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Monetary Policy and Welfare in a Small Open Economy

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Abstract

This paper characterizes welfare in a small open economy and derives the corresponding optimal monetary policy rule. It shows that the utility-based loss function for a small open economy is a quadratic expression in domestic inflation, output gap and real exchange rate. In contrast to previous works, this paper demonstrates that welfare in a small open economy, completely integrated with the rest of the world, is affected by exchange rate variability. Consequently, the optimal policy in a small open economy is not isomorphic to a closed economy and does not prescribe a pure floating exchange rate regime. Domestic inflation targeting is optimal only under a particular parameterization, where the unique relevant distortion in the economy is price stickiness. Under a general specification for preferences and in the presence of inefficient steady state output, exchange rate targeting arises as part of the optimal monetary plan.

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1 Introduction

Should monetary policy target the exchange rate? Numerous papers have analyzed the choice of monetary policy objectives in closed and open economies. In the former, the debate has mainly focused on whether inflation should be the unique policy target. In open economies, the characterization of optimal policy extends beyond policy makers' decision to concentrate on domestic price distortions. More specifically, the role of the exchange rate in the monetary policy framework needs to be considered. This work addresses this particular issue in a small open economy setting. Our results suggest that including the exchange rate as part of the stabilization goals of monetary policy can be welfare improving from a small open economy point of view.

The present work lays out a small open economy model as a limiting case of a two country dynamic general equilibrium framework featuring monopolistic competition and price stickiness. Moreover, the framework assumes no trade frictions (i.e. the law of one price holds) and perfect capital markets (i.e. asset markets are complete). This benchmark specification allows us to focus on the policy implications of the following factors: (a) Calvo-type staggered price setting; (b) monopolistic competition in goods' production and the resulting inefficient level of output; (c) trade imbalances; and (d) deviations from purchasing power parity that arise from the home bias specification. The framework presented here encompasses, as special cases, the closed economy setting (as in Benigno and Woodford, 2003) and the small open economy case with a specific degree of monopolistic competition and no trade imbalances (as in Gali and Monacelli, 2005).

The small open economy representation prevents domestic policy from affecting the rest of the world and, therefore, permits us to abstract from strategic interactions between countries. We focus on understanding how monetary authorities should react to fluctuations in internal and external conditions, when these reactions have no feedback effects.

Following the method developed by Benigno and Woodford (2003) and Sutherland (2002), we derive the loss function for a small open economy from the utility of the representative household. We show that the small open economy's loss function is a quadratic expression in domestic producer inflation,

the output gap and the real exchange rate. The weights given to each of these variables depend on structural parameters of the model, and therefore, are determined by the underlying economic inefficiencies. In addition, the policy targets depend on the source of the disturbance affecting the economy, which includes an external shock.

The analytical representation of welfare allows for a precise qualitative analysis of monetary policy in a small open economy. The results obtained show that domestic inflation targeting is optimal only under a particular parameterization of the model. In cases where the economy experiences productivity and foreign shocks, a domestic inflation target is optimal only under a particular parameterization for the coefficient risk aversion and the elasticity of substitution between domestic and foreign goods. Moreover, if fiscal disturbances are also present, the optimality of domestic price stabilization further requires a production subsidy. Conversely, in the general specification of the model, the exchange rate becomes part of monetary policy targets. Therefore, the policy prescription in a small open economy is not isomorphic to a closed economy and does not prescribe a pure floating exchange rate regime. Moreover, the quantitative results show that for a large set of parameter specifications, an exchange rate peg outperforms a strict domestic inflation target. This result is consistent with the findings of Sutherland (2005). The author demonstrates that for high values of the elasticity of substitution between goods a fixed exchange rate regime leads to higher welfare than targeting domestic prices.

The results obtained in this paper add to the rich debate on optimal monetary policy. Studies such as Woodford (2002) and Woodford and Benigno (2003) were pioneer in deriving a quadratic loss function (from the utility of the representative household) for a closed economy. The authors show how distortions created by monopolistic competition affect the optimality of price stabilization. In an open economy setting, Corsetti and Pesenti (2000) were the first to emphasize that a country might benefit from influencing its terms of trade. Benigno and Benigno (2003) illustrate the potential gains from cooperation of monetary policy between countries by analyzing the incentives of individual countries to affect the exchange rate. Gali and Monacelli (2004) derive the loss function, for a particular characterization of a small open economy, which was shown to be analogous to a closed economy. The present work aims at contributing to the literature by characterizing a general loss function for a small

open economy which leads to a better understanding of the international dimension of monetary policy.

