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Forecasting the Turns of German Business Cycle: Dynamic Bi-Factor Model with Markov Switching

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Abstract

In this paper a dynamic bi-factor model with Markov switching is proposed to measure and predict turning points of the German business cycle. It estimates simultaneously the composite leading indicator (CLI) and composite coincident indicator (CCI) together with corresponding probabilities of being in recession. According to the bi-factor model, on average, CLI leads CCI by 3 months at both peaks and troughs. The model-derived recession probabilities of CCI and those of CLI with a lag of 2–3 months capture the turning points of the ECRI's and OECD's reference cycle much better than the dynamic single-factor model with Markov switching.

Keywords: Forecasting turning points; composite coincident indicator; composite leading indicator; dynamic bi-factor model; Markov-switching

JEL classification: E32; C10

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1 Introduction

Burns and Mitchell (1946) defined business cycles as recurrent sequences of cumulative expansions and contractions diffused over a multitude of economic processes. Diebold and Rudebusch (1996) later summarized these classical cycles into two key features: co-movements among many macroeconomic indicators and asymmetry between the cyclical phases.

The first feature allows constructing the composite leading indicator (CLI) and composite coincident indicator (CCI) in order to measure and predict the state of affairs. CLI is used mostly to forecast the changes in CCI, since by its very nature the CLI is leading the CCI and hence the business cycle. The CCI is contemporaneous with the current state of the economy and can be considered as a proxy of the real GDP.

Both features can be jointly analyzed within a single model thanks to the contributions of Stock and Watson (1991), who re-introduced the dynamic factor model in the econometric research, and Hamilton (1989), who proposed a model with Markov-switching dynamics. The resulting dynamic single-factor model with Markov switching was suggested by Kim (1994) and Kim and Yoo (1995) and implemented for the first time by Chauvet (1998). It permits simultaneously capturing the co-movement and the cyclical asymmetry. This model has become already quite a standard tool of analyzing the business cycle. It has been successfully applied to the U.S. data by Chauvet (1998) and Kim and Nelson (1999), to the data of several European economies by Kaufmann (2000), to the Brazilian data by Chauvet (2002), to the Japanese data by Watanabe (2003), and to the Polish and Hungarian data by Bandholz (2005). In Germany the first such a model (although with only two component series, which raises a question of its identifiability) was estimated by Bandholz and Funke (2003).

The first attempt, as far as we know, to build a dynamic bi-factor model with Markov switching, in which the CLI and CCI are estimated simultaneously, was undertaken in Kholodilin (2001) and Kholodilin and Yao (2005). Using this model the turning points can be measured and predicted simultaneously and in a more timely manner. Kholodilin (2001) and Kholodilin and Yao (2005) applied the model to the U.S. data, whereas this paper concentrates on the German data.

The paper is structured as follows. In section 2 a dynamic bi-factor model with Markov-switching is set up. Section 3 estimates the model using the maximum likelihood method based on four leading indicators and five coincident indicators for the German economy. In section 4 the results are evaluated in sample. The last section concludes the paper.

2 Model

Basically, the dynamic factor model decomposes the dynamics of a group of observed time series into two unobserved sources of fluctuations: (1) the common factor or factors, which are common to all the component series or to the particular subgroups of them; (2) the specific, or idiosyncratic, factors — one per each observed series. In fact, the specific factors "explain" the remaining variation, which is left after the common factors were extracted.

The non-linear dynamic factor models, e.g. the model with Markov switching or with smooth transition autoregressive dynamics (see Kholodilin (2002)), in addition, take into account the possible asymmetries arising in the different states, or regimes. Here by the state we mean the phases of business cycle.

Moreover, the parametric dynamic factor models explicitly specify the dynamics of the latent (unobserved) factors. In the model examined in this paper both common and specific factors are modelled as autoregressive (AR) processes.

In the dynamic bi-factor model the set of the n observed variables is split in two disjoint subsets: n_{CLI} leading and n_{CCI} coincident indicators. The common dynamics of the time series belonging to each group are explained by a single common factor: CLI for the first group and CCI for the second group.

Thus, the complete dynamic bi-factor model with Markov switching can be written as a system of the three equations, where the first equation decomposes the observed dynamics into a sum of common and idiosyncratic factors and the last two equations specify the "law of motion" of the latent common and specific factors.

The decomposition of the observed dynamics:

$$\left(\begin{array}{c}\Delta y_t^{CLI}\\ \Delta y_t^{CCI}\end{array}\right)=\left(\begin{array}{cc}\Gamma_{CLI} & O_{n_{CLI}}\\ O_{n_{CCI}} & \Gamma_{CCI}\end{array}\right)\left(\begin{array}{c}\Delta f_t^{CLI}\\ \Delta f_t^{CCI}\end{array}\right)+\left(\begin{array}{c}u_t^{CLI}\\ u_t^{CCI}\end{array}\right) \tag{1}$$

The dynamics of common factors:

$$\begin{aligned}\left(\begin{array}{c}\Delta f_t^{CLI}\\ \Delta f_t^{CCI}\end{array}\right)=\left(\begin{array}{c}\mu^{CLI}(s_t^{CLI})\\ \mu^{CCI}(s_t^{CCI})\end{array}\right)+\left(\begin{array}{cc}\phi_{1.11} & \phi_{1.12}\\ \phi_{1.21} & \phi_{1.22}\end{array}\right)\left(\begin{array}{c}\Delta f_{t-1}^{CLI}\\ \Delta f_{t-1}^{CCI}\end{array}\right)+\\ \ldots+\left(\begin{array}{cc}\phi_{l.11} & \phi_{l.12}\\ \phi_{l.21} & \phi_{l.22}\end{array}\right)\left(\begin{array}{c}\Delta f_{t-l}^{CLI}\\ \Delta f_{t-l}^{CCI}\end{array}\right)+\left(\begin{array}{c}\varepsilon_t^{CLI}\\ \varepsilon_t^{CCI}\end{array}\right)\end{aligned} \tag{2}$$

