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HIRING PROCEDURES TO IMPLEMENT STABLE ALLOCATIONS*

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WP-AD 97-10

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Editor: Instituto Valenciano de Investigaciones Económicas, s.a.
First Edition April 1997
ISBN: 84-482-1479-X
Depósito Legal: V-1309-1997

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* We are grateful to Carmen Beviá and Roberto Serrano for helpful comments. Alcalde's work is partially supported by dgcyt, project PB-1504, and Institut Valencià d'Investigacions Economiques. Pérez-Castrillo and Romero-Medina's work is partially supported by the DGCYT under project PB 92-0590. Romero-Medina also acknowledges financial support from the Spanish Ministerio de Educación y Ciencia.

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ABSTRACT

We implement the core correspondence in Subgame Perfect Equilibrium using a simple sequential mechanism in which firms propose a salary to each worker (in the first stage). Then, each worker accepts at most one proposal (second stage). Moreover, we show that, if agents' preferences are additive, this mechanism implements in Subgame Perfect Equilibrium that firms' optimal correspondence when firms employ undominated strategies. Finally, we construct another simple sequential mechanism to implement workers' optimal correspondence when agents' preferences are additive.

KEYWORDS: Job Matching Markets, Implementation, Mechanism Design.

1. Introduction

In this paper, we provide two mechanisms to implement the stable correspondence (or a proper subset of it) in a job matching market with monetary transfers. The mechanisms proposed are very simple. They are two-stage game form mechanisms. In the first stage, the agents on one side of the market make simultaneous proposals to the members of the other side. Once all the proposals have been made, they are either accepted or rejected. The final matching is determined by the second-stage decisions, while the salaries come out from the first-stage proposals.

In the first hiring mechanism that we provide, firms play first. Each firm proposes the wage for what it is ready to hire each worker. In the second stage, each worker selects between the first step proposals the one maximizing her utility. In quite a general framework, this mechanism implements the core correspondence in subgame perfect Nash equilibrium in pure strategies (SPE). Moreover, if agents' preferences are additive, this mechanism implements in SPE the firms' optimal stable correspondence, when firms use undominated pure strategies.

In the second mechanism, each worker proposes a firm and the wage for what she is ready to be hired by this firm. Then, each firm selects the set of workers among those that have choosen that firm at stage one. Assuming that the preferences of the agents are additive, this mechanism implements the workers' optimal stable allocation in SPE.

As it is apparent from the description of the mechanisms, they mimic simple hiring procedures. In this sense, the paper can also be viewed as an analysis of the behaviour of simple hiring mechanisms. We show that simple procedures do a very good job. In particular, the outcome of the hiring procedures is stable, therefore the final matching is efficient.

There is a long tradition of game theoretic analyses in many-to-one matching markets (see Roth and Sotomayor [7] for an excellent survey of the results in matching models until 1990). In job matching markets, Kelso and Crawford [5] show that stable allocations may fail to exist. This means that, in these markets, the core may be empty. They also provide a property, called the gross-substitute condition, under which the set of stable allocations is non-empty.

In our knowledge, our is the first paper in dealing with the implementation of stable allocations in job markets. For the college admissions problems, that is, for many-to-one matching models without monetary transfers, Kara and Sönmez [4] analyze widely the problem of implementation. In particular, they show that the set of stable allocations is implementable in Nash equilibrium, while no particular subset of the core is Nash implementable. For the same class of models, Alcalde and Romero-Medina [2] present natural sequential mechanisms to implement in SPE the core correspondence. Finaly, in the more general framework of cooperative games with transferable utility, Pérez-Castrillo [6] and Serrano [9] present sequential mechanisms implementing the core of any cooperative game in characteristic form.

The paper is organized as follows. Section 2 introduces the basic model. Section 3 presents a mechanism to implement the core for job markets when monetary transfers are allowed. Section 4 studies implementation of both the firms' and workers' optimal stable correspondences in additive environments. Finally, Section 5 concludes.

