SIMPLE MECHANISM TO IMPLEMENT THE CORE OF COLLEGE ADMISSIONS PROBLEMS*

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Abstract

This paper provides three simple mechanisms to implement allocations in the core of matching markets. We analyze some sequential mechanisms which mimic matching procedures for many-to-one real life matching markets. We show that only core allocations should be attained when agents act strategically faced with these mechanisms. Two mechanisms implement the core correspondence in SPE, whereas the third implements the students' optimal stable solution.

Keywords: College Admissions Problems, Mechanism Design.

1. Introduction

This paper presents mechanisms implementing stable allocations for matching markets. We present three rules that mimic the sequential interaction holding between agents in contractual processes on both sides of the market.

Matching markets have been extensively analyzed from a game-theoretical point of view. (See Roth and Sotomayor [15] for a detailed state of the art until 1990.) In this framework, Roth [12] and Alcalde and Barberà [3] have shown the existence of incentives for agents to misreport their true preferences when faced with some mechanisms selecting allocations to satisfy "desired" properties.

Some authors have concentrated on partial aspects derived from the strategic behavior of the agents. For instance, Gale and Sotomayor [7] and Roth [13] analyzed partial implementation of the core in a marriage market framework.

Alcalde [1] analyzes a particular case of matching problems called the marriage market. He tackles the implementability of two particular solution concepts: the core correspondence and its extreme selections, and provides positive answers. In college admissions problems, the general model we are interested in, Kara and Sönmez [9] show that the core correspondence can be implemented in Nash equilibria. Nevertheless, they do not provide a simple mechanism which can be employed in real life situations. They also show that no subselection of the core is Nash implementable.

In a related framework, Romero-Medina [11] studies the mechanism employed by the Spanish University system to allocate new students to colleges. He shows that this mechanism can select unstable outcomes but, when students act strategically, only core allocations should be reached. The matching procedure studied by Romero-Medina does not allow universities to act strategically. For this reason, his results cannot be applied to the more general framework in which we are interested. Thus designing a useful mechanism to implement stable solutions for job markets is still an open problem.

An interesting feature of the mechanisms we present, is that they constitute reasonable proposals for effective design. Following Jackson [8], the mechanisms used to implement social choice correspondences should have "natural" features. A way to argue when a mechanism is natural is presented by the possibility of being employed in real life situations. We are going to introduce rules employed in real many-to-one matching markets. For instance, the rule to be introduced in Section 5 is used by the Spanish Public Administrations to allocate the workers they hire.[1]

This paper provides some mechanisms implementing core allocations. The rules to be analyzed capture some aspects which hold in real life college admissions problems. Firstly, we model sequential interactions among agents on both sides of the market, reflecting an adjustment process to reach stable allocations. Secondly, agents on one side of the market adopt an "active" role, making offers, whereas the aptitude shown by agents on the other side can be considered as "passive": they only accept or reject the offers they receive. In fact, the mechanisms analyzed below reflect the idea of the classical Gale and Shapley algorithm.

The rest of the paper is organized as follows. Section 2 introduces the basic model. Sections 3 and 4 present two mechanisms to implement the core for job markets in SPE. Section 5 proposes a family of mechanisms implementing a selection of the core, namely the students' optimal stable matching. Conclusions are collected in Section 6.

2. The model

Let $C = \{c_1, ..., c_n\}$ and $S = \{s_1, ..., s_m\}$ be the set of colleges and students, respectively. Each college has preferences $P(c)$ over the set of groups of students.

[1] We are grateful to Carmen Herrero for pointing out this aspect

$P(c)$ is assumed to be a linear order on 2^S. Each student's preferences $P(s)$ is described by a linear order on $C \cup \{s\}$. A college admissions problem is fully described by a triplet $\left\{C, S; \underset{\sim}{P}\right\}$, where $\underset{\sim}{P} = \{P(c_1), ..., P(c_n), P(s_1), ..., P(s_m)\}$ is a list containing a full description of the agents' preferences and is called a profile.

