FIRST-BEST, SECOND-BEST AND PRINCIPAL-AGENT PROBLEMS*

Javier López Cuñat and Jose A. Silva Reus**

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ABSTRACT

In some pure moral hazard situations the principal can implement a first-best allocation using an incentive contract constructed on the basis of a first-best payment scheme. Such a contract relies on the possibility of discriminate actions according to the outcome by imposing a penalty whenever the observed outcome is lower than the admissible ones. The elimination of inefficient behavior depends basically on the outcome function, and we find that the fine is finite in the more interesting cases. The implementation of the first-best solution does not depend on the principal's risk neutrality. Nevertheless, when the principal is risk neutral, the efficient contract is *dichotomous*. Moreover, we prove that the efficient allocation can be reached through such a dichotomous payment scheme if and only if the principal is risk neutral for a certain range of returns.

1.Introduction

In some pure moral hazard situations, under asymmetric information, first-best allocations can be achieved. This occurs, for example, when the principal is able to observe the agent's action, when the state of the nature is ex-post verifiable or when the agent is risk neutral (See Harris-Raviv (1979), Shavell (1979)).In other cases, the agency relationship is efficient if the principal uses a *dichotomous* contract. This situation has been discussed by Mirrlees (1974), Harris-Raviv (1978), Singh (1983), and Brown-Miller-Thornton (1987). The latter two analyze a principal-agent model with a risk neutral principal and an agent's additively separable utility function. Nevertheless, assumptions on the random variable make differences in their models. Singh (1983) assumes an exogenous random environment with a finite number of states, whereas in Brown-Miller-Thornton (1987) an outcome density function exists which is parameterized by the agent's action.

The aim of this paper is to identify conditions which guarantee the achievement of the first-best allocation, in a state-space formulation of the principal-agent problem, without assuming that this state-space is finite, that the agent's utility function is separable and that the principal is risk neutral.

When the principal's preferences exhibit risk neutrality the efficient contract is dichotomous. However a first-best allocation can be achieved by means of such a dichotomous contract if and only if the principal is risk neutral for returns in a certain interval.

In Section 2, we describe our model. In Section 3, we obtain some conditions under which a first-best allocation can be achieved by constructing an incentive mechanism based on the first best payment rule. The implementation of a first-best allocation using a dichotomous contract is analyzed in Section 4. Finally, Section 5 contains our remarks and conclusions.

2.The model

In this paper we formulate the principal-agent relationshipin a state-space way. The agent chooses his action in the set A=[0, k], where $k \in \mathbb{R}$. The state of the system θ is a random variable, with distribution function $F(\theta)$ and compact support denoted by $\Theta = [\underline{\theta},\bar{\theta}] \subset \mathbb{R}$. The realization of the state occurs after the agent makes his choice. We will assume F to be continuously differentiable almost everywhere.

The action $a \in A$ and the system's state $\theta \in \mathbb{R}$ jointly determine an outcome $x=X(a,\theta)$ which we assume to be verifiable. The function X is assumed to be C^1 with $X_a > 0$[1] (Therefore a can be interpreted as effort). We will also assume X to be finitely oscillating, i.e. X has a finite number of maximum and minimum points.

Since A and Θ are compact sets, the following values are well defined

$$\underline{x}(a) = \min_{\theta \in \Theta} X(a,\theta) \ , \ \bar{x}(a) = \max_{\theta \in \Theta} X(a,\theta) \ , \text{ for all } a \in A$$

$$x_o = \min_{(a,\theta) \in A\times\Theta} X(a,\theta) \ , \ x_1 = \max_{(a,\theta) \in A\times\Theta} X(a,\theta)$$

$$\text{Then, for all } a \in A \quad \underline{x}(a), \bar{x}(a) \in [x_0,x_1]$$

Let W(x-s) denote the principal's utility function. The agent's utility function is represented by U(s,a), s being the payment received by the agent. We will assume

$$W \in C^2, \text{ with } W' > 0, \ W'' \le 0$$

$$U \in C^2, \text{ with } U_s > 0, \ U_{ss} < 0, \ U_a < 0$$

1 For the functions with several arguments, subscripts denote partial derivatives as regards respective argument. For functions with only one argument the derivatives are denoted by '.

Let $s = S(x)$ denote the part of x received by the agent. The contract S is agreed before both the agent takes action and θ is realized. The contracts are measurable functions for which the following expectations exist:[2]

$$EU(S,a)=E\{\ U(\ S(X(a,\theta))\ ,\ a)\ \}$$

$$EW(S,a)=E\{\ W(\ X(a,\theta)\ -\ S(X(a,\theta)))\ \}$$

It is assumed that the agent will accept every contract, giving him an expected utility greater than or equal to $\bar{U}$ (his reservation utility).

If the agent's action is verifiable, the principal can impose a penalty when he observes an undesirable action. In this symmetric information case, the principal will solve the program FB below.

$$(FB)\quad \left[\begin{array}{l} \max\limits_{a,S(x)} \quad EW(S,a) \\ s.t.: \\ \quad EU(S,a) \geq \bar{U} \end{array}\right.$$

A solution of such a program will be referred to as a first-best solution. Let v(FB) denote the optimal value of FB.