The dynamics of idiosyncratic factors:

$$\begin{aligned}\left(\begin{array}{c}u_t^{CLI}\\ u_t^{CCI}\end{array}\right)=\left(\begin{array}{cc}\Psi_1^{CLI} & O_{n_{CLI}\times n_{CCI}}\\ O_{n_{CCI}\times n_{CLI}} & \Psi_1^{CCI}\end{array}\right)\left(\begin{array}{c}u_{t-1}^{CLI}\\ u_{t-1}^{CCI}\end{array}\right)+\\ \ldots+\left(\begin{array}{cc}\Psi_m^{CLI} & O_{n_{CLI}\times n_{CCI}}\\ O_{n_{CCI}\times n_{CLI}} & \Psi_m^{CCI}\end{array}\right)\left(\begin{array}{c}u_{t-l}^{CLI}\\ u_{t-l}^{CCI}\end{array}\right)+\left(\begin{array}{c}\eta_t^{CLI}\\ \eta_t^{CCI}\end{array}\right)\end{aligned} \tag{3}$$

where Δy_t^{CLI} and Δy_t^{CCI} are the $n_{CLI}\times 1$ and $n_{CCI}\times 1$ vectors of the observed leading and coincident variables in the first differences; Δf_t^{CLI} and Δf_t^{CCI} are the latent common factors in the first differences; u_t^{CLI} and u_t^{CCI} are the $n_{CLI}\times 1$ and $n_{CCI}\times 1$ vectors of the latent specific factors; ε_t^{CLI} and ε_t^{CCI} are the disturbances of the common factors, whereas η_t^{CLI} and η_t^{CCI} are the $n_{CLI}\times 1$ and $n_{CCI}\times 1$ vectors of disturbances of the specific factors. Γ_{CLI} and Γ_{CCI} are the $n_{CLI}\times 1$ and $n_{CCI}\times 1$ factor loadings vectors linking the observed series to the common factors. $\mu^{CLI}(s_t^{CLI})$ and $\mu^{CCI}(s_t^{CCI})$ are the state-dependent intercepts of CLI and CCI. ϕ_i are the autoregressive coefficients of common factors; and Ψ_i^{CLI} and Ψ_i^{CCI} are the matrices of the autoregressive coefficients of the idiosyncratic factors. O_n and $O_{n\times m}$ are $n\times 1$ vector and $n\times m$ matrix of zeros, correspondingly. Finally, s_t^{CLI} and s_t^{CCI} are the unobserved state variables following a first-order Markov chain process, which is summarized by the transition probabilities matrix, whose characteristic element is $p_{ij} = prob(s_t = j|s_{t-1} = i)$, that is, the probability of being today in regime j given that yesterday's regime was i.

In the two-regime (expansion-recession, or high-low growth rate) case a state variable s_t takes two values: 0 or 1. Depending on the regime, the common factor's intercept assumes different values: low in contractions and high in expansions. Thus, the common factors grow faster during the upswings and slower (or even decline) during the downswings of the economy.

The dynamic bi-factor model with Markov switching described above is based on the following **assumptions**:

- *The common factors' disturbances*, $\varepsilon_t = (\varepsilon_t^{CLI}|\varepsilon_t^{CCI})'$, and *the specific factors' disturbances*, $\eta_t = (\eta_t^{CLI}|\eta_t^{CCI})'$, *are mutually and serially uncorrelated*:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma(s_t) & O_{2\times n} \\ O_{n\times 2} & \Omega \end{pmatrix}\right) \tag{4}$$

where $\Sigma(s_t)$ is the diagonal 2×2 variance-covariance matrix of common factors, with the common factor residual variances on the main diagonal, $\sigma^2_{CLI}(s_t)$ and $\sigma^2_{CCI}(s_t)$, which may be state dependent; Ω is the diagonal $n \times n$ variance-covariance matrix of the idiosyncratic disturbances.

- *There is no Granger causality between the common factors*: $\phi_{i.12} = \phi_{i.21}$ $\forall i \in [1, l]$. This is a bit strong restriction. Together with the previous assumption it implies that the only way the CLI is linked to the CCI is through the intercept, when the state variables of both common factors are interdependent. There can also exist a relationship between the volatilities of two common factors when their residual variances are state dependent and their state variables are related. In principle, this assumption can be relaxed without changing much the outcomes of the model. Here it is used only for the sake of parsimony.

- This assumption specifies the *state variable dynamics*. In fact, we can consider three cases:

  (a) there is a single state variable, s_t, such that $s_t^{CLI} = s_t^{CCI}$, in other words, the non-linear dynamics of the common factors are identical;

(b) s_t^{CLI} and s_t^{CCI} are completely independent;

(c) s_t^{CLI} and s_t^{CCI} are neither identical as in (a) nor independent as in (b) but interdependent.

Now let us consider in more detail the specification of the Markov switching in the non-linear dynamic bi-factor model under inspection. In the case (a) above there is only one state variable and it all boils down to the standard two-regime Markov switching model as in Hamilton (1989). The transition probabilities matrix then looks like:

$$\pi = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix} \tag{5}$$

In the cases (b) and (c) there are two state variables: one per each common factor. This means that each composite indicator has its own recessions and expansions. Therefore to describe the whole process, a compound state variable, comprising both s_t^{CLI} and s_t^{CCI}, should be constructed as it is done in Phillips (1991). This compound variable will have four different states:

Composite	$s_t = 1$	$s_t = 2$	$s_t = 3$	$s_t = 4$
Leading	$s_t^{CLI} = 1$ $\uparrow$	$s_t^{CLI} = 2$ $\downarrow$	$s_t^{CLI} = 1$ $\uparrow$	$s_t^{CLI} = 2$ $\downarrow$
Coincident	$s_t^{CCI} = 1$ $\uparrow$	$s_t^{CCI} = 1$ $\uparrow$	$s_t^{CCI} = 2$ $\downarrow$	$s_t^{CCI} = 2$ $\downarrow$

where the arrows show whether the economy goes up (expansion) or down (recession).

The dimension of the transition probabilities matrix is then 4×4 and its structure depends on which of the cases is assumed: (b) or (c). In the case (b), when both state variables are independent, the transition probabilities matrix of the compound state variable is a Kronecker product of the transition probabilities matrices of the individual state variables: $\pi = \pi^{CLI} \otimes \pi^{CCI}$. This is equivalent to:

$$\pi = \begin{pmatrix} p_{11}^{CLI} p_{11}^{CCI} & (1-p_{11}^{CLI}) p_{11}^{CCI} & p_{11}^{CLI}(1-p_{11}^{CCI}) & (1-p_{11}^{CLI})(1-p_{11}^{CCI}) \\ (1-p_{22}^{CLI}) p_{11}^{CCI} & p_{22}^{CLI} p_{11}^{CCI} & (1-p_{22}^{CLI})(1-p_{11}^{CCI}) & p_{22}^{CLI}(1-p_{11}^{CCI}) \\ p_{11}^{CLI}(1-p_{22}^{CCI}) & (1-p_{11}^{CLI})(1-p_{22}^{CCI}) & p_{11}^{CLI} p_{22}^{CCI} & (1-p_{11}^{CLI}) p_{22}^{CCI} \\ (1-p_{22}^{CLI})(1-p_{22}^{CCI}) & p_{22}^{CLI}(1-p_{22}^{CCI}) & (1-p_{22}^{CLI}) p_{22}^{CCI} & p_{22}^{CLI} p_{22}^{CCI} \end{pmatrix} \tag{6}$$

Under the hypothesis (c) the two individual state variables are assumed to be interrelated in the sense that the CLI is supposed to enter the recessions (expansions) several periods earlier than the CCI. This is a kind of intermediate case between completely independent and identical state variables corresponding to CLI and CCI.

