EFFICIENT SOLUTIONS FOR BARGAINING PROBLEMS WITH CLAIMS*

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A B S T R A C T

Two solution concepts have been proposed for bargaining problems with claims, namely, the proportional solution, introduced by Chun & Thomson (1992), and the extended claim-egalitarian solution, [see Bossert (1993) and Marco (1993)]. Neither of these solutions exhaust all possible gains from cooperation. In this paper we introduce and provide axiomatic characterizations of different lexicographic extensions of the aforementioned proposals, which provide efficient solutions.

Keywords: Bargaining problems with claims; lexicographic extensions

1.- INTRODUCTION

Bargaining problems with claims have been introduced in a recent paper by Chun & Thomson (1992). They propose a generalization of Nash's (1950) bargaining model by adding a third element to the disagreement point and the feasible set. This element is a point representing the claims of the agents, which should be taken into account when selecting an outcome.

These authors following the axiomatic approach, formulate requirements the solutions should meet and search for rules satisfying these requirements; therefore, a solution concept is defended from a normative point of view.

Two solution concepts have been introduced for bargaining problems with claims[1]:

Chun & Thomson (1992) proposed the proportional solution. This solution chooses the point on the weak Pareto frontier of the feasible set for which agents' utility gains are proportional to the utility gains of the claims point.

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1 We do not consider the claim-egalitarian solution, recently presented by Bossert (1993), since it could yield outcomes that do not weakly dominate the disagreement point and so, the agents will not agree on it because it could always guarantee for themselves the disagreement utility level.

Bossert (1993) presented the extended claim-egalitarian solution. which equalizes the losses from the claims point, whenever it represents an acceptable agreement for all agents. If there are some agents who, at the equal-loss point, are below their status-quo, it keeps these agents at their disagreement level, and only equalizes losses from the claims point for those agents who do not reach their disagreement utility level in the equal-decreasing procedure.

Chun & Thomson (1992) provided several characterizations of the proportional solution. The extended claim-egalitarian solution has been characterized by Bossert (1993) and Marco (1993).

Neither the proportional solution nor the extended claim-egalitarian solution exhaust all possible gains from cooperation; that is, they do not necessarily provide Pareto optimal outcomes. In order to ensure full efficiency, we introduce the lexicographic extension of both of them. We call these extensions the lexicographic proportional solution and the lexicographic extended claim-egalitarian solution.

These lexicographic extensions can be viewed as adaptations to the bargaining problem with claims of the lexicographic Kalai-Smorodinsky solution [see Imai (1983)] and the rational lexicographic equal-loss solution [see Herrero & Marco (1993)], respectively.

Guaranteeing *Pareto optimality* of a solution is always done at the cost of several other properties, among which the most obvious one is *Continuity*. In the bargaining with claims case, if we require full efficiency, the property *Boundedness by claims* also fails to be satisfied [see Chun & Thomson (1992)]. This condition asks the solution not to recommend alternatives above the claims point.

In order to obtain solutions that do not violate *Boundedness by claims* and are as efficient as possible, we introduce a partial efficiency condition: *Restricted Pareto optimality*, which demands the solution to exhaust all possible gains from cooperation on the set of alternatives claimed by agents. Neither the proportional nor the extended claim-egalitarian solutions satisfy this condition. We then suggest, to ensure partial efficiency, two new solution concepts: the restricted lexicographic proportional solution and the restricted lexicographic extended claim-egalitarian solution, which simply are the lexicographic extensions of the proportional and the extended claim-egalitarian solutions, respectively, on the part of the feasible set dominated by the claims point.

Section 2 contains some preliminaries. In Section 3 we introduce the lexicographic proportional solution and the lexicographic extended claim-egalitarian solution. Section 4 is devoted to the *restricted Pareto optimality* condition of both the proportional and the extended claim-egalitarian solutions. Section 5 provides some concluding remarks. Proofs are relegated to an Appendix.

2.- PRELIMINARIES

A n-person bargaining problem with claims is a triple (S,d,c) where S is a subset of $\mathbb{R}^n$, d is a point in S and c is a point in $\mathbb{R}^n \setminus int(S)$[2]. $\mathbb{R}^n$ is the utility space, S the feasible set, d the disagreement point and c the claims point. The intended interpretation of (S,d,c) is as follows: the agents can achieve any point in S if they agree on it unanimously; the point d is an alternative at which the agents end up in the case of no agreement; each coordinate of the claims point represents a promise made to the corresponding agent, and they should be taken into account in the determination of the final compromise.

Let $\sum^n$ be the class of n-person bargaining problems with claims (S,d,c) such that:

(i) S is convex, closed and comprehensive[3],

(ii) $\exists\ p \in \mathbb{R}^n_{++},\ r \in \mathbb{R}$ such that $\forall\ x \in S,\ px \leq r$,

(iii) $\exists\ x \in S$ with $x >> d$,

(iv) $c \notin int(S),\ c > d$.

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2 Given $S \subseteq \mathbb{R}^n$, int(S) is the interior of S.

3 Vector inequalities: given $x,y \in \mathbb{R}^n$, $x \geq y$, $x > y$, $x >> y$.
$S \subseteq \mathbb{R}^n$ is comprehensive if, $\forall\ x \in S$ and $\forall\ y \in \mathbb{R}^n$, $y < x$ implies $y \in S$.

The class of problems considered here is more general that the ones introduced before, since they require $c \notin S$ instead of $S \notin int(S)$ [see Chun & Thomson (1992) Bossert (1992, 1993) and Marco (1993)].

For any $(S,d,c) \in \sum^n$, we use IR(S,d) to denote the set of individually rational points, $IR(S,d) = \{ x \in S \mid x \geq d \}$. C(S,c) will be the set of claimed alternatives, $C(S,c) = \{ x \in S \mid x \leq c \}$. PO(S) will denote the set of Pareto optimal points, $PO(S) = \{ x \in S \mid y > x \Rightarrow y \notin S \}$ and WPO(S) the set of weakly Pareto optimal points, $WPO(S) = \{ x \in S \mid y >> x \Rightarrow y \notin S \}$.

We denote by a(S,d) the ideal point, the point that gives each agent the maximal attainable utility level subject to the condition that all agents achieve at least the utility level of the disagreement point, $a_i(S,d) = \max \{ x_i \mid x \in IR(S,d) \} \; \forall i \in N$.

