\Theta = \{\theta_1, \theta_2, ..., \theta_\tau\} \subset \mathbb{R}_+^\tau$. In order to simplify notation, we define $X(\theta) = D(0;\theta) < \infty$ and $P(\theta) = P(0;\theta) < \infty$, for all $\theta \in \Theta$. It will also be useful to denote by $\overline{X} = \max_{\theta \in \Theta} X(\theta)$, the maximum demand at a price of zero over all states $\theta \in \Theta$.

The first three assumptions establish the basic properties of the industry demand function.

Assumption 1 *For each $\theta \in \Theta$ the quantity $X(\theta)$ is such that $\forall q \in [0, X(\theta))$, $P(q;\theta) \in (0,\infty)$ and $\forall q \geq X(\theta)$, $P(q;\theta) = 0$. On $(0, X(\theta))$, $P(q;\theta)$ is twice-continuously differentiable, strictly decreasing and concave in q.*

Assumption 2 *Demand is increasing in $\theta \in \Theta$, such that for all $p \in (0, P(\theta))$, $D(p;\theta') > D(p;\theta)$ if and only if $\theta' > \theta$.*

Assumption 3 *For any $q \in (0, X(\theta))$, the marginal inverse demand is increasing in the demand parameter:*

$$\frac{\partial P(q;\theta')}{\partial q} \geq \frac{\partial P(q;\theta)}{\partial q} \text{ if } \theta' \geq \theta \text{ for all } \theta \text{ and } \theta' \text{ in } \Theta.$$

For all $\theta \in \Theta$,

$$\lim_{q \uparrow X(\theta)} \frac{\partial P(q;\theta)}{\partial q} > -\infty.$$

[9] Similar assumptions about the structure of demand were introduced in Haltiwanger and Harrington (1991), although the assumptions we use are most similar to Fabra (2006).

The next two assumptions specify the properties of the capacity cost function each firm faces.

Assumption 4 *The capacity cost function is homogeneous across firms. The marginal cost is the constant* $c > 0$*, such that each firm's cost function is* $c_i(x_i) = cx_i$ *for* $i = 1, 2$.

Assumption 5 *The industry cost of capacity permits positive profit:*

$$c < \overline{c}(\delta) = \frac{\sum_{t=1}^{\tau} \delta^{t-1} P(0;\theta_t)}{\sum_{t=1}^{\tau} \delta^{t-1}}.$$

In order to construct specific results about pricing patterns, it is helpful to make a regularity assumption on the structure of the demand cycles.

Assumption 6 *The cycle has a single peak:*

$$\theta_1 < \theta_2 < ... < \theta_{\hat{t}} > ... > \theta_\tau > \theta_1.$$

The final assumption is important; this minimal regularity of the demand cycles is required to make clear statements describing collusive pricing patterns. Two important features of this model are the deterministic nature of demand cycles and the exogenous firm structure. This model pertains to industries where entry or exit of firms is unlikely to happen.[10] From this point onward we impose assumptions 1-6, although only results pertaining to pricing patterns over the demand cycle require assumption 6.

3 The non-cooperative equilibrium

The non-cooperative equilibrium of a single cycle serves as a baseline to compare the most-collusive supergame equilibrium. Thus, we first describe the subgame perfect equilibrium of a single cycle stage-game. Since each demand cycle of pricing games is the same, the subgame perfect equilibrium for a single cycle will be all that is needed to construct the non-cooperative subgame perfect equilibrium of the infinitely repeated game. First, we outline the timing of the stage-game.

[10] The deterministic structure of demand has proven itself useful in the case of the Bertrand supergame and provided results conceptually analogous to the uncertain Markov setting where the end of booms and recessions is unknown. Here we are referring to the similarity of results in Haltiwanger and Harrington (1991) with deterministic demand cycles relative to Bagwell and Staiger (1997) with Markov demand cycles.

3.1 The timing of a typical stage-game

The game takes place over a single demand cycle $\theta_1, \theta_2, ..., \theta_\tau$.

Period 1 : At the beginning of the game, each firm $i = 1, 2$ chooses x_i independently and simultaneously. Then each firm $i = 1, 2$, observes the other firms capacity x_j and chooses $p_1(i) \in (0, P(\theta_1)]$ independently and simultaneously. From this point on, j is used as $j = 1, 2$ such that $j \neq i$.

Period 2 : Each firm observes the other firm's choice $p_1(j)$. Then the firms choose $p_2(i) \in (0, P(\theta_2)]$, independently and simultaneously.

$\vdots$

Period τ : Each firm observes the other firms choice of $p_{\tau-1}(j)$. The firms choose $p_\tau(i) \in (0, P(\theta_\tau)]$ independently and simultaneously.

3.2 The pricing stage-games

There are τ pricing stage-games that follow the initial capacity choice. Here we fix the two firms' capacity choices at arbitrary values x_1 and x_2 and examine the pricing in each stage-game over the τ-period cycle.

We assume the demand is rationed using the Surplus Maximizing Rationing rule.[11] Dasgupta and Maskin (1986a, 1986b) prove the existence of the Nash equilibrium in each pricing stage-game. Following Kreps and Scheinkman (1983), the revenue of the pricing subgames can be split into three regions. To help characterize the equilibrium revenue, we define the Cournot best response functions as follows

$$r_c(y;\theta) = \arg \max_{x \in [0, X(\theta)-y]} \{P(x+y;\theta)x - cx\}$$

and

$$r(y;\theta) = \arg \max_{x \in [0, X(\theta)-y]} \{P(x+y;\theta)x\}.$$

[11] We are confident that the character of the most-collusive equilibrium in the paper can be extended to alternate demand rationing rules, particularly, if the Beckman demand rationing rule were assumed. The primary difference in the most-collusive equilibrium would be the possibility of severe price wars in higher demand periods. Davidson and Deneckere (1986) provide a characterization of the non-cooperative pricing subgame with Beckman rationing.

The unique-equilibrium expected revenue function, given capacities x_1 and x_2, is:

$$R_i^n(x_1, x_2; \theta) = \begin{cases} P(x_1 + x_2; \theta)x_i & \text{if } x_1 \leq r(x_2; \theta) \text{ and } x_2 \leq r(x_1; \theta) \\ 0 & \text{if } \min\{x_1, x_2\} \geq X(\theta) \\ R_i^*(x_1, x_2; \theta) & \text{otherwise.} \end{cases}$$

The third region only has a mixed-strategy pricing equilibrium. The expected revenues in this region depend on which firm possesses more capacity,

$$R_i^*(x_1, x_2; \theta) = \begin{cases} R(x_j; \theta) & \text{if } x_i \geq x_j \\ R(x_1, x_2; \theta) \in [\frac{x_i}{x_j} R(x_i; \theta), R(x_i; \theta)] & \text{if } x_i < x_j. \end{cases}$$

The function $R(x_j; \theta)$ is the follower's profit in a zero cost Stackelberg game. Denote by $p_i^n(x_1, x_2; \theta)$ the non-cooperative pricing of firm i given capacities $x_1$and x_2.

The Nash equilibrium of each pricing stage-game constructs the unique non-cooperative subgame perfect equilibrium of the entire pricing cycle game, given fixed capacities. The reasoning is as follows: If in each time, $t \geq 1$, the other firm prices according to the Nash equilibrium of each individual stage-game, the best response is to price Nash in the current and all subsequent stage-games.

3.3 The capacity choice stage-game

Kreps and Scheinkman (1983) prove, in a sequential capacity and price game with one period of pricing, the unique Nash equilibrium is in pure strategies and has the same price and quantity sold as the analogous Cournot game. In our setting, the non-cooperative subgame perfect capacities will generally not be the same as the analogous game with Cournot pricing. Even the existence of an equilibrium with symmetric pure strategy capacities is not guaranteed. This issue is studied in Lepore (2006), where it is shown that there are equilibria that involve pure symmetric capacities in only two cases: (i) the demand cycle has very little variance in demand periods and the cost of capacity is relatively high, or (ii) the demand cycle includes many large similar demand periods, few very small demand periods and the cost of capacity is low enough. For any demand cycle that does not fit the description of (i) or (ii), all non-cooperative equilibrium will generally involve asymmetric capacities.[12]

[12] Lepore (2006) studies a two-stage game where demand is uncertain at the capacity choice stage, but is realized before firms choose prices. This structure nests our single-cycle capacity stage-game as a special case, where the probability measure μ on $[\theta_1, \theta_{\hat{t}}]$ is defined by $\mu(\theta) = \sum_{t \in \{s \in \{1,..,\tau\} | \theta_s = \theta\}} \delta^{t-1} / \sum_{t \in \{1,..,\tau\}} \delta^{t-1}$.

4 The most-collusive equilibrium

In this section, we examine the infinite-time game with collusive pricing. We focus on the basic properties of the most-collusive equilibrium. The *most-collusive equilibrium* is the symmetric subgame perfect equilibrium with the maximal joint profit for the industry.[13] By assumption, the most-collusive equilibrium is always symmetric in terms of pure strategies for capacities, if there is a feasible symmetric equilibrium. If the set of feasible symmetric capacity subgame perfect equilibria is empty, then the most-collusive equilibrium can involve asymmetric capacities. In the primary analysis of the paper, we restrict attention to symmetric capacity collusion.[14]

The most-collusive equilibrium is symmetric in terms of pure strategies for prices as well, unless the non-cooperative equilibrium pricing yields more revenue than any incentive compatible pricing strategy. In that case, the most-collusive pricing is the non-cooperative pricing equilibrium for that periods stage-game and might only be symmetric in terms of mixed strategies and expected profits.

The most-collusive equilibrium may be supported by many punishment strategies if firms are patient enough. As the discount factors fall away from unity, an 'optimal' punishment, as defined by Abreu (1986, 1988), is among the only symmetric punishments that can support the complete set of subgame perfect equilibria for all discounts. The most-collusive equilibrium does not necessarily exist unless an optimal punishment exists to support it. In the case of exogenous capacity constraints, Lambson (1987) proves that the optimal punishment exists and is at the *security level*; the *security level* is the minimum profit that a firm's competitor can force upon it. Reversion to non-cooperative pricing forever after a cheat yields security level payoffs. Accordingly, we use non-cooperative reversion to calculate the optimal punishment, although it is not necessary to think of the firms actually using this non-cooperative threat. The 'optimal' punishment is not unique and, in fact, Lambson has shown that there is a stick-carrot type punishment that is also optimal.

At the upper bound, when the firms are extremely patient (or, analogously, the period length is very short), the most-collusive equilibrium might be identical to the monopoly equilibrium in prices with each firm having half of the monopoly capacity. The unique monopoly capacity choice is labeled $x^m(c, \delta)$. Firms may also be able to sustain monopoly prices if total capacity is greater than the monopoly level. At capacity x, the unique capacity constrained-monopoly price for each period t is given by:

$$p_t^m(x) = \arg\max_{p \in [0, P(\theta)]} \{D(p; \theta_t)p | D(p; \theta_t) \leq x\}.$$

[13] Although the name, most-collusive equilibrium, has been used predominantly in the literature, the more appropriate name might be the "best symmetric equilibrium" used by Kandori (1991).

[14] In Appendix II subsection 10.2, the reasoning behind this restriction is discussed.

We label the constrained-monopoly prices at the unique monopoly capacity as $\overline{p}_t^m(c) = p_t^m(x^m(c, \delta))$ for all $t = \{1, 2, ...\}$.[15]

The solution to the most-collusive equilibrium involves multiple incentive compatibility constraints for each pricing stage-game and the initial capacity choice. We will explain the equilibrium capacities and prices following a backward induction-type approach. First, we describe the way the pricing incentive compatibility constraints dictate equilibrium pricing behavior given fixed symmetric capacities. Then, the capacity incentive compatibility constraints are analyzed taking the most-collusive pricing behavior as given. The next two subsections provide some insight into how the interaction of these constraints dictates the most-collusive capacity levels. The unique most-collusive equilibrium price strategies, given the symmetric capacities $x_1 = x_2 = x$, are denoted by $p_t^c(x, \delta)$ for $t = \{1, 2, ...\}$. The most-collusive capacities and prices are denoted by $x^c(c, \delta)$ and $\overline{p}_t^c(c, \delta) = p_t^c(x^c(c, \delta), \delta)$ for $t = \{1, 2, ...\}$, respectively.[16]

4.1 The pricing constraints

For each period, the pricing stage-game incentive compatibility constraints can naturally be separated into two constraints. The first is the standard collusive pricing constraint for firms under-cutting the collusive price, guaranteeing that the immediate gain from an undercut cannot exceed the future loss from punishment. The second is a by-product of capacity constraints that only applies if the under-cutting constraint is already binding. If the price in any period is bound significantly below the monopoly level by the under-cutting constraint, then the possibility of a higher price deviation also exists. This over-cutting constraint is a feature unique to models with capacity constraints since depending on the rationing rule, a firm may have an incentive to *increase* its price.

