# LAND REFORM AND INDIVIDUAL PROPERTY RIGHTS*

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**A B S T R A C T**

This paper gives a rational to the land reform processes that many latin American countries have experienced during this century. The reform usually consisted of transferes of land, without compensation, from the owners of large estates to the landless peasants. The peasants, however, did not receive the individual ownership of the land. This was the case of Bolivia, México and Perú. This paper suggests that this type of reform was a measure intended to favor not the peasantry, but the landed elite who traditionally has held the political power in these countries. If the rents of the land are decreasing with the total amount of privately owned regime. I develop a model economy in which all the individuals vote on the land to be expropriated to the landed elite; land and labor are complements in the production process. For economies in which land is the relatively abundant factor, the equilibrium features an amount of privately owned land less than the total and no peasant is given individual ownership of the land received.

Keywords: Land reform; Property Rights; Voting; Land Endowments.

## 1. Introduction

This paper gives a rationale to the land reform processes that many Latin American countries have experienced during this century. The specific features of these reforms are two: first, the reforms ended the system of semi-serfdom under which peasants lived up to then.[1] Secondly, a substantial fraction of the land traditionally held by the *hacendados* was expropriated and given to the peasants. The land, however, was given under a special regime: the peasants did not have full rights on the land granted. The beneficiaries of the reform received either a communal right to the land, as in the case of México and Perú, or an individual right to cultivate a plot of land, as in the case of Bolivia. The recipients of communal land were forbidden to hire labor services. The Bolivian land reform law imposed a ceiling on the amount of land they could hold; thus, they were precluded from hiring labor services *de facto.* In all the cases, the peasants could not transfer the land in any manner but to their heirs.

The reforms have changed the structure of land tenancy in Latin America. In México, before the reform, 5 percent of the population held 80 percent of the farmland. México has undergone a steady land reform that started in 1917, after the revolution, and lasted over 80 years. In 1990, the *ejidos,* the peasant communities that received the transfers of land, controlled around 50 percent of the agricultural land.[2] The number of owners of farms whose size is below 5 hectares has remained practically zero.

The evidence shows that the reform sector's performance —that which comprises the recipients of the land— is worse than that of the private sector. Nguyen and Martínez Saldívar (1979), report that the yield per harvested hectare in the *ejidos* was between 50 percent and 80 percent lower than that of comparable private farms. Moreover, during the period 1976-83 the average income of an *ejido* member was only 47.6 percent of the minimum wage; in 1990 74.1 percent of the *ejidatarios* received an income lower than a minimum wage.[3]

[1] In many regions it was known as *colonato*. The name of the system varies depending on the country. The basic system is very similar. The *hacendado* gave a small plot to each peasant working for him. In exchange, the peasants had the obligation to work on the land kept by the *hacendado* a number of days per week. At the beginning of the century, if a peasant left the hacienda, the *hacendado* could call the police to imprison the peasant. See Heath *et al.* (1969).

[2] In Bolivia, 30 percent of of the farmland was redistributed to 35 percent of the peasant families in the late 50's. In Perú, around 50 percent of the agricultural land was transferred to 10 percent of the peasants families in 1973.

[3] As reported by Martínez Hernández (1992) and Téllez Kuénzler (1994), respectively.

Although it is clear why these societies ended the system of semi-serfdom prevailing before the reforms —the pressure of peasant revolts forced the landed elite to change the system— it is not clear why they chose a mixed ownership regime such as the one described, instead of giving the beneficiaries of the reform full rights on the land received. In particular, given the evidence just cited, it is not obvious why the peasants preferred receiving communal rights instead of individual rights or working as hired labor. This paper suggests that this type of reform was a measure intended to favor not the peasantry but the landed elite who traditionally has held the political power in these countries. The basic idea is that if the rents of land are decreasing with the total amount of privately owned land and the number of owners, the landed elite favors giving the peasants land under a restricted ownership regime.

To illustrate this idea, I develop a model economy populated by two types of individuals: aristocrats —who are to resemble the elite of *hacendados* of a typical Latin American country— and peasants. In this paper I take as given that the society has eliminated the system of semi-serfdom. The transformation of the ownership regime is analyzed in two steps: first, I focus on the conflicts of interests between aristocrats and the peasantry that determine the amount of land to be transferred to the peasants. I assume that only aristocrats can own land; thus, the peasants receive land under a common property regime. Secondly, I extend the model allowing the peasants to become landowners —to receive land with full rights. The number of peasants who can own land is decided by the aristocrats.

Land and labor are complementary factors. Thus, for any given number of workers, the rents of land are maximized when the land is used in a specific ratio per worker. This, for a sufficiently small population size, requires to leave some land idle. This feature of the technology gives rise to a conflict of interests between aristocrats —claimants of the rents of land— and peasants: the former prefer to keep idle part of the land, whereas the peasantry want to have all the land cultivated, to increase their wage. The social unrest that forced the government to expropriate the *hacendados'* land is modeled in two ways: first, all land has to be used in the production process. Secondly, all the members of the society can vote on the amount of land to be privately owned. All the votes are equally weighted. The land expropriated from the aristocrats is given to those peasants who want to farm it under a common property regime. I model the evidence on the lower productivity of communal land as if the owners of land had access to a technology with higher productivity.

The labor market is modelled as an institution conditioned by social norms inherited from the times of semi-serfdom: *hacendados* decide the number of peasants who can sell their labor services before voting on the amount of privately owned land takes place. Those who cannot work for an *hacendado* cultivate the expropriated land.

Three factors are key to determine the amount of privately owned land. The first one is the population size, and the second one is the size of the aristocracy relative to the total population. The third factor is the difference of productivity cultivating land communally and cultivating land with full rights. If the three factors are sufficiently small, the amount of privately owned land is less than the total amount, which is the aristocrats' most preferred outcome. The aristocrats prefer to have expropriated some of their land because, otherwise, they would be forced to cultivate more units of land per worker than those that maximize their rents. Since the yield obtained cultivating land communally is lower than the wage paid by the aristocrats, those peasants who can work for a landowner prefer all the land to be cultivated under a private ownership regime. Those who cannot work for a landowner want a small amount of privately owned land. The two groups of peasants have opposite views about the amount of formal land. Given this opposition, the aristocrats arise as the median voter. Thus, the aristocrats do not want any peasants to become landowners, since it would just decrease the amount of privately owned land per landowner. This is the equilibrium that I take to resemble a Latin American land reform.

To understand the effect of population size, I analyze the equilibrium outcome when the population size is sufficiently large. There, the aristocrats cannot offer the peasants a sufficiently high wage; thus, all the peasants prefer to expropriate the land and cultivate it under a common property regime.

The literature on land reform has stressed the link between level of development and a more egalitarian distribution of land across the members of a society, but there have been few attempts to explain why a society decides to undergo a land reform, specially the type of reform experienced by the Latin American countries mentioned.[4]

Grossman (1994) and Horowitz (1993) develop model economies in which a land reform arises in equilibrium as a mean for the individuals holding the political power to preclude the peasantry from expropriating all their land. To obtain this result, they assume that the landless individuals have access to some expropriation technology which is successful with a positive probability. There are two main differences between their approaches and mine: first, in the model I develop not only the distribution of land but the ownership regime is endogenous. Secondly, as opposed to their approach, in my model economy, the distribution of land is decided by a voting process, which has the same role that Grossman's and

[4]See Moene (1992) and Ray and Streufert (1992), for instance, for a discussion of the link between landownership distribution and level of wealth. Both models compare levels of wealth for different distributions of land across individuals.

Horowitz's expropriation technologies. The difference is that the success of my expropriation technology, the voting process, is endogenous, since it depends on the size of the peasantry.

This paper is related to the growing literature that analyzes the link between conflicts of interests within a society and level of development. There are two different approaches. That which uses a voting process mainly focuses on studying the effects of the heterogeneity among the agents on the level of capital taxation and, therefore, on the level of growth. Krusell, Quadrini, and Ríos-Rull (1994) provide an excellent survey of the literature. There is another approach which uses a game-theoretical framework, which studies the relationship between level of wealth and the ownership regime chosen. Benhabib and Rustichini (1996) provide a very good example of this literature.

The rest of the paper is organized as follows: In Section 2, I obtain the amount of land cultivated in a oligarchy, which will help us to understand the landed elite's preferences on privately owned land. Section 3 presents the full model and Section 4 discusses the choice of ownership regime depending on the population size and the ex-ante landownership concentration. In Section 5 I analyze the relationship between population density and choice of ownership regime. Section 6 studies the role of the differences on productivity between privately owned and communal land in determining the land reform. Section 7 concludes.

# 2. Oligarchy, Formal Land, and the *Colonato* Regime

## 2.1. The Environment

To understand the Latin American land reforms, it will be useful to think of the system prevailing before the reforms took place. To do so, I use a one period model. The economy is populated by a measure $N$ of individuals. They value the consumption of a composite commodity, $c$, and do not value leisure. Each individual is endowed with one unit of time. The consumption good is produced using land and the individuals' endowments of time. There is a group of individuals, called *aristocrats,* whose measure is $A$. The rest of the individuals are called *peasants.* The total amount of farmland in this economy is denoted as $\bar{L}$. Each aristocrat has $\bar{L}/A$ units of land at his disposal and $(N-A)/A$ peasants work for him under the *colonato* regime. We can think of this economy as composed of $A$ different regions, whose size is $\bar{L}/A$. One aristocrat and $(N-A)/A$ peasants live in each region and they cannot migrate. The aristocrat of the region owns the land in the region. Before the production takes place, the aristocrat decides the amount of land to be used in the production process, which is going to be called *formal land.* The rest of the land is called *informal land,* and peasants are precluded from using this land in any manner. The peasants in the region work for the aristocrat tilling the formal land and receive, as a payment, their marginal productivity. Each aristocrat have access to the production technology

$$y_F(f,l) = (\gamma l^{\rho} + (1-\gamma) f^{\rho})^{\frac{1}{\rho}}, \tag{2.1}$$

where $l$ is land and $f$ is labor time. The parameter $\rho$ is less than zero; thus, land and labor are complementary factors. For simplicity, I assume that the return to managerial time is zero.

