DOUBLY IMPLEMENTING THE RATIO CORRESPONDENCE WITH A "NATURAL" MECHANISM[1]

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ABSTRACT

In this paper we present a market-based mechanism for allocating public goods which implements the Ratio correspondence in both Nash and Strong Equilibria.

I.- INTRODUCTION

The *implementation problem* (see e.g. Thomson (1985) or Groves & Ledyard (1987)) arises when agents in the society have private information about the true nature of the environment. The planner is represented by a *choice correspondence* that associates with each economy a set of desired outcomes. Given that agents have private information about the true state, it is clear that they may use the information to manipulate the outcome. The planner's problem is to construct a *game form* -or *mechanism*- such that for each possible economy the equilibrium outcomes of the game coincide with those selected by the choice rule. If such a game exists we say that it *implements the choice correspondence*.

Samuelson conjectured that any decentralized ("spontaneous") mechanism for allocating public goods efficiently will be doomed to fail since "... it is in the selfish interest of each person to give false signals" (see Samuelson (1954) p. 388). This intuition can be stated formally by showing that no revelation game can implement a choice correspondence which selects Pareto efficient and individually rational outcomes (see Hurwicz (1972) for the private good case and Ledyard-Roberts (1975) for the public good case).

This is not, fortunately, the only approach to the problem. Using abstract strategy spaces, Hurwicz (1979) and Walker (1981), obtained implementation of the Lindahl correspondence in Nash Equilibrium when there are constant returns to scale (CRS) in the production of the public good and there are, at least, three agents. Such mechanisms, finally settled the question on the possibility of a decentralized solution to the allocation of public goods (an earlier mechanism proposed by Groves and Ledyard (1977) was not entirely satisfactory since individual rationality was not guaranteed). Moreover Walker's mechanism has several technically desirable features; it is continuous and has minimal dimensionality of the strategy space[(2)].

In this paper we study the implementation problem requiring additional properties on 1) the class of admissible economies 2) the form of the mechanism and 3) the solution concept. Let us take these points in turn.

We want to allow more general technologies than CRS. We would like to find a mechanism which implements a subcorrespondence of the core on the domain of all convex and some non-convex technologies. Furthermore it should be able to handle the case of two agents, which is excluded in the usual approach. A good candidate for such a choice correspondence is the Ratio equilibrium (RE) introduced by Kaneko (1977) (see also Moulin (1989) and Mas-Colell & Silvestre (1989)). RE allocations are always in the core as defined by Foley (1970) independent of returns to scale (while Lindahl equilibrium may fail to be in the core with decreasing returns). The set of RE allocations coincides with the set of Lindahl allocations when the technology is CRS, and when the technology exhibits increasing returns, RE may well exist. It is possible to show that a simple modification of Walker's mechanism can implement the ratio equilibrium when the number of agents is at least three[(3)].

2) See Tian (1989), (1990), for some very interesting extensions of Walker mechanism.

3) This mechanism can be described as follows. A strategy for agent i =

Turning to the second issue, we want to consider mechanisms which are simple, requiring a minimal amount of explanation to be understood. Walker's mechanism has minimal dimensionality since a strategy there plays the double role of a price and a quantity. But this may be confusing since information is too compressed, i.e. low dimensionality of the strategy space does not necessarily imply that the mechanism is "handy". Let us give an example which will clarify this point.

For those who live in a market economy there is no need to explain anybody how a price or a budget constraint look like: they know by everyday experience. Also the fact that costs must be covered for the production of any commodity to be positive can be taken as common knowledge. By contrast, in order to fill income tax declaration people need detailed instructions and even so, it is commonly acknowledged that mistakes are possible. This exemplifies that a simple mechanism is not necessarily one which people can understand and play adequately. The fact that the mechanism is natural, i.e. connected with everyday experience, may be crucial.

