A DEMAND FUNCTION FOR PSEUDOTRANSITIVE PREFERENCES*

Josep E. Peris and Begoña Subiza**

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A B S T R A C T

This paper deals with the existence and properties of the demand correspondence when agents' preferences are pseudotransitive. It is shown that a consumption plan belongs to the demand mapping if and only if it is a maximizer of a real-valued weak utility function. Further properties, as hemicontinuity and convex-valuedness of the demand mapping, are also analyzed.

KEYWORDS: Pseudotransitive Preferences, Demand Function, Weak Utility Function.

1. INTRODUCTION

A standard configuration in classical microeconomic theory consists of a consumer whose consumption set X is a compact and convex subset of $\mathbb{R}^n$ ordered by a convex preference relation (the upper contour sets are convex sets), and who has an initial wealth $w \in \mathbb{R}_+$. For a given price vector $p \in \mathbb{R}^n$, the consumption bundles available to the consumer lie in his budget set $\beta(p,w) = \{ x \in X \mid p \cdot x \leq w \}$. In general, the budget set $\beta(p,w)$ could be empty, but this difficulty can be remedied by considering $S = \{ (p,w) \in \mathbb{R}^n \times \mathbb{R} \mid \beta(p,w) \neq \emptyset \}$ and the mapping $\beta\colon S \twoheadrightarrow X$, called the budget correspondence. By construction this mapping is nonempty and compact valued.

For given values of (p,w), the demand set is the set of all maximal elements of $\beta(p,w)$ with respect to the preference relation. Thus the demand correspondence is given by the mapping $\gamma\colon S \twoheadrightarrow X$ defined as

$$\gamma(p,w) = \{x \in \beta(p,w) \mid x \succsim y, \ \forall\, y \in \beta(p,w) \}$$

When the preference-indifference relation $\succsim$ is representable by means of a utility function u(.), this demand can be expressed as

$$\gamma(p,w) = \{ x \in \beta(p,w) \mid u(x) \text{ is maximum on } \beta(p,w) \}$$

and then the consumer's problem could be formulated as

$$\max\ u(x), \quad x \in \beta(p,w).$$

The formulation of the problem turns out to be much easier to handle, when X is a topological space and u(.) is a continuous function which represents a complete preorder. We know that a non-trivial preference relation, defined over a connected subset of $\mathbb{R}^n$, is representable by a continuous utility function if and only if it is transitive and continuous (Debreu [2] shows that a preference relation is representable by a continuous utility function when it is a complete preorder, and Schmeidler [7] proves that a non-trivial preference relation is complete whenever it is transitive and continuous). But, in other case, this utility function doesn't exist.

Some additional properties of the demand correspondence are very important both in themselves and also because they play an important role in the proof of the existence of competitive equilibria. These properties are: upper hemicontinuity, compactness and convexity of the images $\gamma(p,w)$. Next theorem summarizes the chief properties of the demand mapping, when the preference relation is a preorder.

Theorem 1. (Debreu, [3])

Let $\succsim$ be a convex preference-indifference relation defined on a compact and convex subset X of $\mathbb{R}^n$, and suppose that there exists a continuous utility function $u\colon X \longrightarrow \mathbb{R}$ representing $\succsim$. Let the budget function be $\beta\colon S \twoheadrightarrow X$ defined as $\beta(p,w) = \{ x \in X \mid p \cdot x \leq w \}$ where

$$S = \{ (p,w) \in \mathbb{R}^n \times \mathbb{R} \mid \beta(p,w) \neq \emptyset \}.$$

If $\beta(p,w)$ is continuous in (p,w) then the demand function $\gamma: S \twoheadrightarrow X$ defined as

$$\gamma(p,w) = \{ x^* \in \beta(p,w) \mid u(x^*) \text{ is maximum on } \beta(p,w) \}$$

is upper hemicontinuous in (p,w), and $\gamma(p,w)$ is compact, convex and nonempty.

For more general preference relations there exist weaker numerical representations which generalize the utility functions (e.g., interval representations). A weak representation is given by a real function u(.) such that $x \succ y$ implies $u(x) > u(y)$. This kind of representation exists for a wide class of preferences (see for instance [6]). The maxima of this function are also $\succsim$-maximal elements. Nevertheless, in general, not all the $\succsim$-maximal elements are maxima of the function, as we can see with the following example.

EXAMPLE 1.- Let $\succ$ be the relation defined on $X = [0,100]$ as $x \succ y \iff x-1 > y$; the function $u(x) = x$ is a weak representation. Note that the set of $\succsim$-maximal elements is $[99,100]$ while the only point that maximizes the function u(.) is $x^* = 100$.