2. The model

We consider a job market with n workers and m firms. Let $W = \{w_1, \ldots, w_n\}$ and $F = \{f^1, \ldots, f^m\}$ be the set of workers and firms, respectively. Each worker's preferences depend on two variables. The first one is the firm she is working for, whereas the second relevant aspect is the wage that this firm pays to her. Worker w_i preferences are representable by the utility function $U_i(f^j, p_i)$, non-decreasing and continuous in $p_i \in \mathbb{R}$, where p_i is the salary that worker w_i receives and f^j is the firm she is working for. A worker who is not engaged by any firm reaches an utility level $U_i(\emptyset, 0)$. That is, we represent by $f = \emptyset$ the situation in which the worker is not hired by any firm. We also assume that for each firm f^j there is a reservation salary r_i^j such that $U_i\left(f^j, r_i^j\right) = U_i(\emptyset, 0)$ and that $U_i(f^j, p_i)$ has the same limit as p_i tends to $+\infty$ independently of the identity of the firm f^j.[1]

Each firm's profit depends on the set of workers it contracts, say $W^j \subseteq W$, and the salaries it pays to the workers it hires, say $P^j = (p_i^j)_{w_i \in W^j}$. For notational convenience, we will sometimes treat P^j as vector of $\mathbb{R}^n$, $P^j = (p_i^j)_{w_i \in W}$, and assume that π^j does not depends on p_i^j for $w_i \notin W^j$. Let $\pi^j : 2^W \times \mathbb{R}^n \longrightarrow \mathbb{R}$ be the profit function of firm f^j. $\pi^j(W^j, P^j)$ is assumed to be decreasing in salaries p_i^j whenever $w_i \in W^j$. A firm which does not hire any worker obtains $\pi^j(\emptyset, 0)$.

We describe an allocation for job markets by means of two variables. The first one is a vector $P \in \mathbb{R}^n$ representing the wage that each worker gets, where $p_i = 0$ if w_i is not engaged by any firm. The second one is a correspondence, to be called

[1] This condition can be relaxed assuming that the maximum limit of $U_i\left(f^j, p_i\right)$ as p_i tends to $+\infty$ is reached by at least two firms.

matching, that states which firm (if any) hires each worker and vice-versa. More precisely, a matching μ is a correspondence that applies $W \cup F$ into itself such that (a) for each $w_i \in W$, if $\mu(w_i)$ does not belong to F, then it is the empty set; (b) for each f^j in F, $\mu(f^j)$ is contained in W, and (c) for each pair $(w_i, f^j) \in W \times F$, $\mu(w_i) = f^j$ if, and only if, w_i belongs to $\mu(f^j)$.

We are interested in the job market allocations that are stable. Stability of an allocation depends on the possibilities that agents have to improve their utility level (if workers) or their profits (if firms). Since job matching markets can be viewed as a particular class of cooperative games, the stability concept that we are going to consider is the core.

Stability of an allocation can be easily checked in the case of job markets. An allocation is stable if there exist no subset of workers and a firm, or just a set of workers or a firm, that can improve their own utility by themselves. That is, (μ, P) is stable if, and only if, $\nexists(\widehat{W}, f^j) \in 2^W \times (F \cup \emptyset)$, and $\widehat{P} \in \mathbb{R}^n$ such that

(a) $U_i(f^j, \widehat{p}_i) > U_i(\mu(w_i), p_i)$ for all $w_i \in \widehat{W}$, where $\widehat{p}_i = 0$ if $f^j = \emptyset$, and

(b) $\pi^j\left(\widehat{W}, \widehat{P}\right) > \pi^j(\mu(f^j), P)$, where $\widehat{P} = 0$ if $\widehat{W} = \emptyset$.

Notice that our definition of stability includes two main features. The first one is that of individual rationality, namely each agent weakly prefers the payoff that she/it gets in this allocation rather than being unmatched. The second one is that of collective rationality in the following sense. There is no possibility for a firm and a group of workers to form a *new* matching, in such a way that both the firm and the new workers it hires find the new situation profitable.