In other words, we believe that for economies in which markets allocate private goods it is desirable to mimic the market mechanism in the allocation of public goods. Therefore we should focus attention on mechanisms in which strategies are prices and quantities, and in which the outcome function incorporates both a budget constraint for each consumer and the firm. Even

1,...,n is a real number denoted by s(i). The outcome function for the public good (denoted by y) is as in Walker (1981), i.e. $y = \sum_{i=1}^{n} s(i)$. Let us denote by C(y)the cost function of the public good. Then, the tax payed by agent i is C(y) (1/n + s(i+1) - s(i+2)).

though Walker's mechanism incorporates a budget constraint for consumers, the personalized price payed by any single consumer is determined by her two neighbor's strategies. Additionally, the productive sector has no break-even constraint . Therefore it can not qualify as a market mechanism (this is also true for the revised version of Walker's mechanism in footnote 3))

Finally, virtually all known mechanisms implement in Nash equilibrium but do not address the problem of coalition formation (an exception is Schmeidler (1980)). However, it is not clear at all to us how coalitions can be forbidden. If coalitions are considered, and we use *strong equilibrium* as the solution concept, (see Maskin (1985) and Dutta and Sen (1988)), neither Hurwicz nor Walker mechanisms implement the Lindahl correspondence. On the other hand we do not want to impose all coalitions to form. Ideally we would like the simultaneous implementation of the choice correspondence in both Nash and Strong equilibria. This has been termed by Maskin ((1986) p. 201) *double implementation.*

In this paper we present a mechanism which closely resembles Lindahl's original approach and avoids the three unsatisfactory features mentioned above: 1) The mechanism works for any economy with non increasing returns to scale and for some with increasing returns 2) It fits well in everyday life experience -at least for agents living in market economies-, and 3) It double implements RE, i.e. strategic play by coalitions is allowed but not imposed.

In our mechanism a strategy for say consumer i, is a proportion of the total cost of producing public goods to be payed by i and an incremental (or decremental) proposal about the quantity of the public good. Under CRS this proportion is the Lindahl price payed by i. Under non CRS it can be interpreted as a very simple form of a non-linear price (this explains that a market-based mechanism can cope with some form of increasing returns). The outcome function is given by the budget constraint and by the fact that costs must be covered for production of the public good to be positive. It is surprising that a mechanism so close to Lindahl's idea can be proposed to overcome the incentive problem when research in the area began precisely because the incentive properties of those ideas were questioned!. Our approach also settles the question of the possibility of using markets to allocate efficiently public goods when incentives are taken into account. However, our mechanism is neither continuous nor does it guarantee individual feasibility out of equilibrium. We will show that a simple modification of our basic mechanism (doubly) implements continuously the ratio correspondence in a totally feasible way. However the rules of the game in this new mechanism do not correspond exactly with those of a market game. This suggest that a completely satisfactory mechanism requires **some** departure from a market-based society.

Our Proposition 1 says that any RE allocation is an allocation corresponding to the Nash Equilibrium (NE) of our mechanism. The idea behind our proof is very simple. If a consumer attempts to pay a lower cost ratio than in the RE, no production of the public good is undertaken and she will end up being worse. Therefore no one can profitably exploit the monopoly power that the mechanism gives her. This is closely related to limit theorems in Monopolistic Competition. As Hart (1979) proved, in large economies, monopolistic competition is Pareto Efficient because the graph of the demand function is not lower hemicontinuous. Hence, even if firms retain monopoly power, it does not pay to exploit it (see also Hart (1980), Makowsky & Ostroy (1987) and Benassy (1986)).

Proposition 2 provides a converse to Proposition 1 when the public good is essential (i. e. any allocation with positive consumption of the public good is preferred to any allocation with zero consumption of the public good for some agent). Under this assumption we prove that any NE allocation must be a RE. The key to this Proposition is that in order to maintain a positive production of the public good, each agent must act as a ratio-taker. This Proposition can also be compared with a result on the limiting properties of Monopolistic Competition which asserts that if all markets are open, any Monopolistically Competitive equilibrium is a Walrasian Equilibrium.

Next we consider Strong Equilibrium (SE). Proposition 3 asserts that we can sustain the RE allocation as a SE. The logic behind this is as follows. Suppose that some coalition upsets the RE allocation by means of a change in their strategies. However if the complementary coalition keeps their strategies (and therefore their proposed ratios) fixed, in order to maintain the production of the public good they have to propose a new vector of ratios such that they add up to the same amount than in a RE. Since no member of the coalition can pay a higher ratio and to be better off, the new proposal of ratios must be the same than in a RE and therefore no improvement is possible.