As Phillips (1991) remarks, the model with an integer lag exceeding one period would require a Markov process with the order higher than 1. However, the real-valued (positive) lag can be modelled with a first-order Markov process by constructing the following transition probabilities matrix:

$$\pi = \begin{pmatrix} p_{11} & 1-p_{11} & 0 & 0 \\ 0 & 1-\frac{1}{\mathcal{A}} & 0 & \frac{1}{\mathcal{A}} \\ \frac{1}{\mathcal{B}} & 0 & 1-\frac{1}{\mathcal{B}} & 0 \\ 0 & 0 & 1-p_{22} & p_{22} \end{pmatrix} \tag{7}$$

where $\mathcal{A}$ and $\mathcal{B}$ are the expected leads in the recession and expansion, correspondingly. The expected lead of CLI with respect to CCI when entering the low-growth regime ($s_t = 2$) is:

$$\mathcal{A} = 1 + p(s_t = 2|s_{t-1} = 2) + p(s_t = 2|s_{t-1} = 2)^2 + \ldots \tag{8}$$

that is

$$\mathcal{A} = \frac{1}{1 - p(s_t = 2|s_{t-1} = 2)} \tag{9}$$

We are going to examine three models corresponding to the three above stated cases. By comparing these models one can test the underlying hypotheses. Model (c) is an unrestricted version of model (a). Thus, by imposing restrictions on the parameters $\mathcal{A}$ and $\mathcal{B}$ one can test the hypothesis of identical versus interdependent with lead non-linear cyclical dynamics, for example, using the Likelihood-Ratio test. The null of identical state variable implies that $\mathcal{A} = \mathcal{B} = 1$. The formal testing of (a) versus (b) or (c) versus (b) is a more complicated enterprise. Under the null of identical or interdependent state variable the second state variable is not identified. It means that we are confronting the famous nuisance parameter problem similar, for instance, to testing the hypothesis of two regimes versus three regimes.

In order to estimate the above dynamic factor models with Markov switching, they are expressed in the state-space form:

$$\Delta y_t = \mathscr{A} x_t \tag{10}$$

$$x_t = \alpha(s_t) + \mathscr{C} x_{t-1} + v_t \tag{11}$$

where $x_t = (f_t|u_t)'$ is the state vector containing stacked on top of each other the vector of common factors and the vector of specific factors; v_t is the vector of the common and idiosyncratic factors' disturbances with mean zero and variance-covariance matrix $\mathscr{Q}$; $\alpha(s_t) = (\mu_{CLI}(s_t), \mu_{CCI}(s_t), ..., 0)'$ is the state-dependent vector of intercepts. The structure of the system matrix $\mathscr{A}$ is defined as in Kholodilin (2001), while matrix $\mathscr{C}$ has somewhat different structure given the fact that the assumption of Granger causality between the common factors has been removed from this model:

$$\mathscr{C} = \begin{pmatrix} \Phi^{CLI} & O & & & 0 \\ O & \Phi^{CCI} & & & \\ & & \Psi^1 & & \\ & & & \ddots & \\ 0 & & & & \Psi^n \end{pmatrix} \tag{12}$$

where matrices Φ^{CLI}, Φ^{CCI}, and Ψ^i $(i = 1, ..., n)$ are formulated exactly the same way as in Kholodilin (2001).

There are different ways of estimating the unknown parameters and the latent factors (maximum likelihood, EM, MCMC techniques — see Kim and Nelson (1999) for more details). Here we applied the maximum likelihood method with log-likelihood function obtained using Kalman filter recursions. To save space we will not present them here, referring the reader, for instance, to Hamilton (1994) who gives a very clear and systematic explanation of the Kalman filter methodology.

Note that in the case (b) two separate dynamic single-factor models for CLI and CCI can be estimated, instead of a rather cumbersome bi-factor model.

3 Estimation and evaluation

3.1 Estimation

The component series of both CLI and CCI were picked up from a dataset of 147 series representing various sectors of German economy and available at the site of Deutsche Bundesbank in the Internet. The selection criterion was the cross-correlations between the growth rates of these series. The following selection procedure was employed.

First, the coincident series were selected. We assumed that the industrial production is a natural proxy of the monthly composite coincident indicator. Hence as component series of CCI were chosen those variables, for which the cross-correlation with UXNI63 achieves its maximum at zero lag and is the highest compared to the remaining series. Surprisingly the new orders series, which is normally thought to be a leading indicator, turned out to be rather coincident one. This finding contradicts, e.g., the conclusions of Fritsche and Kuzin (2005). However, in the cited article the leading nature of the indicators is determined with respect to a particular reference chronology series and so the conclusions depend on the choice of such a chronology. On the other hand, the evidence of the leading nature of new orders is not very strong. On the other hand, the cross-correlation analysis conducted in Fritsche and Stephan (2000) leads to the same conclusion as ours about the coincident nature of the new orders in manufacturing. The components of CLI are leading the industrial production and CCI and are highly but not too much correlated among themselves. It is important to avoid collinearity of the components and at the same time to guarantee their high enough correlation, which is indispensable for the extraction of the common factor.

The data employed in this study are four leading indicators and five coincident indicators. They are shortly described in Table 1 of Appendix. The component series of CLI are: Ifo expectations indicator (IFO), 12-month interest rate at Frankfurt's money market (SU0253), DAX stock exchange index (WU3141), and HWWA-index of raw materials' prices "Euroland" (YU0516). The components of CCI are: the index of industrial production (UXNI63), new orders in manufacturing (UXA001), retail trade (UXHK87), job vacancies (UUCC04), and exports of goods in FOB prices (EU2001). UUCC04 is a broad measure of the labor input of the overall economy. The remaining four indicators represent three major sectors of German economy: manufacturing (UXNI63 and UXA001), trade (UXHK87), and foreign trade (EU2001), which is especially important for Germany. Together, according to the German national accounting of 2004, the industrial production and internal trade account for about 43% of the German GDP, while the exports represent 37% of the GDP.

All the series are tested for unit roots using the augmented Dickey-Fuller test. Each series is tested for random walk with drift and deterministic trend, random walk with drift, and random walk only. It turns out that all the series have unit root. All the series are also tested for cointegration. The cointegration between the leading as well as between the coincident series was detected. As in Stock and Watson (1991) and Kim and Nelson (1999), the first differences of the logarithms of the original time series are taken and then demeaned and standardized.

