Diskussionspapiere
Discussion Papers

Discussion Paper No. 266

Incentives to retire later - a solution to the social security crisis?

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Berlin, November 2001

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ISSN 1433-0210

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October 2001

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We thank seminar participants at the Martin-Luther-Universität Halle-Wittenberg for helpful comments and suggestions.

Abstract

As one possible solution to the well-known financing crisis of unfunded social security systems, an increase in the retirement age is a popular option. To induce workers to retire later, it has been proposed to strengthen the link between retirement age and benefit level. The present paper is devoted to analyzing the long-run financial implications of such a reform. We show that with actuarial adjustments the long-run contribution rate is an increasing function of the retirement age chosen by workers. Moreover, the implicit tax paid to the pension system by a participant can increase in the long run if the retirement age rises in response to a “steep” adjustment rule. In this sense, the proposed “cure” may worsen the disease. Finally, we propose an alternative adjustment scheme which avoids these negative consequences. Finally, we show how the negative effects can be avoided by forming a capital stock from the additional revenues due to later retirement.

JEL-**classification:** H55, J18.

Keywords: Pay-as-you-go, retirement age, actuarial adjustment.

Zusammenfassung

Als ein möglicher Ausweg aus der drohenden Finanzkrise umlagefinanzierter Rentensysteme wird gegenwärtig eine Anhebung des Rentenzugangsalters von vielen favorisiert. Um allerdings Arbeitnehmern einen Anreiz zur Verlängerung der Lebensarbeitszeit zu geben, muss nach Auffassung der meisten Experten die Beziehung zwischen Beiträgen und Rentenansprüchen gestärkt werden. In dieser Arbeit werden die langfristigen finanziellen Konsequenzen einer solchen Reform analysiert. Wir zeigen, dass bei versicherungsmathematischen Zuschlägen für Mehrarbeit der Beitragssatz langfristig eine steigende Funktion des tatsächlich gewählten Rentenalters ist. Darüber hinaus steigt auch die implizite Steuer, die ein repräsentativer Versicherter an die Rentenkasse zahlt, sofern das Rentenalter in Folge einer ,,steilen” Zuschlagsfunktion zunimmt. In diesem Sinne könnte die vorgeschlagene ,,Behandlung” die diagnostizierte ,,Krankheit” verschlimmern. Abschließend zeigen wir, wie der negative Effekt durch Aufbau eines Kapitalstocks vermieden werden kann.

JEL-**Klassifikation:** H55, J18.

Schlagwörter: Umlageverfahren, Rentenzugangsalter, versicherungsmathematische Zuschläge.

1. Introduction

There is widespread agreement that population aging in most OECD countries has led to a crisis of pay-as-you-go financed of old age pensions. Indicators of the "crisis" are rising contribution or "tax" rates, falling replacement rates and, more sophisticated, rising levels of the implicit tax paid by a representative participant, which is calculated as the present value of all (expected) pension benefits minus all contributions made.

Looking at the fundamental budget equation of every unfunded pension system,[1]

$$\text{Contribution rate} \times \text{workers} = \text{replacement rate} \times \text{pensioners},$$

a straightforward conclusion is that a worsening of the ratio of contribution rate and replacement rate can be prevented if the worker/pensioner-ratio is increased. Moreover, the most effective way to improve this ratio is by increasing average retirement age because this measure both raises the numerator and depresses the denominator of this ratio. One obvious way of achieving this seems to be raising the legal retirement age, as it is gradually being done in the U.S. over the next decade or so.

However, there are two possible objections to this reform strategy: first, raising the retirement age while holding the level of retirement benefits constant is equivalent to cutting the benefit level, and it is hard to see why the implicit benefit cut should be better than the explicit one. Secondly, as long as there are early-retirement options, raising the legal retirement age may be less effective than influencing the factual mean retirement age, which in most countries falls short of the legal retirement age by several years. Factual retirement age, in turn, may depend to a large degree on the adjustment rules which determine how the pension claims vary with the retirement age chosen by the individual.

In the last few years, there has been a tremendous amount of research into the microeconometrics of the retirement decision, most prominently by a joint research project conducted by researchers in eleven countries.[2] Cross-section evidence presented in a volume edited by Gruber and Wise (1999) shows clearly that the factual retirement age is lower the "flatter" the adjustment schedule is. Therefore the editors conclude that "social security program provisions have indeed contributed to the decline in labor force participation of older persons. ... It seems

[1] In this equation, both sides are already divided by the factor "average wages".

[2] See, e.g., Blundell and Johnson (1998), Börsch-Supan and Schnabel (1998), Gruber and Wise (1998) and Kapteyn and de Vos (1998).

evident that, if the trend toward early retirement is to be reversed, a move that will almost surely be dictated by demographic trends, changing the provisions of social security programs that induce early retirement will play a key role" (ibid., p.35).

Put more bluntly, the adjustment of the pension claims with respect to retiring before or after reaching the legal retirement age have to be increased to induce people to work longer and retire later. In particular, some authors propose "age-neutral systems" in which the adjustments follow actuarial rules, i.e. the marginal implicit tax on working longer should be eliminated altogether to facilitate an "undistorted" choice of retirement age (see, e.g., Börsch-Supan and Schnabel (1998), Börsch-Supan (2000)).

The present paper is devoted to analyzing the long-run financial implications of actuarial adjustments of pension claims with respect to retirement age in a pay-as-you-go pension scheme. For a small open economy, we shall ask the following questions:

1. Suppose that to induce people to work longer, the benefit adjustment rule has to be changed to an actuarial rule. After such a change, would the new long-run contribution rate be higher or lower than before the reform?

2. In the same setting, would the implicit tax that a representative worker pays into the system in a new steady state with actuarial adjustment and increased retirement age be higher or lower than before the change?

3. How would the answers to questions 1. and 2. change if it was sufficient for an increase of the retirement age to incorporate the "pay-as-you-go"-return rate, i.e. the growth rate of the wage bill, into the adjustment rule?

4. How will contribution rate and implicit tax rate change over time during an adjustment process from a lower to a higher retirement age?

The first three questions refer to a pure steady-state comparison, while the fourth question looks at the effects of an increase in the retirement age over time, the simplest case of which may be a sudden jump in period t from a low value E_0 to a higher value E_1.