An allocation for such a problem, or *matching*, is a mapping μ from $C \cup S$ into $2^S \cup C$ satisfying

(i) for all $c \in C$, $\mu(c) \in 2^S$,

(ii) for all $s \in S$, $\mu(s) \in C \cup \{s\}$, and

(iii) for each pair $(c, s) \in C \times S$, $[\mu(s) = c \Longleftrightarrow s \in \mu(c)]$.

From now on we will consider C and S to be fixed sets, thus we can identify a colleges admissions problem $\left\{F, W; \underset{\sim}{P}\right\}$ with the preference profile $\underset{\sim}{P}$.[2] Let $\mathcal{M}\left(\underset{\sim}{P}\right)$ be the set of all possible matchings μ in $\underset{\sim}{P}$. Finally, $\underset{\sim}{\mathbb{P}}$ denotes the set of (potential) matching markets.

Let $\underset{\sim}{P}$ be a matching market. Given a set of students $A \subseteq S$, we denote by $Ch_c(A)$ the $P(c)$ maximal element on 2^A.

[2]For the sake of simplicity, we will employ the same notation for preference profiles the and college admissions problem. The context will be made precise if $\underset{\sim}{P}$ denotes a matching problem or simply a preference profile.

Definition 2.1. *A matching μ is said to be individually rational for $\underset{\sim}{P}$ iff*

(i) $Ch_c(\mu(c)) = \mu(c)$ for all $c \in C$, and

(ii) for all $s \in S$, $c \in C$ $\quad [sP(s)c \Longrightarrow s \notin \mu(c)]$.

Definition 2.2. *Let μ be a matching for $\underset{\sim}{P}$. We say that μ is blocked by a pair $(c, s) \in C \times S$ iff*

(i) $c\ P(s)\ \mu(s)$, and

(ii) $s \in Ch_c\left(\mu(c) \cup \{s\}\right)$.

A pair (c, s) which satisfies the above two conditions is called a blocking pair for μ.

Definition 2.3. *Let μ be a matching for $\underset{\sim}{P}$. We say that μ is (pair-wise) stable if it is individually rational and there is no pair blocking it. Let $\mathcal{C}\left(\underset{\sim}{P}\right)$ be the set of stable allocations for the problem $\underset{\sim}{P}$.*

Finally, we assume that colleges' preferences with regard to groups of students are substitutive. That is, for any two students s and s' if s belongs to $Ch_c(A)$, then she will also belong to $Ch_c(A \setminus \{s'\})$. This assumption is quite usual in the literature and guarantees non-emptiness of the set of stable allocations. (Alcalde

[2] provides further arguments in favour of the need of such an assumption.) Notice that when preferences are substitutive, the set of (pair-wise) stable allocations coincides with the core of the related colleges admission problem. That is, given a stable allocation, no group of agents can find a matching to improve the utility of all its members without being matched with agents outside this group.

The concept of implementation we are going to use throughout the paper is well-known in the literature. We next formalize this for both the subgame perfect Nash equilibrium (SPE) and the strong subgame perfect Nash equilibrium (SSPE) cases. Let $\mathcal{E}_k$ be the set of strategies for agent k and let $\mathcal{E} = \underset{x \in C \cup S}{\times} \mathcal{E}_x$ be the set of strategy profiles. Associated to each strategy profile $\tilde{e}$ we can define a message profile $m(\tilde{e})$, or simply $\tilde{m}$, which describes the action taken by each individual when the agents choose such strategies. A matching mechanism is described by the set of strategies allowed to each agent, and an outcome function γ that assigns a matching to each profile of messages. We say that a matching mechanism *implements* a solution concept, say χ, in (strong) subgame perfect Nash equilibria if (i) for any $\tilde{e}$, (strong) subgame perfect equilibrium of the game $\Gamma \equiv \left\{C, S; \underset{\sim}{P}; \gamma\right\}$, $\gamma(\tilde{m}(\tilde{e}))$ belongs to $\chi\left(\underset{\sim}{P}\right)$ and (ii) for each μ in $\chi\left(\underset{\sim}{P}\right)$ there exists a (strong) SPE for Γ, say $\tilde{e}'$, such that $\gamma\left(m\left(\tilde{e}'\right)\right) = \mu$.