In job markets, the set of stable allocations may be empty. This can happens when firms have some kind of increasing returns, that is, workers are complementary in the sense that a set of workers generates more income than the sum of the income from each worker separately (see Roth and Sotomayor [7, pg. 179] for an example). Kelso and Crawford [5] propose a condition, the 'gross substitutes condition', that rules out this possibility, guarantying the existence of stable allocations in the market. The assumption imposes that increases in other workers' salary can never cause a firm to withdraw an offer from a worker whose salary has not risen.

3. The "firms go fishing" mechanism

This section presents a mechanism implementing the stable correspondence in SPE (recall that we only consider pure strategy equilibria). This mechanism reflects a natural form of firms' competition for workers. Firms' behavior will be modeled by a kind of competition à la Bertrand.

Consider the following two-stage game-form mechanism, named Γ^F. In the first stage each firm proposes a vector of salaries (one for each worker). Once the salaries have been announced, each worker selects a firm. The outcome of this game is the following. The matching is the one chosen by the workers at the second stage, whereas the salary to be payed is the one that firms proposed at the first stage.

More precisely, each firm's message space is $\mathbb{R}^n$. A message for firm f^j, $m^j = (m_1^j, ..., m_n^j)$, will be understood as the salary at which it is willing to hire each worker. Firms' messages are stated simultaneously at the first stage. At the second stage, and knowing firms' messages, worker w_i message, m_i, is an element of $F \cup \{\emptyset\}$. Such a message will be understood as the firm she is willing to work for at salary proposed at the first stage. The outcome function $\phi^1(\cdot)$ associates to each set of messages, $\tilde{m} = (m^1, \ldots, m^j, \ldots, m^m, m_1, \ldots, m_i, \ldots, m_n)$ a matching, $\mu^{\tilde{m}}$, and a salaries vector, $P(\tilde{m}) \in \mathbb{R}^n$ such that

(a) for any $w_i \in W$, $\mu^{\tilde{m}}(w_i) = m_i$,

(b) for each $f^j \in F$, $\mu^{\tilde{m}}(f^j) = \{w_i \in W \mid m_i = f^j\}$, and

(c) $p_i(\tilde{m}) = m_i^{\mu^{\tilde{m}}(w_i)}$ if $\mu^{\tilde{m}}(w_i) \in F$ and $p_i(\tilde{m}) = 0$ otherwise.

Next theorem analyzes the SPE of the mechanism Γ^F.

Theorem 3.1. *The mechanism Γ^F implements in SPE the stable social choice correspondence.*

Proof. We first prove that each SPE outcome is stable. Let $\tilde{m}$ be the vector of messages that agents state at a certain SPE. Let suppose that $\phi^1(\tilde{m}) = (\mu^{\tilde{m}}, P(\tilde{m}))$ is not stable. Then, at least one of the following situations happens. Either a worker gets less utility than $U_i(\emptyset, 0)$, or a firm profits are lower than $\pi^j(\emptyset, 0)$, or there is a profitable deviation concerning one firm and a proper subset of workers. Consider that the third possibility holds, the others are easier to analyze. Thus, assume that there are a firm f^k, a set of workers W^k, and a vector of salaries $\bar{P}$, such that

(a) $U_i\left(f^k, \overline{p}_i\right) > U_i\left(\mu^{\tilde{m}}\left(w_i\right), p_i\left(\tilde{m}\right)\right)$ for all w_i in W^k, and

(b) $\pi^k\left(W^k, \bar{P}\right) > \pi^k\left(\mu^{\tilde{m}}\left(f^k\right), P\left(\tilde{m}\right)\right)$.