We estimated three dynamic bi-factor models with Markov-switching corresponding to the hypotheses (a), (b), and (c) mentioned in the previous section. Recall that these hypotheses are defined by the following equations:

(a) Hypothesis of identical Markov-switching dynamics of CLI and CCI — equations (1) – (4) and (5). The model, in which CLI and CCI enter the recessions and expansions simultaneously, without any leads.

(b) Hypothesis of independent Markov-switching dynamics of CLI and CCI — equations (1) – (4) and (6). This model can be alternatively estimated as two separate dynamic single-factor models with two-regime switching based on coincident and leading indicators correspondingly. Each of the separate dynamic single-factor models is identical to that of Chauvet (1998) and Kim and Nelson (1998).

(c) Hypothesis of interdependent Markov-switching dynamics of CLI and CCI — equations (1) – (4) and (7). A dynamic bi-factor model with interdependent cyclical dynamics that results in four-regime switching: two regimes for the leading indicator and two regimes for the coincident indicator. The composite leading factor (or CLI) switches between its regimes earlier than the composite coincident indicator (or CCI).

We determined the lag structure by balancing two requirements: on the one hand, our composite indicators should have some dynamics, that is, the lag order must be higher than zero; on the other hand, due to the short sample the lag order cannot be too high. Therefore both the common and idiosyncratic factors are specified as AR(1). For the identification purposes, the first loading factor in all the models is normalized to 1, i.e. the loading factor of IFO for leading indicators and that of UXNI63 for coincident indicators.

The parameter estimates and their standard errors corresponding to single-factor models of CLI and CCI and to bi-factor models are reported in Tables 2 through 5. The parameters of linear and Markov-switching models are reported. Both MS models of composite leading and coincident indicators clearly distinguish between two regimes of positive and negative growth rates.

ΔCLI has a high positive autoregressive coefficient varying for different models in the interval between 0.462 and 0.752, whereas ΔCCI has a negative, not always significant autoregressive coefficient varying between -0.228 and -0.113. Correspondingly, as one can see on Figure 1, the CLI's profile is much smoother than that of CCI.

Factor loadings are positive and mostly significant, except for that of UUCC04. They are somewhat higher in the bi-factor model. The magnitudes of the factor loadings are roughly the same, which is an indirect indicator of the approximately equal weights of the components in both CLI and CCI.

The estimates of the single- and bi-factor models, linear and Markov-switching models do not differ very much. The estimated lead-time of CLI over CCI, which can be estimated under the hypothesis (c), is approximately 3 for both peaks and troughs. For the CLI estimated as a single-factor model with Markov switching the transition probability of being today in expansion given that yesterday was expansion, $p_{11} = 0.917$ is much higher than the transition probability of being today in recession given that yesterday was recession, $p_{22} = 1 - p_{12} = 0.728$, implying that the expected duration of expansions is approximately equal to 12 months and is greater than the expected duration of recessions, which is equal to 4 months. These durations are a little bit too small, especially the expected duration of recessions. The estimates of the dynamic single-factor model with Markov switching for CCI (see Table 3) suggest that the expected duration of CCI's expansions is about 53 months and the expected duration of CCI's recessions is 8 months. The same durations result from the bi-factor model with Markov switching corresponding to the hypothesis (a) — see Table 4. In fact, the conditional recession probabilities obtained from this model almost coincide with those obtained from the single-factor model of CCI. Finally, according to the dynamic bi-factor model with Markov switching corresponding to the hypothesis (c) — Table 5 — the expected duration of expansions is 17 months (more than 5 quarters, or roughly 1.5 years) and the expected duration of recessions is about 4 months (1 quarter).

We can also test which of the three hypotheses fits the data best. At least this can be done for the models (a) and (c), where for the comparison the likelihood ratio test can be employed. According to the likelihood ratio test, with two degrees of freedom and the test statistic equal to $LR = 37.3 > LR_{0.01}(2) = 9.21$ the model (c) is significantly different from the model (a), and hence the null of completely identical non-linear cyclical dynamics can be rejected in favor of CLI leading the CCI.

3.2 Evaluation

Unlike for the USA, we do not have any generally accepted business cycle chronology for Germany. Among the few available alternative chronologies[1] we selected two, to which the recession probabilities of our non-linear models will be compared, namely: ECRI's growth cycle dating[2] and the OECD's dating. Both datings are reported in the first four columns of Table 6.

The first chronology relies upon the growth rates. This dating seems to be more appropriate to the case of Germany than the so-called ECRI's classical cycle chronology, because over the whole sample, 1991–2005, the absolute decline in level of GDP was a rather rare event, whereas the deceleration of the growth rates happens much more frequently and is an event, which is worth predicting.

The profiles of CLI and CCI estimated from the dynamic single- and bi-factor models with and without Markov switching are plotted on Figure 1.

[1] We do not consider here the chronology obtained by Heilemann and Münch (1999), since it ends in 1994, although their approach appears to be fruitful and can be applied in the future research for a sample covering more recent past.

[2] An important disadvantage of this chronology is that it was updated for the last time in November 2003.

The filtered and smoothed conditional probabilities of recessions corresponding to these models are plotted in Figures 2 and 3 respectively. We do not display the conditional recession probabilities of the model (a), because they are almost identical to those of the single-factor model for CCI.

In the bi-factor models with interdependent non-linear dynamics (two state variables), there are four regimes: two for the common leading factor and the other two for the common coincident factor. In the former case, the CLI's recession probabilities are obtained as the sum of the conditional probabilities of regimes 2 and 4 ("low leading factor and high coincident factor" and "low leading factor and low coincident factor"), while CCI's recession probabilities are the sum of the probabilities of regimes 3 and 4. The recession probabilities stemming from the three models — (a), (b), and (c) — are very different. Whereas the conditional recession probabilities derived from the model (a) are almost identical with those corresponding to the single-factor model of CCI (model (b) estimated as two separate single-factor models), the estimated conditional probabilities derived from model (c) differ from those of model (a) and (b).

Under the hypothesis of completely independent cycles of CLI and CCI (model (b)), the conditional recession probabilities of CLI are quite volatile even after smoothing — see the upper panel of Figure 2. The shaded areas represent the recessionary phases of both official chronologies. The CLI's recession probabilities signal five recessions, the first four of which roughly coincide with the ECRI's growth cycle, whereas all the CLI's recessions fall into the shaded areas of the OECD's dating. The CCI's conditional recession probabilities, plotted on the bottom panel of Figure 2, give only two signals of downswings: in 1992 and in 2001. The second signal is rather weak being lower than 0.5. The CCI's probabilities are thus detecting a kind of classical cycle and not the growth cycle.