We will first answer these questions for changes of the adjustment rule that leave the level of retirement benefits at the current *factual* retirement age constant. However, the proposals mentioned before can also be given a different interpretation, viz. increasing the reduction of retirement benefits for early retirement, holding

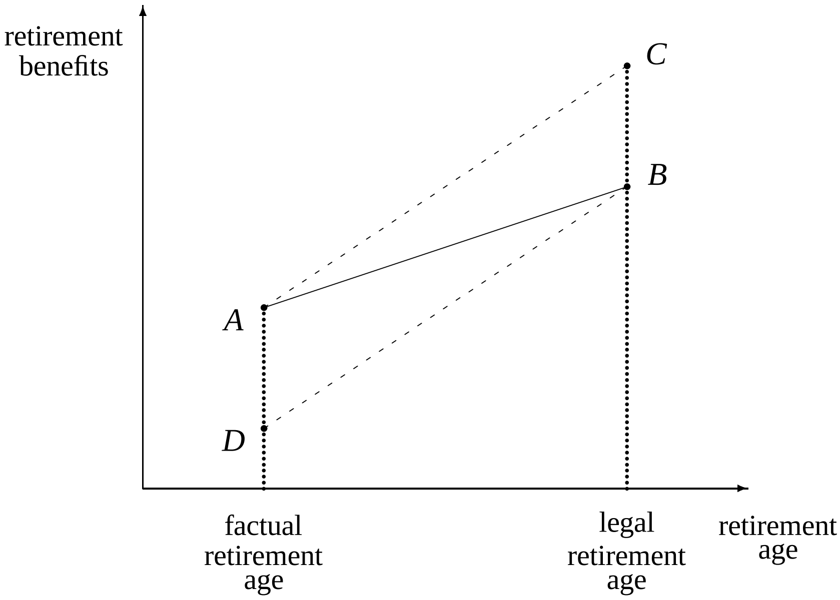


Figure 1: *Changes in the adjustment rule*

the benefit level at the *legal* retirement age constant. As the current factual retirement age is smaller than the legal retirement age, such a move can be decomposed into two separate steps: (i) lowering the benefit level at the current retirement age, and (ii) making the benefit adjustment "steeper". The first step by itself obviously lowers both the long-run contribution rate and the long-run implicit tax, but it is still interesting to know whether this also holds for the two steps combined. Therefore, we analyze this policy change as well.

In Figure 1, the two changes of the adjustment rule are illustrated. Initially, the retirement benefits schedule is given by the line *AB*. If the level of retirement benefits is kept constant at the factual retirement age and the adjustment rule is made steeper, the schedule turns counterclockwise around point *A*. The new schedule is given by the line *AC*. If the benefit level is held constant at the legal retirement age, the schedule turns counterclockwise around point *B* leading to the new schedule *DB*. As a consequence, retirement benefits fall at the factual retirement age.

The paper is structured as follows. In Section 2 we briefly review the previous literature. In Section 3 we formulate a continuous-time model of a steady state economy in which different rules of adjustment of pension claims to the number of working years can be compared. In Section 4, we shall analyze the consequences of changing the adjustment rule when the benefit level at the current factual retirement age is kept constant. We first answer questions 1. through 3. by holding the new adjustment rule fixed and comparing steady states that differ in the retirement age. Subsequently, we shall answer question 4 by giving a few numerical examples of typical transition paths from an old to the new steady state with higher retirement age. Section 5 is devoted to the case in which the benefit level at the legal retirement age is kept constant. Section 6 discusses an alternative system in which the redistributive effects found in the previous sections are avoided. Policy implications of our results will be discussed in Section 7.

2. Previous Results

A first analysis of some of the questions addressed in this paper was performed by the present authors in Breyer et al. (1997). In a discrete-time model the special case of a zero interest rate and an actuarial adjustment rule was examined.[3] In the present paper all these limitations will be removed.

The related question of an "optimal" rate of return to social security contributions has been addressed before by Hassler and Lindbeck (1997), (1999) within an OLG framework in which each individual lives for two periods and works only in the first. So the problem of choosing the retirement age is not addressed directly but only indirectly through the general labor supply decision. The authors show that the adjustment rule which maximizes steady-state utility of a representative individual equalizes marginal and average return to social security contributions. In other words, the optimal adjustment rule is not actuarial fairness (rate of return equal to the interest rate) but "pay-as-you-go fairness" (rate of return equal to the growth rate of GDP).

However, the two-period-OLG framework in these papers is not suited to analyze the distributional consequences on the different cohorts of introducing a particular adjustment rule if the several cohorts work at the same time. Furthermore, the analysis is confined to the case of a fixed contribution rate whereas holding the utility of the "old" generation constant at the introduction of a new adjustment rule requires fixing the benefit level.

[3] In addition, the paper appeared in German and thus is not easily accessible to an English-speaking audience.

3. The Model

3.1. Demography

In a continuous-time model, we examine a small open economy in which the interest rate r, the rate of population growth m and the rate of wage growth g are all constant. All individuals are assumed to have a life expectancy of exactly T. They work during the first E periods and retire for the remaining periods $T-E$. $N(x)$ denotes the number of individuals entering the labor force at time $t = x$. Therefore the size of the labor force at t is given by

$$L(t) = \int_{t-E}^{t} N(x)dx.$$

Likewise

$$P(t) = \int_{t-T}^{t-E} N(x)dx$$

will be the number of retired persons where the size of a cohort at time t is

$$N(t) = N(0)e^{mt}$$

with $N(0) > 0$. Thus, in a steady state, the number of workers $L(t)$ and retired $P(t)$ simplifies to

$$L(t) = \begin{cases} \frac{N(0)}{m}\left(e^{mt} - e^{m(t-E)}\right) & \text{if} \quad m \neq 0 \\ N(0)E & \text{if} \quad m = 0 \end{cases},$$

$$P(t) = \begin{cases} \frac{N(0)}{m}\left(e^{m(t-E)} - e^{m(t-T)}\right) & \text{if} \quad m \neq 0 \\ N(0)(T-E) & \text{if} \quad m = 0 \end{cases}.$$

The dependency ratio q is defined as the number of pensioners per worker. Hence, the steady state dependency ratio corresponds to

$$q(m,T,E) \equiv \frac{P(t)}{L(t)} = \begin{cases} \frac{e^{m(T-E)} - 1}{e^{mT} - e^{m(T-E)}} & \text{if} \quad m \neq 0 \\ \frac{T-E}{E} & \text{if} \quad m = 0 \end{cases} \tag{1}$$

and is independent of t in a steady state.[4]

[4] Throughout the analysis, we concentrate on the more general case $m \neq 0$. The extension to $m = 0$ is straightforward.

Wages per worker grow at the constant rate g so that the wage at time t is given by

$$w(t) = w(0)e^{gt}. \tag{2}$$

3.2. The pension system

The pay-as-you-go pension systems is characterized by time paths of average pensions $p(t)$, and contribution rates, $b(t)$. The balanced budget condition for the pension system is

$$L(t)b(t)w(t) = P(t)p(t). \tag{3}$$

The contribution rate is the same for every worker. In contrast, the pension level is determined for each pensioner i individually and takes account of his earnings history. In particular, the individual replacement rate, can be expressed as a function of the length of the working life and thus of the retirement age of individual i. This function, which we denote by n,

$$\frac{p^i(t)}{w(t)} = n(E^i) \tag{4}$$

is assumed to have the following properties:

a) There is a "standard" retirement age E^* such that $n(E^*) = n^*$. This standard can be either

  a1) a minimum retirement age, before which retirement benefits can not be collected, or

  a2) a "legal" retirement age, from which the individual can in principle deviate in both directions.

b) If individual i retires at age E_s, then his/her replacement rate is equal to n_s, where n_s is related to E_s by the following "pension adjustment formula":

$$\int_{t+E^*}^{t+E_s} e^{-z(x-t)}\left(b(x)+n^*\right)w(x)dx = \int_{t+E_s}^{t+T} e^{-z(x-t)}\left(n_s-n^*\right)w(x)dx. \tag{5}$$

z is the rate of return on extra working years within the pension system:

- if individual i retires at age $E_s > E^*$, then the left-hand side of (5) is the present value of extra contributions and foregone benefits in the extra working years between age E^* and E_s based on the discount rate z. Likewise, the right-hand side of (5) is equal to the corresponding present value of the increase in pensions from n^* to n_s. Notice that the compensation through higher pension benefits is actuarially fair if z is equal to interest rate r.