1) The incentive compatibility constraint for under-cutting. The arbitrary price vector $p = (p_t)_{t=1}^{\infty}$ is in the set of under-cutting incentive compatible collusive prices if it is such that,[17]

$$G_t^u(p_t, x_1, x_2)(i) \leq L(p, x_1, x_2; \delta)(i), \text{ for all } i = 1, 2, \text{ and } t = 1, 2, ...$$

where,

[15] The proof of both existence and uniqueness of the monopoly prices and capacity is provided in Appendix II.

[16] The existence and uniqueness of the most-collusive equilibrium prices and a discussion of the conditions guaranteeing incentive compatible most-collusive capacities is the primary subject of Appendix II.

[17] Note that $D_i(p_t; \theta_t)$ depends on the specification of a particular collusive demand rationing rule. See the Appendix II for a discussion of the range of possible rules.

$$G_t^u(p_t, x_1, x_2)(i) = p_t(\min\{x_i, D(p_t;\theta_t)\} - D_i(p_t;\theta_t)),$$
$$L_t(p, x_1, x_2; \delta)(i) = \textstyle\sum_{s=t+1}^{\infty} \delta^{s-1}\left(p_s D_i(p_s;\theta_s) - R_i^n(x_1, x_2; \theta_s)\right).$$

2) The incentive compatibility constraint for over-cutting. The arbitrary price vector $p = (p_t)_{t=1}^{\infty}$ is in the set of over-cutting incentive compatible collusive prices if it is such that,

$$G_t^o(p_t, x_1, x_2)(i) \leq L_t(p, x_1, x_2; \delta)(i), \text{ for all } i = 1, 2, \text{ and } t = 1, 2, ...$$

where,

$$G_t^o(p_t, x_1, x_2)(i) = \max_{\rho_t \in [0, P(\theta_t 0]} \{\rho_t \min\{D(\rho_t;\theta_t), x_j - D(p_t;\theta_t)\}\} - p_t D_i(p_t;\theta_t).$$

In some cases, it is possible that there is no pure strategy price vector that satisfies both the over-cutting and under-cutting constraints. In this case, the highest incentive compatible expected revenue comes from non-cooperative mixed-strategy pricing.

4.2 The most-collusive pricing determined by the constraints

The most-collusive pricing in each period is determined by whether the first constraint is binding, both are binding, or neither binds. The most-collusive revenue in each period depends on the fixed capacity levels, x_1 and x_2, and the two constraints to subgame perfect pricing. At this point, we impose symmetry in the firms' capacity choices so that $x_1 = x_2 = x$. In any period, there are three basic most-collusive pricing patterns. Which of the three pricing patterns takes place, depends on how the demand parameter of a given time period relates to the most-collusive capacity.[18]

Region 1 ***(constrained-monopoly pricing)*** The collusive price for a period of demand in this region is sustainable at or above the single-period unconstrained-monopoly level. We denote by p_t^{um} the unconstrained-monopoly price for the period θ_t, formally,

$$p_t^{um} = \arg\max_{p \in [0, P(\theta_t)]} \{pD(p;\theta_t)\}.$$

If the capacity constraint binds so strongly that no undercut can increase the demand for both firms, then the most-collusive price is above the single-period unconstrained-monopoly level . This is where

[18] The characterization of the pricing regions follows similar lines to Staiger and Wolak (1992) with the exception that they assume linear demand.

θ_t is such that $D(p_t^{um};\theta_t) \geq 2x$. The revenue of each firm in this region of demand parameters is $P(2x;\theta_t)x$. Price is higher than the single-period unconstrained-monopoly level for time period t, i.e. $\overline{p}_t^c(c,\delta) > p_t^{um}$. The prices are at the unconstrained-monopoly level if $D(p_t^{um};\theta_t) \leq 2x$, thereby permitting sustainable joint surplus maximizing revenue. When capacities are sufficient to meet demand at the unconstrained-monopoly price, each firm earns half the unconstrained-monopoly profit for that single state: $p_t^{um}D(p_t^{um};\theta_t)/2$.

Region 2 ***(Mild price wars)*** These are periods such that each firm's under-cutting incentive compatibility constraint binds at the constrained-monopoly price, but the over-cutting constraint does not play a roll. In this region, $\overline{p}_t^c(c,\delta)$ is lower than $p_t^m(2x^c(c,\delta))$ and since $p_tD(p_t;\theta_t)/2$ and $L_s(p^c,x;\delta)$ for $s \neq t$ are increasing in p_t on $(0,p_t^{um})$, the most-collusive price in this region is the highest price that is under-cutting incentive compatible. More precisely, $\overline{p}_t^c(c,\delta)$ is the largest price less than the unconstrained-monopoly price such that the incentive constraint holds with equality for period t.

Region 3 ***(non-cooperative pricing)*** In this region, the highest revenue level that satisfies the incentive compatibility constraints is from the non-cooperative pricing strategies. Thus, the pricing and revenue will be at the non-cooperative levels.

A *severe* mixed-strategy price war period must be within this region. In particular, mixed-strategy price wars occur for demand periods where, for at least one firm i, $x_i \in (r(\theta_t;x_j),X(\theta_t))$ and for all p that are incentive compatible, $pD(p;\theta_t)/2 < R^n(x,\theta_t)$.

4.3 The capacity constraint

The most-collusive capacity choices involve incentive compatibility constraints to insure no deviation from the collusive capacity level is beneficial to either firm. We define discounted profits, at the most-collusive prices, given capacities $x_1 = x_2 = x$, as $\Pi^c(x,c,\delta)$. The profit from deviating from x is given as $\Pi^n(x_i,x,c,\delta)$, where the prices are non-cooperative. Formally,

$$\begin{aligned}
\Pi^c(x,c,\delta) &= \textstyle\sum_{t=1}^{\infty}\delta^{t-1}\left(\overline{p}_t^c(c,\delta)\left(\min\left\{x,\dfrac{D(\overline{p}_t^c(c,\delta);\theta_t)}{2}\right\}\right)-cx\right),\\
\Pi^n(x_i,x,c,\delta) &= \textstyle\sum_{t=1}^{\infty}\delta^{t-1}\left(R^n(x_i,x;\theta_t)-cx_i\right).
\end{aligned}$$

The incentive compatibility constraint: Capacities An arbitrary capacity $x \in [0, \overline{X}]$ is in the set of symmetric incentive compatible collusive capacity levels if:

$$\Pi^c(x, c, \delta) \geq \max_{x_i \in [0, \overline{X}]} \{\Pi^n(x_i, x, c, \delta)\}.$$

This incentive constraint is more complex than the pricing incentive constraints. If the discount factor is far enough away from one, then the most-collusive prices interact in substantive ways with the capacity choices.

4.4 The most-collusive capacities determined by the constraint

Region 1 ***(Monopoly(ish) capacities)*** If the capacity incentive constraint does not bind at the joint profit maximizing capacities, then $x^c(c, \delta) = \widehat{x}^c(c, \delta)$ is the most-collusive equilibrium. $\widehat{x}^c(c, \delta)$ is defined as:

$$\widehat{x}^c(c, \delta) \in \arg \max_{\xi \in [0, \overline{X}]} \{\Pi^c(\xi, c, \delta)\}.$$

This is half the capacity a monopolist would choose given the most-collusive prices. In spite of the fact that the capacity incentive constraint does not bind, the most-collusive capacities are not necessarily half the monopolists capacity. The most-collusive prices might be different than the constrained-monopoly prices which is likely to lead to a different choice of joint profit maximizing capacities. Hence, we denote the most-collusive capacities in this context as *monopoly-ish capacities*. If the most-collusive prices are all monopoly prices, then $2\widehat{x}^c(c, \delta) = x^m(c, \delta)$. In this region, capacities are always symmetric.

Region 2 ***(Symmetric Capacities)*** In this region, the capacity incentive constraint binds strongly that $\widehat{x}^c(c, \delta)$ is not incentive compatible, but the set of symmetric pure strategy capacities that satisfy the incentive constraint is non-empty. The only capacities that are in the incentive compatible set are capacities larger than $\widehat{x}^c(c, \delta)$.[19] In this region, the most-collusive capacities can range greatly, lying anywhere inside the interval $[\widehat{x}^c(c, \delta), \overline{X}]$. If the cost of capacity is low enough, the capacities will be close to the upper bound $\overline{X}$ for most discounts. It is also possible

[19] At any symmetric capacity level $x < \widehat{x}^c(c, \delta)$, the most-collusive profit is lower than at $\widehat{x}^c(c, \delta)$. Based on Proposition 3 in Benoit and Krishna (1987), for each θ_t, $R_i^n(z, x; \theta_t) \geq R_i^n(z, \widehat{x}^c(c, \delta); \theta_t)$ $\forall$ $z \in [0, X]$, hence $\Pi^n(z, x, c, \delta) \geq \Pi^n(z, \widehat{x}^c(c, \delta), c, \delta)$ $\forall z \in [0, X]$, $\delta \in (0, 1)$ and $c \in (0, \overline{c}(\delta))$. These two facts together imply the incentive constraint for the most-collusive capacity choice is violated for all $x < \widehat{x}^c(c, \delta)$.

that the marginal cost of capacity is high enough that the capacities will never get very large and the capacities will barely stray from $\widehat{x}^c(c,\delta)$.[20]

Region 3 ***(Asymmetric capacities)*** In this region, the incentive constraint binds so harshly that the only admissible capacities are the non-cooperative capacities given the most-collusive revenue. In this case, an equilibrium with symmetric pure strategy capacities is not likely to exist. There is no collusion in capacities, instead they are the result of competition with most-collusive prices. For much of the subsequent analysis, we do not consider discount-cost combinations in this region.

5 Dynamic pricing patterns

Here we present some general properties of the most-collusive equilibrium of the infinite time game. As in most collusion games, the level of collusive profits that can be maintained by the optimal punishment varies with the discount factor of the two firms. If the discount factor is close enough to one, monopoly prices are sustainable and most-collusive capacities are symmetric.

Proposition 1 *There exists* $\overline{\delta}(c) \in (0,1)$ *such that for all* $\delta \in [\overline{\delta}(c), 1)$, $\overline{p}^c_t(c,\delta) = p^m_t(2x^c(c,\delta))$ *for all* $t \in \{1, ..., \tau\}$.

The proof of all propositions and theorems presented in the body of the text are located in Appendix I. The proof of Proposition 1 is based on first establishing that there is a discount factor where all collusive prices are at the monopoly level. This is essentially a Folk theorem for capacity constrained pricing in this specific class of models. To show this for a single period t, we fix capacities at an arbitrary level and all other prices at the constrained-monopoly levels. The constrained-monopoly price level is sustainable in period t, if the discount factor is one. The fact that the future loss is continuous and goes to zero as δ goes to zero is used to prove that the monopoly price is sustainable if and only if the discount is greater or equal to $\overline{\delta}(t,x) < 1$. The discount factor $\overline{\delta}(c)$, is the smallest discount such that all most-collusive prices are at constrained-monopoly levels at the most-collusive capacities, for all $\delta \in [\overline{\delta}(c), 1]$.

In contrast, there is a discount factor such that, for any discount factor above this level the most-collusive equilibrium involves pure strategy capacities and is *not* the non-cooperative equilibrium.

[20] In Section 6, the high cost ($c = 0.05$) numerical example involves capacity levels fairly close to half the monopolist capacities for all discounts where collusive pricing is sustainable.

Proposition 2 *There exists $\underline{\delta}(c) \in (0, \overline{\delta}(c)]$, such that for all $\delta \in (\underline{\delta}(c), 1)$ the most-collusive equilibrium involves pure symmetric capacities, $x^c(c, \delta)$ and $\overline{p}_t^c(c, \delta) \neq p^n(x^c(c, \delta); \theta_t)$ for at least one $t \in \{1, .., \tau\}$.*

In all that follows, the range of discount factors where the most-collusive equilibrium is primarily studied, lie between $\underline{\delta}(c)$ and $\overline{\delta}(c)$. In our numerical examples, the upper bound discount $\overline{\delta}(c)$ is approximately 0.64 in the lowest cost example, and approximately 0.94 when costs are extremely high. The lower bound discount $\underline{\delta}(c)$ ranges from just above 0.5 at the lowest cost, to approximately 0.76 when costs are high. Both discount factors tend to increase as the cost of capacity increases so that the interval of discount factors between $\underline{\delta}(c)$ and $\overline{\delta}(c)$ has a fairly wide range across different levels of c.