The situation changes drastically when model (c) — stating that the cyclical dynamics of CCI are lagging behind those of CLI — is estimated. As can be seen on Figure 3, the conditional recession probabilities of CLI underwent rather minor change. By contrast, the recession probabilities of CCI started resembling those of CLI with a clearly visible lag. Now both CLI and CCI signal five recessions.

The dates of the two official reference chronologies can also be compared to the dates derived from the non-linear models considered in this paper — see Table 6. There the model-derived dates are obtained using the the algorithm of Chauvet and Piger (2003), which is applied here to the smoothed recession probabilities. Both the reference chronologies identify four recessions during the period under inspection. These recessions, save for the last one, are coinciding quite well. The recession probabilities of CLI and especially of CCI derived from the model (c) have an improved concordance to the reference cycles compared to those derived from the model (b).

When the predicted reference chronology is known, the above comparison of the in-sample forecasting performance can be formalized using some criterion measuring the difference between the reference chronology and the model-derived dating. We use the Quadratic Probability Score (QPS) proposed in Brier (1950), which is based on probabilities derived from each model. Let P_t be the conditional probability that the economy is in recession, estimated from the model; let R_t be the ECRI- or OECD-defined chronology (1 if recession, 0 otherwise), and the slightly modified version of QPS is given by:

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(P_{t-\tau} - R_t) \tag{13}$$

where τ is the time shift accounting for the possibly leading character of the recession probabilities and P_t. QPS varies between 0 and 1, with a score of 0 corresponding to perfect accuracy. This is the unique proper scoring rule that is only a function of the discrepancy between realizations and model-derived probabilities (see Diebold and Rudebusch (1989) for further discussion). In order to compare the forecasting accuracy of different models; that is, to test whether the differences in the QPS of each model are significant, a test statistic developed by Diebold and Mariano (1995) was used. The null hypothesis states no difference between the predicted accuracy of the pair of models being compared. Given a sample path $\{d_t\}_{t=1}^{T}$ of a loss differential series ($P_{t-\tau} - R_t$ in the above definition), we have

$$\sqrt{T}(\bar{d} - \mu) \xrightarrow{d} N(0, 2\pi\widehat{f}_d(0)) \tag{14}$$

where $\bar{d}$ is the sample mean loss differential, $f_d(0)$ is the spectral density of the loss differential at frequency zero, and $\gamma_d(l) = E((d_t - \mu)(d_{t-l} - \mu))$. The

Diebold-Mariano (DM) statistic for testing the H0 of equal forecast accuracy is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \widehat{f}_d(0)}{T}}} \tag{15}$$

which is standardized and hence asymptotically distributed as $N(0,1)$. The DM statistics are calculated with a rectangular window of length 15.

The in-sample predicting performance of the non-linear models estimated in this paper is reported in Table 7. It is measured by the QPS computed with respect to two above mentioned alternative reference chronologies. The in-sample performance is evaluated for different lags τ (see equation (13)) in order to account for the fact that CLI may lead the reference chronologies. In fact, the QPS is computed between the non-negative lags of the recession probabilities and binary reference datings.

The overall in-sample forecasting accuracy of all the examined models is far from being satisfactory — the QPS are very high. On average the model-derived chronologies correspond better to the ECRI's growth cycle. This is mainly due to the differences in the reference chronologies in the last few years of the sample. It is the recession probabilities of CCI estimated in model (c) that are characterized by the best fit. According to the DM-statistics, which were computed but are not presented here to save the space, the chronologies of CCI derived from the model (c) have *significantly* better in-sample forecasting accuracy than the chronologies of CCI derived from the model (b). This means that imposing condition of CLI's cycle leading the CCI's cycle, compared to the model where these two indicators are completely independent, pays off. The minima of QPS for the CLI's chronology derived from models (b) and (c) are achieved at lags 2 and 3, which perfectly corresponds to the estimates of the expected leads in recession and expansion. The performance of CLI is higher than that of model (a) and of CCI in model (b).

Given the uncertainty about the reference chronology, the out-of-sample forecasting exercise is hardly possible. Of course, we can make the forecasts with our models but their eventual performance will be affected both by the forecasting errors and by the fact that it is not sure whether the selected reference chronologies reflect well the turning points of the German business cycle.

Finally, let us make a few remarks about the developments in the nearest future that can be inferred from the recession probabilities in the very end of the sample. Both from Figure 2 and 3 one can see that the conditional probability of getting into recession has increased in the last few months. This increase is especially pronounced in the probabilities derived from the single-factor model of CLI. It is, however, substantially less noticeable when the bi-factor model is considered. In any case the recession probabilities never exceed 0.5. Therefore for the moment we may expect an outbreak of recession soon or it can well be just a false alarm. To confirm or reject the signal a few more observations are needed.

4 Summary

The paper estimates a dynamic bi-factor model with Markov-switching based on four selected leading indicators and five coincident indicators of the German economy. This enables us to measure and predict the turning points of CLI and CCI simultaneously. Three alternative hypotheses were examined: (a) switches between the recessionary and expansionary phases of CLI and CCI are identical; (b) these switches happen independently; and (c) the switches of the CLI precede those of CCI with some positive lead. In the latter case the model estimates the expected lead time of CLI over CCI as approximately 3 months at both peaks and troughs. The turning point dates derived from bi-factor models based on coincident indicators have a correspondence with ECRI- and OECD-defined recessions.

The test of in-sample performance of single-factor and bi-factor models relative to ECRI- and OECD-defined turning points is conducted using the modified quadratic probability score and DM test statistics. CCI in model (c) has the lowest QPS and hence the highest conformity to the reference chronologies. Since CLIs lead the reference cycles, they have a high QPS and low conformity with the ECRI's and OECD's recessions at zero lag. The estimated CLIs have the closest match with the latter when they are moved 2–3 months forward. Therefore, this suggests that the estimated CLI and its probability of recession can be used as a predictor of the reference dates with an average lead of 3 months.

