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Abstract

This paper studies how differences in the size of barriers to capital accumulation can account for differences in long run economic development paths. In this model barriers affect both the beginning date and the pace of the modern economic growth. A fundamental property of the model is that cross-country income differences matches the inverted U-shape pattern over time as observed in the data, hence implies a substantial fraction of existing income differences is really a transitional phenomenon. Relative to papers that model this as steady state phenomenon, my model requires a smaller size of barriers to account for current disparities. Another important finding is that this transitional effect increases significantly when I include the fact that today's low-income countries have had higher population growth rates during their early development stage than did the currently rich countries. In a quantitative exercise I find that given the beginning dates of modern growth, the model accounts for a significant portion of current income differences.

JEL Classification: D58, O11, O14, O42
Keywords : Industrialization, income disparity, distortion

This paper was produced as part of the Centre's Technology and Growth Programme

Acknowledgements

This is a chapter of my dissertation at the University of Pennsylvania. I would like to thank Richard Rogerson for his encouragement and valuable suggestions. I also benefited from discussion with Nobu Kiyotaki, John Knowles, Ichiro Obara, Stephen Parente, Christopher Pissarides, Randall Wright, Chun-Seng Yip, and seminar participants at the 2000 Annual Meeting of Society of Economics Dynamics and the 2000 World Congress of the Econometric Society. All errors are mine.

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Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

© L. Rachel Ngai, submitted October 2002

ISBN 0 7530 1596 X

Individual copy price: £5

Barriers and the Transition to Modern Growth

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January 2003

The Centre for Economic Performance is financed by the Economic and Social Research Council

1 Introduction

Why do some countries produce so much more output per worker than others? This paper addresses this question by focusing on the properties of long-run development paths found in economic data. Long run economic data demonstrate three important development facts. First, all countries that have experienced a sustained increase in per capita output also experienced a long period of stagnation before it. Second, countries entered modern growth at different points in time, referred to by Reynolds(1985) as its turning point. Third, income differences between early and later developers exhibit an inverted U-shape pattern over time, a feature of the data emphasized by Lucas (1998, 2000) and Pritchett (1997).

Models of international income differences usually compare steady states, ignoring these important development facts.[1] A parallel literature studies development paths but with no reference to international income differences either during the transition or in steady state.[2] In this paper, I bring elements from both literatures and study the international income differences implied by differences in development paths. I do this by extending the Hansen and Prescott (1999) model. In their model, there are two technologies with exogenous technological improvement. The first technology is the Malthusian technology which uses land, labor and capital. The second technology is the Solow technology which uses labor and capital only. They show that when the level of total factor productivity in the Solow technology is sufficiently low, only the Malthus technology is operated and there exists a balanced growth path in which stagnation results. As the level of total factor productivity in the Solow technology increases, it becomes profitable to use the Solow technology and the turning point is being reached. The economy will then asymptotically converge

[1] This literature generally focuses on policies that distort capital accumulation (Mankiw, Romer and Weil(1992), Chari, Kehoe and McGrattan (1996), Parente, Rogerson and Wright (2000)), technology adoption (Parente and Prescott(1994)), and level of total factor productivity (Hall and Jones(1999), Prescott(1998) , and Parente and Prescott(1999)). See McGrattan and Schmit(1998) for a survey of papers on cross country income differences.

[2] An exception is the work of Lucas (2000) which uses the model by Tamura (1996) to study the evolution of the relative income distribution by assigning turning points exogenously, and finds that income inequality exhibits an inverted U-shape. I study the same issue but with the turning point endogenously determined. Models on transition from stagnation to modern growth includes Becker, Murphy and Tamura (1990), Goodfriend and McDermott (1995), Galor and Weil (1998), Jones (1999) and Hansen and Prescott (1999). These models differ in several aspects regarding the driving forces of the transition to modern growth and whether such transition is inevitable or not.

to a Solow balanced growth path, in which the Malthus technology is not used at all. The turning point is determined endogenously and depends on initial conditions - capital stock and population, and the two technologies - input shares and total factor productivity levels and growth rates.

I extend the Hansen-Prescott model by introducing policies which act as barriers to discourage capital accumulation in the Solow technology. Barriers in my model lower the level of income along the balanced growth path and, more importantly, delay the turning point. Because of this second effect, cross-country income differences exhibit an inverted U-shape pattern over time, and hence my model accounts for the third development fact. A key implication of my model is that a substantial fraction of existing income differences is transitional. I show that relative to papers that model this as a steady-state phenomenon, my model requires a smaller barrier to account for current income disparities. Another important finding is that the transitional effect increases significantly when I include the fact that today's low-income countries have higher population growth rates during their early development stage than did the currently rich countries when they were growing.

I consider two empirical case studies to illustrate the strength of this model as a development model. These case studies are the development experiences of Africa and Japan. In both cases, I am interested in their experiences relative to the UK which is the first country to experience an industrial revolution. I use the actual difference in turning points to determine their relative sizes of barriers. In the case of Africa and the UK, I find that barriers that account for the difference in turning points can account for more than 70 percent of their current income differences. Moreover, my model predicts relative income in Africa will continue to worsen until the year 2045 even if its relative size of barriers remains unchanged. In the case of Japan, I show that its postwar miracle experience is a result of a reduction in barriers. Moreover, I find that its slowdown during the 70s is not necessarily a result of an increase in its relative size of barriers as argued by Parente and Prescott (1994).

The remainder of the paper is organized as follows. Section 2 documents the three long run development facts as a motivation for this paper. Section 3 presents the model. The model's implication for international income differences are studied in section 4. Section 5 discusses the role of the population profile in the model. The two cases studies are considered in section 6. A conclusion is given in section 7.

2 Motivation

This section documents three important long run development facts in the data. (1) All countries experienced a long period of stagnation before experiencing modern economic growth (sustained increase in per capita GDP). (2) Countries enter into modern growth at different points in time. (3) The income difference between the early developers and the later developers exhibits an inverted U-shape pattern. The data used in this paper are reported in Lucas (1998). Figure (1) demonstrates that per capita income for all five different regions in the world had been stagnant before the 19th century and started to grow at different times for different regions.[3] This stagnation is not because the world experienced no growth in total output but, rather because the increase in population offset the increases in output. The Malthusian theory therefore matches the experiences of the world fairly well prior to 1750. However, countries subsequently started to leave this type of stagnation and enter the modern growth regime. For instance, as suggested by Reynolds (1985) and shown in Figure (2), the turning point (the time at which modern growth begins) for the UK is around 1800 while the turning points for Japan and Africa are around 1900 and for China and the Indian Subcontinent around 1950.[4] As a result, income disparities across groups increase after 1800. For the same group of countries, Figure (3) plots the GDP per capita ratio between the UK and the rest of the countries. For most countries, the path of relative GDP resembles an inverted U-shape pattern over time.

The message from the data is clear: in order to understand current income differences, we should not overlook the fact that countries have different turning points. To proceed, I study a

[3] Region I includes UK, US, Canada, Australia and New Zealand. Japan and Western Europe are region II and III respectively. Region IV includes Latin America, Eastern Europe and Soviet Union. Finally, region V includes Africa and Asia(except Japan).

[4] Africa includes all of Africa except Morocco, Algeria, Tunisia, Libya and Egypt. Indian Subcontinent includes Pakistan, India, Bangladesh, Sri Lanka, Nepal and Bhutan. According to Reynolds, modern growth began in Africa in late nineteen century due to colonization.

version of Hansen and Prescott (1999) model. The Hansen-Prescott model has the advantage that it determines the turning point endogenously and behaves asymptotically like the standard Solow model. In their model, the reason for different turning points across countries is that there are differences in the unit efficiency of land per worker and differences in the level of technologies. However, such differences cannot be big enough to explain the observed large differences in turning points. Such differences also cause disparities in income prior to the turning points and the data show that the income differences between the richest and the poorest countries pre-1800 were small. I argue that different institutions for investment incentives can reconcile both small income differences prior to turning points and large differences in turning points, and as a result large current income differences. The intuition is straightforward, because capital has a small role to play prior to modern growth, differences in investment incentives do not have an important role to play in the determination of income disparities along the Malthusian path. But they can delay the adoption of the capital-intensive Solow technology and so explain large disparities in income post-development as a result of the differences in the turning points.

3 The Model

I use barriers to capital accumulation as an explanation for why countries are poor and, in the context of this paper, why modern growth begins later in some countries. Barriers can take the form of taxes on investment goods, corruption or other institutional factors that increase the relative price of investment goods, which in turn discourages capital accumulation. In this paper, I follow Parente and Prescott and model barriers by assuming that they reduce the efficiency of transforming forgone consumption goods into usable capital goods.

3.1 The Economy

Technology Output in this economy can be produced using either one of two technologies, the Malthus and the Solow technologies. The Malthus technology features constant return to scale in capital, labor and land. In contrast, the Solow technology features constant return to scale in capital and labor only. Both technologies are subject to exogenous technological change. The two production functions are as follows:

$$Y_{mt} = A_m \gamma_m^t K_{mt}^{\phi} N_{mt}^{\mu} L_{mt}^{1-\mu-\phi} \tag{1}$$

$$Y_{st} = A_s \gamma_s^t K_{st}^{\theta} N_{st}^{1-\theta} \tag{2}$$

where K_{it}, N_{it} and L_{it} denote capital, labor and land used in technology i at time $t = 0, 1......$, $\phi \in (0,1)$ is the capital share, $\mu \in (0,1)$ is the labor share and $1-\mu-\phi \in (0,1)$ is the land share for the Malthus technology, $\theta \in (0,1)$ is the capital share for the Solow technology, $\gamma_m > 1$ and $\gamma_s > 1$ are the growth rates while A_m and A_s are the initial level of total factor productivity (TFP) for the Malthus and Solow technologies. I will interpret the Malthus sector as the agricultural sector and the Solow sector as the industrial sector.

Physical capital is assumed to depreciate completely each period.[5] Land is a fixed factor. Output of the two sectors are identical, and can be used either for consumption or investment. Hence, feasibility requires:

$$C_t + X_{mt} + X_{st} = Y_{mt} + Y_{st} \tag{3}$$

where C_t is aggregate consumption, while X_{mt} and X_{st} are the aggregate investments in the Malthus and Solow capital stock in period t.

Firms in each sector are assumed to behave competitively and rent all factors of production from households. A representative firm in sector j takes the wage rate and rental rates for capital and land as given, and chooses labor, capital and land input to maximize profits.

$$\underset{N_{jt},K_{jt},L_{jt}}{Max}\ Y_{jt} - w_t N_{jt} - r_{Kjt} K_{jt} - r_{Lt} L_{jt} \qquad j = m, s$$

$$s.t.(1)\ and\ (2)$$

Household Sector The population structure is that of a two period overlapping generations model. In the beginning of each period, the current old agents give birth to young agents. Following

[5] In the quantitative work carried out later a period will be interpreted to be 35 years, so this assumption is empirically reasonable.

Hansen and Prescott, the number of children an old agent has depends on his living standard when young. Letting N_t be the number of young agents in period t, and c_{1t} be the consumption level for young agents in period t, the population dynamics are given by:

$$N_{t+1} = g(c_{1t})N_t$$

where $g(.)$ is an exogenous function that will be specified in more detail when the model is calibrated.