In this case then, with numerical representations such as these, when is it possible to express the demand function as in Theorem 1 and to obtain a similar result for weaker preferences?

This paper provides an answer to this question for pseudotransitive preferences by using a weak numerical representation obtained from the interval representation presented in [1]. Pseudotransitive preferences were introduced by Fishburn [4] to generalize the notion of semiorder. Both kinds of preferences deal with the idea of imperfect discrimination. The example provided by Luce [5] about a cup of coffee with different, but similar amount of sugar, suggests that, if the indifference were transitive, then a subject would be unable to any sugar concentration difference.

The paper is organized as follows. Section 2 presents the necessary definitions and the standard result for pseudotransitive preferences. Section 3 provides us with a weak representation which nevertheless characterizes maximal elements. A few final comments in Section 4 conclude the work.

2. DEFINITIONS

Let X denote a set of alternatives; let $\succ$ denote a preference relation [i.e., an asymmetric binary relation] on X; and let $\succsim$ be the completion of $\succ$ [i.e., $x \succsim y$ means that $y \succ x$ does not hold]. We say that $\succ$ is **pseudotransitive** when $x \succ y \succsim z \succ t \Rightarrow x \succ t$.

A real function $u: X \longrightarrow \mathbb{R}$ is a **weak utility function** for the relation $\succ$ if $x \succ y \Rightarrow u(x) > u(y)$.

From a binary relation $\succ$ it is possible to derive two new binary relations on X as follows:

$$x \succ_1 y \iff \exists\, z \in X \mid x \succsim z \succ y$$

$$x \succ_2 y \iff \exists\, z \in X \mid x \succ z \succsim y$$

Observe that, when the initial relation is pseudotransitive, both relations $\succsim_1$ and $\succsim_2$ are preorders.

A binary relation defined on a topological space X is **continuous** if for all $x \in X$ the lower and upper contour sets, $L(x) = \{ y \in X \mid x \succ y \}$, $U(x) = \{ y \in X \mid y \succ x \}$, are open.

A relation defined on a convex set X is said to be **convex** if for all $x \in X$ the set $\{ y \in X \mid y \succsim x \}$ is a convex set.

The relation $\succ$ defined on X is called **strongly separable** (see [1]) if there exists a countable set $A \subseteq X$ such that

$$\forall\, x,y \in X \mid x \succ y \Rightarrow \exists\, a,b \in A : x \succ a \succsim b \succ y$$

The next result gives us necessary and sufficient conditions for the existence of a continuous numerical representation of pseudotransitive relations by using two real functions.

Theorem 2. (Chateauneuf, [1])

Let X be a connected topological space endowed with an asymmetric binary relation $\succ$. Then two continuous functions $u,v: X \longrightarrow \mathbb{R}$ exist such that $x \succ y \iff v(x) > u(y)$, if and only if,

(1) $\succ$ is pseudotransitive

(2) $\succ_1$ and $\succ_2$ are continuous

(3) $\succ$ is strongly separable

Two continuous real-valued functions, like those in the above theorem, are called an interval continuous representation of $\succ$. Note that, to each element $x \in X$ can be associated a real interval $I(x) = [v(x),u(x)]$ in such a way that $I(x)$ and $I(y)$ intersect if an only if these elements are indifferent and when $x \succ y$ then the interval $I(x)$ is on the right of the interval $I(y)$.

An additional important fact is that when there exist representations such as these, then one of them can be chosen so that u(.) and v(.) are utility functions for, respectively, the complete preorders $\succsim_1$ and $\succsim_2$ (see [1]). Finally note that these functions are also weak utility functions for the relation $\succ$. Nevertheless, the maxima of these functions do not characterize the maximal elements (as can be see in the example 1), and this representation must be modified in order to achieve this goal.

3. A WEAK UTILITY FUNCTION CHARACTERIZING MAXIMAL ELEMENTS

In the next result we use the previous numerical representation in order to prove the existence of a continuous weak utility function characterizing the $\succsim$-maximal elements.

Theorem 3.

