FORWARD INDUCTION IN A WAGE REPEATED NEGOTIATION*

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A B S T R A C T

Abstract

We present a finitely repeated bargaining game with complete information. The stage game is a simultaneous demand game with a fall-back position for both parties, in which we allow one party (say, the union) to establish a credible commitment to strike if it is not offered a determined wage. We try to refine the equilibrium set of the repeated game using a formulation of Forward Induction. In particular, we say that a path of Nash Equilibria in the repeated game is Consistent with Forward Induction (CFI) if for all period t the cost of deviation (if it is strictly positive) is greater or equal than the maximal net gain in CFI paths with $t-1$ horizon.

We present several cases in which the average payoff for the union in any CFI path, when the horizon tends to infinity, is his preferred wage. These results are similar to those obtained with the reputation effects approach and reveal some connection between the FI notion and the approach consisting of perturbing the game with some incomplete information.

Keywords: Repeated Bargaining, forward induction, commitment.

1 Introduction.

Suppose that two agents are engaged in a long-run relationship and, negotiate periodically how to divide a surplus. Many negotiations in real life belong to this kind of situation and a paradigmatic example is the wage bargaining between a firm and a union. The problem is that in this repeated bargaining game, there is typically an important problem of multiplicity of equilibria.

A quite well-known way of reducing this multiplicity problem and getting, under some conditions, an equilibrium selection, consists of perturbing the repeated game with a particular kind of incomplete information. This is called, the reputation effects approach. In some previous work (Calabuig and Olcina, 1996a,b), we have applied this sort of analysis to a repeated wage bargaining situation.

But, there is another alternative way of alleviating the multiplicity problem: the refinements approach. Namely, some equilibria of the complete information repeated game might not be consistent with some notion of Forward (and/or Backward) Induction.

In this paper we present and analyze a finitely repeated bargaining game with complete information. The stage game is a simultaneous demand game with a fall-back position for both parties, in which we allow one party (say, the union) to establish credible commitments to strike if it is not offered a determined wage. In this constituent game, there is a multiplicity of Nash Equilibria (NE), and the same happens with the repeated game.

We can refine the equilibrium set of the repeated game using a formulation of Forward Induction related with previous proposals made by Osborne (1989), Ponssard (1991) and Al-Najjar (1995). In particular, we say that a path of NE in the repeated game is Consistent with Forward Induction (CFI) if for all period t the cost of a deviation (if it is strictly positive) is greater or equal than the maximal net gain in CFI paths with $t-1$ horizon.

But, our main motivation and purpose in this paper is not the refinement problem *per se*. We are basically interested in analyzing under which conditions it is possible to obtain through the FI logic some strong selection results (in the line of the reputation effects).

Therefore, we assume some particular restrictions on the parameters that reflect an asymmetric situation more favorable for the union. On one hand, the cost for the firm of deviating and suffering a strike (in case the union demands for the highest possible wage) is greater or equal than the maximal net gain or benefit of this deviation. And, on the other hand, the cost for the union of a strike (in case the firm offers the lowest possible wage) is lower than the maximal net gain of doing so.

Obviously, these restrictions favor the signalling incentives of the union as compared to those of the firm. We are interested in analyzing under which conditions, the union gets, as average payoff, his preferred wage in all the CFI paths of NE, as the horizon tends to infinity.

We show first, that, with the restrictions imposed above, if the union is able to establish a credible commitment to strike linked only with the highest possible wage, he would get the latter as average payoff. In some sense, this is a negative result because in many circumstances it would not be credible or possible to commit to strike *only* with the preferred wage.

If we allow a less demanding commitment to strike, then it is shown that the union can get his preferred outcome if the incentives to strike are not only positive, but sufficiently high, or alternatively the incentive of the firm to resist the strikes are low enough. In particular, we prove that there is a trade-off between these incentives.

These results reflect that there is some connection between the notion of FI and the reputation effects approach, i.e. the equilibrium selection obtained perturbing the repeated game, in particular allowing the existence of some "committed" type. In our analysis, the potential signalling capacity of the union is what plays the key role in obtaining the selection results.

The rest of the paper is organized as follows. Section 2 describes the repeated wage bargaining game. Section 3 analyzes the concept of equilibrium consistent with forward induction and its application to the repeated bargaining games with finite horizon. In section 4 we show the main results, in which the union, under certain conditions, can get his preferred wage making use of the implicit signalling ability in the notion of FI. Finally, in Section 5, we briefly discuss the problem involved with other configurations of the parameters of the model and we comment possible lines of future research.

2 The wage bargaining model.

In this section we model the long-run relationship between two agents, (say a union U and a firm, F) as a finitely repeated game with horizon T. In each period t, the players negotiate how to share a surplus of size b.