If the agent's action is not observable (asymmetric information case), the principal must solve the program SB below.

$$(SB)\quad \left[\begin{array}{ll} \max\limits_{a,S(x)} \quad EW(S,a) & \\ s.t.: & \\ \quad EU(S,a) \geq \bar{U} & [1] \\ \quad a \in \operatorname{Arg}\max\limits_{e\in A} EU(S,e) & [2] \end{array}\right.$$

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2 E(.) denotes the expectation operator according to the probability distribution of θ.

Condition [2] is the incentive compatibiliyy constraint.

We will call a solution of SB second-best solution. Let v(SB) denote the optimal value of SB.

Obviously, v(FB) ≥ v(SB), and in many cases the equality is not achieved (see Shavell (1979), Holmström (1979)). However, if the agent is risk neutral then v(FB) = v(SB) (see, for instance, Harris & Raviv (1979)). Another situation, where second best contracts are efficient, occurs when the state of nature is ex-post verifiable (see Harris-Raviv 1978). However, in some not-so-extreme cases, the first-best is also implementable under conditions on the production technology and the principal and agent's utility functions. Singh (1983) and Brown-Miller-Thornton (1987) are two examples of such a situation in which the principal is assumed to be risk, neutral and the agent's utility function to be additively separable.

The aim of this paper is to determine conditions which guarentee the existence of a contract such that v(FB)=v(SB), using a more general framework than the one analyzed in previous literature.

We will assume that a first-best solution (S*,a*) exists such that $S^* \in X$, $a^* \in \text{int } A$, where X is the following normed linear space :

$$X = \{ S : [x_0, x_1] \longrightarrow \mathbb{R} \;/\; S \text{ is } C^1 \text{ in } [x_0, x_1]\}$$

Using the Fréchet derivative of functionals EU and EW and the generalized Kuhn-Tucker Theorem (see Luenberger (1969)), we obtain the necessary conditions on (S*,a*), such as

(NC1) $$\lambda\{ EU(S^*,a^*) - \bar{U} \} = 0 \qquad (\lambda \geq 0)$$

(NC2) $W'(X(a^*,\theta)-S^*(X(a^*,\theta))) = \lambda \; U_s(S^*(X(a^*,\theta)) , a^*)$ for θ such that F is discontinuous at θ (the probability of θ is positive) or $\exists$ $F'(\theta)>0$ (a positive density function exists at θ)

Therefore, $EU(S^*,a^*)=\bar{U}$ holds. (P1)

When F admits a positive density function in Θ, by (NC2) the following is true

$$0 \le S^{*\prime}(x) < 1, \quad \forall x \in [\underline{x}(a^*\bar{x}(a^*)] \qquad \text{(P2)}$$

Otherwise $S^{*\prime}$ is not completely characterized in $[\underline{x}(a^*), \bar{x}(a^*)]$. However, we will assume (P2) to always be true, because S^* is not completely characterized by the necessary condition (NC2) of program FB.[3]

By (P2), the following property holds.

$$-\infty < \frac{d}{da} EU(S^*,a^*) < 0 \qquad \text{(P3)}$$

If (P3) holds, any small desviation of the agent's action regardig to the first-best action a* will imply that the agent's expected utility decreases.

We will use (S*,a*) to construct the optimal contract.

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3 Under some conditions on U and W (for instance when they exhibit the property of the absolute risk aversion constant (CARA)) and assuming the existence of a first-best solution, a continuously differentiable first-best contract can be obtained. This contract will satisfy (P2). The property (P2) is used in the proof of Proposition 1.

3. A second-best solution with first-best value.

A way to force the agent to take an action (effort) greater than or equal to a*, is to offer a contract which penalizes any outcome that signals without error that the effort made is lower than the efficient one. If this is possible, the outcome x can be interpreted as a signal that allows to distinguish the agent's individual actions. As it is assumed that $X_a>0$, we can get the former. We are sure that the agent has taken an action less than a* when we observe an outcome less than $\underline{x}(a^*)$ (See figure 1).

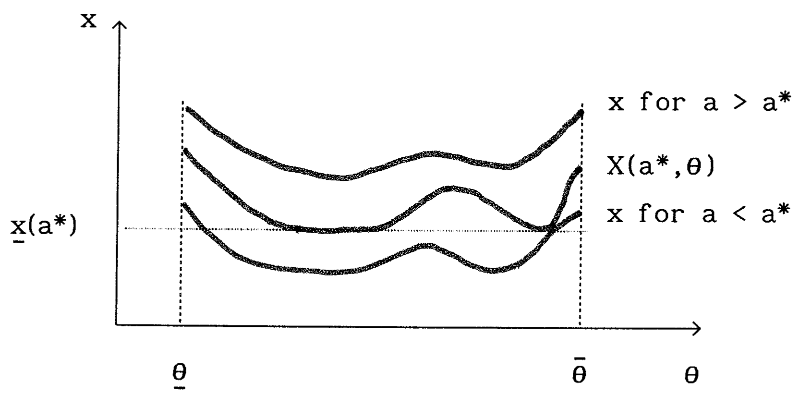

Figure 1

We propose the following contract[4]

$$\hat{S}(x)=\begin{cases} S^{*\prime}(\bar{x}(a^*))(x-\bar{x}(a^*)) + S^*(\bar{x}(a^*)) & ;\ x \geq \bar{x}(a^*). \\ S^*(x) & ;\ \underline{x}(a^*) \leq x \leq \bar{x}(a^*) \\ m & ;\ x < \underline{x}(a^*). \end{cases} \quad [3]$$

[4] If S* is constant, the contract $\hat{S}$ becomes dichotomous. Therefore $\hat{S}$ has as a particular case the payment rule proposed by Singh (1983).