### 2.2. The Amount of Formal Land in an Oligarchic Society

The marginal productivity of labor in any given region is

$$w\left(\frac{N-A}{A}, l\right) = (1-\gamma)\left(\frac{N-A}{A}\right)^{\rho-1}\left(\gamma l^{\rho} + (1-\gamma)\left(\frac{N-A}{A}\right)^{\rho}\right)^{\frac{1}{\rho}-1}, \tag{2.2}$$

where $l$ denotes a given amount of formal land and $(N-A)/A$ is the number of peasants in the region. Each aristocrat chooses the amount of formal land that maximizes his rents; thus, it solves

$$\begin{aligned} &\max_{l} \quad \left(\gamma l^{\rho} + (1-\gamma)\left(\tfrac{N-A}{A}\right)^{\rho}\right)^{\frac{1}{\rho}} - w\left(l, \tfrac{N-A}{A}\right)\tfrac{N-A}{A} \\ &s.t. \quad l \in \left[0, \tfrac{\bar{L}}{A}\right]. \end{aligned} \tag{2.3}$$

The aggregate amount of formal land is

$$L = \min\left\{\bar{L}, \left(\frac{-\gamma}{\rho(1-\gamma)}\right)^{\frac{-1}{\rho}}(N-A)\right\}. \tag{2.4}$$

The previous expression shows that the amount of cultivated land is less than the total amount when the population size is sufficiently small —when land is sufficiently abundant. Given the complementarity between land and labor, if land is too abundant not only the marginal productivity of land is low, but also the absolute rents of land. Therefore, the aristocrats are better off when not all the land is cultivated. Actually, the aristocrat maximizes his rents when the formal land-labor ratio is $\left(\frac{-\gamma}{\rho(1-\gamma)}\right)^{\frac{-1}{\rho}}$, for any given number of peasants. This model gives a rationale to a common behavior of the landed elite: they did not cultivate all their farmland. The reason might rest on a complementarity between land and labor, the latter being the relatively scarce factor. This result depends crucially on assuming that peasants are precluded from invading the informal land. This simple model also gives a rational to the *colonato* regime: in the absence of this regime, the peasants would be free to sell their labor services to any aristocrat; thus, their marginal productivity would be determined by the economy-wide land-labor ratio. In such a case, the wage and, hence, the labor rents would not depend on the land a single aristocrat cultivates, consequently, he would not keep any land uncultivated and his total rents would be lower.[5]

### 2.3. The Weakening of the *Colonato* Regime: Land Invasions and the Amount of Formal Land

The social tensions that led to a land reform were usually expressed as land invasions. The peasants in the region invaded the aristocrat's uncultivated land, the informal land, and farmed it.[6] The power of the aristocrats eroded, so the society evolved to a *de facto* situation: land could not be left unused. If the aristocrats did not cultivate the land, some peasants would invade it. This situation was institutionalized in the land reform laws: for the aristocrats to keep their land they had to cultivate it; otherwise, it was expropriated.[7] To understand the effect of this law on the behavior of the aristocrats, let us think of an oligarchic society where the system of *colonato* is weakly enforced: the aristocrats do not have the power to prohibit land invasions, but peasants cannot move across regions.[8] Furthermore, since we are analyzing economies in which land is the relatively abundant factor, let us assume that the size of the population is sufficiently small,

$$N < \frac{\bar{L}}{\Gamma} + A, \tag{2.5}$$

where

$$\Gamma = \left(\frac{-\gamma}{\rho(1-\gamma)}\right)^{\frac{-1}{\rho}}. \tag{2.6}$$

[5]This is the rationale given in the development theory literature to the serfdom systems prevailing in the feudal land-abundant periods in Western Europe, the systems of semi-serfdom in Latin American countries and to the slavery system established in places like the US Southeast. See Bisnwanger *et al.* (1992) and De Janvry (1981).

[6]There is much informal evidence in this respect, but the literature does not provide rigorous estimates of the amount of land invaded and the number of peasants involved. See De Janvry (1981), De Soto (1989), Heath *et al.* (1969), Kay (1982) , Wilkie (1974) and Wilkie (1993).

[7]The land reforms laws in Bolivia, Mexico, Peru and Venezuela forced the holders of land to cultivated it; otherwise, it was given to someone else. See De Janvry (1981), Heath *et al.* (1969), Ireson (1987), Kay (1982), Wilkie (1974).

[8]There are two possible justifications to this assumption: first, migration is very costly. Secondly, under a semi-serfdom regime, the aristocrats had the power to imprison the peasant who left the *hacienda.* This last argument holds if we assume that there exists an implicit agreement among the aristocrats to enforce that law, which I take as given throughout the paper.

Let us assume that the aristocrat in the representative region decides to cultivate $l$ units of land. The rest, $\bar{L}/A - l$, is informal land. The aristocrat cannot prevent the peasants from invading this land. Let us call *formal workers* those peasants who work for the aristocrat and *informal workers* those who invade the informal land. The informal workers are assumed to farm the land communally and the yield they obtain is

$$y_I(f,l) = \theta\left(\gamma\left(\frac{\bar{L}}{A} - l\right)^{\rho} + (1-\gamma)\left(\frac{N-A}{A} - f\right)^{\rho}\right)^{\frac{1}{\rho}}, \tag{2.7}$$

where $f$ is a given number of formal workers. The parameter $\theta$ satisfies $0 < \theta < 1$, and it is meant to capture the evidence on the lower productivity of the communally operated farms. Each informal worker receives the average yield, thus,

$$C_I(f,l) = \frac{1}{\frac{N-A}{A} - f} \cdot y_I(f,l). \tag{2.8}$$

The number of formal workers is such that the wage paid by the aristocrat equals the average yield farming communal land; otherwise, all peasants would either work for the aristocrat or invade the informal land. Thus, the number of formal workers is a function of the amount of formal land,[9]

$$f = \eta(l). \tag{2.9}$$

Let us assume that the aristocrat decides to cultivate $\Gamma(N-A)/A$ units of his land, as he would choose if the peasants could not invade the informal land. The minimum amount of land needed for no peasant to invade the informal land is $L_M$, as it is shown in the Lemma 1.1 in the Appendix. If the productivity parameter satisfies $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$, $L_M$ is greater than the amount the aristocrat wants to cultivate, $\Gamma(N-A)/A$; thus, some peasants invade the informal land.[10] Since the aristocrat can foresee peasants' behavior, he chooses the amount of formal land that solves the problem

$$\begin{array}{ll} \max & C_A(\eta(l), l) \\ s.t. & l \in \left[0, \frac{\bar{L}}{A}\right] \end{array}, \tag{2.10}$$

where $C_A(f,l)$ denotes the aristocrat's rent and is equal to $(\gamma l^{\rho} + (1-\gamma)f^{\rho})^{\frac{1}{\rho}} - w(f,l)f$. The solution to this problem is an amount of formal land $\tilde{l}$ per region. The number of formal workers is $\eta(\tilde{l})$. Given $\eta(\tilde{l})$, the amount of formal land that

[9]The properties of the function $\eta$ are shown in the Lemma 1.1 in the Appendix.

[10]Shown in the Proposition 1.5 in the Appendix.

maximizes the aristocrat's rents is $\Gamma\eta(\tilde{l})$, which is less than $\tilde{l}$, if $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$. Thus, the aristocrat is forced to cultivate more land for the peasants to be willing to work for him. His rents are decreased because he is forced to use land and labor in a ratio different from $\Gamma$.[11]

The main conclusion from this analysis is that in societies in which land is the relatively abundant factor the claimants of the land rents are better off when the amount of cultivated land is less than the total amount; specifically, given any number of workers, the rents of land are maximized if the land-labor ratio is $\Gamma$. For the cultivated land to be less than the total either the peasants must be prevented from invading the idle land and from migrating or the wage the aristocrats pay is greater than the average yield in the communal lands.

## 3. Labor Market Institutions and Land Reform

This section uses the framework discussed previously to analyze how a mixed ownership regime emerges in a society. By a mixed ownership regime is meant a regime under which only a fraction of the farmland is privately owned. A land reform is modelled as a reallocation of land. The social conflicts that led the Latin American countries to undertake a land reform are modelled as if all the individuals voted on the amount of formal land to be kept by the aristocrats. For simplicity, I have assumed that all votes are equally weighted.[12] The existence of social conflicts implies that land cannot be left uncultivated; otherwise, some individuals would invade it, as it was shown in the previous Section. The land kept by the aristocrats is called formal land. The rest of the land is called informal land. A peasant can either be hired by an aristocrat to farm formal land, in which case he is a formal worker, or cultivate the informal land, along with other peasants, under a common property regime. In this last case he receives the average yield of the land, as in the previous Section the peasants who invaded land.

The countries mentioned in the Introduction share three features: labor was the relatively scarce production factor, the reforms were triggered by intense social conflicts, and the reforms not only encompassed expropriations of land but also the end of the semi-serfdom regime under which most of the peasants lived. After the reforms, the peasants were free to sell their labor services; nevertheless, there is evidence that suggests it was a somewhat restricted freedom. Heath *et al.* (1969) give a detailed description of the patterns of work in Bolivia. The most common practice for a peasant was to become a member of the syndicate of the region. The

[11] Shown in Proposition 1.6 in the Appendix.

[12] To capture the fact that aristocrats and peasants have different political power I could have assumed that an aristocrat's vote is worth two peasants' vote, for instance. The assumption does not change the result. It will be discussed in the conclusions.

landowners used to make collective contracts through the syndicate. Only those peasants who were members of the syndicate could work for a landowner. Similar were the cases of México and Perú. This observation suggests that the reforms encompassed a "segmented" labor market. The segmentation comes from an implicit agreement between the landowners, those who I have named aristocrats, that I take as given throughout the paper.

I incorporate this institutional feature of the Latin American countries to the model developed in the previous Section in the following way: peasants are free to move across regions, but only those named formal workers can work for an aristocrat. The group of the aristocrats, before voting takes place, collectively decides the number of formal workers. This timing has a rationale: land reforms are lumpy events that occur, perhaps, once every ten years, whereas social norms, as the one described, are very persistent over time. The model is organized in the following stages:

1. *Differentiation Stage*: The aristocrats decide the number of formal workers, $F$, who are chosen randomly across all peasants in all region. The rest of the peasants are informal workers.[13]

2. *Voting Stage*: All the individuals vote on the amount of formal land, $L$.

3. *Production Stage:* Each aristocrat is expropriated, without compensation, $(\bar{L} - L)/A$ units of land. The formal workers decide whether to work for an aristocrat or cultivate communal land along with the informal workers. Individuals produce and consume.[14]

I am going to analyze the equilibrium of the model assuming that no peasant can become landowner, and extract conclusions about the nature of the land reform. Throughout the paper, "formal sector" is the sector in which land is privately owned, comprised by aristocrats and those formal workers who work for the aristocrats. The "informal sector" is the sector in which no individual has individual rights to the land, composed by those peasants who cultivate informal land. In the rest of this Section I describe the actions the agents take at each stage and define an equilibrium for this model economy.[15]

[13] Alternatively, we could assume that once the aggregate number has been decided, $F/A$ peasants, drawn randomly across the peasants in the region are made members of the syndicate of the region. The result would be the same since the peasants can migrate.

[14] I could have assumed that the individuals vote on the amount of formal land in their region. Given the simmetry of the model, the results would be identical. In either case I need to assume that, in the production stage, the group of aristocrats can commit to not hire informal workers.

[15] I could have assumed that each aristocrat chooses the number of formal workers in the region and only these formal workers are allowed to work for him. Naturally, in each region individuals

### 3.1. The Production Stage

At this stage, the number of formal workers $F$ and the amount of formal land $L$ have been decided. Since formal workers can move freely across regions and the technology displays constant returns to scale in land and labor, the wage is determined by the aggregate variables

$$w\left(F, L, \varphi(F, L)\right) = (1-\gamma)\varphi(F, L)^{\rho-1}\left(\gamma L^{\rho} + (1-\gamma)\varphi(F, L)^{\rho}\right)^{\frac{1}{\rho}-1}. \tag{3.1}$$

For any given number of formal workers, $F$, and formal land, $L$, $\varphi(F, L)$ denotes the number of formal workers that stay in the formal sector.[16] Each aristocrat receives a fraction $1/A$ of the amount of formal land and, hence, the same fraction of the aggregate rents from formal land. Therefore, their consumption is

$$C_A\left(F, L, \varphi(F, L)\right) = \frac{1}{A}\gamma L^{\rho}\left(\gamma L^{\rho} + (1-\gamma)\varphi(F, L)^{\rho}\right)^{\frac{1}{\rho}-1}. \tag{3.2}$$

Obviously, $\varphi(F, L) \leq F$, since the aristocrats have committed to hire only those formal workers who want to work for them. Therefore, in equilibrium, the wage is always greater than or equal to the informal worker's income, which is

$$C_I\left(F, L, \varphi(F, L)\right) = \theta\left(\gamma\left(\frac{\bar{L}-L}{N-A-\varphi(F, L)}\right)^{\rho} + (1-\gamma)\right)^{\frac{1}{\rho}}. \tag{3.3}$$

For any given amount of formal land $L$, let us define $\eta(L)$ as the number of formal workers who would work for the aristocrats if all peasants could sell their labor services freely. Thus, if $\eta(L)$ satisfies $0 < \eta(L) < N-A$, the wage and the informal income satisfy[17]

$$w\left(N-A, L, \eta(L)\right) = C_I\left(N-A, L, \eta(L)\right). \tag{3.4}$$

Otherwise, all the peasants would stay in one sector. The function $\varphi$ satisfies

$$\varphi(F, L) = \min\left\{F, \eta(L)\right\}. \tag{3.5}$$

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vote on the amount of formal land in the region. This model also needs of the agreement among aristocrats to not hire formal workers from other regions. Thus, in this case, migration is more restricted. Nevertheless, the symmetry of the model implies that no one migrates in equilibrium. The equilibrium for this economy is identical to that of the economy I describe.