Let us consider the following strategy for f^k: $\overline{m}^k = \hat{P}$, with $\widehat{p}_i = \overline{p}_i$ if $w_i \in W^k$ and $\widehat{p}_i = -\infty$ otherwise, keeping constant the strategies m^j, for $f^j \neq f^k$. At the second stage each agent in W has to choose her employer in order to maximize her utility level. Because of condition (a) above, the message of any worker w_i in W^k will be $m_i = f^k$. Notice that $U_i\left(f^k, \overline{p}_i\right) > U_i\left(\mu^{\tilde{m}}\left(w_i\right), p_i\left(\tilde{m}\right)\right)$ and $U_i\left(\mu^{\tilde{m}}\left(w_i\right), p_i\left(\tilde{m}\right)\right) \geq U_i\left(f^j, m_i^j\right)$ for any f^j since $\mu^{\tilde{m}}\left(w_i\right)$ was the optimal choice given $\tilde{m}$. Moreover, no worker w_h outside W^k will send a message such as $m_h = f^k$. Therefore, f^k is interested in deviating, so we find a contradiction.

We now prove that each stable allocation can be supported by a SPE. Let (μ, P) be a stable allocation. Take w_i such that $\mu\left(w_i\right) \in F$, and denote $\widehat{p}_i^j$ the salary that firm f^j would have to pay to worker w_i for her to be indifferent between working in firm f^j at a salary $\widehat{p}_i^j$ and working in firm $\mu\left(w_i\right)$ at a salary p_i. Note that $\widehat{p}_i^j$ exists because $U_i\left(f^k, p_i\right)$ has the same limit as p_i tends to $+\infty$ independently of f^k

Consider the following strategies. For each firm f^j, its message is m^j, where

$$m_i^j = \begin{cases} \widehat{p}_i^j & \text{if } \mu\left(w_i\right) \in F \\ r_i^j & \text{otherwise.} \end{cases}$$

When confronted to the previous firms' strategies, worker i's strategy is $m_i = \mu\left(w_i\right)$. For any other possible firms' strategies, she makes any choice that maximizes her utility. These strategies constitute a SPE yielding the desired allocation. ■

We can read Theorem 3.1 in two ways. First, the theorem shows that it is possible to implement the core correspondence in quite a general model using very simple mechanisms. Although we already knew that the stable set (the core) was implementable, it is interesting to show that the implementation can be achieved through "natural" mechanisms. Second, the result tells us that simple hiring procedures do a good job. They lead to stable, and hence efficient, allocations. In this sense, our theorem can be understood as the characterization of the outcome of a particular hiring mechanism.

Theorem 3.1 establishes the equivalence between the stable set of the job matching market and the SPE of Γ^F. However, we have already remarked that the stable set may be empty. In the situations where this happens, we know that Γ^F has no SPE in pure strategies. Unhopefully, analyzing mixed strategies of Γ^F is very difficult, so we cannot characterize the outcome of the mechanism in those games where the stable set is empty.

4. Fishing in additive environments

Sometimes, we would like to have the possibility of selecting among the set of stable allocations, which can be large enough, some particular allocations. Hence, this section is devoted to the analysis of the implementation of particular subsets of the stable correspondence. More precisely, we show first that, if firms employ undominated strategies, the mechanism Γ^F implements in SPE the firms' optimal stable correspondence. Second, we design a mechanism implementing the workers' optimal stable correspondence in SPE. However, in order to do it, we are forced to restrict attention to additive environments. Therefore, let us recall what an additive environment is and the characteristics of two particular stable selections, namely the firms' and the workers' optimal stable correspondences.

4.1. Additive Job Matching Markets

We assume that worker w_i preferences are representable by the following utility function, $U_i\left(f^j, p_i\right) = p_i - r_i^j$, where r_i^j is the reservation salary of worker w_i when she is working for firm f^j. We also assume that there is a function $g^j : 2^W \longrightarrow \mathbb{R}$ which represents the income that firm f^j raises whenever it engages a certain set of workers. Thus, if firm f^j hires workers in W^j at salaries $P^j = (p_i^j)_{w_i \in W^j}$, its profit is $\pi^j\left(W^j, P^j\right) = g^j\left(W^j\right) - \sum_{w_i \in W^j} p_i^j$. Moreover, we also assume that it is possible to identify the value of a worker to a firm, this value being independent of the worker's mates. That is, there are numbers g_i^j, for $i = 1, ..., n$ and $j = 1, ..., m$ such that $g^j\left(W^j\right) = \sum_{w_i \in W^j} g_i^j$. Therefore, $\pi^j\left(W^j, P^j\right) = \sum_{w_i \in W^j} \left(g_i^j - p_i^j\right)$. With this assumption, we rule out the possibility of complementarity or substituability among workers.