These two propositions construct the outline of the most-collusive pricing picture shown in Figure 1. We will expand on this diagram throughout the section to complete the picture of the most-collusive equilibrium pricing behavior.

5.1 Pricing patterns

The goal of this section is to understand the pricing patterns of the most-collusive equilibrium when monopoly prices are not sustainable in all periods. Towards this end, there are two classes of results: locations of mixed-strategy price wars and general properties of prices in booms versus recessions. Both sets of results depend on the cost of capacity.

The presence of capacity constraints alters both the gains and losses from defecting at constrained-monopoly prices. When demand is sufficiently high, a defecting firm is unable to capture the entire market. Similarly, during high near-term demand periods, the severity of the punishment is reduced since non-cooperative prices are no longer zero. The relative magnitude of these two countervailing incentives drives the pricing patterns we would expect to see under collusion. To establish a concrete example of the effect of changes in demand levels on incentives of the firms, we graph the single-period expected gains and losses from cheating at the monopoly price. The figures illustrate the single-period gains and losses from optimal defection at unconstrained monopoly prices when demand is of the form, $D(p; \theta) = \theta - p$.

In any period with demand parameter θ, the profit from deviating is determined by its relationship to capacity. There are three distinct forms of the gains relative to θ, depending on the fixed symmetric capacities. For low demand states, the gains from defection rise with demand as the firm is able to undercut its competitor and capture the entire market. There is a point, however, where the firm is not able to meet all of the additional demand from defection and the gains from cheating fall with θ. At some point, an individual firm is producing at capacity under collusion and there are no gains from cheating. Specifically, the short term gain in any state θ given x is:

$$G(\theta, x) = \begin{cases} \frac{\theta^2}{8} & \text{if } \theta \leq 2x, \\ \frac{\theta}{2}x - \frac{\theta^2}{8} & \text{if } \theta \in (2x, 4x), \\ 0 & \text{if } \theta \geq 4x. \end{cases}$$

Figure 2 plots $G(\theta, x)$ with the capacity fixed at $x = 2$ in terms of the demand parameter. The gains are non-monotonic in θ and if the demand parameter is large enough relative to the capacity constraints, then the largest gain from deviation will be zero. In contrast, without capacity constraints, as the demand parameter θ increases, the gain from deviating increases indefinitely.

The losses in the first period following defection can also be characterized by the period's value of θ relative to capacity. As with the gains, these losses initially rise and then begin to fall with θ. When the capacity levels are sufficiently large, such that prices fall to zero, defection implies a loss of half of the monopoly profits. If the firms are unable to commit to zero prices upon defection, then the losses from defection are reduced by an amount that depends on the installed capacity. Finally, if demand is sufficiently large such that the combined capacity base cannot meet demand at the monopoly price, then there is no penalty from defecting. We can characterize the losses in the first period following defection as:

$$L(\theta, x) = \begin{cases} \frac{\theta^2}{8} & \text{if } \theta \leq x, \\ \frac{\theta^2}{8} - \frac{(\theta - x)^2}{4} & \text{if } \theta \in (x, 3x], \\ \frac{\theta^2}{8} - (\theta - 2x)\, x & \text{if } \theta \in (3x, 4x), \\ 0 & \text{if } \theta \geq 4x. \end{cases}$$

In Figure 3 we plot the first period's losses with demand parameter θ, after a deviation, when $x = 2$.

What will drive our results with respect to pricing along the cycle is that for any two time periods on opposite sides of the cycle s and t, such that $\theta_s = \theta_t$, the gains from defection are the same, but the discounted losses are different since the sequence of demand states that follow are not identical. For equal demand levels, whether prices are higher in the boom or the recession, will depend on where next period's demand falls in relation to Figure 3. This in turn depends on the endogenously chosen capacity levels. In the two following subsections, we establish some pricing properties of the most-collusive equilibrium that are based on understanding how the immediate gain from a deviation, along with the discounted future losses from the deviation, vary over the demand cycle.

5.1.1 Severe price wars

The results in this subsection apply to the demand range where severe price wars can occur. For all demand cycles, mixed-strategy price wars can occur in a period with demand parameter θ if and only if the capacity is between the Cournot duopoly equilibrium quantity with zero costs and the demand for the good when price is zero, i.e. $x \in (r^*(\theta), X(\theta))$. Both bounds on the capacity are strictly increasing in the demand parameter θ. As capacity increases, the range of demand parameters such that mixed-strategy price wars can occur in that period increases as well.

Theorem 1 *For all $\theta \in \Theta$, there exists $c_l(\theta)$ and $c_h(\theta)$ such that:*

(i) a mixed-strategy price war can occur in a period with demand parameter θ if and only if $c \in [c_l(\theta), c_h(\theta)]$;

(ii) $c_l(\theta') \leq c_l(\theta)$ and $c_h(\theta') \leq c_h(\theta)$ if and only if $\theta' \geq \theta$, for all $\theta, \theta' \in \Theta$.

Although not stated in the theorem itself, it is possible that $c_l(\theta) \leq 0$ or $c_h(\theta)$ exceeds the highest cost that permits positive profit for some $\theta \in \Theta$. If this is the case, then under no circumstances can that period of demand have a mixed-strategy price war.

5.1.2 Cost of capacity and prices in booms versus recessions

In the pricing model without capacity constraints studied in Haltiwanger and Harrington (1991), one of the strongest implications is that prices in two periods with the same demand, one in the boom and one in the recession, are always weakly greater in the boom. This result has lent itself nicely to empirical tests.[21]

In our model, pricing will not be consistently higher in booms or recessions for all marginal costs of capacity. There are, in fact, marginal costs where a consistent inequality between prices in booms and recessions cannot be established. However, when capacity costs in an industry are high enough, or low enough, there are strong testable predictions for most-collusive pricing patterns. Theorem 2

[21] Two empirical papers that utilize the predictions of Haltiwanger and Harrington (1991) are Borenstein and Shepard (1996) with US retail gasoline data and Rosenbaum and Sukharomana (2001) with US cement industry data.

establishes the fact that there is a lowest marginal cost, c_b, such that if the industry's marginal cost exceeds this level, all prices will have a predictable pattern where prices in the boom are *no greater* than prices of periods with the same demand parameter in the recession. This result implies that the Haltiwanger and Harrington pricing pattern is reversed if capacity costs are high enough. Formally, denote the most collusive revenue for period t by $R_t^c(c,\delta) = \overline{p}_t^c(c,\delta)\min\{x^c(c,\delta), D(\overline{p}_t^c(c,\delta);\theta_t)/2\}$, then:

Theorem 2 *There exists a smallest c_b such that, for all $c > c_b$ if $\theta_{t'} = \theta_{t''}$ where $1 \leq t' < \widehat{t} < t'' \leq \tau$, then:*

1. $R_{t'}^c(c,\delta) \leq R_{t''}^c(c,\delta)$ *for all* $\delta \in (0,1)$*;*
2. *If* $\overline{p}_{t''}^c(c,\delta) \neq \overline{p}_{t''}^n(x^c(c,\delta))$ *and* $\overline{p}_{t'}^c(c,\delta) < p_{t'}^m(2x^c(c,\delta))$*, then* $R_{t'}^c(c,\delta) < R_{t''}^c(c,\delta)$ *for all* $\delta \in (\underline{\delta}(c), \overline{\delta}(c))$*;*
3. *If* $\overline{p}_{t'}^c(c,\delta) \neq \overline{p}_{t'}^n(x^c(c,\delta))$ *and* $\overline{p}_{t'}^c(c,\delta) < p_{t'}^m(2x^c(c,\delta))$*, then* $\overline{p}_{t'}^c(c,\delta) < \overline{p}_{t''}^c(c,\delta)$ *for all* $\delta \in (\underline{\delta}(c), \overline{\delta}(c))$*;*
4. *If* $\overline{p}_{t'}^c(c,\delta) = \overline{p}_{t'}^n(x^c(c,\delta))$*, then* $\overline{p}_{t''}^c(c,\delta) = \overline{p}_{t''}^n(x^c(c,\delta))$ *for all* $\delta \in (0,1)$*.*

While the proof of Theorem 2 is presented in the Appendix I, the heuristic explanation of the proof follows from the examples of the gains and losses at the beginning of this section. If marginal cost of capacity is high enough, then the firms will always choose to collude with capacities that are small relative to the demand cycle. At small enough capacity levels, the punishment incurred after a defection will be lowest in the highest demand periods. Now take the two demand periods on either side of the peak. At any given price, the punishment from defecting in the period that precedes the peak will be lower than the losses in the period following the peak. Since, at any fixed price, the gain from a deviation in these two periods is the same, the incentive compatibility constraint in the boom period binds more strongly than in the recession. Hence, both higher revenue and higher prices are sustainable in the comparable recession period.

In a similar vain, consistent pricing patterns can also exists in industries with low marginal cost of capacity. Pricing properties in this case are slightly more delicate than the high cost case.[22]

Theorem 3 *There exists $x_r \in (x_b, \overline{X})$ such that for all $x^c(c,\delta) > x_r$, if $\theta_{t'} = \theta_{t''}$ where $1 \leq t' < \widehat{t} < t'' \leq \tau$, then:*

[22]Not all models will exhibit the pricing behavior specified in Theorem 3. Take the example: $D(p_t;\theta_t) = \theta_t(12 - p_t)$ where the demand cycle is $\Theta = \left\{\frac{1}{3.01}, \frac{1}{3}, 1, \frac{1}{3}\right\}$. For all cost specifications the most-collusive equilibrium price comparison of period 2 versus period 4 follows the pattern of Theorem 2.

1. $R^c_{t''}(c,\delta) \leq R^c_{t'}(c,\delta)$ *for all* $\delta \in (0,1)$*;*

2. *If* $\overline{p}^c_{t'}(c,\delta) \neq \overline{p}^n_{t'}(x^c(c,\delta))$ *and* $\overline{p}^c_{t''}(c,\delta) < p^m_{t''}(2x^c(c,\delta))$*, then* $R^c_{t''}(c,\delta) < R^c_{t'}(c,\delta)$ *for all* $\delta \in (\underline{\delta}(c), \overline{\delta}(c))$*;*

3. *If* $\overline{p}^c_{t''}(c,\delta) \neq \overline{p}^n_{t''}(x^c(c,\delta))$ *and* $\overline{p}^c_{t''}(c,\delta) < p^m_{t''}(2x^c(c,\delta))$*, then* $\overline{p}^c_{t''}(c,\delta) < \overline{p}^c_{t'}(c,\delta)$ *for all* $\delta \in (\underline{\delta}(c), \overline{\delta}(c))$*;*

4. *If* $\overline{p}^c_{t''}(c,\delta) = \overline{p}^n_{t''}(x^c(c,\delta))$*, then* $\overline{p}^c_{t'}(c,\delta) = \overline{p}^n_{t'}(x^c(c,\delta))$ *for all* $\delta \in (0,1)$.

The intuition behind Theorem 3 is straight forward: If the most-collusive capacity is large enough, relative to the demand cycle, both the immediate gain from a defection and the individual period loss after a defection are increasing in the demand parameter. Therefore, the future loss from deviating is greater in comparable boom periods than recession periods. At equal current demand levels, the gain from a defection is the same for these two periods. Hence, the incentive constraint binds first and more strongly in the recession period. Both the expected revenues and the prices are higher in comparable boom periods.

Figure 4 summarizes the implications of Theorems 2 and 3. The complexity of the model places a limit on the character of most-collusive pricing we can prove analytically. In particular, the precise range in terms of marginal costs and discount factors for each of the three most-collusive patterns is left open. In order to establish stronger patterns, we calculate numerical examples under various demand cycles.

6 Numerical examples

6.1 Prices

To provide concrete examples of how capacity constraints affect market equilibria, we parameterize the model and conduct numerical simulations of the most-collusive equilibrium. These numerical examples give a more detailed picture of the collusive pricing patterns over the demand cycle. In doing so, we adopt the functional form assumptions used by Haltiwanger and Harrington (1991), so that the results of our model can be easily compared to those described in their paper.[23] Specifically, demand is parameterized as: $D(p_t;\theta_t) = \theta_t - 400p_t$ and we use an eight-period cycle which varies in terms of the intercept θ_t, given as $\Theta = \{100, 125, 150, 175, 200, 175, 150, 125\}$. We analyze equilibria

[23] Although Haltiwanger and Harrington analyze a market with three firms, whereas we focus on a duopoly. The details involved with solving the constrained maximization problems is discussed in Appendix III.

under three capacity marginal cost levels. For each marginal cost, the discount factor is varied within the range $[\underline{\delta}(c), \overline{\delta}(c)]$. We focus our attention on (a) the relative pricing during booms and busts, (b) the cyclicality of prices, and (c) the benefits from endogenizing capacity choices.