We assume that both players have a participation constraint (or reservation payoff) that sets bounds to the set of feasible wage offers. Namely, we suppose that the firm has to guarantee herself a minimal profitability. This will be equivalent to the surplus minus the highest wage that the union can get, i.e. $b - w^*$. On the other hand, the union has a reservation wage, w_o. No player can be forced down to accept a lower payoff in each period, because otherwise he could leave this long-run relationship and get his reservation utility.

The stage game is a simultaneous demand game. In each period, both players, firm and union, make simultaneous offers, w_f and w_u, respectively. Both offers have to belong to the finite set, with n elements, $M = \{w_o, w_1, w_2, ...$ $w_{n-2}, w^*\}$, where $0 < w_o < .....w^* < b$. We also assume that Δ is the distance between two consecutive wage offers and, therefore $\Delta = \frac{w^* - w_o}{n-1}$.

We introduce two distinct features with respect to the standard simultaneous demand game.

Firstly, we assume that before starting the game (and as part of its rules), the union establishes a binding commitment to strike from some certain wage level $\overline{w}$, whenever the firm offers him a lower wage than the one he demands. Notice that the standard simultaneous demand game is a particular case, in which $\overline{w} = w_1$. In any case, we want to stress that this binding commitment is not part of the union's strategy, but instead, it is given exogenously, that is, it is a rule of the game. In other words, it is more accurate to think of this work as the analysis of a family of stage games, parametrized by $\overline{w}$.

Secondly, as we will see next, we allow for the possibility of an asymmetric cost of striking.

If the firm's offer is greater or equal than the wage the union demands, the agreed wage will be w_f, and the negotiation finishes in that period with payoffs $g_f(w_u, w_f) = b - w_f$ for the firm and $g_u(w_u, w_f) = w_f$ for the union. If the firm's wage offer is smaller than the union's demand, but the latter is not linked with the strike commitment, then the agreed wage is also the one offered by the firm, w_f. Finally, if the firm's offer is smaller than the union's demand, but the latter is linked with a strike commitment, then the payoff for both players, will be cero for the union and $(-c)$ for the firm, where $c \geq 0$. These payoffs reflect the possibility that the cost of a strike is asymmetric between the agents. In particular, if c is strictly positive, we suppose that in addition to the loss of surplus due to the absence of production, the firm has to bear some added "material costs" as the loss of customers, the costs of stopping the machines, etc....

We will assume in what follows, that there exists a wage $w_s \in M$, such that $c = (w^* - w_s)$.

In this stage game with n wage offers, where $\overline{w} = w_j$, there will be n Nash equilibria (NE) in pure strategies. Among these n Nash equilibria, j of them will have identical payoffs: $(w_o, b - w_o)$. In these equilibria, the firm offers w_o and the union demands any wage lower than w_j. We will denote any of these equilibria by W_o. In the remaining $(n - j)$ equilibria, the union demands w_h and the firm also offers w_h , where $w_j \leq w_h \leq w^*$. The payoffs will be $(w_h, b - w_h)$ for the union and the firm respectively. We will denote these equilibria by W_h.

In figure 1 we depict the payoff matrix of this stage game with an arbitrary $\overline{w}$. The row player is the union and the firm is the column player.

	w_o	w_1	$\dots$	w_j	$\dots$	w^*
w_o	$w_o, b - w_o$	$w_1, b - w_1$	$\cdots$	$w_j, b - w_j$	$\cdots$	$w^*, b - w^*$
w_1	$w_o, b - w_o$	$w_1, b - w_1$	$\cdots$	$w_j, b - w_j$	$\cdots$	$w^*, b - w^*$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\overline{w} = w_j$	$0, -c$	$0, -c$	$\cdots$	$w_j, b - w_j$	$\cdots$	$w^*, b - w^*$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$
w^*	$0, -c$	$0, -c$	$\cdots$	$0, -c$	$\cdots$	$w^*, b - w^*$

Figure 1

Two particular cases deserve a comment. In the first one, $\overline{w} = w_1$ and there are n Nash equilibria. The n equilibrium payoffs cover all the the Pareto efficient frontier, that is, $[(w_o, b - w_o), (w_1, b - w_1), (w_2, b - w_2)........(w^*, b - w^*)]$. In the second one, with $\overline{w} = w^*$, there are two Nash equilibrium "outcomes", W_o, with payoffs $(w_o, b - w_o)$ and W^* with payoffs $(w^*, b - w^*)$.

Let G^T denote the stage game repeated T times, where T is finite. We will assume, basically for simplicity, that there is no discounting. The overall payoff for player i, in the repeated game, is simply the sum of the payoffs in each period.

The commitment to strike starting from $\overline{w}$ is fixed for the repeated game. An analysis in which the union can revise in every period such commitment will be part of future research.