In period 0, there are N_{-1} old agents and N_0 young agents. Each initial old agent is endowed with $\frac{K_0}{N_{-1}}$ units of capital and $\frac{L}{N_{-1}}$ units of land. Young agents are endowed with one unit of time, which they supply inelastically. Old agents are assumed to be unable to work. Young agents make a consumption-saving decision by deciding how much land and capital to purchase. An old agent receives income from renting land and capital to firms and by selling land to the next generation.[6] The size of barriers, π, is modelled as a policy parameter that discourages young agents from investing in Solow capital. More specifically, for every unit of consumption good that a young agent gives up, he can get $\frac{1}{\pi}$ units of Solow capital. In equilibrium, π will be the relative price of Solow capital goods to consumption goods. In my international income comparison that follows, π is allowed to vary across countries.[7]

For each generation t, young agents maximize the following lifetime utility, by choosing consumption (c_{1t}, c_{2t+1}) and investment portfolio $(x_{mt}, x_{st}, l_{t+1})$:

$$U(c_{1t}, c_{2t}) = u(c_{1t}) + \beta u(c_{2t+1})$$

subject to the budget constraints

$$c_{1t} + x_{mt} + x_{st} + q_t l_{t+1} = w_t \tag{4}$$

$$c_{2t+1} = r_{kmt+1} x_{mt} + r_{kst+1} \frac{x_{st}}{\pi} + (q_{t+1} + r_{Lt+1}) l_{t+1} \tag{5}$$

[6] More generally, if capital did not depreciate completely, the old agent would also sell capital to next generation.

[7] Note that putting barriers in the Malthus sector does not change the main results of the paper as will be shown later.

where β is the discount factor and q_t is the price of land in period t. Assuming log utility, $u(c) = \ln c$, the optimal decision for the young is to consume a constant fraction of their income

$$c_{1t} = \frac{w_t}{1+\beta} \tag{6}$$

and equalize the return to assets

$$\begin{aligned} R_{t+1} &= \frac{q_{t+1} + r_{Lt+1}}{q_t} \qquad if\ l_t > 0 \\ &= \frac{r_{kst+1}}{\pi} \qquad\qquad if\ x_{st} > 0 \\ &= r_{kmt+1} \qquad\qquad if\ x_{mt} > 0 \end{aligned} \tag{7}$$

3.2 Competitive Equilibrium

Given π, N_0, K_0 and L, the total land of the economy, a competitive equilibrium for this economy consists of sequences for $t \geq 0$ of prices $\{q_t, w_t, r_{Kmt}, r_{Kst}, r_{Lt}\}$; firm allocations, $\{K_{mt},\ K_{st},\ N_{mt},\ N_{st},\ L_{mt},\ Y_{mt},\ Y_{st}\}$; and household allocations, $\{c_{1t}, c_{2t+1}, x_{mt}, x_{st}, l_{t+1}\}$, such that:

1. Given the sequence of prices, household allocations solve the utility maximization problem.
2. Given the sequence of prices, firm allocations solve the firm's profit maximization problem
3. All markets clear:

$$Y_{mt} + Y_{st} = N_t c_{1t} + N_{t-1} c_{2t} + N_t x_t$$

$$N_{mt} + N_{st} = N_t$$

$$K_{mt} + K_{st} = K_t$$

$$L_{mt} = L = N_{t-1} l_t$$

where N_t and K_t denotes the aggregate labor and capital in this economy.

4. The following laws of motion hold:

$$K_{mt+1} = N_t x_{mt}$$

$$K_{st+1} = N_t \frac{x_{st}}{\pi}$$

$$N_{t+1} = g(c_{1t}) N_t$$

3.3 Dynamics of the Model

The first question I address is under what circumstances are the two technologies operated. Because land is always supplied inelastically, in equilibrium it is always profitable to operate the Malthus technology[8]. This, however, is not necessarily true for the Solow technology. However, I will show that for sufficiently high TFP in the Solow technology, it will also be operated. When the Solow technology is not operated, I call this the Malthus-only economy. When the Solow technology is used, I say that the economy is in transition to modern economic growth.

I now proceed as follows. First, I characterize the Malthus-only economy. Second, I find the condition for the Solow technology to be operated. Third, I describe the asymptotic behavior of the economy.

3.3.1 Malthus-only Economy

When the Solow technology is not used, the firm's optimization problem implies

$$r_{Kt} = \phi \frac{y_{mt}}{k_{mt}} \tag{8}$$

$$w_t = \mu y_{mt} \tag{9}$$

$$r_{Lt} = (1 - \phi - \mu) \frac{Y_{mt}}{L} \tag{10}$$

where y_{mt} and k_{mt} are the output and capital per worker in Malthus-only economy.

One can look for a balanced growth path in the Malthus-only economy. To do this, I need to put some restrictions on the population growth function $g(.)$. As the model is motivated to reproduce the fact that output per worker is stagnant before the industrial revolution, $g(c_{1t})$ is chosen such that the population growth rate is the same as the growth rate of output along the balanced growth path in the Malthus-only economy. Following Hansen-Prescott, I now show that the population growth function $g(.)$ can be chosen to ensure this. Letting $\hat{y}_m$ and $\hat{k}_m$ be the stagnant levels of

[8] Suppose r_{Lt}, r_{kmt}, r_{Kst} and w_t are equilibrium prices such that the Malthus technology is not operated. Then since land can only be used in the Malthus technology, there is an excess supply of land, which implies that these prices cannot be an equilibrium.

output and capital per worker respectively, the Malthus production function implies:

$$\hat{y}_m = A_m \gamma_m^t \hat{k}_m^{\phi} (\frac{L}{N_t})^{1-\mu-\phi}$$

Thus, along the Malthus-only balanced growth path, output per worker is stagnant if

$$g(\hat{c}_{1m}) = \gamma_m^{1/(1-\phi-\mu)}$$

and

$$g(c_1) > g(c_{1m}) \quad \forall c_1 \in [c_{1m}, c_{1m} + \epsilon] \textit{ where } \epsilon > 0$$

which I henceforth assume.

Under this restriction, equations (1), (8), (9) and (10) together with the market clearing conditions imply that, along a Malthus-only balanced growth path, aggregate output, capital, the price of land and the rental rate of land all grow at the same rate as does population. The wage rate, the rental rate of capital, output per capita, capital per capita, capital-output ratio and consumption of the young and old are all constant.

3.3.2 Transition

Given N_0, I can choose K_0 such that the economy begins on the Malthus-only balanced growth path in period 0.[9] I can then determine when the Solow technology will be used.

Proposition 1 *Assume the economy is on the Malthus-only balanced growth path in period 0. The Solow technology is used when*

$$t > \frac{\ln \frac{A_m}{A_s} + \ln B_0 + \theta \ln \pi}{\ln \gamma_s} \tag{11}$$

where $B_0 = (\frac{\phi}{\theta})^{\theta} (\frac{\mu}{1-\theta})^{1-\theta} \left(\frac{K_0}{N_0}\right)^{\phi-\theta} \left(\frac{L}{N_0}\right)^{1-\mu-\phi}$

[9] One can solve for the constant capital-output ratio along the the Malthus-only balanced growth path using the market clearing condition

$$K_{t+1} = N_t (w_t - c_{1t}) - q_t L$$

Proof. First note that if the Solow technology were to be used, profit maximization implies that the capital to labor ratio would be:

$$\frac{K_{st}}{N_{st}} = \frac{\theta w_t}{(1-\theta)r_{kst}}$$

The profit function for a firm in the Solow sector in period t is:

$$\Psi(r_{kmt}, w_t) = \max_{K_{st},N_{st}} A_s\gamma_s^t K_{st}^\theta N_{st}^{1-\theta} - r_{kst}K_{st} - w_t N_{st}$$

which is equivalent to:

$$\Psi(r_{kmt}, w_t) = \max_{N_{st}} \left[A_s\gamma_s^t (\frac{\theta w_t}{(1-\theta)r_{kst}})^\theta - \frac{w_t}{1-\theta} \right] N_{st}$$

If both technologies were used, we must have $\frac{r_{kst}}{\pi} = r_{kmt}$, which implies

$$\Psi(r_{kmt}, w_t) = \max_{N_{st}} \left[A_{st} (\frac{\theta w_t}{(1-\theta)\pi r_{kmt}})^\theta - \frac{w_t}{1-\theta} \right] N_{st}$$

Let $\hat{r}_m$ and $\hat{w}_m$ be the constant rental rate of capital and wage along the Malthus balanced growth path. It is profitable to start operating the Solow technology if $\Psi(\hat{r}_m, \hat{w}_m)$ is positive,

$$A_s\gamma_s^t > (\frac{\pi\hat{r}_m}{\theta})^\theta (\frac{\hat{w}_m}{1-\theta})^{1-\theta}$$

By assumption, the economy is on the Malthus-only balanced growth path in period 0,

$$A_s\gamma_s^t > \pi^\theta A_m B_0$$

It follows that the Solow technology is first used in period t_π^*, where t_π^* is the minimum integer that satisfies:

$$t > \frac{\ln\frac{A_m}{A_s} + \ln B_0 + \theta\ln\pi}{\ln\gamma_s}$$

■

Once the Solow technology is used, output per worker starts to grow. This is precisely the turning point when modern growth begins, or the process of Industrial Revolution starts. In what follows I will refer to t_π^* as the turning point. Note that the Solow technology is used independently of the relative size of γ_m and γ_s. Since the right hand side of the equation (11) is just a constant, the Solow technology will be used at some point as long as $\gamma_s > 1$. Therefore, the model predicts that modern growth is inevitable in all countries, but that it's starting point depends on the level of barriers, the relative level of total factor productivity of the two technologies, input shares, and the initial quality of land, labor force and capital. Note that the population growth function will not affect the turning point as it only takes effect after consumption exceeds the level along the Malthus balanced growth path, at which point the economy has already passed its turning point.

To characterize the equilibrium when both technologies are used, the profit maximization conditions imply:

$$\frac{r_{kst}}{\pi} = \frac{\theta Y_{st}}{\pi K_{st}} = \frac{\phi Y_{mt}}{K_{mt}} = r_{kmt} \tag{12}$$

$$w_t = (1-\theta)\frac{Y_{st}}{N_{st}} = \mu \frac{Y_{mt}}{N_{mt}} \tag{13}$$

$$r_{Lt} = (1-\phi-\mu)\frac{Y_{mt}}{L} \tag{14}$$

Note that, as implied by (12) and (13), when both technologies are operated, marginal products are equal across technologies. This, together with the market clearing conditions, determines the labor and capital allocated to each sector.

Lemma 2 *Assume $\theta \geq \phi$, $\gamma_s \geq \gamma_m$, and one of the following is true: (i) $(1-\theta) \geq \mu$ and $g(c_{1t}) > 1$, or (ii) $(1-\theta) > \mu$ and exist $\bar{t}$ and sufficiently small $n \geq 1$ such that $g(c_{1t}) \leq n \; \forall \; t > \bar{t}$. Then, the equilibrium $\frac{N_{mt}}{N_t}$ is converging to zero.*

Proof. See Appendix 1. ■

This lemma simply says that if capital is more productive in the faster-growing Solow technology, and if one of the following is true: (i) the labor share is higher in the Solow sector and labor supply is growing, or (ii) labor share is higher in the Malthus sector and the growth in labor supply is small enough after a certain date. Then, the Malthus sector will eventually disappear.