The remainder of the paper is structured as follows. Section 2 introduces the model and derives the small open economy dynamics. Section 3 is dedicated to the derivation of welfare and the quadratic loss function. Section 4 analyses the optimal plan and the performance of standard policy rule. Section 5 concludes.

2 The Model

The framework consists of a two-country dynamic general equilibrium model with complete asset markets. Deviations from purchasing power parity arise from the existence of home bias in consumption. The dimension of this bias depends on the degree of openness and the relative size of the economy. This specification allows us to characterize the small open economy by taking the limit of the home size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two country model. After the limit is taken, the two countries Home and Foreign represent the small open economy and the rest of the world, respectively.

Monopolistic competition and sticky prices are introduced in the small open economy in order to address issues of monetary policy. We further assume that home price setting follows a Calvo-type contract, which introduces richer dynamic effects of monetary policy than a setup where prices are set one period in advance. Moreover, we abstract from monetary frictions by considering a cashless economy, as in Woodford (2003, Chapter 2).

2.1 Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F . The utility function of a consumer j

in country H is given by¹:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s(h), \varepsilon_{Y,s})], \quad (1)$$

Households obtain utility from consumption $U(C)$ and contribute to the production of a differentiated good $y(h)$ attaining disutility $V(y(h), \varepsilon_{Y,s})$ ². Productivity shocks are denoted by $\varepsilon_{Y,s}$. Moreover, C is a Dixit-Stiglitz aggregator of home and foreign goods as

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (2)$$

where $\theta > 0$ is the intratemporal elasticity of substitution and C_H and C_F are the two consumption sub-indices that refer to the consumption of home-produced and foreign-produced goods, respectively. The parameter determining home consumers' preference for foreign goods, $(1-v)$, is a function of the relative size of the foreign economy, $1-n$, and of the degree of openness, λ ; more specifically, $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the rest of the world:

$$C = \left[v^*{}^{\frac{1}{\theta}} C_H^*{}^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^*{}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

with $v^* = n\lambda$. That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of v and v^* generates a home bias in consumption, as in Sutherland (2002).

The sub-indices C_H (C_H^*) and C_F (C_F^*) are Home (Foreign) consumption of the differentiated prod-

¹In the next sections we assume the following isoelastic functional forms: $U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$ and $V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{-\eta} y_t^{1+\eta}}{1+\eta}$. Where ρ is the coefficient of relative risk aversion and η is equivalent to the inverse of the elasticity of labor production.

²This specification would be equivalent to one in which the labour market is decentralized. These firms employ workers who have disutility of supplying labour and this disutility is separable from the consumption utility.

ucts produced in countries H and F . These are defined as follows:

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad (4)$$

$$C_H^* = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad C_F^* = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad (5)$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by:

$$P = \left[v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \theta > 0 \quad (6)$$

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*) (P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \theta > 0, \quad (7)$$

where P_H (P_H^*) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and P_F (P_F^*) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency.

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad (8)$$

$$P_H^* = \left[\left(\frac{1}{n} \right) \int_0^n p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad P_F^* = \left[\left(\frac{1}{1-n} \right) \int_n^1 p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}. \quad (9)$$

Moreover, we assume that the law of one price holds, therefore:

$$p(h) = S p^*(h) \text{ and } p(f) = S p^*(f), \quad (10)$$

where the nominal exchange rate S_t denotes the price of foreign currency in terms of domestic

currency. Therefore, equations (6), (7) together with condition (10), imply that $P_H = SP_H^*$ and $P_F = SP_F^*$. However, as equations (8) and (9) illustrate, the home bias specification leads to deviations from purchasing power parity, that is, $P \neq SP^*$ ³. For this reason, we define the real exchange rate as $RS = \frac{SP^*}{P}$.

From consumers' preferences, we can derive the total demand for a generic good h , produced in country H, and the demand for a good f , produced in country F:

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[vC + \frac{v^*(1-n)}{n} \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G \right\} \quad (11)$$

$$y^d(f) = \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left\{ \left[\frac{P_F}{P} \right]^{-\theta} \left[\frac{(1-v)n}{1-n} C + (1-v^*) \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G^* \right\}, \quad (12)$$

where G and G^* are country-specific government shocks. We assume that the public sector in the Home (Foreign) economy only consumes Home (Foreign) goods and has preferences for differentiated goods analogous to the ones of the private sector (given by equations 4 and 5). The government budget constraints in the Home and Foreign economy are respectively given by:

$$\tau_t \int_0^n p_t(h) y_t(h) dh = n P_{H,t} (G_t + Tr_t) \quad (13)$$

$$\tau_t \int_n^1 p_t^*(f) y_t^*(f) dh = (1-n) P_{F,t} (G_t^* + Tr_t^*). \quad (14)$$

We consider the case in which fluctuations in proportional taxes τ_t (τ_t^*) or government spending G_t (G_t^*) are exogenous and completely financed by lump-sum transfers Tr_t (Tr_t^*), made in the form of domestic (foreign) goods.