[16] I assume that the wage the formal workers receive is their marginal productivity. We could think they receive the marginal productivity minus a fixed amount. What is essential to this argument is that their wage is an increasing function of the amount of formal land. Sadoulet (1992) shows that in model economies that share the main characteristics of the Latin American countries mentioned, an efficient contract would feature a wage increasing with the amount of land.

[17] Defined in Lemma 1.1 in the Appendix.

The wage and the informal income satisfy

$$w(F, L, \varphi(F, L)) \geq C_I(F, L, \varphi(F, L)). \tag{3.6}$$

Figures 1, 2, and 3 show the shape of $\eta$ and $\varphi$ as functions of the amount of formal land, given a number of formal workers $F$. They show the shape of the functions $\eta$ and $\varphi$ for different population sizes. In each case, there exists an amount of formal land, $\lambda(F)$, for which the wage is equal to the informal income and no formal worker joins the informal sector. It satisfies $F = \eta(\lambda(F))$. For any amount of formal land less than $\lambda(F)$, some formal workers leave the formal sector. The main difference between the three cases is the shape of the function $\eta$. If the population size is small, $N < \Psi + A$, the land-labor ratio in both sectors, $L/\eta(L)$, and $(\bar{L} - L)/(N - A - \eta(L))$, respectively, are increasing with the amount of formal land.[18] This implies that the wage and the informal income would be increasing functions of the amount of formal land if the number of formal workers were $\eta(L)$ —if the peasants could move freely across sectors. If $N = \Psi + A$, both ratios are constant and equal to the economy-wide ratio $\bar{L}/(N - A)$. If population size is large, the ratios and, hence, the wage and the informal income would be decreasing with the amount of formal land. This different behavior will determine the peasants' preferences on the amount of formal land.[19]

## 3.2. The Individuals' Preferences on Formal Land and the Differentiation Stage

The amount of formal land is decided at the voting stage. The amount of formal land chosen is the one that defeats all others in pair wise comparisons. In any comparison, each individual ranks both amounts and votes for the amount that he or she prefers the most. The individuals' preferences on the amount of formal land are given by their level of consumption $C_j(F, l, \varphi(F, l))$, where $j$ denotes the type of the individual, $j \in \{A, F, I\}$. $C_F(F, l, \varphi(F, l))$ denotes the level of consumption of a formal worker, which is either the wage or the average yield in the informal sector. Let $L_j(F)$ be the maximum amount of formal land that maximizes $C_j(F, l, \varphi(F, l))$, for each $j \in \{A, F, I\}$. We will show in the following sections that the voting outcome $L(F)$ satisfies

$$L(F) = L_m(F), \tag{3.7}$$

[18] The parameter $\Psi$ is equal to $\frac{\bar{L}}{\Gamma}\left(\frac{1-\theta}{-\rho\theta}\right)^{\frac{-1}{\rho}}$.

[19] The proofs of these results are found in Lemma 1.2, Lemma 1.3 and Proposition 1.4 in the Appendix.

where $m$ denotes the type of the median voter. At the differentiation stage, the aristocrats choose the number of formal workers $F$ that solves

$$\begin{array}{ll} \max_{f} & C_A(f, L(f), \varphi(f, L(f)) \\ & f \in [0, N-A] \end{array} . \tag{3.8}$$

**Definition 3.1.** *An equilibrium for this economy is a voting outcome function* $L : [0, N-A] \to [0, \bar{L}]$ *and a number of formal workers* $F$ *that satisfy*

*1.* $L(F) = \arg\max_{l \in [0,\bar{L}]} C_m(F, l, \varphi(F, l))$ *where* $m \in \{A, F, I\}$ *denotes the median voter type and* $C_A$, $C_F$, $C_I$ *satisfy (3.1)–(3.6).*

*2.* $F = \arg\max_{f \in [0,N-A]} C_A(f, L(f), \varphi(f, L(f)))$

## 4. A Latin American Land Reform

This Section focuses on the conflicts of interests that determine the amount of privately owned land, which I have called formal land. In the following Section we will study the link between population size and the conflicting interests that arise within a society. The analysis in this Section is restricted to the region of the parameters space determined by the following assumptions:

ASSUMPTIONS:

A1. $A < N/2$.

A2. $\bar{\rho} \leq \rho < 0, \quad \bar{\rho} < -1$.

A3. $0 < \theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$.

A3. $N < \bar{L}/\Gamma + A$.

Assumption 1 rules out the equilibrium in which the aristocrats are the majority of the population.[20] Assumption 2 imposes a lower bound to the elasticity of substitution between land and labor, this bound being less than -1.[21] Assumption 3 implies that cultivating land communally is very inefficient. This assumption is made to simplify the analysis, and it is relaxed in Section 6. Assumption 3 tells us that land is the relatively abundant factor, as in the Latin American countries we are focusing on. This assumption also implies that under an oligarchic regime

[20] I do not discuss this case because the aristocrats are to resemble the landed elite in a typical Latin American country, who owned most of the farmland and usually comprised a very small fraction of the population. In the countries mentioned in the Introducion the landed elite comprised about 5 percent of the population.

[21] The lower bound $\bar{\rho}$ is set so $2\bar{L}/\Gamma \leq \Psi$ for all $\theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$. Recall that $\Psi = \frac{\bar{L}}{\Gamma}\left(\frac{1-\theta}{-\rho\theta}\right)^{\frac{-1}{\rho}}$. This assumption implies that any population size that satisfies $N \leq 2\bar{L}/\Gamma$ also satisfies $N \leq \Psi + A$, which is sufficient to guarantee that formal worker's preferences are single peaked.

only $\Gamma(N-A)$ units of land would be cultivated. Assumption 1 and Assumption 3 imply that the population size satisfies $N < 2\bar{L}/\Gamma$. This inequality and Assumption 2 ensure that the population size satisfies $N < \Psi + A$, where $\Psi$ is a combination of parameters defined in Lemma 1.2 in the Appendix. Thus, we are in the case in which for any amount of formal land that satisfies $l < \lambda(F)$ the land-labor ratios in both sectors are increasing functions of the amount of formal land —which is sufficient to guarantee that formal workers' preferences are single peaked.

## 4.1. The Individuals' Preferences on Formal Land and the Voting Outcome

Figure 4 shows the individual preferences on formal land for a given number of formal workers that satisfies $0 < F < N - A$. The proofs of the single-peakedness of the preferences are shown in Propositions 2.1, 2.2, and 2.3 in the Appendix. Figure 4 shows that the aristocrats' consumption is maximized for an amount of formal land $\Gamma F$, which, given Assumption 4, is less than the total amount of land. The informal workers' preferences have a maximum at $l = \lambda(F)$, which is the amount for which the wage equals the informal income and no formal worker joins the informal sector. Informal workers do not want the aristocrats to keep more than $\lambda(F)$ units of land since the amount of land per informal worker and, hence, the average yield would decrease. They do not want the aristocrats to keep less than $\lambda(F)$ units because, otherwise, some formal workers would join the informal sector. Since the population size satisfies $N < \Psi + A$, the land-labor ratio and, thus, the consumption in the informal sector would be lower than those obtained if $L = \lambda(F)$. Formal workers prefer all the land to be formal to increase their wage. Assumption 3, which imposes an upper bound on the parameter $\theta$, implies that whenever the land-labor ratio in the formal sector is $\Gamma$, the wage is greater than the informal income; thus $\lambda(F) < \Gamma F$, for any $F$.[22] The preferences of the three types of agents are single peaked; thus, we can apply the median voter theorem. The amount of formal land $L(F)$ satisfies

$$L(F) = \begin{cases} \lambda(F) & \text{if } F < \frac{N}{2} - A, \\ \Gamma F & \text{if } \frac{N}{2} - A \leq F \leq \frac{N}{2}, \\ \bar{L} & \text{if } F > \frac{N}{2}. \end{cases} . \tag{4.1}$$

The three candidates to be the voting outcome satisfy $\lambda(F) \leq \Gamma F \leq \bar{L}$. Since preferences are single peaked, it follows that informal workers prefer $\Gamma F$ to $\bar{L}$, and formal workers prefer $\Gamma F$ to $\lambda(F)$. If informal workers are the majority of

[22] If $\theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$, for any given $F$ that satisfies $0 < F < N - A$ it follows that $w(F, \Gamma F, F) > C_I(F, \Gamma F, F)$.

the population —$F + A < N/2$— $\lambda(F)$ units of land are formal. The rest is expropriated. If formal workers are the majority of the population, the aristocrats keep all the land, but they have to cultivate it, as it was imposed in the land reform laws in Bolivia, México, and Perú. If neither group is the majority, the aristocrats keep $\Gamma F$ units of land, and the rest is expropriated and given as informal land to those peasants who want to farm it. Notice that $\Gamma F$ can be the voting outcome only if $\lambda(F) \leq \Gamma F$, which is the case here, but we will see other scenarios where the reverse is true and, hence, the aristocrats cannot be the median voter.

## 4.2. The Number of Formal Workers

To determine the number of formal workers, the aristocrats solve the problem shown in the expression (3.8) at the differentiation stage. For any number of formal workers that satisfies $F < N/2 - A$, the amount of formal land is $\lambda(F)$, for which the wage is equal to the informal income and no formal worker joins the informal sector. The assumption on the parameter $\theta$ implies that $\lambda(F) < \Gamma F$, for any $F$ that satisfies $0 < F < N - A$. This implies that the aristocrats' consumption is an increasing function of the number of formal workers for any $0 < F < N/2 - A$.[23] Thus, the number of formal workers always satisfies $N/2 - A \leq F < N - A$. For any number of formal workers that satisfies $N/2 - A \leq F \leq N/2$, aristocrats are the median voter and their equilibrium consumption is

$$C_A\left(F, \Gamma F, \varphi(F, \Gamma F)\right) = -\rho \frac{F}{A} \left(1-\gamma\right)^{\frac{1}{\rho}} \left(1-\rho\right)^{\frac{1-\rho}{\rho}}, \tag{4.2}$$

which is a strictly increasing function of the number of formal workers. If they choose a number of workers greater than half of the population, all land becomes formal and their consumption is

$$C_A\left(F, \bar{L}, \varphi(F, \bar{L})\right) = \frac{1}{A}\gamma \bar{L}^{\rho} \left(\gamma \bar{L}^{\rho} + (1-\gamma)F^{\rho}\right)^{\frac{1}{\rho}-1}, \tag{4.3}$$

which is also strictly increasing in the number of formal workers. It follows that the choice is in the set $\{N/2, N - A\}$. Thus, the aristocrats choose $F = N/2$ or $F = N - A$, depending on which associated level of consumption is highest, which in its turn depends on the size of the population and the relative size of the aristocracy.