- if the individual i retires at age $E_s < E^*$, which is possible in case a2) but not in a1), then the left-hand side of (5) corresponds to the present value of avoided contributions and additional benefits in the time period between E_s and E^* based on the discount rate z. The right-hand side of (5) is equal to the corresponding present value of the cut in pensions from n^* to n_s. Again, the adjustment is actuarially fair if $z = r$.

A steady state $s = 0,1$ is characterized by a uniform value of the retirement age, $E_s^i = E_s$ chosen by all individuals i at all times. In a steady state, the replacement rate n_s, the contribution rate b_s, and the dependency ratio q_s are all constant over time, and they are related by

$$b_s = n_s q_s = n_s q(m, T, E_s). \tag{6}$$

In a steady state, we obtain for the relationship between the replacement and the contribution rate and the rate of return on extra working time, z:

Proposition 1: *Suppose the rate of return on extra working time is increased. Then, we have in the new steady state for a fixed retirement age E_f:*

$$\frac{\partial n_s}{\partial z} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textit{and} \quad \frac{\partial b_s}{\partial z} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textit{if and only if} \quad E_f \begin{matrix} > \\ = \\ < \end{matrix} E^*.$$

Proof: See Appendix.

Clearly, the steady state effects of an increase in rate on return on extra working time depend on whether the factual retirement age is below or above the legal retirement age. In the first case, individuals are "punished" for working less than E^*. Hence, the replacement rate and the contribution rate are lower in the steady state. In the latter case, individuals receive additional compensation for time they contribute in excess of E^* which leads to a higher replacement and contribution rate in the steady state.

3.3. The effects of an increase in the retirement age

3.3.1. Contribution rates

Examining the relationship between retirement age and contribution rate in a steady state, we obtain the following result:

Proposition 2: *Suppose steady state 1 is characterized by a retirement age* $E_1 > E_0$*. Then we have*

$$b_1 \begin{matrix} > \\ = \\ < \end{matrix} b_0 \quad \textit{if and only if} \quad z \begin{matrix} > \\ = \\ < \end{matrix} m+g.$$

Proof: See Appendix.

The intuition for the result in Proposition 2 is the following: In a pay-as-you-go pension system, all retirement benefits of a given cohort, including the compensation for having worked longer, have to be financed by later cohorts of workers. Now suppose everybody works exactly one period longer. Then there are two effects on the contribution rate that have to be paid by subsequent cohorts:

(i) total contributions rise and total retirement benefits fall by the amounts paid or not received, respectively, by those who retire later. This effect alone would depress the contribution rate,

(ii) all pensioners now receive higher pensions as a compensation. This effect alone would raise the contribution rate.

In assessing the relative sizes of the two effects, two facts must be taken into account: Since the compensation accrues later than additional contributions, it can be financed from an increased wage bill. On the other hand, the compensation itself is bigger than the extra payment by the return paid on it. Now it becomes clear that if the rate or return equals the growth rate of the wage bill ($m+g$), the two effects exactly offset each other, and the contribution rate stays the same. However, if the rate of return fall short of $m+g$, the first effect is dominant and the contribution must fall. Finally, if the rate of return exceeds $m+g$, the second effect is larger in size than the first one, and the contribution rate must go up. In particular, this is the case if the adjustment is actuarially fair, i.e. if $z=r$. In this case the contribution rate must rise in the long run because the pay-as-you-go system can only generate a smaller rate of return.

3.3.2. Implicit taxes

We measure the intergenerational distribution effects due to the unfunded pension system by the implicit taxes $\mathcal{T}(t)$ of the individuals born at time t. These correspond to the difference between the present value of contributions and the present value of pension payments. Under pay-as-you-go-pension systems with a constant replacement rate, b_s and n_s are constant in steady state s. Given a steady state interest rate of r, the implicit taxes paid by an individual born at t are therefore given by

$$\mathcal{T}_s(t) \equiv \int_t^{t+E_s} e^{-r(x-t)} b_s w(x)dx - \int_{t+E_s}^{t+T} e^{-r(x-t)} n_s w(x)dx. \tag{7}$$

Using (2) and (6) and assuming $r \neq g$,[5] this equation can thus be simplified to

$$\mathcal{T}_s(t) \quad = \quad b_s w(t) \left\{ \int_t^{t+E_s} e^{(g-r)(x-t)} dx - q_s^{-1} \int_{t+E_s}^{t+T} e^{(g-r)(x-t)} dx \right\}$$

Finally, substituting for the dependency ratio from equation (1) and assuming $m \neq 0$ yields

$$\mathcal{T}_s(t) = b_s \frac{w(t)}{g-r} \left\{ e^{(g-r)E_s} - 1 - \frac{e^{mT} - e^{m(T-E_s)}}{e^{m(T-E_s)} - 1} \left(e^{(g-r)T} - e^{(g-r)E_s} \right) \right\} \tag{8}$$

In a steady state, implicit taxes $\mathcal{T}_s(t)$ are positive if and only if $r > m + g$ (see Kifmann and Schindler (2000)). In the following, we assume that this condition is fulfilled.

Comparing steady states with different levels of the retirement age, we obtain the following result:

Proposition 3: *If $E_1 > E_0$ and $z \geq m + g$, then $\mathcal{T}_1(t) > \mathcal{T}_0(t)$.*

Proof: See Appendix.

The intuition for this result is straightforward if Proposition 2 is taken into account. Suppose that the pension benefit adjustment formula provides a rate of

[5] The extension to $r = g$ is straightforward.

return to the additional contributions which is equal to the growth rate of the wage bill, $m+g$. From Proposition 2 we know that in this case the steady-state contribution rate (per period of time) is independent of the retirement age. But this means that by working longer, everybody pays a higher total contribution to a system with an internal rate of return which is lower than the interest rate. This means that in present value terms the loss incurred by participating in the system is increased precisely because the size of the investment is increased.

Once it is established that the implicit taxes rise with an increase in the retirement age even if the contribution rate remains the same, it is easy to see that taxes must rise even more if the contribution rate increases, which (by Proposition 2) is the case for $z > m+g$. In this case the size of the investment in a low-yield system rises for two reasons: both the investment per period (contribution rate) and the length of the contribution period (working life) rise.

Finally, notice that $z \geq m+g$ is only a sufficient condition for implicit taxes to rise in a steady state upon an increase in the retirement age. Even if $z < m+g$, it is possible that implicit taxes increase. In this case, the effect of a longer period of time in which individuals contribute to the pension system overcompensates the fall in the contribution rate.