Given the hypotheses made on workers' utility functions and on firms' profits, the market satisfies the gross-substitutes condition by Kelso and Crawford [5]. Therefore, stable matchings always exist. Moreover, they are easy to characterize. A matching (P, μ) is stable if

$$\mu(w_i) = f^j \Rightarrow p_i - r_i^j \geq \max\left\{ \left\{ g_i^k - r_i^k \right\}_{k \neq j}, 0 \right\} \text{ and } p_i \leq g_i^j,$$

$$\mu(w_i) = \emptyset \Rightarrow \max_k \left\{ g_i^k - r_i^k \right\} \leq 0.$$

Among the set of stable allocations, it is possible to identify the subset of allocations most preferred by firms. This is the firms' optimal stable set.

For a stable allocation to be firms' optimal, the necessary and sufficient condition is that

$$\mu(w_i) = f^j \Rightarrow p_i = \max\left\{ \left\{ g_i^k - r_i^k + r_i^j \right\}_{k \neq j}, r_i^j \right\}.$$

Finally, the necessary and sufficient condition for a stable allocation to be workers' optimal is

$$\mu(w_i) = f^j \Rightarrow p_i = g_i^j.$$

4.2. Firms' fishing

In order to introduce the next result, let us consider the following very simple market. There are two firms f^1 and f^2 and one worker, with $g_1^1 = 5$, $g_1^2 = 2$, and $r_1^1 = r_1^2 = 0$.

The stable allocations match the worker with f^1 at a salary $p \in [2, 5]$. Consider the stable allocation with $p = 4$. The only strategies that implement this allocation in the mechanism Γ^F are $m^1 = m^2 = 4$. However, it is easy to remark that the strategy $m^2 = 4$ is weakly dominated by the strategy $m^2 = 2$. That is, in the proof of the Theorem 3.1 we are sometimes using dominated strategies.

Theorem 4.1 analyses the outcome of the mechanism Γ^F in additive environments when firms use undominated strategies.

Theorem 4.1. *In additive environments the mechanism Γ^F implements in SPE the firms' optimal stable correspondence when firms' strategies are undominated.*

Proof. By Theorem 3.1 we know that every SPE outcome is stable. Let $\tilde{m}$ be the vector of messages that agents state in a certain SPE for this game. Assume that $\phi^1(\tilde{m}) = (\mu^{\tilde{m}}, P(\tilde{m}))$ does not belong to the firms' optimal stable correspondence. We are going to show that at least one firm is using a dominated strategy.

Since $(\mu^{\tilde{m}}, P(\tilde{m}))$ is stable but it is not optimal from the point of view of firms, there should be i and j such that

$$\mu^{\tilde{m}}(w_i) = f^j \text{ with } p_i(\tilde{m}) > \max\left\{ \left\{ g_i^k - r_i^k + r_i^j \right\}_{k \neq j}, r_i^j \right\}.$$

Assume that $\max \left\{ \{g_i^k - r_i^k + r_i^j\}_{k \neq j}, r_i^j \right\} = g_i^h - r_i^h + r_i^j$ for some $h \neq j$ (the other case is similar). Given that the allocation is the outcome of a SPE, it cannot be the case that $p_i(\tilde{m}) = m_i^j > m_i^k - r_i^k + r_i^j$ for every $k \neq j$. If it was the case, firm j would have a profitable deviation decreasing m_i^j while keeping it above $m_i^k - r_i^k + r_i^j$ for every $k \neq j$. Thus, $\exists\ k \neq j$ such that $p_i(\tilde{m}) = m_i^k - r_i^k + r_i^j$ and $p_i(\tilde{m}) > g_i^k - r_i^k + r_i^j$. Then, $m_i^k > g_i^k$. But it is a dominated strategy for a firm to make offers greater than its valuation, since in the best possible case it does not hire the worker so it gets zero from this offer, while it could be the case that its offer is the best for the agent so it loses money with her. The strategy m_i^k is dominated by $\overline{m}_i^k = g_i^k$.