Figure 5 is the low cost example; prices follow the predictions of Theorem 3; at the same demand level prices are lower in the recession than in the boom for all discounts plotted. This result is driven by the fact that capacity is so cheap that it is most productive for the cartel to hold excess capacity to increase the severity of punishment. These results mirror those in Haltiwanger and Harrington (1991) where firms are without capacity constraints. The high capacity levels imply that punishment is greatest during high demand periods, reducing the incentive to deviate during booms, relative to recessions. This pricing pattern holds for almost all discount factors, only very close to $\overline{\delta}(c)$ is there a difference. Just as in the limitless capacity setting, prices are strongly pro-cyclical at high discounts, but become counter-cyclical for low enough discount factors. The counter-cyclicality stems from the fact that as the discount factor falls, firms reduce prices in the highest demand periods to sustain collusion because the largest one-shot gains are in these periods.

Figure 6 plots the most-collusive equilibrium prices for a high cost example. The pricing pattern is starkly different than the low cost case; the prices follow the predictions of Theorem 2 and are always strongly pro-cyclical. When the price of capacity is high, equilibrium capacity levels are low. This increases the incentive to deviate during boom periods, relative to an equal demand period in the recession. To counteract this, firms lower prices during booms, relative to an equal demand period in the recession. Despite this, prices never become counter-cyclical because capacity is too expensive to hold for punishment. Instead, the firms keep capacities small to lock in the high profits from the highest demand periods.

Another interesting feature of the high cost equilibrium is that in period 1, at the discount factor 0.8, the collusive pricing is the same as the non-cooperative mixed-strategy pricing given the equilibrium capacities. This is an example of the mixed-strategy price wars detailed in Theorem 1. In this demand period, the two firms will almost surely name different prices in equilibrium; this would have the appearance of a single-period undercutting price war that occurs at the beginning of every demand cycle. In the figure, the line from prices 0.05625 to 0.075 represents the continuous support of the mixed-strategy pricing. This severe price war does not occur in the equilibrium for the lower discount factors 0.78 and 0.76. The firms instead find it optimal to choose larger capacities because their additional impatience significantly lowers sustainable price levels at low capacity levels.

Figure 7 plots the medium cost case. In this example, both pricing patterns of Theorems 2 and

3 are evident. For the two largest discount factors, the prices follow the pattern of Theorem 2; the three lowest discount factors show pricing patterns consistent with Theorem 3. The collusive pricing at the intermediate discount factor 0.68 does not follow either theorem. Instead, we see that prices are lower in the boom for the higher demand levels and lower in the recession for the demand level 125.

These numerical examples underline the importance of including capacity as a strategic tool when analyzing and testing for collusion. Previous empirical papers test for collusion using the Haltiwanger and Harrington result that, conditional on current demand, prices will be higher if demand is expected to grow. Our results suggest that this may not be a powerful test since collusion may exist even if prices don't follow the predictions of Haltiwanger and Harrington. Furthermore, the medium cost case implies that these inequalities may change along the demand cycle. This suggests that empirical tests may want to focus on periods around exogenous changes in the cost of capacity and explicitly test for an inequality reversal.

6.2 Profits

By endogenizing capacity, firms are able to cater capacity choices to best facilitate collusion. Next we compare the benefits of capacity as a strategic tool by calculating equilibrium profits under four scenarios: monopoly, colluding in both capacity and prices, colluding in prices but having non-cooperative capacities, and non-cooperative behavior.[24] The third scenario represents markets where collusion takes place after capacity choices have been made; Davidson and Deneckere (1990) refer to this as *semi-collusion*. We calculate profits, relative to monopoly levels for each of the discount/capacity cost combinations discussed above. The results are striking.

While it is not surprising that there are large differences between the non-cooperative profits and the other three scenarios, we find that including capacity as a strategic tool can have large effects on profits levels, especially when capacity costs are low. As the discount factor in the low cost scenario drops below 0.56, semi-collusive profits fall dramatically, while profits when firms collude both in prices and capacities remain near monopoly levels.

[24] This calculation is the profit at the non-cooperative symmetric candidate capacities. This provides an approximate average profit from asymmetric equilibria. These capacities are also used for the calculation of most-collusive prices at the non-cooperative capacities. See Lepore (2006) for the characterization of the symmetric candidate capacities.

7 Extending the model: Allowing for capacity flexibility

The model can be extended to allow for flexible capacity levels without a substantive alteration of the main results. We allow the capacity to be flexible in a specific way. That is, there is both a time lag to alter capacity and a fixed cost to adjust capacity. The time lag is denoted as $T \in \{1, 2, ...\}$ and is the number of pricing periods it takes to alter capacity. The adjustment cost $d \in \mathbb{R}_+$ is the fixed cost for any change from the current capacity level.

Proposition 3 *For any $T \in \{1, 2, ...\}$, there exists $d^*(T) \in \mathbb{R}_{++} \setminus \{\infty\}$ such that if $d \geq d^*(T)$, then the most-collusive equilibrium of the flexible capacity game involves capacities $x^c(c, \delta)$ and the prices $(p^c_t(c, \delta))^{\infty}_{t=1}$ for all $\delta \in (\underline{\delta}(c), 1)$.*

The argument is based on first showing that for all positive time lags, there is a finite fixed cost such that the optimal punishment after a deviation (price or capacity) does not involve altering of capacity levels. Second, we show that for all positive time lags, there exists a finite fixed cost such that no joint change in capacity levels from initial levels is ever profitable. The combination of these two facts guarantees that, for all d greater than the maximum of the two fixed costs: (i) at each pricing period and the initial capacity choice the optimization problems are identical to the inflexible capacity game, and (ii) at each period after the initial period it is not profitable to alter capacities to enhance collusive profits.

8 Conclusion

We establish a predictive theory of collusive pricing over demand cycles for homogeneous product industries with endogenous long run capacity and short run price competition. Two key features drive the results in our model: (i) because of the capacity constraints, gains from deviating from collusive prices do not increase monotonically with demand; and (ii) the loss after a deviation is different for two periods of identical demand, if they differ in location on the business cycle. The most-collusive pricing predictions depend on the capacity costs and fall into two categories. Our main pricing result is with regards to how prices compare on either side of the demand peak. If the marginal cost of capacity is high enough, pricing in two periods with the same demand will be at least as small (much of the time smaller) in the boom than in the recession. While, if the marginal cost of capacity is low enough, pricing in two periods with the same demand will be at least as large (much of the time larger) in the boom than in the recession; the interval of discounts where this is true grows towards the unit interval as the cost of capacity decreases towards zero. Finally, there is the possibility of a third region of costs in the middle where no blanket pricing patterns are true when comparing booms and recession.

The Bertrand price competition model in Haltiwanger and Harrington predicts that collusive prices are weakly lower in similar demand periods in recessions than in booms. The fact that, in our model, all pricing implications are not the same for all capacity costs, highlights the importance of this feature as a determinant of collusive pricing in any industry. The findings in this paper endorse the idea that cyclical variation in pricing is dependent on the expectation of future demand as suggested by Haltiwanger and Harrington, with the caveat that how these prices vary over the cycle depends heavily on the long run capacity cost in an industry.

9 Appendix I: Proof of primary results

Proof of Proposition 1. First we show that given the capacity is fixed at x, there exists $\overline{\delta}_p(x) \in (0,1)$ such that $(p_t^c(x,\delta))_{t=1}^{\tau} = (p_t^m(x))_{t=1}^{\tau}$ if and only if $\delta \in [\overline{\delta}_p(x), 1)$. We fix x, and analyze how the discount affects the gains $G_t^u(p_t^m(x), x)$ and the losses $L_t(p^m(x), x; \delta)$, for each period t. It is clear that $L_t(p^m(x), x; 0) = 0 \leq G_t^u(p_t^m(x), x)$, and $\lim_{\delta \to 1} L_t(p^m(x), x; \delta) \to \infty > G_t^u(p_t^m(x), x)$. Note the derivatives of the loss and gain function, in terms of δ, have the following characterizations; $\frac{\partial L_t(p^m(x), x;\delta)}{\partial \delta} > 0$ and $\frac{\partial G_t^u(p_t^m(x), x)}{\partial \delta} = 0$. Based on the continuity of $L_t(p^m(x), x; \delta)$ in δ, there exists a $\overline{\delta}_p(t)$ such that $L_t(p^m(x), x; \delta) \geq G_t^u(p_t^m(x), x)$ if and only if $\delta \in [\overline{\delta}_p(t, x), 1)$ for each t. This holds for all periods over a single cycle $t \in \{1, 2, ..., \tau\}$ if and only if $\delta \in [\overline{\delta}_p, 1)$, where $\overline{\delta}_p(x) = \max_{t \in \{1,..,\tau\}} \{\overline{\delta}_p(t, x)\}$. Define $\overline{\delta} = \max_{x \in [0,X]} \{\overline{\delta}_p(x)\}$. Therefore, for any $\delta \in [\overline{\delta}, 1)$ $\overline{p}_t^c(c, \delta) = p_t^m(x^c(c, \delta))$ for $t \in \{1, 2, ..., \tau\}$. Define the set[25]

$$\overline{\Sigma}(c) = \left\{\delta \in [0,1] \mid \forall \delta' \geq \delta,\ \Phi(c, \delta') \neq \varnothing \text{ and } (p_t^m)_{t=1}^{\infty} \in \Delta(x^c(c, \delta'), \delta')\right\}.$$

Note that by construction $[\overline{\delta}, 1] \subseteq \overline{\Sigma}(c) \subseteq [0, 1]$, and the constraint sets are closed in δ. A closed subset of a compact set is compact, therefore $\overline{\Sigma}(c)$ is compact and the $\min \overline{\Sigma}(c)$ exists. Define the discount factor $\overline{\delta}(c) = \min \overline{\Sigma}(c)$. This discount must weakly less than $\overline{\delta}$, thus $\overline{\delta}(c) \leq \overline{\delta} < 1$ and $\overline{\delta}(c)$ satisfies the statement of the proposition. ■

Proof of Proposition 2. Define the set of discount factors

$$\underline{\Sigma}(c) = \max\left\{\delta \in [0, \overline{\delta}(c)] \;\middle|\; \begin{array}{l} \forall \delta' \geq \delta,\ \Phi(c, \delta') \neq \varnothing \text{ and} \\ 0 \leq \min\limits_{t \in \{1,2,...,\tau\}} L_t((\overline{p}_t^c(c, \delta'))_{t=1}^{\infty}, x^c(c, \delta'); \delta') \end{array}\right\}.$$

$\underline{\Sigma}(c)$ is non-empty because, from the first proposition, $\overline{\delta}(c)$ must be in the set. Both constraints are continuous in δ, hence the constraint set is closed. $\underline{\Sigma}(c) \subseteq [0, \overline{\delta}(c)]$ a compact set, a closed subset of a compact set is compact, therefore $\underline{\Sigma}(c)$ is compact. Define the discount $\underline{\delta}(c) = \min \underline{\Sigma}(c)$ and notice that it meets the criteria of the statement of the proposition. ■

Proof of Theorem 1. The mixed strategy price wars can only occur in a demand period with parameter θ if $x^c(c, \delta) \in (r^*(\theta), X(\theta))$. For all cost $c \in (0, \overline{c})$ where $\overline{c} = \min\left\{c(\delta) \mid \delta \in \left[\inf \underline{\delta}(c), \sup \overline{\delta}(c)\right]\right\}$ the range of $x^c(c, \delta)$ is limited by c. If we denote

$$c_l(\theta) = \inf\{c \in (0, \overline{c}) | x^c(c, \delta) \in (r^*(\theta), X(\theta)) \text{ for some } \delta \in (\underline{\delta}(c), 1]\}$$

[25] The set $\Delta(x, \delta)$ is formally defined in section 10.2.2 equation 9. The set $\Phi(c, \delta)$ is formally defined in section 10.2.3 equation 10.

and

$$c_h(\theta) = \sup\left\{c \in (0,\overline{c}) | x^c(c,\delta) \in (r^*(\theta), X(\theta)) \text{ for some } \delta \in (\underline{\delta}(c), 1]\right\},$$

then (i) is immediate.