Finally, to portray our small open economy we use the definition of v and v^* and take the limit for

³The literature investigating the empirical evidences of Purchasing Power Parity is vast and has shown that the short run deviations from PPP are large and volatile (as documented in Rogoff (1996)). Even though our model specification is in accordance with these findings, it dismisses the evidences of failures of the law of one price.

$n \rightarrow 0$. Consequently, conditions (11) and (12) can be rewritten as:

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[(1-\lambda)C + \lambda \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G_t \right\} \quad (15)$$

$$y^d(f) = \left[\frac{p^*(f)}{P_F^*} \right]^{-\sigma} \left\{ \left[\frac{P_F^*}{P^*} \right]^{-\theta} C^* + G^* \right\}. \quad (16)$$

Equations (15) and (16) show how external changes in consumption affect the small open economy, but the reverse is not true. Moreover, movements in the real exchange rate do not modify the rest of the world's demand.

2.2 The asset market structure

We assume that, as in Chari et al. (2002), markets are complete domestically and internationally. In each period t , the economy faces one of the finitely many events, $s^t \in \Upsilon$ (where Υ is the set of finitely many states). We denote the history of events up to and including period t by x^t . Looking ahead from period t , the conditional probability of occurrence of state s^{t+1} is $\mu(s^{t+1} | x^t)$. The initial realization s^0 is given. We represent the asset structure by having complete contingent one period nominal bonds denominated in the home currency. We let $B^j(s^{t+1})$ denote the home consumer's holdings of this bond, which pays one unit of the home currency if state s^{t+1} occurs and 0 otherwise, and we let $Q(s^{t+1} | x^t)$ denote the price of one unit of such a bond at date t and state s^t in units of domestic currency. Therefore, consumer j faces a sequence of budget constraints given by:

$$P(s^t)C^j(s^t) + \sum_{s^{t+1} \in \Upsilon} Q(s^{t+1} | x^t)B^j(s^{t+1}) \leq B^j(s^t) + (1 - \tau_t)p^j(s^t)y^j(s^t)dh + P_H(s^t)Tr(s^t). \quad (17)$$

A similar expression can be derived for the foreign economy. Households at home maximize (1) subject to (17) and their optimal allocation of wealth across the different state contingent bonds implies that

$$Q(s^{t+1} | x^t) = \beta \mu(s^{t+1} | x^t) \frac{U_C(C(s^{t+1}))}{U_C(C(s^t))} \frac{P(s^t)}{P(s^{t+1})}. \quad (18)$$

Similarly for the foreign economy:

$$Q(s^{t+1} | x^t) = \beta \mu(s^{t+1} | x^t) \frac{U_C(C^*(s^{t+1}))}{U_C(C^*(s^t))} \frac{S(s^t)P^*(s^t)}{S(s^{t+1})P^*(s^{t+1})}. \quad (19)$$

Therefore, the optimal risk sharing setting implies that the intertemporal marginal rate of substitution (in nominal terms) is equalized across countries.

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}}. \quad (20)$$

Equation (20) holds in all states of nature. This specification for the asset market implies that the risk arising from movements in agent's nominal wealth is shared with the rest of the world. However, because of deviations from purchasing power parity, real exchange rate movements may lead to differences between home and foreign real income and, consequently, differences in the evolution of consumption across borders.

2.3 Price-setting Mechanism

Prices follow a partial adjustment rule a la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions (given by equations (15) and (16)), and maximize profits taking the overall market prices and products as given. In each period a fraction $\alpha \in [0, 1)$ of randomly chosen producers is not allowed to change the nominal price of the good it produces. The remaining fraction of firms, given by $(1 - \alpha)$, chooses prices optimally by maximizing the expected discounted value of profits⁴. Therefore, the optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T

⁴All households within a country (that can modify their prices at a certain time) face the same discounted value of the streams of current and future marginal costs. Thus, they choose to set the same price.

is:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(j)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(j)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma}{(1-\tau_T)(\sigma-1)} \frac{V_y(\tilde{y}_{t,T}(j), \varepsilon_{Y,T})}{U_c(C_T)} \right] \right\} = 0 \quad (21)$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by $\frac{\sigma}{(1-\tau_t)(\sigma-1)}$ ⁵. We allow for fluctuations on this wedge by assuming a time varying proportional tax τ_t . Hereafter, we refer to these fluctuations as mark up shocks μ_t , where $\mu_t = \frac{\sigma}{(1-\tau_t)(\sigma-1)}$.