The nature of the contract $\hat{S}$, for outcomes greater than $\underline{x}(a^*)$, makes good use of the optimal properties of S^*, discouraging the agent to take actions greater than a^*. On the other hand, the fine m will avoid actions lower than a^* from being preferred by the agent.

Notice that $\hat{S}$ is continuously differentiable except at point $x = \underline{x}(a^*)$. Moreover by P1 and P2, for every m, it follows that

$$\hat{S}'(x) \in [0,1[\quad \forall\, x \geq \underline{x}(a^*)$$

$$EU(\hat{S},a^*) = EU(S^*,a^*) = \bar{U}$$

$$EW(\hat{S},a^*) = EW(S^*,a^*) = v(FB)$$

If we select m such that $(\hat{S},a^*)$ is a feasible point of program SB ,then we have $EW(\hat{S},a^*) = v(SB) = v(FB)$. In the rest of this section we will obtain and analyze the conditions which guarantee the fact that $(\hat{S},a^*)$ is a feasible point of SB. These conditions must imply that, for some contract such as $\hat{S}$, the agent prefers the first-best action a^* to both greater and lower actions. We assume the hypotheses presented in Section 2.

Under contract $\hat{S}$, for every m, the agent prefers a^* to greater actions.

Lemma 1. *For all* $a > a^*$ $\quad EU(\hat{S},a) < \bar{U} = EU(\hat{S},a^*)$.

Proof:□ Assume that $\exists\, \bar{a} > a^*$ such that $EU(\hat{S},\bar{a}) \geq \bar{U}$.

Let us consider the following contract

$$\bar{S}(x)=\begin{cases} S^{*\prime}(\bar{x}(a^*))(x-\bar{x}(a^*)) + S^*(\bar{x}(a^*)) & ;\ x \geq \bar{x}(a^*). \\ S^*(x) & ;\ \underline{x}(a^*) \leq x \leq \bar{x}(a^*) \\ S^{*\prime}(\underline{x}(a^*))(x-\underline{x}(a^*)) + S^*(\underline{x}(a^*)) & ;\ x \leq \underline{x}(a^*). \end{cases}$$

By construction $\bar{S} \in X$ and $\bar{S}'(x) \in [0,1[$ for all x (See P2).

Since $\bar{a} > a^*$ and $X_a > 0$ it follows that

$$X(\bar{a},\theta) > X(a^*,\theta) \geq \underline{x}(a^*) \quad \text{for all } \theta \in \Theta.$$

Consequently, $EU(\bar{S},\bar{a}) = EU(\hat{S},\bar{a}) \geq \bar{U}$ and then $(\bar{S},\bar{a})$ is a feasible point of program FB.

On the other hand we have that

$$EW(\bar{S},\bar{a})-EW(S^*,a^*) = EW(\bar{S},\bar{a})-EW(\bar{S},a^*) =$$

$$= E\{ W(X(\bar{a},\theta)-\bar{S}(X(\bar{a},\theta))) - W(X(a^*,\theta)-\bar{S}(X(a^*,\theta)))\}$$

By the mean value Theorem,
$\forall\ \theta$, $\exists\ \psi(\theta)$ between $X(\bar{a},\theta)-\bar{S}(X(\bar{a},\theta))$ and $X(a^*,\theta)-\bar{S}(X(\bar{a},\theta))$ such that

$$EW(\bar{S},\bar{a})-EW(S^*,a^*)=$$
$$=E\{ W'(\psi(\theta)).[X(\bar{a},\theta)-X(a^*,\theta) + \bar{S}(X(a^*,\theta)) - \bar{S}(X(\bar{a},\theta))] \}$$

But also, $\forall\ \theta$, $\exists\ \varphi(\theta)$ between $X(a^*,\theta)$ and $X(\bar{a},\theta)$ such that

$$EW(\bar{S},\bar{a})-EW(S^*,a^*)=$$
$$=E\{ W'(\psi(\theta)).[X(\bar{a},\theta)-X(a^*,\theta)].[1-\bar{S}'(\varphi(\theta))] \}$$

Since $W'>0$, $X(\bar{a},\theta) > X(a^*,\theta)\ \forall\ \theta$ and $1-\bar{S}'(x) > 0\ \ \forall\ x$, we conclude that $EW(\bar{S},\bar{a}) > EW(S^*,a^*)$ is impossible because (S^*,a^*) is a first-best solution.■

Therefore, if the principal wishes the agent to take a* under the contract $\hat{S}$, he must select m such that $\forall a<a^*$ $EU(\hat{S},a)<\bar{U}$. The existence of such an m is associated with the existence of a value C that we will interpret as a penalty on utilities.

For every a<a*, we define

$$A(a)=\{\ \theta \in \Theta\ /\ X(a,\theta) \geq \underline{x}(a^*)\ \}$$

the set of states generating outcomes greater than $\underline{x}(a^*)$;

$$p(a)=Pr(\ X(a,\theta) < \underline{x}(a^*)\ )$$

the probability of observing an outcome lower than $\underline{x}(a^*)$ if the agent's effort is a; and

$$H(a)=\int_{A(a)} U(\ \hat{S}(X(a,\theta)),\ a)\ dF(\theta) = \int_{A(a)} U(\ S^*(X(a,\theta)),\ a)\ dF(\theta)$$

the agent's expected utility, conditional to a, over the set of outcomes above $\underline{x}(a^*)$.