The second statement of the theorem, (ii) is true based on showing that both $r^*(\theta)$ and $X(\theta)$ are non-decreasing in θ. $X(\theta)$ is non-decreasing in θ based on assumption 2. We show that $r^*(\theta)$ is increasing θ by contradiction. Suppose that $\theta' > \theta$ and $r^*(\theta') < r^*(\theta)$.At $r^*(\theta)$ the following first order condition must hold for the Cournot game with profit function $P(q;\theta)q$:

$$\left.\frac{\partial P(q+r^*(\theta);\theta)}{\partial q}\right|_{q=r^*(\theta)} r^*(\theta) + P(2r^*(\theta);\theta) = 0.$$

Note that based on assumption 2 and 3, $P(2r^*(\theta);\theta') \geq P(2r^*(\theta);\theta)$ and $\left.\frac{\partial P(q+r^*(\theta);\theta')}{\partial q}\right|_{q=r^*(\theta)} \geq \left.\frac{\partial P(q+r^*(\theta);\theta)}{\partial q}\right|_{q=r^*(\theta)}$, respectively. Hence each firm's first order for $P(q + r^*(\theta);\theta')q$ at $r^*(\theta)$ is positive

$$\left.\frac{\partial P(q+r^*(\theta);\theta')}{\partial q}\right|_{q=r^*(\theta)} r^*(\theta) + P(2r^*(\theta);\theta') \geq 0.$$

Based on the strict concavity of $P(q + r^*(\theta);\theta')q$ the optimal choice $r(r^*(\theta);\theta')$ for the firm will exceed $r^*(\theta)$. The equilibrium quantity $r^*(\theta')$ is such that $r(r^*(\theta');\theta') - r^*(\theta') = 0$, hence $r(r^*(\theta);\theta') - r^*(\theta) \geq r(r^*(\theta');\theta') - r^*(\theta')$. Since, based on Lemma 1, Kreps and Scheinkman (1983), $r(q;\theta')$ is non-increasing in q, and $r(q;\theta') - q$ is decreasing in q, thus, $r^*(\theta') \geq r^*(\theta)$ contradict the previous inequality. ■

Proof of Theorem 2. First we prove that $\exists x_b \in [0,\overline{X}]$ such that $\forall x' > x_b$ if $\theta_{t'} = \theta_{t''}$ where $1 \leq t' < \widehat{t} < t'' \leq \tau$, then: 1-4. For all Θ, if $x = D(p_\tau^{um};\theta_\tau)$, then at or bellow this capacity the above statement is satisfied, because all pairs of comparable periods across the cycle have equal revenue at the constrained monopoly level. Therefore, the set of such capacities X_b is non-empty. Denote by x_b the least upper bound of X_b. Next we must show that for each $\delta \in (0,1)$ there exists a cost $c \in (0, c(\delta))$ such that $x^c(c,\delta) < x_b$. Denote by $\Xi(c,\delta)$ the choke capacity; the largest capacity such that

$$\sum_{t=1}^{\infty} \delta^{t-1}\left(p_t^m(x)\min\left\{x, \frac{D(p_t^m(x);\theta_t)}{2}\right\} - cx\right) = 0. \tag{2}$$

Note that (2) will hold at zero and $\Xi(c,\delta) > 0$ for all $c \in (0, c(\delta))$. The monopoly profit is strictly concave in x hence $\Xi(c,\delta)$ is a function.[26] $\Xi(c,\delta) = 0$ at $c = c(\delta)$ and $\Xi(c,\delta) \to +\infty$ as $c \to 0$. The monopoly profit is jointly continuous in c and x therefore $\Xi(c,\delta)$ is continuous in c. Based on the median value theorem there must be a cost $c \in (0, c(\delta))$ such that $\Xi(c,\delta) < x_b$. Clearly,

[26] See the proof of proposition 4 is Appendix II for proof of this statement.

$x^c(c,\delta) \leq \Xi(c,\delta)$ therefore the set of cost $\zeta_b = \{c \in (0, c(\delta)) \mid x^c(c,\delta) < x_b\}$ is non-empty. Denote by $c_b = \inf\{\zeta_b\}$, the smallest cost such that the statements 1-4 are true. ■

Proof of Theorem 3. In term of the exogenous capacity subgame, we need to show there exists a lowest capacity such that for all capacities greater pricing patterns follow statements 1-4. The proof of this is straightforward, first we show there exists an example of a capacity where the statements are true for it and all capacities greater. The capacity $x = \overline{X}$ is an example. Notice that at $x \geq \overline{X}$ the gain from defection and non-discounted loss from punishment is the exact same for all periods as the repeated Bertrand pricing model. Therefore the implications of Theorem 7 of Haltiwanger and Harrington (1991) applies to this model, and the statements of this theorem are true. So the set of capacities $X_r = \{x | \forall x' \geq x\ 1-4 \text{ are true}\}$ is not empty. The set is bounded, so the infimum of the set exists and we define it as $x_r = \inf X_r$.

Take $x_1 = x_2 = \overline{X}$ and an arbitrary discount factor $\delta \in (0,1)$ and we consider the possibility that (i) incentive constraint might not bind for some periods, (ii) only the under-cutting constraint might bind for some periods and (iii) the price might be bound to non-cooperative pricing in other periods.

(i) Consider two arbitrary comparable periods t' and t'' such that $1 \leq t' < \widehat{t} < t'' \leq \tau$ and $\theta_{t'} = \theta_{t''}$. Suppose the two comparable periods are such the constraint binds at least the recession period at $x_1 = x_2 = \overline{X}$, and look at property 2. $R^c_{t''}(\overline{X},\delta) < R^c_{t'}(\overline{X},\delta)$ for all $\delta \in (\underline{\delta}_p(\overline{X}), \overline{\delta}_p(\overline{X}))$ and based on the continuity of the two functions, there exists $\epsilon > 0$ small enough such that the revenues will still be ordered $R^c_{t''}(\overline{X}-\epsilon,\delta) < R^c_{t'}(\overline{X}-\epsilon,\delta)$ for $\overline{X}-\epsilon$.

(ii) Next consider any period with monopoly pricing, where both the incentive constraints are slack. Based on the continuity of the gains and losses in prices and capacities, there exists $\epsilon > 0$ small enough such that if

$$L_t(p^c(\overline{X},\delta), \overline{X}; \delta) > G^u_t(p^c_t(\overline{X},\delta), \overline{X})$$

for a period t, then

$$L_t(p^c(\overline{X}-\epsilon,\delta), \overline{X}-\epsilon; \delta) > G^u_t(p^c_t(\overline{X}-\epsilon,\delta), \overline{X}-\epsilon)$$

for $\overline{X}-\epsilon$ as well.

(iii) At the capacity levels $\overline{X}$ the non-cooperative pricing is $p^n_t(\overline{X}) = 0$ for all t. For all $\delta \in (1/2, 1)$ prices are above non-cooperative levels at capacities $\overline{X}$. Hence, by a continuity argument analogous to those above there exists $\epsilon > 0$ such that this must also be true for $\overline{X}-\epsilon$.

Therefore, we know that there exists x_r that satisfies the proposition and is less than $\overline{X}$. ■

Proof of Proposition 3. Throughout the proof, we fix T as an arbitrary positive integer. Denote by the single period profits $\pi_t^n(x,y) = R^n(x,y;\theta_t) - cx$.

First, we prove that non-cooperative equilibrium pricing in each period stage-game combined with not altering the initial capacity capacities is a subgame perfect equilibrium of the game with time lag T and fixed adjustment cost d. This is the case if, $\forall k \geq 1$, $x \in [\widehat{x}^c(c,\delta), \overline{X}]$, $(x',y) \in [0,\overline{X}] \times [0,\overline{X}]$ and $t = \{2,...\}$,

$$d \geq \sum_{s=t+T+1}^{T+t+k+1} \delta^{s-t}(\pi_s^n(x',y) - \pi_s^n(x,y)). \tag{3}$$

The y is permitted to be any capacity; this insures staying at the collusive capacity is a subgame perfect equilibrium for the non-deviator after any capacity cheat. The right hand side of (3) is bounded so there is a cost d such that (3) is true. Define the interval of such d where (3) is true as $D_1(T)$.

For a firm to credibly commit to increasing its capacity upon defection, it must be in its interest to do so, conditional on a defection. For this to be the case, the increase in discounted profits from a capacity change must be greater than the adjustment costs associated with this change. If not, the firm would rather punish using its collusive capacity level. A firm will only alter capacity if it is profitable to do so over some period of time $k \geq 1$. Condition (3) guarantees that no change from the most-collusive capacity levels will be beneficial to either firm, for any length of time during punishment.

There is also an interval of costs such that the firms will not benefit from jointly moving to a new capacity on the most-collusive equilibrium path. The same argument as above applies so that we can restrict ourselves to negating a benefit from a single deviation. Thus, if $\forall k \geq 1, \forall x \in [\widehat{x}^c(c,\delta), \overline{X}], \forall y \in [\widehat{x}^c(c,\delta), \overline{X}]$ and $\forall t = \{1,2,...\}$

$$d \geq \sum_{s=t+T+1}^{T+t+k+1} \delta^{s-t}(\pi_s^c(y,\delta) - \pi_s^c(x,\delta)). \tag{4}$$

The difference on the right hand side is finite. Hence, there exists a set finite real numbers that satisfy (4). Label this set of adjustment costs $D_2(T)$.

Next we show that staying with the most-collusive capacity is the optimal punishment. If we look at carrot/stick punishments in terms of capacities, then the optimal penal code is to not change capacity $x \in [\widehat{x}^c(c,\delta), \overline{X}]$, if for all carrots: $\widetilde{x}$, sticks: $\overline{x}$, and $t = \{2, ...\}$,

$$d \geq \frac{\delta^{T+1}}{(1+\delta)} \left(\pi^n_{t+T+1}(\overline{x}, \overline{x}) - \pi^n_{t+T+1}(x, \overline{x}) \right) + \sum_{s=t+T+2}^{\infty} \delta^{s-t} \left(\pi^c_s(\widetilde{x}, \widetilde{x}) - \pi^c_s(x, \overline{x}) \right) \tag{5}$$

If d is large enough, then there is no $\widetilde{x}$ and $\overline{x}$ such that it is optimal to alter x for punishment. In essence, the condition (3) guarantees that staying with x is part of the an optimal penal code. For all fixed T, the right hand side is bounded, therefore there exists an interval of costs such that (3) is true. Denote this interval by $D_3(T)$.

Define

$$d^*(T) = \min \left\{ d \in \mathbb{R}_+ \mid d \in \cap_{i=1}^3 D_i(T) \right\}. \tag{6}$$

The minimum is well defined because the constraint set is non-empty, closed and bounded below. Notice that $\forall d \geq d^*(T)$, the incentive constraints are the same for the the most-collusive equilibrium at all periods for pricing and initially for capacity. No deviation in capacities or prices, joint or singular, will be beneficial therefore, $\forall T \in \mathbb{N}$ and $\forall d \geq d^*(T)$ the outcomes $x^c(c,\delta)$ and $(\overline{p}^c_t(c,\delta))_{t=1}^{\infty}$ will also be the most-collusive solution to the flexible capacity game. ◾

10 Appendix II: Existence issues

10.1 Monopoly prices and capacities

Proposition 4 *There exists a unique Monopoly solution $x_i = x^m(c,\delta)$ and $(\overline{p}^m_t(c,\delta))_{t=1}^{T}$ for all $c, \delta,$ and Θ.*

Proof. The proof is broken down into four steps.

First we characterize each maximization problem of the pricing subgames. Based on the assumptions A.1-4 we show there is a unique solution to the problem constrained maximization problem

$$p^m_t(x) = \arg \max_{p \in [0, P(\theta)]} \{pD(p;\theta_t) | D(p;\theta_t) \leq x\} \text{ for all } \theta_t \in \Theta \tag{7}$$

for any subgame. If θ_t is such that $D(p;\theta_t) < x$, then the solution to the maximization (7) problem is the solution to the unconstrained problem $p^{um}_t = \arg\max_{p \in [0,P(\theta)]} \{pD(p;\theta_t)\}$ and the revenue is uniquely equal to $p^{um}_t D(p^{um}_t;\theta_t)$. If the constraint binds, then $D(p^m_t(x);\theta_t) = x$ and the price is uniquely determined by this equality by inverting the demand function $p^m_t(x) = P(x;\theta_t)$. Thus, the monopoly revenue of any period t is given by

$$R^m(x;\theta_t) = \begin{cases} p_t^{um} D(p_t^{um};\theta_t) & \text{if } D(p_t^{um};\theta_t) < x; \\ P(x;\theta_t)x & \text{if } D(p_t^{um};\theta_t) \geq x. \end{cases}$$

Second, we prove the monopoly revenue of each period is continuous, quasi-concave (strictly on the relevant region) and bounded.