3. The "colleges-propose-and-students-choose" mechanism

This section is devoted to analysing a matching mechanism that mimics matching procedures which hold in real life. The mechanism we are going to introduce is simple in the following sense. The message space of each agent can be straightforwardly obtained from its own preferences. The outcome function can be very easily evaluated at any profile of messages. Thus, any individual is able to evaluate the consequences of her strategy without using a sophisticated analysis of the mechanism.

We next introduce a mechanism to implement the core correspondence in SPE. We are going to employ a natural two-stage game form mechanism. In the first stage each college makes proposals to a set (possibly empty) of students. In the second stage each student chooses the college she prefers. The outcome of the game is a matching where by each student is enrolled by the college she selected whenever she has received a proposal from this college.

More formally, in the first stage, colleges have to decide simultaneously. Each college message space coincides with the set of potential teams of students, 2^S. In the second stage students, knowing the colleges' messages, select simultaneously the college in which they want to study. Thus, each student message space coincides with $C \cup \{s\}$. Let $m(k)$ denote the message by agent $k \in C \cup S$, and $\tilde{m}$ be an ordered vector containing the messages of all the agents.

The outcome function, denoted by Φ^{CS}, selects a matching which is defined as follows:

$\Phi^{CS}(\tilde{m}) = \mu_{\tilde{m}}$, where for any s in S,

$$\mu_{\tilde{m}}(s) = \begin{cases} m(s) & \text{if } s \in m(m(s)) \\ s & \text{otherwise} \end{cases}$$

and, for each c in C,

$$\mu_{\tilde{m}}(c) = \{s \in m(c) \mid c = m(s)\}$$

Theorem 3.1. *The mechanism described above implements in SPE the core correspondence.*

Proof. First, we prove that every SPE outcome is in $\mathcal{C}\left(\underset{\sim}{P}\right)$. Let $\tilde{m}'$ be a SPE[3] for $\Gamma^{CS} := \left\{C, S; \underset{\sim}{P}; \Phi^{CS}\right\}$. One can check that, at the second stage, each student s has a dominant strategy, namely $m'(s) = \arg\max P(s)$ on $\{c \in C \text{ s.t. } s \in m'(c)\} \cup$

[3]Strictly speaking, $\tilde{m}'$ is the ordered vector of messages that result in a subgame perfect equilibrium. We abuse on the notation throughout the paper identifying messages at a SPE with subgame perfect equilibria.

$\{s\}$. Thus, $\Phi^{CS}(\tilde{m}')$ should be an individually rational matching for $\underset{\sim}{P}$.

Let us suppose that $\Phi^{CS}(\tilde{m}')$ is not in $\mathcal{C}\left(\underset{\sim}{P}\right)$, then there should be a blocking pair, say (c,s) in $C \times S$. Since all the colleges play simultaneously, this can not be the case, because college c can reach higher utility by playing $m''(c) = Ch_c\left(\mu_{\tilde{m}'}(c) \cup \{s\}\right)$. Notice that, at the second stage the message stated by student s has to be $m''(s) = c$. A contradiction.

On the other hand, let μ be a stable matching for $\underset{\sim}{P}$. Let us consider the following strategies for the agents. Each college message (and strategy) is $m(c) = \mu(c)$. At the second stage any student's strategy is her dominant strategy. Thus, her message is $m(s) = \mu(s)$. It is very easy to see that this constitutes a SPE for the related game whose outcome coincides with μ, which yields the desired result. ∎

The solution concept we implemented involves a high level of cooperation among agents. For this reason one is tempted to analyze the consequences of agents cooperation when faced with our mechanism. As Example 3.2 shows, cooperation among agents does not reduce the set of outcomes one can expect.