Given the Calvo-type setup, the price index evolves according to the following law of motion:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) (\tilde{p}_t(h))^{1-\sigma}. \quad (22)$$

The rest of the world has an analogous price setting mechanism.

2.4 A log-linear representation of the model

In this section we present a log linear version of the model. This is done in order to obtain a simple representation of the optimality conditions derived above and illustrate the dynamic properties of the model. Moreover, we later solve the log-linearized model numerically using the algorithm of King and Watson (1998) and present a quantitative analysis of the model. We approximate the model around a steady state in which the exogenous variables $(\varepsilon_{y,t}, G_t, \mu_t)$ assume the values $\bar{\varepsilon}_y \geq 0$, $\bar{G} = 0$, $\mu \geq 1$, and producer price inflation is set as $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1} = 1$. In addition, in this steady-state $\bar{R}\bar{S} = 1$, $\bar{C} = \bar{C}^*$, $\bar{Y} = \bar{Y}^*$ and $\bar{U}_C(\bar{C}) = \mu \bar{V}_y(\bar{Y}, 0)$ ⁶. Log deviations from the steady state are denoted with a hat.

The small open economy system of equilibrium conditions derived from log linearizing equations (6), (15), (20) and (21) is:

⁵Note that if there are no proportional taxation and an infinitely elastic demand $\mu = 1$. This specification characterizes the perfect competition case.

⁶This specification implies a specific level of initial distribution of wealth across countries. Appendix A contains the full characterization of the steady state.

$$(1 - \lambda)\hat{p}_H + \lambda\widehat{RS} = 0 \quad (23)$$

$$\hat{Y}_t = -\theta\hat{p}_H + (1 - \lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\widehat{RS}_t + \hat{g}_t \quad (24)$$

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho}\widehat{RS}_t \quad (25)$$

$$\hat{\pi}_t^H = k \left(\rho\hat{C}_t + \eta\hat{Y}_t - \hat{p}_H + \hat{\mu}_t - \eta\hat{\varepsilon}_{Y,t} \right) + \beta E_t \hat{\pi}_{t+1}^H \quad (26)$$

Equation (23) describes the relationship between domestic relative prices ($p_{H,t} = P_{H,t}/P_t$) and the real exchange rate. Equation (24) characterizes the demand for domestic goods, with \hat{g}_t defined as $\frac{G_t - \bar{G}}{\bar{Y}}$. The risk sharing condition is described in equation (25). Finally, the last equation represents the small open economy Phillips Curve. We define $k = (1 - \alpha\beta)(1 - \alpha)/\alpha(1 + \sigma\eta)$ and π_t^H denotes the producer price inflation, i.e. $\pi_{H,t} \equiv \ln(P_{H,t}/P_{H,t-1})$. Moreover, it is clear from equation (26), that a policy of pure domestic price stabilization, that sets $\hat{\pi}_t^H = 0$ in every state, leads to the same equilibrium allocation as the case in which prices are perfectly flexible, i.e. when $\alpha = 0$, and therefore $k \rightarrow \infty$.

The system of structural equilibrium conditions is closed by specifying the monetary policy rule. In this paper we consider the case in which monetary policy follows an optimal monetary policy. We represent the optimal plan in the form of a targeting rule. Targeting rules, as expressed in Svensson (2005), are a description of "goal directed monetary policy". Contrary to Taylor rules, an explicit expression for the evolution of the monetary policy instrument (i.e. the nominal interest rate) is not specified⁷. Gianonni and Woodford (2003) describe these rules as flexible inflation targets. Following this class of rules, the central bank stabilizes movements in the target variables in order to implement the most efficient allocation of resources (i.e. targeting rules are derived from a microfounded welfare

⁷For further discussion on targeting rules and instrumental rules see McCallum and Nelson (2005).

maximization problem). Moreover, apart from the case in which monetary policy is represented by an optimal targeting rule, we also consider the case in which the central bank follows standard policy rules. In particular, we analyze the performance of a producer price index (PPI) inflation targeting regime, an exchange rate peg, and a consumer price index (CPI) inflation target.

Therefore, the dynamics of \widehat{Y}_t , \widehat{RS}_t , \widehat{C}_t , $\widehat{\pi}_t^H$ and $\widehat{p}_{H,t}$ are determined by equations (23) to (26) together with the specified monetary policy, given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}$, \widehat{g}_t , $\widehat{\mu}_t$ and the external shock \widehat{C}_t^* ⁸.