A solution on $\sum^n$ is a function $F: \sum^n \longrightarrow \mathbb{R}^n$ that associates to each $(S,d,c) \in \sum^n$ a unique point of S, F(S,d,c), called the solution outcome of (S,d,c). A solution can be interpreted as an agreement made by the agents or, alternatively, as a recommendation of an impartial arbitrator.

The following definitions present the solution concepts which have been considered for the class $\sum^n$.

Definition 1 [Chun & Thomson (1992)]: For all $(S,d,c) \in \sum^n$, <u>the proportional solution</u>, P, is the maximal point of S on the segment connecting d and c.

This solution is closely related to the Kalai-Smorodinsky solution for the classical bargaining problem [Kalai & Smorodinsky (1975)].

The next solution is an adaptation to the class of bargaining problems with claims of the rational equal-loss solution, introduced by Herrero & Marco (1993) for the classical bargaining problem. In order to present it we need some additional notation.

For $A \subseteq \mathbb{R}^n$, Com(A) denotes the comprehensive hull of set A, and ComCo(A) the comprehensive and convex hull of set A. Given $(S,d,c) \in \Sigma^n$, let $S^* = \text{Com}(IR(S,d))$, and let E(S,d,c) be the alternative in S which minimizes agents' losses from their claims, subject to these losses being equal for all agents.

Definition 2: [Marco (1993)]: For all $(S,d,c) \in \Sigma^n$, <u>the extended claim-egalitarian solution</u>, EE, chooses the alternative with coordinates:

$$EE_i(S,d,c) = \begin{cases} d_i & \text{if} \quad E_i(S^*,d,c) < d_i \\ E_i(S^*,d,c) & \text{if} \quad E_i(S^*,d,c) \geq d_i \end{cases}$$

It is easy to check that this definition is equivalent to that appearing in Bossert (1993).

The usual way in which solutions are defended is by looking for those ones that satisfy appealing properties. The following axioms are borrowed from Chun & Thomson (1992), and most of them are just slight reformulations of the corresponding axioms for classical bargaining problems.

(WPO) *Weak Pareto optimality.* For all $(S,d,c) \in \sum^n$, $F(S,d,c) \in WPO(S)$.

(PO) *Pareto optimality.* For all $(S,d,c) \in \sum^n$, $F(S,d,c) \in PO(S)$.

(CONT) *Continuity.* For all sequences $\{(S^\upsilon,d^\upsilon,c^\upsilon)\}$ of elements of $\sum^n$ and for all $(S,d,c) \in \sum^n$, if $S^\upsilon \longrightarrow S$, $d^\upsilon \longrightarrow d$ and $c^\upsilon \longrightarrow c$, then $F(S^\upsilon,d^\upsilon,c^\upsilon) \longrightarrow F(S,d,c)$, (convergence of a sequence of sets is evaluated in the Hausdorff topology).

(AN) *Anonymity.* For all $(S,d,c) \in \sum^n$, and for all permutations π of N, $F[\pi(S),\pi(d),\pi(c)] = \pi(F(S,d,c))$.

(T.INV) *Translation invariance.* For all $(S,d,c) \in \sum^n$, and for all $t \in \mathbb{R}^n$, $F(S + \{t\}, d + t, c + t) = F(S,d,c) + t$.

Let Λ^n be the class of transformations $\lambda: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ defined as follows: for each $i \in N$, there exists $a_i \in \mathbb{R}_{++}$ and $b_i \in R$ such that for all $x \in \mathbb{R}^n$, $\lambda_i(x) = a_i x_i + b_i$.

(S.INV) *Scale invariance.* For all $(S,d,c) \in \sum^n$, and for all $\lambda \in \Lambda^n$, $F(\lambda(S),\lambda(d),\lambda(c)) = \lambda(F(S,d,c))$.

(IR) *Individual rationality.* For all $(S,d,c) \in \sum^n$, $F(S,d,c) \geq d$.

(BC) *Boundedness by claims.* For all $(S,d,c) \in \sum^n$, $F(S,d,c) \leq c$.

(IIIA) *Independence of individually irrational alternatives.* For all (S,d,c), $(S',d',c') \in \sum^n$, if $IR(S,d) = IR(S',d')$, $F(S,d,c) = F(S',d',c')$.

(IUA) *Independence of unclaimed alternatives.* For all (S,d,c), $(S',d',c') \in \sum^n$, if $C(S,c) = C(S',c')$, $F(S,d,c) = F(S',d',c')$.

WPO requires that there is no element in S at which all agents are better off than they are at the solution outcome. PO demands the solution outcome to exhaust all gains from cooperation. CONT says that "small changes" in the problem cause "small changes" in the solution outcome. AN says that the names of the agents do not affect the solution outcome. T.INV requires the choice of origin of the utility functions to be irrelevant. S.INV says that subjecting a problem to a positive linear transformation independent for each agent leads to a new problem that should be solved at the image under this transformation of the original problem's solution outcome. IR says that no agent is worse off at the solution outcome than at the disagreement point. BC requires that no agent to be better off at the solution outcome than at the claims point. IIIA asks the solution not to take the individually irrational points into account. IUA requires that the feasible points with coordinates above the corresponding coordinate of the claims point be ignored.

3.- THE LEXICOGRAPHIC EXTENSIONS OF P AND EE

As Chun & Thomson (1992) show, the proportional solution is not, in general, Pareto optimal. With the aim of guaranteeing full Pareto optimality we consider its lexicographic extension.

Let M be a nonempty subset of N. We denote by e_M the n-dimensional vector which ith coordinate is 1 if $i \in M$ and 0 otherwise.

Let $>^{\ell}$ denote the lexicographic ordering on $\mathbb{R}^n$, i.e., $x >^{\ell} y$ $(x,y \in \mathbb{R}^n)$ if there is an $i \in N$ with $x_i > y_i$ and $x_j = y_j$ for all $j < i$. Let $\alpha: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a function such that for each $x \in \mathbb{R}^n$ there is a permutation π of N with $\alpha(x) = \pi(x)$ and $\alpha_1(x) \leq \alpha_2(x) \leq \ldots \leq \alpha_n(x)$. Then the lexicographic maximin ordering $>^{\ell m}$ on $\mathbb{R}^n$ is defined by $x >^{\ell m} y$ $(x,y \in \mathbb{R}^n)$ if $\alpha(x) >^{\ell} \alpha(y)$.