Example 3.2. *Let consider the following five students and three colleges market.*

$P(s_1) = c_1$ $\qquad$ $P(s_2) = c_3c_1c_2$

$P(s_3) = c_3c_1c_2$ $\qquad$ $P(s_4) = c_2c_1c_3$

$P(s_5) = c_2c_1c_3$ $\qquad$ $P(c_1) = s_1s_2s_3s_4s_5$

$P(c_2) = (s_2s_3)(s_4s_5)s_1s_2s_3s_4s_5$ $\qquad$ $P(c_3) = (s_4s_5)(s_2s_3)s_1s_2s_3s_4s_5$

It is straightforward to see that there is a strong subgame perfect Nash equilibrium yielding each stable matching. For instance, the matching μ^S *in which* $\mu^S(c_1) = s_1$, $\mu^S(c_2) = (s_4s_5)$ *and* $\mu^S(c_3) = (s_2s_3)$ *can be supported in SSPE by strategies* $e(c_1) = (s_1s_2s_3s_4s_5)$, $e(c_2) = (s_4s_5)$, $e(c_3) = (s_2s_3)$ *and, for each student* s, $e(s) = \arg\max P(s)$ *on* $\{c \in C \text{ s.t. } s \in m(c)\} \cup \{s\}$, *where* $m(c)$ *is the message of college* c. *In a similar way, we can support the colleges' optimal stable matching* μ^C *in which* $\mu^C(c_1) = s_1$, $\mu^C(c_2) = (s_2s_3)$ *and* $\mu^S(c_3) = (s_4s_5)$ *by a SSPE described by strategies* $e(c_1) = s_1$, $e(c_2) = (s_2s_3)$, $e(c_3) = (s_4s_5)$ *and, for each student* s, $e(s) = \arg\max P(s)$ *on* $\{c \in C \text{ s.t. } s \in m(c)\} \cup \{s\}$.

4. The "students-propose-and-colleges-choose" mechanism

This section introduces a second mechanism implementing the core correspondence of college admissions problems. The idea underlying this mechanism is very similar to the previous one. In this case offers are made by students and each college selects the best set of students, from the proposals it receives. That is, the main formal difference between this mechanism and the one studied in Section 3 is that we shift the order in which agents on both sides of the market make their decisions.

Let us introduce the mechanism. This is a two-stage game-form mechanism. In the first stage, students have to decide. Each student message space coincides with the set of colleges and her being unmatched option, $C \cup \{s\}$. In the second stage, colleges which know students' messages, select the set of students that they want to admit. Thus, each college's message space coincides with 2^S. Let $m(k)$ denote the message of agent $k \in C \cup S$, and $\tilde{m}$ be an ordered vector containing the messages of all the agents.

The outcome function, denoted by Φ^{SC}, selects a matching which is defined as follows:

$\Phi^{SC}(\tilde{m}) = \mu_{\tilde{m}}$, where for any s in S,

$$\mu_{\tilde{m}}(s) = \begin{cases} m(s) & \text{if } s \in m(m(s)) \\ s & \text{otherwise} \end{cases}$$

and, for each c in C,

$$\mu_{\tilde{m}}(c) = \{s \in m(c) \mid c = m(s)\}$$

Theorem 4.1. *The "students-propose-and-colleges-choose" mechanism implements in SPE the core correspondence of college admissions problems.*

Proof. First, we show that every SPE outcome is a stable matching relative to agents' preferences. For, let $\tilde{m}'$ be a SPE for $\Gamma^{SC} := \left\{C, S; \underset{\sim}{P}; \Phi^{SC}\right\}$. One can check that, at the second stage, each college has a dominant strategy, namely $m'(c) = Ch_c(\{s \in S \mid c = m'(s)\})$. Thus, $\Phi^{SC}(\tilde{m}')$ should be an individually rational matching for $\underset{\sim}{P}$.