4. Changing the adjustment rule with a constant benefit level at the initial retirement age

In this section, we examine the consequences of creating incentives to retire later by increasing the rate of return z on extra working years in the pension formula (5). The initial benefit level is kept constant which is equivalent to the assumption that the initial retirement age E_0 is equal to the legal retirement age E^*. We assume in line with the empirical evidence that an increase in z will lead to an increase in the retirement age. In the new steady state, we therefore have $E_1 > E_0$.

If the initial retirement age E_0 corresponds to the standard retirement age E^*, then we know from Proposition 1 that the replacement rate n_0 and the contribution rate b_0 remain unaffected by an increase in z at retirement age $E_0 = E^*$. The steady state consequences of an increase in the retirement age on the contribution rate and on implicit taxes therefore follow straightforwardly from Propositions 2 and 3:

- by Proposition 2, the contribution rate rises if and only if the new value of z is larger than $m+g$. In particular, the contribution rate increases if the adjustment rule is changed to an actuarial rule. If the rate on return on extra working time is set equal to the growth rate of the wage bill, $m+g$, then the long-run contribution rate is not affected.

- by Proposition 3, $z \geq m+g$ is a sufficient condition for implicit taxes to be higher in the new steady. Thus, setting the rate on return on extra working time equal to the growth rate of the wage bill already puts a larger burden on future generations. Switching to an actuarial rule yields even higher long-run implicit taxes.

Once the results for different steady states are derived, it is easy to make qualitative statements on how the contribution rate and the implicit taxes behave in a transition from a steady state with a lower to one with a higher retirement age. Suppose that all cohorts who enter the labor force before some period t' retire after E_0 years of working, whereas all later-born generations retire after E_1 years with $E_1 > E_0$. The short-term effect on the contribution rate after period $t' + E_0$ is clearly dampening because ceteris paribus more persons work and contribute and fewer are pensioners, whereas the level of retirement benefits per pensioners, which is determined by their former labor supply behavior, remains unaffected.

On the other hand we know from Proposition 2 that - at least in the case of $z \geq m+g$, - the long-run contribution rate must be larger than or equal to the

Figure 2: *Contribution rate if z = r*

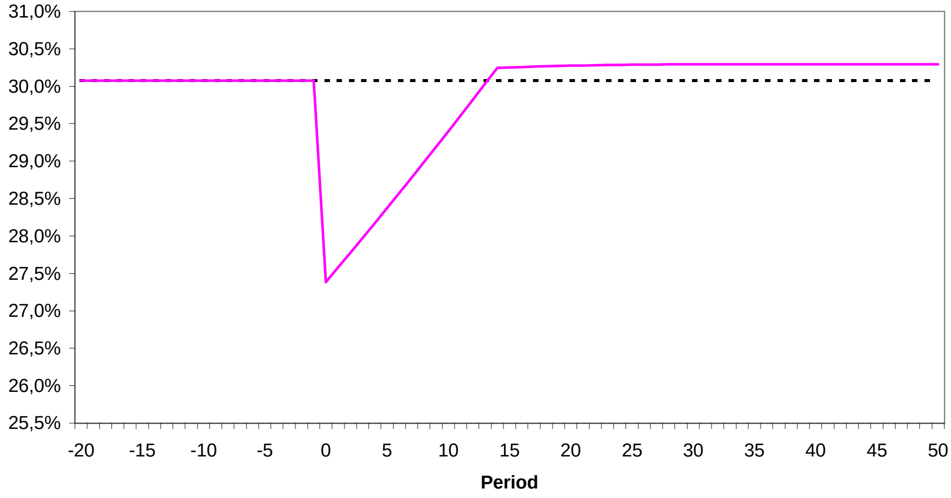
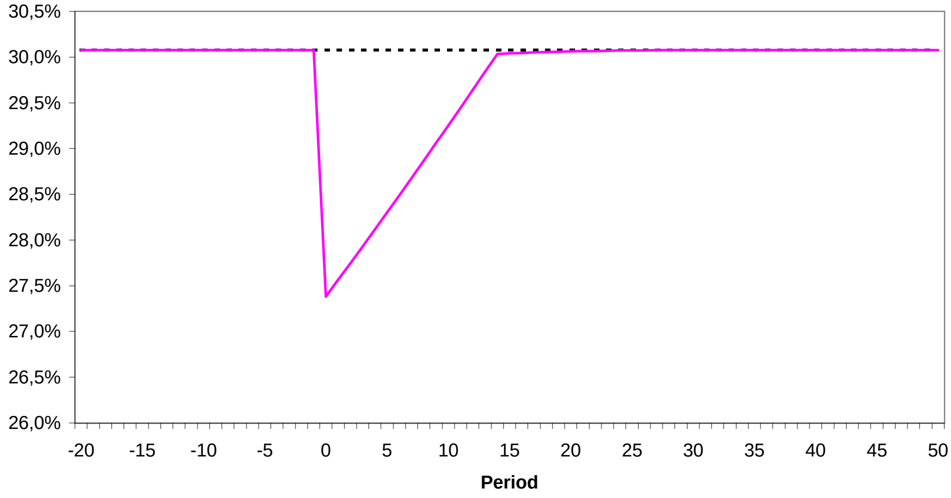


Figure 3: *Contribution rate if z = m + g*



initial level, b_0. This proves that the behavior of the contribution rate over time must follow a V-shaped curve. Figures 2 and 3 illustrate this pattern for a specific numerical example in which the exogenous parameters were set at the following values: $T = 55, m = -0,5\%, g = 0,5\%, r = 1\%$. All individuals born before period -40 retire at age $E_0 = 40$ while everyone born in period -40 or afterwards retires at age $E_1 = 41$. Therefore, the dependency ratio falls in period 0. The replacement rate is $n_0 = 70\%$ which leads to an contribution rate of $b_0 = 30.08\%$.

With respect to implicit taxes, it is well known (for a proof see, e.g., Sinn (2000)) that in a dynamically efficient economy, total discounted net payments of all presently living and future cohorts into an unfunded pension system are exogenously given and determined by the accumulated net gains of all past cohorts. This establishes that in the transition to a new steady state with higher retirement age, there must be cohorts whose implicit taxes fall relative to the case in which the old steady state is maintained. This is precisely because by Proposition 3, the net payments of later generations increase if z is at least equal to $m+g$.

Again, this general pattern is illustrated for our numerical example by Figures 4 and 5, in which the abscissa refers to the cohort entering the labor force at a certain date. As predicted, implicit taxes are higher in the new steady state. The increase is larger in Figure 4 in which the adjustment is actuarial. In this case, the first generation who works longer is unaffected because it is exactly compensated for its additional contributions and foregone benefits. The subsequent generations who work longer are better off at the expense of later-born generations because they profit from the fall in the contribution rate (see Figure 2). The later-born generations who face the new increased contribution rate, however, are worse off. In Figure 5 in which $z = m+g$ the pattern is similar. The main difference to Figure 4 is that the first generations who work longer face an increase in implicit taxes because their compensation falls short of an actuarial adjustment. The subsequent generations are better off, however, because they benefit from a lower contribution rate. Again, the later-born generations who face the same contribution rate as in the old steady state (see Figure 3) are worse off since they contribute for a longer time to the pay-as-you-go pension system.