Comparing the expressions (4.2) and (4.3) it is easy to show that there exists $\chi > 1$, such that for any population size that satisfies $N \leq \bar{L}/(\Gamma\chi)$, regardless of their group size, $A$, the aristocrats choose $F = N/2$. In that case the amount

[23] Recall that, for any given number of workers $F$, $\Gamma F$ is the amount of formal land that maximizes the land rents, i.e. $\frac{\partial C_A(F, \Gamma F, F)}{\partial l} = 0$. Thus, for any $l < \Gamma F$, $\frac{\partial C_A(F, l, F)}{\partial l} > 0$.

of formal land is $\Gamma N/2$. If $N > \bar{L}/(\Gamma\chi)$, the choice depends on the size of the aristocracy. Again, from direct comparison of the expressions (4.2) and (4.3), we obtain a critical level for the number of aristocrats, $\alpha_{l(N)}$, which is larger the larger is $N$. If the number of aristocrats satisfies $A \geq \alpha_{l(N)}$, the aristocrats choose $F = N/2$. Otherwise, they choose $F = N - A$, case in which they keep all the land and cultivate it. The proof of this result is found in Proposition 2.4 in the Appendix. Thus, the expropriation of lands takes place if the population size is sufficiently small, $N \leq \bar{L}/(\Gamma\chi)$, or if the size of the peasantry is small relative to the amount of land: $A \geq \alpha_{l(N)}$ and $\bar{L}/(\Gamma\chi) < N < \bar{L}/\Gamma + A$. In those cases the aristocrats are better off hiring $F = N/2$ which implies an amount of formal land equal to $\Gamma N/2$.

The conflict in the society is the following: land cannot be left idle; unused land is taken away from the owner and given to someone else. That is, aristocrats can only keep the land if they cultivate it. If all peasants were free to sell their labor services anywhere —$F$ were equal to $N - A$— peasants would not want to expropriate the aristocrats' lands; they would force the aristocrats to cultivate the land. This would imply a land-labor ratio in the formal sector larger than $\Gamma$, the ratio for which the aristocrats' rents are maximized. Thus, aristocrats prefer some of the land to be expropriated; specifically, they want to keep $\Gamma F$ units of land. The amount $\Gamma F$ is a voting outcome only if neither formal workers or informal workers are the majority of the population and formal workers prefer this outcome to $\lambda(F)$, the outcome most preferred by informal workers. This is the case only if formal workers are ensured a higher wage when the amount of formal land is $\Gamma F$, which is implied by Assumption 3. Hence, the choice aristocrats face is to allow all peasants to sell their labor services in the formal sector and cultivate all the land —more than $\Gamma$ units per worker— or to have a land-labor ratio equal to $\Gamma$ at the price of prohibiting some peasants to work in the formal sector. The choice depends on the population size and the relative size of the peasantry.

### 4.3. On the Number of Landowners

So far, I have assumed that only aristocrats can own land. I have modelled private ownership as being able to hire workers. Let us assume now that the peasants can become landowners. I am going to model it as if the aristocrats can decide, before voting takes place, the number of peasants who are going to receive land with total rights. I am going to call them *small owners* and their number is denoted as $S$. All individuals vote on the amount of land to be privately owned, $L$. Once it has been voted, each aristocrat is expropriated $\bar{L}/A - L/(A+S)$ units of land. Thus, an aristocrat keeps $L/(A+S)$ units and each small owner is given $L/(A+S)$ units. As the aristocrats, the group of small owners commit, at the production stage, to hire only formal workers.

The timing is as follows: at the differentiation stage, the aristocrats choose which workers are to be formal and which peasants are to be small owners. The other two stages are identical to those of the previous version of the model. At the voting stage and the production stage, small owners and aristocrats behave identically in every respect. As opposed to formal workers, just for simplicity, small owners cannot join the informal sector.

For any given number of small owners, $S$, formal workers, $F$, and formal land, $L$, the consumption of formal workers, aristocrats, small owners and informal workers is shown in the expressions (3.1), (3.2), and (3.3). I am going to analyze how the aristocrats' consumption changes as a function of $S$. Since aristocrats and small owners behave identically, the number of formal workers and amount of formal land chosen in a economy in which there are $A$ aristocrats and $S$ small owners are equal to those chosen in an economy in which the number of aristocrats is $A+S$ and the peasants cannot own land. The individuals' preferences on land are identical to those shown in Figure 4.

The aristocrats always choose $S$ such that the number of landowners is less than or equal to $N/2$, since for $S \geq N/2-A$ the landowners are always the median voter. To analyze the equilibrium outcome in this model economy, I am going to assume, first, that the number of small owners is fixed, and I will show that the aristocrats' rent is a decreasing function of the number of small owners.

For any population size that satisfies either $N \leq \bar{L}/(\Gamma\chi)$, or $\bar{L}/(\Gamma\chi) < N < 2\bar{L}/\Gamma$ and $A+S \geq \alpha_{l(N)}$, the number of formal workers is $N/2$ and the amount of formal land is $\Gamma N/2$. The aristocrats' rent is

$$-\rho\frac{N/2}{A+S}(1-\gamma)^{\frac{1}{\rho}}(1-\rho)^{\frac{1-\rho}{\rho}} \tag{4.4}$$

which is decreasing in the number of small owners. If $\bar{L}/(\Gamma\chi) < N < 2\bar{L}/\Gamma$ and $A+S < \alpha_{l(N)}$, the aristocrats' rent is

$$\frac{\gamma}{A+S}\bar{L}^{\rho}\left(\gamma\bar{L}^{\rho}+(1-\gamma)(N-A-S)^{\rho}\right)^{\frac{1-\rho}{\rho}} \tag{4.5}$$

which is also decreasing in the number of small owners. Thus, the number of small owners is zero. Thus, if the *hacendados* have the power to decide the nature of the land reform, they never allow the peasants to own land. The *hacendados* would allow the peasants to become owners if they needed enough constituency to be the median voter. Being land abundant and productivity in the informal sector very low, the aristocrats obtain enough support from formal workers, who prefer $\Gamma F$ to the outcome most preferred by the informal workers, $\lambda(F)$. Thus, the *hacendados* do not need the small owners and they are better off whenever the group of landowners is a very small fraction of the population.

We can use this model to understand the Mexican reform started in 1917. Then, a minority of the population, smaller than $\alpha_{l(N)}$, controlled practically all the farmland. The revolution forced a change of the institutions and the political system. A land reform was enacted. According to the evidence shown in the Introduction, peasants would have been better off if they had received the ownership right to the land granted. Actually, total product would have been higher if all the land were cultivated under a private property regime. Why would a society choose an inefficient outcome? The reason must be that some group might benefit from it.[24]

## 5. Population Size and the Ownership Regime

In the previous Section we have seen that the aristocrats, by means of dividing the peasants in formal and informal workers, split them in two groups with opposite preferences on formal land. Given this opposition, the landowners arise as the median voter. This "divide and conquer" policy it is not always possible. If the size of the population is sufficiently large, formal and informal workers' preferences are maximized when all land is informal. Thus, in this section I am going to explore the equilibrium for economies that satisfy $N \geq \bar{L}/\Gamma + A$ and the following assumptions:

ASSUMPTIONS:

A1. $A < N/2$.

A2. $0 < \theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$.

A3. $\bar{\rho} \leq \rho < -1, \;\; \bar{\rho} < -1$.

In the previous Section I assumed the parameter $\rho$ to satisfy $\bar{\rho} \leq \rho < 0$; here I restrict the set in which it can take values to $[\bar{\rho}, -1)$ to simplify the analysis. I am going to assume, as in the basic model, that no peasant can become a landowner.

[24] There is no formal evidence on this hypothesis for the Mexican case, but there are some anecdotic episodes of the beginning of the agrarian reform in Perú in the early 1070s narrated by De Soto (1989). Rigth before the reform was enacted, owners of land in the outskirts of Lima engaged with settlers in organizing fictitious invasions of the land receiving, in exchange, "more money than expropriation would have brought but less than the normal price" (pg. 30). If the owners were better off having the land invaded instead of selling it, it must be because after the sale the land remains a commodity, a transferable good, whereas the invaded land does not. Thus, given that they were going to have their land expropriated, the Perovians landowners preferred the peasants —if we think of the invasion as a *de facto* expropriation— not to receive the ownership to the land transferred; otherwise, they would have sold it or let it to be expropriated.

In economies in which the size of the population satisfies $\bar{L}/\Gamma + A \leq N < 2\bar{L}/\Gamma$, the equilibrium outcome is identical to the one discussed in the previous section. The amount of formal land is less than the total amount of land whenever the size of the aristocracy is sufficiently small, that is, if $A \leq \alpha_{l(N)}$.

If the population size satisfies $2\bar{L}/\Gamma \leq N < \Psi + A$, the formal and informal workers' preferences on formal land are identical to those in the cases where the population size is smaller than $2\bar{L}/\Gamma$. The amount of formal land that maximizes the aristocrats consumption is, as shown in Prop 2.3 in the Appendix, either $\bar{L}$ or $\Gamma F$, depending on which amount is the lowest. Figure 5 shows the individual preferences for any given number of formal workers that satisfy $F \geq N/2$. In this case, formal workers' and aristocrats' preferences are maximized for $L = \bar{L}$. Thus, given that the voting outcome is always $L = \bar{L}$, the aristocrats choose all peasants to be formal workers. Thus, if the population size satisfies $2\bar{L}/\Gamma \leq N < \Psi + A$, the aristocrats maximize their rents cultivating all the land. The proof of this result is found in Proposition 3.1 in the Appendix.

The scenario is very different for any economy in which $N > \Psi + A$. Figure 6 shows the individuals' preferences on formal land for a number of formal workers that satisfies $0 < F < N - A$. The wage and the informal income are decreasing functions of the amount of formal land, for any amount less than $\lambda(F)$, whereas they were increasing in the case in which $N < \Psi + A$. The formal workers preferences' on land are not single-peaked in this case. They prefer all the land to be formal or informal, depending on the size of their group relative to the size of the population. The informal workers prefer to have all the land informal. Thus, a voting outcome does not exists unless the aristocrats' most preferred outcome is $\bar{L}$. Regardless of formal workers' preferences, if the aristocrats choose $F < N/2 - A$, all the land becomes informal; thus, the aristocrats always choose $F \geq N/2 - A$. For any population size that satisfies $N > 2\bar{L}/\Gamma + 2A$ , if $F \geq N/2 - A$, the aristocrats' rents are maximized for $L = \bar{L}$.[25] Hence, in this case, a voting outcome is well defined: it is either $\bar{L}$ or zero. It depends on the size of the group of formal workers. If all land becomes informal, the peasants receive the average yield of the informal land

$$C_I = \theta \left( \gamma \left( \frac{\bar{L}}{N - A} \right)^{\rho} + (1 - \gamma) \right)^{\frac{1}{\rho}}. \tag{5.1}$$

If, for a given number of formal workers $F$, all land becomes formal, the formal workers receive

[25] Since $\theta < (1 - \rho)^{\frac{1-\rho}{\rho}}$ it implies that $\Gamma F > \lambda(F)$, for all $F$. Thus, the aristocrats' most preferred outcome is equal to $\min \left\{ \bar{L}, \Gamma F \right\}$ . For any population size that satisfies $N > 2\bar{L}/\Gamma + 2A$ the most preferred outcome is $\bar{L}$.

$$w(F) = \left(\gamma\left(\frac{\bar{L}}{F}\right)^{\rho} + (1-\gamma)\right)^{\frac{1}{\rho}-1}. \tag{5.2}$$

$\bar{L}$ maximizes the formal workers' preferences if $w(F) \geq C_I$. Since $w(F)$ is decreasing with the number of formal workers, it follows that there exists a maximum number of formal workers, which I am going to call $\zeta_{(N,A)}$, for which formal and informal workers' preferences on the amount of formal land are opposed. Thus, if the number of formal workers is greater than $\zeta_{(N,A)}$, both types of peasants prefer all the land to be informal. If the number of formal workers is less than or equal to $\zeta_{(N,A)}$, the formal workers' preferences are maximized when all the land is formal. Thus, formal and informal workers have opposite preferences on the amount of formal land whenever $F \leq \zeta_{(N,A)}$. This bound increases with the size of the population and the ratio $\zeta_{(N,A)}/N$ decreases monotonically, given that the parameter $\rho$ has been assumed to be less than $-1$. Therefore, there exists a threshold, $\Upsilon$, such that for any $N \leq \Upsilon$ it is satisfied that $\zeta_{(N,A)} + A \geq N/2$; for any $N > \Upsilon$, the inequality is reversed. Thus, if $N \leq \Upsilon$, the number of formal workers equals $\zeta_{(N,A)}$ and no land is expropriated from the aristocrats. If $N > \Upsilon$, regardless of the number of formal workers, all land is expropriated and given to the peasants under a common property regime. Propositions 3.2 and 3.3 show the formal and informal workers' preferences on formal land. Proposition 3.4 contains the proof of the equilibrium.