Foreign dynamics are governed by the foreign Phillips curve and foreign demand:

$$\widehat{\pi}_t^* = k \left(\rho \widehat{C}_t^* + \eta \widehat{Y}_t^* + \widehat{\mu}_t^* - \eta \widehat{\varepsilon}_{Y,t}^* \right) + \beta E_t \widehat{\pi}_{t+1}^* \quad (27)$$

$$\widehat{Y}_t^* = \widehat{C}_t^* + \widehat{g}_t^*. \quad (28)$$

The specification of the foreign policy rule completes the system of equilibrium conditions, which determine the evolution of \widehat{Y}_t^* , \widehat{C}_t^* and $\widehat{\pi}_t^*$. We should note that the dynamics of the rest of the world is not affected by *Home* variables. Therefore, the small open economy can treat C_t^* as exogenous. Moreover, the policy choice of the rest of the world modifies the way in which foreign structural shocks affect C_t^{*9} but does not influence how the latter affects the small open economy.

3 Welfare

The advantage of a microfounded model is that agents' discounted sum of expected utility provides a precise measure for welfare. That is, the small open economy objective function can be obtained from equation (1). We follow the method developed by Woodford (2003) and Benigno and Woodford (2003) and obtain a quadratic expression for equation (1). This allows us to represent the policy

⁸In order to retrieve the value of the nominal exchange rate and interest rate we can use households' intertemporal choice (i.e. the Euler equation) and the definition of the real exchange rate.

⁹For example, if the foreign authority is following a strict inflation target the evolution of domestic consumption is given by: $(\rho + \eta)\widehat{C}_t^* = \eta\widehat{\varepsilon}_{Y,t}^* - \eta\widehat{g}_t^* - \widehat{\mu}_t^*$

problem in a comprehensive manner. That is, policy makers minimize a quadratic loss function subject to linear constraints. Moreover, the resulting optimal monetary policy can be expressed analytically. Alternative approaches to welfare evaluation include the computational methods described in Schmitt-Grohe and Uribe (2004), Collard and Juillard (2000) and Kim et al. (2003). These techniques are based on perturbation methods and deliver a numerical evaluation of the optimal policy problem.

We should note that the linear quadratic approach presented here take into account the effect of second moments in the mean of the endogenous variables. As discussed in Benigno and Woodford (2004), this ensures that the method delivers an accurate (local) welfare evaluation tool. Another important contribution that emphasizes the relevance of second order effects on the mean of variables is Obstfeld and Rogoff (1998).

In the appendix we derive a second order approximation to equation (1). In order to eliminate the discounted linear terms in the Taylor expansion, we use a second order approximation to some of the structural equilibrium conditions and obtain a complete second order solution for the evolution of the endogenous variables of interest. It follows that the final expression for the small open economy loss function can be written as a quadratic function of \widehat{Y}_t , \widehat{RS}_t , and $\widehat{\pi}_t^H$:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] + t.i.p, \quad (29)$$

where the policy targets \widehat{Y}_t^T and \widehat{RS}_t^T are functions of the various shocks and, in general, do not coincide with the flexible price allocation for output and the real exchange rate. Moreover, the weights of inflation, output and the real exchange rate gap in welfare losses, Φ_π , Φ_Y and Φ_{RS} , all depend on the structural parameters of the model. The expressions for these variables are specified in the appendix. Moreover, the term *t.i.p* stands for *terms independent of policy*.

What are the economic forces behind these welfare losses? The small open economy specification presented in this work is characterized by two economic inefficiencies: price rigidity and monopolistic competition in production. In addition, in an open economy, domestic consumption is not necessarily equal to domestic production. In particular, movements in international relative prices can create differences between the marginal utility of consumption and the marginal disutility of production that

directly affect welfare¹⁰. These three factors create different policy incentives. The presence of staggered prices brings in gains from minimizing relative price fluctuations (justifying the presence $\Phi_\pi(\widehat{\pi}_t^H)^2$ in equation (29)). Moreover, monopolistic competition in production implies a suboptimal level of steady state output and introduces an incentive to reduce the steady state production inefficiency. Finally, there may be incentives to manage fluctuations in the exchange rate in order to affect the wedge between the marginal utility of consumption and the marginal disutility of production (hereafter this is referred as the " U_c/V_y gap"). The last two factors imply that optimal monetary policy might deviate from price stability (and are responsible for the presence of the terms $\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2$ and $\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T)^2$ in equation 29).

