MONOPOLY EXPERIMENTATION[*]

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MONOPOLY EXPERIMENTATION

Abstract

This paper considers a firm facing an uncertain demand curve. The firm can experimentally adjust its output in order to gain information that will increase expected future profits. We examine two basic questions. Under what conditions is it worthwhile for the firm to experiment? How does the firm adjust its output away from the myopic optimum to exploit its ability to experiment? Two necessary conditions are established for experimentation to occur, involving requirements that experimentation be informative and that information be valuable. Conditions are then established for experimentation to induce the firm to increase or decrease quantity. These results, which contain several previous analyses as special cases, provide an understanding of experimentation that will be useful in a number of applications.

MONOPOLY EXPERIMENTATION

I. Introduction

A growing strand of the literature on the economics of information deals with experimentation, or the incentive for economic agents to adjust present actions in order to acquire information that increases expected future returns. Analyses of experimentation have appeared in macroeconomics, consumer theory, and monopoly and duopoly theory. In addition, the potential for further applications is rich. In light of this, it would be useful to identify the key factors in the economics of experimentation.

In this paper we examine experimentation in the simplest and most accessible terms possible. We study a monopoly which faces a random demand curve with an uncertain mean demand. The monopolist has a prior distribution of the (two) possible means. The monopoly is assumed to maximize profits over a two-period horizon. Incentives to experiment arise because the firm can strategically vary output in the initial period in order to collect information concerning the true mean of the demand function. This information will increase expected profits in the second period.

We examine two basic questions. Under what conditions is it worthwhile for the firm to experiment? How does the firm adjust its output level away from the single-period or myopic optimum in the initial period to exploit its ability to obtain information by experimenting?

Three approaches to the question of experimentation have appeared in the literature. The common technique is to model demand (or, alternatively, the utility function of a consumer) as being subject to random fluctuations which are independent of all endogenous variables, so that the experimenting agent gains imprecise information in each period concerning the underlying demand or utility parameters. In this context, one approach involves formulating an infinite horizon model in which attention naturally turns to the limiting expectation of the firm. In particular, one can ask whether the firm will eventually learn the true state of demand with certainty. Examples of this type of analysis include MacLennan (1984), Easley and Kiefer (1988a, 1988b), Kihlstrom, Mirman and Postlewaite (1984), Rothschild (1974), and Kiefer (1987, 1988a,b). A basic result in many of these models is that with positive probability the agent may converge to an incorrect expectation. A second approach addresses the issue of how the ability to collect information by experimentation affects the firm's behavior. The analysis is restricted to two periods and attention is focussed on how the opportunity for experimentation affects the period-one output or consumption level. Examples include Prescott (1972), Grossman, Kihlstrom and Mirman (1977), Fusselman and Mirman (1988) and Mirman and Urbano (1988). The basic result, established by Prescott (1972), Grossman, Kihlstrom and Mirman (1977) and Fusselman and Mirman (1988), is that with a demand curve with known vertical intercept and unknown slope, the agent will experiment by expanding first-period output to collect information.[1] Mirman and Urbano (1988) have recently studied the forces behind this result by showing that experimentation does not occur in a duopoly model when the uncertainty concerns only the demand curve intercept.[2]

In this paper we follow the second approach in examining a two-period model and focus on the effect of experimentation on first-period output levels.[3] Our first set of results establish two necessary conditions for experimentation. Intuitively, they are that information must be generated by experimentation and that the information that is generated must be useful. To illustrate these conditions, we show that if the slopes of the two possible mean demand curves are equal at every quantity, then the information content of all quantity decisions is equal and no information is generated by experimentation. Hence there is no incentive to experiment and the one-period (myopic) output remains optimal.[4] We next show that if one mean demand curve is a multiple of the other, then information has no value and there is again no experimentation. The key to this last result is to show that the monopolists' optimal quantity does not depend on the firm's information in this case, rendering the latter valueless.

These results provide some insight into two previous findings. First, this type of analysis has its roots in Grossman, Kihlstrom and Mirman (1977) (GKM). GKM examine two models of an experimenting monopolist. Quantity is the choice variable in the first model and price is the choice variable in the second model. It is shown that the quantity-setting firm increases output due to experimentation while the price-setting firm increases prices and reduces quantity. These results may initially appear paradoxical because it should seemingly make no difference whether the monopoly chooses price or quantity. GKM explain that these results are not contradictory since, in the course of switching from quantity to price-setting firms, the structure of demand is altered. Somewhat surprisingly, we show that if one considers the alternatives of a quantity and price-setting monopoly <u>without</u> altering the demand structure, then the quantity-setting firm will experiment by increasing quantity while the price-setting firm will not experiment. This occurs because information is valuable in the quantity-setting case but not in the price-setting case. Hence, when learning through experimentation is possible the optimal action <u>does</u> depend upon whether the firm chooses price or quantity. Second, it is interesting to note that our analysis of the conditions under which experimentation will not occur is reminiscent of an example of Fusselman and Mirman (1988) (FM) in which a Cobb-Douglas consumer does not experiment. To confirm the consistency of these results we note that information is again valueless in the FM example. In particular, the fact that information is valuable turns up in the convexity of the period-two value function. In both our examples and the Fusselman-Mirman example in which information is not valuable, the value function is linear.

Having established conditions under which experimentation will occur, our attention turns to the form which it takes. Does the opportunity to experiment induce the firm to increase or decrease period-one quantity? Our second set of results establish sufficient conditions for each case. Intuitively, the firm adjusts its period-one quantity to push the mean demand curves further apart. This spreads apart the distributions from which the random variable, price, might be drawn and makes price a more informative signal of the true distribution.

In the course of this analysis, we provide a new proof for the special case generally examined in the literature: a monopoly faced with two demand curves, the more favorable of which is also less steeply sloped, will experimentally increase quantity. The proof that we provide is an alternative to Fusselman and Mirman's (1988) generalization of Grossman, Kihlstrom and Mirman (1977) and is of special significance because it is analytically more tractable than previous analyses and thus presents the first prospect of generalizing several of the results contained in this literature. For example, this proof appears to readily generalize to encompass a finite number of possible values for the unknown parameter. Moreover it can be used to derive results under more general assumptions on the specification of the random demand curve, including the possibility that the shape as well as the mean of the distribution of price may depend on the value of output chosen. The latter generalization is important because letting the shape of the distribution depend on output changes the informational content of experimentation and may change the direction of experimentation (see Creane (1989)).

In the following section the basic model is constructed. Section III establishes necessary conditions for experimentation to occur, including conditions for information to be valuable and obtainable via experimentation. Sections IV and V examine conditions under which experimentation induces the firm to increase and decrease quantity, respectively. The forces which drive these results are examined in Section VI.