Thus, the aristocrats' ability to "divide and conquer" depends on the population density. If population is too large, the aristocrats cannot offer high enough wages for the peasants to prefer the wage instead of the average yield in the informal sector. Since I have assumed the size of the aristocracy to be less than half of the population, all land becomes informal. In this case, although it would go beyond the scope of this paper, it would benefit the aristocrats to give some peasants the right to own land privately to build enough constituency so they would keep some of the land.

## 6. Differences on Productivity

In the scenarios studied, in any economy in which the amount of formal land is less than the total amount, the equilibrium wage is strictly greater than the informal income. This implication matches the Mexican evidence on minimum wages and average income in the *ejidal* land, as it was mentioned in the Introduction, but it has been obtained using a very strong assumption on the productivity in the informal sector, that is, $\theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$. This assumption, although not necessary to obtain the results, was used to simplify the analysis. In Section 2, I justified the assumption that no land could be left idle assuming $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$. Thus, I am going to show that the result holds for $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$. It is shown in Lemma 4.1 in the Appendix the existence of $\theta^*$ that satisfies $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta^* < (1-\rho)^{-1}$, such that if $\theta$ satisfies $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$, the equilibrium features an amount of formal land less than the total amount and the equilibrium wage is strictly greater than the informal income. The results are summarized in the following table:

$(1\text{-}\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$
$N \leq 2\bar{L}/(\Gamma(1+$

Table 1

| | | |
|---|---|---|
| $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$ | | |
| | $2\bar{L}/(\Gamma(1+\kappa)) < N \leq \bar{L}/(\Gamma\chi)$ | |
| | $A \leq \alpha_{u(N)}$ | $A > \alpha_{u(N)}$ |
| $F \leq N/2$<br>$L = \lambda(F)$ | $F = N/2$<br>$L = \Gamma N/2$ | $F \leq N/2$<br>$L = \lambda(F)$ |

For any population size that satisfies $2\bar{L}/(\Gamma(1+\kappa)) < N \leq \bar{L}/(\Gamma\chi)$, only if the number of aristocrats is small, $A \leq \alpha_{u(N)}$, the wage is greater than or equal to the informal income for $F = N/2$ and $L = \Gamma N/2$. This is the equilibrium that I take to resemble a Latin American land reform. This result is proved in Lemma 4.2 and Proposition 4.3 in the Appendix.[26]

For any population size that satisfies $N \leq 2\bar{L}/(\Gamma(1+\kappa))$, $\Gamma F$ is less than $\lambda(F)$, regardless of the number of formal workers. It implies that if the amount of formal land is $\Gamma F$, the average yield in the informal sector is larger than the wage; thus, $L = \Gamma F$ cannot be a voting outcome. The voting outcome is either $\lambda(F)$ or $\bar{L}$, depending on the number of formal workers. Since land is very abundant, the aristocrats prefer to have expropriated some of their land, instead of cultivating it, which would imply a much larger land-labor ratio in the formal sector and, hence, a greater wage and lower land rents.

If the population size satisfies $2\bar{L}/(\Gamma(1+\kappa)) < N \leq \bar{L}/(\Gamma\chi)$, and the size of the aristocracy satisfies $A > \alpha_{u(N)}$, the amount of land $\Gamma N/2$ cannot be a voting outcome, because the assumption on $\theta$ implies that $\Gamma N/2 < \lambda(N/2)$. For $\Gamma F$ to be greater than $\lambda(F)$ and, hence, to be a voting outcome, the number of formal

[26]The parameter $\kappa$ is equal to $\left(\frac{\theta^{-\rho}(1-\rho)^{1-\rho}-1}{-\rho}\right)^{\frac{1}{\rho}}$.

workers should be less than a number $\hat{F}$, that is less than $N/2$. This number satisfies $w(\hat{F}, \Gamma\hat{F}, \hat{F}) = C_I(\hat{F}, \Gamma\hat{F}, \hat{F})$. Nevertheless, the aristocrats prefer to hire a number of formal workers that satisfies $\hat{F} < F \leq N/2$. If they chose $F = \hat{F}$, the increase of rents encompassed by using a land-labor ratio equal to $\Gamma$ is lower than the loss occurred by hiring too few workers. Thus, they choose a number of formal workers $\tilde{F}$ for which $\Gamma\tilde{F} < \lambda(\tilde{F})$. The proof of these results are found in Lemma 4.4, Proposition 4.5, and Proposition 4.6 in the Appendix.

The most interesting feature of this type of equilibrium is that the aristocrats' rents are maximized for $L = \lambda(\tilde{F})$, the informal workers' most preferred outcome. If $\lambda(\tilde{F})$ were not the maximizer, it would imply that $\tilde{F}$ does not maximize the aristocrats' rents at the differentiation stage.

For any population size and number of aristocrats that satisfy $\bar{L}/(\Gamma\chi) < N < \Psi + A$ and $A < \alpha_{l(N)}$ the equilibrium coincides with the one obtained earlier in the case in which $\theta \leq (1-\rho)^{\frac{1-\rho}{\rho}}$ : all the land becomes formal.

The result points out that landownership concentration —measured as the number of aristocrats— is relevant to determine the type of land reform chosen. Nguyen and Martinez Saldívar (1979) estimated the shares of the ejidal farms in the total cultivated Mexican area. In 1959, these shares oscillate from 11 percent in Baja California Sur to 88 percent in the state of Morelos. Thus, although it is out of the scope of this paper, it is possible to study the interaction between the parameter that measures the differences in productivity, $\theta$, and landownership concentration, measured as $A$, to understand the wide differences in land reform across regions within a same country.

## 7. Conclusions

This paper gives a rational to the land reform processes that many Latin American countries have experienced during this century. It focuses on the conflicts of interests that arise within a society to understand why a mixed ownership regime on farmland would be chosen. Two assumptions are key for such conflicts to arise: land and labor are assumed to be complementary factors and land is the relatively abundant factor. The first assumption is not unreasonable. Cornia (1985) estimates output elasticity of land for a cross section of countries and finds it to be decreasing with the land-labor ratio. An output elasticity decreasing in this ratio is satisfied by a C.E.S. production function only if the elasticity substitution parameter $\rho$ is negative. The third key element to obtain the result is the structure of the labor market, justified as a social norm inherited from the times of semi-serfdom, which implies that the peasantry is fractionated in two groups with opposite interests. The social conflicts that determine land reform are modelled as a voting process. I have assumed, for simplicity, that all votes are equally weighted. Alternatively, to capture the fact of the peasants' lower political power, that one *hacendado's* vote is equivalent to two peasants's votes. The main feature of the result would not change: if land is sufficiently abundant, a mixed ownership regime arises.

Differences in productivity of privately and commonly owned land, modelled as $\theta$ being less than one, can be studied to understand the wide differences in land reform across regions within the same countries. In Mexico, for instance, the fraction of private land is larger in the northern states, where irrigation is more extended and where the differences of productivity between private farms and *ejidos* is smallest, than in the southern states that mostly grow rain-fed crops.[27] Differences in $\theta$ across regions might help us to understand such variations.

The model abstracts from capital accumulation; modelling it explicitly we would obtain differences in productivity between the private and communal farms endogenously and we would see the evolution of the land reform depending on the level of wealth of the economy. The result would give us some insight about the evolution of the property rights system chosen by the society over time.

The model can be used to obtain implications about the dynamics of land reform: as population increases, the amount of privately owned land increases. In Mexico, the amount of privately owned land has decreased to be 50 percent of the farm land in the 1980's. Nevertheless, as Yates (1981) points out, this evidence hides the fact that a substantial and increasing fraction of the *ejidal* land —larger than 50 percent in the irrigated districts— was being rented out to private operators illegally, which suggests that the *ejido* members were, *de facto,* acquiring rights to the land. In 1992, the Mexican government changed the Constitution so that the *ejido* members were allowed to divide the *ejidal* land and become owners. In the terms of this model, they were made small owners. To model this evolution of the reform, we should take in account the existence of an urban sector that wants to have cheap agricultural goods; thus, they favor giving the peasants the ownership of the land, since it increases the aggregate level of productivity in the agricultural sector.

[27] Reported by Nguyen and Martínez Saldívar (1979).

# Appendix

## 1. The Weakening of the *Colonato* Regime: Land Invasions and the Amount of Formal Land

**Lemma 1.1.** *Let* $L_M = \min\left\{\frac{\bar{L}}{A}, \left(\frac{-\gamma}{\rho(1-\gamma)}\right)^{\frac{-1}{\rho}} \left(\frac{-\rho}{\theta^{\frac{\rho}{1-\rho}}-1}\right)^{\frac{-1}{\rho}} \frac{N-A}{A}\right\}$. *The function* $\eta : \left[0, \frac{\bar{L}}{A}\right] \to \left[0, \frac{N-A}{A}\right]$ *satisfies :* $\eta(0) = 0, \eta(.)$ *is strictly increasing in the interval* $(0, L_M)$, *and* $\eta(l) = \frac{N-A}{A}$ *for* $l \in \left[L_M, \frac{\bar{L}}{A}\right]$.

**Proof.** The existence of the function $\eta$ in the interval $(0, L_M)$ is ensured by the Implicit Function Theorem. $L_M$ is the amount of formal land that satisfies $\lim_{f \to N-A} w(f, L_M) = \lim_{f \to N-A} C_I(f, L_M)$.

**Lemma 1.2.** *Let us denote* $l_F(x) \equiv \frac{x}{\eta(x)}$ *and* $l_I(x) \equiv \frac{\bar{L}-x}{N-A-\eta(x)}$, *where* $x$ *denotes the amount of formal land and let us define* $l \equiv \frac{\bar{L}}{N-A}$. *For any positive amount of formal land* $x$ *less than* $L_M$, *the land-labor ratios in both sectors satisfy*

1. $l_F(x) < l_I(x)$ *if* $N - A < \Psi$.
2. $l_F(x) = l_I(x)$ *if* $N - A = \Psi$.
3. $l_F(x) > l_I(x)$ *if* $N - A > \Psi$.

Where $\Psi = \frac{\bar{L}}{\Gamma}\left(\frac{1-\theta}{-\rho\theta}\right)^{\frac{-1}{\rho}}$.

**Proof.** Let us assume that $N - A < \Psi$ and $l_F(x) \geq l_I(x)$. Then, $\frac{w(x,\eta(x))}{C_I(x,\eta(x))} = \frac{w(l_F(x),1)}{C_I(l_I(x),1)} \geq \frac{w(l,1)}{C_I(l,1)}$, which is greater than 1 since $N - A < \Psi$, which contradicts that $w(x, \eta(x)) = C_I(x, \eta(x))$. Thus, $l_F(x) < l_I(x)$. The other cases are proved in similar way.