A necessary condition for the existence of m such that the agent prefers the first-best action a* to lower actions, under the contract $\hat{S}$, is that the supreme

$$C = \sup_{a\in[0,a^*[} \left\{ \frac{H(a) - \bar{U}}{p(a)} \right\}$$

exists.[5]

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[5] The result proved in the appendix is the following
Proposition 1. *If $\exists\ m \in \mathbb{R}$ such that $\forall\ a < a^*$ $EU(\hat{S},a) < \bar{U}$ then C exist.*

Let us discuss this condition. If the supreme C exists then it follows that $H(a) \leq \bar{U} + C.p(a) \quad \forall a<a^*$. Therefore

$$E\{U(S^*(X(a,\theta)), a) \; / \; x \geq \underline{x}(a^*)\}.(1-p(a))-C.p(a) \leq \bar{U} \quad \forall a<a^* \qquad [4]$$

Under the contract $\hat{S}$, the agent always obtains an expected utility of $\bar{U}$ taking the action a^*. We can interpret C as a penalty on utilities imposed by the principal when he observes $x < \underline{x}(a^*)$. The inequality [4] indicates that the agent will never deceive the principal by taking $a < a^*$.

If the penalty C is represented by a payoff m, then the principal forces the agent to choose the efficient effort by means of the contract $\hat{S}$. The following result provides a sufficient condition.

Lemma 2. *Assume that*

$$\exists \; C = \sup_{a \in [0,a^*[} \left\{ \frac{H(a) - \bar{U}}{p(a)} \right\} \qquad [5]$$

$$\exists \; m \in \mathbb{R} \; / \; U(m,0) = -C$$

then the contract $\hat{S}$ *in [3] is such that*

$$a^* \in \underset{a \in A}{\operatorname{argmax}} \; EU(\hat{S},a)$$

Moreover a^* *is the unique maximizer of* $EU(\hat{S},a)$ *if the supreme* C *is not achieved at* $a=0$.

Proof: See Appendix.

C always exists, at least in non-degenerated cases. We will use the following results to obtain conditions guaranteeing the existence of C. Beforehand, we give some definitions.

Let $R^*=\{ \theta \in \Theta \ / \ X(a^*,\theta) = \underline{x}(a^*) \}$ be the set of minimum points of the function $X(a^*,\theta)$. Since $X(a,\theta)$ is finitely oscillating, the set R^* is a finite union of isolated points and/or closed intervals. A particular case is represented in Figure 2.

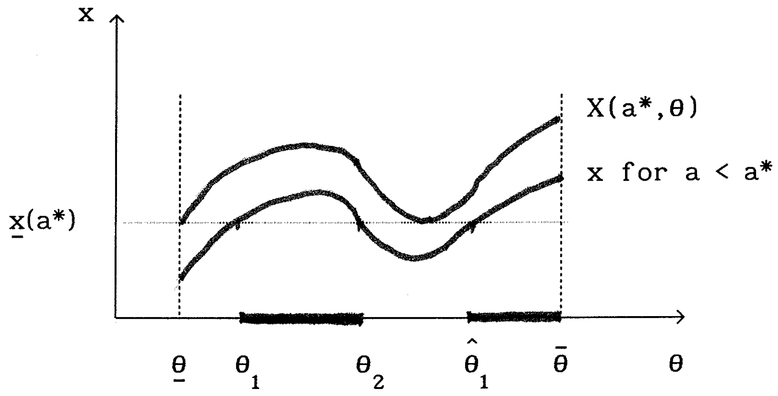

Figure 2

We will prove that each one of the following conditions implies the fact that C exists.

(C1) F is discontinuous at some point of R^*

(C2) $\exists \ \theta^* \in R^* \ / \ F$ is C^1 in a neighborhood of θ^* with $F'(\theta) > 0$

Lemma 3. *Under (C2),*

$$p'_-(a^*) = \lim_{a \longrightarrow a^{*-}} \frac{p(a) - p(a^*)}{a - a^*} < 0$$

Proof: See Appendix.

Lemma 4. *Let* $Q = \{ (x,y) \in \mathbb{R}^2 \ / \ c \le y \le d,\ a(y) \le x \le b(y) \}$ *where* $c,d \in \mathbb{R}$ *and a and b are real functions* C^1 *in* $[c\ ,\ d]$ *with* $a' \le 0,\ b' \le 0$.

Let $f(x,y)$ *be a continuous function in Q such that* f_y *is also continuous in Q.*

Let α *be a function of bounded variation such that it is continuous and differentiable on the left at* $a(y_0)$, $b(y_0)$, *where* $y_0 \in [c\ ,\ d]$.

Let $F(y) = \displaystyle\int_{a(y)}^{b(y)} f(x,y)\, d\alpha(x)$. *Then*

$$\lim_{y \to y_0^-} \frac{F(y) - F(y_0)}{y - y_0} = \int_{a(y_0)}^{b(y_0)} f_y(x,y_0)\, d\alpha(x) \ +$$

$$+ f(b(y_0),y_0).\alpha'_-(b(y_0)).b'(y_0) - f(a(y_0),y_0).\alpha'_-(a(y_0)).a'(y_0)$$

Lemma 5. *Each one of the conditions (C1) and (C2) implies the existence of C.*

Proof:□ The function $H(a)-\bar{U}$ is bounded and continuous in $[0\ ,\ a^*[$. The function $p(a)$ is upper semicontinuous and bounded in $[0\ ,\ a^*[$.