Figure 4: *Implicit Taxes if z = r*

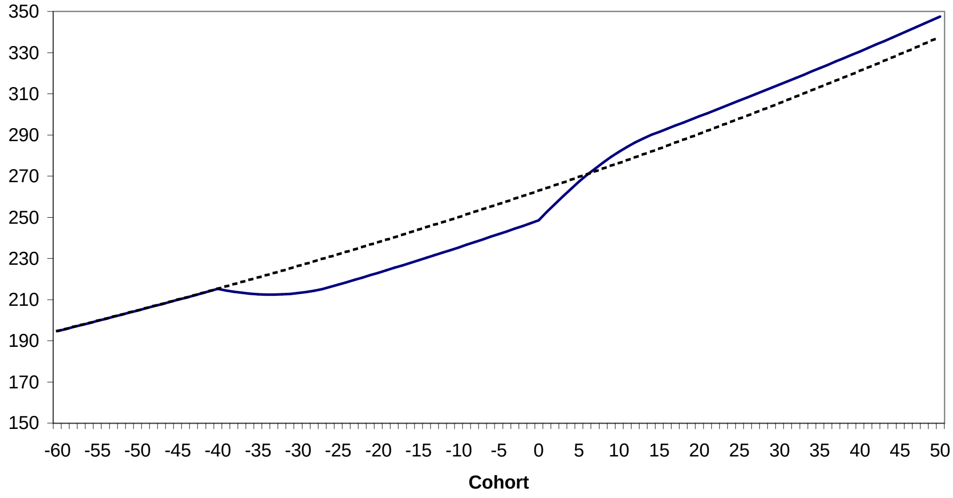
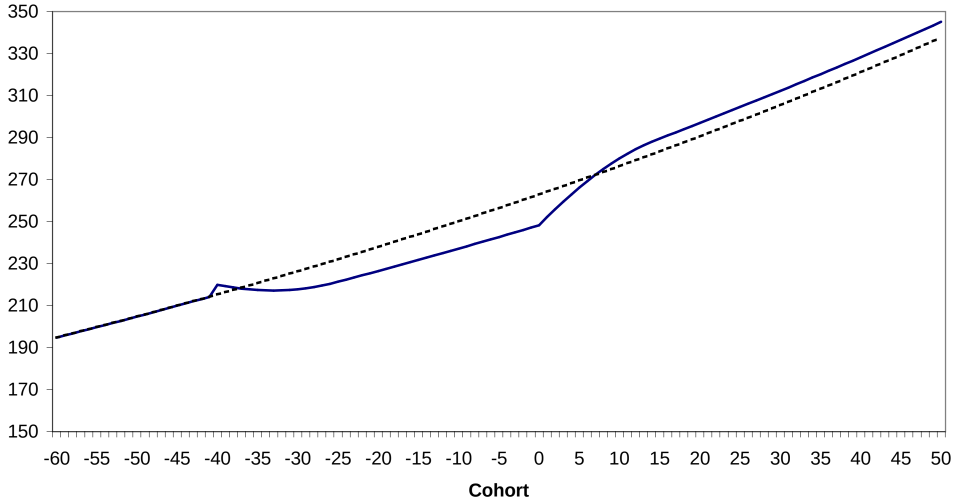


Figure 5: *Implicit Taxes if z = m + g*



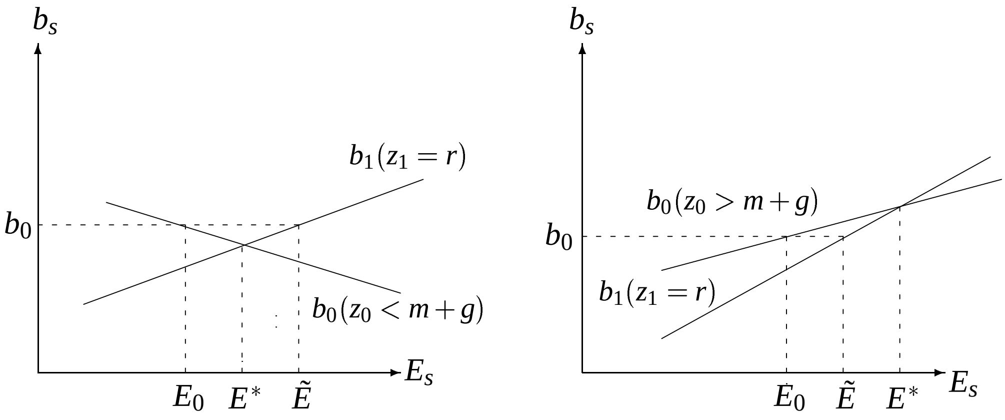


Figure 6: *Steady state contribution rates*

5. Cutting the benefit level at the initial retirement age

If the initial retirement age E_0 falls short of the standard retirement age E^*, then the replacement rate n_0 and the contribution rate b_0 at retirement age E_0 both fall if z is increased. Thus, there are two opposing effects with respect to the contribution rate and implicit taxes. On the one hand, the cut in the benefit level for a given retirement age tends to decrease both the contribution rate and implicit taxes in the new steady state (see Proposition 1). On the other hand, the increase in the retirement age works in the opposite direction if $z \geq m+g$. Which effect is dominant depends on the extent to which the retirement increases.

Figure 6a and 6b illustrate the opposing effects on the contribution rate. They show the steady state contributions rates b_s as a function of the steady state retirement age E_s. The rate of return on extra working time is increased from z_0 to $z_1 = r$ which turns the curve around the point $(E^*, b_s(E^*))$ (see Proposition 1). Since the new return of return z_1 is actuarial, the steady state contribution rate b_1 rises with the steady state retirement age E_s by Proposition 2. The two figures differ with respect to the initial rate of return on extra working time, z_0:

- in Figure 6a, the initial rate of return on extra working time is smaller than $m+g$. By Proposition 2, the steady state contribution rate b_0 thus decreases with E_s. Consequently, the new steady state contribution rate is lower than b_0 if $E_1 < \tilde{E}$ and higher than b_0 if $E_1 < \tilde{E}$. Since $z_0 < m+g$, we have $\tilde{E} > E^*$.
- in Figure 6b, z_0 is larger than $m+g$. By Proposition 2, the steady state contribution rate b_0 thus increases with E_s. Again, the new steady state contribution rate is lower than b_0 if $E_1 < \tilde{E}$ and higher than b_0 if $E_1 < \tilde{E}$. Since $z_0 > m+g$, we have $\tilde{E} < E^*$.

z_0	-1.5 %	- 1 %	- 0.5 %	0 %	0.5 %
$\tilde{E}$	52.1	49.8	47.4	45.0	42.5
$\hat{E}$	42.7	42.2	41.6	41.1	40.6