3.3.3 Solow-only Economy

Assume now that the Solow technology has a higher growth rate of TFP. As already noted, this condition is not necessary for the Solow technology to be used. Assuming also that the population growth rate eventually decreases and converges to a constant, the previous lemma implies labor and capital allocated to the Malthus sector will converge to zero. Equation (14) then implies that the rental rate of land relative to the price of output will also converge to zero.[10] Hence, asymptotically, the economy behaves the same as a standard Solow growth economy and will converge to a balanced growth path. In particular, the firm's problem implies:

$$r_{kt} = \theta \frac{\hat{y}_{\pi st}}{\hat{k}_{\pi st}} \tag{15}$$

$$w_t = (1-\theta)\hat{y}_{\pi st} \tag{16}$$

where $\hat{y}_{\pi st}$ and $\hat{k}_{\pi st}$ are the output and capital per worker along the asymptotic Solow balanced growth path for an economy with size of barriers equal to π.

Along the asymptotic balanced growth path, output and capital per worker grow at the same constant rate. The Solow technology production function implies:

$$\hat{y}_{\pi st} = A_s \gamma_s^t \hat{k}_{\pi st}^\theta$$

Thus, both output and capital per worker grow at the rate $(\gamma_s^{1/(1-\theta)} - 1)$ along the asymptotic balanced growth path. Equations(2), (10),(15) and (16), together with the market clearing conditions then imply that output per worker, capital per worker, consumption per young and old, and the wage rate all grow at $(\gamma_s^{1/(1-\theta)} - 1)$.

The dynamics of the model, therefore, capture the experiences of rich countries. The economy starts off with stagnant output per worker. Modern growth then begins with an increase in labor being allocated to the industrial sector. Finally, the economy converges to a Solow balanced growth path.

4 International Income Differences

This section studies the potential of the model to account for international income differences. For this purpose, I consider two economies that are identical except for the level of their barriers.

[10] A test for this result should be compared to the value of farmland in the data, as land in this model is only used for the Malthus sector. Hansen and Prescott (1999) document that value of farmland relative to the value of GNP has declined from 88% in 1870 to 9 % in 1990.

4.1 Analytical Results

Proposition 3 *Assume $\gamma_s \geq \gamma_m$ and $g(.) \to g$. Consider two economies that differ only in their levels of barriers. Let $\hat{y}_{\pi_i st}$ denote the output per worker along the asymptotic Solow-only balanced growth path for country i. Then*

$$\frac{\hat{y}_{\pi_1 st}}{\hat{y}_{\pi_2 st}} = \left(\frac{\pi_2}{\pi_1}\right)^{\theta/(1-\theta)} \tag{17}$$

The proof consists essentially of showing that the ratio of these two economies' capital-output ratios is equal to $\frac{\pi_2}{\pi_1}$(See appendix 2). This model thus generates the same long run income differences as the standard one sector barrier model.[11]

The interesting point of this model, however, is its implications for different turning points as a result of different levels of barriers. Proposition 1 implies two main analytical results.

Lemma 4 *An Industrial Revolution is inevitable in both economies which means there is no absolute poverty trap.*

Lemma 5 *The relationship between the turning points $t^*_{\pi_1}$ and $t^*_{\pi_2}$ is as follow:*

$$t^*_{\pi_2} = t^*_{\pi_1} + \theta \frac{\ln\left(\frac{\pi_2}{\pi_1}\right)}{\ln \gamma_s} \tag{18}$$

[11] By the standard barrier model, we mean the following:

There is a representative infinitely-lived agent with preferences

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ and c_t is agent's consumption in period t.

The production function is:

$$Y_t = A\gamma^t K_t^{\theta} N_t^{1-\theta}$$

where γ is the total factor productivity growth, K_t, and N_t are capital and labor inputs at period t.

The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + \frac{X_t}{\pi}$$

where δ is depreciation rate for capital, and X_t is aggregate investment at period t. Feasibility requires:

$$C_t + X_t = Y_t$$

where C_t is aggregate consumption at period t.

Thus, the turning point for the economy with a larger π occurs $\theta\frac{\ln\pi}{\ln\gamma_s}$ periods later. Note that a higher capital share for the Solow technology not only increases the income difference along the Solow balanced growth path, but also increases the difference in turning points between economies. The intuition is as follow. The turning point is reached when the Solow technology is used which implies investment in Solow capital is positive. On the other hand, the effect of barriers is to reduce investment in Solow capital. As θ increases, the role of capital in the Solow technology becomes more important. Thus, given the TFP growth rate for the Solow technology, a given size of barriers causes longer delay in turning point when θ is increased.[12]

4.2 Quantitative Results

4.2.1 Calibration and Computation

The economy with π equal to one is calibrated to match the development experience of England before 1800 and the postwar development experience of the industrialized countries. The year 1800 is taken as the time at which modern growth began for the English economy, and will map to my endogenously determined variable t_1^*. A period in this economy is interpreted to be 35 years in real time, which as noted earlier, justifies the assumption that capital fully depreciates after one period. Agents in this economy will therefore live for 70 years working for the first 35 years of their life-span. The postwar period will therefore be interpreted as t_1^*+5 in my model. The initial conditions, A_m, A_s, L and N_0 are set to be one arbitrarily. Given N_0, K_0 is chosen such that the economy is initially on the Malthus-only economy.[13] As the calibration strategy is the same as Hansen and Prescott (1999), I will only briefly review their steps. The population growth rate for the pre-1800 period in the UK is used to calibrate the productivity growth rate of the Malthus technology, and the relationship between the population growth rate and the GDP per capita for

[12] Since capital has a very small role in Malthus sector, putting barriers there does not change the main results. In general, if barriers affect both sectors, the income differences along Malthus-only balanced growth path is $\pi^{\phi/(1-\phi)}$, along the Solow-only balanced growth path is $\pi^{\theta/(1-\theta)}$ and the turning point is delayed by $\frac{\theta-\phi}{1-\phi}\frac{\ln\pi}{\ln\gamma_s}$ where by assuumption $\theta>\phi$.

[13] The capital-output ratio along the Malthus-only balanced growth path is $\frac{K}{Y}=\frac{1+\beta-\mu-\sqrt{(1+\beta-\mu)^2-4\mu\phi\beta(1+\beta)}}{2(1+\beta)\gamma_m^{1/(1-\mu-\phi)}}$ which implies $K_0=\left[N_0^{\mu}L^{1-\phi-\mu}\left(\frac{K}{Y}\right)\right]^{1/(1-\phi)}$

the industrial economies is used to calibrate the population growth function $g(.)$. A general pattern in the long run population data presented in Lucas (1998) can be summarized by the $g(c_1)$ function in Figure (4), which is also similar to Figure II in Kremer (1993). It says population growth rate first increases until the living standard is x_1 times its Malthusian level and the decreases to a constant level when the living standard is x_2 times its Malthusian level. The population growth function is then calibrated to this shape with $x_1 = 2$, $x_2 = 18$ and $m = 2$ where $m = 2$ corresponds to a 2% average annual population growth rate. Finally, the postwar economic development of the industrial economies is used to calibrate the productivity growth rate of the Solow technology and the discount factor. To summarize, the parameters values are:

θ	μ	ϕ	γ_m	γ_s	β
0.4	0.6	0.1	1.03	1.52	1

The main issue in solving for the equilibrium in this model is to find the equilibrium price of land. Given L, N_0 and K_0, the equilibrium price for land is solved using the shooting algorithm described in Hansen and Prescott (1999).

4.2.2 Results

With the same calibrated parameters, I then compute the equilibrium path of another economy with a π equal to 4 as a benchmark case. Jones (1994) studies the Summer and Hetson data set and finds that the maximum relative machinery price to that of the US for the period 1960-85 is equal to 4. More recently, using the same data set, Restuccia and Urrutia (2001) construct a panel for the relative price of aggregate investment to consumption over the period 1960-85. They found that the relative price differences across countries are large. In particular, the ratio between the average of the top and bottom five percent of the distribution of relative prices is 11.3 in 1960 and 6.5 in 1985. Therefore, I will also report the results of using higher values of π later in this section.

Figures (5) - (9) summarize the quantitative results for the case in which π equals 4. Figures (5) and (6) show that while the UK starts to allocate labor and capital inputs to the Solow sector in 1800, the Solow technology is still inactive in the distorted economy until 1870. The fraction of labor and capital allocated to Malthus sector decrease because of the diminishing returns in

Malthus technology associated with the fixed supply of land. The first and the second development facts are replicated in Figure (7). It shows that output per worker starts to grow in 1800 for the UK and in 1870 for the distorted economy. The model predicts that in 1975, output per worker for the UK is 18 times higher than its level in 1765 while it's only 7 times higher for the distorted economy.

The third development fact is also captured by the model in Figure (8). The model predicts that relative output per worker will increase from 1 to a maximum of 3.2 before declining to 2.5 once both economies have reached their Solow-only balanced growth path. This pattern of relative income differences generated by the model closely resembles the data in Figure (3). Moreover, a bigger income difference is obtained (a 26 percent increase) relative to the balanced growth path level.

As show in Figure (9), the growth rate is not monotonic as in the standard Solow growth model as an economy converges to the balanced growth path. In particular, it first increases and then decreases to its balanced growth path rate. The increasing growth is a feature of the data emphasized by Romer (1986).[14] It is interesting to note that this model can produce such an outcome with two constant return to scale technologies.

The growth rate dynamics and the inverted U-shape income differences are intimately tied. In the standard Solow model, growth rates fall monotonically since the capital-output ratio increases monotonically along the transition. However, the capital-output ratio in this two-sector model first decreases then increases. Two elements are important for this result: (1) perfect mobility of capital and labor across sectors, i.e. equation(12) and (13) imply:

$$\left(\frac{K_{st}}{Y_{st}}\right)^{\frac{(\theta-\phi)}{(1-\theta)(1-\phi)}} = \frac{A_{mt}^{1/(1-\phi)}}{A_{st}^{1/(1-\theta)}}\left(\frac{\pi\phi}{\theta}\right)^{\phi/(1-\phi)}\left(\frac{\mu}{1-\theta}\right)\left(\frac{L}{N_{mt}}\right)^{(1-\mu-\phi)/(1-\phi)} \tag{19}$$

and (2)positive growth in labor supply at the initial phase of transition.