Definition 3 : For all $(S,d,c) \in \sum^n$, <u>the lexicographic proportional solution</u> is given by $LP(S,d,c) = \lambda^{-1}(x^*)$, where:

$\lambda \in \Lambda^n$ is such that $(S',d',c') = (\lambda(S),\lambda(d),\lambda(c))$ is a problem with $c' = e_N$ and $d' = 0$, and x^* is the maximal element of S' with respect to $>^{\ell m}$.

Notice that (S',d',c') is obtained by choosing an affine positive transformation of the utility functions which is independent for each agent, and in such a way that the proportional solution, P, will be the maximal feasible point with equal coordinates. Now, by starting from this solution outcome, the lexicographic proportional solution will improve as many agents as possible without damaging the rest.

The procedure to find LP(S,d,c) is the following: firstly, increase the utilities of the n agents in $N^1 = N$ from the disagreement point along $d + \zeta^1(c - d)$, $(\zeta^1 > 0)$, until a boundary point is reached, that is x^1. If $x^1 \in PO(S)$, then $x^* \equiv x^1$. Otherwise, let $N^2 \subset N$ be the largest possible subset of agents whose utilities can be equally increased from x^1 along $x^1 + \zeta^2 e_{N^2}$, $(\zeta^2 > 0)$. Let x^2 be the maximal point of S in this direction. If $x^2 \in PO(S)$, then $x^* \equiv x^2$; otherwise continue along $x^2 + \zeta^3 e_{N^3}$ $(\zeta^3 > 0)$, where $N^3 \subset N^2$ is the largest possible subset of agents which utility can be increased along $x^2 + \zeta^3 e_{N^3}$ etc.. After a finite number of steps this procedure yields a point $x^* \in PO(S)$. It can be shown that $x^* = LP(S,d,c)$ by adapting lemma 3 in Imai (1983) to our context.

Let us consider the following notation in order to introduce some properties which we use to characterize axiomatically the lexicographic proportional solution.

For $x \in \mathbb{R}^n$ and $i \in N$, let x_{-i} be the (n-1)-dimensional vector obtained by deleting the ith component of x. For $(S,d,c) \in \sum^n$, let S_{d-i} be the closure of $\{ x_{-i} \mid x \in S,\ x \leq a(S,d) \}$.

(W.MON) *Weak monotonicity*. For all (S,d,c), (S',d',c') $\in \sum^n$, if $S \subseteq S'$, $d = d'$, $c = c'$ and $S_{d-i} = S'_{d'-i}$ for all i, then $F(S',d',c') \geq F(S,d,c)$.

W.MON says that whenever the disagreement point and the claims point remain fixed, if the feasible set expands in such a way that the projection of the feasible points which are below the ideal point to payoffs of players in $N\setminus\{i\}$ does not change for any $j \neq i$, then no agent may be worse off. This axiom is an adaptation of the property "*Combined individual monotonicity*" [introduced by Imai (1983) for the classical bargaining problem] to the class of bargaining problems with claims.

(IADC) *Independence of alternatives other than the disagreement and the claims points*. For all (S,d,c), (S',d',c') $\in \sum^n$, if $S' \subseteq S$, $d = d'$, $c = c'$, and $F(S,d,c) \in S'$, then $F(S',d',c') = F(S,d,c)$.

IADC requires that whenever the disagreement point and the claims point are fixed, if the feasible set shrinks and the solution outcome for the original problem is still feasible for the smaller problem, then the solution outcome for the smaller problem should coincide with that of the original problem. This axiom is an adaptation of the property "*Independence of alternatives other than the disagreement point and the ideal point*" [introduced by Roth (1977) for the classical bargaining problem] to the class of bargaining problems with claims.

The following result is our axiomatic characterization of LP:

Theorem 1 : *The lexicographic proportional solution is the only solution on $\sum^n$ satisfying Pareta optimality, anonymity, scale invariance, weak monotonicity and independence of alternatives other than the disagreement and the claims points.*

It is worth noticing the independence between W.MON and IADC in the presence of the rest of the axioms characterizing LP: it is immediate to show that the lexicographic Kalai-Smorodinsky solution [see Imai (1983)] satisfies PO, AN, S.INV and W.MON, but does not satisfy IADC; on the other hand, PO, AN, S.INV and IADC hold for the Nash solution [see Nash (1950)], whereas W.MON does not.

It is also true that the extended claim-egalitarian solution does not exhaust all possible gains from cooperation either. It only satisfies *Weak Pareta optimality*. In order to ensure *Pareta optimality* we propose its lexicographic extension.

Definition 4 : For all $(S,d,c) \in \sum^n$, the lexicographic extended claim-egalitarian solution is given by $LEE(S,d,c) = z^* - t$, where:

$t \in \mathbb{R}^n$ is such that $(S',d',c')=(S+\{t\},d+t,c+t)$ is a problem with $EE_i(S',d',c') = k \ \forall \ i \in N$ and $c'_i = 0 \ \forall \ i \in N \mid E_i(S,d,c) \geq d_i$, and z^* is the maximal element of S' with respect to $>^{\ell m}$.

Notice that (S',d',c') is obtained by choosing the origin of the utility functions in such a way that the extended claim-egalitarian solution, EE, is the maximal feasible point with equal coordinates. Now, by starting from this, the lexicographic extended claim-egalitarian solution will improve as many agents as possible without damaging the rest.

We can find LEE(S,d,c) with a similar procedure to that used to find LP(S,d,c), but taking S* and starting from E(S*,d,c). $\{z^t\} \subset S$ and $\{Q^t\} \subseteq N$ for t =1,.,T will denote the sequences which appear in this way. It is not hard to show that z*= LEE(S,d,c) by adapting lemma 3 in Imai (1983).

In order to characterize axiomatically the lexicographic extended claim-egalitarian solution, we consider the following condition:

(RIAC) *Rational Independence of alternatives other than the claims point.* For all (S,d,c), (S',d',c') $\in \sum^n$, if $S' \subseteq S$, c = c' and $d \leq d'$, F(S,d,c) $\in$ IR(S',d') implies F(S',d',c') = F(S,d,c).

RIAC requires that if there is no change in the claims point, when the feasible set shrinks and the disagreement point increases in such a way that the solution outcome for the original problem is still feasible and individually rational for the smaller problem, then the solution outcome for the smaller problem should be the same as for the original one. This axiom is a straightforward extension of the *Rational independence of alternatives other than the ideal point* condition [introduced by Herrero & Marco (1993) for classical bargaining problems] to the class of bargaining problems with claims.

Our characterization result for the lexicographic extended claim-egalitarian solution is summarized in the following theorem:

Theorem 2 : *The lexicographic extended claim-egalitarian solution is the only solution on $\sum^n$ satisfying Pareto optimality, anonymity, translation invariance, independence of individually irrational alternatives, weak monotonicity and rational independence of alternatives other than the claims point.*

It is interesting to notice the independence between IIIA and RIAC in the presence of the rest of the axioms characterizing LEE: it is immediate to check that the lexicographic egalitarian solution [see Imai (1983)] satisfies PO, AN, T.INV, W.MON and IIIA, but does not satisfy RIAC; on the other hand, PO, AN, T.INV, W.MON and IIIA hold for the lexicographic claim-egalitarian solution[4], whereas IIIA does not.

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4 This solution has not been proposed for the class $\sum^n$, but it could be done using the standard lexicographic procedure from the claim-egalitarian solution in order to get efficiency. A parallel solution concept to this for the classical bargaining problem is the lexicographic equal-loss solution [see Chun & Peters (1991)].

4.- THE RESTRICTED LEXICOGRAPHIC EXTENSIONS OF P AND EE

Both the lexicographic proportional and the lexicographic extended claim-egalitarian solutions could recommend, to some agents, utility levels above their claims. As Chun & Thomson (1992) pointed out, there is no solution satisfying *Pareto optimality* and *Boundedness by claims* for the class of n-person bargaining problems analyzed here.

Nonetheless, it is straightforward to check in the bargaining problem with claims where $S = ComCo\{(1,4)\}$, $d = (0,0)$ and $c = (2,3)$ that the proportional and the extended claim-egalitarian solutions do not exhaust all possible gains from cooperation on the set of alternatives dominated by the claims point.

As a way of finding "a compromise" between efficiency and *Boundedness by claims*, we suggest two new proposals.

In order to introduce these solution concepts $\hat{S}$ will be used to denote the set of elements claimed by the agents, $\hat{S} = C(S,c) = \{ x \in S \mid x \leq c \}$.

Definition 5 : For all $(S,d,c) \in \sum^n$, <u>the restricted lexicographic proportional solution</u>, LP*, is the lexicographic proportional solution on the set of alternatives claimed by the agents, $LP^*(S,d,c) = LP(\hat{S},d,c)$.

For each problem $(S,d,c) \in \Sigma^n$, in order to find LP*(S,d,c), first we take $\hat{S}$, and we use the process defined to find LP on $(\hat{S},d,c)$. Denote by $\{y^t\} \subset \hat{S}$ and $\{I^t\} \subseteq N$ for $t=1,..,T$ the sequences which appear in this procedure.

Definition 6 : For all $(S,d,c) \in \Sigma^n$, <u>the restricted lexicographic extended claim-egalitarian solution,</u> LEE*, is the lexicographic extended claim-egalitarian solution on the set of alternatives claimed by the agents, $LEE^*(S,d,c) = LEE(\hat{S},d,c)$.

For each problem $(S,d,c) \in \Sigma^n$, we can find LEE*(S,d,c) by using the process defined to find LEE on $(\hat{S},d,c)$. Let $\{v^t\} \subset \hat{S}$ and $\{J^t\} \subseteq N$ for $t = 1,..,T$ denote the sequences which appear in this procedure.

These solutions could be interpreted as "second best" outcomes, since they are Pareto optimal on one part of the feasible set, whereas the lexicographic proportional and the lexicographic extended claim-egalitarian solutions could be viewed as "first best" recommendations because they are Pareto optimal on the entire feasible set.

In order to characterize LP* and LEE* we provide the following axioms:

(POR) *Restricted Pareto optimality.* For all $(S,d,c) \in \Sigma^n$, $F(S,d,c) \in WPO(S)$ and there is no alternative x in $\hat{S}$ such that $x > F(S,d,c)$.

POR requires that there is no feasible point at which all agents are better off than they are at the solution outcome and that the solution outcome exhausts all gains from cooperation on the set of claimed points.

For $(S,d,c) \in \sum^n$, let $\hat{S}_{-i}$ be the closure of $\{ x_{-i} \mid x \in C(S,c) \}$.

(R.MON) *Restricted monotonicity.* For all (S,d,c), $(S',d',c') \in \sum^n$, if $S \subseteq S'$, $d = d'$, $c = c'$ and $\hat{S}_{-i} = \hat{S}'_{-i}$ for all $i \in N$, then $F(S',d',c') \geq F(S,d,c)$.

R.MON says that if the feasible set expands in such a way that the projection of $\hat{S}$ to payoffs of players in $N\setminus\{i\}$ does not change for any $j \neq i$, then no agent may be worse off, as long as the disagreement point and the claims point remain fixed.

The axiomatic characterization of the restricted lexicographic proportional solution is presented in the following theorem:

Theorem 3 : *The restricted lexicographic proportional solution is the only solution on $\sum^n$ satisfying restricted Pareto optimality, anonymity, scale invariance, restricted monotonicity, independence of alternatives other than the disagreement and the claims points and boundedness by claims.*

Notice that we characterize LP (theorem 1) by means of five axioms: PO, AN, S.INV, W.MON and IADC. Our characterization of LP* is made by introducing *Boundedness by claims* , substituting *Pareto optimality* with a weaker requirement, namely, *Restricted Pareto optimality* and asking for *Restricted monotonicity* instead of *Weak monotonicity* .

Our axiomatic characterization of the restricted lexicographic extended claim-egalitarian solution is contained in the following theorem:

Theorem 4 : *The restricted lexicographic extended claim-egalitarian solution is the only solution on* $\sum^n$ *satisfying restricted Pareto optimality, anonymity, translation invariance, independence of individually irrational alternatives, restricted monotonicity, rational independence of alternatives other than the claims point and boundedness by claims.*

We characterize LEE (theorem 2) by means of six axioms: PO, AN, T.INV, IIIA, W.MON and RIAC. In order to characterize LEE* apart from AN, T.INV, IIIA and RIAC, we also consider *Restricted Pareto optimality* and *Restricted monotonicity*.

5.- CONCLUDING REMARKS

In this paper we have proposed and axiomatically characterized, for the bargaining problems with claims, the standard and the restricted lexicographic extensions of both the proportional and the extended claim-egalitarian solutions.

The standard lexicographic extension of a solution ensures efficiency and could recommend the agents utility levels above their claims, while the restricted lexicographic extension of a solution is bounded by the claims and partially efficient.

The following example shows that both extensions might represent a great deal of real situations.

An abandoned house which is in terrible conditions is occupied unlawfully by a group of people who suit it for living. We find different situations as a consequence of this action:

(1) The owner of the house allows the occupants to live in it.

(2) The owner asks the occupants to leave the house and they do it peaceably.

(3) The occupants are forced to go , and they destroy all the improvements they had made in the house before leaving.

In the first situation both parts obtain a benefit: the house is in good condition and the occupants make use of it; clearly, this agreement is above the claims of the occupants and it could be viewed as the lexicographic extension of a solution. In the second case, which could be interpreted as the restricted lexicographic extension of a solution, nobody receives more than they claim: the owner has the right to the improvements of the house and the occupants are not entitled to use it. The last standing represents the disagreement.

The aforementioned example does not correspond to an isolated situation, in fact, it is a somehow generalized way of acting nowadays. This tendency has become so important that the people who follow it have given a new sense to the word " squatters ".

Table 1 summarizes the axiomatic properties of all solutions introduced in this paper, and indicates their characterization results.

	LP	LEE	LP*	LEE*
PO	yes*	yes*	no	no
POR	yes	yes	yes*	yes*
WPO	yes	yes	yes	yes
CONT	no	no	no	no
AN	yes*	yes*	yes*	yes*
S. INV	yes*	no	yes*	no
T. INV	yes	yes*	yes	yes*
IIIA + PO	yes	yes*	yes	yes*
IR	yes	yes	yes	yes
IUA	no	no	yes	yes
BC	no	no	yes*	yes*
W. MON	yes*	yes*	no	no
R. MON	no	no	yes*	yes*
RIAC + IR	no	yes*	no	yes*
IADC	yes*	yes	yes*	yes
RESULTS	Th. 1	Th. 2	Th. 3	Th. 4

* Axioms that are used in characterization results.

TABLE 1

Axiomatic properties discussed in the paper of the lexicographic and the restricted lexicographic extensions of both the proportional and the extended claim-egalitarian solutions

APPENDIX

Proof of theorem 1

It is straightforward to prove that LP satisfies PO, AN and S.INV. Lemmas 1 and 2 prove that it also verifies W.MON and IIAC.

Lemma 1 : LP *satisfies weak monotonicity.*

Proof: Let (S,d,c), $(\bar{S},\bar{d},\bar{c}) \in \sum^n$ be such that $S \subseteq \bar{S}$, $d = \bar{d}$, $c = \bar{c}$ and $S_{d-i} = \bar{S}_{d-i}$ for all $i \in N$. Since LP satisfies S.INV, we may assume that $c = e_N$ and $d = 0$. The proof is done with the help of two steps, which require additional notation.

For $y \in S$, $N(S,y) \equiv \{i \in N|\ y + \zeta e_{(i)} \in S$ for some $\zeta > 0\ \}$. $N(S,y)$ denotes the largest subset of players of N, where utilities could be increased equally from y in S. Let ζ^* be the minimal number such that for all $\zeta > \zeta^*$, $y + \zeta e_{N(S,y)} \notin S$. Finally, let $z(S,y) \equiv y + \zeta^* e_{N(S,y)}$.

step 1: For all $y \in S$, if $N(S,y) \neq \emptyset$ then $N(S,y) = N(\bar{S},y)$.

proof: see lemma 2, claim 1 in Chun & Peters (1991).

step 2: Let $T > 1$ be the final step in finding LP(S,d,c). Also, let $\{x^t\}$ and $\{\bar{x}^t\}$ be the two sequences as defined in the process of finding LP(S,d,c) and $LP(\bar{S},\bar{d},\bar{c})$ respectively. Then for all $t = 1,..,T-1$ $x^t = \bar{x}^t$.

proof: first, we will consider the case where $t = 1$. Since $S \subseteq \bar{S}$, $c = \bar{c}$ and $d = \bar{d}$, $x^1 \leq \bar{x}^1$. We need to show that $x^1 \geq \bar{x}^1$, conclusion that we get by reasoning in the same way as lemma 2, claim 2 in Chun & Peters (1991).

The proofs for $t = 2,..,T-1$ are analogous, using step 1.

Finally, by combining the results of the steps 1 and 2, it follows that $x^T = LP(S,d,c) \leq \bar{x}^T \leq LP(\bar{S},\bar{d},\bar{c})$. Therefore, LP satisfies W.MON.■

Lemma 2 : LP *satisfies independence of alternatives other than the disagreement and the claims points.*

Proof[(5)]: let (S,d,c), $(\bar{S},\bar{d},\bar{c}) \in \sum^n$ be two problems satisfying the hypotheses of IADC, that is, $\bar{S} \subseteq S$, $\bar{d} = d$, $\bar{c} = c$, and $F(S,d,c) \in \bar{S}$. Also let $\{x^t\} \subset S$ be the sequence as defined in the process of finding $LP(S,d,c) = x^T$. Since $x^T \in \bar{S}$, $x^t \leq x^T$ for all t, and $\bar{S}$ is comprehensive, we have $x^t \in \bar{S}$ for all t. Now we construct the sequence $\{\bar{x}^t\} \subset \bar{S}$ to find $LP(\bar{S}.\bar{d},\bar{c})$. Since $\bar{S} \subseteq S$, $\bar{c} = c$ and $x^t \in \bar{S}$ for all t, $\bar{x}^t = x^t$ for all t. Therefore, we conclude that $LP(\bar{S},\bar{d},\bar{c}) = x^T = LP(S,d,c)$.■

Now, we need some notation. Given $p \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$, $H(p,py) \equiv \{ x \in \mathbb{R}^n \mid px \leq py \}$.

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5 This proof is an adaptation of lemma 1 in Chun and Peters (1991) to our context.

Suppose F is a solution satisfying axioms PO, AN, S.INV, W.MON and IADC.

Let $(S',d,c) \in \sum^n$ be given. By S.INV, we assume that $c = e_N$ and $d = 0$. Let $S \equiv \{ x \in S' \mid x \leq a(S',d) \}$, then $a(S',d) = a(S,d)$. Now let $\{x^t\}_{t=1}^T$ and $\{N^t\}_{t=1}^T$ be the sequences as defined in the process of finding LP(S,d,c). We will show that $F(S,d,c) = x^T$. The theorem, then follows from axioms W.MON and PO.

First we construct elementary problems. Let $M^t = N \backslash N^t$ and $p^t = e_{M^t}$ for $t = 1,..,T$ (where $M^1 = \emptyset$ and $p^1 = 0$), and let a^* be the greatest coordinate of a(S,d). Define:

$$\bar{S}^{1,t} \equiv H(e_N, \Sigma x_i^t) \cap \left[\bigcap_{k=1} H(p^k, p^k x^k) \right] \cap \mathrm{Com}\{(a^* e_N)\} \qquad \text{for } t = 1,..,T$$

$$\bar{S}^{2,t} \equiv \bar{S}^{1,t} \cap H(p^{t+1}, p^{t+1} x^{t+1}) \qquad \text{for } t = 1,..,T-1$$

$$S^{1,t} \equiv H(e_N, \Sigma x_i^t) \cap \left[\bigcap_{k=1}^{t} H(p^k, p^k x^k) \right] \cap \mathrm{Com}\{(a(S,d))\} \qquad \text{for } t = 1,..,T$$

$$S^{2,t} \equiv S^{1,t} \cap H(p^{t+1}, p^{t+1} x^{t+1}) \qquad \text{for } t = 1,..,T-1$$

$$S^{3,t} \equiv H(e_N, \Sigma x_i^t) \cap S \qquad \text{for } t = 1,..,T$$

$$S^{4,t} \equiv S^{1,t} \cap S \qquad \text{for } t = 1,..,T$$

The proof is done with the help of the following eight steps:

step 1: $x^t \in S^{r,t}$ and $d \in \mathrm{int}\,(S^{r,t})$ for all $r = 1,..,4$ and for all t.

proof: similar to claim 1 of the main result in Chun & Peters (1991).

step 2: $x^t \in \bar{S}^{r,t}$ and $d \in int(\bar{S}^{r,t})$ for $r = 1,2$ and for all t.

proof: since $S^{r,t} \subseteq \bar{S}^{r,t}$ for $r = 1,2$ and for all t, the desired conclusion follows from step 1.

step 3: $a(\bar{S}^{r,t},d) = a^* e_N$ for $r = 1,2$ and for all t.

proof: For all $i \in N$, let y^i be such that $y^i_i = a^*$ and $y^i_j = 0 \ \forall \ j \neq i$. It is enough to show that all y^i's belong to the half-spaces defined above to construct $\bar{S}^{r,t}$ for $r = 1,2$ and for all t. By means of identical reasoning as in claim 2 of the main result in Chun & Peters (1991), with the only substitution being a^* with δ, we obtain the desired conclusion.

step 4: $a(S^{3,t},d) = a(S,d)$ for all t.

proof: For all $i \in N$, let q^i be such that $q^i_i = a_i(S,d)$ and $q^i_j = 0 \ \forall \ j \neq i$. Since S is comprehensive $q^i \in S \ \forall i$. Now it is enough to show that all q^i's belong to the half-spaces defined above to construct $S^{3,t}$ for all t. Similarly to claim 2 of the main result in Chun & Peters (1991) by substituting δ by a^*, we can obtain $y^i \in H(e_N, \Sigma x^t_i)$ for all $i \in N$ and for all t (y^i as in step 3). Then, since $q^i \leq y^i \ \forall \ i \in N$ we get $q^i \in H(e_N, \Sigma x^t_i)$ for all $i \in N$ and for all t.

step 5: $\bar{S}^{1,t+1}_{d-i} = \bar{S}^{2,t}_{d-i}$ and $S^{3,t+1}_{d-i} = S^{3,t}_{d-i}$ for all $i = 1,.,n$ and for all $t = 1,.,T-1$ and $S^{3,T}_{d-i} = S_{d-i}$ for all $i = 1,.,n$.

proof: similar to claim 3 of the main result in Chun & Peters (1991) by using steps 3 and 4.

step 6: $F(S^{r,1},d,c) = x^1$ for all $r = 1,.,4$.

proof: note that $\bar{S}^{1,1} \equiv H(e_N, \Sigma x_i^1) \cap Com\{(a^*e_N)\}$. Therefore by PO and AN $F(\bar{S}^{1,1},d,c) = x^1$. Then by IADC,

$$F(S^{1,1},d,c) = F(S^{2,1},d,c) = F(S^{3,1},d,c) = F(S^{4,1},d,c) = x^1.$$

step 7: $F(S^{r,t},d,c) = x^t$ for all $r = 1,.,4$ and for all t.

proof: we use induction on t based on step 6. Suppose as an induction hypothesis, that the conclusion of step 7 holds for all $t = 1,.,h-1$. Now we consider the case when $t = h$. We will use step 5 several times, without explicit mentioning. By applying W.MON between $(\bar{S}^{2,h-1},d,c)$ and $(\bar{S}^{1,h},d,c)$, $F(\bar{S}^{1,h},d,c) \geq F(\bar{S}^{2,h-1},d,c) = x^{h-1}$. Therefore by PO and AN $F(\bar{S}^{1,h},d,c) = x^h$. By IADC $F(S^{1,h},d,c) = F(\bar{S}^{1,h},d,c) = x^h$. Again by IADC, $F(S^{4,h},d,c) = F(S^{1,h},d,c) = x^h$. By applying W.MON between $(S^{3,h-1},d,c)$ and $(S^{3,h},d,c)$, $F(S^{3,h},d,c) \geq F(S^{3,h-1},d,c) = x^{h-1}$. Note that $x \in S^{3,h}$ and $x \geq x^{h-1}$ implies that $x_i = x_i^{h-1}$ for all $i \in M^h$. Then $p^t x = p^t x^h = p^t x^t$ for all $t = 1,..,h$ and, consequently $x \in S^{4,h}$. Since $F(S^{3,h},d,c) \geq x^{h-1}$, $F(S^{3,h},d,c) \in S^{4,h}$. Therefore, by applying IADC between $(S^{3,h},d,c)$ and $(S^{4,h},d,c)$, $F(S^{3,h},d,c) = F(S^{4,h},d,c) = x^h$. Finally, again by IADC between $(S^{1,h},d,c)$ and $(S^{2,h},d,c)$, $F(S^{2,h},d,c) = x^h$ (this last step does not apply for $h = T$).

step 8: $F(S,d,c) = LP(S,d,c) = x^T$

proof: By applying W.MON between $(S^{3,T},d,c)$ and (S,d,c), $F(S,d,c) \geq F(S^{3,T},d,c) = x^T$, where the equality follows from step 7. Since $x^T \in PO(S)$, $F(S,d,c) = x^T$.■

Proof of theorem 2

LEE satisfies PO, AN, T.INV and IIIA. Lemmas 3 and 4 prove that it also verifies W.MON and RIAC.

Lemma 3 : LEE *satisfies weak monotonicity.*

proof: let (S,d,c), $(\bar{S},\bar{d},\bar{c}) \in \sum^n$ be such that $S \subseteq \bar{S}$, $d = \bar{d}$, $c = \bar{c}$ and $S_{d-i} = \bar{S}_{d-i}$ for all i in N. Since LEE satisfies T.INV, we may assume that $c = e_N$. Now, by means of an identical argument as in lemma 1, with the only substitution being to take $\{z^t\}$ and $\{\bar{z}^t\}$, i.e., the two sequences as defined in the process of finding LEE(S,d,c) and LEE($\bar{S},\bar{d},\bar{c}$) respectively, instead of $\{x^t\}$ and $\{\bar{x}^t\}$, we get LEE(S,d,c) $\leq$ LEE($\bar{S},\bar{d},\bar{c}$).■

Lemma 4 : LEE *satisfies rational independence of alternatives other than the claims point.*

proof: let (S,d,c), $(\bar{S},\bar{d},\bar{c}) \in \sum^n$ be two problems satisfying the hypotheses of RIAC: $\bar{S} \subseteq S$, $c = \bar{c}$, $d \leq \bar{d}$ and LEE(S,d,c) $\in IR(\bar{S},\bar{d})$. Also let $\{z^t\}$ be the sequence as defined in the process of finding LEE(S,d,c) $= z^T$, therefore $\{z^t\} \subset S^*$. Since $z^T \in IR(\bar{S},\bar{d})\}$, $z^t \leq z^T$ for all t, we have $z^t \in \bar{S}^*$ for all t. Now we construct the sequence $\{\bar{z}^t\} \subset \bar{S}^*$ to find LEE($\bar{S},\bar{d},\bar{c}$). Since $\bar{S}^* \subseteq S^*$, $\bar{c} = c$, and $z^t \in \bar{S}^*$ for all t, $\bar{z}^t = z^t$ for all t. Therefore, we conclude that LEE($\bar{S},\bar{d},\bar{c}$) $= z^T =$ LEE(S,d,c).■

Suppose F is a solution satisfying PO, AN, T.INV, IIIA, W.MON and RIAC.

Now, let $(\bar{S},\bar{d},\bar{c}) \in \sum^n$ be given. By T.INV we may assume that $\bar{c} = e_N$. Let $S = \bar{S}^*$. Let $d' \leq \bar{d}$, $d' \in int(S)$ such that $d'_i = d'_j = 1 - \varepsilon$ $\forall i,j \in N$. Equivalently, we may take, by T.INV, $d' = 0$, and $\bar{c} = \varepsilon e_N$, $\varepsilon > 0$, then $a(S,d') = a(\bar{S},\bar{d})$. Now, let $\{z^t\}_{t=1}^T$ and $\{Q^t\}_{t=1}^T$ be the sequences as defined in the process of finding LEE$(S,d',\bar{c})$. Then we define auxiliary problems constructed in the same way as in theorem 1 but taking $M^t = N \backslash Q^t$ for all t, a* as the greatest coordinate of $a(S,d')$ and substituting z^t for x^t for all t and $a(S,d')$ for $a(S,d)$. Then we prove that $F(S,d',\bar{c}) = z^T$ by replacing IADC by RIAC, d by d' and c by $\bar{c}$ and reasoning in the same way as in theorem 1. Now, by PO $F(S,d',\bar{c}) \in IR(S,\bar{d})$, and by RIAC $F(S,\bar{d},\bar{c}) = z^T$. Finally by IIIA, $F(\bar{S},\bar{d},\bar{c}) = z^T$.■

Proof of theorem 3

It is straightforward to prove that LP* satisfies POR, AN, S.INV and BC. Lemmas 5 and 6 show that it also verifies R.MON and IIAC.

Lemma 5 : LP* *satisfies restricted monotonicity.*

Proof: let (S,d,c), $(S',d',c') \in \sum^n$ be such that $S \subseteq S'$, $d = d'$, c c' and $\hat{S}_{-i} = \hat{S}'_{-i}$ for all $i \in N$. By reasoning in a similar way as lemma 1 we get:

step 1: For all $y = \hat{S}$, if $N(\hat{S},y) \neq \emptyset$ then $N(\hat{S},y) = N(\hat{S}',y)$.

step 2: Let $T > 1$ be the final step in finding LP*(S,d,c). Also, let $\{y^t\}$ and $\{y'^t\}$ be the two sequences as defined in the process of finding LP*(S,d,c) and LP*(S',d',c') respectively. Then for all $t = 1,..,T-1$ $y^t = y'^t$.

By combining the results of steps 1 and 2, it follows that $y^T = LP^*(S,d,c) \leq y'^T \leq LP^*(S',d',c')$.■

Lemma 6: LP* *satisfies independence of alternatives other than the disagreement and the the claims points.*

Proof: let $(S,d,c), (S',d',c') \in \sum^n$ be two problems satisfying the hypotheses of IADC, that is, $S' \subseteq S$, $d' = d$, $c' = c$ and $LP^*(S,d,c) \in S'$. Then, $\hat{S}' \subseteq \hat{S}$, and $LP^*(\hat{S},d,c) \in \hat{S}'$, since $LP^*(\hat{S},d,c) = LP^*(S,d,c)$ by definition 5, and LP* satisfies BC. Now, $LP(\hat{S}',d',c') = LP(\hat{S},d,c)$ by lemma 2, and again taking definition 5 into account we get $LP^*(S',d',c') = LP^*(S,d,c)$.■

Now suppose F is a solution satisfying axioms POR, AN, S.INV, R.MON, IADC and BC.

Let $(S,d,c) \in \sum^n$ be given. By S.INV, we assume $c = e_N$ and $d = 0$. Now let $\{y^t\}_{t=1}^T$ and $\{I^t\}_{t=1}^T$ be the sequences defined in the process of finding $LP^*(S,d,c)$. We will show that $F(\hat{S},d,c) = y^T$. The theorem then follows from axioms R.MON and BC.

First we construct elementary problems. Let $M^t = N \setminus I^t$ and $p^t = e_{M^t}$ for $t = 1,..,T$ (where $M^1 = \emptyset$ and $p^1 = 0$). Define:

$$S^{1,t} \equiv H(e_N, \Sigma y_i^t) \cap \left[\bigcap_{k=1}^{t} H(p^k, p^k y^k) \right] \cap Com\{(c)\} \qquad \text{for } t = 1,..,T$$

$$S^{2,t} \equiv S^{1,t} \cap H(p^{t+1}, p^{t+1} y^{t+1}) \qquad \text{for } t = 1,..,T\text{-}1$$

$$S^{3,t} \equiv H(e_N, \Sigma y_i^t) \cap \hat{S} \qquad \text{for } t = 1,..,T$$

$$S^{4,t} \equiv S^{1,t} \cap \hat{S} \qquad \text{for } t = 1,..,T$$

The proof is done with the help of the following five steps.

step 1: $y^t \in S^{r,t}$ and $d \in \text{int}(S^{r,t})$ for all $r = 1,..,4$ and for all t.

proof: see step 1 of theorem 1.

step 2: $\hat{S}^{1,t+1}_{-i} = \hat{S}^{2,t}_{-i}$ and $\hat{S}^{3,t+1}_{-i} = \hat{S}^{3,t}_{-i}$ for all $i = 1,.,n$ and for all $t = 1,.,T-1$ and $\hat{S}^{3,T}_{-i} = \hat{S}_{-i}$ for all $i = 1,.,n$.

proof: similar to claim 3 of the main result in Chun & Peters (1991).

step 3: $F(S^{r,1},d,c) = y^1$ for all $r = 1,.,4$.

proof: note that $S^{1,1} \equiv H(e_N, \Sigma y^1_i) \cap \text{Com}\{(c)\}$. Therefore by POR and AN $F(S^{1,1},d,c) = y^1$. Then by IADC, $F(S^{2,1},d,c) = F(S^{3,1},d,c) = F(S^{4,1},d,c) = y^1$.

step 4: $F(S^{r,t},d,c) = x^t$ for all $r = 1,.,4$ and for all t.

proof: we use induction on t based on step 3. Suppose as an induction hypothesis, that the conclusion of step 4 holds for all $t = 1,.,h-1$. Now we consider the case where $t = h$. We will use 2 several times, without explicit mentioning. By applying R.MON between $(S^{2,h-1},d,c)$ and $(S^{1,h},d,c)$, $F(S^{1,h},d,c) \geq F(S^{2,h-1},d,c) = y^{h-1}$. Therefore by POR and AN $F(S^{1,h},d,c) = y^h$. By IADC $F(S^{4,h},d,c) = F(S^{1,h},d,c) = y^h$. Now reasoning in the same way as step 7 in theorem 1, with the only substitutions being W.MON with R.MON, x^k with y^k for $k = h-1$, h, and x^t with y^t we get the desired conclusion.

step 5: $F(\hat{S},d,c) = LP^*(S,d,c) = y^T$

proof: By applying R.MON between $(S^{3,T},d,c)$ and $(\hat{S},d,c)$, $F(\hat{S},d,c) \geq F(S^{3,T},d,c) = y^T$, where the equality follows from step 4. Since $y^T \in PO(\hat{S})$, $F(\hat{S},d,c) = y^T$.■

Proof of theorem 4

It is easy to check that LEE* satisfies POR, AN, T.INV, IIIA and BC. Lemmas 7 and 8 show that it also verifies R.MON and RIAC.

Lemma 7: LEE* *satisfies restricted monotonicity.*

Proof: let (S,d,c), $(S',d',c') \in \sum^n$ such that $S \subseteq S'$, $d = d'$ and $\hat{S}_{-i} = \hat{S}'_{-i}$ for all $i \in N$. Since LEE* satisfies T.INV, we may assume that $c = e_N$. Now by means of an identical argument as in lemma 5, with the only substitution being $\{y^t\}$ and $\{y'^t\}$ with $\{v^t\}$ and $\{v'^t\}$, i.e., the two sequences as defined in the process of finding $LEE^*(S,d,c)$ and $LEE^*(S',d',c')$ respectively, we get $LEE^*(S,d,c) \leq LEE^*(S',d',c')$.■

Lemma 8: LEE* *satisfies rational independence of alternatives other than the claims point.*

Proof: let (S,d,c), $(S',d',c') \in \sum^n$ be two problems satisfying the hypotheses of RIAC, that is, $S' \subseteq S$, $c' = c$, $d \leq d'$ and $LEE^*(S,d,c) \in IR(S',d')$.Now, $\hat{S}' \subseteq \hat{S}$ and $LEE^*(\hat{S},d,c) \in IR(\hat{S}',d')$, since $LEE^*(\hat{S},d,c) = LEE^*(S,d,c)$ by definition 6 and LEE* verifies BC. Now, by lemma 4, $LEE(\hat{S}',d',c') = LEE(\hat{S},d,c)$ and again taking definition 6 into account we get $LEE^*(S',d',c') = LEE^*(S,d,c)$.■

Now, suppose F is a solution satisfying axioms POR, AN, T.INV, IIIA, R.MON, RIAC and BC.

Let $(S',d',c') \in \sum^n$ be given. By T.INV we may assume that $c' = e_N$. Consider now S'^* and $\hat{S}'^*$. Let $d^* \leq d'$, $d^* \in int(S'^*)$ be such that $d^*_i = d^*_j = 1-\varepsilon$ $\forall i,j \in N$. Equivalently, we may take, by T.INV, $d^* = 0$ and $c' = \varepsilon e_N$, $\varepsilon > 0$. Now let $\{v^t\}_{t=1}^T$ and $\{J^t\}_{t=1}^T$ be the sequences defined in the process of finding $LEE^*(S'^*,d^*,c')$. In order to prove that $F(\hat{S}'^*,d^*,c') = v^T$ we construct identical auxiliary problems as in theorem 3, by taking $M^t = N\backslash J^t$ and substituting c with c', y^t with v^t and $\hat{S}$ with $\hat{S}'^*$. Then the desired conclusion is obtained by reasoning in the same way as in theorem 3, by substituting IADC with RIAC, d with d* and c with c'. Now, by POR $F(\hat{S}'^*,d^*,c') \in IR(\hat{S}'^*,d')$, and by RIAC $F(\hat{S}'^*,d',c') = v^T$. By R.MON and BC $F(S'^*,d',c') = v^T$. Finally by IIIA, $F(S',d',c') = v^T$.■

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