Table 1: Critical values of E as a function of z_0

The logic with respect to implicit taxes $\mathcal{T}_s(t,E)$ is analogous. From Proposition 3, we know that a sufficient condition for $\mathcal{T}_s(t,E)$ to be increasing in E is $z \geq m+g$. Furthermore, for a given retirement age, $\mathcal{T}_s(t,E)$ is proportional to b_s (see equation (8)). Hence, an increase in z turns the implicit tax curve counterclockwise around the point $(E^*, \mathcal{T}_0(t,E^*))$ because the contribution rate falls if $E < E^*$ and rises if $E > E^*$. For $z_1 = r$, there therefore exists a retirement age $\hat{E}$ such that the new steady state level of implicit taxes is lower than $\mathcal{T}_0(t,E)$ if $E_1 < \hat{E}$ and higher than $\mathcal{T}_0(t,E)$ if $E_1 > \hat{E}$. Whether $\hat{E}$ is smaller or larger than E^* depends on z_0:

- if $z_0 \geq m+g$, then $\hat{E} < E^*$ because $\mathcal{T}_0(t,E)$ is increasing in E due to Proposition 3.
- if $z_0 < m+g$, then it is not clear whether $\hat{E}$ is smaller or larger than E^* because $z_0 \geq m+g$ is only a sufficient but not a necessary condition for $\mathcal{T}_0(t,E)$ to be increasing in E. If z_0 is sufficiently large, then it is possible that $\hat{E} < E^*$.

Table 1 shows the values of $\tilde{E}$ and $\hat{E}$ depending on the initial rate of return z_0. The exogenous parameters were set at the following values: $T = 55, E_0 = 40, E^* = 45, m = -0,5\%, g = 0,5\%, r = z_1 = 1\%$. Both critical values of E are decreasing in z_0 because the cut in benefits at the initial retirement age is less pronounced the larger z_0. The critical value for the steady state contribution rate, $\tilde{E}$, is equal to E^* if $z_0 = m+g = 0\%$, and larger (smaller) than E^* if $z_0 < (>) m+g$. The critical value for implicit taxes, $\hat{E}$, is always smaller than E^* and $\tilde{E}$. Thus, the example shows that even though there is a cut in benefits as consequence of an increase in z, a relatively small increase in the retirement age may be sufficiently large to lead to higher implicit taxes in the new steady state. Furthermore, if $\hat{E} < E_1 < \tilde{E}$, implicit taxes may be higher in the new steady state although the contribution rate has fallen.

6. A non-redistributing adjustment scheme

Up to know, we have assumed that the adjustments of benefits to changes in lifetime contributions have to be made within a pure pay-as-you-go pension system, i.e. a system whose budget must be balanced in each period. In this section, we analyse an alternative scheme which relies on partial funding. Under this scheme, the level of retirement benefits is actuarially adjusted. Partial funding is used to avoid intergenerational redistribution if the retirement age changes. The scheme operates as follows:

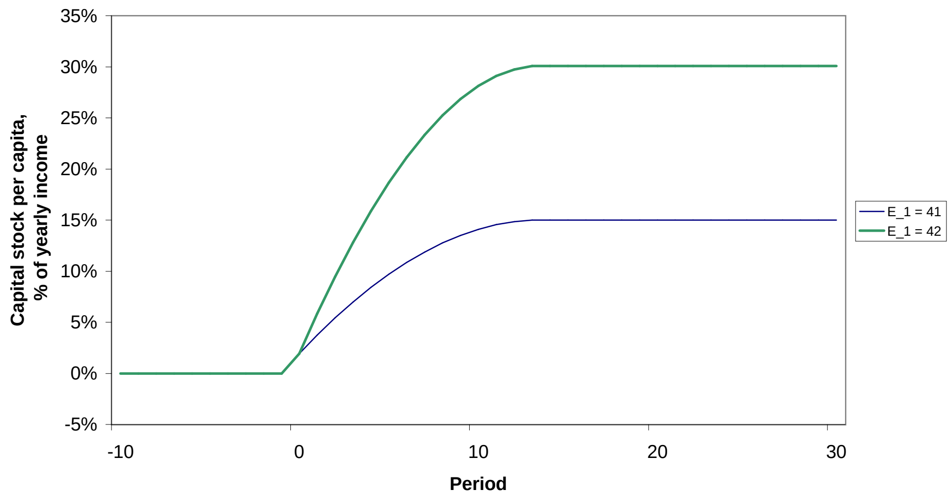
- the contribution rate is fixed to its initial level b_0.
- individuals can retire whenever they want.
- the replacement rate of each individual is calculated such that implicit taxes remain a certain predetermined level regardless of the retirement age. With $\mathcal{T}_d(t)$ as the desired level of implicit taxes for individuals entering the labor force at t, life expectancy T and retirement age $E(t)$, the replacement rate $n(t)$ can be calculated by solving (7) for n. This yields

$$n(t,E(t)) = \frac{b \int_{t}^{E(t)+t} e^{r(t-x)} w(x) dx - \mathcal{T}_d(t)}{\int_{E(t)+t}^{T+t} e^{r(t-x)} w(x) dx}. \tag{9}$$

- the capital market is used as a buffer if the budget of the pay-as-you-go pension scheme is not balanced.

For example, this scheme allows to switch to an actuarial adjustment of pension claims while fixing implicit taxes at their intial level $\mathcal{T}_0(t)$. For the parameters above, Figure 7 shows the resulting capital stock per capita as a percentage of yearly income if the retirement age increases to 41 or 42 periods for generations entering the labor force in period -40 and later. As a consequence, the pension system runs a surplus from period 0 to period 13 because the dependency ratio falls and earlier-born generations still obtain a pension according to the initial replacement rate $n_0 = 70\%$. The surplus is higher, the larger the increase in the retirement age. The surplus turns into a deficit after period 14 because then all living generations receive a higher pensions since the replacement rate rises to $n(t,41) = 77.4\%$ and $n(t,42) = 86.0\%$ according to formula (9). The fund, on the other hand, generates interest income out of the accumulated savings. In total, the capital stock per capita grows at the same rate as wages after period 14 because

Figure 7: *A non-redistributing adjustment scheme*



pensions grow at this rate as well. Clearly, it might be tempting to use this steady state capital stock for other purposes. Therefore, strict rules have to be formulated and obeyed as to the use of the accumulated funds for "additional" benefits.

7. Policy conclusions

In summarizing the main results of our paper, we can say that within a pure pay-as-you-go system the short-run and the long-run financial implications of inducing people to work longer can be quite different: If the inducement necessary to achieve a higher average retirement age involves making the pension adjustment formula actuarial (or at least paying a higher return to the extra contributions than to the average lifetime contribution) while leaving the benefit level at the initial retirement age unchanged, then the short-run drop in the contribution rate will be followed by a long-run increase in this rate. Even worse, if one thinks that the true measure of welfare loss from contributing to an unfunded pension system is the implicit tax, the same time pattern with respect to this indicator (gains for early cohorts, losses for later cohorts) prevails under even less restrictive conditions. A sufficient condition for a rising long-run implicit tax is that the adjustment formula uses the growth rate of the wage bill as the internal rate of return.

If the factual retirement age falls short of the legal retirement age, then making the pension adjustment formula actuarial has an additional effect. At the initial retirement age, the benefit level is cut which tends to lower the long-run contribution rate and the level of implicit taxes. However, this effect can be dominated by the consequences of an increased retirement age. In particular, relatively small increases in the retirement age may be sufficient to lead to a higher level of implicit taxes.