Finally Proposition 4 shows that any SE must yield a RE allocation. This follows from the fact that a SE must be a NE which in turn (by Proposition 2) yields a RE allocation. We remark that Propositions 3 and 4 do not need the additional assumption on preferences needed to prove Proposition 2 and that convexity of preferences is not needed in any of our results.

An outline of the paper is as follows. Section 2 presents the basic model and the main definitions. Section 3 presents our main results. Finally in Section 4 we show that a straightforward modification of our mechanism yields a new mechanism which is completely feasible and continuous and implements the ratio correspondence.

II.- THE MODEL

We consider an economy with $N = \{1,2,\ldots,n\}$ agents. Each consumer has preferences defined on R^2_+, representable by a utility function $U_i(x_i,y)$ strictly increasing in each argument, where x_i is agent i's allocation of the private good, and y the level of the public good. The initial endowment of the private good in the hands of i is $w_i \in R_{++}$. The technology for producing the public good is represented by a cost function C where C(y) is the amount of the private good required to obtain y units of public good. We will say that $(x_i,y)_{i=1,\ldots,n}$ is a *balanced allocation* if $C(y) \leq \sum_{i=1}^{n} (w_i - x_i)$.

__DEFINITION 1.-__ *A tuple* $(x_i,r_i,y)_{i=1,\ldots,n} \in R^{2n+1}_+$ *is a Ratio Equilibrium (RE) if it is balanced and* $\forall i \in N$

a) (x_i,y) *maximizes* $U_i(x'_i,y')$ *subject to* $x'_i \leq w_i - r_i C(y')$

b) $\sum_{i=1}^{n} r_i = 1.$

We will refer to the pair $(x_i,y)_{i=1,\ldots n}$ as a *Ratio allocation.* The *Ratio Correspondence R* selects for each economy in the domain, the set of RE allocations of that economy. The reader is reminded that on the domain of CRS economies RE is just a Lindahl Equilibrium where the Lindahl price for agent i is given by $p_i = r_i.c$. We now introduce the notion of the cost share mechanism.

DEFINITION 2.- *The Cost Share Mechanism is specified by the following components*

a) *Strategy sets* $S_i = [0,1] \times R \quad \forall i \in N$ *with generic element* $s_i = (r_i, y_i)$.

b) *Outcome function,* $V: S^n \longrightarrow R^{n+1}$ *where,*

$V(s_1, \ldots, s_n) = \left((w_1 - r_1 C(\sum y_i)), \ldots, (w_n - r_n C(\sum y_i)), \sum y_i\right)$ *if* $\sum_{i=1}^{n} r_i \geq 1$, *and*

$V(s_1, \ldots, s_n) = \left((w_1, 0), \ldots, (w_n, 0), 0\right)$ *otherwise.*

We interpret the first component of an agent's strategy space as the proportion of public expense she is willing to bear, and the second component as a incremental change in the level of public good. The first n components of the image of the outcome function are the consumption of agents 1,...,n of the private good and the last component is the consumption of the public good. Notice that if the shared proposed by agents add up at least to one, any agent can get as much public good as she likes. However if the sum of shares is less than one, no production of the public good is undertaken.

This mechanism, has a weak Individual Feasibility property since for any $(r_j, y_j)_{j=1}^{n}$ consumer i can achieve a bundle inside his consumption set by means of some strategy $(r_i, y_i) \in S_i$. However the mechanism is not Individually Feasible and therefore we have to assume that any point inside the consumption set is preferred to any point outside the consumption set (see Hurwicz (1979)). Finally the above mechanism is balanced since for every possible

2n-tuple of strategies the allocation in the image of the outcome function is balanced (this follows from adding up budget constraints).

When the meaning is clear we will use y for $\sum_{i=1}^{n} y_i$. Given a *strategy profile* $s = (s_1,\dots, s_n)$, we will employ the notation $U_i(s'_i,s_{-i})$ for $U_i\left(V_i(s_1,\dots, s_n)\right)$. Finally let us define our equilibrium concepts.