II. The Model

Consider a monopoly which maximizes the (undiscounted, for convenience) sum of profits from period-one and period-two sales. For simplicity, we assume there are no costs. The demand curve is given by $P = g(\gamma,Q) + \varepsilon$, where P is price, Q is quantity, γ is a parameter unknown to the monopolist, and ε is a random variable. The parameter γ can take on two values, $\gamma \in \{\underline{\gamma},\bar{\gamma}\}$. Intuitively, we can think of $\bar{\gamma}$ as representing a state of good demand. The a priori probability that $\gamma = \bar{\gamma}$ is $\rho_0 = \text{Prob}\{\gamma = \bar{\gamma}\}$. The random variable ε is characterized by density $f(\varepsilon)$. We assume that the expected value of ε is zero ($E(\varepsilon) = 0$) and (for convenience) that $f(\varepsilon)$ has full support on the entire real line, $\mathbb{R}$. Finally, we assume that $f(\varepsilon)$ satisfies the monotone likelihood ratio property (MLRP). In particular, let

$$L(\varepsilon) = \frac{f'(\varepsilon)}{f(\varepsilon)} .$$

Then f has the MLRP iff L is a continuous and nonincreasing function.

The firm chooses a quantity Q in period one and observes the price $P = g(\gamma,Q) + \varepsilon$. The randomness of ε prevents the firm from inferring the value of

γ from its price observation. Instead, the firm uses Bayes' rule to construct a posterior probability, denoted ρ, that $\gamma = \bar{\gamma}$. The firm then chooses its period-two quantity Q_2 and receives price $P_2 = g(\gamma, Q_2) + \varepsilon$. The firm maximizes the sum of period-one and period-two profits. An incentive to experiment arises because the firm may be able to adjust its period-one quantity to make its observation of period-one price more informative concerning γ and then use this information to increase expected profits in period two.

The firm's second-period problem is straightforward. Let[5]

$$V(\rho) \equiv \max_{Q_2} (\rho Q_2 g(\bar{\gamma}, Q_2) + (1-\rho) Q_2 g(\underline{\gamma}, Q_2)).$$

$V(\rho)$ gives the maximized value of period-two profits as a function of the (posterior) probability ρ. Note that because this is the final period the issue of experimentation does not arise.

In the first period, the firm's problem is to find Q^E such that[6]

$$Q^E \in \operatorname{argmax} \; \{\rho_0 Q g(\bar{\gamma}, Q) + (1-\rho_0) Q g(\underline{\gamma}, Q) + E_{\varepsilon,\gamma} V(\rho(Q, \gamma, \varepsilon))\}. \tag{2.1}$$

Here ρ is calculated as a function of Q, γ, and ε via Bayes' rule, or

$$\rho(Q, \bar{\gamma}, \varepsilon) = \frac{\rho_0 f(\varepsilon)}{\rho_0 f(\varepsilon) + (1-\rho_0) f(g(\bar{\gamma}, Q) + \varepsilon - g(\underline{\gamma}, Q))},$$

and

$$\rho(Q, \underline{\gamma}, \varepsilon) = \frac{\rho_0 f(g(\underline{\gamma}, Q) + \varepsilon - g(\bar{\gamma}, Q))}{\rho_0 f(g(\underline{\gamma}, Q) + \varepsilon - g(\bar{\gamma}, Q)) + (1-\rho_0) f(\varepsilon)}.$$

The firm does not experiment if it sets a quantity equal to

$$Q^{NE} = \operatorname{argmax} \; \rho_0 Q g(\bar{\gamma}, Q) + (1-\rho_0) Q g(\underline{\gamma}, Q). \tag{2.2}$$

In this case the period-one quantity is set so as to maximize period-one (myopic) profits and does not take account of the future.

We are interested in whether the firm will set a period-one quantity, Q^E, that differs from Q^{NE} in order to collect information and, if so, how this quantity will compare with Q^{NE}. We immediately obtain:

<u>Lemma 2.1</u>

$$\frac{dE_{\gamma,\varepsilon}\{V(\rho(Q^E,\gamma,\varepsilon))\}}{dQ} >(<)\ 0 \Rightarrow Q^E >(<)\ Q^{NE}. \tag{2.3}$$

This follows from (2.1) and (2.2) and the observation that $d[\rho_0 Q^{NE} g(\bar{\gamma},Q^{NE}) + (1-\rho_0)Q^{NE}g(\underline{\gamma},Q^{NE})]/dQ = 0$. Intuitively, (2.3) states that if the firm can increase period-two expected profits by increasing its period-one quantity (and hence collecting information) then it will produce a higher output than that which maximizes period-one profits. First-period profit losses from increased output are balanced against the expected second-period gains provided on average by more accurate information. Figure 1 illustrates the forces behind this result ($\pi(Q,\gamma,\varepsilon) = Qg(\gamma,Q) + \varepsilon$ is period-one profits).

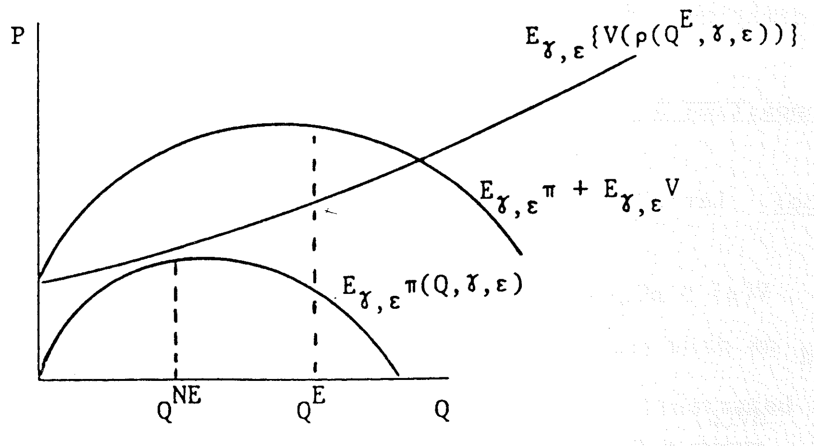

Figure 1

III. The Incentives to Experiment

This section establishes two necessary conditions for experimentation to occur. First, information must be useful. Second, adjustments in quantity must be capable of increasing the informativeness of price.

Consider first the conditions under which information is useful. Let $Q_2(\rho)$ be the optimal period-two output given that the posterior expectation is given by ρ. Then

Definition 3.1 Information is useless if

$$\frac{dQ_2(\rho)}{d\rho} = 0 \tag{3.1}$$

for all $\rho \varepsilon [0,1]$ and is useful if $Q_2(\rho)$ is not constant in ρ.

Information is thus useless if the monopoly's optimal period-two output does not depend upon its posterior expectation of the state of demand, ρ. In particular, the optimal period-two output in state $\bar{\gamma}$ (denoted $Q_2(1)$) equals the optimal output given $\underline{\gamma}$ (denoted $Q_2(0)$). In this case actions do not depend on information and hence the firm will not incur costs to acquire information. Formally,

Proposition 3.1. If information is useless the firm will not experiment.