The main conclusion which follows from these results is that a policy supposed to bring about efficiency gains may lead to considerable intergenerational redistribution. In fact, making the adjustment formula more responsive to retirement age may shift even more of the implicit tax burden in the pay-as-you-go pension system to later-born generations. In this sense, the “pension crisis” is not solved by increasing the retirement age.

We have also shown that later generations can be insulated from any negative effects of changes in the retirement age of earlier generations if the adjustments are self-financing, using a capital stock in which the additional benefits and saved benefits are accumulated. However, such a scheme can only operate if strict rules are formulated and enforced with respect to the use of the accumulated funds. Furthermore, it has to be emphasized that the mixed system resulting from such a reform has nothing to do with the ”partial funding” proposals that are nowadays all too popular (see, e.g. Feldstein and Liebman (2001)) and whose merits are discussed elsewhere (Breyer (2001)). As opposed to these proposals, we do not claim to bring about “efficiency gains” by using partial funding. The objective of our scheme is to avoid undesirable intergenerational redistribution.

Appendix

Proof of Proposition 1

We prove Proposition 1 for $\frac{\partial n_s}{\partial z}$. The result for $\frac{\partial b_s}{\partial z}$ follows straightforward because $b_s = n_s q(m,T,E_f)$. First note that in a steady state, we obtain from (5)

$$(b_s + n^*) \int\limits_{t+E^*}^{t+E} e^{-z(x-t)} w(x) dx = (n_s - n^*) \int\limits_{t+E_s}^{t+T} e^{-z(x-t)} w(x) dx.$$

Substituting $b_s = n_s q(m,T,E_f)$ and assuming $z \neq g$[6], this simplifies to

$$n_s = n^* \frac{\int\limits_{t+E^*}^{t+T} e^{(g-z)(x-t)} dx}{\int\limits_{t+E_f}^{t+T} e^{(g-z)(x-t)} dx - q(m,T,E_f) \int\limits_{t+E^*}^{t+E_f} e^{(g-z)(x-t)} dx}.$$

Simplifying the integrals, this equation can be transformed to

$$n_s = n^* \frac{e^{(g-z)T} - e^{(g-z)E^*}}{\left(e^{(g-z)T} - e^{(g-z)E_f}\right) - q(m,T,E_f)\left(e^{(g-z)E_f} - e^{(g-z)E^*}\right)}.$$

Differentiating with respect to z yields

$$\frac{\partial n_s}{\partial z} = \frac{\overbrace{(T-E_f)e^{(g-z)(T+E_f)} - (E^* - E_f)e^{(g-z)(E_f+E^*)} - (T-E^*)e^{(g-z)(T+E^*)}}^{g(E_f)}}{\left(\left(e^{(g-z)T} - e^{(g-z)E_f}\right) - q(m,T,E_f)\left(e^{(g-z)E_f} - e^{(g-z)E^*}\right)\right)^2}.$$

Clearly, $\frac{\partial n_s}{\partial z} = 0$ for $E_f = E^*$. Furthermore, $g(E_f) < 0$ is $E_f < E^*$: Since

$$\frac{T-E^*}{T-E_f}\left\{(g-z)(T+E^*)\right\} + \frac{E^*-E_f}{T-E_f}\left\{(g-z)(E_f+E^*)\right\} > (g-z)(T+E_f),$$

the convexity of the exponential function implies

[6] The extension to $z = g$ is straighforward

$$\frac{T-E^*}{T-E_f}e^{(g-z)(T+E^*)}+\frac{E^*-E_f}{T-E_f}e^{(g-z)(E_f+E^*)}>e^{(g-z)(T+E_f)}$$

and thus

$$(T-E^*)e^{(g-z)(T+E^*)}+(E^*-E_f)e^{(g-z)(E_f+E^*)}>(T-E_f)e^{(g-z)(T+E_f)}$$

which is equivalent to $g(E_f)<0$. Thus, $\frac{\partial n_s}{\partial z}<0$ for $E_f<E^*$. Likewise, the convexity of the exponential function implies $\frac{\partial n_s}{\partial z}>0$ for $E_f>E^*$.□

Proof of Proposition 2

From (5), we obtain

$$\int_{t+E_1}^{t+E_0} e^{-z(x-t)}\left(b(x)+n_0\right)w(x)dx = \int_{t+E_1}^{t+T} e^{-z(x-t)}\left(n_1-n_0\right)w(x)dx.$$

Substituting $n_0 = b_0/q_0$, this equation simplifies to

$$b_1 = b_0 \frac{\frac{1}{q_0}\int_{t+E_0}^{t+T} e^{(g-z)(x-t)}dx}{\frac{1}{q_1}\int_{t+E_1}^{t+T} e^{(g-z)(x-t)}dx - \int_{t+E_0}^{t+E_1} e^{(g-z)(x-t)}dx}.$$

Using (1) and simplifying the integrals, we obtain

$$b_1 = b_0 \frac{\frac{e^{mT}-e^{m(T-E_0)}}{e^{m(T-E_0)}-1}\left(e^{(g-z)T}-e^{(g-z)E_0}\right)}{\frac{e^{mT}-e^{m(T-E_1)}}{e^{m(T-E_1)}-1}\left(e^{(g-z)T}-e^{(g-z)E_1}\right)-\left(e^{(g-z)E_1}-e^{(g-z)E_0}\right)}.$$

If $z < g$, then numerator and denominator are both positive. Thus $b_1 > b_0$ is equivalent to

$$\frac{e^{mT}-e^{m(T-E_0)}}{e^{m(T-E_0)}-1}\left(e^{(g-z)T}-e^{(g-z)E_0}\right) > \frac{e^{mT}-e^{m(T-E_1)}}{e^{m(T-E_1)}-1}\left(e^{(g-z)T}-e^{(g-z)E_1}\right)-\left(e^{(g-z)E_1}-e^{(g-z)E_0}\right)$$

which is equivalent to

$$e^{(g-z)E_0}-\frac{e^{mT}-e^{m(T-E_0)}}{e^{m(T-E_0)}-1}\left(e^{(g-z)T}-e^{(g-z)E_0}\right) < e^{(g-z)E_1}-\frac{e^{mT}-e^{m(T-E_1)}}{e^{m(T-E_1)}-1}\left(e^{(g-z)T}-e^{(g-z)E_1}\right).$$

Below it is shown that if $T > E$ and $a > m$, then the function

$$f(T,E,m,a) = e^{-aE}-\frac{e^{mT}-e^{m(T-E)}}{e^{m(T-E)}-1}\left(e^{-aT}-e^{-aE}\right)$$

is decreasing in E if $a > 0$ and increasing in E if $a < 0$. If $a = m$, then f does not depend on E. Furthermore, if $T > E$ and $a < m$, then $f(T,E,m,a)$ is increasing in E if $a > 0$ and decreasing in E if $a < 0$.

Thus, we can derive the following results which prove Proposition 2:

- If $a = z - g < 0$, we know that the function $f(T,E,m,z-g)$ is increasing in E if $a = z - g > m$ which is equivalent to $z > m + g$. Thus, for $z < g$, we have established that if $z > m + g$, then $b_1 > b_0$.