DEFINITION 3.- *A Nash equilibrium (NE) of the Cost Share Mechanism is a strategy profile* $(s_i)_{i=1,\dots,n}$, *such that given* s_{-i}, s_i *maximizes* $U_i(s'_i,s_{-i})$ *for each i.*

We will refer to an allocation that is the outcome of a NE as a Nash equilibrium allocation. Finally we define a Strong Equilibrium. A *coalition* is a non-empty subset of N.

DEFINITION 4.- *A strategy profile* $(s_i)_{i=1,..,n}$ *is a Strong Equilibrium (SE) of the Cost Share Mechanism if there does not exist a coalition S and a list of strategies* $(s^d_i)_{i\in S}$ *such that*

if $s'_i = s^d_i$ *for* $i \in S$ *and* $s'_i = s_i$ *for all* $i \notin S$, *then* $U_i(s'_i,s'_{-i}) \geq U_i(s_i,s_{-i})$ *for all* $i \in S$, *with strict inequality for some i.*

III.- RESULTS

PROPOSITION 1.- *Any Ratio equilibrium allocation is a Nash equilibrium outcome of the Cost Share Game.*

Proof: *Suppose* $(x'_i,y',r'_i)_{i=1,\ldots,n}$ *is a RE. Consider the strategy profile* $s_i = (r'_i,0)$ *for all agents except k, and* $s_k = (r'_k,y')$. *Then* $V(s) = (x'_1,\ldots,x'_n,y')$. *We claim that the above strategies are a NE. Consider a deviation by agent k. If she declares* $r_k < r'_k$, *then as* $\sum r_i < 1$, $V_i(s) = (w_i,0)$ *which by the individual rationality of RE allocations is (weakly) inferior to* (x'_i,y'). *Furthermore any feasible y can be attained by the strategy* (r'_k,y) *subject to* $s_k \leq w_k - r'_k C(y)$. *However, by the definition of a RE allocation the solution to this problem yields* $y = y'$. *The same reasoning holds for each of the other agent's strategies. Thus the above strategies constitute a NE yielding the RE allocation.*■

The converse to Proposition 1, namely that each NE outcome is a RE allocation is not always true. For instance assume a constant marginal cost c and that all players except i announce a zero ratio. Then i must pay c in order to have some public good, and for some preferences this may lead to choose zero quantity of the public good even though at a RE the quantity of the public good is positive. Repeating the argument for each consumer we get than there is a NE with no production of the public good. However we have the following partial converse to Proposition 1.

PROPOSITION 2.- *For any Nash equilibrium, $(s_1, \ldots, s_n)$ which leads to an allocation with $y > 0$, the announced ratios and NE allocation, $(s_i,r_i,y)_{i=1,\ldots,n}$ is a Ratio equilibrium.*

Proof: *If $y > 0$ then from the definition of V, $\sum_{i\in N} r_i^a \geq 1$. Then by the monotonicity of each U_i in private goods, and the requirements of a NE, $\sum_{i\in N} r_i^a = 1$. Furthermore by the definition of a NE, (x_i,y) maximizes $U_i(x_i,y)$ subject to $x_i + r_iC(y) \leq w_i$. Thus the triple $(x_i,r_i^a,y)_{i=1,\ldots,n}$ is a ratio equilibrium.* ■

The key fact driving the above results is that the outcome function V forces each agent to act as a ratio-taker where $r_j = 1-\sum_{i\neq j} r_i$. In a CRS economy, this reduces to being a price-taker, hence agents act as if they were competitive.

There are several ways to dispose of this kind of inefficient NE. First notice that if we assume that indifference curves do not meet the y-axis, the quantity of the public good must be positive at any NE. A slightly stronger assumption has been used by Tian (1989) in the context of completely feasible implementation(4). Second, it may be argued that any NE with no public good produced at all, is Pareto Dominated by RE which by Proposition 1 is also a NE Therefore it is unlikely that agents play the first NE (i.e. the efficient NE is a focal point). Third, the inefficient NE is not robust to the introduction of certain kind of trembles. This route has been taken by Bagnoli and Lipman

4) A difference with Tian is that our mechanism is not individually feasible. However, as we said above, a minor modification of our mechanism yields a new mechanism which is individually feasible (see Section 4). Therefore it is not surprising that we need such an assumption.