Intuitively, as total labor supply is growing during the early phase of the transition, labor

[14] Romer (1986) tests the trend of the growth rate using raw data from Maddison (1979) for countries with data no later than 1870. These countries include: United Kingdom, France, Denmark, United States, Germany, Sweden, Italy, Australia, Norway, Japan and Canada. He rejects the null hypothesis that there is a nonpositive trend in the growth rate for 8 out of the 11 countries at the 10 percent level.

allocated to Malthus sector is not decreasing at a significant rate even though the fraction allocated to the Malthus sector may be. Since the Solow sector is growing at a faster rate, the capital-output ratio must be decreasing from equation (19). Labor allocated to the Malthus sector eventually decreases significantly when the growth in labor supply slows down and causes an increase in the capital-output ratio. Therefore, the growth rate in this two-sector model exhibit an inverted U-shape pattern. The implied income differences mimic this pattern. As the barrier delays the turning point for the distorted economy, the growth rate for the undistorted economy is higher than that of the distorted economy before it starts to decrease. Thus, income differences increase during this period. After this point, the model predicts faster growth in the distorted economy so that the income difference decreases. Income differences converge to a constant when both economies converge to the Solow-only balanced growth path.

In this model income differences across countries are generated by differences in balanced growth path levels and differences in turning points. It follows that the balanced growth income differences are smaller than income differences along the transition from Malthus to Solow. Table (1) reports the balanced growth income differences and the maximum income differences along the transition between the undistorted economy and the distorted economy for varies value of π. It shows that as π increases, the percentage difference between maximum income difference and the balanced growth income differences increases. This is partly due to the longer delay of modern growth. For example, when π is increased from 8 to 16, the delay in modern growth increases from 2 to 3 periods. Thus, the percentage increase in the income difference rises from 33 percent to 40 percent.

To address the factor 30 income differences in the data, Table (2) reports the corresponding combination of capita shares and barriers that can generate maximum differences of this magnitude. Note that, to be consistent with my calibration procedure, γ_s and b have to be adjusted when θ is increased. Note, therefore, that increasing θ need not necessarily increase the delay in modern growth as noted earlier in section 4.1. Table (2) shows that by considering different turning points, the required size of barriers needed for a factor 30 income difference is much lower than the size along the balanced growth path. For example, for θ equals to 0.4, π is reduced by 40 percent.

The reduction holds true for other levels of θ as well. Note that a factor 30 income difference is associated with a three- or four-period delay in the model. In other words, given the UK entered into modern growth in 1800, the model predicts that a country 30 times poorer by today's standard is more likely to be the one that entered into modern growth in 1940.

Finally, I consider the exercise of increasing θ holding π fixed. Given π equals to 4, Table (3) illustrates that the percentage difference between maximum income difference and the balanced growth difference is increasing in the value of capital's shares. In particular, when θ equals 0.5, the delay in modern growth increases the income differences by 45 percent. Thus, as θ increases, the ability of the model to account for the current income disparity improves. It improves because as θ increases the maximum income differences increases by more than the balanced growth path difference.[15]

5 Population Profile

In the previous quantitative exercise I assumed the population profile is the same for both distorted and undistorted economies. My focus there was to study the effect of barriers holding other factors constant. I find that the income difference between these two economies first increases, reaching a maximum equal to 3.2 (when $\pi = 4$), then decreases to its balanced growth path level of 2.5. In the sensitivity analysis (Appendix 3), I find that this result is sensitive to the change in the population profile. In particular, when the maximum population growth rate is increased from 2% to 3% ($m = 2$ to $m = 2.8$) for both economies, the maximum income difference increases from 3.2 to 3.5, a nearly 10 percent increase. In view of this, it is of interest to see what the data imply for the population profiles for a broader set of countries.

As shown in Figure (10), the data suggest that whereas the shapes of the population profile are

[15] Alternatively, some have argued that some countries are poor because there are barriers that deter technology adoption which in terms lower the level of total factor productivity. Thus, an alternative way to incorporate barriers into this model is as $Y_s = \frac{A_{st}}{\pi_2} K_{st}^{\theta} N_{st}^{1-\theta}$. At a general level, these two types of models are isomorphic in that one can choose the size of barriers such that they imply the same output per worker ratio along the balanced growth path for the two models. In particular, set $\pi_2 = \pi_1^{\theta}$,where π_1 and π_2 are the barriers to capital accumulation and technology adoption respectively. Then, the delay in turning points implied by these two models are the same and same quantitative results apply.

similar across countries, the peaks are very different. More precisely, late developers have much higher peaks than early developers.

Why does the population growth rate increase during the early development stage of an economy? One may think that this is solely due to the decline in the mortality rate. However, as Coale (1979) has documented for the case of Europe, and Dyson and Murphy (1985) have documented for the case of other countries, the total fertility rate was also increasing during this period. This increase in the total fertility rate can be decomposed into changes in marriage behavior and changes in marital fertility. Wrigley and Schofield (1981) provide evidence that in England, the marriage rate increased and age of first marriage decreased during the initial stage of industrialization. Evidence from the demography literature (see Dyson and Murphy (1985)), suggests that marital fertility was increasing during the early development stage and that this increase was mainly due to changes in postpartum sexual abstinence and duration of breast-feeding. In addition, Livi-Bacci (1997) shows that mortality levels at the early development stage in developing countries are more or less the same as European mortality rates. However, the fertility rates in developing countries are considerably larger than those experienced in European countries. Hence, the available literature suggests that the difference in the peaks of population profiles in Figure (10) is due mainly to differential fertility rates. Cultural, religious and policy differences that affect the fertility decision may all be important for understanding Figure (10). While understanding what accounts for these differences is of interest in its own right, I will simply take these differences as exogenous and examine their consequences for development.

Given the difference in population profiles, I now ask what is the implication of the model if I allow the distorted economy to also have a population profile with a bigger m. Specifically, I assume the maximum growth rate for the distorted economy ($\pi = 4$) is 3 percent, compared to 2 percent of the undistorted economy. As discussed earlier, this change in the maximum growth rate will not affect the turning point.

The effect on relative income is shown in Figure (11). The lower line in Figure (11) is the same as Figure (8) which plots the relative income paths for two economies that are identical in every aspect except for the level of their barriers. The upper line in Figure (11) corresponds to the case in which the maximum population growth rate in the distorted economy is higher than in the undistorted economy. The maximum income difference increases from 3.2 to 4, which is a 25 percent increase. Moreover, the income differences from 1940 to 2045 increase by more than 20 percent. Therefore, the model confirms the intuition that the difference in population profiles between the early and the later developers is important in accounting for their income differences.

6 Applications

In this section, I consider two case studies to illustrate the strength of this model. These two cases are Japan and Africa. In particular, they demonstrate two interesting and important development facts: (1) the current disparities in income observed across countries is high and (2) the countries that have undergone a growth miracle are those countries that are initially among the bottom of the world. In the case study of Africa, I show that a size of barriers that accounts for the delay in the turning point can also account for the subsequent behavior of Africa's relative income. In the case study of Japan, I find that the model can generate both the miracle and subsequent slowdown in the growth of income.

6.1 Application I: Africa

The long run data presented in Lucas (1998) shows that the UK's income was only two times higher than Africa's in 1750. This factor increased to 14 in 1990. Moreover, the data also indicate that the turning point for Africa was around 1900 as suggested by Reynolds (1985). In this section, I address the following question: can barriers that account for the difference in the turning points between two economies also account for the subsequent path of their income disparities?

We learn from Figure (8) that this model can generate the pattern of increasing income difference for a given π in Africa. More importantly, the model can determine $\pi = \gamma_s^{\frac{t^*_{Africa} - t^*_{UK}}{\theta}}$ from equation (18) using the turning points of the UK and Africa.

Assume that the values of all parameters except π are the same for the UK and the Africa.

Specifically, $\theta = 0.5$ and $\gamma_s = 1.43$. This value of θ is larger than that in section 4.2. This is in accordance with many authors, e.g. Parente and Prescott, who have argued that capital's share should be higher than the canonical values because of unmeasured investment.[16] Given these values, π must be between 5 and 9 to generate a three-period delay. In what follows I assume that $\pi = 8$ for Africa. This is a plausible value as shown in Restuccia and Urrutia (2001).

With this size of barriers, the model predicts that the UK's income relative to Africa will reach a maximum of 12 in 2045 and then decrease to its balanced growth path level of 8.[17] Figure (12) shows that the model replicates the increasing trend of the income difference between the UK and Africa and accounts for around 70 percent of the income difference in 1970 and 1980. By matching the turning points, I have shown that the subsequent paths generated by the model are quite close to the data.

The above calculation assumed no differences between the UK and Africa other than the size of barriers. In what follows I analyze how incorporating other sources of heterogeneity may improve on the model's predictions. The first element I consider is initial conditions. As mentioned earlier, even before the turning point of the UK, output per capita in the UK was almost double it's corresponding value in Africa. I assume all parameters as before but different land per worker in period 0. The ratio of their outputs per capita along the Malthus balanced growth path is equal to $(\frac{l_{UK}}{l_{Africa}})^{(1-\mu-\phi)/(1-\phi)}$ where l denotes land per worker in period 0. I then choose the relative value of land per worker in the two economies to match the initial income difference.[18] Figure (13) shows the model's prediction for this scenario. With this adjustment in initial conditions, the model now accounts for around 90 percent of the income difference in 1970 and 1980. However, the model did not perform well for the income difference in 1990.

The second source of heterogeneity that I consider is differences in population profiles. As discussed earlier, the population profiles for Africa and the UK are quite different. Specifically, Africa had a maximum population growth rate of 4% whereas the UK had a maximum level of

[16] The choice of θ will mainly affect the level of π but not the main results.

[17] I use linear interpolation between the periods in the model to compare the model with the data.

[18] Of course it is not literally land per person that matters, but rather efficiency units of land per person from the perspective of the technology.

1.5%. In order to assess the impact of these differences, I set $m = 4$ for Africa according to the calibration for the population profile described in section 4.2.1. As in section 5, I will simply take these fertility differences as exogenous.

Figure (14) shows that the model implies a much higher income difference for the period 1960-1990. Moreover, the model predicts that if the relative size of barriers in Africa remains unchanged, the UK's income relative to Africa will increase to 24 in 2045 before decreasing towards its balanced growth path level. Moreover, note that the model now replicates closely the income difference in 1990.

To sum up, the case of Africa illustrates some interesting predictions of the model. First, the size of barriers that accounts for the delay in the turning point for Africa relative to the UK can account for 70 percent of the current income difference. Second, in contrast with the standard balanced growth path approach, the model predicts that income disparities between Africa and the UK will continue to worsen even if relative barriers are unchanged. Last but not the least, the high peaked population profile in Africa implies the current income difference will double in fifty years.

6.2 Application II: Japan

Japan is an interesting case study because it underwent a development miracle experience. Modern growth began in Japan around the end of 19th century, 100 years later than the UK. However, Japan's GDP per capita exceeded that of the UK in 1990, only 90 years after its period of modern growth began. This rapid rate of catch up can be seen in Figure (15). Its GDP per capita growth rate was 7.5% in 1950-60 and 9.5% in 1960-70, compared to a 2.5% for the UK in 1950-70. However, the growth rate in Japan dropped to 3.5% in 1970-90.

Within a version of the neoclassical growth model, Parente and Prescott (1994) interpret the miracle in Japan as a reduction in its size of barriers to less than that of the US, while the subsequent slowdown is associated with an increase in its relative size. They argue that Japan is converging to three different balanced growth paths corresponding to the period before the miracle, during the miracle, and the slowdown after the miracle.