- If $a = z - g > 0$, the function $f(T,E,m,z-g)$ is decreasing in E if $a = z - g > m \Leftrightarrow z > m + g$. Thus, also for $z > g$, $z > m + g$, implies $b_1 > b_0$.

- If $a = z - g < 0$, we know that the function $f(T,E,m,z-g)$ is decreasing in E if $a = z - g < m$ which is equivalent to $z < m + g$. Thus, for $z < g$, we have established that if $z < m + g$, then $b_1 < b_0$.

- If $a = z - g > 0$, the function $f(T,E,m,z-g)$ is decreasing in E if $a = z - g < m \Leftrightarrow z > m + g$. Thus, also for $z > g$, $z < m + g$, implies $b_1 < b_0$.

- Finally, if $a = m \Leftrightarrow z = m + g$ and therefore $f(T,E,m,z-g)$ does not depend on E, then it is straightforward to show that $b_1 = b_0$.

Properties of the function $f(T,E,m,a)$

To prove the properties of the function $f(T,E,m,a)$, we derive the partial derivative of f with respect to E

$$\frac{\partial f}{\partial E} = \frac{e^{mT} - 1}{\left(e^{m(T-E)} - 1\right)^2} \underbrace{\left[ae^{-aE} - me^{m(T-E)-aT} - (a-m)e^{m(T-E)-aE}\right]}_{k(T,E,a,m)}.$$

This function is continuous in a if $m \neq 0$ and $T \neq E$.

The function $k(T,E,a,m)$ has the following properties:

- If $a = 0$ or $a = m$, then $k(T,E,a,m) = 0$ which implies $\dfrac{\partial f}{\partial E} = 0$. Thus, f does not depend on E if $a = m$.

- If $T = E$, then $k(T,E,a,m) = 0$.

- The partial derivative of k with respect to T is

$$\begin{aligned}\frac{\partial k}{\partial T} &= -m(m-a)e^{m(T-E)-aT} - m(a-m)e^{m(T-E)-aE} \\ &= m(a-m)e^{m(T-E)}\left[e^{-aT} - e^{-aE}\right].\end{aligned}$$

  For $a > m > 0$, we have $\frac{\partial k}{\partial T} < 0$ and, since $e^{mT} > 1$ for $m > 0$, $\frac{\partial f}{\partial E} < 0$ for $T > E$.

  For $a > m$ and $m < 0$, we need to distinguish two cases:

  - If $a > 0$, then $\frac{\partial k}{\partial T} > 0$ and, since $e^{mT} < 1$ for $m < 0$, $\frac{\partial f}{\partial E} < 0$ for $T > E$.
  - If $a < 0$, then $\frac{\partial k}{\partial T} < 0$ and $\frac{\partial f}{\partial E} > 0$ for $T > E$.

  For $a < m < 0$, we have $\frac{\partial k}{\partial T} > 0$ and, since $e^{mT} < 1$ for $m < 0$, $\frac{\partial f}{\partial E} < 0$ for $T > E$.

  For $a < m$ and $m > 0$, we need to distinguish two cases:

  - If $a > 0$, then $\frac{\partial k}{\partial T} > 0$ and, since $e^{mT} > 1$ for $m > 0$, $\frac{\partial f}{\partial E} > 0$ for $T > E$.
  - If $a < 0$, then $\frac{\partial k}{\partial T} < 0$ and $\frac{\partial f}{\partial E} < 0$ for $T > E$.

Therefore, the function f is decreasing in E if $a > 0$ and increasing in E if $a < 0$ if $T > E$ and $a > m$ and increasing in E if $a > 0$ and decreasing in E if $a < 0$ if $T > E$ and $a < m$.□

Proof of Proposition 3

For the implicit taxes of generation t, $\mathcal{T}_1(t)$, we obtain

$$\mathcal{T}_1(t) = b_1 \frac{w(0)}{g-r} \left\{ e^{(g-r)E_1} - 1 - \frac{e^{mT} - e^{m(T-E_1)}}{e^{m(T-E_1)} - 1} \left(e^{(g-r)T} - e^{(g-r)E_1} \right) \right\} \quad \text{(A.1)}$$

¿From Proposition 2, we have $b_1 > b_0$ if If $E_1 > E_0$ and $z > m+g$ and $b_1 = b_0$ if $z = m+g$. Since we assumed $r > m+g$, we know from the properties of the function $f(T,E,m,a)$ (see the proof of Proposition 2), that the term in brackets is decreasing in E if $a = r-g > 0 \Leftrightarrow r > g$. In this case $\frac{1}{g-r} < 0$. Therefore the terms besides b_1 are increasing in E. This implies $\mathcal{T}_1(t) > \mathcal{T}_0(t)$. Likewise, if $r < g$, the term in brackets is increasing in E and $\frac{1}{g-r} > 0$. Again, the terms besides b_1 are increasing in E and therefore $\mathcal{T}_1(t) > \mathcal{T}_0(t)$. $\square$

References

Blundell, R. and P. Johnson (1998), Pensions and Labor-Market Participation in the United Kingdom, *American Economic Review, Papers and Proceedings* 88, 168-172.

Börsch-Supan, A. and R. Schnabel (1998), Social Security and Declining Labor-Force Participation in Germany, *American Economic Review, Papers and Proceedings* 88, 173-178.

Börsch-Supan, A. (2000), A Blue Print for Germany's Pension Reform, Discussion Paper No. 581, Institut für Volkswirtschaftslehre und Statistik, Universität Mannheim.

Breyer, F. (2001), Why Funding is not a Solution to the "Social Security Crisis", Discussion Paper No. 254, DIW Berlin.

Breyer, F., M. Kifmann and K. Stolte (1997), Rentenzugangsalter und Beitragssatz zur Rentenversicherung, *FinanzArchiv* 54, 187 - 202.

Feldstein, M.S. and J.B. Liebman (2001), Social Security, N.B.E.R. Working Paper 8451, September (forthcoming in: Handbook of Public Economics).

Gruber, J. and D. Wise (1998), Social Security and Retirement: An International Comparison, *American Economic Review, Papers and Proceedings* 88, 158-163.

Gruber, J. and D. Wise, eds. (1999), *International Comparison of Social Security Systems*, Chicago.

Hassler, J. and A. Lindbeck (1997), Optimal Actuarial Fairness in Pension Systems: A Note, *Economics Letters* 55, 251-255.

Hassler, J. and A. Lindbeck (1999), Can and Should a Pay-as-you-go Pension System Mimic a Funded System?, mimeo, Institute for International Economic Studies, Stockholm University, June.

Kapteyn, A. and K. de Vos (1998), Social Security and Labor-Force Participation in the Netherlands, *American Economic Review, Papers and Proceedings* 88, 164-167.

Kifmann, M., and D. Schindler (2001), Smoothing the Implicit Tax Rate in a Pay-as-you-go Pension System, *FinanzArchiv* 57, 261 - 283.

Sinn, H.-W. (2000), Why a Funded Pension System is Needed and Why it is not Needed, *International Tax and Public Finance* 7, 389 - 410.