Then, the value C exists if and only if

$$\exists \lim_{a \to a^{*-}} \left\{ \frac{H(a) - \bar{U}}{p(a)} \right\} \in \mathbb{R} \qquad (1)$$

Let us suppose (C1). Then it follows that

$$p(a) \ge Pr(R^*) > 0 \qquad \forall\ a < a^*$$

and (1) is true.

Let us suppose (C2). Assume that F is continuous at every point in R^*. In this case we cannot assure $\lim_{a \to a^{*-}} p(a) > 0$, but we have that

$$\frac{H(a) - \bar{U}}{p(a)} = \frac{H(a) - H(a^*)}{a - a^*} \Bigg/ \frac{p(a) - p(a^*)}{a - a^*}$$

Moreover, by Lemma 3 $p'_-(a^*) < 0$. Therefore if we prove that

$$\exists \lim_{a \longrightarrow a^*-} \frac{H(a) - H(a^*)}{a - a^*} \in \mathbb{R} \qquad (2)$$

(1) will be true.

Let us prove (2) when F is continuous at every point in R*. For $a < a^*$, being sufficiently close to a*, the set A(a) is a finite union of intervals with the following forms

$$[\underline{\theta}\ ,\ \theta_2(a)] \quad , \quad [\theta_1(a)\ ,\ \theta_2(a)] \quad , \quad [\theta_1(a)\ ,\ \bar{\theta}]$$

where θ_1 and θ_2 are continuously differentiable functions such that

$$\theta_1' < 0, \quad \theta_2' > 0, \quad \lim_{a \to a^*-} \theta_1(a) = \theta_1^* \in R^*, \quad \lim_{a \to a^*-} \theta_2(a) = \theta_2^* \in R^*$$

A particular case is represented in Figure 2.

Then, H(a) will be a finite addition of terms with the following forms

$$\int_{\underline{\theta}}^{\theta_2(a)} U(\ S^*(X(a,\theta)),\ a)\ dF(\theta)$$

$$\int_{\theta_1(a)}^{\theta_2(a)} U(\ S^*(X(a,\theta)),\ a)\ dF(\theta)$$

$$\int_{\theta_1(a)}^{\bar{\theta}} U(S^*(X(a,\theta)), a)\ dF(\theta)$$

Notice that , since F is continuous in R*, F is differentiable on the left in Θ and F is a function of bounded variation (F is a distribution function), Lemma 4 can be applied.

By Lemma 4, $\lim_{a \longrightarrow a^{*-}} \frac{H(a) - H(a^*)}{a - a^*}$ is a finite addition of terms with the following forms

$$\int_{\underline{\theta}}^{\theta_2^*} \frac{d}{da} U(S^*(X(a^*,\theta)) , a^*)\ dF(\theta) + U(S^*(X(a^*,\theta_2^*)) , a^*).F'_{-}(\theta_2^*).\theta'_2(a^*)$$

$$\int_{\theta_1^*}^{\theta_2^*} \frac{d}{da} U(S^*(X(a^*,\theta)) , a^*)\ dF(\theta) + U(S^*(X(a^*,\theta_2^*)) , a^*).F'_{-}(\theta_2^*).\theta'_2(a^*)-$$

$$-U(S^*(X(a^*,\theta_1^*)) , a^*).F'_{-}(\theta_1^*).\theta'_1(a^*)$$

$$\int_{\theta_1^*}^{\bar{\theta}} \frac{d}{da} U(S^*(X(a^*,\theta)) , a^*)\ dF(\theta) - U(S^*(X(a^*,\theta_1^*)) , a^*).F'_{-}(\theta_1^*).\theta'_1(a^*)$$

and then, (2) holds by P3. ■

The following theorem summarizes the results in this section.

Theorem 1. *Under (C1) or (C2), the contract* $\hat{S}$ *in [3] with*

$$m \in \mathbb{R} \;/\; U(m,0)=-C, \quad \text{where} \quad C = \sup_{a\in[0,a^*[} \left\{ \frac{H(a) - \bar{U}}{p(a)} \right\} \in \mathbb{R}$$

is such that

The point $(\hat{S},a^*)$ *is feasible for program SB, with value v(FB), and then* $EW(\hat{S},a^*) = v(SB) = v(FB)$.

Remarks.

1. In (C2) the assumption $F'(\theta^*) > 0$ is indispensable. For instance if the action set is $A=[0\ ,\ 2]$, the outcome function is $X(a,\theta) = a + \theta^2$, the agent's utility function is $U(s,a)=-\exp(1-s)-a$, $\bar{U}=-2$, the principal is risk neutral and $a^*=1$, for the state density function

$$f(\theta)=3\theta^2 \ ;\ 0\leq \theta \leq 1, \qquad f(\theta)=0 \quad \text{otherwise},$$

then we have that $C=+\infty$.

2. In the principal-agent literature a usual assumption is that

$$\lim_{s \to \bar{s}+} U(s,a)=-\infty \qquad \forall\ a\in A \quad (\bar{s} \text{ may be } -\infty).$$

Under this assumption it is true that $\exists\ m \in \mathbb{R} \;/\; U(m,0)=-C$.