Let X be a connected topological space endowed with a preference relation $\succ$ such that

(1) $\succ$ is pseudotransitive

(2) $\succ_1$ and $\succ_2$ are continuous

(3) $\succ$ is strongly separable

Then for any compact subset $D \subseteq X$, there exists a continuous weak utility function $u_D: D \longrightarrow \mathbb{R}$ such that

$$x^* \in D \text{ is } \succsim\text{-maximal on } D \quad \Longleftrightarrow \quad x^* \text{ maximizes } u_D(x) \text{ on } D$$

Proof:

From Theorem 2, there is a continuous representation $u,v: X \longrightarrow \mathbb{R}$ such that $u(.)$ and $v(.)$ are utility functions for the preorders $\succsim_1$ and $\succsim_2$ respectively. If we take the set $S_D = \{ y \in D \mid \exists\, z \in D \,,\, z \succ y \}$ as $\succ$ is transitive and continuous (see Chateauneuf, [1]), the set of $\succsim$-maximal elements on D, $M_D = D - S_D$, is compact and nonempty. To see this, note first that $S_D = \bigcup_{z \in D} \{ y \in D \mid z \succ y \}$ is open in D. If $M_D = \emptyset$, then $D = S_D$ and

$$D = \bigcup_{i=1}^{n} \{ y \in D \mid z_i \succ y \}$$

since D is compact. For each z_i there is z_j such that $z_j \succ z_i$, which is a contradiction with the transitivity of the relation. Then, being M_D compact and non empty, there exists $y^* \in D$ such that

$$u(y^*) = \text{minimum } u(y), \ y \in M_D$$

Let now $u_D: D \longrightarrow \mathbb{R}$ be the function defined as

$$u_D(y) = \begin{cases} u(y) & \text{if } y \in S_D \\ u(y^*) & \text{if } y \in M_D \end{cases}$$

To prove that this function characterizes the maximal elements on D, notice first that max $u_D(y) = u(y^*)$ and if $x^* \in D$ is a $\succsim$-maximal element on D, $u_D(x^*) = u(y^*)$. On the other hand, if $y \in D$ is not $\succsim$-maximal, there exists some $z \in D$ such that $z \succ y$, and then $y^* \succsim z \succ y$. Thus,

$$y^* \succ_1 y \text{ and } u(y^*) > u(y) = u_D(y)$$

so the function $u_D(.)$ characterizes the $\succsim$-maximal elements on D. This function is obviously a weak utility function. To see that $u_D(.)$ is continuous, let $\alpha \in \mathbb{R}$, then

If $\alpha \leq u(y^*)$

$$u_D^{-1}(\leftarrow,\alpha) = \{ y \in D \mid u_D(y) < \alpha \} =$$
$$= \{ y \in D \mid u(y) < \alpha \} = \{ x \in X \mid u(x) < \alpha \} \cap D$$

If $\alpha > u(y^*)$

$$u_D^{-1}(\leftarrow,\alpha) = D$$

If $\alpha \geq u(y^*)$

$$u_D^{-1}(\alpha,\rightarrow) = \{ y \in D \mid u_D(y) > \alpha \} = \emptyset$$

If $\alpha < u(y^*)$,

$$u_D^{-1}(\alpha,\rightarrow) = \{ y \in D \mid u(y) > \alpha \} = \{ x \in X \mid u(x) > \alpha \} \cap D$$

all of them are open sets in D because u(.) is a continuous function. Thus $u_D(.)$ is continuous.

■

Note that we obtain a numerical representation characterizing the maximal elements in any compact subset D, but the functions $u_D(.)$ and $u_C(.)$ are, in general, different when D and C are two different compact subsets of X. The function $u_D(.)$ coincides with u(.) except for the maximal elements in which case $u_D(.)$ is constant. Thus, $u_D(.)$ and $u_C(.)$ coincide for all x such that x is not maximal on these sets.

Also note that under the hypotheses of Theorem 3, the consumer's problem can be expressed as max $u_X(x)$, $x \in X$, provided that X is a compact set. We can then obtain a result similar to that in Theorem 1 for pseudotransitive preferences.

By using the weak utility function which characterizes the maximal elements obtained in Theorem 3, the next result proves that in this case the demand mapping has identical properties to those in the preorders' case.

Theorem 4.

Let $X \subseteq \mathbb{R}^n$ be a convex and compact set endowed with a convex preference relation $\succ$ such that this relation has a continuous representation by means of two functions u,v: $X \longrightarrow \mathbb{R}$. If the budget correspondence β: $S \twoheadrightarrow X$ is continuous in $(p,w) \in S$, then the demand function γ: $S \twoheadrightarrow X$ defined as

$$\gamma(p,w) = \{ x \in \beta(p,w) \mid x \text{ is } \succ\text{-maximal on } \beta(p,w) \}$$

satisfies

a) $\gamma(p,w)$ is a nonempty and compact set

b) γ is upper hemicontinuous

c) $\gamma(p,w)$ is a convex set

Proof:

a) Under the above hypotheses the relation $\succ$ is transitive and continuous (see [1]), so that $\gamma(p,w)$ is nonempty and compact.

b) In order to see that γ is upper hemicontinuous it is sufficient to prove that if $\{(p^k,w^k)\}$ is a sequence convergent to (p,w), and if $x^k \in \gamma((p^k,w^k))$ then there is a subsequence of $\{x^k\}$ whose limit belongs to $\gamma(p,w)$.