As we have checked, there is multiplicity of equilibria in the stage game and, obviously, we have the same problem in the repeated game G^T. For example, every path of Nash equilibria of the stage game, is a Nash equilibrium of the repeated game G^T and, furthermore, it is Subgame Perfect Equilibrium (SPE). Evidently, this implies that Backward Induction has no cutting power in order to refine the equilibrium set in this game. As we will see in the next sections, the notion of the Forward Induction is indeed able to alleviate the multiplicity problem, discarding as unreasonables, some equilibria path of G^T.

3 Consistency with Forward Induction.

The intuition behind the FI concept can be clearly observed through a simple example. Consider the repeated bargaining game, where $T = 2$, $M = \{w_o, w^*\}$, $w^* > 2w_o$ and $\overline{w} = w^*$. The payoff matrix of this stage game is shown in figure 2, where the union is the row player and the firm is the column player.

	w_o	w^*
w_o	$w_o, b - w_o$	$w^*, b - w^*$
w^*	$0, -c$	$w^*, b - w^*$

Figure 2

In this stage game there are two Nash equilibria in pure strategies: (W_o) and (W^*) (Recall that (W_o), means that both players offer w_o). Roughly speaking, the FI notion requires an equilibrium to be consistent with deductions based on rational behavior of the players in the past. In other words, we have to look for a rational explanation of possible deviations from the equilibrium. A NE that does not satisfy this condition must be considered as "unreasonable" as a possible prediction for the play among rational players.

Let us examine whether the NE path $\{(W_o), (W_o)\}$ is a "reasonable" prediction. In particular, we want to analyze the interpretation we should give to the fact that the union deviates from W_o in the first period, demanding w^* and striking. By doing so, he would obtain a payoff of zero. The only rational explanation (whenever $w^* > 2w_o$), is that he tries to obtain w^* in the second period. The cost of deviation, w_o, is smaller than the potential gain of doing it $(w^* - w_o)$,(in case the union succeeds in convincing the firm that he is going to play w^*). But, obviously, it does not make sense for the firm to think that the union will play w_o in the second period. On the contrary, as we have seen, there is a rational explanation for playing W^*. In other words, if we require a rational explanation for such a deviation, then the equilibrium path $\{(W_o), (W_o)\}$ is "destabilized", in the sense that there is a deviation that would be carried out by rational players. Therefore $\{(W_o), (W_o)\}$ is not a reasonable prediction for G^2. In what follows, we will say that such kind of paths are not consistent with Forward Induction (CFI).

The formalization of the FI requisite has shown to be elusive. There are many proposals of refinements and there is not a clear agreement in the literature about which is the right one.

Dekel (1990) works with the concept of strategic stability of Kolhberg and Mertens (1986) trying to refine the equilibrium set in a model of sequential bargaining and gets some results based on the FI logic. But, the FI properties of stable equilibria, as for example, the property of "never a weak best response", is only a by-product of perturbing the strategic form of the game instead of perturbing the normal-agent form as Selten does. But it is not clear that this indirect way of obtaining FI properties guarantees that all the implications of the FI notion are captured.

In fact, authors as Van Damme (1989) propose an alternative formulation of FI that stress the notion of consistency of the signals that players send. Our definition of FI is highly inspired in this latter approach. Several authors have made proposals of definitions of FI, in this direction, for the class of repeated two-person games with finite horizon and complete information.

Osborne (1990) shows that every outcome of a path of the repeated game, that can be "upset" by a convincing deviation, is not stable in the sense of Kolhberg and Mertens. We say that a path P can be "upset" by a convincing deviation if after a deviation of player i of such path in period t, there exists a unique continuation path Q in which the payoff for player i since period t onwards, is greater than his payoff in P. Furthermore, it is necessary that the opponent j can not gain by deviating from Q. A drawback of the Osborne's definition is that it does not require the continuation to be consistent with Forward Induction.

Ponssard (1991) and Al-Najjar (1995) propose definitions of FI for repeated games, similar to the Osborne's one, but they incorporate a consistency requisite: the continuations, after a deviation, must also satisfy the proposed definition of FI. Thus, the paths consistent with FI, have to be computed using Backward Induction.

Our definition of FI for repeated games, would state that whenever there exists a deviation for a player, with a strictly positive cost, in a NE path, such that this cost is strictly smaller than the maximal net gain over all the possible continuations consistent with Forward Induction, then we will say that such a path will not be reasonable or more formally, that is not Consistent with Forward Induction (CFI).

As we saw in the above example, this definition tries to capture the idea that such a deviation has a rational explanation which "destabilizes" the given equilibrium path.