Notice that for all θ_t, $R^m(x;\theta_t)$ is bounded for all $x \geq 0$ between 0 and $p_t^{um} D(p_t^{um};\theta_t)$ and non-decreasing in x. We first prove directly it is concave for all $x \geq 0$. This implies that for all $x \geq 0$

$$R^m(\lambda x + (1-\lambda)x';\theta_t) \geq \lambda R^m(x;\theta_t) + (1-\lambda) R^m(x';\theta_t) \text{ for all } \lambda \in [0,1]. \tag{8}$$

This is trivially true when $\min\{x,x'\} > D(p_t^{um};\theta_t)$ or $\max\{x,x'\} \leq D(p_t^{um};\theta_t)$. We are left to verify concavity when $x \leq D(p_t^{um};\theta_t)$ and $x' > D(p_t^{um};\theta_t)$. First suppose $x \leq D(p_t^{um};\theta_t)$, $x' > D(p_t^{um};\theta_t)$ and $\lambda x + (1-\lambda)x' \leq D(p_t^{um};\theta_t)$. Define $x_t^m = D(p_t^{um};\theta_t)$, then for all $\lambda \in [0,1]$,

$$\begin{aligned} P(\lambda x + (1-\lambda)x';\theta_t)\left(\lambda x + (1-\lambda)x'\right) &> P(\lambda x + (1-\lambda)x_t^m;\theta_t)\left(\lambda x + (1-\lambda)x_t^m\right) \\ &\geq \lambda P(x;\theta_t)x + (1-\lambda)P(x_t^m;\theta_t)x_t^m \\ &= \lambda P(x;\theta_t)x + (1-\lambda)p_t^{um}D(p_t^{um};\theta_t). \end{aligned}$$

If $x \leq D(p_t^{um};\theta_t)$, $x' > D(p_t^{um};\theta_t)$ and $\lambda x + (1-\lambda)x' > D(p_t^{um};\theta_t)$, then $p_t^{um}D(p_t^{um};\theta_t) \geq \lambda P(x;\theta_t)x + (1-\lambda)p_t^{um}D(p_t^{um};\theta_t)$ is true for all $\lambda \in [0,1]$. Hence, $R^m(x;\theta_t)$ is concave for all $x \geq 0$. We can further conclude that $R^m(x;\theta_t)$ is strictly concave on $x \in [0, x_t^m]$.

Next we prove the monopoly revenue of each cycle is continuous, quasi-concave (strictly on the relevant region) and bounded.

We denote $R^m(x,\delta) = \sum_{t=1}^{\tau} \delta^{t-1} R^m(x;\theta_t)$, because $\delta^{t-1} \in (0,1)$ for all t, and each $R^m(x;\theta_t)$ is bounded the weighted sum $R^m(x,\delta)$ is also bounded. The weighted sum of concave functions on is a concave function, therefore $R^m(x,\delta)$ is concave on $x \geq 0$. We need to show that $R^m(x,\delta)$ is strictly concave on $x \in [0, x_{\hat{t}}^m]$, where $x_{\hat{t}}^m = D(p_{\hat{t}}^{um};\theta_{\hat{t}})$. Define, $R_{-\hat{t}}^m(x,\delta) = \sum_{t\in\{1,2,\ldots\tau\}\setminus\hat{t}} \delta^{t-1} R^m(x;\theta_t)$, and note that it is concave for all $x \geq 0$. $R^m(x,\delta)$ is strictly concave on $x \in [0, x_{\hat{t}}^m]$, if

$$R^m(\lambda x + (1-\lambda)x',\delta) > \lambda R^m(x,\delta) + (1-\lambda)R^m(x',\delta) \text{ for all } x, x' \in [0, x_{\hat{t}}^m]$$

We can re-write this expression as

$$\begin{aligned} & R^m(\lambda x + (1-\lambda)x'; \theta_{\widehat{t}}) + R^m_{-\widehat{t}}(\lambda x + (1-\lambda)x', \delta) \\ > \; & \lambda R^m_{-\widehat{t}}(x, \delta) + \lambda R^m(x; \theta_{\widehat{t}}) + (1-\lambda) R^m_{-\widehat{t}}(x', \delta) + (1-\lambda) R^m(x'; \theta_{\widehat{t}}). \end{aligned}$$

Based on the concavity of $R^m_{-\widehat{t}}(x, \delta)$ on $[0, x^m_{\widehat{t}}]$, the expression reduces to

$$R^m(\lambda x + (1-\lambda)x'; \theta_{\widehat{t}}) > \lambda R^m(x; \theta_{\widehat{t}}) + (1-\lambda) R^m(x'; \theta_{\widehat{t}}) \text{ for all } x, x' \in [0, x^m_{\widehat{t}}]$$

which is true by the strict concavity of $P(x; \theta_{\widehat{t}})x$.

Finally, we show that the profit function of the cycle is strictly concave and bounded, i.e., the maximum exists and is uniquely labeled as $(x^m(c, \delta), (p^m_t)^{\tau}_{t=1})$.

We define the costs over a single cycle of x units of capacity as $C(\delta) = \sum_{t=1}^{\tau} \delta^{t-1} c$. The single cycle monopoly profit $\Pi^m(x, c, \delta) = R^m(x, \delta) - C(\delta)x$ is concave and bounded on all $x \geq 0$. This is enough to guarantee the existence of a maximum, but for uniqueness we first show that any $x > x^m_{\widehat{t}}$ is not a maximum of $\Pi^m(x, c, \delta)$. Suppose that $\widetilde{x} > x^m_{\widehat{t}}$ is a maximum of $\Pi^m(x, c, \delta)$, then $\Pi^m(\widetilde{x}, c, \delta) \geq \Pi^m(x^m_{\widehat{t}}, c, \delta)$, which is the same expression as $R^m(\widetilde{x}, \delta) - C(\delta)\widetilde{x} \geq R^m(x^m_{\widehat{t}}, \delta) - C(\delta)(x^m_{\widehat{t}})$. Note that $R^m(\widetilde{x}, \delta) = R^m(x^m_{\widehat{t}}, \delta)$, thus that cost function must be such that $C(\delta)\widetilde{x} \leq C(\delta)x^m_{\widehat{t}}$. This is a contradiction because, $C(\delta)x$ is an increasing function.

The monopoly capacity choice problem can be reduced to

$$x^m(c, \delta) = \arg\max \left\{ \Pi^m(x, c, \delta) | x \in [0, x^m_{\widehat{t}}] \right\}.$$

On $x \in [0, x^m_{\widehat{t}}]$, $\Pi^m(x, c, \delta)$ is strictly concave and the constraint defines a convex and compact (closed and bounded subset of $\mathbb{R}$) set. Therefore, the maximum $x^m(c, \delta)$ is unique, and from the initial argument of the proof, there is a unique monopoly price determined by $x^m(c, \delta)$, $\overline{p}^m_t(c, \delta) = p^m_t(x^m(c, \delta))$ for all t. ■

10.2 Most-collusive equilibrium

In this section, we characterize the optimal symmetric collusive capacities and prices of the two firms.

10.2.1 Symmetric capacities

Throughout the paper the analysis is restricted to the case of symmetric capacities. This is done primarily because, it is not obvious how to establish a focal most-collusive price when capacities are asymmetric without the explicit use of a collusive demand rationing rule. In the case of symmetric capacities, when the collusive price is such that $D(p_t;\theta_t) < 2x$, an implicit assumption is that half of the demand is given to each firm; a standard assumption of collusive models. When capacities are asymmetric there are many collusive demand rationing rules that reduce to $D_i(p_t;\theta_t) = D(p_t;\theta_t)/2$ for symmetric capacities. One example, without loss of generality take $x_1 \geq x_2$, firm 2's demand is

$$D_2(p_t;\theta_t) = \begin{cases} D(p_t;\theta_t)/2 & \text{if } x_2 \geq D(p_t;\theta_t)/2 \\ x_2 & \text{otherwise} \end{cases}$$

and while the demand of firm 1 is

$$D_1(p_t;\theta_t) = \begin{cases} D(p_t;\theta_t) - D_2(p_t;\theta_t) & \text{if } x_1 \geq D(p_t;\theta_t) - D_2(p_t;\theta_t) \\ x_1 & \text{otherwise} \end{cases}.$$

The above collusive rationing rule reduces to the one half-one half rule when $x_1 = x_2$. The problem is that under this rule the two firm can most often disagree on the profit maximizing collusive price. There is only one rule that reduces to the one half-one half rule, when capacities are symmetric, where the optimal collusive price is always the same for both firms. The demand is giving to the colluding firms proportionally to their capacities:

$$D_i(p_t;\theta_t) = \begin{cases} \frac{x_i}{x_1+x_2} D(p_t;\theta_t) & \text{if } x_i \geq \frac{x_i}{x_1+x_2} D(p_t;\theta_t) \\ x_i & \text{other wise} \end{cases}.$$

10.2.2 Characterization of optimal price choice

Denote by $\mathcal{P}_t$, the Borel probability measures on the price space $[0, P(\theta_t)]$, which are endow with the topology of weak convergence. Denote by $\mathcal{P}$, the cross product of the mixed strategy spaces for each t, $\mathcal{P} = \prod_{t=1}^{\infty} \mathcal{P}_t$ and denote by σ, an element of the set $\mathcal{P}$. We define the set of prices that satisfy the constraint set at a given capacity and discount factor as $\Delta(x, \delta)$. Formally,

$$\Delta(x,\delta) = \left\{ \sigma \in \mathcal{P} \mid \begin{array}{l} \min\left\{G_t^u(\sigma_t, x, x)(i), G_t^o(\sigma_t, x, x)(i)\right\} \\ \leq L_t(\sigma, x, x; \delta)(i)\ \forall i = \{1, 2\}\ \ \forall t = \{1, 2, ...\} \end{array} \right\}. \tag{9}$$

We assume that randomization devices are not publicly observable ex post and that firms only utilize trigger strategies where selecting price outside the support of the most-collusive pricing strategy is considered a cheat. This effectively eliminates mixed strategies as optimal choices unless they are non-cooperative equilibrium of that periods stage game.

Proposition 5 *$\forall x \in [0, \overline{X}]$ and $\forall \delta \in (0,1)$, there exists a unique most-collusive pricing solution $\sigma^c(x,\delta) = (p_t^c(x,\delta))_{t=1}^{\infty}$ such that*

$$\sigma^c(x,\delta) = \arg\max \left\{ \sum_{t=1}^{\infty} \delta^{t-1} \sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\} \middle| \sigma \in \Delta(x,\delta) \right\}.$$

Proof. Existence: First note that each function $\sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\}$ is both bounded and continuous on $\mathcal{P}_t$. Then, $\forall \delta \in (0,1)$ the function

$$\sum_{t=1}^{\infty} \delta^{t-1} \sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\}$$

is bounded and continuous on $\mathcal{P}$. The set of probability measures on a compact metric space is compact, hence each set $\mathcal{P}_t$ is compact . By Tychonoff's product theorem the set $\mathcal{P}$ is compact. The constraints are continuos in $\sigma \in \mathcal{P}$, hence the constraint set $\Delta(x,\delta)$ is closed. By construction $\Delta(x,\delta)$ is a subset of the space $\mathcal{P}$. $\Delta(x,\delta)$ is a closed subset of a compact space and therefore is compact. Since $(p_t^n(x))_{t=1}^{\infty} \in \Delta(x,\delta)$, the set $\Delta(x,\delta)$ is non-empty. A continuous function on a non-empty compact set always attains a maximum, therefore $\sigma^c(x,\delta)$ exists.