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Appendix

Table 1: Component series of German CLI and CCI, monthly data 1991:1–2005:3

Code	Description
	Composite leading indicator
IFO	Das Ifo Geschäftsklima für die Gewerbliche Wirtschaft, Erwartungen (R3), 2000=100, SA)
SU0253	Geldmarktsätze am Frankfurter Bankplatz, Dreimonatsgeld, Monatsdurchschnitt
WU3141	DAX-Index, 1987 = 1000
YU0516	HWWA-Rohstoffpreisindex "Euroland"
	Composite coincident indicator
UXNI63	Produktion Industrie
UXA001	Auftragseingang in Verarbeitendes Gewerbe, Werte, arbeitstäglich bereinigt
UXHK87	Einzelhandelumsatz, Volumen, kalenderbereinigt
UUCC04	Offene Stellen Insgesamt
EU2001	Außenhandel, Warenhandel, Ausfuhr (fob)

Source: Database of Deutsche Bundesbank

Table 2: Estimates of the parameters of German single-factor linear and Markov-switching model of CLI, monthly data 1991:1–2005:3

	Linear LL=-909.9			Markov-switching LL=-907.5		
	Coeff	t-stat	p-value	Coeff	t-stat	p-value
p_{11}	—	—	—	0.917	13.1	0.0
p_{12}	—	—	—	0.272	1.60	0.111
μ_1	—	—	—	0.170	2.60	0.010
μ_2	—	—	—	-0.554	-2.28	0.024
γ_{SU0253}	0.550	2.07	0.040	0.698	2.32	0.022
γ_{WU3141}	0.631	2.76	0.006	0.865	2.93	0.004
γ_{YU0516}	0.580	2.20	0.029	0.657	2.29	0.023
ϕ_{CLI}	0.752	9.33	0.0	0.443	1.39	0.167
ψ_{IFO}	0.081	0.58	0.561	0.179	1.52	0.131
ψ_{SU0253}	0.375	4.92	0.0	0.370	4.80	0.0
ψ_{WU3141}	-0.162	-1.80	0.074	-0.211	-2.48	0.014
ψ_{YU0516}	0.294	3.78	0.0	0.305	3.97	0.0
σ_{CLI}	0.148	1.87	0.064	0.018	0.41	0.684
σ_{IFO}	0.648	5.10	0.0	0.737	6.74	0.0
σ_{SU0253}	0.749	8.49	0.0	0.747	8.66	0.0
σ_{WU3141}	0.837	7.79	0.0	0.783	7.76	0.0
σ_{YU0516}	0.802	8.48	0.0	0.814	8.73	0.0

Note: LL is the log-likelihood function value

Table 3: Estimates of the parameters of German single-factor linear and Markov-switching model of CCI, monthly data 1991:1–2005:3

	Linear LL=-1102.4			Markov-switching LL=-1097.3		
	Coeff	t-stat	p-value	Coeff	t-stat	p-value
p_{11}	—	—	—	0.981	42.6	0.0
p_{12}	—	—	—	0.121	1.26	0.208
μ_1	—	—	—	0.134	1.41	0.161
μ_2	—	—	—	-0.923	-3.01	0.003
γ_{UXA001}	0.658	2.12	0.035	0.789	3.90	0.0
γ_{UXHK87}	0.281	2.58	0.011	0.273	2.30	0.023
γ_{UUCC04}	0.096	0.99	0.323	0.119	1.06	0.289
γ_{EU2001}	0.414	2.11	0.037	0.482	3.25	0.001
ϕ_{CCI}	-0.131	-0.68	0.498	-0.263	-1.87	0.063
ψ_{UXNI63}	-0.373	-3.09	0.002	-0.362	-3.60	0.0
ψ_{UXA001}	-0.353	-3.96	0.0	-0.371	-4.41	0.0
ψ_{UXHK87}	-0.385	-5.37	0.0	-0.383	-5.34	0.0
ψ_{UUCC04}	0.635	10.1	0.0	0.634	10.0	0.0
ψ_{EU2001}	-0.319	-4.09	0.0	-0.329	-4.34	0.0
σ_{CCI}	0.580	1.88	0.062	0.369	2.57	0.011
σ_{UXNI63}	0.334	1.22	0.223	0.433	3.32	0.001
σ_{UXA001}	0.669	4.71	0.0	0.616	6.03	0.0
σ_{UXHK87}	0.799	8.91	0.0	0.809	8.94	0.0
σ_{UUCC04}	0.603	9.11	0.0	0.601	9.05	0.0
σ_{EU2001}	0.815	8.23	0.0	0.802	8.51	0.0

Note: LL is the log-likelihood function value

Table 4: Estimates of the parameters of German bi-factor Markov-switching model (a), monthly data 1991:1–2005:3

	Linear LL=-2016.4			Markov-switching LL=-2011.0		
	Coeff	t-stat	p-value	Coeff	t-stat	p-value
p_{11}	—	—	—	0.983	42.7	0.0
p_{12}	—	—	—	0.122	1.22	0.225
μ_1^{CLI}	—	—	—	0.008	0.25	0.806
μ_2^{CLI}	—	—	—	-0.058	-0.44	0.658
μ_1^{CCI}	—	—	—	0.126	1.35	0.178
μ_2^{CCI}	—	—	—	-0.951	-2.80	0.006
γ_{SU0253}	0.503	1.84	0.067	0.502	1.92	0.058
γ_{WU3141}	0.669	2.53	0.013	0.662	2.80	0.006
γ_{YU0516}	0.615	2.06	0.041	0.610	2.25	0.026
γ_{UXA001}	0.683	2.20	0.029	0.819	3.94	0.0
γ_{UXHK87}	0.281	2.56	0.012	0.271	2.24	0.027
γ_{UUCC04}	0.100	0.96	0.340	0.124	1.08	0.282
γ_{EU2001}	0.422	2.15	0.033	0.487	3.24	0.002
ϕ_{CLI}	0.748	9.02	0.0	0.732	7.96	0.0
ϕ_{CCI}	-0.118	-0.62	0.540	-0.253	-1.79	0.076
ψ_{IFO}	0.102	0.71	0.480	0.100	0.80	0.426
ψ_{SU0253}	0.344	4.54	0.0	0.343	4.52	0.0
ψ_{WU3141}	-0.168	-1.79	0.075	-0.167	-1.85	0.066
ψ_{YU0516}	0.295	3.78	0.0	0.296	3.79	0.0
ψ_{UXNI63}	-0.373	-3.18	0.002	-0.361	-3.66	0.0
ψ_{UXA001}	-0.357	-3.89	0.0	-0.376	-4.39	0.0
ψ_{UXHK87}	-0.385	-5.38	0.0	-0.383	-5.36	0.0
ψ_{UUCC04}	0.632	9.99	0.0	0.630	9.93	0.0
ψ_{EU2001}	-0.320	-4.11	0.0	-0.330	-4.34	0.0
σ_{CLI}	0.141	1.74	0.084	0.146	1.91	0.059
σ_{CCI}	0.563	1.90	0.059	0.359	2.56	0.012
σ_{IFO}	0.665	5.07	0.0	0.661	5.44	0.0
σ_{SU0253}	0.793	8.67	0.0	0.792	8.65	0.0
σ_{WU3141}	0.828	7.54	0.0	0.830	7.68	0.0
σ_{YU0516}	0.796	8.37	0.0	0.796	8.39	0.0
σ_{UXNI63}	0.349	1.34	0.182	0.447	3.47	0.0
σ_{UXA001}	0.656	4.60	0.0	0.600	5.83	0.0
σ_{UXHK87}	0.799	8.90	0.0	0.810	8.95	0.0
σ_{UUCC04}	0.606	9.10	0.0	0.605	9.04	0.0
σ_{EU2001}	0.815	8.20	0.0	0.803	8.52	0.0