Notice that we only consider deviations with a strictly positive cost. In other words, the deviation has to be costly in order to be credible as a signal that the player tries to get a bigger payoff. On the other hand, we ask these continuations to be CFI themselves as a minimal consistency requirement.

We need some additional notation, before stating the formal definition of our proposal.

Let $\{W^1, W^2.....W^T\}$ be a NE path in G^T. Recall that $g_i(W^t) = g_i(w^t, w^t)$ is the payoff for player i in period t, where $g_u(W^t) = w^t$ and $g_f(W^t) = b - w^t$.

Let $\overline{V}_i^K$ be the maximal payoff for player i among all the CFI paths in a repeated game with horizon K.

Definition 1 *The NE path $\{W^1, W^2, ...W^T\}$ is a Consistent with Forward Induction (CFI) path, in G^T if and only if $\{W^2, W^3, ...W^T\}$ is a CFI path in G^{T-1}, and for all i, $j = u, f$, $i \neq j$ and $\widehat{w}_i \neq w_i^1$, such that*

$g_i(W^1) - g_i(\ \widehat{w}_i, w_j^1) > 0$, *the next condition holds:*

$$g_i(W^1) - g_i(\widehat{w}_i, w_j^1) \geq \overline{V}_i^{T-1} - \sum_{t=2}^{T} g_i(W^t).$$

Notice that the set of CFI paths can be computed using Backward Induction, beginning by the last stage of the game. Likewise, it can be shown that it is enough to check the deviation with the smallest cost. Observe also that for $T = 1$, the CFI paths coincide with the Nash equilibria of G.

In the next section, we will apply this definition of Forward Induction to the repeated bargaining game.

4 Commmitment and Forward Induction.

We are interested, in this section, in analyzing the set of CFI paths of the repeated wage bargaining game. In particular, we want to examine whether with a long enough horizon T, any party (or both) can obtain a more favorable outcome making use of the signalling capacity implicit in the notion of FI.

At first, it does not seem possible to obtain general results for all the range of parameters of the model, therefore we will have to restrict the set of feasible values. A "natural" restriction, that captures an interesting case, would be to assume that:

* $(w_o < w^* - w_o)$. That is, the cost of the strike for the union (in case the firm offers the smallest possible wage) is less than his maximal net gain, and on the other hand,

* $(b - w^* \geq w^* - w_o)$. That is, for the firm the cost of suffering the strike (in case the union claims the highest wage) is greater or equal than her maximal potential gain. Note that we distinguish between the costs of not receiving the share that she would have obtained in the negotiation of the surplus, and another sort of costs of a strike (those we mentioned in section 2). In this case, obviously, $(b - w^* + c) \geq (w^* - w_o) + (w^* - w_s)$. Notice that it might be the case that $(b - w^*) < (w^* - w_o)$, but $(b - w^* + c) \geq$

$(w^* - w_o) + (w^* - w_j)$, with $w_j > w_s$.

We will not distinguish these different possibilities in what follows and we will work with the restriction $(b - w^* + c) \geq (w^* - w_o) + (w^* - w_s)$.

Obviously, with these restrictions on the parameters, we are giving more signalling possibilities to the union as compared to those of the firm. Therefore, intuition suggests that perhaps, for a sufficiently long horizon, some NE paths unfavorable for the union would disappear from the set of CFI paths. The reason is that the latter would be able to destabilize those paths with credible deviations. On the contrary, it is more costly and difficult for the firm to carry out this behavior. In fact, Dekel (1990), in the short existing literature of FI on bargaining models, obtains how the FI logic eliminates "very biased" equilibria for any of the players, in a symmetric situation.

But the question we want to address in this work is under which conditions the union can get his preferred wage as average payoff in all CFI paths of NE, as the horizon tends to infinity. That is, under which circumstances the union is able to exploit to the maximum the signalling advantage we have given to him.

The first proposition shows that only in the game where the union commits to strike just with the highest possible wage, that is, $\overline{w} = w^*$, he obtains as average payoff in any CFI path, his preferred wage w^*, if the horizon is long enough.

Denote by $\underline{V}_U^T$ the lowest CFI payoff of the union for an horizon T.

Proposition 1 *Let G^T be a repeated wage bargaining game. Assume that $w^* > 2w_o$ and $(b - w^*) \geq w^* - w_o$. Let $\overline{w} = w^*$. Then:*
$\overline{V}_U^T = Tw_o + T(n-1)\Delta$.

$\underline{V}_U^T = Tw_o + (T-1)(n-1)\Delta$.

Proof.

As we said above, w_o stands for the reservation wage of the union and w^* is the highest wage that the union can reach, and $w^* = w_o + (n-1)\Delta$. Where Δ is the distance between two consecutive wage offers. Recall that there are n wage offers, but only two Nash equilibria payoffs in the stage game.