To better understand the argument presented above, we first characterize a closed economy by setting $\lambda = 0$. In this case, equation (29) can be written as:

$$L_{to}^c = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y^c (\widehat{Y}_t - \widehat{Y}_t^{T,c})^2 + \frac{1}{2} \Phi_\pi^c (\widehat{\pi}_t^H)^2 \right] + t.i.p, \quad (30)$$

where the subscript c denotes the closed economy. The policy maker's problem in a closed economy can be illustrated by the relative weight of inflation with respect to output Φ_π/Φ_Y and by the difference between \widehat{Y}_t^T and \widehat{Y}_t^{Flex} (where \widehat{Y}_t^{Flex} represents the flexible price allocation for output).

$$\frac{\Phi_\pi^c}{\Phi_Y^c} = \frac{\sigma}{k(\eta + \rho)} \quad (31)$$

$$\widehat{Y}_t^{T,c} = \frac{\eta \varepsilon_{Y,t}}{(\eta + \rho)} - \frac{(\mu - 1)(\eta + 1)\mu_t}{(\eta + \rho)(\mu\eta + \rho + (\mu - 1))} + \frac{\rho(\eta\mu + \rho)g_t}{(\eta + \rho)(\mu\eta + \rho + (\mu - 1))} \quad (32)$$

$$\widehat{Y}_t^{Flex,c} = \frac{\eta \varepsilon_{Y,t} - \mu_t + \rho g_t}{(\eta + \rho)}. \quad (33)$$

As the above expressions show, $\widehat{Y}_t^{T,c} \neq \widehat{Y}_t^{Flex,c}$, that is, a policy of strict inflation targeting (which

¹⁰This is represented by the term $U_c \bar{C} (\widehat{C}_t - \widehat{Y}_t/\mu)$ in the Taylor expansion of the utility function (shown in the appendix). Note that in our linear-quadratic approach this term is expressed in terms of second moments. In particular, it can be written as a function of the variance of the real exchange rate and output gap.

mimics the flexible price allocation) does not close the welfare relevant output gap. Therefore, there is a trade-off between stabilizing inflation and output. Moreover, the weight of inflation relative to output in the loss function depends essentially on the degree of market power σ and the degree of price rigidity α (which determines the parameter k). When the elasticity of substitution between goods is infinite, i.e. the market is competitive, then the relative weight on the output gap vanishes. On the other hand, when $\alpha \rightarrow 0$ (and consequently $k \rightarrow \infty$), the relative weight on inflation fades away as there are no distortions associated with price rigidity. Furthermore, the steady state level of mark up, μ , and mark up fluctuations, μ_t , imply differences between \widehat{Y}_t^T and \widehat{Y}_t^{Flex} . Whenever the steady state level of production is efficient, i.e. $\mu = 1$, and there are no mark up fluctuations, we have $\widehat{Y}_t^T = \widehat{Y}_t^{Flex}$.

On the other hand, in a small open economy, in addition to domestic prices and output fluctuations, real exchange rate movements also affect welfare. This is because the exchange rate can generate fluctuations in the so called " U_c/V_y gap". As shown in equation (24) and (23), the real exchange rate influences the relative price of Home produced goods and modifies the small open economy's demand. Secondly, in a world where purchasing power parity does not hold, real exchange rate movements generate real wealth variations, which, in turn, create fluctuations in households' spending and consumption (this can be seen by inspection of equation (25)). It follows that the impact of the real exchange rate on output and consumption affects the wedge between marginal utility of consumption and marginal disutility of production. And fluctuations in this gap have an effect on the small open economy's welfare.

The value of intertemporal and intratemporal elasticities of substitution, $1/\rho$ and θ , determine the real exchange rate effect on consumption and output through the risk sharing and demand channels explained above. Therefore, the weight of the real exchange rate in the loss function, Φ_{RS} , depends crucially on these parameters. More specifically, when $\rho\theta = 1$ the real exchange rate does not affect the " U_c/V_y gap" and $\Phi_{RS} = 0$. Section 4.1 explores this special case in detail. In addition, when the economy is relatively close, the welfare implications of real exchange rate movements are small (as expected, when $\lambda \rightarrow 0$, $\Phi_{RS} \rightarrow 0$).