Let us suppose that $\Phi^{SC}(\tilde{m}')$ is not in $\mathcal{C}\left(\underset{\sim}{P}\right)$, then there should be a blocking pair, say (c, s), in $C \times S$. Since all the students play simultaneously, this can not be the case, because student s can reach higher utility by playing $m''(s) = c$. Notice that, at the second stage c's message has to include such a student. A contradiction.

On the other hand, let μ be a stable matching for $\underset{\sim}{P}$. Let us consider the following strategies for the agents. Each student message (and strategy) is $m(s) = \mu(s)$. At the second stage any college's strategy is its dominant strategy. Its message is $m(c) = \mu(c)$. It is very easy to see that this constitutes a SPE for the related game whose outcome coincides with μ, which yields the desired result. ∎

Since the Social Choice Correspondence that we study is the core, we are tempted to analyze the influence of agents' behavior on the expected outcome when their commitment is allowed for. In such a case strong subgame Nash equilibrium seems to be a minimal requirement to be fulfilled by our predictions. The analysis of such an equilibrium concept is the aim of our Theorem 4.2.

Theorem 4.2. *The students-propose-and-colleges-choose mechanism implements in SSPE the students optimal stable allocation.*

Proof. First, we are going to show that the students' optimal stable matching can be supported by a SSPE. Let $\underset{\sim}{P}$ be a matching market, and μ^S be its students' optimal stable allocation. Consider the following strategies. For any s in S, $e(s) = \mu^S(s)$ and, for each c in C, $e(c) = \arg\max P(c)$ on $\{s \in S \text{ s.t. } c = e(s)\}$. As the reader can see these strategies constitute a SSPE whose outcome is μ^S.

On the other hand, let $\tilde{e}'$ be a SSPE yielding $\mu \neq \mu^S$ as outcome. We will show that it cannot be possible. Notice that every SSPE is a SPE. Thus, by Theorem 4.1, μ has to be stable. Since $\mu \neq \mu^S$, there is a set of students, say S', preferring their mate under μ^S rather than that assigned to them by μ. Let $S' = \left\{s \in S : \mu^S(s)\, P(s)\, \mu(s)\right\}$. And consider the following strategies. For all s in S', $e''(s) = \mu^S(s)$, and any s in $S \backslash S'$ plays her strategy $e'(s)$. Because of the latticial structure of the core, it holds that $e'(s) = \mu^S(s)$ for all s not in S' (Roth and Sotomayor [15, Theorem 5.31]). Given that colleges play their dominant strategies (see proof of our Theorem 4.1 above), the outcome when agents in S' shift their strategy and play $e''(s)$ yield μ^S as outcome. A contradiction. ∎

5. The "students-sequentially-propose-and-colleges-choose" mechanism

This section introduces a modified version of the "students-propose-and-colleges-choose" mechanism. The allocation rule that we are going to analyze differs from that studied in Section 4 because students' decisions are made sequentially. In fact, we are going to introduce a family of mechanisms (each for any different order in which students have to decide). Nevertheless the expected outcome does not depend on such an order.

Conclusions of Theorem 3.1 do not remain valid when such a sequentiality is introduced in the mechanism. Theorem 5.1 shows that our capacity of prediction (when no commitment is allowed) increases. The outcome we attain is still stable, but no any stable outcome can be reached by a subgame perfect equilibrium of this mechanism. In fact, there is only one outcome implemented by this mechanism, namely the optimal stable matching from the point of view of students.

In some sense, the sequentiality (in students' decisions) plays a role similar to the students ability to commit themselves to the strategies to be played. Because of the latticial structure of the core such a commitment will lead to the best stable allocation that students can reach.