For $T = 1$, we know that $\overline{V}_U^1 = w_o + (n-1)\Delta$ and $\underline{V}_U^1 = w_o$. In this case the set of CFI paths coincides with the set of Nash Equilibria (NE) of G.

Assume that $T = 2$.

* The maximal CFI payoff for the union is $\overline{V}_U^2 = 2w^* = 2w_o + 2(n-1)\Delta$.

This is due to the fact that the firm will not deviate from the path (W^*, W^*), because the cost of deviating, $(b - w^*)$, is greater than the maximal net gain of doing it, $(w^* - w_o)$. The union, obviously, will not deviate of this path.

* The minimal CFI payoff of the union is: $\underline{V}_U^2 = 2w_o + (n-1)\Delta$.

The path $\{ W_o, W_o\}$ is not CFI because the union has a cost of deviation of w_o and a maximal net gain of $(w^* - w_o)$.

Obviously, the path $\{W^*, W_o\}$ is CFI, because the firm has her best continuation in $T = 1$ and the union does not have any deviation with a strictly positive cost.

Summarizing, in $T = 2$:

$\overline{V}_U^2 = 2w_o + 2(n-1)\Delta$.

$\underline{V}_U^2 = 2w_o + (n-1)\Delta$.

Assume that the proposition holds for $T = K$, we will show then that holds for $T = K + 1$.

* The maximal CFI payoff of the union in $T = K+1$, $\overline{V}_U^{K+1} = (K+1)w_o + (K+1)(n-1)\Delta$.

We know that the continuation in K with a payoff of $(Kw_o + K(n-1)\Delta)$ is CFI with certainty. We check that the path started with w^* followed by this continuation will be CFI because the firm will not deviate, since the cost for her is $(b - w^*)$ which is greater than her net gain of $(n-1)\Delta$. The union will also not deviate because he does not have a deviation with a strictly positive cost. Therefore, $\overline{V}_U^{K+1} = (K+1)w_o + (K+1)(n-1)\Delta$.

* The minimal CFI payoff of the union in $T = K+1$, $\underline{V}_U^{K+1} = (K+1)w_o + K(n-1)\Delta$.

We can take the CFI continuation with certainty in $T = K$ with a payoff for the union of $Kw_o + (K-1)(n-1)\Delta$. We can check that a path starting with w_o, followed by this continuation is not a CFI path, because the union will deviate. It can also be checked with similar arguments to those used above that, a path initiated in $K+1$ with w^* and with this continuation is a CFI path, because the firm has his best continuation and will not deviate. The union will also not deviate because is a path started with the wage w^*.

Therefore, $\underline{V}_U^{K+1} = (K+1)w_o + K(n-1)\Delta$.

■

It follows, under the restrictions imposed on the parameters, that the best option for the union if this latter could choose, would be to link his commitment to strike only with his preferred wage. We have not yet analyzed what the union can get with other commitments, but in any case, it is evident that he could obtain at best the same result in payoffs.

Nevertheless, this result has a negative character in some sense, because in many circumstances it would not be credible or possible to commit to strike *only* with w^*. If we want to obtain a similar result for different games, i.e., games characterized by $\overline{w} \neq w^*$, then we need to impose additional restrictions on the parameters.

The next result we present deals with a situation in which the incentives to strike in order to get higher wages by the union are not only positives but also sufficiently high or, alternatively, the incentives of the firm to resist the strikes are low enough. In particular, assume that $(b - w^* + c) \geq (w^* - w_o) + (w^* - w_s)$ and $(w^* - w_{s-1}) > w_o$.

This last restriction corresponds with a situation in which, the reservation wage is very low or alternatively, the preferred wage, derived from the reservation profitability of the firm, is relatively very high. That is, $w^* > (2w_o + (s-1)\Delta)$ instead of $w^* > 2w_o$.

In this case, if the union links his commitment to strike to the wage $\overline{w} = w_s$, gets also his preferred wage as average payoff in any CFI path, for a horizon high enough.

Proposition 2 *Let G^T be a repeated wage bargaining game. Assume that $w^* > (2w_o + (s-1)\Delta)$ and $(b - w^* + c) \geq (w^* - w_o) + (w^* - w_s)$, where $s \leq (n-2)$. Let $\overline{w} = w_s$. Then,*

$$\overline{V}_U^T = T \cdot w_o + T \cdot (n-1) \cdot \Delta.$$

$$\underline{V}_U^T \geq T \cdot w_o + (T-2) \cdot (n-1) \cdot \Delta + s \cdot \Delta \quad \textit{for } T \geq 2.$$

$$\underline{V}_U^T \leq T \cdot w_o + (T-1) \cdot (n-1) \cdot \Delta.$$

Proof.