Table 5: Estimates of the parameters of German bi-factor Markov-switching model (c), monthly data 1991:1–2005:3

	LL=-1992.3		
	Coeff	St.error	p-value
p_{11}	0.941	35.6	0.0
p_{22}	0.729	6.39	0.0
A	2.87	2.13	0.035
B	3.29	1.75	0.082
μ_1^{CLI}	0.153	2.92	0.004
μ_2^{CLI}	-0.483	-3.41	0.001
μ_1^{CCI}	0.212	2.49	0.014
μ_2^{CCI}	-0.574	-3.37	0.001
γ_{SU0253}	0.815	2.82	0.006
γ_{WU3141}	0.837	3.96	0.0
γ_{YU0516}	0.645	2.56	0.012
γ_{UXA001}	0.802	4.45	0.0
γ_{UXHK87}	0.235	2.01	0.046
γ_{UUCC04}	0.100	1.00	0.317
γ_{EU2001}	0.508	3.68	0.0
ϕ_{CLI}	0.462	3.11	0.002
ϕ_{CCI}	-0.228	-1.75	0.083
ψ_{IFO}	0.184	2.20	0.029
ψ_{SU0253}	0.351	4.59	0.0
ψ_{WU3141}	-0.198	-2.39	0.018
ψ_{YU0516}	0.301	4.04	0.0
ψ_{UXNI63}	-0.359	-3.68	0.0
ψ_{UXA001}	-0.389	-4.76	0.0
ψ_{UXHK87}	-0.385	-5.37	0.0
ψ_{UUCC04}	0.664	9.38	0.0
ψ_{EU2001}	-0.373	-5.84	0.0
σ_{CLI}	0.017	0.51	0.610
σ_{CCI}	0.364	3.27	0.001
σ_{IFO}	0.747	8.10	0.0
σ_{SU0253}	0.736	8.78	0.0
σ_{WU3141}	0.806	8.35	0.0
σ_{YU0516}	0.818	8.94	0.0
σ_{UXNI63}	0.438	3.78	0.0
σ_{UXA001}	0.599	6.24	0.0
σ_{UXHK87}	0.813	9.00	0.0
σ_{UUCC04}	0.572	8.29	0.0
σ_{EU2001}	0.699	8.04	0.0

Table 6: Official and model-derived chronologies of German business cycle, monthly data 1991:2–2005:3

				Model (b)				Model (c)			
ECRI		OECD		CLI		CCI		CLI		CCI	
P	T	P	T	P	T	P	T	P	T	P	T
1991:1	1993:1	1991:2	1993:7				1991:5				1991:5
				1992:5	1992:10	1992:3	1993:4	1992:2	1992:10	1992:4	1993:4
1994:12	1997:1	1994:12	1996:2	1994:10	1995:4			1994:10	1995:4	1994:12	1996:2
1998:3	1999:2	1998:3	1999:2					1998:6	1998:10	1998:7	1998:12
				2000:10	2001:9			2000:10	2001:9	2001:1	2001:11
2000:5	2002:3	2000:5	2005:3	2002:3	2003:2			2002:3	2003:2	2002:9	2003:5

Note: P is peak, T is trough; ”ECRI” stands for the ECRI’s growth cycle chronology for Germany as of November 2003; ”OECD” stands for the OECD’s reference cycle chronology for Germany as of May 2005; ”Model (b)” is the model with independent Markov switching of CLI and CCI; ”Model (c)” is the model with interdependent Markov switching of CLI and CCI, when CLI is leading CCI. Both model-derived chronologies are based on the smoothed probabilities.
Source: ECRI, OECD, own calculations.

Table 7: The in-sample predicting performance of the dynamic factor models with Markov switching (measured by the Quadratic Probability Score), monthly data 1991:2–2005:3

Model	Lag 0	Lag 1	Lag 2	Lag 3
ECRI's growth reference cycle				
Model (a) filtered prob.	0.372	0.381	0.389	0.398
Model (a) smoothed prob.	0.370	0.378	0.386	0.395
Model (b) CLI filtered prob.	0.312	0.306	0.302	0.303
Model (b) CLI smoothed prob.	0.314	0.302	0.293	0.292
Model (b) CCI filtered prob.	0.366	0.375	0.384	0.394
Model (b) CCI smoothed prob.	0.363	0.371	0.379	0.388
Model (c) CLI filtered prob.	0.326	0.322	0.319	0.321
Model (c) CLI smoothed prob.	0.319	0.305	0.296	0.296
Model (c) CCI filtered prob.	0.266	0.278	0.290	0.308
Model (c) CCI smoothed prob.	0.257	0.267	0.280	0.296
OECD's reference cycle				
Model (a) filtered prob.	0.496	0.501	0.506	0.507
Model (a) smoothed prob.	0.512	0.511	0.512	0.512
Model (b) CLI filtered prob.	0.355	0.356	0.358	0.358
Model (b) CLI smoothed prob.	0.343	0.341	0.339	0.339
Model (b) CCI filtered prob.	0.485	0.492	0.498	0.500
Model (b) CCI smoothed prob.	0.500	0.500	0.502	0.504
Model (c) CLI filtered prob.	0.379	0.384	0.389	0.389
Model (c) CLI smoothed prob.	0.371	0.363	0.360	0.364
Model (c) CCI filtered prob.	0.309	0.321	0.330	0.339
Model (c) CCI smoothed prob.	0.317	0.321	0.334	0.347