Being X a compact set, there is a convergent subsequence, namely $\{x^r\}$, such that $\lim x^r = x \in X$. From the continuity of β, $x \in \beta(p,w)$. Suppose now that $x \notin \gamma(p,w)$; then there is some $y \in \beta(p,w)$ such that $y \succ x$ and therefore $v(y) > u(x)$.

On the other hand, as β is lower hemicontinuous, there is a sequence $\{y^k\}$ convergent to y, such that $y^k \in \beta((p^k,w^k))$. The continuity of functions u(.) and v(.) implies that $\lim u(x^r) = u(x)$ and $\lim v(y^k) = v(y)$. It is possible then to find some r_0 such that, for all $r \geq r_0$, $v(y^r) > u(x^r)$ in spite of the fact that $x^r \in \gamma((p^r,w^r))$. Thus $x \in \gamma(p,w)$ and γ is upper hemicontinuous on S.

c) First, we shall prove that the preorder $\succsim_1$ is a convex relation; in order to see this, note that if $x \succsim_1 y$ ($x \succsim_1 y \Leftrightarrow \forall$ w such that $y \succsim w$ then

$x \succsim w$), and $\lambda \in [0,1]$, then $\lambda x + (1-\lambda)y \succsim_1 y$, because for any $w \in X$ such that $y \succsim w$ then, $x \succsim w$ and by the convexity of the relation $\lambda x + (1-\lambda)y \succsim w$ which implies $\lambda x + (1-\lambda)y \succsim_1 y$. Now, let $x,y,w \in X$ be such that $x \succsim_1 w$, $y \succsim_1 w$. Suppose, without loss of generality, that $x \succsim_1 y$. Then, for all $\lambda \in [0,1]$, $\lambda x + (1-\lambda)y \succsim_1 y \succsim_1 w$, with $\succsim_1$ being a preorder, this implies that

$$\lambda x + (1-\lambda)y \succsim_1 w$$

and $\succsim_1$ is a convex relation.

From Theorem 3, the set $\gamma(p,w)$ can be written as

$$\gamma(p,w) = M_D = \{ y \in D \mid y \text{ maximizes the function } u_D(x) \text{ on } D \}$$

where $D = \beta(p,w)$. Then, $M_D = \{ x \in X \mid u(x) \geq u(y*) \} \cap D$ and

$$u(y*) = \text{minimum of } u(y),\ y \in M_D$$

Thus in order to see the convexity, it is enough to prove that the function u(.) is quasiconcave on X, which is evident because it is the utility function of the convex preorder $\succsim_1$.

■

Note that, if $\succsim$ is a preorder, then $\succsim = \succsim_1 = \succsim_2$ and then u(.) and v(.) are utility functions which represent $\succsim$. Thus, Theorem 4 is a generalization of Theorem 1.

The hemicontinuity of the demand correspondence γ can also be obtained by using the result in Walker [8], in which for all open binary relations, the demand mapping turns out to be upper hemicontinuous and compact-valued, although in this case $\gamma(p,w)$ may be empty.

4. FINAL COMMENTS

In this paper we have found, for a pseudotransitive relation $\succ$ defined on X, a weak utility function u(.) such that in any compact subset D there is a function u_D: $D \longrightarrow \mathbb{R}$ satisfying the following conditions:

(a) $u_D(.)$ characterizes the maximal elements in D

(b) if $z \in D$ is not maximal then $u_D(z) = u(z)$

(c) for any $x \in D$, $u\ (x) \leq u(x)$

Let us finally address the following question: since weak utility functions exist for quasitransitive relations (see [6]), would it be possible to construct in this case a function $u_D(.)$ in any compact subset satisfying conditions (a), (b) and (c)?. The answer is no, unless $\succ$ is pseudotransitive. To see this, let x, y, z, t $\in$ X such that

$$x \succ y \succsim z \succ t$$

Then, if $t \succ x$ the only possibility, being $\succ$ quasitransitive, is

$$x \sim t,\ x \sim z,\ y \sim t,\ y \sim z$$

Taking the set D = {x,y,t}, the elements x and t are maximals, and then

$$u(y) = u_D(y) < u_D(t) \leq u(t)$$

Taking C = {y,z,t}, the elements y and z are maximals, and then

$$u(t) = u_C(t) < u_C(y) \leq u(y)$$

which is a contradiction. Thus, $x \succ t$, and the relation $\succ$ is pseudotransitive.

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