Instead of studying this postwar development as an isolated experience, I look at it as a part of the long run economic development of Japan. I find that the slowdown of the Japanese economy after its miracle can be obtained without increasing its relative size of barriers. The difference in our results highlight the difference in my approach and the standard balanced growth path approach in accounting for international income differences.

As Japan also experienced a three-period delay compared to the UK, I assume π equal to 8 in Japan along the Malthus-only balanced growth path. The historical record suggests two episodes that significantly lowered barriers in Japan. They are the Meiji Restoration in 1868 which ended Shogunate Japan, and the postwar economic and institutional reforms.

According to Yamamura (1977), the new Meiji government adopted policies to encourage the absorption and dissemination of western technologies and skills, and help the growth of private industries. Following these policy changes, the fraction of workers employed in industry by both private and public firms increased significantly in 1907.

Postwar Japan underwent many major reforms such as introducing numerous tax-exemptions or tax-reliefs for investment; industry-financing program; allowing the purchase of new foreign patents; dissolving the *zaibatsu* system[19] and the deconcentration of many *zaibatsu* subsidiaries; and trade liberalization.[20] According to Ohkawa and Rosovsky (1963), these reforms led to a steep rise in the rate of private investment, a rapid decline in the agricultural sector, an acceleration of the introduction of new technologies, and a 38% increase in the productivity level of the manufacturing sector.

These reductions in barriers are also consistent with the data reported in Collins and Williamson (1999) and Jones (1994). Based on the data in their Tables (1a) – (2b), Figure (16) plots the data for Japan only while Figure (17) plots the data for Japan relative to the UK. Figure (16) illustrates two consistent facts for Japan during the period 1750 – 1950. First, the price of capital goods

[19] The "*zaibatsu*" is referred to a relatively small number of family-dominated company systems holding assets through large segments of the Japanese economy. These groups had become a major force in Japanese economic and political life before the World War II.

[20] There are many references for these reforms. For examples, Tsuru (1961), Ohkawa and Rosovsky (1963) and Rotwein (1964).

relative to consumer goods decreased drastically between 1875/79 to 1880/84 and remained fairly stable thereafter. Second, the price of equipment relative to consumer goods fell by 63 percent between 1875/79 and 1880/84 and continued to fall steadily during the period 1880 – 1950. Figure (17) plots the ratio of these relative prices between Japan and UK. The ratio of the relative price of capital goods in Japan to the UK dropped drastically between 1875/79 and 1880/84 and remained fairly stable until 1945. Similarly, the relative price of equipment in Japan to the UK dropped by 32 percent between 1875/79 and 1880/84, then fell steadily to a ratio of 2 in 1910, and remained fairly stable until 1945. This evidence is consistent with the view that barriers in Japan were reduced after the Meiji Restoration. According to Collins and Williamson, the relative price of equipment in Japan was 1.9 times that of the US in 1950. For the period 1960-1985, figures in Jones (1994) demonstrate that the relative price of equipment in Japan relative to the UK is equal to 0.6. Therefore, the data also supports a further reduction in π for the postwar period.

In view of these facts, I carry out the following exercise to account for the experience of Japan. Initially, π equal 8. In 1905, π is reduced by half. This I do because Figure (17) shows the ratio of relative equipment price between UK and Japan is reduced by half from 1870 to 1905. While I am not limiting my interpretation of barriers to this one dimension, I think this magnitude of reduction is at least a useful benchmark. Finally, based on the evidence in Jones (1994), I assume π is reduced to 0.6 for the postwar period. When solving for the model's equilibrium, I assume these changes in π are unexpected to the household.

Figure (18) shows the model's predictions. As seen, the model predicts that Japan catches up with the UK. There are two interesting points to note. First, the income difference for the period 1875 to 1940 is fairly stable though π is reduced by half in 1905. This is because the model predicts an inverted U-shape (see Figure (17)) for the time path of income differences for a given level of π. Therefore, if π is reduced before the maximum income difference is reached, it will only cause the income difference to increase at a smaller rate but not necessarily reduce it. This is an interesting property of the model and is consistent with the finding of Restuccia and Urrutia (2001) that the range of the relative price of investment is decreasing for the period 1960-85 while the magnitude of income differences is not.

Second, as shown in Figure (19), the model replicates both the Japanese miracle and the slowdown.[21] In contrast to Parente and Prescott (1994), in my model both the slowdown and miracle are consistent with a single change in π at the end of WWII. This result is closely related to the hump-shaped growth dynamics generated by this model.

I close this section with a remark. Reynolds (1985) documents that turning points for many countries have been associated with major political reform. In the context of this model, political reform (a permanent reduction in the level of barriers) is not necessary to generate a turning point, as shown in proposition 1. However, it can speed up the process of shifting input from the Malthus sector to the Solow sector. Moreover, as in the standard barrier model, it moves the economy to a higher balanced growth path. As shown by Japan's example, political reform increases the growth rate significantly.

7 Conclusion

Recent studies have emphasized differences in the cost of capital accumulation as a determinant of cross-country income differences, but they have generally focused on steady states. In this paper I focus on the role of the cost of capital accumulation in determining the beginning date and pace of modern economic growth. A fundamental property of the model is that cross-country income differences exhibit an inverted U-shape pattern over time, an important feature of long run economic data. A key implication of my model is that a substantial fraction of existing income differences is really a transitional phenomenon. This transitional effect increases significantly when I include the fact that today's low-income countries have had higher population growth rates during the early development stage than did the currently rich countries. I find interesting results in two case studies. The case of Africa relative to the UK demonstrates that the size of barriers that accounts for the differences in turning points also accounts for the path of relative income. The case of Japan

[21] The removal of barriers can only partly replicate the postwar miracle of Japan as the destruction of the capital during the war is also an important factor.

relative to the UK illustrates how the model can generate both the growth miracle and slowdown along the same development path.

There still remain other interesting questions. We have seen from Figure (10) that population profiles are different between early and late developers. While the population profiles of these countries do not affect their turning points, they have significant effects on the path of relative income. In this paper, I have treated the differences in population profiles across countries as exogenous. Endogenizing these differences is certainly an interesting topics. Doepke (1999) endogenizes the fertility dynamics for the Hansen-Prescott model I consider here. By assuming countries have the same population growth rate at their common turning point, the differences in the peaks of the population growth rates cannot be addressed in his model. Thus, we are still in search for a theory to accounts for such differences.

This model abstracts from the fact that home production (non-market sector) plays an important role in the early development stage of economy. Parente, Rogerson and Wright (2000) extend the standard barrier model to include home production. They find that the measured income disparity along the balanced growth path increases significantly if market and home produced goods are close substitutes and the capital share of the home production technology is small. Incorporating home production in this model is expected to work in a similar way as in their model.

Another interesting extension is to allow for mortality risk and human capital accumulation. One well-known development fact is that average years of schooling and life expectancy both increase over time. Moreover, the data clearly suggest a positive relationship between life expectancy at birth and GDP per capita for a given country. Intuitively, these features may be important because higher mortality may dampen the incentives for both human capital and physical capital accumulation, thereby slowing down the transition studied here. Incorporating these features into my model, modern growth begins later in a country with higher cost of capital accumulation, and so does its improvement in life expectancy. This provides a simple reason for why child labor is more prevalent in low-income countries, since acquiring education becomes less attractive relative to working if the return to education is decreased by a short life. Thus the model predicts both low levels of schooling and high mortality risk in low-income countries, a prediction which is supported by McGrattan and Schmitz (1998). They find a strong correlation between GDP per worker and the capital to output ratio, GDP per worker and primary school enrollment, and GDP per worker and secondary school enrollment in 1985.

Appendix 1.

Given q_{t-1}, N_t,L, and $I_t \equiv N_{t-1}(w_{t-1} - c_{1t-1}) - q_{t-1}L$, the total value of capital goods available at time t,and the fact that Solow technology is used, the fraction of labor and capital input allocated to each sectors can be determined. Profit and utility maximization conditions imply

$$k_{mt}= \psi \pi k_{st}$$

where $k_{mt} = \frac{K_{mt}}{N_{mt}}, k_{st} = \frac{K_{st}}{N_{st}}, \psi = \frac{(1-\theta)\phi}{\theta\mu} < 1$. Use the market clearing conditions

$$k_{mt} = \frac{\psi I_t / N_t}{1 - (1-\psi) m_t}$$

where $m_t = \frac{N_{mt}}{N_t}$ denotes the fraction of worker allocated to the Malthus sector. For labor to be indifferent across sectors

$$k_{mt}^{\theta-\phi} = \frac{\mu}{1-\theta} \frac{A_{mt}}{A_{st}} (\psi\pi)^{\theta} \left(\frac{L}{N_{mt}}\right)^{1-\phi-\mu}$$

Combining these two conditions, m_t solve

$$f(m_t; I_t, N_t) = \frac{\mu}{1-\theta} \pi^{\theta} \psi^{\phi} (1 - (1-\psi) m_t)^{\theta-\phi} - \frac{A_{st}}{A_{mt}} I_t^{\theta-\phi} N_t^{1-\theta-\mu} m_t^{1-\phi-\mu} = 0$$

Note that f is strictly decreasing in m_t if positive land share in the Malthus technology(1-$\mu - \phi > 0$) and capital is more important for the Solow technology($\theta \geq \phi$). Together with $f(0) > 0$ and $f(1) < 0$ (since $t \geq t_{\pi}^{*}$), there is an unique $m_t \in [0, 1)$ solve $f(m_t; I_t, N_t) = 0$. If $\frac{A_{st}}{A_{mt}} I_t^{\theta-\phi} N_t^{1-\mu-\theta}$ is increasing in t, then m_t converges to zero. This condition holds if $\gamma_s \geq \gamma_m$ and either $(i) 1-\theta \geq \mu$ and $g(c_{1t}) \geq 1$ or (ii) 1-$\theta < \mu$ and exist $\bar{t}$ and sufficiently small $n \geq 1$ *s.t.* $g(c_{1t}) \leq n \ \forall t > \bar{t}$.

Appendix 2.

This appendix shows that relative output per worker converge to $\left(\frac{\pi_1}{\pi_2}\right)^{\theta/(1-\theta)}$. As shown previously, young consume $\frac{1}{1+\beta}$ fraction of wage income. Since price of land converges to zero, feasibility implies

$$K_{t+1} = N_t (w_t - c_{1t}) = N_t \frac{\beta w_t}{\pi (1+\beta)}$$

Using firm's profit maximization and the condition that $g(c_{1t}) \to g$, the constant capital-output ratio is

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{\beta (1-\theta)}{\pi (1+\beta) g \gamma_s^{1/(1-\theta)}}$$

Thus,

$$\frac{\hat{y}_{\pi_1 st}}{\hat{y}_{\pi_2 st}} = \left(\frac{\pi_2}{\pi_1}\right)^{\theta/(1-\theta)}$$

Appendix 3. Sensitivity Analysis

I examine the robustness of the shape of Figure (8) with respect to changes in parameters of the model. These parameters are initial population, initial capital stock, quality of land, initial TFP levels for the Malthus and Solow technologies, input shares for Malthus technology, population growth rate along the Malthus balanced growth path, and the population growth function $g(c_1)$.