For the reverse implication, let (μ, P) be a firms' optimal stable allocation. Consider the following strategies. The message (and strategy) of firm f^j is $m^j = \widehat{P}^j$, where

$$\widehat{p}_i = \begin{cases} p_i & \text{if } w_i \in \mu(f^j) \\ g_i^j & \text{otherwise.} \end{cases}$$

When confronted to the previous firms' strategies, worker i's strategy is $m_i = \mu(w_i)$. For any other possible firms' strategies, she makes any choice that maximizes her utility. These strategies for firms and workers constitute a SPE leading to (μ, P). Moreover, no agent uses dominated strategies. ■

Notice that the mechanism Γ^F generates a sort of Bertrand competition among firms. Nevertheless, as Theorem 4.1 states, the outcomes that are expected from the agents' strategical behaviour are the optimal stable allocations from the firms' point of view. Hence, firms' competition is strong enough to make them reach stable outcomes, but the fact that they are playing first allows them to reach the most profitable ones.

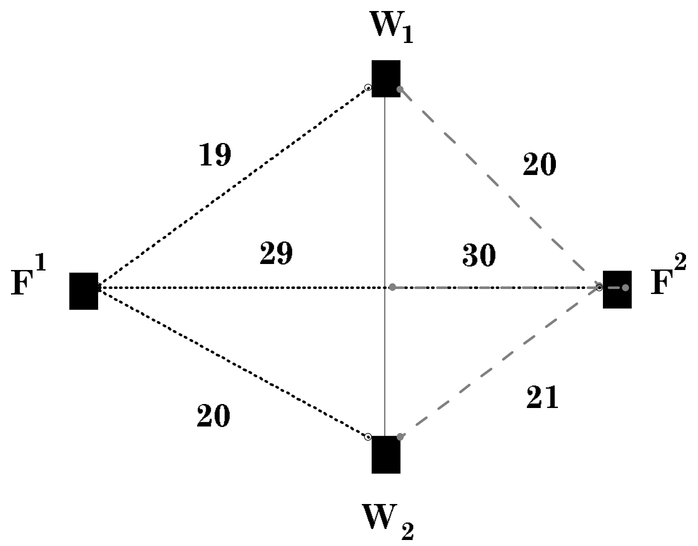
The result that Bertrand competition among firms do not lead to workers' optimal allocations is not surprising in our framework. The reason for this comes from the following argument. The firms know that, at the second stage, each worker has a dominant strategy, namely to select the firm with whom she maximizes her utility. Given that, firms can obtain the maximum benefit from strategical behaviour. Similar results in matching markets are due to Alcalde [1] (in one-to-one matching markets) and Schummer [8] and Demange and Gale [3] for the assignment problem. In Serrano [9], where the core of convex games is implemented, it also happens that the player to act first announcing the vector of prices (the broker, in Serrano's terminology) gets her best core payoff.

Unfortunately, it is not possible to generalize Theorem 4.1 to more general environments. The following example presents a market with non-empty core, where the mechanism Γ^F implements in non-dominated strategies a set of outcomes larger than the firms' optimal stable allocations.