Figure 1. German CLI and CCI in levels, 1991:2−2005:3

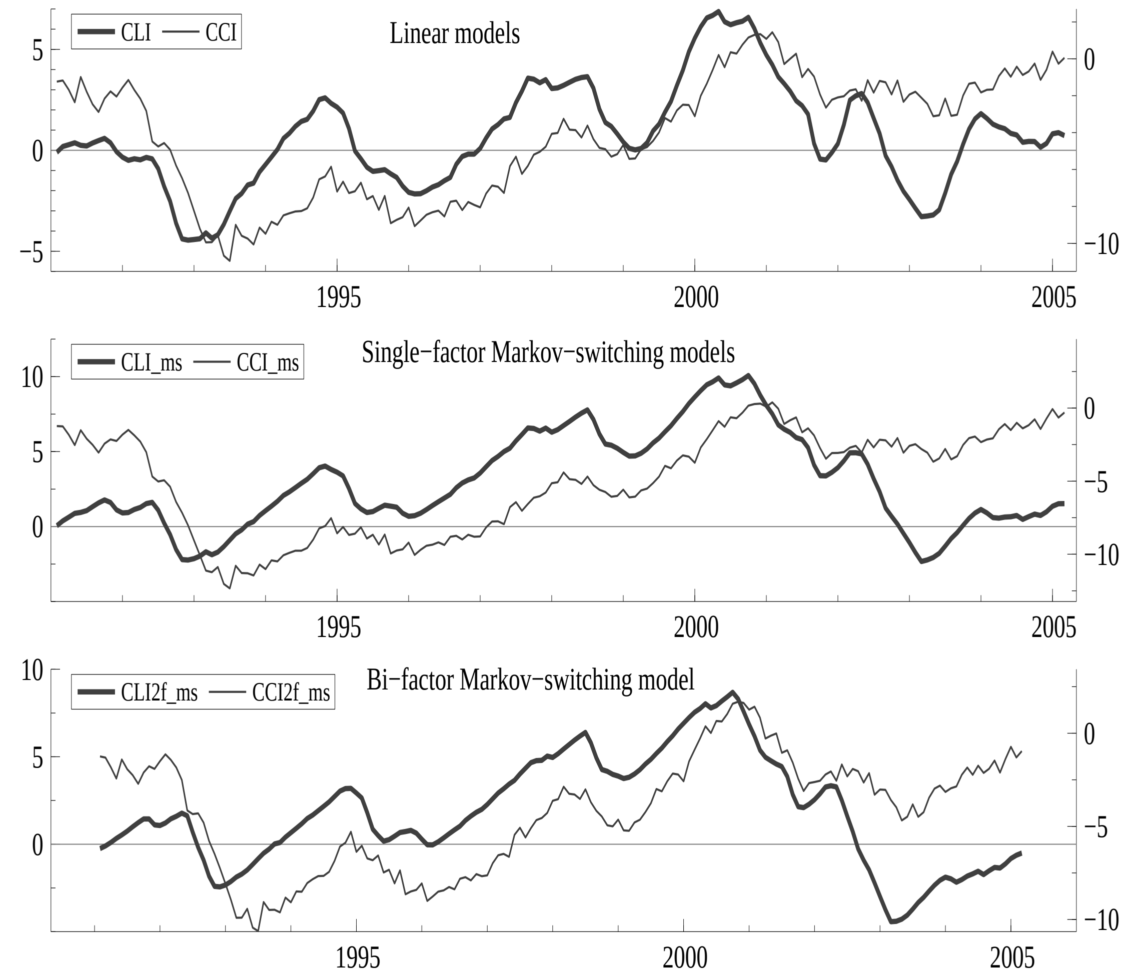

Figure 2. Recession probabilities of two single−factor models for Germany vs. two alternative cyclical chronologies, 1991:1−2005:3

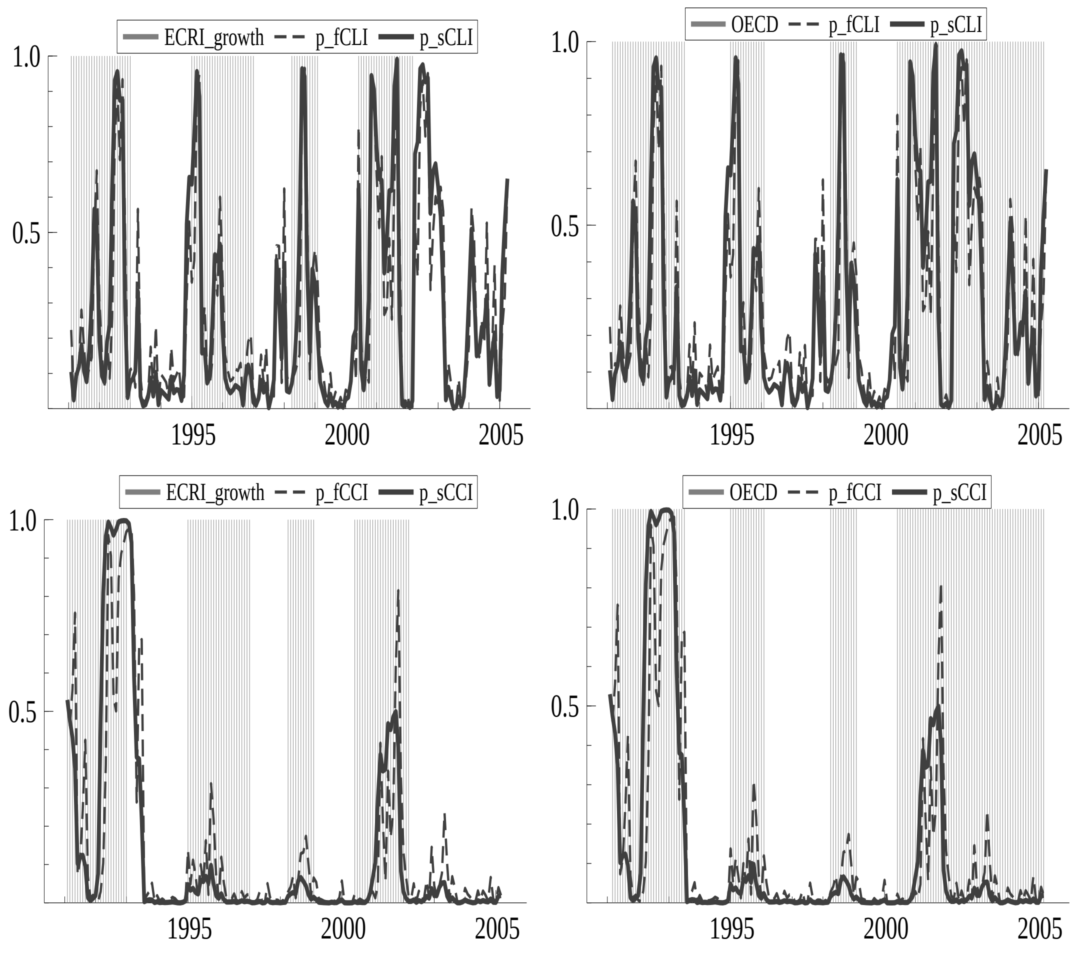

Figure 3. Recession probabilities of the bi−factor model for Germany vs. two alternative cyclical chronologies, 1991:1−2005:3