Sometimes we will use w^*_{-i} to denote the wage w_{n-i-1}.

Suppose $T = 1$.

Recall that in this case, the set of CFI paths coincides with the set of Nash Equilibria (NE) of G.

Then, $\overline{V}^1_U = w^* = w_o + (n-1)\Delta$, and $\underline{V}^1_U = w_o$.

Assume $T = 2$.

* The maximal CFI payoff of the union satisfies $\overline{V}^2_U = 2w_o + 2(n-1)\Delta$.

This is due to the fact that the firm will not deviate of the path $\{W^*, W^*\}$, because the cost of deviating, $(b - w^* + c)$, is greater than the maximal net gain of doing it, $(w^* - w_o)$. The union, evidently, will also not deviate because it is his best possible payoff.

* The minimal CFI payoff of the union satisfies $\underline{V}^2_U \geq 2w_o + s\Delta$.

We know that the path $\{W_o, W_o\}$ is not CFI, because the union has a cost from deviating of w_o, while the net gain is $(w^* - w_o)$.

Analyze next the path $\{W_s, W_o\}$. Notice that the firm will not deviate since has the best possible continuation. On the other hand, the cost of a deviation for the union is $w_o + s\Delta$, as contrasted with a net gain of $w^* - w_o$, that is $(n-1)\Delta$. As $(n-s-1)\Delta < (n-s)\Delta = (w^* - w_{s-1})$ we can not discard that this path is CFI. Therefore, $\underline{V}^2_U \geq 2w_o + s\Delta$.

* The minimal CFI payoff of the union satisfies $\underline{V}^2_U \leq 2w_o + (n-1)\Delta$.

Obviously, the path $\{W^*, W_o\}$ is CFI, since the firm has her best continuation in $T = 1$ and the union does not have a deviation with a strictly positive cost. It can be easily checked that a CFI path with a lower payoff for the union does not exist with certainty, because those with a continuation of W_o, the union might have incentives to deviate and in those paths with a continuation different from W_o, the firm will deviate with certainty.

Summarizing, in $T = 2$:

$\overline{V}_U^2 = 2w_o + 2(n-1)\Delta.$

$\underline{V}_U^2 \geq 2w_o + s \cdot \Delta.$

$\underline{V}_U^2 \leq 2w_o + (n-1)\Delta.$

Now, we apply the inductive hypothesis. Assume that the proposition holds for $T = K$. We will demonstrate that it holds for $T = K + 1$.

* The maximal CFI payoff of the union in $T = K + 1$, $\overline{V}_U^{K+1} = (K + 1)w_o + (K+1)(n-1)\Delta$.

We know that the continuation in K with a payoff of $(Kw_o + K(n-1)\Delta)$ is CFI with certainty. We check that the path starting in $K + 1$ with the wage w^* and followed by this continuation payoff, is also a CFI path. The firm has a deviation cost of $b - w^* + c$ and the net gain, at best, is $Kw_o + K(n-1)\Delta - [Kw_o + (K-2)(n-1)\Delta + s\Delta] =$

$(2(n-1)\Delta - s\Delta) = (2n - s - 2)\Delta.$

As $(w^* - w_o) + (w^* - w_s) = (2n - s - 1)\Delta$ is strictly greater than $(2n - 2 - 2s)\Delta$, this path is CFI with certainty.

Therefore, $\overline{V}_U^{K+1} = (K+1)w_o + (K+1)(n-1)\Delta.$

* The minimal CFI payoff of the union in $K + 1$ satisfies $\underline{V}_U^{K+1} \leq (K + 1)w_o + K(n-1)\Delta.$

We can take the CFI continuation with certainty in $T = K$ with the lowest wage bill, that is, the one with a payoff for the union of $Kw_o + (K-1)(n-1)\Delta$. It can checked with similar arguments to those used above that, a path initiated with w^* and with this continuation is a CFI path, and furthermore, there is no another path with a lower wage bill that is CFI with certainty.

* The minimal CFI payoff in $K+1$ for the union satisfies $\underline{V}_U^{K+1} \geq (K+1)w_o + (K-1)(n-1)\Delta + s\Delta$.

Take a continuation in $T = K$ that might be CFI with the lowest possible wage bill. That is, a continuation payoff $Kw_o + (K-2)(n-1)\Delta + s\Delta$.

Start with a initial wage of w_o and this continuation. The net gain of the union from deviating is $Kw_o + K(n-1)\Delta - [Kw_o + (K-2)(n-1)\Delta + s\Delta] = (2n-s-2)\Delta$. This gain is greater or equal than $(n-s)\Delta$. Therefore, this path is not CFI.