(1989) in a paper which bears some similarities with our approach[5]. In this paper we will take the first approach. In particular we will assume:

Assumption 1. $\forall\ i \in N\ \forall\ (x_i,y) \in \mathbb{R}^2_{++}\ U_i(x_i,y) > U_i(x'_i,0)\ \forall\ x'_i \in \mathbb{R}_+$

Under this assumption is easy to prove that at each NE the quantity of the public good is positive. Therefore under assumption 1 Proposition 2 implies that any Nash equilibrium is a RE. Formally

COROLLARY 1.- *Under Assumption 1 any Nash Equilibrium outcome is a Ratio Equilibrium allocation.*

The last two Propositions are devoted to show the equivalence between Strong equilibrium and RE.

PROPOSITION 3.- *Any Ratio Equilibrium allocation is a Strong Equilibrium outcome of the Cost Share Game.*

Proof: *Let $(x_i,y,r_i)_{i=1,\dots,n}$ is a RE. Consider the strategy profile s_i = $(r_i,0)$ for all agents except k, and $s_k = (r_k,y)$. Then $V(s) = (x_1,\dots,x_n,y)$. We claim that the above strategies are a SE. Suppose there exists some coalition S and strategies $(r^d_i,y^d_i)_{i\in S}$ such that if $s'_i = s^d_i$ for $i \in S$ and $s'_i = s_i$ for all $i \notin S$, and $U_i(s'_i,s'_{-i}) \geq U_i(s_i,s_{-i})$ for all $i \in S$, with strict inequality for some i. By the reasoning in Proposition 1, it we must have $\sum_{i\in S} r^d_i = \sum_{i\in S} r_i$. Suppose for some $i \in S$ $r^d_i > r'_i$, then the hypothesis that i benefits by playing*

5) Both papers focus on "natural" mechanisms in a market context. However, there are some differences: Their technology is of the (0,1) type, only quasi-linear utility functions are allowed and coalitions are not considered.

s'_i contradicts Proposition 1. Thus $r^d_i = r'_i$ $\forall i \in S$. Then by the definition of a RE there cannot exist a level of y such that any agent in S strictly benefits.■

PROPOSITION 4.- *For any Strong equilibrium, $(r^a_i,y^a_i)_{i=1,\dots,n}$ the announced ratios and subsequent allocation are a Ratio equilibrium.*

Proof: *Let $(r^s_i,y^s_i)_{i=1,\dots,n}$ be a SE. As any SE is a NE, if the SE allocation involves positive production then we apply proposition 2 to prove the result. Consider the case of a SE with $y^s = 0$. Then there are two cases,*

a) $(w_i,0)_{i=1,\dots,n}$ is a RE allocation, and we are done, or

b) $(w_i,0)_{i=1,\dots,n}$ is not a RE allocation. Let $(x'_i,r'_i,y)_{i=1,\dots,n}$ be a RE then we claim for some i $U_i(x'_i,y') > U_i(w_i,0)$. By the hypothesis that $(w_i,0)_{i=1,\dots,n}$ is not a RE allocation, for some i $\exists(x_i,y)$ such that i) $U_i(x_i,y) > U_i(x'_i,y')$ and ii) $x_i + r'_iC(y) \leq w_i$. If not we obtain a contradiction because $(w_i,r'_i,0)_{i=1,\dots,n}$ is also a RE. But then the coalition N, by playing strategies that yield the RE allocation, contradicts the hypothesis that $(r^s_i,y^s_i)_{i+1,\dots,n}$ is a SE.■

We end this Section by noting that Propositions 1-4 remain valid if the technology were of the (0,1) type -i.e. a public project of fixed size- and that many public goods can be considered. Both extensions can be carried out easily. The second requires additively separable cost functions.

IV.- FINAL COMMENTS

In this paper we have shown that a very simple and natural mechanism doubly implements the ratio correspondence even when there are only two agents. However, there are, at least, two unsatisfactory features of our mechanism: It is discontinuous and it is not individually feasible[6].