Let us introduce the mechanism. First, fix the order in which students are going to play. Without loss of generality, let us assume that s_1 is the first to play, s_2 is the second and so on. This is a m+1 stage game form. At stage i-th, i = 1, ..., m, student s_i selects a college. Thus, each student message space coincides with the set of colleges (and her being unmatched option). At stage m+1-th, the last stage, colleges simultaneously select the set of students they want to admit, one set of students for each college. Thus, each college message space coincides with 2^S. Finally, the outcome function, denoted by Φ^{SsC}, selects the matching defined as follows:

$\Phi^{SsC}(\tilde{m}) = \mu_{\tilde{m}}$, where for any s in S,

$$\mu_{\tilde{m}}(s) = \begin{cases} m(s) & \text{if } s \in m(m(s)) \\ s & \text{otherwise} \end{cases}$$

and, for each c in C,

$$\mu_{\tilde{m}}(c) = \{s \in m(c) \mid c = m(s)\}$$

where $\tilde{m}$ is a list containing a full description of agents' messages.

We next state the main result for this section. The students-sequentially-propose-and-colleges-choose mechanism implements in SPE the stable solution which is optimal from the point of view of students.

Theorem 5.1. *Let $\tilde{e}$ be a SPE for $\Gamma^{SsC} := \left\{C, S; \underset{\sim}{P}; \Phi^{SsC}\right\}$, and $\tilde{m}$ be the vector of messages that agents state in $\tilde{e}$. Then $\mu_{\tilde{m}} = \mu^S$, the optimal stable allocation from the point of view of students.*

Proof. We will proceed to show this result in a constructive way. First, we will present some properties that any SPE has to satisfy. Then we will argue that agents' messages will lead to the optimal students' stable matching.

In order to characterize the set of SPE, we will apply backward induction. At stage m+1-th, given students messages, each college c has a best response, namely, $m^*(c) = \arg\max P(c)$ on $\{s \mid m(s) = c\}$[4]. At stage m-th, given messages for

[4]Notice that such a strategy is not the unique best response. In fact, the set of best responses for college c is the union of such a set with any set of students S' such that $c \neq m(s')$ for all

students other than s_m, $(\bar{m}(s_1), \ldots, \bar{m}(s_{m-1}))$, and knowing colleges' behavior, this agent's best reply is

$$m^*(s_m) = \arg\max P(s_m) \text{ on}$$
$$\{c \mid s_m \in Ch_c(\{s \in S \setminus \{s_m\} \mid \bar{m}(s) = c\} \cup \{s_m\})\}.$$

Notice that such a message coincides with $\mu^S(s_m)$, when any student s_i in $S \setminus \{s_m\}$ has preferences such that $m(s_i)$ is the only college which is preferred to her being unmatched option[5], and $P'(x) = P(x)$ for agents in $W \cup \{s_m\}$.

In order to apply an inductive argument, let the strategy of student s_k, $\hat{m}^i(s_k)$ be defined in a recursive way by $\bar{m}^i(s_k)$ if $k < i$ and $m^*(s_k)$ whenever $k > i$. Thus

$$m^*(s_i) = \arg\max P(s_i) \text{ on the set}$$
$$\{c \mid s_i \in Ch_c(\{s_k \in S \setminus \{s_i\} \mid \hat{m}^i(s_k) = c\} \cup \{s_i\})\} =$$
$$= \mu^S(s_i), \text{ when agents preferences are } P'(x) = m(x) \text{ if } x \in S \setminus \cup_{j>i}\{s_j\}$$
$$\text{and } P'(x) = P(x) \text{ otherwise.}$$

Finally, given messages that students other than s_1 have to be $m^*(s_i) =$

s' in S'. Nevertheless, all these messages are strategically equivalent. Since we are interested in equilibrium payoffs rather than equilibria strategies, we do not pay attention to these strategies. The same argument applies to the proof of Theorem 4.1.

[5]For simplicity, we identify student' s_i preferences with college c_j, i.e. $P(s_i) = c_j$, whenever such a college is the only for which $c_j P(s_i) s_i$ holds.