3. The results of this section are independent of the principal risk aversion.

4. Efficiency of the second-best dichotomous contract

When the principal is risk neutral the contract S* of the previous section becomes constant in $[\underline{x}(a^*),\bar{x}(a^*)]$ and then the contract $\hat{S}$ is dichotomous:

$$\hat{S}(x)=\begin{cases} s & ; x \geq \underline{x}(a^*) \\ m & ; x < \underline{x}(a^*) \end{cases} \qquad [6]$$

It may be asked whether or not a first-best allocation can be achieved by means of the contract $\hat{S}$ in [6]; we will prove that it can if and only if W(x) is linear for x at a certain interval.

In this section we will consider a slightly more general case in which $U_{ss} \leq 0$ (the agent may be not risk averse). We will obtain conditions under which the agent will be forced to take the first-best action a* with a dichotomous contract as [6] and thereafter we will analyze when the value of such a contract coincides with v(FB).

Let S*(x) , a* $\in$ int A a first-best solution. In this case it is not necessary for S* $\in$ X.

By particularizing Lemma 2 and Lemma 5, we obtain the following result.

Lemma 6. *Under assumptions (C1), or (C2), if the contract*

$$\hat{S}(x)=\begin{cases} s & ; x \geq \underline{x}(a^*) \\ m & ; x < \underline{x}(a^*) \end{cases}$$

is such that

$$U(s,a^*)=\bar{U}$$

$$U(m,0)=\bar{U}-d, \quad \text{where} \quad d=\sup_{a\in[0,a^*[}\left\{\frac{U(s,a)-\bar{U}}{p(a)}\right\}$$

then, a^* is the unique maximizer of $EU(\hat{S},a)$ with $EU(\hat{S},a^*)=\bar{U}$.

This result is true without assumming the concavity of U in s. The existence of d is implicated by $p'_{-}(a^*) < 0$ in Lemma 3.

The following result provides a necessary and sufficient condition for the contract in proposition 4 to be simultaneously a first-best and second-best solution.

Theorem 2. *Under assumptions of Lemma 6.*

$$EW(\hat{S},a^*) = EW(S^*,a^*) \quad (\text{i.e. } v(FB)=v(SB)\ ) \text{ if and only if}$$
$$W(x) \text{ is linear in } x \in[\underline{x}(a^*)-s\ ,\ \bar{x}(a^*)-s].$$

Proof.□ Since W is concave and smooth we have that

$$W(x-s)-W(x-S^*(x)) \geq W'(x-s).[\ S^*(x)-s\ ]$$

Then, it follows that

$$0 \geq EW(S,a^*)-EW(S^*,a^*)=E\{\ W(x-s)-W(x-S^*(x))\ \} \geq$$

$$\geq E\{\ W'(x-s).[\ S^*(x)-s\ ]\ \} \qquad (1)$$

where $x=X(a^*,\theta)$.

From the concavity of U in y, it follows that

$$U(\ E\{S^*(x)\},\ a^*) \geq E\{\ U(\ S^*(x),\ a^*)\ \} \geq \bar{U}=U(s,a^*)$$

Then $$E\{\ S^*(x)\ \} \geq s \qquad (2)$$

If W is linear in $[\underline{x}(a^*)-s\ ,\ \bar{x}(a^*)-s]$ then $W'(X(a^*,\theta)-s)$ will be constant for θ and from (1) and (2) it follows that $EW(S^*,a^*)=EW(S,a^*)$.

In order to prove sufficiency, suppose $EW(S^*,a^*)=EW(S,a^*)$.

Let us consider the constant contract $\bar{S}(x)\equiv s$. Since

$$EW(\bar{S},a^*)=EW(\hat{S},a^*) \text{ and further } EU(\bar{S},e^*)=U(s,a^*)=\bar{U},$$

the point $(\bar{S},a^*)$ is a first-best solution.

Consequently, $(\bar{S},a^*)$ will satisfy the optimum's necessary conditions of program FB, and therefore

$$-W'(x-s\ )+\lambda U_y(s\ ,a^*)=0$$
$$\lambda[\ E\{U(s\ ,a^*)\}\ -\ \bar{U}\ ]=0, \qquad \text{with } x=X(a^*,\theta)$$

Then, one obtains

$$\lambda>0, \qquad U(s\ ,a^*)=\bar{U}, \qquad W'(x-s\ )=\lambda U_y(s\ ,a^*) \quad \forall x = x(a^*,\theta)$$

This implies that $W'(x-s)$ is constant for $x = X(a^*,\theta)$ and then, $W(x)$ is linear in $[\underline{x}(a^*)-s\ ,\ \bar{x}(a^*)-s]$.■

Note that $[\underline{x}(a^*)-s\ ,\ \bar{x}(a^*)-s]$ is the attainable principal's return set under contract $\hat{S}$.

5. Final remarks and conclusions.

In this paper we study the conditions which guarantee the implementation of the first-best allocation in a principal-agent situation under asymmetric information. Notice that, in our model, since $X_a > 0$ the outcome $x=X(a,\theta)$ is a random variable whose support changes with action.

Assuming that the outcome support is constant, any contract which depends only on x can be dominated by a contract depending on x and a verifiable independent random variable y whose density function changes with action (see Holmström (1979), Harris-Raviv (1979)).