Discontinuity is usually taken to be bad because small mistakes when choosing strategies will result in allocations which are far away from the N.E.[7]. On the one hand it is known that no smooth mechanism can implement LE with CRS in NE when there are two agents only (see Vega (1985), see also Aghion (1985) for smooth and efficient market games). However continuous implementation still possible (see Kwan & Nakamura (1987)). On the other hand implementation in strong equilibrium has been carried out in discontinuous games (see Schmeidler (1980), see also Dutta & Sen (1988) for implementation by means of a modulo game and Jackson (1989) for a criticism of this procedure).

Also, recall that we extended the domain of agent's utility functions by postulating that any element in the consumption set is strictly preferred to any element outside it. This is clearly an artificial construction and the interpretation of negative consumptions is problematic. An alternative approach is to require that the mechanism results in an outcome that is *individually feasible*, that is in every agent's consumption set for every

6) It also requires double dimensionality than Walker's mechanism.

7) In a cooperative framework mistakes may be not frequent since (small) errors will be more easily spotted by coalitions.

possible strategy profile. This approach has been called *implementation by a completely feasible mechanism.* We will show that a small modification of our game doubly implements the RE correspondence with a continuous outcome function in a totally feasible way.

Let $r'_i = r_i$ *if* $\sum_{j\in N} r_j \geq 1$ and $r'_i = 1 - \sum_{j\neq i} r_j + (1- \sum_{j\in N} r_j)/n$ otherwise.

Let us define $y(s)$ as follows: If $x_i = w_i - r'_i C(\sum y_i) \geq 0 \ \forall i \in N$, $y = \sum y_i$. Otherwise $y(s)$ is arbitrary continuous function with range in the individually feasible set. Let our new game Γ_2 be as follows,

a) Strategy sets $S_i = [0,1] \times R \quad \forall i \in N$ *with generic element* $s_i = (r_i, y)$.

b) Outcome function, $V: S^n \longrightarrow R^{n+1}$ *where,*

$$V(s_1, \ldots, s_n) = \Big((w_1 - r'_1 C(y(s)), \ldots, (w_n - r'_n C(y(s)), y(s)\Big)$$

where $r'_i = r_i$ *if* $\sum_{j\in N} r_j \geq 1$ *and* $r'_i = 1 - \sum_{j\neq i} r_j + (1- \sum_{j\in N} r_j)/n$ *otherwise*

The idea of the mechanism is that if the proposed shares add up at least to one, any trader can get as much public good as she wishes subject to individual feasibility and the quantity of the private good is given by the budget constraint. However if proposed shares do not add up to one she is penalized and has to give up some additional quantity of the public good. This mechanism is balanced and continuous.

The mechanism is some kind of modified market game in which the two bad features of our cost-share mechanism are dispensed off at the cost of some complications. This mechanism can be regarded as a moderate reform of existing institutions. Notice that $\sum r'_i \geq 1$ always so costs are always covered. If strict inequality occurs the mechanism wastes some resources out of equilibrium. Since this mechanism works for the case of n=2 this is an unavoidable feature (see Kwan and Nakamura (1987)). Then, this game implements the Ratio correspondence in both Nash and Strong Equilibrium if all goods are essential, i.e. if $(x_1,y) >> 0$ $U_i(x_1,y) > U_i(x'_1,y')$ if either x'_1, or y' are equal to zero. This is a stronger assumption than A1.

Now we provide a sketch of the proofs of the equivalents to Propositions 1-4.

Proposition 1'. Any REA is a NEO of Γ_2.

Proof. Give to agents the CRE strategies. Suppose someone deviates. If the new declared ratio is greater or smaller than before her opportunity set shrinks. And by moving y_i alone no improvement is possible,

Proposition 2'. Any interior NEO of Γ_2 is a REA.

Proof. Ratios add up to one. And since the allocation is interior, any agent acts as a ratio-taker.

Proposition 3'. Any REA is a SEO of Γ_2.

Proof. Give to agents the RE strategies. Suppose that a coalition deviates. By the same reasoning as in Proposition 1 they must play the same ratios than in a RE and by moving the corresponding y_i alone no improvement is possible.

Proposition 4'. Any SEO of Γ_2 is a REA.

Proof. Under essentiality the NEO is interior. Then Proposition 2 yields the result.

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