There are two firms, f^1 and f^2, and two workers, w_1 and w_2. Workers maximize salary, so they do not have any preference for firms, and their reservation salary is zero. Firms' profits are given by $\pi^j\left(W^j, P^j\right) = g^j\left(W^j\right) - \sum_{w_i \in W^j} p_i^j$, where:

$$g^1\left(\{w_1\}\right) = 19,\ g^1\left(\{w_2\}\right) = 20,\ g^1\left(\{w_1, w_2\}\right) = 29$$

$$g^2\left(\{w_1\}\right) = 20,\ g^2\left(\{w_2\}\right) = 21,\ g^2\left(\{w_1, w_2\}\right) = 30.$$



The set of stable allocations consists of any matching of one firm with one worker, with salaries satisfying $p_1 \in [9, 19]$ and $p_2 = p_1 + 1$. Notice that the firms' optimal stable allocations involve salaries $p_1 = 9$ and $p_2 = 10$. However, the undominated strategies:

$$m^1 = (18, 19), m^2 = (18, 19), m_1 = f^1, m_2 = f^2$$

(with any optimal workers' response for out-of-equilibrium firms' strategies) lead to the following stable allocation:

$$\mu(w_1) = f^1, \mu(w_2) = f^2, p_1 = 18, p_2 = 19$$

which is not firms' optimal.

4.3. Workers' fishing

This section presents a two-stage mechanism implementing the workers' optimal stable allocation in SPE. Let us refer to it by Γ^W. In the first stage, workers play simultaneously. Each of them announces the salary and the firm for which she is willing to work. Each worker can only announce a salary and a firm, so she will only get hired if she is offered at least this salary by this firm. Once the demands of salary to the firms have been announced, firms simultaneously (or sequentially) select their set of workers. The outcome is determined as follows. The matching is the one chosen by the firms at the second stage, whereas the salary to be payed is determined by the demands of the workers.

More precisely, each worker's message space is $\mathbb{R} \times \{F \cup \emptyset\}$. A message for worker w_i, $m_i = (p_i, f_i)$, will be understood as the salary p_i at what she is willing to work for a certain firm f_i. Knowing workers' messages, firm f^j message, m^j, is a subset of $\{w_i \in W \mid f_i = f^j\}$. Such a message will be understood as the set of workers that firm f^j is willing to hire given the salaries they asked for at the first stage. The outcome function $\phi^2(\cdot)$ associates to each message, $\tilde{m} = (m_1, \ldots, m_i, \ldots, m_n, m^1, \ldots, m^j, \ldots, m^m)$ a matching, $\mu^{\tilde{m}}$, and a salaries vector, $P(\tilde{m}) \in \mathbb{R}^n$ such that

- (a) for each $f^j \in F$, $\mu^{\tilde{m}}(f^j) = m^j$,
- (b) for each $w_i \in W$, $\mu^{\tilde{m}}(w_i) = f_i$ if $w_i \in \mu^{\tilde{m}}(f_i)$, and $\mu^{\tilde{m}}(w_i) = \emptyset$ otherwise, and
- (c) $p_i(\tilde{m}) = p_i$ if $\mu^{\tilde{m}}(w_i) \in F$, or $p_i(\tilde{m}) = 0$ otherwise.

We next introduce our main result in this section.

Theorem 4.2. *In additive environments the mechanism Γ^W implements in SPE the workers' optimal stable allocations.*

Proof. First, a firm f^j is willing to satisfy any demand of a worker whose value is under her demand. Therefore, in any SPE, it is the case that $w_i \in m^j$ whenever $f_i = f^j$ and $p_i < g_i^j$, while $w_i \notin m^j$ whenever $p_i > g_i^j$. Second, for a given firm, the worker is interested in maximizing the salary. Then, assume that worker w_i is sending the message $m_i = (p_i, f_i)$ in equilibrium. It is necessarily the case that $p_i = g_i^{f_i}$. A message with $p_i > g_i^{f_j}$ would lead to w_i remaining unmatched, while if $p_i < g_i^{f_j}$ and $g_i^{f_j} > r_i^{f_j}$, then worker w_i could increase her demand of salary, still being sure that her offer would be accepted. Finally, worker w_i can choose among the set of firms (or remaining unmatched), knowing the maximum demand each firm is ready to accept. She will choose the situation that maximizes her utility, that is, in equilibrium it is the case that

$f_i \in F \Rightarrow f_i \in \arg\max_k \left\{g_i^k - r_i^k\right\}$, and
$f_i = \emptyset \Rightarrow \max_k \left\{g_i^k - r_i^k\right\} \leq 0.$

Moreover, in equilibrium, $p_i = g_i^{f_i}$. Therefore, the equilibrium leads necessarily to a workers' optimal allocation.