Initial Conditions Figures (20) and (21) demonstrate that doubling initial population, initial capital, quality of land and $\frac{A_m}{A_s}$ all have insignificant effects on the shape of the income difference curve.

Input Shares of the Malthus Technology Conditioning on the fact that the input shares does not affect the turning points, changing both the capital and land shares of the Malthus technology have an insignificant effect on the income difference. This is not surprising given Figure (5); the economy is almost in a Solow-only economy three periods after modern growth begins. Therefore input shares of the Malthus technology are not important in determining the income difference along the transition path.

Population Growth Rate Along the Malthus Balanced Growth Path Doubling the population growth rate along the Malthus balanced growth path from 0.3 percent to 0.6 percent will increase γ_m from 1.03 to 1.07. This will not have an effect on the turning point according to the equation (11). Moreover, γ_m does not enter into $g(c_1)$ when consumption is more than double its Malthus steady level. And, Figure (7) illustrates that consumption is doubled two periods after the transition. Therefore, γ_m is insignificant in determining the income difference once modern growth begins.

Population Dynamics I check the robustness of shape of income difference by varying x_1, x_2 and m. Figures (22) and (23) show that both x1 and x2 have an insignificant effect on the maximum income difference but m has a significant effect. By increasing the maximum annual population growth rate from 2% to 3% ($m = 2$ to $m = 2.81$), the maximum income difference is increased from 3.2 to 3.5 (a nearly 10 percent increase).

References

[1] Becker, Gary S., Kevin M. Murphy, and Robert Tamura. 1990. "Human Capital, Fertility, and Economic Growth." Journal of Political Economy, 98: S12-S37.

[2] Chari, V.V., Patrick Kehoe, and Ellen McGrattan. 1997. "The Poverty of Nations: A Quantitative Investigation." Working Paper, Federal Reserve Bank of Minneapolis.

[3] Coale, Ansley and Roy Treadway. 1979. " Summary of the Changing Distribution of Overall Fertility, Marital Fertility, and the Proportion Married in the Provinces of Europe." in *The Decline of Fertility in Europe: The Revised Proceedings of a Conference on the Princeton European Fertility Project.* Edited by Ansley J. Coale and Susan Cotts Watkins. Princeton University Press.

[4] Collins, William., Jeffrey G. Williamson. 1999. "Capital Goods Prices, Global Capital Markets and Accumulation: 1870-1950." NBER Working Paper No. 7145.

[5] Doepke, Matthias. 1999. "Growth and Fertility in the Long Run." The University of Chicago Working Paper.

[6] Dyson, Tim and Mike Murphy. 1985. "The Onset of Fertility Transition." Population and Development Review, 11(3): 399-440.

[7] Galor, Oded and David N.Weil. 1998. "Population, Technology, and Growth: From the Malthus Regime to the Demographic Transition." NBER Working Paper No. 6811.

[8] Goodfriend, Marvin, and John McDermott. 1995. "Early Development." American Economic Review, 85: 116-133.

[9] Hall, Robert E., and Charles I. Jones. 1998. "Why do Some Countries Produce So Much More Output per Worker than Others?" Quarterly Journal of Economics, 83-116.

[10] Hansen, Gary D., and Edward C. Prescott. 1999. "Malthus to Solow" Federal Reserve Bank of Minneapolis Staff Report #257.

[11] Jones, Charles I. 1994. "Economic Growth and the Relative Price of Capital." Journal of Monetary Economics, 34: 359-382.

[12] Jones, Charles I. 1999. "Was an Industrial Revolution Inevitable? Economic Growth over the Very Long Run." Working Paper, Stanford University.

[13] Kremer, Michael. 1993. "Population Growth and Technological Change: One Million B.C. to 1990." Quarterly Journal of Economics, 108(3): 681-716.

[14] Livi-Bacci, M., 1997. A Concise History of World Population. Second Edition. Blackwell, Oxford.

[15] Lucas, Robert E., Jr. 1988. "On the Mechanics of Economic Development." Journal of Monetary Economics, 22: 3-42.

[16] Lucas, Robert E., Jr. 1998, *The Industrial Revolution: Past and Future*, mimeo., University of Chicago, Department of Economics.

[17] Lucas, Robert E., Jr. 2000. "Some Macroeconomics for the 21st Century." Journal of Economic Perspectives, 14: 159-168.

[18] Mankiw, N. Gregory, David Romer, and David Weil. 1992. "A Contribution to the Empirics of Economic Growth." Quarterly Journal of Economics, 107:407-438.

[19] McGrattan, Ellen R., and James A. Schmitz, Jr. 1998. "Explaining Cross-Country Income Differences." Federal Reserve Bank of Minneapolis Staff Report #250.

[20] Ohkawa, Kazushi, and Henry Rosovsky. 1963. "Recent Japanese Growth in the Historical Perspective." American Economic Review, 53(2): 578-588.

[21] Parente, Stephen L., and Edward C. Prescott. 1994 "Barriers to Technology Adoption and Development." Journal of Political Economy, 102: 298-321.

[22] Parente, Stephen L., and Edward C. Prescott. 1999 "Monopoly Rights: A Barrier to Riches." American Economic Review 89: 1216-1233

[23] Parente, Stephen L., Richard Rogerson, and Randall Wright. 2000. "Homework In Development Economics: Household Production and the Wealth of Nations." Journal of Political Economy, 108(4): 680-688.

[24] Prescott, Edward C. 1998. "Needed: A Theory of Total Factor Productivity." International Economic Review, 39: 525-549.

[25] Pritchett, Lant. 1997. "Divergence, Big Time." Journal of Economic Perspectives, 11(3):3-18.

[26] Reynolds, L. George. Economic growth in the Third World, 1850-1980 (Yale University Press, 1985)

[27] Restuccia, Diego, and Carlos Urrutia. 2001. "Relative Prices and Investment Rates." Journal of Monetary Economics, 47: 93-121.

[28] Rotwein, Eugene. 1964. "Economic Concentration and Monopoly in Japan." Journal of Political Economy, 72 (3): 262-277.

[29] Tamura, Robert. 1996. "From Decay to Growth: A Demographic Transition to Economic Growth." Journal of Economic Dynamics and Control, 20: 1237-1261.

[30] Tsuru, Shigeto. 1961. "Growth and Stability of the Postwar Japanese Economy." American Economic Review, 51(2): 400-411.

[31] Wrigley, E.A., and R.S. Schofield. 1981. "The Population History of England 1541-1871: A Reconstruction." Harvard University Press.

[32] Yamamura, Kozo. 1977. "Success Illgotten? The Role of Meiji Militarism in Japan's Technological Progress." Journal of Economic History, 37(1): 113-135.

Table 1: Relative output per worker with θ equal to 0.4

π	Delay	BGP Difference	Maximum Difference	Percent Increased
2	1	1.6	1.8	18%
4	2	2.5	3.2	26%
8	2	4	5.3	33%
16	3	6.3	8.8	40%
32	4	10	14.1	41%
64	4	16	23	44%

Table 2: Combinations of θ and π for factor 30 income differences

θ	Delay	π (BGP)	π (Transition)	Percent Reduced
0.33	4	900	500	44%
0.4	4	164	100	39%
0.45	4	64	40	37%
0.5	4	30	18	40%
0.55	3	16	10	38%
0.6	3	10	6.5	35%

Table 3: Relative output per worker with π equals to 4

θ	Delay	BGP Level	Maximum Level	Percent Increased
0.33	1	2.0	2.3	15%
0.4	2	2.5	3.2	26%
0.45	2	3.1	4.1	31%
0.5	2	4	5.8	45%
0.55	2	5.4	8.1	50%
0.6	2	8	13	63%

Figure 1: GDP per Capita for 5 Regions

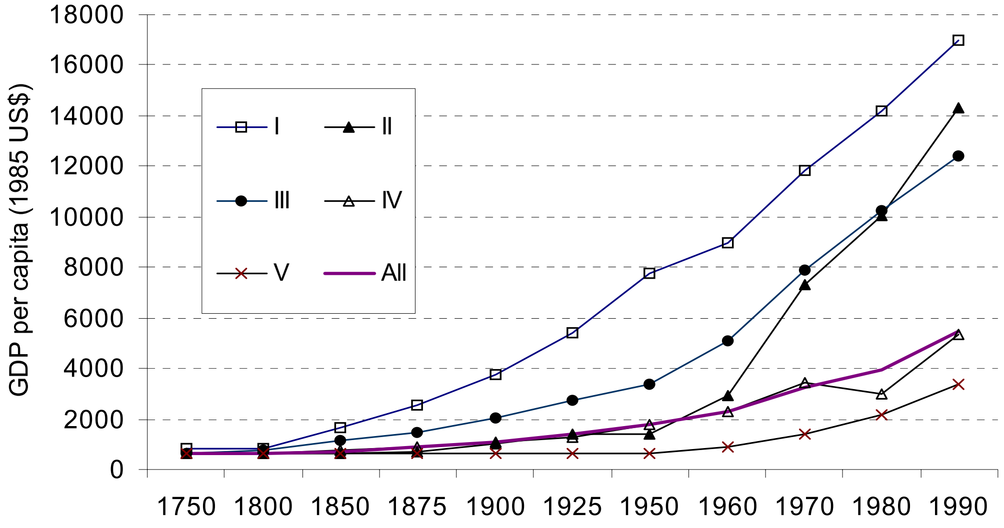


Figure 2: Turning Points

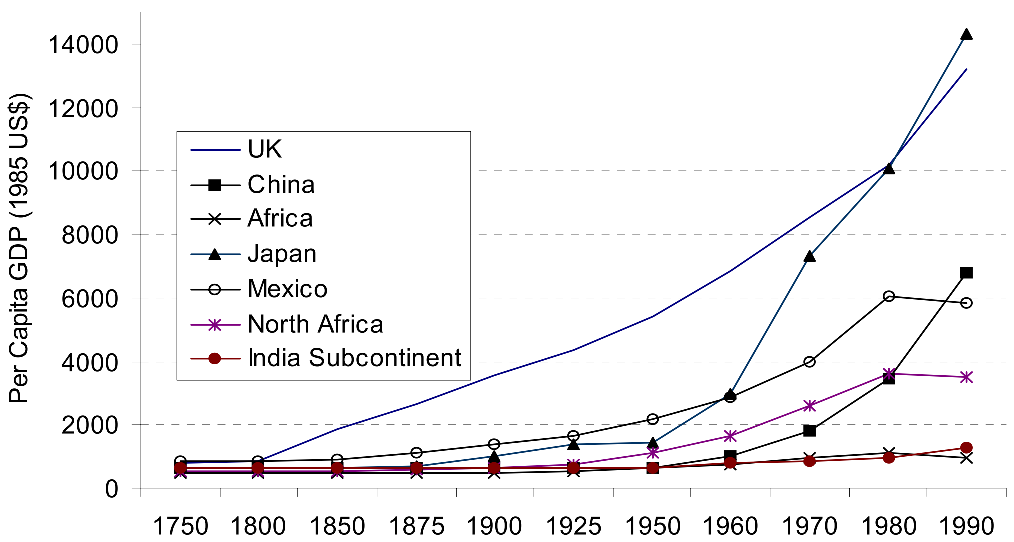


Figure 3: Relative GDP per Capita ($\frac{y_{Uk}}{y_i}$)