Proof. Let $dQ_2(\rho)/d\rho = 0$. Then

$$V(\rho) = \rho Q_2(\rho) g(\bar{\gamma}, Q_2(\rho)) + (1-\rho) Q_2(\rho) g(\underline{\gamma}, Q_2(\rho))$$

and

$$\frac{dV(\rho)}{d\rho} = Q_2(\rho)[g(\bar{\gamma}, Q_2(\rho)) - g(\underline{\gamma}, Q_2(\rho))]$$

does not vary in ρ. Then

$$E_{\gamma,\varepsilon}\{V(\rho(Q,\gamma,\varepsilon))\} = \int (\rho_0 V(\rho(Q,\bar{\gamma},\varepsilon))f(\varepsilon) + (1-\rho_0)V(\rho(Q,\underline{\gamma},\varepsilon))f(\varepsilon))d\varepsilon$$

and

$$\frac{d\, E_{\gamma,\varepsilon}\{V(\rho(Q,\gamma,\varepsilon))\}}{dQ} = \int V'(\rho)\left(\rho_0 \frac{d\rho(Q,\bar{\gamma},\varepsilon)}{dQ} + (1-\rho_0)\frac{d\rho(Q,\underline{\gamma},\varepsilon)}{dQ}\right)f(\varepsilon)d\varepsilon$$

$$= V'(\rho)\frac{d\, E_{\gamma,\varepsilon}\{\rho(Q,\gamma,\varepsilon)\}}{dQ}$$

$$= 0. \qquad ||$$

The key step in this proof is that since $V'(\rho)$ is constant, the expected value at $V(\rho)$ can be changed only if variations in Q can affect the expected value of ρ. Given that ρ is obtained via Bayesian updating, this is impossible.

This result is straightforward. If actions do not depend on information, the latter is not valuable. The following three examples illustrate when information is useless.

<u>Example 3.1</u> Let $g(\bar{\gamma},Q) = \theta g(\underline{\gamma},Q)$ for some $\theta > 1$. Then $Q_2(\rho)$ solves $\rho[Q_2g'(\underline{\gamma},Q_2) + g(\underline{\gamma},Q_2)] + (1-\rho)[Q_2g'(\bar{\gamma},Q_2) + g(\bar{\gamma},Q_2)] = [\rho+(1-\rho)\theta][Q_2g'(\underline{\gamma},Q_2) + g(\underline{\gamma},Q_2)] = 0 = Q_2g'(\underline{\gamma},Q_2) + g(\underline{\gamma},Q_2)$ giving $dQ_2(\rho)/d\rho = 0$. Information is thus useless.

<u>Example 3.2</u> The conditions of the previous example hold if

$$g(\bar{\gamma},Q) = \bar{a} - \bar{b}Q$$

$$g(\underline{\gamma},Q) = \underline{a} - \underline{b}Q$$

where $\bar{a}/\bar{b} = \underline{a}/\underline{b}$. This is the case of linear mean demand curves with an intersection on the horizontal axis. These demand curves are illustrated in

Figure 2. The second-period optimization problem of the firm in this case gives (cf. (2.7))

$$Q_2 = \frac{\hat{a}}{2\hat{b}}$$

where $\hat{a} = \rho\bar{a} + (1-\rho)\underline{a}$ and $\hat{b} = \rho\bar{b} + (1-\rho)\underline{b}$. The assumption $\bar{a}/\bar{b} = \underline{a}/\underline{b}$ then suffices to give $dQ_2/d\rho = 0$, so that information is useless. The optimal period-two quantity thus does not depend upon expectations concerning the demand curve and in this case the firm will not experiment to gain information.

Example 3.3 In the previous example, information is useless because the marginal revenue curves associated with the two demand curves intersect the marginal cost curve (given by the horizontal axis) at the same point. This in turn occurs because the marginal cost curve is horizontal and the two demand curves intersect at a point on this curve. This leads to the following generalization. If two linear demand curves (and their marginal revenue curves) intersect at

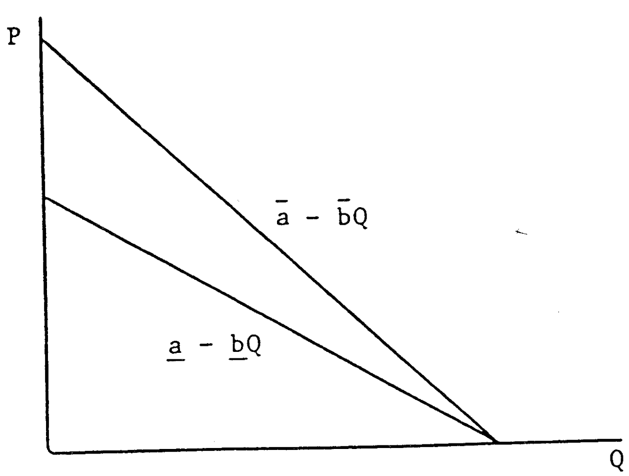

Figure 2

$$P^* = \frac{\bar{b}\underline{a}-\underline{b}\bar{a}}{\bar{b}-\underline{b}} > 0.$$

and the point

$$\left(\frac{\bar{b}\underline{a}-\underline{b}\bar{a}}{\bar{b}-\underline{b}},\ \frac{\bar{a}-\underline{a}}{2(\bar{b}-\underline{b})}\right)$$

is on the marginal cost curve, information is useless and the firm will not experiment. This is illustrated in figure 3. Note that this is true whether $MC = P^*$ or if the MC curve is increasing and meets the MR curves at P^*.

The second condition required for experimentation is that altering quantity must affect the informativeness of price. For a given period-one quantity Q, the value of the posterior expectation ρ is a random variable. Let $\theta(\rho,Q)$ be the probability density of this random variable.