On the other hand, let $(\hat{\mu}, \hat{P})$ be a workers' optimal stable allocation. Let consider the following strategies. For each worker w_i her message (strategy) is $m_i = (p_i, f_i)$, where

$$(p_i, f_i) = (\hat{p}_i, \hat{\mu}(w_i)) \quad \text{if } \hat{\mu}(w_i) \in F \text{ and}$$

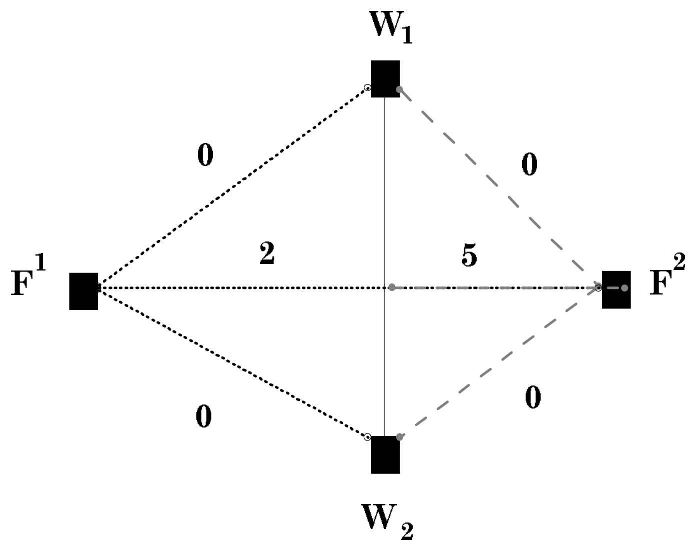
$$(p_i, f_i) = (0, \emptyset) \quad \text{if } \hat{\mu}(w_i) \notin F.$$

Each firm f^j message is

$$m^j = \left\{w_i \in W / p_i \leq g_i^j\right\}.$$

These strategies yield a SPE whose outcome is the allocation $(\hat{\mu}, \hat{P})$. ■

Notice that, when job matching markets are not additive, unstable allocations can be supported by mechanism Γ^W SPE, as is shown in the next example.



Let $F = \{f^1, f^2\}$ and $W = \{w_1, w_2\}$ with $r_i^j = g^j(\{w_i\}) = 0$ for all i and j, $g^1(W) = 2$, $g^2(W) = 5$. In such a case, strategies $m_1 = m_2 = (f^1, 1)$, $m^1 = \{w_1, w_2\}$ and $m^2 = \emptyset$ is a SPE for Γ^W. Note that the outcome of such an equilibrium is not stable because it can be blocked by f^2 and W by stating salaries $p_i = 2$.

Let us finally remark that we can state Theorem 4.2 also in non-dominated strategies, since we have not used dominated strategies in the proof. That is, the mechanism Γ^W implements in SPE the workers' optimal stable allocations when workers' strategies are undominated.

5. Conclusions

We construct two non-cooperative mechanisms to implement stable correspondences in job markets. First, in a general many-to-one matching framework, we implement the stable correspondence using a two-stage mechanism in which firms make offers to the workers for them afterwards to choose the best place to work. Second, we implement the firms' and workers' optimal stable correspondences in additive environments.

One important message of our paper is that simple mechanisms can lead to good allocations. The mechanisms we use mimic simple hiring procedures. Therefore, our results show that the outcome of some real-lifewise hiring mechanisms have interesting properties of stability in spite of their non-cooperative nature.

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