Examine next a path that begins with the wage w_s, followed by the former continuation payoff. The cost of deviating for the union is $w_o + s\Delta$, and the net gain is $(2n-2-s)\Delta$. Thus, this path is not CFI, since: $(2n-2-2s)\Delta \geq (n-s)\Delta$. But, this is equivalent to $n \geq s+2$.

It can be checked that in paths initiated with wages $w \neq w^*$ followed by this continuation the union would deviate. For example, with an initial wage of w^*_{-1}, the deviation cost is $w_o + (n-2)\Delta$, then, given that $((2n-2-s)\Delta - (n-2))\Delta = (n-s)\Delta$, this path is not CFI with certainty.

But, the path that begins with w^* followed by the continuation which we are working with, may be CFI , because the firm has her best continuation from those that might be CFI.

It is easy to confirm that another paths with lower wage bill, constructed with a slight greater continuation payoff, are not CFI.

Therefore, we conclude: $\underline{V}_U^{K+1} \geq (K+1)w_o + (K-1)(n-1)\Delta + s\Delta$. ■

It follows as a corollary from this proposition, that if the horizon is long enough, the union can guarantee himself his preferred payoff as average payoff, in any CFI path.

Corollary 1 *The minimal average payoff for the union in any CFI path of G^T (which satisfies the restrictions imposed in proposition 2) converges to w^* as $T \to \infty$.*

These results show how under certain conditions, the union using the signalling ability implicit in the FI notion is able to obtain his preferred payoff, as if he could credibly commit to play his "Stackelberg strategy" (that is, "always demand w^*"). These results are similar to those obtained with the reputation effects approach (Fudenberg and Levine, 1989), consisting of perturbing the game G^T with some particular kind of incomplete information. Namely, allowing for the existence of a type of union committed to the Stackelberg strategy.

Notice that in the condition of Proposition 2, there is implicitly a clear trade-off between the incentives to strike of the union and those to resist the strike of the firm. In order to get strong results (in terms of the proposition), whenever the incentives to resist strikes of the firm are relatively low (for example, because of very high material costs, c), then we do not need relatively high incentives to strike of the union, and viceversa.

In particular, for the game $\overline{w} = w_1$, that is, the standard simultaneous demand stage game, the restriction concerning the union's incentives is again $w^* > 2w_o$, but then, we need the lowest possible incentives to resist strikes for the firm.

Proposition 3 *Let G^T be a repeated wage bargaining game. Assume that,*

$w^* > 2w_o$, and, $(b - w^* + c) \geq (w^* - w_o) + (w^* - w_1)$. Let $\overline{w} = w_1$. Then:

$$\overline{V}_U^T = T \cdot w_o + T \cdot (n-1) \cdot \Delta.$$

$$\underline{V}_U^T \geq T \cdot w_o + (T-2) \cdot (n-1) \cdot \Delta + \Delta \quad \textit{for } T \geq 2.$$

$$\underline{V}_U^T \leq T \cdot w_o + (T-1) \cdot (n-1) \cdot \Delta.$$

Proof.

We omit it because it is completely similar to the one of Proposition 2.

Therefore, for any particular stage game, characterized by a concrete $\overline{w}$, we need different restrictions on the parameters in order to get strong "commitment" effects. From another point of view, let us assume that the union can choose his commitment wage level, $\overline{w}$, once for all before the repeated game starts. Then, our results characterize the optimal choice of $\overline{w}$ for all possible values of the parameters within the basic restrictions established at the beginning of this section.

In all the previous propositions, the FI logic strongly works to the advantage of the union, and we obtain very similar effects to the so-called reputation effects. A natural question would be to analyze what happens in the cases not contemplated by the restrictions of the propositions. This is what we discuss in the next section

5 Final Comments and Conclusions.

We will present in this section some general comments about the kind of mechanisms that work out when the restrictions imposed in the above propositions do not hold. We want to know which restrictions impose the FI notion when the following conditions are satisfied:

* $b - w^* + c \geq (w^* - w_o) + (w^* - w_s)$

* $w^* - w_o > w_o$,

* $\overline{w} = w_s$ and $w_s \neq w_1, w^*$.

A first conclusion is that it is impossible to obtain strong and favorable results for the union as those presented in the last section. That is, it can not be shown that in any CFI path, the union gets the best wage w^* as average payoff or comes close to it, when the horizon is sufficiently long. This is due to the fact that the union's incentives to signal (and to strike) are not strong enough and/or those of the firm to resist the strikes are not sufficiently weak.

Formally, the former causes that the lower bound of the minimal payoff for the union is not sufficiently high. This is because, although the net gain for striking is greater than his cost, it is not high enough for the union to be able to destabilize paths with relatively low wage bills. This leads to an increase in the upper bound of the CFI maximal payoff for the firm, which at the same time increases the incentives of the firm to deviate from those paths more favorable for the union (as for example, those composed of equilibria W^*exclusively).