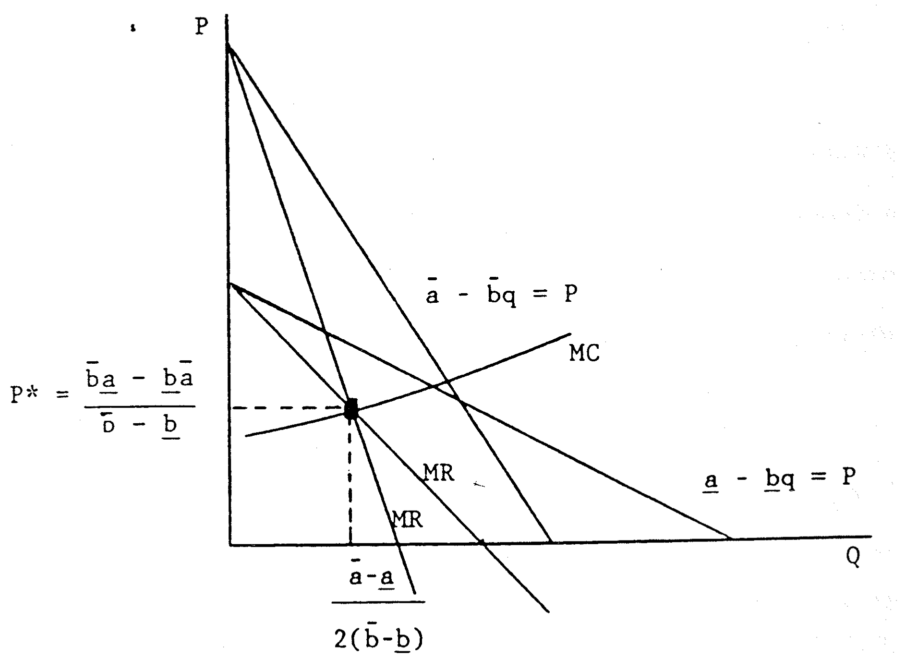

Figure 3

Definition 3.2. Quantity cannot affect the informativeness of price if

$$\frac{d\theta(\rho,Q)}{dQ} = 0$$

for all ρ and Q.

Proposition 3.2 If quantity cannot affect the informativeness of price then the firm will not experiment.

Proof. If $d\theta(\rho,Q)/dQ = 0$ for all Q, then period-one quantity does not affect the probability of ρ and hence cannot affect period-two expectations. There is accordingly no incentive to move quantity away from Q^{NE}. ||

Example 3.4 If

$$\frac{dg(\bar{\gamma},Q)}{dQ} = \frac{dg(\underline{\gamma},Q)}{dQ} \tag{3.2}$$

for all Q, then quantity cannot affect the informativeness of price and the monopoly will not experiment. Notice that (3.2) requires the slopes of the two mean demand curves to be equal at every quantity, though the levels of the mean demand curves may differ and the curves need not be linear. To verify this, let $\bar{\varepsilon}(\rho,Q)$ and $\underline{\varepsilon}(\rho,Q)$ solve, respectively,

$$\rho = \frac{\rho_0 f(\bar{\varepsilon}(\rho,Q))}{\rho_0 f(\bar{\varepsilon}(\rho,Q)) + (1-\rho_0)f(\bar{\varepsilon}(\rho,Q) + g(\bar{\gamma},Q) - g(\underline{\gamma},Q))} \tag{3.3}$$

and

$$\rho = \frac{\rho_0 f(\underline{\varepsilon}(\rho,Q) + g(\underline{\gamma},Q) - g(\bar{\gamma},Q))}{\rho_0 f(\underline{\varepsilon}(\rho,Q) + g(\underline{\gamma},Q) - g(\bar{\gamma},Q)) + (1-\rho_0)f(\underline{\varepsilon}(\rho,Q))}. \tag{3.4}$$

Here, $\bar{\varepsilon}(\rho,Q)$ is the value of ε that must appear if the period-two posterior is ρ, the quantity produced in period one is Q, and the true state of demand is $\bar{\gamma}$, and $\underline{\varepsilon}(\rho,Q)$ is analogous for a true state of demand of $\underline{\gamma}$. Then we can write

$$\theta(\rho,Q) = \rho_0 f(\bar{\varepsilon}(\rho,Q)) + (1-\rho_0)f(\underline{\varepsilon}(\rho,Q)). \tag{3.5}$$

Expressions (3.2), (3.4), and (3.5) immediately give

$$\frac{d\bar{\varepsilon}(\rho,Q)}{dQ} = \frac{d\underline{\varepsilon}(\rho,Q)}{dQ} = 0$$

and (3.5) then gives

$$\frac{d\theta(\rho,Q)}{dQ} = 0,$$

for all ρ and Q, completing the result.

Example 3.5. Condition 3.2 will hold, and the firm will not experiment because variations in quantity cannot affect the informativeness of price, if

$$g(\bar{\gamma},Q) = \bar{a} - bQ$$
$$g(\underline{\gamma},Q) = \underline{a} - bQ,$$

with $\bar{a} > \underline{a}$. This is the case of linear demand curves with uncertain vertical intercept.

We thus find that a monopoly faced with uncertainty concerning the intercept of a linear demand curve, or more generally with mean demand curves satisfying (3.2), will not experiment. The key to this finding is the demonstration that experimentation does not alter the second-period beliefs of the firm. The intuition behind the result is that altering quantity makes price a more informative signal if it spreads apart the means of the distributions (corresponding to $\underline{\gamma}$ and $\bar{\gamma}$) from which price is potentially drawn. If (3.2) holds, altering output does nothing but shift the two distributions of possible prices without altering their relative position. The failure to affect their relative position prevents price signals from becoming more informative and hence precludes any gain from experimentation.

IV. Quantity--Increasing Experimentation

Attention now turns to the question of how experimentation alters the firm's period-one quantity, given that the firm experiments. In this section we present conditions under which the monopoly experiments and does so by increasing its output. We initially assume:

Assumption 4.1. For all $Q > 0$,

$$g(\bar{\gamma},Q) > g(\underline{\gamma},Q) \tag{4.1}$$

$$0 > \frac{dg(\bar{\gamma},Q)}{dQ} > \frac{dg(\bar{\gamma},Q)}{dQ}. \tag{4.2}$$

Assumption 4.2. The revenue functions $Qg(\bar{\gamma},Q)$ and $Qg(\underline{\gamma},Q)$ are strictly concave.

Assumption 4.1 indicates that the demand curve corresponding to $\gamma = \bar{\gamma}$ lies above and is flatter than that corresponding to $\gamma = \underline{\gamma}$. Assumption 4.2 is a familiar second-order-condition assumption.

Proposition 4.1. Under assumptions 4.1 - 4.2,

$$\frac{d\, E_{\varepsilon,\gamma} V(\rho(Q^{NE},\gamma,\varepsilon))}{dQ} > 0, \tag{4.3}$$

so that the firm experiments by increasing quantity.

The intuition behind Proposition 4.1 is that by increasing output, the firm drives the means of the two distributions from which the market price might have been drawn farther apart. This makes the price a more informative signal, yielding more accurate period-two beliefs. This in turn allows the firm to choose a more appropriate period-two quantity and raises the period-two expected value. The firm thus trades off the first-period loss associated with raising output above the non-experimentation optimum in order to secure the second-period gain associated with better information.

The proof of Proposition 4.1, given below, exploits this intuition directly and is likely to be of independent interest. A similar result was first obtained by Grossman, Kihlstrom and Mirman (1977) in a model with restrictions on the density $f(\varepsilon)$. This was generalized to the case in which $f(\varepsilon)$ satisfies the MLRP by Fusselman and Mirman (1988) (FM). However, the FM proof seems much less likely to be useful for applications and generalizations of these results than that given below.