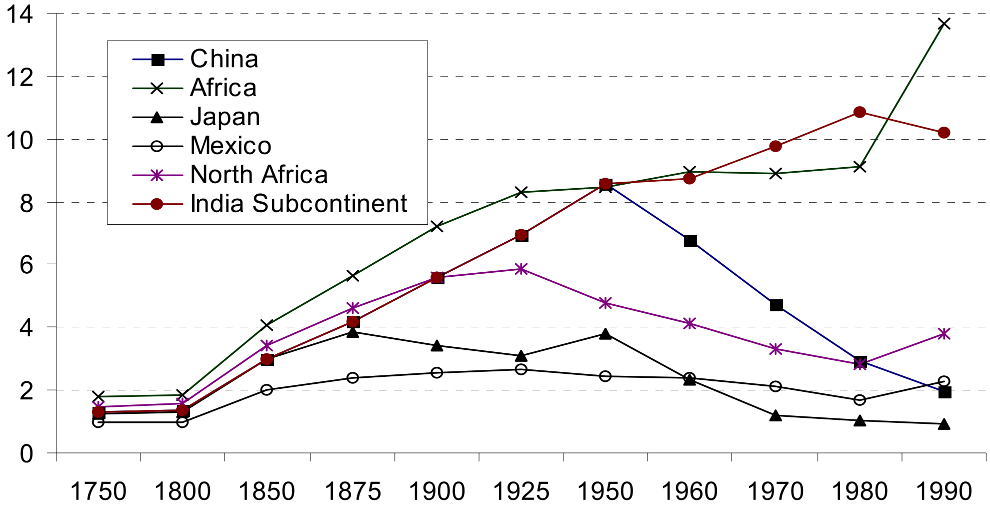


Figure 4: Population Growth Function

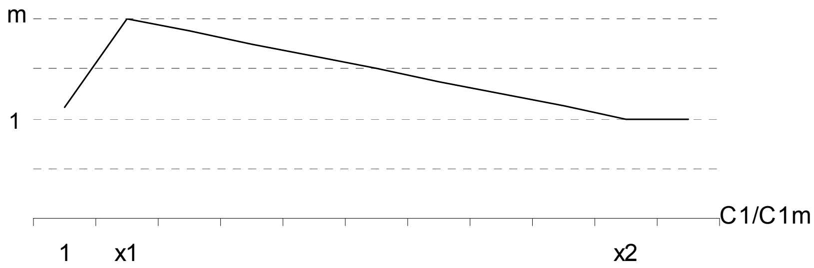


Figure 5: Fraction of Labor Allocated to the Malthus Sector

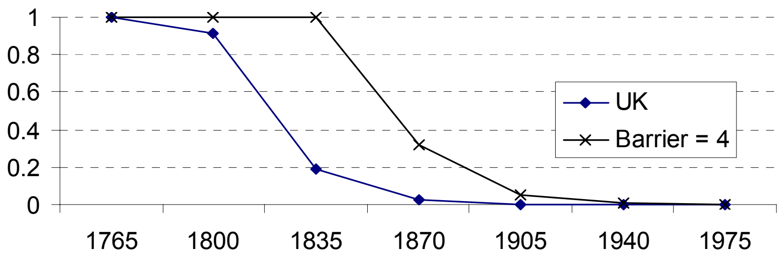


Figure 6: Fraction of Capital Allocated to the Malthus Sector

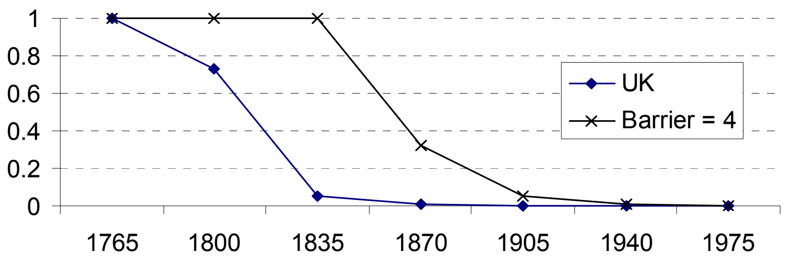


Figure 7: Normalized Output per Worker

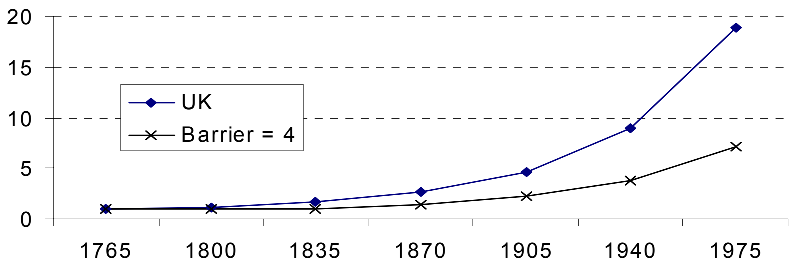


Figure 8: Relative Output per Worker

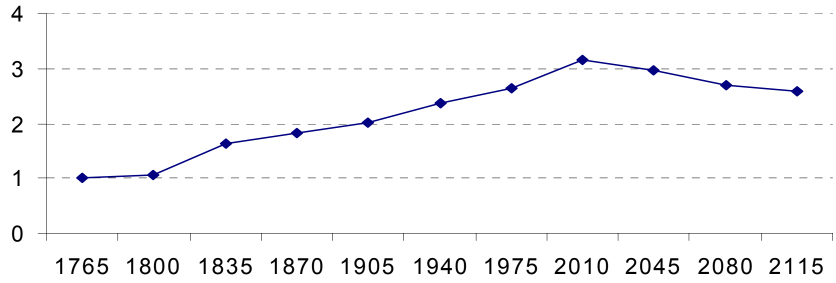


Figure 9: Growth rate of Output per Worker

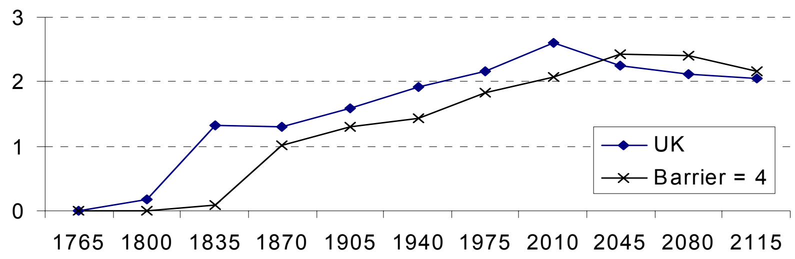


Figure 10: Average Annual Population Growth Rate

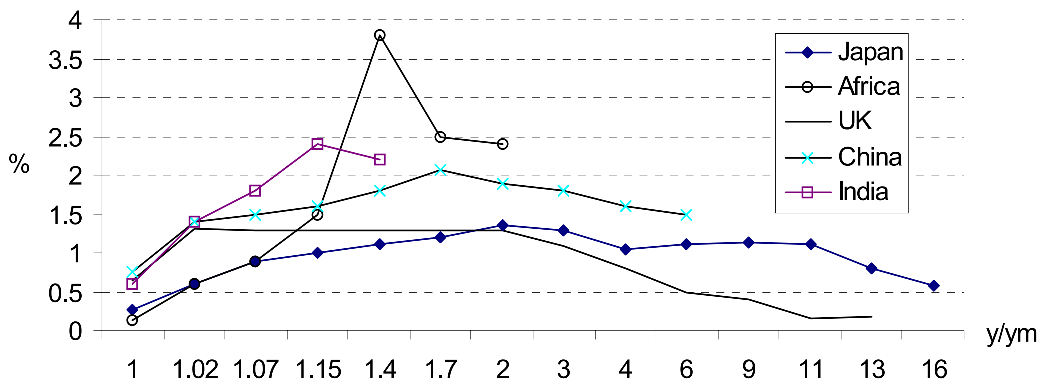


Figure 11: Relative Output per Worker (Different Population Profiles)

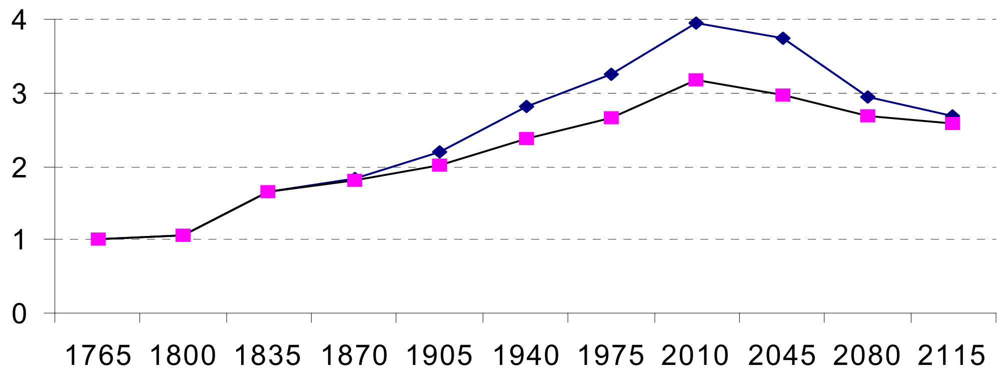


Figure 12: The UK's Income Relative to Africa

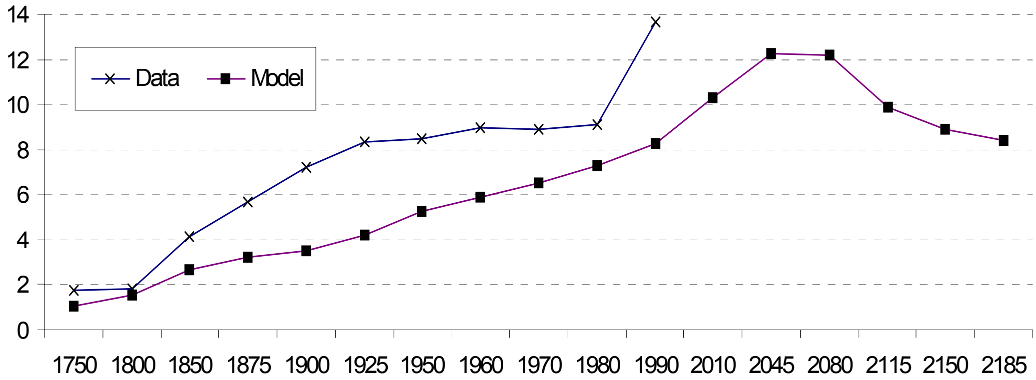


Figure 13: The UK's Income Relative to Africa (Adjust Initial Condition)