Therefore, the payoff of the path with a higher wage bill, that we could know with certainty that is CFI, diminishes. Namely, it goes away from the path in which all its components are W^*. But, at the same time, this reinforces, in paths with longer horizons, the tendency of the lower bound of the minimum payoff of the union to be relatively low, given that the net gain of which the union can be sure is smaller (reducing the incentives to deviate from paths with low wages).

Summarizing, the fact that the incentives to signal of both parties are not in one case sufficiently high and, in the other, low enough, produces a mechanism that feeds back reciprocally, holding the low bound of the CFI minimum average payoff of the union far from w^*as average.

All the results obtained until now refer to a situation defined by the restrictions imposed in the last section. As we said above, such restrictions characterize an *a priori* "favorable" situation for the union. Similarly, we might analyze the inverse situation, "profitable" for the firm in this case. That is, the one in which the cost of resisting the strike for the firm is less than the net gain of doing it, and on the contrary, the cost of striking for the union is greater than his net gain. Formally, $(b - w^* + c) < (w^* - w_o)$ and $w_o \geq (w^* - w_o)$.

More interesting seems a situation in which both parties have *a priori* a positive incentive to signal (to strike and to resist the strike, respectively). In this case, $w^* - w_o > w_o$ and $w^* - w_o > b - w^*$.

A more detailed analysis of this case belongs to future research. But, one would expect that the signalling ability of both agents would make disappear as CFI paths, those results excessively biased and that only would "survive" intermediate wages. This is the sort of results obtained in the FI literature as in the repeated Battle of Sexes, for example.

Just to illustrate this intuition, take as constituent game the one of figure 2 and assume that the above restrictions on the parameters are satisfied. It is easy to check, that for $T = 2$, the path $\{W^*, W^*\}$ would not be CFI, and neither the path $\{W_o, W_o\}$. On the contrary, the path $\{W^*, W_o\}$, for example, is CFI. In particular, the only CFI payoff for the union is $(w_o + w^*)$.

Likewise, in $T = 3$, the set of CFI payoff is $\{2w_o + w^*, 2w^* + w_o\}$. But in $T = 4$, the set of CFI payoffs collapses in a unique value: $(2w_o + 2w^*)$. In short, the FI logic, in this case, tends to produce intermediate average wages.

Finally, we want to emphasize that there is another situation in which the effects of FI are very strong and clear, in the sense that one party obtains his preferred payoff. This is the case, when, for example, the union faces a sequence of firms in a repeated negotiation. Each one of these firms negotiates only in one period and knows the previous history of the repeated game. This situation is known in the literature on reputation effects as the case of a long-run agent against a sequence of short-run or myopic agents (Fudenberg and Levine (1989)).

In this case, the union guarantees himself as average payoff his preferred wage in any CFI path with sufficiently long horizon. The reason is simple: only the union has incentives to signal. The firms lack of them, because they are short-run agents. The next proposition establishes an upper bound on the number of periods in which the union does not obtain his preferred wage.

Proposition 4 *Let G^T be a repeated wage bargaining game. Assume $M = \{w_o, w^*\}$ and $c \geq 0$. Then, there exists a positive integer K, such that in any CFI path it is played, at most, K equilibria different from W^*.*

Proof.

Define K as the smallest integer such that $K > w_o/(w^* - w_o)$. Suppose that $T > K$ and there is a CFI path in which is played $K + 1$ times the equilibrium W_o. Take the first period τ in which is played W_o. Obviously, in the continuation it will be play K times different from W^*.

Notice that, trivially, any path composed totally of equilibria W^* is CFI, because the firms can not deviate because they live only one period. Then, the maximal CFI payoff for the union in a game with an horizon T, is $T \cdot w^*$.

Therefore, the cost of deviating for the union, in the period τ, is w_o and the maximum net gain will be: $(T-\tau-1)\cdot w^* - [(T-\tau-1-K)\cdot w^* + K\cdot w_o] = K \cdot (w^* - w_o)$.

Then, for the proposed path to be CFI, it must satisfy $K \leq \frac{w_o}{w^*-w_o}$, which is a contradiction. ■

Notice that this result is valid without necessity of imposing the kind of restrictions on the parameters used above.

In summary, we have presented in this work, a repeated wage bargaining game with complete information, where there is an important problem of multiplicity of equilibria. We have shown how, under some "natural" restrictions, the FI arguments are sufficient to obtain "reputation effects". In particular, it has been specified the concrete conditions for one party (for example, the union) to obtain his preferred wage as average payoff in any CFI equilibrium path of the repeated game. Likewise, we have discussed the problems arisen, under weaker restrictions, in order to the signalling ability to be more effective and we have pointed future lines of research.

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