We prove Proposition 4.1 with the help of two lemmas. The first shows that if the period-two value function is strictly convex, then the firm will experiment by increasing quantity. The second shows that the period-two value function is strictly convex.

<u>Lemma 4.1</u> If assumptions 4.1 - 4.2 hold and $V''(\rho) > 0$, then

$$\frac{d\, E_{\varepsilon,\gamma} V(\rho(Q^{NE},\gamma,\varepsilon))}{dQ} > 0. \tag{4.3}$$

Proof. The proof proceeds in two steps. The first investigates the links between the first and second periods created by the model's information flows. The second exploits this flow of information to evaluate the effect of period-one quantity on the value function and establish that period-one quantity will be increased.

[Step 1]. We investigate the link between the first and second periods provided by the model's information flow. In particular, we examine how ρ depends on first-period quantity and price. Let

$$\bar{f} = f(P-g(\bar{\gamma},Q)), \quad \underline{f} = f(P-g(\underline{\gamma},Q)).$$

Then we can define a function $\rho(Q,P)$ by Bayes' rule,

$$\rho(P,Q) = \text{prob}(\bar{\gamma}|P,Q) = \frac{\text{prob}(P,Q|\bar{\gamma})\ \text{prob}(\bar{\gamma})}{\text{prob}(P,Q|\bar{\gamma})\ \text{prob}(\bar{\gamma}) + \text{prob}(P,Q|\underline{\gamma})\ \text{prob}(\underline{\gamma})}$$

$$= \frac{f(P-g(\bar{\gamma},Q))\rho_0}{f(P-g(\bar{\gamma},Q))\rho_0 + f(P-g(\underline{\gamma},Q))(1-\rho_0)} = \frac{\bar{f}\rho_0}{\bar{f}\rho_0 + \underline{f}(1-\rho_0)}. \tag{4.4}$$

Let the denominator in (4.4) be denoted D, so $D = \bar{f}\rho_0 + \underline{f}(1-\rho_0)$. Then

$$\frac{d\rho(P,Q)}{dP} = \frac{1}{D^2}[D\bar{f}'\rho_0 - \bar{f}\rho_0(\bar{f}'\rho_0 + \underline{f}'(1-\rho_0))]$$

$$= \frac{\rho_0(1-\rho_0)}{D^2}[\underline{f}\bar{f}' - \bar{f}\underline{f}'] \geq 0,$$

where the inequality follows from the fact that

$$\frac{\bar{f}'}{\bar{f}} = \frac{f'(P-g(\bar{\gamma},Q))}{f(P-g(\bar{\gamma},Q))} \geq \frac{f'(\rho-g(\underline{\gamma},Q))}{f'(\rho-g(\underline{\gamma},Q))} = \frac{\underline{f}'}{\underline{f}}. \tag{4.5}$$

(Because $g(\bar{\gamma},Q) > g(\underline{\gamma},Q)$, we have $P-g(\bar{\gamma},Q) < P-g(\underline{\gamma},Q)$ and (4.5) is equivalent to the imposition of the monotone likelihood ratio property.) Hence, higher prices, because they are more likely to have come from a high demand curve, increase the expectation that demand is favorable.

Next, we examine how the second-period expectation depends on period-one quantity. This result will be needed below. Let $\bar{g}' \equiv dg(\bar{\gamma},Q)/dQ$ and $\underline{g}' \equiv dg(\underline{\gamma},Q)/dQ$. Then

$$\frac{d\rho}{dQ} = -\frac{1}{D^2}[D\rho_0\bar{f}'\bar{g}' - \bar{f}\rho_0(\rho_0\bar{f}'\bar{g}' + (1-\rho_0)\underline{f}'\underline{g}')]$$

$$= -\bar{g}'\frac{d\rho}{dP} - \frac{\rho_0(1-\rho_0)}{D^2}(\bar{g}'-\underline{g}')\bar{f}\underline{f}'. \tag{4.6}$$

$$= -\underline{g}'\frac{d\rho}{dP} - \frac{\rho_0(1-\rho_0)}{D^2}(\bar{g}'-\underline{g}')\bar{f}'\underline{f}. \tag{4.7}$$

The first term in (4.6) and (4.7) is positive; the second takes the sign of $\bar{f}'$ or $\underline{f}'$.[7]

[Step 2]. We now have the information available to evaluate (4.3). Notice that

$$E[V(\rho(Q,P))] = \int V(\rho(Q,P))h(P,Q)dP \tag{4.8}$$

where

$$h(P,Q) = f(P-g(\bar{\gamma},Q))\rho_0 + f(P-g(\underline{\gamma},Q))(1-\rho_0)$$

giving

$$\Gamma \equiv \frac{dE\{V(\rho(Q,P))\}}{dQ} = \int\left(V'(\rho)\frac{d\rho}{dQ}(\rho_0\bar{f}+(1-\rho_0)\underline{f}) - V(\rho)(\rho_0\bar{g}'\bar{f}'+(1-\rho_0)\underline{g}'\underline{f}')\right)dP. \tag{4.9}$$

We require $\Gamma > 0$. First, we integrate (4.9) by parts. To do this, notice that for either $g'f = \bar{g}'\bar{f}$ or $g'f = \underline{g}'\underline{f}$,

$$\int V\frac{df}{dQ}dP = V(\rho)g'f]_{-\infty}^{\infty} - \int V'\frac{d\rho}{dP}g'fdP,$$

with the first term on the right equaling 0, because f has support on the entire real line. Then performing integration by parts on (4.9) yields

$$\Gamma = \int V'\frac{d\rho}{dQ}(\rho_0\bar{f}+(1-\rho_0)\underline{f})dP + V'(\rho_0\bar{g}'\ \bar{f} + \underline{g}'\underline{f}(1-\rho_0))\frac{d\rho}{dP}\ dP$$

$$= \int V'(\frac{d\rho}{dQ} + \bar{g}'\frac{d\rho}{dP})\rho_0\bar{f}dP + \int V'(\frac{d\rho}{dQ} + \underline{g}'\frac{d\rho}{dP})(1-\rho_0)\underline{f}dP. \quad (4.10)$$

Using (4.6) and (4.7), expression (4.10) becomes

$$\Gamma = -[\int V'\frac{\rho_0(1-\rho_0)}{D^2}(\bar{g}'-\underline{g}')\bar{f}\ \underline{f}'\bar{f}\rho_0dP + \int V'\frac{\rho_0(1-\rho_0)}{D^2}(\bar{g}'-\underline{g}')\bar{f}'\underline{f}\ \underline{f}(1-\rho_0)dP]. \quad (4.11)$$

Because $\rho = \bar{f}\rho_0/D$ and $(1-\rho) = \underline{f}(1-\rho_0)/D$, (4.11) becomes

$$\Gamma = -(\bar{g}'-\underline{g}')\ [\int V'\rho^2(1-\rho_0)\underline{f}'dP + \int V'(1-\rho)^2\rho_0\bar{f}'dP]. \quad (4.12)$$