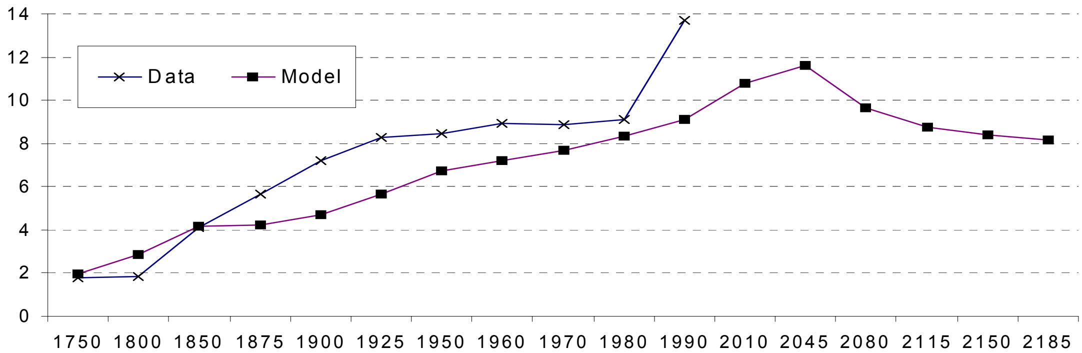


Figure 14: The UK's Income Relative to Africa (Adjust Population Profile)

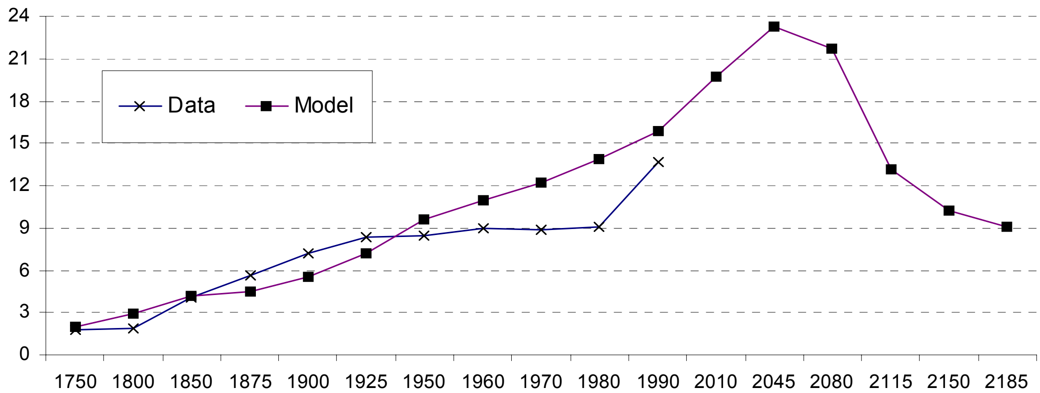


Figure 15: Per Capita GDP Growth Rate in the Data

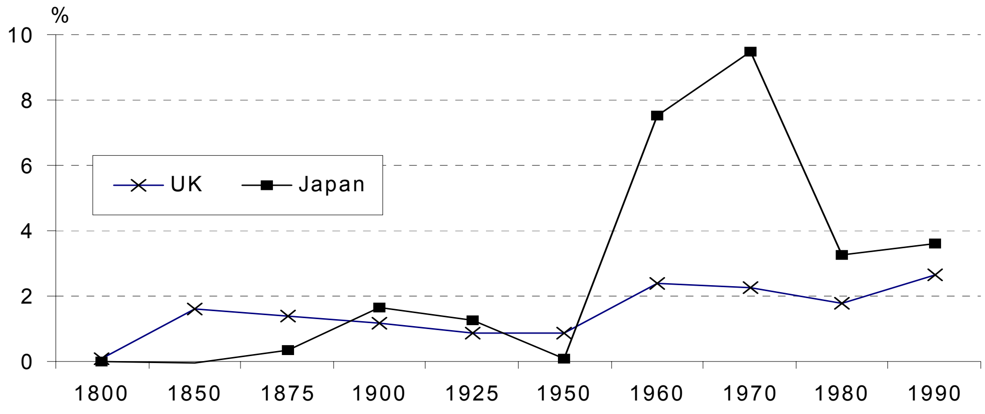


Figure 16: Relative Prices in Japan

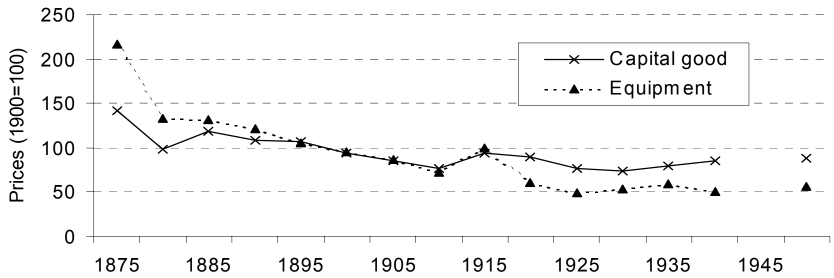


Figure 17: Ratio of Relative Prices (Japan/UK)

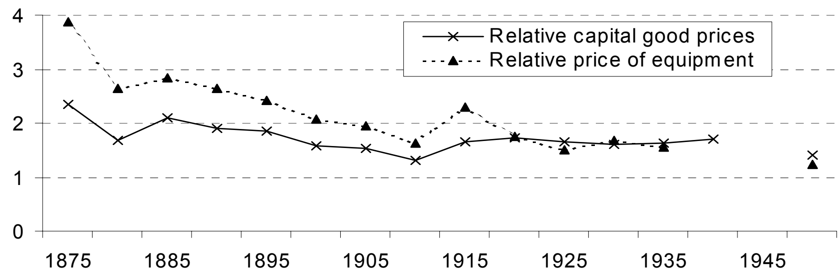


Figure 18: The UK's Income Relative to Japan

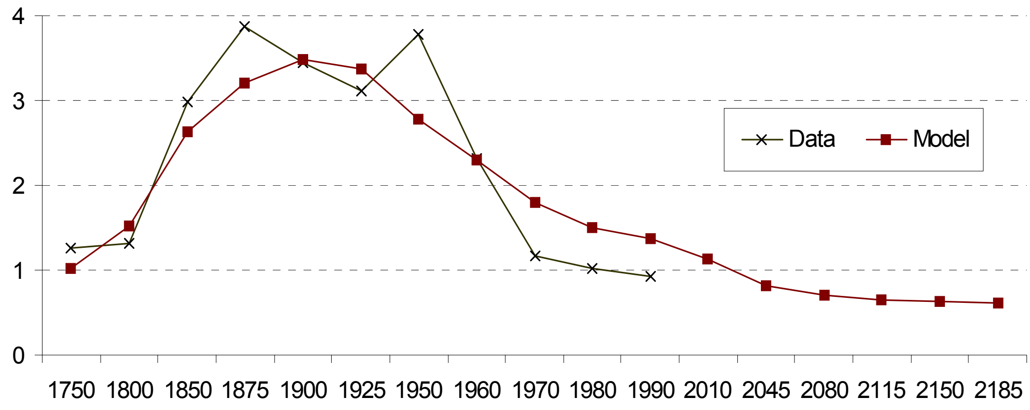


Figure 19: Per Capita Output Growth Rate in the Model

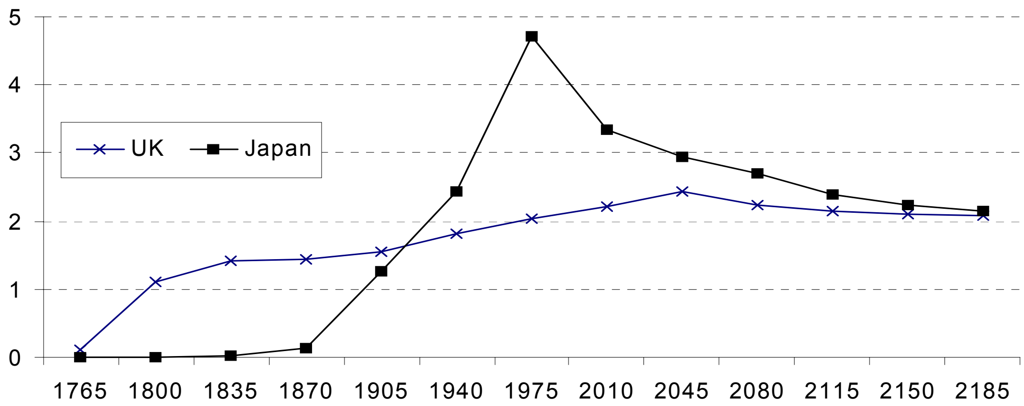


Figure 20: Initial Capital and Labor

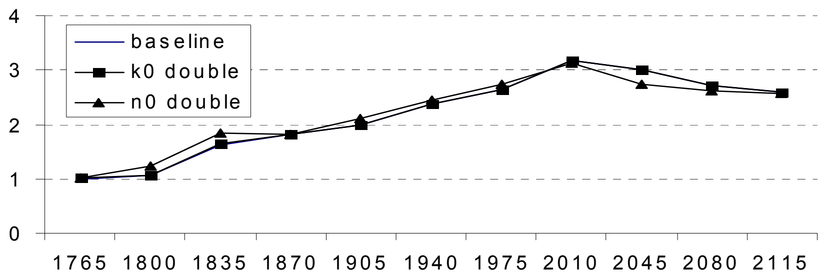


Figure 21: Initial TFP and Land

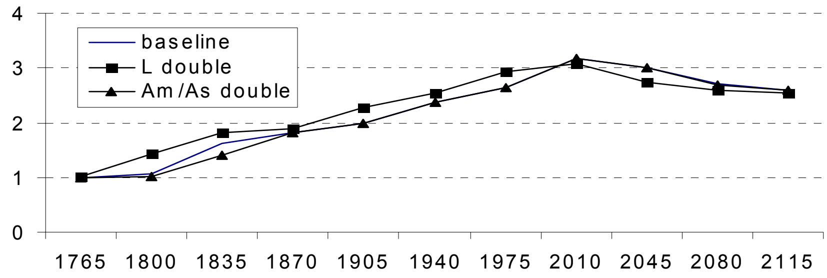


Figure 22: Population Profile (a)

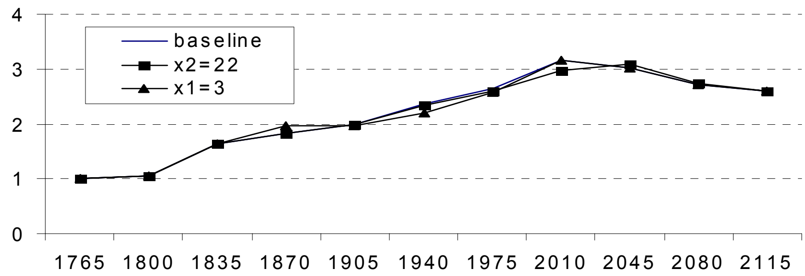
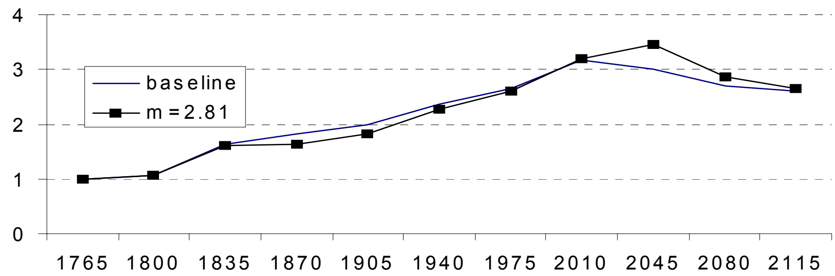


Figure 23: Population Profile (b)



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