4 Optimal Monetary Policy

After characterizing the policy objective, we now turn to the constraints of the policy problem. The first constraint the policy maker faces is given by the Phillips Curve:

$$\widehat{\pi}_t^H = k \left(\eta(\widehat{Y}_t - \widehat{Y}_t^T) + (1 - \lambda)^{-1}(\widehat{RS}_t - \widehat{RS}_t^T) + u_t \right) + \beta E_t \widehat{\pi}_{t+1}^H, \quad (34)$$

where u_t is a linear combination of the shocks defined in the appendix. The policy problem is further constrained by the small open economy aggregate demand equation (24) and the risk sharing condition (20). Combining these two conditions, the following relationship between output and the real exchange rate arises:

$$(\widehat{Y}_t - \widehat{Y}_t^T) = (\widehat{RS}_t - \widehat{RS}_t^T) \frac{(1 + l)}{\rho(1 - \lambda)} + \chi u_t, \quad (35)$$

where $l = (\rho\theta - 1)\lambda(2 - \lambda)$ and χ is a vector whose elements depend on the structural parameters (as shown in the appendix). From equation (34) we can see that the policy targets \widehat{Y}_t^T and \widehat{RS}_t^T are not necessarily the flexible price allocations of output and the real exchange rate. That is, the targets do not coincide with the allocations that would prevail if $\alpha = 0$ (and consequently $k \rightarrow \infty$). Moreover, equation (35) shows that closing the “output gap” does not eliminate the “real exchange rate gap”.

We proceed by characterizing the optimal plan under the assumption that policy makers can commit to maximizing the economy’s welfare. We lay out the Ramsey problem and derive the optimal policy response to the different shocks. The policy problem consists of choosing a path for $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t\}$ in order to minimize (29), subject to the constraints (34) and (35), and given the initial conditions $\widehat{\pi}_{t_0}$ and \widehat{Y}_{t_0} . This last condition implies that the resulting set of first order conditions does not internalize the effect of previous expectations of the initial policy. This method follows Woodford’s (1999) timeless perspective approach, and thereby ensures that the policy prescription does not constitute a time inconsistent problem. In effect, the constraints on the initial conditions impose that the first order conditions to the problem are time invariant¹¹. The multipliers associated with (34) and (35) are respectively φ_1 and φ_2 . Thus, the first order conditions with respect to $\widehat{\pi}_t^H$, \widehat{Y}_t and \widehat{RS}_t are given by:

¹¹For a discussion on the timeless perspective of optimal rule see Woodford, 2003.

$$(\varphi_{1,t} - \varphi_{1,t-1}) = k\Phi_\pi \widehat{\pi}_t^H \quad (36)$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) \quad (37)$$

$$-\varphi_{2,t} - \frac{\rho}{(1+l)}\varphi_{1,t} = \frac{\rho(1-\lambda)}{(1+l)}\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T). \quad (38)$$

Combining equations (36), (37), and (38), we obtain the following expression:

$$(1+l)\Phi_Y\Delta(\widehat{Y}_t - \widehat{Y}_t^T) + \rho(1-\lambda)\Phi_{RS}\Delta(\widehat{RS}_t - \widehat{RS}_t^T) + (\rho + \eta(1+l))k\Phi_\pi(\widehat{\pi}_t^H) = 0, \quad (39)$$

where Δ denotes the first difference operator. The above expression characterizes the small open economy optimal targeting rule. It prescribes responding to movements in inflation, output and the real exchange rate¹². Moreover, equation (39) stipulates how monetary policy should respond to the different shocks, according to the composition of \widehat{Y}_t^T and \widehat{RS}_t^T . When following this policy rule, the central bank may allow some variability in inflation in order to respond to costly movements in other variables. Equation (39) indicates the policy maker's behavior that minimizes welfare losses is associated with such fluctuations. It implements the most efficient allocation of resources, conditional on the structural characteristics of the economy.

In the appendix we show which parametric restrictions are needed for the above first order conditions to lead to a determinate equilibrium. The appendix also contains an analysis of whether the above first order conditions indeed characterize an optimal policy. That is, section D of the appendix investigates if there is any alternative random policy that could improve welfare. As shown in Benigno and Woodford (2003), this approach coincides with the investigation of whether the second order conditions of the minimization problem are satisfied. It follows that some parameter specifications violate these conditions (those are shown in table 13 and 14).

¹²Even if we express equation (39) as a function of Consumer Price Index inflation instead of producer price inflation $\widehat{\pi}_t^H$, the targeting rule still includes the term $\Delta(\widehat{RS}_t - \widehat{RS}_t^T)$.

We now turn to the analysis of some special cases of the optimal plan. Moreover, we explore how certain economic characteristics influence the optimal monetary policy.