We now delete $(\bar{g}'-\underline{g}')$ (> 0, see assumption 4.1), and implicitly redefine Γ, since we are concerned only with signs. We can use the fact that $(1-\rho)^2 = (1-\rho) - \rho(1-\rho)$ to write (4.12) as

$$\Gamma = -[\int V'[\rho^2(1-\rho_0)\underline{f}' - \rho_0\rho(1-\rho)\bar{f}']dP + \int V'(1-\rho)\rho_0\bar{f}'dP]$$

$$= -[\int V'\rho[\rho(1-\rho_0)\underline{f}' - (1-\rho)\rho_0\bar{f}']dP + \int V'(1-\rho)\rho_0\bar{f}'dP]. \quad (4.13)$$

We now work on the expression $[\rho(1-\rho_0)\underline{f}' - (1-\rho)\rho_0\bar{f}']$. Expression (4.4) yields

$$\rho[\bar{f}\rho_0 + \underline{f}(1-\rho_0)] = \bar{f}\rho_0. \tag{4.14}$$

Differentiating (4.14) in P and rearranging gives,

$$\underline{f}'\rho(1-\rho_0) - \bar{f}'\rho_0(1-\rho) = -\frac{d\rho}{dP}[\bar{f}\rho_0 + \underline{f}(1-\rho_0)]. \tag{4.15}$$

Inserting (4.15) in (4.13) yields,

$$\Gamma = \int V'\rho\frac{d\rho}{dP}(\bar{f}\rho_0+\underline{f}(1-\rho_0))dP - \int V'(1-\rho)\rho_0\bar{f}'dP$$

$$= \int V'\rho\frac{d\rho}{dP}\bar{f}\rho_0 dP + \int V'\rho\frac{d\rho}{dP}\underline{f}(1-\rho_0)dP - \int V'(1-\rho)\rho_0\bar{f}'dP. \tag{4.16}$$

From (4.4), we have $\underline{f}(1-\rho_0)\rho = \bar{f}\rho_0(1-\rho)$, so (4.16) becomes

$$\Gamma = \int V'\rho\frac{d\rho}{dP}\bar{f}\rho_0 dP + \int V'\frac{d\rho}{dP}\bar{f}\rho_0(1-\rho)dP - \int V'(1-\rho)\rho_0\bar{f}'dP$$

$$= \int V'\frac{d\rho}{dP}\bar{f}\rho_0 dP - \int V'(1-\rho)\rho_0\bar{f}'dP. \tag{4.17}$$

Now we integrate the second term in (4.17) by parts. This yields,

$$\rho_0\int V'(1-\rho)\bar{f}'dP = \rho_0 V'(1-\rho)\bar{f}]_{-\infty}^{\infty} - \rho_0\int [V''(1-\rho)\frac{d\rho}{dP} - V'\frac{d\rho}{dP}]\bar{f}dP$$

$$= \int -V''(1-\rho)\rho_0\frac{d\rho}{dP}\bar{f}dP + \int V'\rho_0\frac{d\rho}{dP}\bar{f}dP. \tag{4.18}$$

Inserting (4.18) in (4.17) gives

$$\Gamma = \int V' \frac{d\rho}{dP} \bar{f}\rho_0 dP + \int V''(1-\rho)\rho_0 \frac{d\rho}{dP} \bar{f}dP - \int V'\rho_0 \frac{d\rho}{dP} \bar{f}dP$$

$$= \int V''(1-\rho)\rho_0 \frac{d\rho}{dP} \bar{f}dP. \tag{4.19}$$

Because $d\rho/dP \geq 0$, the desired result follows from the fact that $V'' > 0$. ||

Lemma 4.2. If assumptions 4.1 - 4.2 hold then $V''(\rho) > 0$.

Proof. Let $Q_2(\rho)$ be the optimal period-two quantity given probability ρ. Fix ρ^*. Define

$$F(\rho) = \rho g(\bar{\gamma}, Q_2(\rho^*))Q_2(\rho^*) + (1-\rho)g(\underline{\gamma}, Q_2(\rho^*))Q_2(\rho^*).$$

Then $F(\rho)$ is linear in ρ and

$$V(\rho) \geq F(\rho)$$

with equality of $\rho = \rho^*$. Furthermore,

$$\frac{dV(p^*)}{d\rho} = \frac{dF(p^*)}{d\rho}.$$

Strict convexity follows if we can show $V(\rho) > F(\rho)$ when $\rho \neq \rho^*$. Since $V(\rho) = \rho g(\bar{\gamma}, Q_2(\rho))Q_2(\rho) + (1-\rho)g(\underline{\gamma}, Q_2(\rho))Q_2(\rho)$ (assumption 4.2 ensures $Q_2(\rho)$ is uniquely defined), assumption 4.1 gives $dQ_2(\rho)/d\rho > 0$, yielding $Q_2(\rho) \neq Q_2(\rho^*)$ and hence $V(\rho) > F(\rho)$ for $\rho \neq \rho^*$. ||

This proves Proposition 4.1 and establishes the result that the monopoly increases first-period output in order to collect additional information concerning the state of demand.

The intuition behind both lemmas is straightforward. Lemma 4.1 states that *if* information is useful, then the firm will experiment by increasing quantity. (The presumption that information is useful is implicitly contained in the statement of Lemma 4.1 in the strict convexity of the function $V(\rho)$.) The key to the proof is showing that increasing quantity yields more information. Lemma 4.2 then establishes that information is useful by showing that $V(\rho)$ is strictly convex. Notice that these steps reflect the conditions for experimentation established in Section III. Notice also that in each of examples 3.1 - 3.3, in which information is useless, $V(\rho)$ is linear.

Lemma 4.2 establishes the strict convexity of the value function. It is interesting to note that Fusselman and Mirman (1988) are forced to resort to quite sophisticated arguments to establish this convexity. In contrast, we use only elementary arguments. If one has more specific information concerning the function $g(\gamma,Q)$, it is often possible to use even simpler arguments to directly derive the convexity of $V(\rho)$.