In fact, in assuming a constant outcome support, one can use a contract depending on x and on a random variable z, independent with x, whose support $[\underline{z}(a),\bar{z}(a)]$ is such that $\underline{z}'(a)>0$, to achieve a first-best allocation. This assertion can be proved with similar techniques to those used in this paper. If W is linear the optimal contract $\hat{S}$ is dichotomous.

In the continuous case, when θ is a continuous random variable with a positive density function at some point in R^*,the penalty C always exists. This due to condition $p_{\_}'(a^*) < 0$ in Lemma 3.

This condition can be interpreted in a way which helps to generalize and clarify the result in Brown-Miller-Thornton (1987). They assume an outcome's density function f(x;a), with distribution function F(x;a), such that $\underline{x}'(a)>0$, where $\underline{x}(a)$ is the support lowest point of F(x;a). They also assume that the agent's utility function is additively separable i.e. G(s)-V(a) with $G'>0$, $G''<0$, $V'>0$, and the principal is risk neutral. In this case, if $-V'(a) / F_a(\underline{x}(a^*);a)$ is bounded from above then a first-best allocation can be achieved.

The preceding condition implies $p_{\_}'(a^*)<0$ if, in our model, we assume that $F(x;a)=Pr(X(a,\theta) \leq x)$ allows for a density function f(x;a). In fact, $f(\underline{x}(a^*);a^*)>0$ and $\underline{x}'(a^*)>0$ both implys the existence of C in the continuous case.

Finally, let us summarize the results of this paper.

In some pure moral hazard situations, a first-best allocation can be achieved.

In our model, the outcome is a function of the agent's action and the random state. Because of this, the principal is able to force the agent's decision by means of a contract constructed via a first-best solution. By offering such a contract, actions can be discriminated according to the outcome.

In practice, the payment rules we analyze are , in terms of utility, such that the principal always pays the agent his reservation utility, but imposes a penalty when he observes a less-than-desirable outcome. The existence of the penalty depends basically on the outcome function. Such a mechanism provides an efficient outcome even if the principal is risk adverse.

When the principal is risk neutral, the efficient contract is dichotomous. However, we prove that a first-best allocation can be achieved through such a dichotomous contract if and only if the principal is risk neutral for returns in a specific interval.

APPENDIX

Proof of Proposition 1.

□ ∀ a<a* we have that $EU(\hat{S},a) = \int_{A(a)} U(\hat{S}(X(a,\theta)) dF(\theta) + p(a)U(m,a) =$

$$= H(a) + p(a)U(m,a) < \bar{U}. \text{ Then}$$

$$\frac{H(a)-\bar{U}}{p(a)} < -U(m,a) \ \forall \ a \in [0,a^*[$$ and it follows that C exist.■

Proof of Lemma 2.

□ From Lemma 1 it follows that $\forall a>a^* \quad EU(\hat{S},a)< \bar{U} = EU(\hat{S},a^*)$.

For a<a* we have that

$$EU(\hat{S},a) = H(a) + p(a)U(m,a) \le H(a) + p(a)U(m,0) = H(a)-p(a)C \le$$

$$\le H(a) - p(a)\frac{H(a)-\bar{U}}{p(a)} = \bar{U}.■$$

Proof of Lemma 3.

□ Let us suppose that (C2) is true: $\exists \ \theta^* \in R^*$ such that F is C^1 in a neighborhood of θ^*, with $F'(\theta^*) > 0$.

Only three cases are possible:

Case 1. $\exists \ [\theta_1,\theta_2] \subset \Theta \ / \ \theta^* \in [\theta_1,\theta_2] \subset R^*$

Case 2. $\exists \ \eta>0 \ / \ X_\theta(a^*,\theta) < 0 \quad \forall \ \theta \in \ ]\theta^*-\eta \ , \ \theta^*[$
$\exists \ F'(\theta) > 0 \quad \forall \ \theta \in \ ]\theta^*-\eta \ , \ \theta^*[$

Case 3. $\exists \ \eta>0 \ / \ X_\theta(a^*,\theta) > 0 \quad \forall \ \theta \in \ ]\theta^* \ , \ \theta^*+\eta[$
$\exists \ F'(\theta) > 0 \quad \forall \ \theta \in \ ]\theta^* \ , \ \theta^*+\eta[$

Assume that *Case 1* holds.

Then, $\forall\ a<a^*$ $\quad X(a,\theta) < X(a^*,\theta) \quad \forall\ \theta \in [\theta_1,\theta_2]$

It follows that $p(a) \geq Pr(\ \theta_1 \leq \theta \leq \theta_2) = F(\theta_1)-F(\theta_2) > 0 \quad \forall\ a<a^*$.

Then $p'_-(a^*)=-\infty < 0$ holds.

Assume that *Case 2* is true.

Let $\{a_n\}$ be a strictly increasing sequence with limit a*. It is true that

$$\exists\ n_0\ /\ \forall\ n\geq n_0 \quad \exists\ \theta \in\ ]\ \theta^*-\eta\ ,\ \theta^*\ [\ :\ X(a_n,\theta)=\underline{x}(a^*)$$

For $n\geq n_0$ let be $\theta_n=\max\{\ \theta \in\ ]\ \theta^*-\eta\ ,\ \theta^*\ [\ \ /\ \ X(a_n,\theta)=\underline{x}(a^*)\ \}$

Since $\theta_n < \theta^*$ $\quad \forall\ n\geq n_0$ $\quad$ it follows that

$$\forall\ n\geq n_0 \quad \text{if} \quad \theta_n < \theta < \theta^* \quad \text{then} \quad X(a_n,\theta) < \underline{x}(a^*)$$

and moreover $\quad \{\theta_n\}\uparrow\theta^*$ for $n\geq n_0$.