Uniqueness: Note that based on Proposition 4 the revenue function for each t has the unique maximum $p_t^m(2x)$. Also note that for any $x \in [0, \overline{X}]$,

$$\sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\} > \sigma_t'(\min\left\{x, \frac{D(\sigma_t';\theta_t)}{2}\right\} \text{ iff } \begin{array}{l} \sigma_t' < \sigma_t \le p_t^m(2x) \text{ or} \\ \sigma_t' > \sigma_t \ge p_t^m(2x). \end{array}$$

Now suppose that there are multiple maximizers. Denote by $\widehat{P}$ the set of maximizers and by $\widehat{P}_t$ the set of all prices for period t that are part of a maximizing price vector $\forall t = \{1,2,...\}$. Denote by $(\widehat{p}_t)_{t=1}^{\infty}$ the price vector where each $\widehat{p}_t$ is defined by

$$\widehat{p}_t = \arg\max \left\{ \sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\} \middle| \sigma_t \in \widehat{P}_t \right\}.$$

The higher the price is in the subgame at time t the harsher the loss from defection is in every other pricing period. The price vector $(\widehat{p}_t)_{t=1}^{\infty}$ yields the at least as much future loss $\forall t = \{1,2,...\}$

of all the price vectors in $\widehat{P}_t$, and hence the incentive constraints for each t must hold $\forall\sigma \in \widehat{P}$, therefore $(\widehat{p}_t)_{t=1}^{\infty} \in \Delta(x,\delta)$. By the construction of $(\widehat{p}_t)_{t=1}^{\infty}$, the total revenue in each period is maximal among $\widehat{P}_t$ because the functions are single peaked and strictly decreasing away from the peak. Thus, $\widehat{p}_t(\min\left\{x, \frac{D(\widehat{p}_t;\theta_t)}{2}\right\} > \sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\}$ $\forall\sigma_t \in \widehat{P}_t \setminus \widehat{p}_t$. Therefore, the total revenue of $(\widehat{p}_t)_{t=1}^{\infty}$ exceeds that of any other $\sigma \in \widehat{P}$:

$$\sum_{t=1}^{\infty}\delta^{t-1}\widehat{p}_t(\min\left\{x, \frac{D(\widehat{p}_t;\theta_t)}{2}\right\} > \sum_{t=1}^{\infty}\delta^{t-1}\sigma_t(\min\left\{x, \frac{D(\sigma_t;\theta_t)}{2}\right\} \quad \forall\sigma \in \widehat{P} \setminus (\widehat{p}_t)_{t=1}^{\infty}.$$

This contradicts the optimality of any non-singular set of strategies $\widehat{P}$. ■

10.2.3 Characterization of optimal capacity choice

The capacity pure strategy space is $\left[0, \overline{X}\right]$, where $\overline{X} = \max_{\theta\in\Theta} X(\theta)$. The most-collusive capacity incentive compatibility constraint is:

$$\Phi(c,\delta) = \left\{\xi \in \left[0, \overline{X}\right] \mid \Pi^c(\xi, c, \delta) \geq \max_{x_i}\left\{\Pi_i^n(x_i, \xi, c, \delta)\right\}\right\}. \tag{10}$$

The most-collusive must be a solution to the constrained maximization problem,

$$\xi^c(c,\delta) \in \arg\max\left\{\Pi^c(\xi, c, \delta) \mid \xi \in \Phi(c,\delta)\right\}.$$

We denote by $x^c(c,\delta)$ the most-collusive capacity choice; which is the smallest capacity in the set $\xi^c(c,\delta)$ when it is not a singleton.

Proposition 6 *If $\Phi(c,\delta)$ non-empty, then a solution $x^c(c,\delta)$ exists.*

Proof. Based on Proposition 5 we know that for any $\delta \in (0,1)$, $\sigma^c(x,\delta)$ is a function on $\left[0, \overline{X}\right]$. The first step is to prove that the function $\sigma^c(x,\delta)$ is continuous in x on $\left[0, \overline{X}\right]$. A function $p(x)$ is *continuous* at $\overline{x}$ if whenever $x^k \to \overline{x}$, $p^k = p(x^k)$ for all k and $\overline{p} = \lim_{k\to\infty} p^k$, we have that $\overline{p} \in p(\overline{x})$. To verify the continuity of each function $\sigma^c(\overline{x},\delta)$ in x, we suppose that there is a sequence $x^k \to \overline{x}$ and a sequence $p^k = \sigma^c(x^k,\delta)$ for all k such that $p^k \to \widetilde{p}$ and $\widetilde{p} \neq \sigma^c(\overline{x},\delta)$. Because the constraints hold for all k, taking the limit as $k \to \infty$ we can conclude that , $\widetilde{p} \in \Delta(x,\delta)$. Thus, $\widetilde{p}$ is a feasible price choice. However, since it is not an optimal choice in this set it must be that there exists $\overline{p}$ such that

$$\sum_{t=1}^{\infty}\delta^{t-1}\overline{p}_t(\min\left\{x, \frac{D(\overline{p}_t;\theta_t)}{2}\right\} > \sum_{t=1}^{\infty}\delta^{t-1}\widetilde{p}_t(\min\left\{x, \frac{D(\widetilde{p}_t;\theta_t)}{2}\right\}$$

and $\overline{p} \in \Delta(x,\delta)$. By the continuity of the function $\sum_{t=1}^{\infty} \delta^{t-1} p_t(\min\left\{x, \frac{D(p_t;\theta_t)}{2}\right\}$ there is a ρ arbitrarily close to $\overline{p}$ such that $\rho \in \Delta(x,\delta)$. If k is large enough, then $\rho \in \Delta(x^k,\delta)$, and it must be the case that

$$\sum_{t=1}^{\infty} \delta^{t-1} p_t^k(\min\left\{x, \frac{D(p_t^k;\theta_t)}{2}\right\} > \sum_{t=1}^{\infty} \delta^{t-1} \rho_t(\min\left\{x, \frac{D(\rho_t;\theta_t)}{2}\right\}$$

because, $p^k = \sigma^c(x^k,\delta)$. Next we take the limit as $k \to \infty$, and based on the continuity of the revenue function it must be that

$$\sum_{t=1}^{\infty} \delta^{t-1} \widetilde{p}_t(\min\left\{x, \frac{D(\widetilde{p}_t;\theta_t)}{2}\right\} > \sum_{t=1}^{\infty} \delta^{t-1} \rho_t(\min\left\{x, \frac{D(\rho_t;\theta_t)}{2}\right\}.$$

This is a contradiction, so this establishes that $\sigma^c(x,\delta)$ is continuous in x on $\left[0,\overline{X}\right]$. The sum of continuous functions is itself continuous and the cost function cx is continuous. Thus,

$$\Pi^c(x,c,\delta) = \sum_{t=1}^{\infty} \delta^{t-1} \left(p_t^c(x,\delta) \min\left\{x, \frac{D(p_t^c(x,\delta);\theta_t)}{2}\right\} - cx \right)$$

is continuous. Both $\Pi^c(\xi,c,\delta)$ and $\max_{x_i} \{\Pi_i^n(x_i,\xi,c,\delta)\}$ are continuous functions of ξ on $\left[0,\overline{X}\right]$. Thus, $\Phi(c,\delta)$ is a closed subset of $\left[0,\overline{X}\right]$. By construction $\Phi(c,\delta) \subseteq \left[0,\overline{X}\right]$ and hence is a closed subset of a compact set, therefore $\Phi(c,\delta)$ is compact. A continuous function on a non-empty compact set attains a maximum; if $\Phi(c,\delta) \neq \varnothing$, then $x^c(c,\delta)$ exists. ∎

11 Appendix III: Methodology for calculations

The problem is estimated using a backward induction approach. The most-collusive equilibrium pricing solution does not have a closed form when the capacity is not given a specific value. Therefore, at each discount factor a discrete grid of capacities is used and the most-collusive prices are found for each capacity. In the capacity stage the problem is a discrete maximization subject to the capacity incentive constraint.

11.1 Pricing stage

Given a fixed discount factor $\delta \in \left((\underline{\delta}(c), \overline{\delta}(c))\right)$ we use a discrete grid of capacities between one half the monopoly capacity at δ and 200.

Define the terms

$$z_t = p_t\left(\min\left\{\frac{\theta_t - 400p_t}{2}, x\right\}\right) \text{ for } t = 1, ..., 8,$$
$$y_t = p_t\left(\min\left\{\theta_t - 400p_t, x\right\}\right) \text{ for } t = 1, ..., 8,$$
$$r_t = \begin{cases} 0 & \text{if } x \geq \theta_t \\ \frac{(\theta_t - x)^2}{1600} & \text{if } x \in [\frac{\theta_t}{3}, \theta_t] \\ \frac{(\theta_t - 2x)}{400}x & \text{if } x \leq \frac{\theta_t}{3} \end{cases} .$$

For each capacity in the grid we solve the constrained optimization problem using penalty functions in the objective

$$(p_t^c)_{t=1}^8 = \arg \max_{\prod_{t=1}^8 [p_t^l, p_t^m(x)]} \left\{ \sum_{t=1}^{8} \left(\delta^{t-1} z_t - f\left(y_t - z_t - \sum_{s=t+1}^{t+8} \delta^{s-t}\left(z_s - r_s\right) \right) \right) \right\}. \qquad (11)$$

Where $f(\cdot)$ is a convex penalty function, with $f(0) = 0$. The price p_t^l is the pure strategy price that give the same expected revenue as the non-cooperative equilibrium pricing in period t. In the case that $p_t^c = p_t^l$ the program is re-run substituting r_t for both y_t and z_t and only optimizing in terms of the other period prices. We denote by $R^c(x, \delta)$, the revenue that comes from the solution of the program (11). This program is solved for the entire grid of capacities.

11.2 Capacity stage

For the same δ and a given cost c, we take the discrete set of capacities and solve the following problem. Define $R^n(z, x, \delta)$ as the non-cooperative equilibrium for a firm when it plays z and its rival plays x. Denote the most-collusive profit for the first cycle by

$$\Pi_8^c(x, c, \delta) = R^c(x, \delta) - \textstyle\sum_{t=1}^8 \delta^{t-1} cx.$$

Denote the maximal deviation non-cooperative profit for the first cycle by

$$\widehat{\Pi}_8^n(x, c, \delta) = \max_{z \in [0,200]} \left\{ R^n(z, x, \delta) - \sum_{t=1}^{8} \delta^{t-1} cz \right\}.$$

The most collusive capacity solves the discrete maximization problem:

$$\Pi^c(c, \delta) = \max_{x \in \{0,\ldots,200\}} \left\{ \Pi_8^c(x, c, \delta) \;\middle|\; \Pi_8^c(x, c, \delta) \geq \widehat{\Pi}_8^n(x, c, \delta) \right\}.$$

The method here is to run a program that starts at the most profitable symmetric capacity pair and checks if it is incentive compatible, then moves to the next most profitable capacity pair and so on. The first capacity pair that is incentive compatible is the most-collusive equilibrium capacities.

A Figures and Tables

A.1 Figures

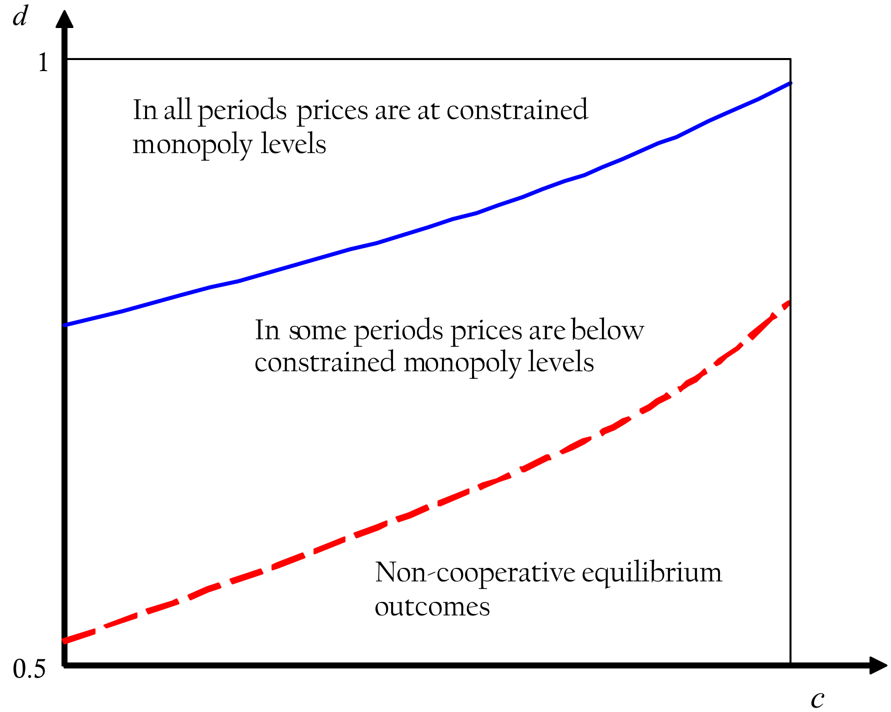

Figure 1: Basic most-collusive pricing regions, by discount and cost.