**Lemma 1.3.** *The expression* $l'_j(x)$ *denotes the derivative with respect to the amount of formal land. The derivatives of both ratios satisfy* $l'_F(x) \cdot l'_I(x) \geq 0$. *If* $l'_F(x) \cdot l'_I(x) = 0$, *then* $l'_F(x) = l'_I(x) = 0$.

**Proof.** Since the wage and the informal income depend only on the land-labor ratio, both ratios have to move always in the same direction.

**Proposition 1.4.** *Let us consider any* $x \in \left(0, \bar{L}\right)$. *If* $N - A < \Psi$, *the ratio* $l_F(x)$ *is a strictly increasing function of* $x$. *If* $N - A = \Psi$, *then* $l_F(x) = l_I(x) = l$. *If* $N - A > \Psi$, *then* $l_I(x)$ *is a strictly decreasing function of* $x$.

**Proof.** Assume that $N - A < \Psi$. Let us assume that $l'_F(x) \leq 0$ in the neighborhood of some $x$. By definition, the ratios satisfy $l_F(x) \cdot x + l_I(x) \cdot (\bar{L} - x) = N - A$. Differentiating both sides with respect to $x$ and taking in account that, by Lemma 1.2, $l_F(x) < l_I(x)$, we obtain $l'_I(x) > 0$, which contradicts Lemma 1.3. The other two cases are proved in the same way.

**Proposition 1.5.** *If $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$ and $l = \Gamma\left(\frac{N-A}{A}\right)$, then $\frac{N-A}{A} - \eta(l)$ peasants invade the land left idle $\frac{\bar{L}}{A} - l$.*

**Proof.** The assumption on $\theta$ implies that $L_M$ is greater than $\Gamma\left(\frac{N-A}{A}\right)$; thus, for any number of peasants arbitrarily close to $\frac{N-A}{A}$, the wage and the informal income satisfy $w\left(\frac{N-A}{A}, \Gamma\left(\frac{N-A}{A}\right)\right) < C_I\left(\frac{N-A}{A}, \Gamma\left(\frac{N-A}{A}\right)\right)$. Hence, if the amount of formal land is $\Gamma\left(\frac{N-A}{A}\right)$, only $\eta\left(\Gamma\left(\frac{N-A}{A}\right)\right)$ peasants remain working for the aristocrat. Since $\eta$ is strictly increasing, $\eta\left(\Gamma\left(\frac{N-A}{A}\right)\right) < \eta(L_M) \equiv \frac{N-A}{A}$.

**Proposition 1.6.** *Let $\tilde{l} = \arg\max_{l \in [0, \frac{\bar{L}}{A}]} C_A\left(\eta(l), l\right)$. If $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$ then, $\tilde{l} > \Gamma\eta(\tilde{l})$ and $C_A(\eta(\tilde{l}), \tilde{l}) < C_A\left(\frac{N-A}{A}, \Gamma\frac{N-A}{A}\right)$.*

**Proof.** Since $\theta > (1-\rho)^{\frac{1-\rho}{\rho}}$, then $L_M > \Gamma\left(\frac{N-A}{A}\right)$, which implies that $\frac{L_M}{\eta(L_M)} \equiv \frac{L_M}{\frac{N-A}{A}}$ is greater than $\Gamma$. Since $N < \Psi + A$ Proposition 1.4 ensures that the ratio $\frac{l}{\eta(l)}$ increases with $l$; thus there exists at most one $\hat{l} \in \left(0, \Gamma\frac{N-A}{A}\right)$ for which $\frac{\hat{l}}{\eta(\hat{l})}$ $= \Gamma$. This value $\hat{l}$ satisfies $\frac{\partial C_A(\eta(\hat{l}), \hat{l})}{\partial l} = 0$; thus, $\frac{dC_A(\eta(\hat{l}), \hat{l})}{dl} = \frac{\partial C_A(\eta(\hat{l}), \hat{l})}{\partial f}\eta'(\hat{l}) > 0$. It follows that $\hat{l}$ cannot solve $\max_{l \in [0, \frac{\bar{L}}{A}]} C_A\left(\eta(l), l\right)$. Therefore, $\tilde{l} > \hat{l}$. Proposition 1.4 ensures that $\frac{\tilde{l}}{\eta(\tilde{l})} > \frac{\hat{l}}{\eta(\hat{l})} = \Gamma$. If $\hat{l}$ does not exist it implies that $l > \Gamma\eta(l)$, for all $l$. Then, $C_A(\eta(\tilde{l}), \tilde{l}) < C_A(\eta(\tilde{l}), \Gamma\eta(\tilde{l})) < C_A\left(\frac{N-A}{A}, \Gamma\frac{N-A}{A}\right)$.

# 2. A Latin American Land Reform

## 2.1. The Individuals' Preferences on Formal Land and the Voting Outcome

**Proposition 2.1.** *If $N < \Psi + A$, the formal workers' preferences are single-peaked and their most preferred voting outcome is $\bar{L}$, for all $F$.*

**Proof.** Let us denote as $l_F(F,l,\varphi(F,l))$ the land-labor ratio in the formal sector, given $F$ formal workers and $l$ units of formal land. We should keep in mind that $sign\left(\frac{dw(\cdot)}{dl}\right) = sign\left(\frac{dl_F(F,l,\varphi(F,l))}{dy}\right)$. For any amount of formal land $l \geq \lambda(F)$, $l_F(F,l,\varphi(F,l)) = \frac{l}{F}$; thus $\frac{dl_F(F,l,\varphi(F,l))}{dl} = \frac{\partial l_F(F,l,\varphi(F,l))}{\partial l} > 0$. If $l < \lambda(F)$, $l_F(F,l,\varphi(F,l)) = \frac{l}{\eta(l)}$, and Proposition 1.4 implies that $\frac{dl_F(F,l,\varphi(F,l))}{dl} > 0$. Thus, the preferences are single-peaked and their most preferred outcome is $\bar{L}$, for all $F$.

**Proposition 2.2.** *If $N < \Psi + A$, the informal workers' preferences are single-peaked and their most preferred voting outcome is $\lambda(F) \in (0,\bar{L})$, for any $F \in (0, N-A)$.*

**Proof.** The informal income is $C_I(F,l,\varphi(F,l)) \equiv \theta\left(\gamma l_I(F,l,\varphi(F,l))^{\rho} + (1-\gamma)\right)^{\frac{1}{\rho}}$. The expression $l_I(F,l,\varphi(F,l))$ denotes the informal land-labor ratio. For any $N - A \leq \Psi$, if $l > \lambda(F)$, $l_I(F,l,\varphi(F,l)) = \frac{\bar{L}-l}{N-A-F}$, $\frac{dl_I(F,l,\varphi(F,l))}{dl} < 0$; thus, $C_I$ is a decreasing function of $l$, for any $l > \lambda(F)$. If $l \leq \lambda(F)$, $l_I(F,l,\varphi(F,l)) = \frac{\bar{L}-l}{N-A-\eta(l)}$, and it follows from Proposition 1.4 that $\frac{\bar{L}-l}{N-A-\eta(l)}$ is increasing with $l$. Thus, the informal workers most preferred outcome is $\lambda(F)$.

**Proposition 2.3.** *If $N < \Psi + A$, the aristocrats' preferences are single peaked in the interval $\left[0,\bar{L}\right]$ and their most preferred outcome is equal to* $\min\left\{\bar{L},\xi(F)\right\}$, *where*

$$\xi(F) = \begin{cases} \Gamma F & if\ \Gamma(F) \geq \lambda(F) \\ some\ l \in (\Gamma F, \lambda(F)], & otherwise. \end{cases}$$

**Proof.** $C_A$ is a strictly concave function of land. $\Gamma F$ satisfies $\frac{\partial C_A(F,\Gamma F,F)}{\partial l} = 0$ for any $F$.

Let us assume that $\Gamma F \geq \lambda(F)$, then $\varphi(F,\Gamma F) = F$. Proposition 1.4 ensures that $\Gamma f > \lambda(f)$, for all $f < F$; that is, $\Gamma\eta(l) > l$, where $l = \lambda(f)$, $f < F$. Concavity of $C_A$ with respect to the amount of formal land ensures that $\frac{dC_A(F,l,\varphi(F,l))}{dl} = \frac{\partial C_A(F,l,\eta(l))}{\partial l} > \frac{\partial C_A(F,\Gamma\eta(l),\eta(l))}{\partial l} = 0$, for any $l < \lambda(F)$. For any $l > \lambda(F)$ we know that $\frac{dC_A(F,l,\varphi(F,l))}{dl} = \frac{\partial C_A(F,l,F)}{\partial l}$, which is positive for $l < \Gamma F$ and negative for $l > \Gamma F$. Therefore, $\xi(F) = \Gamma F$.

Let us assume that $\Gamma F < \lambda(F)$. Then, for any $l > \lambda(F)$, $\frac{dC_A(F,l,\varphi(F,l))}{dl} = \frac{\partial C_A(F,l,F)}{\partial l} < 0$, since $\varphi'(F,\Gamma F) = 0$, because $\varphi(F,l) = F$ and concavity of $C_A$ ensures that $\frac{\partial C_A(F,l,F)}{\partial l} < \frac{\partial C_A(F,\lambda(F),F)}{\partial l} < \frac{\partial C_A(F,\Gamma F,F)}{\partial l} = 0$. For any $l \leq \Gamma F$ it is satisfied $\frac{dC_A(F,l,\varphi(F,l))}{dl} = \frac{\partial C_A(F,l,\varphi(F,l))}{\partial l} + \frac{\partial C_A(F,l,\varphi(F,l))}{\partial f} \cdot \varphi'(F,l) > 0$, $\varphi'(F,l) \equiv \eta'(l)$. Thus, continuity of $\frac{dC_A(F,l,\varphi(F,l))}{dl}$ ensures that there exists $\xi(F) \in (\Gamma F, \lambda(F)]$ that maximizes $C_A$.

### 2.2. The Number of Formal Workers

**Proposition 2.4.** *$C_A(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) \geq C_A(N-A, \bar{L}, N-A)$ if $A \geq \alpha_{l(N)}$, where $\alpha_{l(N)} = N - \left((1-\rho)\left(\frac{N}{2}\right)^{\frac{\rho}{1-\rho}}\left(\frac{\bar{L}}{\Gamma}\right)^{\frac{-\rho^2}{1-\rho}} + \rho\left(\frac{\bar{L}}{\Gamma}\right)^{\rho}\right)^{\frac{1}{\rho}}$. There exists $\chi > 1$ such that for all $N \geq \frac{\bar{L}}{\Gamma\chi}$, $\alpha_{l(N)}$ satisfies $0 \leq \alpha_{l(N)} < \frac{N}{2}$ and it monotonically increases with the size of the population.*

**Proof.** The expression for $\alpha_{l(N)}$ satisfies $C_A(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) = C_A(N-\alpha_{l(N)}, \bar{L}, N-\alpha_{l(N)})$. It is non negative if and only if $(1-\rho)2^{\frac{-\rho}{1-\rho}}\left(\frac{\bar{L}}{\Gamma N}\right)^{\frac{-\rho^2}{1-\rho}} \geq 1-\rho\left(\frac{\bar{L}}{\Gamma N}\right)^{\rho}$. If $\frac{\bar{L}}{\Gamma N} = 1$, the inequality is strict and it is reversed if $\frac{\bar{L}}{\Gamma N}$ becomes arbitrarily large. Hence, there exists $\chi > 1$ such that for any $\frac{\bar{L}}{\Gamma N} \leq \chi$, $\alpha_{l(N)} \geq 0$. It is straightforward to prove that $\alpha_{l(N)}$ is monotonically increasing with $N$ and has an upper bound at $\frac{N}{2}$, for any $N \geq \frac{\bar{L}}{\Gamma\chi}$.