This situation is represented in Figure 3.

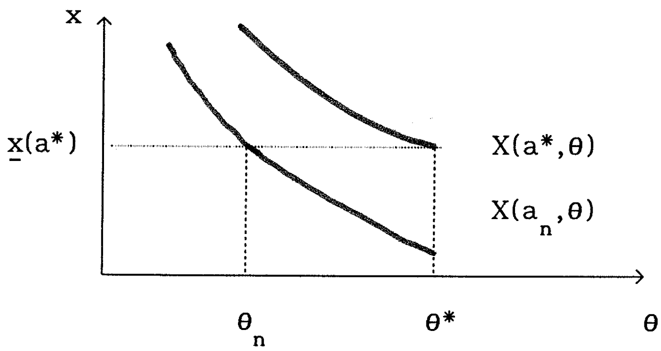

Figure 3 (Case 2)

Then, $\quad \forall\ n\geq n_0 \quad p(a_n) \geq Pr(\ \theta_n < \theta < \theta^*\ )=$

$$=F(\theta^*)-F(\theta_n) = F'(\bar{\theta}_n).(\theta^*-\theta_n) \quad \text{with} \quad \bar{\theta}_n \in\ ]\ \theta_n,\ \theta^*\ [$$

and we have that

$$\frac{p(a_n)}{a_n - a^*} \leq F'(\bar{\theta}_n)\frac{\theta^* - \theta_n}{a_n - a^*} \qquad \forall\ n \geq n_0 \tag{1}$$

By Taylor's Theorem it follows that

$$\forall\ n\geq n_0 \quad \exists\ \hat{a}_n \in\ ]\ a_n,\ a^*\ [,\quad \exists\ \hat{\theta}_n \in\ ]\ \theta_n,\ \theta^*\ [\quad \text{such that}$$

$$0 = X(a_n,\theta_n)-X(a^*,\theta^*) = X_a(\hat{a}_n,\hat{\theta}_n).(a_n-a^*) + X_\theta(\hat{a}_n,\hat{\theta}_n).(\theta_n-\theta^*)$$

This implies $\quad X_\theta(\hat{a}_n,\hat{\theta}_n) < 0 \quad \forall\ n\geq n_0 \quad$ and

$$\frac{\theta^* - \theta_n}{a_n - a^*} = \frac{X_a(\hat{a}_n,\hat{\theta}_n)}{X_\theta(\hat{a}_n,\hat{\theta}_n)} \tag{2}$$

Since $\lim_n X_a(\hat{a}_n,\hat{\theta}_n)=X_a(a^*,\theta^*) > 0$ and $\lim_n X_\theta(\hat{a}_n,\hat{\theta}_n)=X_\theta(a^*,\theta^*) \leq 0$

from (2), it follows that $\lim_n \dfrac{\theta^* - \theta_n}{a_n - a^*}$ is a negative real number or is $-\infty$.

And from (1), it is true that $\quad \lim_n \dfrac{p(a_n)}{a_n - a^*} < 0.$

Since the sequence $\{a_n\}$ is arbitrary , it follows that

$$p'_-(a^*) < 0.$$

If *Case 3* holds, a similar line of argument proves $p'_-(a^*)<0$. This situation is represented in Figure 4.■

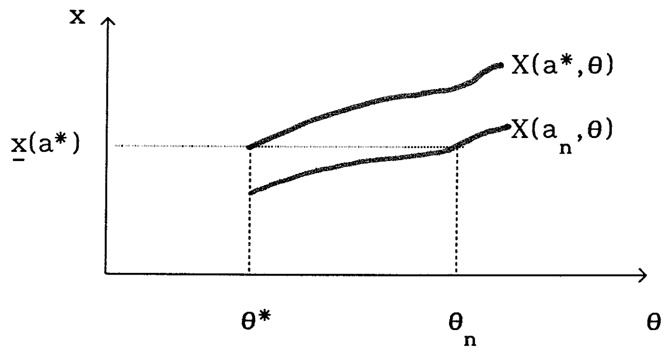

Figure 4 (Case 3)

REFERENCES

Brown-Miller-Thornton (1987). *An Optimal Incentive Scheme for Planning with Targets.* Journal of Comparative Economics,11,596-600.

Harris-Raviv (1979). *Optimal Incentive Contracts with Imperfect Information.* Journal of Economic Theory, 20, 231-259.

Harris-Raviv (1978). *Some results on incentive contracts with application to education and employment, health insurance and law enforcement.* Amer. Econ. Rev. 68, 20-30.

Mirrlees (1974). *Notes on Wefare Economics, Information and Uncertainty,* in Balch, McFaden and Wu eds. Essays on Economic Behavior under Uncertainty, Amsterdam North-Holland Publishing Co.

Holmström (1979). *Moral Hazard and Observability.* Bell Journal of Economics, Vol.10,No.1, 74-91.

Luenberger (1969). **Optimization by vector space methods.** John Wiley & Sons, Inc.

Shavell (1979). *Risk Sharing and Incentives in the Principal-Agent relationship.* Bell Journal of Economics, 10, 55-73.

Singh (1984). *Moral Hazard with a finite number of states.* Economics Letters, 16, 45-51.

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