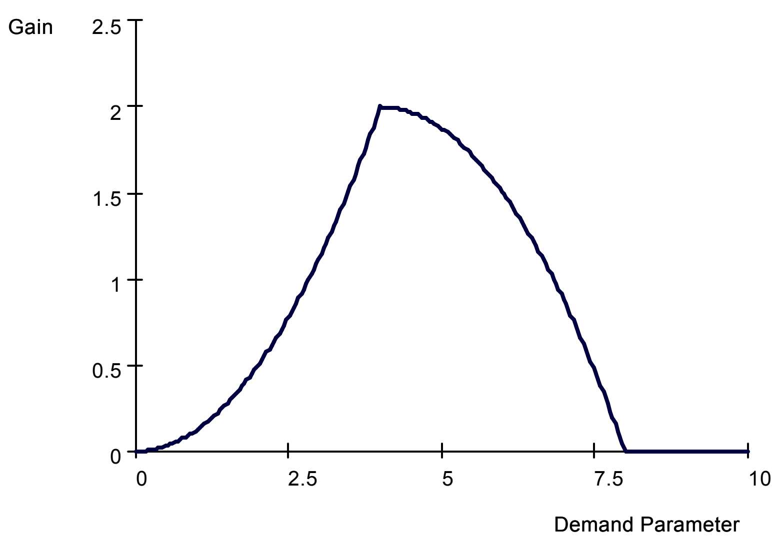

Figure 2: The immediate gain from a defection with capacity fixed.

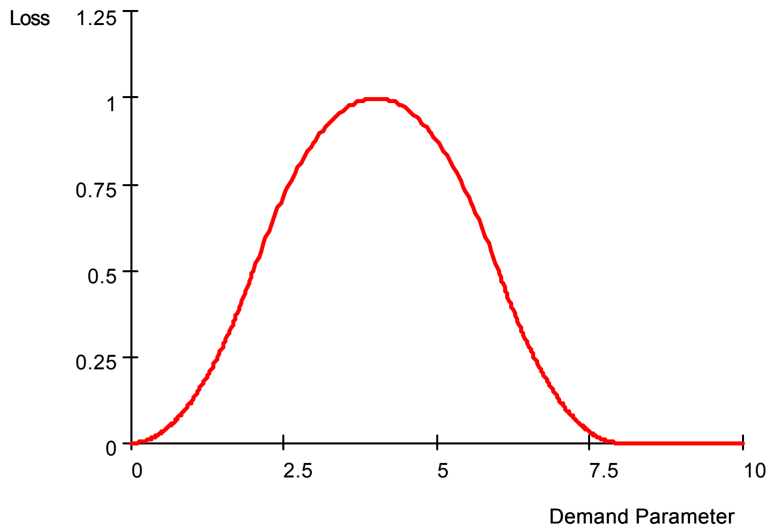

Figure 3: The single period loss after a defection with fixed capacity.

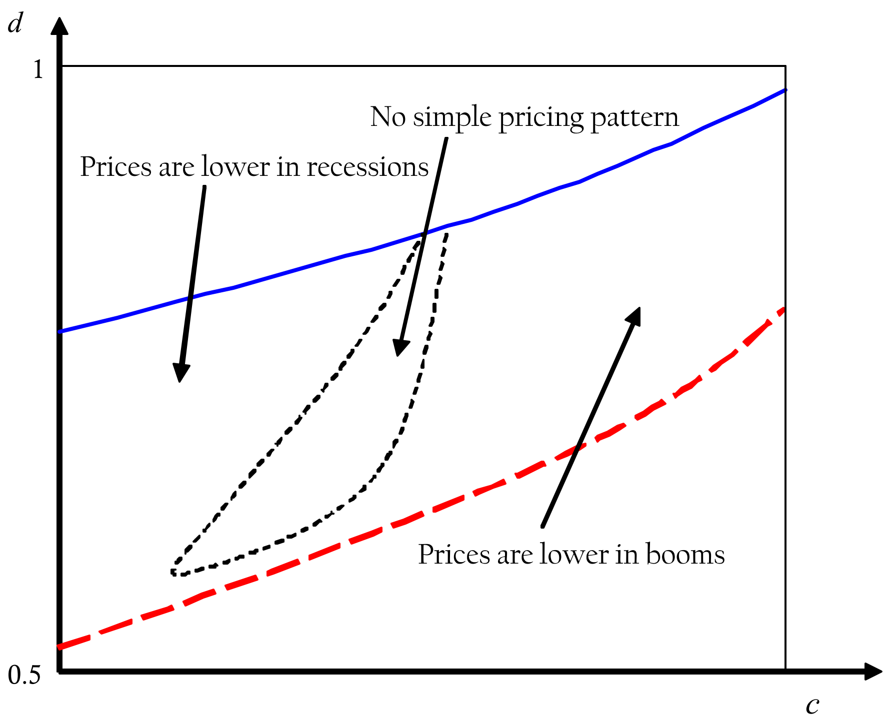

Figure 4: An example of most-collusive pricing patterns.

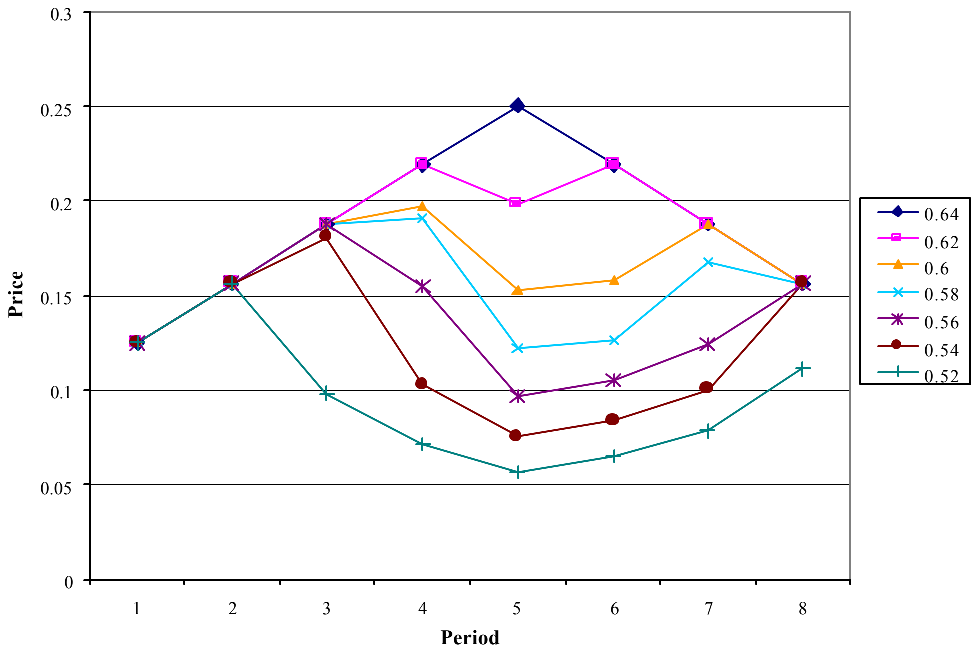

Figure 5: Most-collusive equilibrium pricing for low costs, $c = 0.000001$.

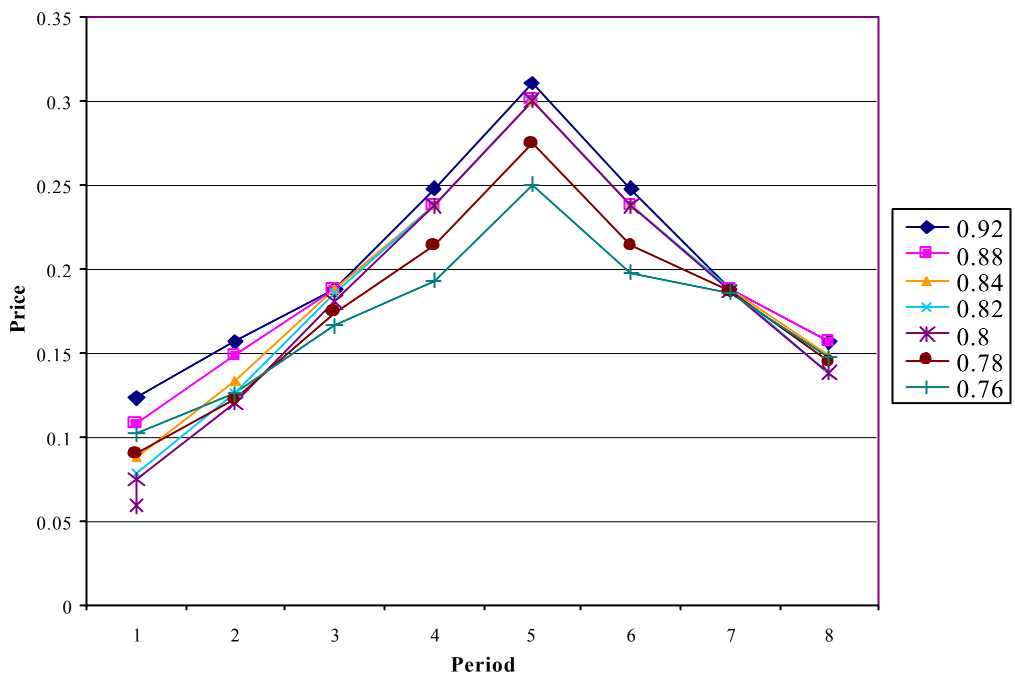

Figure 6: Most-collusive pricing for high costs, $c = 0.05$.

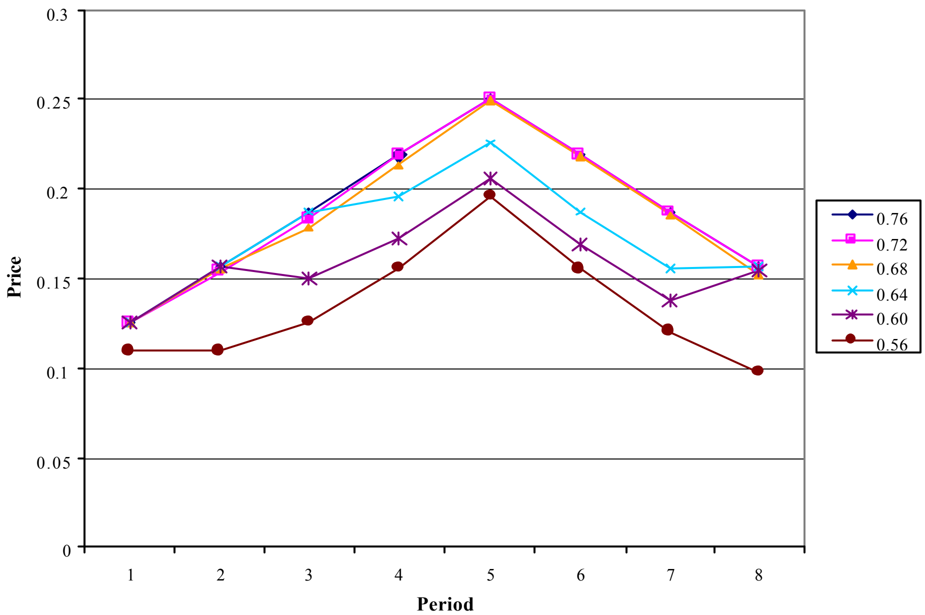

Figure 7: Most-collusive pricing for median costs, $c = 0.001$.

A.2 Tables

Table 1: Relative Expected Profits for Low Capacity Costs

			Collusion with	
Discount	Monopoly	Most-Collusive	Non-cooperative capacities	Non-cooperative
0.64	1	1	0.959	0.569
0.62	1	0.995	0.935	0.559
0.60	1	0.976	0.920	0.548
0.58	1	0.959	0.892	0.537
0.56	1	0.934	0.841	0.525
0.54	1	0.896	0.514	0.514
0.52	1	0.825	0.501	0.501

Table 2: Relative Expected Profits for High Capacity Costs

			Collusion with	
Discount	Monopoly	Most-Collusive	Non-cooperative capacities	Non-cooperative
0.92	1	0.982	0.945	0.839
0.88	1	0.977	0.941	0.839
0.84	1	0.961	0.935	0.839
0.82	1	0.947	0.928	0.840
0.80	1	0.937	0.918	0.840
0.78	1	0.903	0.903	0.842
0.76	1	0.849	0.845	0.845

Table 3: Relative Expected Profits for Medium Capacity Costs

			Collusion with	
Discount	Monopoly	Most-Collusive	Non-cooperative capacities	Non-cooperative
0.76	1	0.998	0.998	0.637
0.72	1	0.998	0.997	0.624
0.68	1	0.996	0.987	0.614
0.64	1	0.988	0.956	0.597
0.60	1	0.970	0.917	0.580
0.56	1	0.924	0.837	0.564

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