## 3. Population Size and the Ownership Regime

**Proposition 3.1.** *Let us assume that $\theta$ is less than $(1-\rho)^{\frac{1-\rho}{\rho}}$. For any $N$ such that $\frac{2\bar{L}}{\Gamma} \leq N < \Psi + A$, the number of formal workers is $F = \frac{N}{2}$ and the amount of formal land is $L = \bar{L}$.*

**Proof.** For any given number of formal workers, $F$, the formal and informal workers' most preferred voting outcome are $\bar{L}$ and $\lambda(F)$, respectively. The aristocrats' most preferred outcome is $\min\left\{\bar{L}, \Gamma F\right\}$, as it follows from Proposition 2.3. The aristocrats always choose $F \geq \frac{N}{2} - A$. For any $F$ in the interval $\left[\frac{N}{2} - A, \frac{N}{2}\right]$ the voting outcome is either $\Gamma F$ or $\bar{L}$. In either case $C_A$ is an increasing function of the number of formal workers. Thus, the aristocrats choose $F$ in the set $\left\{\frac{1}{2}N, N-A\right\}$. Since $\frac{2\bar{L}}{\Gamma} \leq N$, it follows that $\bar{L} < \Gamma\frac{1}{2}N$; therefore, for any $F \in \left\{\frac{1}{2}N, N-A\right\}$ the voting outcome is $L(F) = \bar{L}$. Given that the voting outcome does not depend on the number of formal workers, and since $C_A$ is strictly increasing in labor, it follows that $F = N - A$.

**Proposition 3.2.** *For any $N > \Psi + A$, for any $F \in (0, N-A)$, the formal worker's preferences have a minimum at $\lambda(F)$. The formal worker's most preferred outcome is $L(F) = 0$ if $F > \zeta_{(N,A)}$ and it is $L(F) = \bar{L}$ otherwise.*

**Proof.** Since the function $\eta$ takes values in the entire interval $[0, N-A]$, for any $F \in (0, N-A)$ there exists $\lambda(F)$, such that $F = \eta(\lambda(F))$. For any amount of land $l < \lambda(F)$, $\varphi(F,l) = \eta(l)$, which is less than $F$. It follows from Proposition 1.4 that the wage decreases with the amount of formal land in the interval $[0, \lambda(F))$. For any amount of land $l > \lambda(F)$, $\varphi(F,l) = F$; it implies that the wage strictly increases with the amount of formal land in the interval $\left(\lambda(F), \bar{L}\right)$. The formal worker's most preferred outcome is either 0 or $\bar{L}$, depending on which associated level of consumption is highest. Continuity of the wage and the informal income ensures that there exists a number of formal workers

$$\zeta_{(N,A)} = \bar{L}\left(\frac{\left(\frac{\theta}{1-\gamma}\right)^{\frac{\rho}{1-\rho}}\left(\gamma\left(\frac{\bar{L}}{N-A}\right)^{\rho} + (1-\gamma)\right)^{\frac{1}{1-\rho}} - (1-\gamma)}{\gamma}\right)^{\frac{-1}{\rho}} \tag{3.1}$$

such that if $F \leq \zeta_{(N,A)}$, the formal workers preferences are maximized at $\bar{L}$. If $F > \zeta_{(N,A)}$, they are maximized at 0.

**Proposition 3.3.** *For any $N > \Psi + A$, the informal worker's preferences are monotonically decreasing in the amount of formal land.*

**Proof.** For any amount of formal land $0 < l < \lambda(F)$, it follows from Proposition 1.4 that the informal income is strictly decreasing with the amount of formal land. For any $\lambda(F) < l < \bar{L}$, all the formal workers stay in the formal sector, $\varphi(F,l) = F$ and the informal income strictly decreases with $l$.

**Proposition 3.4.** *Let the population size satisfy $N > \max\left\{\Psi + A, \frac{2\bar{L}}{\Gamma} + 2A\right\}$. There exists $\Upsilon > \Psi + A$, which satisfies $\zeta_{(\Upsilon,A)} = \frac{N}{2} - A$, such that if $N \leq \Upsilon$, the number of formal workers is $F = \zeta_{(N,A)}$ and the amount of formal land is $L = \bar{L}$. If $N > \Upsilon$, the number of formal workers lies in the interval $[0, N-A]$ and the amount of formal land is $L = 0$.*

**Proof.** Since the informal workers' most preferred outcome is zero, the aristocrats choose $F$ such that $F + A \geq \frac{N}{2}$, which, given the assumption on the size of the population implies that the aristocrats' most preferred outcome is $\bar{L}$. The aristocrats do not choose $F > \zeta_{(N,A)}$, since all the peasants' most preferred outcome would be zero; therefore, $F$ is chosen to belong to $\left[\frac{N}{2} - A, \zeta_{(N,A)}\right]$, which requires that $\frac{N}{2} - A \leq \zeta_{(N,A)}$. The ratio $\frac{\zeta_{(N,A)}+A}{N}$ decreases monotonically with $N$, since $\rho$ is assumed to be less than -1. Thus, there exists a size of the population $\Upsilon$ such that for all $N \leq \Upsilon$, $\frac{\zeta_{(N,A)}+A}{N} \geq \frac{1}{2}$. Thus, if $N \leq \Upsilon$, $F = \zeta_{(N,A)}$ and $L = \bar{L}$. If $N < \Upsilon$, $L = 0$, regardless of the number of formal workers.

## 4. Differences in Productivity

**Lemma 4.1.** *Let $\chi$ be the number for which the aristocrat's consumption satisfies $C_A(\frac{1}{2}N, \Gamma\frac{N}{2}, \frac{1}{2}N) \geq C_A(N-A, \bar{L}, N-A)$, for any number of aristocrats $A \geq 0$, if the population size satisfies $N \geq \frac{\bar{L}}{\Gamma\chi}$,. Let $\kappa = \left(\frac{\theta^{-\rho}(1-\rho)^{1-\rho}-1}{-\rho}\right)^{\frac{1}{\rho}}$. There exists $\theta^*$ $\in \left((1-\rho)^{\frac{1-\rho}{\rho}}, (1-\rho)^{-1}\right)$ such that for any $\theta \leq \theta^*$ then, $\frac{2\bar{L}}{\Gamma(1+\kappa)} \leq \frac{\bar{L}}{\Gamma\chi}$.*

**Proof.** If $\theta$ is arbitrarily close to $(1-\rho)^{\frac{1-\rho}{\rho}}$, $\kappa$ becomes arbitrarily large. For $\theta = (1-\rho)^{-1}$, $\kappa = 1$. Thus, since $\chi > 1$, there exists $\theta^*$ in the interior of the interval for which $\frac{1+\kappa}{2} = \chi$.

**Lemma 4.2.** *Let $\theta$ satisfy $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$. There exists an upper bound $\alpha_{u(N)} = N\frac{1+\kappa}{2\kappa} - \frac{\bar{L}}{\Gamma\kappa}$ such that if $A \leq \alpha_{u(N)}$, the wage and the informal income satisfy $w(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) \geq C_I(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2})$.*

**Proof.** The equality $w(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) = C_I(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2})$ is satisfied if the number of aristocrats equals $N\frac{1+\kappa}{2\kappa} - \frac{\bar{L}}{\Gamma\kappa}$, which I call $\alpha_{u(N)}$. This number is non negative if $N \geq \frac{2\bar{L}}{\Gamma(1+\kappa)}$. Therefore, if $A \leq \alpha_{u(N)}$, the inequality $w(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) \geq C_I(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2})$ is satisfied.

**Proposition 4.3.** *Let $\theta$ satisfy $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$. For any population size that satisfies $\frac{2\bar{L}}{\Gamma(1+\kappa)} < N \leq \frac{\bar{L}}{\Gamma\chi}$ and $A \leq \alpha_{u(N)}$, the equilibrium number of workers is $F = \frac{N}{2}$ and the amount of formal land is $L = \Gamma\frac{N}{2}$.*

**Proof.** Individuals' preferences on formal land are shown in Propositions 2.1, 2.2 and 2.3. Lemma 4.2 ensures that $w(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) \geq C_I(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2})$; in other words, $\lambda(\frac{N}{2}) < \Gamma\frac{N}{2}$. It follows that $\Gamma\frac{N}{2}$ is the voting outcome if $F = \frac{N}{2}$. Proposition 2.4 ensures that for any $N \leq \frac{\bar{L}}{\Gamma\chi}$, the aristocrats choose $F = \frac{N}{2}$, regardless of their group size, measured as $A$.

**Lemma 4.4.** *Let $A < \frac{N}{2}$, $N < \Psi + A$ and $F \leq \frac{N}{2}$. If $w(F, \Gamma F, F) < C_I(F, \Gamma F, F)$, then the voting outcome is $L(F) = \lambda(F)$.*

**Proof.** If $w(F, \Gamma F, F) < C_I(F, \Gamma F, F)$ then, $\lambda(F) > \Gamma F$; otherwise, the inequality would be reversed. Individuals preferences are shown in Propositions 2.1, 2.2 and 2.3. The three candidates to be a voting outcome satisfy $\xi(F) \leq \lambda(F) < \bar{L}$. Let us assume that $F = \frac{N}{2}$. Since preferences are single peaked, it follows that $\bar{L}$ cannot be a voting outcome, because informal workers and aristocrats are the majority and $\xi(F)$ cannot be the voting outcome either, because formal and informal workers prefer $\lambda(F)$ to $\xi(F)$. Thus, if $F = \frac{N}{2}$ the amount of formal land is $\lambda(F)$. Proposition 2.4 ensures that for any $N \leq \frac{\bar{L}}{\Gamma\chi}$ the aristocrats choose $F = \frac{N}{2}$, regardless of their group size, $A$.

**Proposition 4.5.** *Let $\theta$ satisfy $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$. Let $\tilde{F}$ solve the problem*

$$\arg \max_{f \in [0,N/2]} C_A(f, L(f), \varphi(f, L(f))).$$

The voting outcome is $L(\tilde{F}) = \lambda(\tilde{F})$ if the number of formal workers is equal to $\tilde{F}$ and the population size satisfies $N \leq \frac{2\bar{L}}{\Gamma(1+\kappa)}$, or $\frac{2\bar{L}}{\Gamma(1+\kappa)} < N \leq \frac{\bar{L}}{\Gamma\chi}$ and $A > \alpha$.

**Proof.** I prove the proposition proceeding in several steps.

**1.** Let us assume that $\frac{2\bar{L}}{\Gamma(1+\kappa)} < N \leq \frac{\bar{L}}{\Gamma\chi}$ and $A > \alpha_{u(N)}$. If $N$ and $A$ satisfy these assumptions is easy to check that $w(F, \Gamma F, F) < C_I(F, \Gamma F, F)$, for any $F$ sufficiently close to $\frac{N}{2}$. In other words, $\lambda(F) > \Gamma F$ in a neighborhood of $\frac{N}{2}$. Let us divide the region of the parameter space.

**1.1.** Let us assume that $\bar{L}/(\Gamma\kappa) + A < N \leq \frac{\bar{L}}{\Gamma\chi}$ and $A > \alpha_{u(N)}$. Then, there exists $\hat{F} \in (0, \frac{N}{2})$ such that $\lambda(F) \leq \Gamma F$, for all $F \leq \hat{F}$, and $\lambda(F) > \Gamma F$ for $F \in \left(\hat{F}, \frac{N}{2}\right]$. The voting outcome is as follows:

If $\hat{F} \leq \frac{N}{2} - A$, it follows from Lemma 4.4 that $L(F) = \lambda(F)$ for all $F \leq \frac{N}{2}$. Thus, $L(\tilde{F}) = \lambda(\tilde{F})$.