Example 4.1. Let

$$g(\bar{\gamma},Q) = a + \bar{b}Q$$

$$g(\underline{\gamma},Q) = a + \underline{b}Q$$

where $0 > \bar{b} > \underline{b}$. Then

$$V(\rho) = \max_{Q_2}\{\rho(a+\bar{b}Q_2)Q_2 + (1-\rho)(a+\underline{b}Q_2)Q_2\} = \max_{Q_2} aQ_2 + (Q_2)^2\hat{b} \tag{4.20}$$

where $\hat{b} = \rho\bar{b} + (1-\rho)\underline{b}$. Differentiating with respect to Q_2, the first-order condition for this maximization problem is

$$a + 2\hat{b}Q_2 = 0 \tag{4.21}$$

giving a solution of

$$Q_2 = -\frac{a}{2\hat{b}}. \tag{4.22}$$

Inserting (4.22) into the objective in (4.20) gives an optimal value of

$$V(\rho) = -\rho\left(a - \frac{\bar{b}a}{2\hat{b}}\right)\frac{a}{2\hat{b}} - (1-\rho)\left(a - \frac{\underline{b}a}{2\hat{b}}\right)\frac{a}{2\hat{b}} = -\frac{a^2}{2\hat{b}}\left[\frac{2\hat{b}-\hat{b}}{2\hat{b}}\right] = -\frac{a^2}{4\hat{b}}.$$

Then

$$V'(\rho) = \frac{a^2 4(\bar{b}-\underline{b})}{16\hat{b}^2} = \frac{a^2(\bar{b}-\underline{b})}{4\hat{b}^2} > 0. \tag{4.23}$$

Hence, a flatter demand curve is more profitable. Moreover,

$$V''(\rho) = -\frac{a^2(\bar{b}-\underline{b})^2}{2\hat{b}^3} > 0. \tag{4.24}$$

Thus in this example V is a strictly convex function of ρ.

V. Quantity--Decreasing Experimentation

In this section we give an example in which it is optimal to decrease output in order to take advantage of the information flow.

Proposition 5.1. Let

$$g(\bar{\gamma},Q) > g(\underline{\gamma},Q) \tag{5.1}$$

$$\frac{dg(\bar{\gamma},Q)}{dQ} < \frac{dg(\underline{\gamma},Q)}{dQ} < 0 \tag{5.2}$$

for all $Q \geq 0$ and let $Qg(\bar{\gamma},Q)$ and $Qg(\underline{\gamma},Q)$ be strictly concave. Then the firm experiments by decreasing quantity, i.e., $Q^E < Q^{NE}$.

Proof. The proof follows precisely that of Lemma 4.1, except that the value $(\bar{g}' - g')$ that is factored out (see (4.12)) is now negative (cf. (5.2)). This gives

$$\frac{d\, E_{\varepsilon,\gamma}\{V(\rho(Q^{NE},\gamma,\varepsilon))\}}{dQ} < 0,$$

yielding the result. ||

Example 5.1. We can again provide a simple example with the advantage that the strict convexity of the value function can be easily derived. Let

$$g(\bar{\gamma},Q) = \bar{a} + \bar{b}Q$$

$$g(\underline{\gamma},Q) = \underline{a} + \underline{b}Q$$

where

$$\bar{a} > \underline{a}$$

$$\bar{b} < \underline{b} < 0$$

$$\frac{\bar{a}}{\bar{b}} < \frac{\underline{a}}{\underline{b}}.$$

This is the case of linear demand curves which intersect in the fourth quadrant, as shown in Figure 4. (Notice that $V(\rho)$ would not be

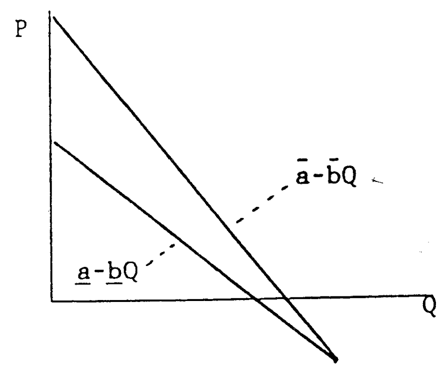

Figure 4

strictly convex if they intersected on the horizontal axis, as shown in Example 3.2) Solving the period-two problem gives

$$Q = \frac{\hat{a}}{2\hat{b}} \quad , \qquad V(\rho) = \frac{\hat{a}^2}{4\hat{b}} \tag{5.3}$$

where $\hat{a} = \rho\bar{a} + (1-\rho)\underline{a}$ and $\hat{b} = \rho\bar{b} + (1-\rho)\underline{b}$. Then

$$V'(\rho) = \frac{2\hat{a}\hat{b}(\bar{a}-\underline{a}) - \hat{a}^2(\bar{b}-\underline{b})}{4\hat{b}^2} > 0, \tag{5.4}$$

where a sufficient condition for the positive sign is $\bar{a}/\bar{b} < \underline{a}/\underline{b}$, as assumed. Hence $V'(\rho) > 0$. We can now calculate

$$-V''(\rho) = \left(\frac{1}{8\hat{b}^4}\right)(4\hat{b}^2(2\hat{b}(\bar{a}-\underline{a})^2 + 2\hat{a}(\bar{b}-\underline{b})(\bar{a}-\underline{a}) - 2\hat{a}(\bar{a}-\underline{a})(\bar{b}-\underline{b}))$$

$$- (2\hat{a}\hat{b}(\bar{a}-\underline{a}) - \hat{a}^2(\bar{b}-\underline{b}))8\hat{b}(\bar{b}-\underline{b}))$$

$$= \left(\frac{1}{8\hat{b}^4}\right)(8\hat{b})(\hat{b}^2(\bar{a}-\underline{a})^2 - 2\hat{a}\hat{b}(\bar{a}-\underline{a})(\bar{b}-\underline{b}) + \hat{a}^2(\bar{b}-\underline{b})^2)$$

$$= \frac{1}{\hat{b}^3}(\hat{b}(\bar{a}-\underline{a}) - \hat{a}(\bar{b}-\underline{b}))^2 < 0$$

where the strict inequality holds if $\bar{a}/\bar{b} < \underline{a}/\underline{b}$, as assumed. Hence, $V''(\rho) > 0$.

We can summarize the results of examples 3.2, 4.1, and 5.1. Let marginal cost be fixed at zero (perhaps by normalizing price to be net of marginal cost) and let there be two possible linear mean demand curves. If the point at which these curves intersects is in the second quadrant, then experimentation leads the firm to increase quantity. If the intersection is in the interior of the fourth quadrant, experimentation leads the firm to reduce its quantity. If the intersection falls on the vertical axis, no experimentation will occur.

We might now be tempted to continue with the statement that if the intersection is in the first quadrant, then experimentation will lead the firm to alter its first-period quantity away from the myopic optimum so as to move it away from the point of intersection. This is not immediately obvious, however. There is one local optimum which involves such a movement. However, another local optimum may exist which involves moving to the other side of the intersection, and additional conditions are required to establish the global optimality of the former.

VI. Conclusion

We have examined the incentives for a firm faced with an uncertain demand curve to experimentally adjust output in order to collect information concerning demand. We have established necessary conditions for experimentation to occur as well as conditions under which the result of experimentation will be an increase or decrease in quantity.

There are two steps involved in examining experimentation. The first is to establish that the monopoly will find it optimal to incur a cost to collect information and the second is to ascertain what period-one quantity adjustment allows more information to be collected. Intuitively, we find that it is optimal to incur costs to obtain information if the single-period optimal actions under the various expectations concerning the mean demand curves differ. Next, period-one adjustments allow more information to be collected if the firm can undertake a quantity adjustment which increases the spread between the price distributions corresponding to the two demand curves.

Similar considerations provide the key to determining whether experimentation induces the firm to increase or decrease quantity: the firm will adjust quantity so as to increase the spread between mean demand curves.

Five comments can be made. First, it is interesting to note that our results for a firm facing linear mean demand curves with known intercept and unknown slope (that quantity will be increased to collect information) contrast with those derived for the case of uncertain utility in Grossman, Kihlstrom and Mirman (1977), Kihlstrom, Mirman and Postlewaite (1984), and Fusselman and Mirman (1988). The latter papers establish the direction that experimentation will take if it occurs but cannot preclude cases in which it does not occur.

Second, our analysis depends upon the distribution $f(\varepsilon)$ having full support on the real line. This analysis then implicitly assumes that prices are not truncated at zero, so that very small realizations of ε can yield negative prices. This assumption significantly simplifies the analysis. An alternative would be to assume that prices cannot be negative. This would not only complicate the analysis but would affect the results. For example, if prices are truncated at zero then incentives to experiment can arise in the case of linear demand curves with uncertain intercept. This occurs because shifting quantity now affects the relative positions of the price distributions by shifting the relative positions of the truncation points.

Third, we have found that the possibility of experimentation produces some results that initially appear counterintuitive. For example, linear demand curves with a known vertical intercept but uncertain slope yield an experimental increase in quantity for a quantity-setting monopolist. However, if price rather than quantity is the firm's choice variable or if the known intercept lies on the horizontal axis, no experimentation occurs.

Fourth, we have assumed throughout that the distribution of the random variable ε is independent of any endogenous variables in the model. The assumption is reflected in the specification of demand, $g(\gamma,Q) + \varepsilon$, with an additive error term. An interesting issue concerns the possibility that quantity choices may affect the distribution of $f(\varepsilon)$, so that the demand curve $g(\gamma,Q,\varepsilon)$ may not be separable. An advantage of the proofs we have established is that they readily generalize to such a case (see, for example, Creane (1989)).

In the process of considering such a generalization, some additional insight is gained. Grossman, Kihlstrom and Mirman (1977) suggest that experimentation does not occur in the case of a linear demand curve with uncertain intercept, or $P = a + bQ + \varepsilon$ where $a = \bar{a}$ or $a = \underline{a}$, because the variable observed by the firm is essentially $a + \varepsilon$ $(= (P-bQ))$. Since $a + \varepsilon$ is unaffected by Q, no experimentation occurs. This intuition is reinforced by the observations that experimentation <u>does</u> occur if $P = a + bQ + \varepsilon$ with $b = \bar{b}$ or $b = \underline{b}$, where the firm observes $b + \varepsilon/Q$ $(= (P - a)/Q)$, which is affected by Q; and experimentation does not occur if $P = a + bQ + Q\varepsilon$ with $b = \bar{b}$ or $\underline{b}$, where the firm observes $b + \varepsilon$ $(= (P - a)/Q)$ (see Creane (1989)). However, our results suggest that this intuition is misleading in two respects. First, cases arise such as linear demand curves which intersect on the horizontal axis in which the random variable observed by the firm <u>is</u> affected by the firm's quantity but the firm does not experiment because the resulting information is not valuable. Second, our results suggest that even when information is valuable, the ability of the firm to profitably experiment depends not on a single distribution such as $a + \varepsilon$ but on the <u>relative</u> positions of distributions such as $\bar{a} + \varepsilon$ and $\underline{a} + \varepsilon$ which correspond to the various possible parameter values. The key to experimentation is the ability to affect these relative positions by altering Q.[8]

Finally, the question naturally arises as to how these results would be affected if the model were extended to allow more than one firm. Experimentation would still potentially arise in this case but there would now also be opportunities for firms to affect the observations and hence information of other firms. We will address this case in future papers.

FOOTNOTES

[1]More generally, one might expect that when faced with linear demand curves with a known point of intersection and unknown slopes, the firm will adjust period-one output away from the single-period optimum in that direction which increases the gap between the demand curves, though we will find that some qualifications of this statement are needed. MacLennan (1984) touches on similar issues in an infinite horizon model. Aghion, Espinosa, and Jullien (1988) also obtain a similar result. They work with general demand curves but assume that the model satisfies a condition referred to as information being fully valuable as well as a condition specifying which output adjustments make price more informative. Much of our analysis is concerned with deriving the counterpart of these conditions from the structure of the model.

[2]Both of these approaches differ significantly from that of a third approach in which demand is not subject to random perturbations. Examples include Aghion, Bolton and Jullien (1988), Alpern and Snower (1987a,b), and Reyniers (1987a,b,c).

[3]Gal-Or (1988) examines experimentation on the part of duopolists in a two-period model. Her analysis differs from ours in that the uncertainty concerns cost rather than demand. More importantly, the mechanism by which quantities are transformed into information in her model is exogenously specified as part of the structure of the model rather than being derived from the underlying specification of the uncertainty and optimal decisions via Bayes' rule. Thus, it is simply assumed that larger quantities yield more precise information.

[4]Mirman and Urbano (1988) find that firms in a duopolistic setting facing linear demand curves with intercept uncertainty do not experiment. Our results show that information is not valuable in this case.

[5]We find it most convenient to attach the subscript 2 to period-two prices and quantities and to leave period-one variables without subscripts.

[6]Note that the objective function in (2.1) need not be concave, although it is continuous, so that there might be multiple solutions.

[7]Clearly, (4.6) and (4.7) show that the MLRP alone is insufficient to give $d\rho(P,Q)/dQ < 0$. As Q increases for fixed P, two things happen. First, the means of the two (identical) distributions from which P might have been drawn are shifted downward. This shift and the MLRP would suffice to increase the expectation that P was drawn from the higher distribution if the distance between the means of the two distributions remained constant. However, this distance increases, and additional conditions (for example, that one of $\bar{f}'$ or $\underline{f}'$ be negative) are then needed to ensure that this does not disrupt the increased likelihood that P is drawn from the higher distribution.

[8]The intuition concerning the relative positions of the two distributions, rather than considerations of whether the random variable

observed by the firm is affected by Q, appears most appropriate. For example, Creane (1989) has shown that if $P = a + bQ + g(Q)\varepsilon$ with $b = \bar{b}$ or $\underline{b}$ and if $g(Q)$ has unitary elasticity at the quantity which maximizes period-one profits, information is valuable but then the firm will not experiment. The firm observes $b + \varepsilon\ g(Q)/Q$ in this case, which is affected by Q, but the effect of Q on the relative positions of the distributions corresponding to $\underline{b}$ and $\bar{b}$ is such that experimentation is not optimal.

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