$\mu^S(w_i)$, when agents' preferences are $\underset{\sim}{P}^*$, where $P^*(x) = P(x)$ for all agent other than s_1, the best option for such a student is $m^*(s_1) = \mu^S(w_1)$, when agents preferences are $\underset{\sim}{P}$, their true preferences. ∎

Remark 1. *Notice that in the proof the existence of a unique stable matching which is individually rational and weakly Pareto efficient from the point of view of students is very important to our result. Since such a property does not hold for colleges (See Roth [14]), we cannot guarantee that a symmetric result can be reached for colleges. In fact our Example 6.1 (see Section 6) shows that sequential extension for the "colleges-propose-and-students-choose" mechanism well might produce unstable SPE allocations.*

6. Final Remarks

This paper introduces two mechanisms implementing the core correspondence of matching markets. The results it provides solve two essential questions. First, the core of such games can be implemented in subgame perfect equilibria. And, second, it provides simple mechanisms to implement such a solution concept.

We also provide a mechanism to implement a particular selection of the core, namely the students' optimal stable matching. Thus, this paper also provides a positive answer to the implementability of a selection of the core in matching markets. Notice that Kara and Sönmez [9] prove that no selection of the core can be implemented in Nash equilibria.

Unfortunately a symmetric result cannot be provided for the set of colleges. This result points out (as Roth [14] did) the asymmetry holding among both sets of the market. Moreover, we can also state, in the words of Roth, that "*the college admissions problem is not equivalent to the marriage problem.*" Note that, in the particular case of marriage markets (colleges have only one position each), a symmetrical result for Theorem 5.1 can be stated by exchanging the role of students and colleges.

Let us conclude the paper providing an example to show the asymmetry above mentioned.

Example 6.1. *Let be* $\left\{C, S; \underset{\sim}{P}\right\}$ *a three colleges-four students market. Following* *table summarizes agents preferences.*

$P(c_1)$	$P(c_2)$	$P(c_3)$	$P(s_1)$	$P(s_2)$	$P(s_3)$	$P(s_4)$
$\{s_3, s_4\}$	$\{s_4\}$	$\{s_3\}$	c_3	c_2	c_2	c_3
$\{s_2, s_4\}$	$\{s_3\}$	$\{s_4\}$	c_2	c_1	c_1	c_1
$\{s_2, s_3\}$	$\{s_2\}$	$\{s_1\}$	c_1	c_3	c_3	c_2
$\{s_1, s_4\}$	$\{s_1\}$	$\{s_2\}$	s_1	s_2	s_3	s_4
$\{s_1, s_3\}$	$\emptyset$	$\emptyset$				
$\{s_1, s_2\}$						
$\{s_4\}$						
$\{s_3\}$						
$\{s_2\}$						
$\{s_1\}$						
$\emptyset$						

Let us consider the "colleges-sequentially-propose-and-students-choose" mechanism, Φ^{CsS}. This is a symmetrical version for the mechanism proposed in Section 5 where colleges are to make proposals in a sequential way. We will see that two *interesting features which are satisfied by the family of mechanisms* Φ^{SsC} *are not satisfied by mechanism in* Φ^{CsS}. *First, some SPE outcome can be unstable relative to agents preferences. In order to show that, let us suppose that the order in which colleges sequentially decide is* c_1 , c_2 *and* c_3. *There is a SPE with messages* $m(c_1) = \{s_1, s_2\}$, $m(c_2) = \{s_4\}$, $m(c_3) = \{s_3\}$, $m(s_1) = \{c_1\}$, $m(s_2) = \{c_1\}$, $m(s_3) = \{c_3\}$ *and* $m(s_4) = \{c_2\}$. *Notice that* $\Phi^{CsS}(\tilde{m})$ *is unstable because the pair* $\{c_1, s_3\}$ *blocks it. Secondly, the SPE outcomes set depends upon the order in which colleges make their decisions. Indeed, let us consider the order for colleges in which first* c_2 *proposes, then* c_3, *and finally* c_1 *is the last to make a proposal. In such a case the unique SPE outcome is* $\mu(c_1) = \{s_1, s_2\}$, $\mu(c_2) = \{s_3\}$ *and* $\mu(c_3) = \{s_4\}$, *which is different from the SPE outcome when firm* c_1 *is the first to play.*

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