If $\hat{F} > \frac{N}{2} - A$, the amount of formal land is $L(F) = \lambda(F)$ if $F < \frac{N}{2} - A$, $L(F) = \Gamma F$ if $\frac{N}{2} - A \leq F \leq \hat{F}$, and $L(F) = \lambda(F)$ if $\hat{F} < F \leq \frac{N}{2}$. For any $F \leq \hat{F}$ the aristocrats' consumption $C_A(f, L(f), \varphi(f, L(f))$ increases monotonically with the number of formal workers since the corresponding voting outcome is always less than or equal to the amount $\Gamma F$. If $\tilde{F}$ satisfied $\tilde{F} \leq \hat{F}$ then $\tilde{F}$ could not solve the problem (*),which contradicts our assumption; therefore $\tilde{F} \in \left(\hat{F}, \frac{N}{2}\right]$. Then, $L(\tilde{F}) = \lambda(\tilde{F})$.

**1.2.** Let us assume that $N \leq \bar{L}/(\Gamma\kappa) + A$ and $A > \alpha_{u(N)}$. Then, $\lambda(F) > \Gamma F$, for all $F \leq \frac{N}{2}$. It follows, from Lemma 4.4 that $L(\tilde{F}) = \lambda(\tilde{F})$.

**2.** Let us assume that $N \leq \frac{2\bar{L}}{\Gamma(1+\kappa)}$. Then, it is easy to check that $w(F, \Gamma F, F) < C_I(F, \Gamma F, F)$, for any $F \leq \frac{N}{2}$. In other words, $\lambda(F) > \Gamma F$, for all $F \leq \frac{N}{2}$. Thus, by Lemma 4.4, $L(\tilde{F}) = \lambda(\tilde{F})$.

**Proposition 4.6.** *Let $\theta$ satisfy $(1-\rho)^{\frac{1-\rho}{\rho}} < \theta \leq \theta^*$. If the population size satisfies $N \leq \frac{2\bar{L}}{\Gamma(1+\kappa)}$, or $\frac{2\bar{L}}{\Gamma(1+\kappa)} < N \leq \frac{\bar{L}}{\Gamma\chi}$ and $A > \alpha_{u(N)}$, the equilibrium number of workers is $\tilde{F} \leq \frac{N}{2}$ and the voting outcome is $\lambda(\tilde{F})$, where $\tilde{F}$ is defined in the previous Proposition*

**Proof.** I prove the proposition proceeding in several steps.

**1.** I am going to prove that $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F}) > C_A(N-A, \bar{L}, N-A)$, for all $A > \alpha_{u(N)}$.

**1.1**. Let us assume that $A = \alpha_{u(N)}$; Lemma 4.2 ensures that $w(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2}) = C_I(\frac{N}{2}, \Gamma\frac{N}{2}, \frac{N}{2})$, in other words, $\lambda(\frac{N}{2}) = \Gamma\frac{N}{2}$ Thus, $\tilde{F} = \frac{1}{2}N$. Since $N \leq \bar{L}/(\Gamma\chi)$ it follows that $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F}) > C_A(N-A, \bar{L}, N-A)$.

**1.2**. Let us assume now that $A = \frac{N}{2}$; in this case, since $\lambda(\frac{N}{2}) < \bar{L}$, the following inequalities hold: $C_A(N-A, \bar{L}, N-A) = C_A(\frac{N}{2}, \bar{L}, \frac{N}{2}) < C_A(\frac{N}{2}, \lambda(\frac{N}{2}), \frac{N}{2}) < C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F})$.

**1.3**. We know that $C_A(N-A, \bar{L}, N-A)$ decreases monotonically as the number of aristocrats increases. I need to prove that $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F})$ is also strictly decreasing. To do so, I am going to assume that the number of aristocrats can vary. I am going to keep the same notation for the sake of simplicity, but we should bear in mind that we should write now $\tilde{F}(A)$ and $\lambda(F, A)$. $\tilde{F}$ satisfies $\frac{\partial C_A(\tilde{F},\lambda(\tilde{F}),\tilde{F})}{\partial l}\frac{\partial \lambda(\tilde{F})}{\partial f} + \frac{\partial C_A(\tilde{F},\lambda(\tilde{F}),\tilde{F})}{\partial f} \geq 0$, where $\frac{\partial C_A(\tilde{F},\lambda(\tilde{F}),\tilde{F})}{\partial f} < 0$. If $\tilde{F}$ is an interior solution, it is easy to check that it decreases as the aristocracy size increases, for any $A \in \left[\alpha_{u(N)}, \frac{N}{2}\right]$. If $\tilde{F} = \frac{N}{2}$, then it is non increasing with the number of aristocrats. To show that $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F})$ decreases with the number of aristocrats I take the derivative with respect to $A$. Simplifying the notation it can be written as $\frac{dC_A}{dA} = \left(\frac{\partial C_A}{\partial l}\frac{\partial \lambda}{\partial f} + \frac{\partial C_A}{\partial f}\right)\tilde{F}' + \frac{\partial C_A}{\partial l}\frac{\partial \lambda}{\partial A}$. If $\tilde{F}$ is an interior solution, $\frac{dC_A}{dA} = \frac{\partial C_A}{\partial l}\frac{\partial \lambda}{\partial A}$, which is negative since $\frac{\partial C_A}{\partial l}$ is negative because $\lambda(\tilde{F}) > \Gamma\tilde{F}$. If $\tilde{F} = \frac{N}{2}$, the expression in the brackets is positive but $\tilde{F}' \leq 0$; thus, $\frac{dC_A}{dA} < 0$, for all $A \in \left[\alpha_{u(N)}, \frac{N}{2}\right]$. Thus, $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F})$ never crosses $C_A(N-A, \bar{L}, N-A)$, for any $A > \alpha_{u(N)}$. It follows that $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F}) > C_A(N-A, \bar{L}, N-A)$.

**2.** So far, I have not specified the size of the population. For any economy that satisfies $\frac{2\bar{L}}{\Gamma(1+\kappa)} < N \leq \frac{\bar{L}}{\Gamma\chi}$ the bound $\alpha_{u(N)}$ is positive. For any $A > \alpha_{u(N)}$, $C_A(\tilde{F}, \lambda(\tilde{F}), \tilde{F})$ is greater than $C_A(N-A, \bar{L}, N-A)$; thus, it implies that the aristocrats choose $F = \tilde{F}$ and the voting outcome is $\lambda(\tilde{F})$. If the population size satisfies $N \leq \frac{2\bar{L}}{\Gamma(1+\kappa)}$ the bound $\alpha_{u(N)}$ is less than or equal to zero. The proof above applies for any $A \in \left[\alpha_{u(N)}, \frac{N}{2}\right]$; thus, it applies for $A \in \left[0, \frac{N}{2}\right]$. It implies that, if $N \leq \frac{2\bar{L}}{\Gamma(1+\kappa)}$, the aristocrats choose $F = \tilde{F}$ and the voting outcome is $\lambda(\tilde{F})$.

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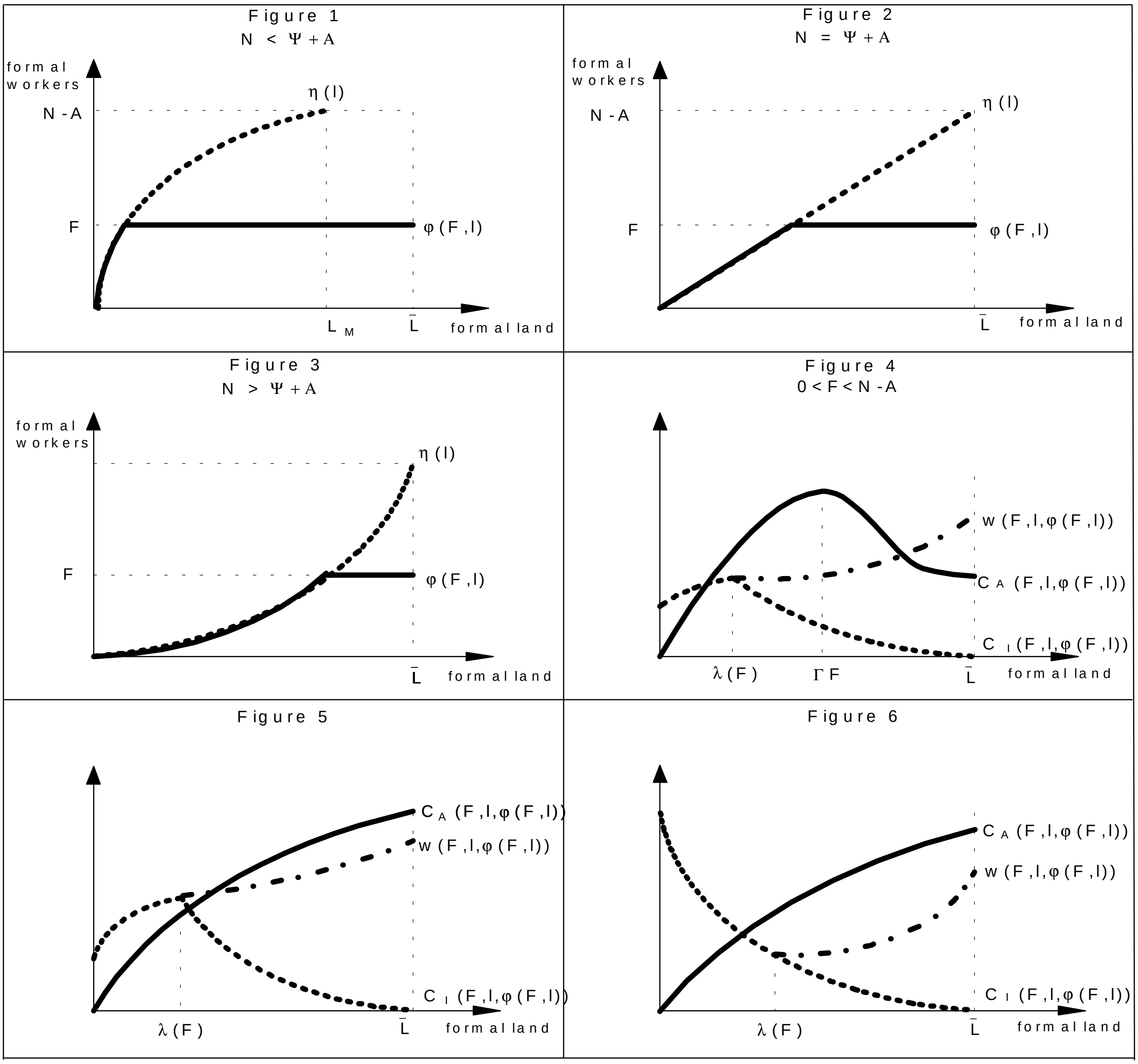
Figure 1
N < Ψ + A
formal workers
η(l)
N - A
F
φ(F,l)
L M
L̄
formal land
Figure 2
N = Ψ + A
formal workers
η(l)
N - A
F
φ(F,l)
L̄
formal land
Figure 3
N > Ψ + A
formal workers
η(l)
F
φ(F,l)
L̄
formal land
Figure 4
0 < F < N - A
w(F,l,φ(F,l))
C A (F,l,φ(F,l))
C I (F,l,φ(F,l))
λ(F)
Γ F
L̄
formal land
Figure 5
C A (F,l,φ(F,l))
w(F,l,φ(F,l))
C I (F,l,φ(F,l))
λ(F)
L̄
formal land
Figure 6
C A (F,l,φ(F,l))
w(F,l,φ(F,l))
C I (F,l,φ(F,l))
λ(F)
L̄
formal land

Figure 4.1: