A CHARACTERIZATION OF ACYCLIC PREFERENCES ON COUNTABLE SETS*

Carmen Herrero and Begoña Subiza**

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ABSTRACT

In this paper a new numerical representation of preferences (by means of set-valued real functions) is proposed. Our representation extends the usual utility function (in case preferences are preorder-type) as well as the pairwise representation (in case preferences are interval-order type). Then, we provide a characterization of acyclic preference relations on countable sets as those admitting a set-valued numerical representation.

I. INTRODUCTION.

Consider an agent who has to choose an element within a possibility set X, according to a binary relation $\succ$ defined on X, such that $x \succ y$ means that x is strictly preferred to y. Relation $\succ$ is called a *preference relation* whenever it is *asymmetric*, that is, in case $x \succ y$, then $(y \not\succ x)$.

In case the binary relation $\succ$ is a preference relation, then two other binary relations on X are defined in a standard way: the *preference indiference relation*, $\succsim$, such that $x \succsim y$ if and only if not$(y \succ x)$, (which turns out *reflexive* and *complete*), and the *indiference relation*, $\sim$, such that $x \sim y$ if and only if $x \succsim y$, and $y \succsim x$, simultaneously (which is *reflexive and symmetric*).

The agent's optimization problem is solved by looking for a $\succ$-maximal element in X, that is, an element $x^* \in X$ such that there is no $y \in X$, with $y \succ x^*$.

It is quite frequent to assume that agent's preferences are representable by means of a real function $u:X \to \mathbb{R}$, in such a way that $x \succ y$ if and only if $u(x) > u(y)$. Function u is referred to as a *utility function*, and then, an element is $\succ$-maximal if and only if it is a maximum of u. Assuming that agent's preferences are representable by a utility function ensures that both $\succ$ and $\sim$ are *transitive* [i.e., if $x \succ y$, and y ≻ z, then x ≻ z, and if x ~ y, y ~ z, then x ~ z]. In this case the preference relation is called *preorder*.

In a more general way, Fishburn (1970) studies the case in which the preference relation ≻ on **X** is representable by means of two real functions, $u,v:X \to \mathbb{R}$, $v(x) \geq u(x)$ $\forall$ $x \in X$, in such a way that x ≻ y if and only if $u(x) > v(y)$. If such a pair of real functions exist, and x* is a maximum of u (respectively, a maximum of v), then x* is a ≻-maximal.

The existence of previous pairwise numerical representation implies that the preference relation is *pseudotransitive*, that is,

$$x \succ y,\ y \succsim z,\ z \succ t, \text{ then } x \succ t$$

Notice that, under pseudotransitivity, the indiference relation, ~ is not necessarily transitive, but the strict preference relation, ≻ is transitive. Examples of pseudotransitive preference relations are *semiorders* [see Luce (1956), Jamison & Lau (1977), or Ng (1975)].

A pseudotransitive preference relation is usually called an *interval-order*.

Transitivity of both the strict preference and the indifference relations have been criticized since the early fifties as being unrealistically strong [see, for instance Armstrong (1950), May (1954) or Luce (1956)].

A property which is less restrictive than pseudotransitivity is *acyclicity*. The preference relation $\succ$ is acyclic if whenever we have finitely many elements, $x_1, x_2, \ldots, x_n$ in X such that $x_1 \succ x_2$; $x_2 \succ x_3$, ..., $x_{n-1} \succ x_n$, then $x_n \not\succ x_1$. Under acyclicity, neither $\succ$, nor $\sim$ are necessarily transitive. Moreover, both preorders and interval-orders are particular cases of acyclic relations.

Acyclic preferences have been widely studied in economic contexts of choice: consumer behaviour [see, for instance, Bergstrom (1975) and Walker (1977)], decision under risk [Rubinstein (1988)], social choice [Sen (1970)], etc. It is worth mentioning the results on existence of maximal elements on compact sets for acyclic preference relations [Sloss (1971), Brown (1973)].

Numerical representations of preferences have some obvious advantages. Aside from the fact that such numerical representation might throw in being interpreted as a welfare (or utility) index or an efficiency index, there can also be substantial economies of representation, storage and communication. Moreover, in dealing with the problem of characterizing $\succ$-maximal elements, we can move to the problem of getting maximum elements of a single-valued function, which is an easier one.

Our main concern is to provide numerical representations of preferences (or *utility correspondences*) for acyclic preference relations, extending the utility function representation (for the transitive case), as well as the interval representation (for the pseudotransitive case). In Section II, we present the concept of utility correspondence, and Section III is devoted to the main result, in which we characterize acyclic preference relations on countable sets as those admitting a utility correspondence. Section IV, with some final remarks, closes the paper.

II. UTILITY CORRESPONDENCES.

Let us consider the following definition:

Definition 1.- Let $\succ$ be a preference relation defined on a set **X**, and Φ:**X** ->> $\mathbb{R}$ a correspondence. We shall say that Φ is a *utility correspondence* for the preference relation $\succ$ if:

(a) $\forall\ x \in X$, $\Phi(x)$ is bounded, and

(b) $x \succ y \iff \Phi(x) \cap \Phi(y) = \emptyset$, and $\sup \Phi(x) > \sup \Phi(y)$.

It is obvious that, in case u:**X** -> $\mathbb{R}$ is a utility function representing the preference relation $\succ$, then u is a utility correspondence, which, in this case turns out single-valued. On the other hand, if the preference relation $\succ$ is an interval order, representable by functions u,v: X -> $\mathbb{R}$, then $\Phi(x) = [u(x),v(x)]$ turns out a utility correspondence for $\succ$.

The existence of a utility correspondence $\Phi:X \twoheadrightarrow \mathbb{R}$ representing the preference relation $\succ$, has some immediate consequences:

Proposition 1.- Let $\succ$ be a preference relation on X, for which a utility correspondence exists. Then, $\succ$ is acyclic.

Proof:

Suppose that $\Phi:X \twoheadrightarrow \mathbb{R}$ is the utility correspondence for $\succ$. We only need to observe that, in case there were a finite set $x_1, x_2, \dots, x_n$ in X, such that

$$x_1 \succ x_2 \succ \dots \succ x_n \succ x_1$$

then, $\sup \Phi(x_1) > \sup \Phi(x_2) > \dots > \sup \Phi(x_n) > \sup \Phi(x_1)$, which is a contradiction. Thus, $\succ$ is acyclic.

$\blacksquare$

Proposition 2.- Let $\succ$ be a preference relation on X for which an utility correspondence $\Phi:X \twoheadrightarrow \mathbb{R}$ exists. Then, $x^* \in X$ is $\succ$-maximal if and only if $\sup \Phi(x^*) \geq \sup \Phi(y)$, $\forall\ y \in X$ such that $\Phi(x^*) \cap \Phi(y) = \emptyset$.

Proof: Immediate.

III. A CHARACTERIZATION OF ACYCLIC PREFERENCES ON COUNTABLE SETS.

Proposition 1 ensures that, whenever a utility correspondence exists, then the preference relation is acyclic. In this section we investigate the possibility of obtaining a converse result.

Notice that it is not possible to obtain a pure converse result *for every set* X. Consider the following example: Let $X = \mathbb{R}\times\{0,1\}$, and let $\succ$ be the lexicographic order on X [$(a,i) \succ (b,j)$ if $a > b$, $\forall\ i,j = 0,1$, and $(a,1) \succ (a,0)$, $\forall\ a \in \mathbb{R}$]. Since $\succ$ is a complete preorder on X, $\succ$ is an acyclic preference relation on X. Nevertheless, $\succ$ is not representable by means of a utility correspondence:

Suppose there is a correspondence $\Phi: X \twoheadrightarrow \mathbb{R}$ representing $\succ$, then, $\sup \Phi(b,0) < \sup \Phi(b,1)$, $\forall\ b \in \mathbb{R}$.

In this case, we can construct a function $g: \mathbb{R} \to \mathbb{Q}$, by associating to every $b \in \mathbb{R}$, a rational number $g(b)$ in such a way that

$$\sup \Phi(b,0) < g(b) < \sup \Phi(b,1).$$

Notice that function g constructed in the previous way turns out injective, since

$$\sup \Phi(b + \delta,1) > \sup \Phi(b + \delta,0) > \sup \Phi(b,1) > \sup \Phi(b,0),$$

$\forall\ \delta > 0$, and $\forall\ b \in \mathbb{R}$.

But this leads to a contradiction, since $\mathbb{Q}$ is a countable set, and $\mathbb{R}$ is not.

It is interesting to observe that, similarly, in case $\succ$ is a preorder or an interval-order, additional conditions are needed whenever X is neither a finite set nor a countable one in order to get a representation by means of a (single-valued) utility function and a pair of functions, respectively. See Debreu (1954) (1964), Fishburn (1970) (1983) or Monteiro (1987) for the preorder case, and Bridges (1983a) (1985) or Chateauneuf (1987), for the interval-order case.

Nevertheless, in case X is a finite or countable set, we obtain the following result:

Proposition 3.- Let X be a finite set, and $\succ$ an acyclic preference relation on X. Then, there exists a utility correspondence for $\succ$.

Proof:

Since X is a finite set and $\succ$ is an acyclic preference relation, any subset of X has, at least a $\succ$-minimal element, that is for any $Y \subset X$ there exists $y^* \in Y$ such that there is no $y \in Y$, $y^* \succ y$. Then, we may consider the elements of X in the following way:

$$x_1 = \succ\text{-minimal in } X$$

$$x_2 = \succ\text{-minimal in } (X - \{x_1\})$$

....

$$x_{k+1} = \succ\text{-minimal in } (X - \{x_1,\dots,x_k\})$$

In such a way, we consider now $X = \{x_1,\dots,x_p\}$, as obtained before. Then, if $x_i \succ x_j$, we conclude that $i > j$.

For every $i = 1,\dots,p$, let $a_i = \sum_{n\leq i} \frac{1}{3^n}$, and define the following function Φ: $\mathbf{X} \twoheadrightarrow \mathbb{R}$,

$$\Phi(x_i) = \{(a_i + a_k) \text{ if } x_i \sim x_k,\ x_i \neq x_k\} \cup \{3 - \tfrac{1}{i}\}$$

Notice that, for every $x_i \in X$, $\Phi(x_i) \subset [0,3]$, so $\Phi(x_i)$ is a bounded set.

We claim that Φ is a utility correspondence for $\succ$:

First, notice that, in the definition of $\Phi(x_i)$, we consider two different parts: the set A_i of elements $(a_i + a_k)$, for $x_i \sim x_k$, contained in $]0,1]$, and the element $\sup \Phi(x_i) = 3 - \frac{1}{i}$, which is in $[2,3[$. As a consequence, in order to check condition $\Phi(x_i) \cap \Phi(x_j) = \varnothing$, we only need to check that $A_i \cap A_j = \varnothing$, and $\sup \Phi(x_i) \neq \sup \Phi(x_j)$.

It is clear that, in case $x_i \sim x_j$, then $A_i \cap A_j \neq \varnothing$.

Suppose now that $x_i \succ x_j$, then $i > j$, and $3 - \frac{1}{i} > 3 - \frac{1}{j}$, that is, $\sup \Phi(x_i) > \sup \phi(x_j)$. Moreover, $(a_i + a_k) \neq (a_j + a_h)$, for every k,h such that $x_i \sim x_k$, $x_j \sim x_h$, that is, $A_i \cap A_j = \varnothing$. Then we can conclude that $\Phi(x_i) \cap \Phi(x_j) = \varnothing$.

Conversely, suppose that

$$\Phi(x_i) \cap \Phi(x_j) = \varnothing, \text{ and } \sup \Phi(x_i) > \sup \phi(x_j)$$

then, since $\Phi(x_i) \cap \Phi(x_j) = \varnothing$, x_i and x_j are not indifferent; on the other hand, $\sup \Phi(x_i) > \sup \phi(x_j)$ implies that $i > j$ and then $x_j \succ x_i$. So, we can conclude that $x_i \succ x_j$, and Φ is a utility correspondence for $\succ$.

∎

Proposition 4.- Let X be a countable set, and $\succ$ an acyclic preference relation on X. Then, there exists a utility correspondence for $\succ$.

Proof:

Let $X = \{x_1, x_2, \ldots\}$. For every $i \in \mathbb{N}$, we consider $a_i = \sum_{n \le i} \frac{1}{3^n}$, and $A_i = \{-(a_i + a_k) \text{ if } x_i \sim x_k,\ x_i \neq x_k\}$.

We shall write $x \succ\succ y$ $(x,y \in X)$ if there exist finitely many elements $x_1 = x, \ldots, x_n = y$ of X such that $x_1 \succ \ldots \succ x_n$.

For $x \in X$, let $S(x) = \{n \in \mathbb{N} \mid x \succ\succ x_n\}$, and define

$$u(x) = \begin{cases} \sum_{n \in S(x)} \frac{1}{2^n} & \text{if } S(x) \neq \emptyset \\ 0 & \text{if } S(x) = \emptyset \end{cases}$$

Consider now $\Phi : X \twoheadrightarrow \mathbb{R}$ such that $\Phi(x_i) = A_i \cup \{u(x_i)\}$. For any $x \in X$ $\Phi(x)$ is bounded ($\Phi(x) \subset [-1,1]$). Let us now to check that Φ is a utility correspondence for $\succ$:

Notice that $\forall$ i, $A_i \subset [-1,0[$, and $u(x_i) \in [0,1]$.

Thus, $u(x_i) = \sup \Phi(x_i)$. and $\Phi(x_i) \cap \Phi(x_j) = \emptyset$ if and only if $A_i \cap A_j = \emptyset$, and $u(x_i) \neq u(x_j)$.

Suppose $x_i \succ x_j$. Then, $A_i \cap A_j = \emptyset$, and $S(x_i) \supset S(x_j)$. In this case, $u(x_i) \ge u(x_j)$. Also, $j \in S(x_i)$, but, as $\succ$ is acyclic, $j \notin S(j)$. Thus, $u(x_i) > u(x_j)$.

Conversely, if $\Phi(x_i) \cap \Phi(x_j) = \emptyset$, and $u(x_i) > u(x_j)$, then x_i and x_j are not indifferent. Moreover, in case $x_j \succ x_i$, we have $i \in S(x_j)$, and $u(x_j) > u(x_i)$, against the hypothesis. Thus, $x_i \succ x_j$, and we conclude that Φ is a utility correspondence for $\succ$.

■

From Propositions 1, 2 and 3 we obtain the following characterization theorem:

Theorem 1.- Let X be a finite or countable set, and $\succ$ a preference relation on X. Then, the following conditions are equivalent:

(a) $\succ$ is acyclic

(b) there exists a utility correspondence for $\succ$.

Since utility functions and interval representations are particular cases of utility correspondences, Theorem 1 can be seen as an extension of the analogous representation results, in the countable case, for preorders [see Debreu (1954)] and interval-orders [see Fishburn (1970) or Bridges (1983b)].

IV. FINAL REMARKS.

In this paper we propose a new concept of utility function extending previous numerical representations of preference relations (as single-valued utility functions, in case of transitive preference relations, and pairwise function representations, in case of pseudotransitive preference relations). Then, in case the opportunity set X is either finite or countable, we prove that a preference relation $\succ$ on X is acyclic if and only if it is representable by means of a utility correspondence.

Although the assumption of countable set X is relevant in some specific contexts (e.g., social choice), theoretical economists usually deal with the case where X is a closed, convex subset of $\mathbb{R}^n$. For such X, a representation theorem requires additional assumptions. See Debreu (1954), (1964), Fishburn (1970) (1983) or Monteiro (1987) for the preorder case, and Bridges (1983a) (1985) or Chateauneuf (1987), for the interval-order case.

Finally notice that, in the case whereby the preference relation $\succ$ (on X) is representable by means of a utility correspondence Φ:X ->> $\mathbb{R}$, then $x^* \in X$ is maximal if and only if sup $\Phi(x^*) \geq$ sup $\Phi(x)$, $\forall\ x \in X$ such that $\Phi(x) \cap \Phi(x^*) = \emptyset$. Therefore, we may use function sup Φ in order to get maximal elements of $\succ$.

- EN BLANCO -

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