4.1 Producer Price Inflation Target

Under certain circumstances, the loss function approximation leads to clear cut results in terms of optimal policy. In this section we analyze when optimal policy consists of PPI (producer price index) inflation targeting, i.e., it prescribes complete domestic price stability. The conditions under which this is true involve assumptions on the parameter values of the model and on the source of the shock present in the economy.

In order to understand the inefficiency of the flexible price allocation in the general case and the special cases where a strict inflation target is optimal, it is useful to characterize the social planner's optimal allocation. In this section we step back from the linear quadratic analysis and compare our non-linear equilibrium conditions with the social planner's optimal allocation. The planner's objective is to maximize agents utility U_t subject to the small open economy's demand and the risk sharing condition, given by equations (11) and (20), and is further completed by the determination of relative prices (equation (6)). Moreover, the small open economy's social planner takes external conditions as given. Hence, the social planner's first order condition can be written as

$$P_{H,t}^e U_c(C_t^e) = q_t^e V_y(Y_t^e, \varepsilon_{Y,t}), \quad (40)$$

where $q_t^e = \left[1 + \lambda(\rho\theta - 1) (RS_t^e)^{\theta-1} + \frac{\lambda}{1-\lambda} \rho\theta (RS_t^e)^{\frac{\rho\theta-1}{\rho}}\right]$ and the superscript e denotes the efficient allocation. The full characterization of the efficient allocation can be obtained by combining the above equation with the constraints of the policy problem (i.e. equations (6), (11), and (20)). Furthermore, in steady state we have

$$U_c(\bar{Y}^e) = \frac{1}{(1-\lambda)} V_y(\bar{Y}^e). \quad (41)$$

On the other hand, in the decentralized problem, the equilibrium condition implied by monopolistic

competition and price stickiness is given by the price setting equation (21). If we assume, however, that prices are flexible, the equilibrium condition (21) becomes

$$p_{H,t}^{Flex} U_c(C_t^{Flex}) = \mu_t V_y(Y_t^{Flex}, \varepsilon_{Y,T}), \quad (42)$$

and in steady state

$$U_c(\bar{Y}^{Flex}) = \mu V_y(\bar{Y}^{Flex}). \quad (43)$$

Therefore, comparing conditions (40) and (42), it is clear that even with perfectly flexible prices, mark up shocks and movements in the real exchange rate generate inefficient fluctuations in the ratio of marginal disutility of production and marginal utility of consumption. In addition, unless $\mu = 1/(1-\lambda)$, the small open economy steady state output is inefficient (this can be seen by inspection of equations (41) and (43)). That is, in general, a policy of domestic price stabilization that mimics the flexible price allocation does not implement an efficient allocation.

Nevertheless, if we impose that $\rho\theta = 1$, the efficiency condition (40) and the decentralized flexible price allocation (42) can be written as follows

$$(1 - \lambda)(Y_t^e - G_t)^{-\rho} = \varepsilon_{Y,t}^{-\eta} (Y_t^e)^\eta \quad (44)$$

$$\frac{1}{\mu_t} (Y_t^{Flex} - G_t)^{-\rho} = \varepsilon_{Y,t}^{-\eta} (Y_t^{Flex})^\eta. \quad (45)$$

The above expressions are illustrated in Figure 1, where $f(Y_t, \varepsilon_t) = \varepsilon_{Y,t}^{-\eta} Y_t^\eta$, $g^{flex}(Y_t, \mu_t, G_t) = \frac{1}{\mu_t} (Y_t - G_t)^{-\rho}$, and $g^e(Y_t, G_t) = (1 - \lambda)(Y_t - G_t)^{-\rho}$. The inefficiency of the steady state flexible price allocation is represented by the location of \bar{Y}^{flex} below \bar{Y}^e . Moreover, apart from the steady state distortion, fluctuations in the wedge between g^e and g^{flex} characterize a departure from the efficient allocation given by (44) and also represent distortions present in the flexible price equilibrium.

Figure 2 illustrates how mark up shocks impact the wedge between g^e and g^{flex} . It shows that

even with $\rho\theta = 1$ and flexible prices, mark up shocks generate distortions that affect welfare. Hence, there is an incentive to stabilize these shocks and depart from the flexible price equilibrium (i.e. a strict domestic inflation target is not optimal).

Figure 3 shows how productivity shocks affect efficiency. In the case of $\rho\theta = 1$, the equilibrium flexible price allocation and the efficient allocation move proportionally to each other. This leaves the welfare relevant wedge unchanged. Therefore, there are no welfare losses under price flexibility and a producer price inflation targeting characterizes the optimal plan. The same result holds for the case of foreign shocks. External disturbances do not appear in the expressions for $f(\cdot)$, $g^{flex}(\cdot)$ or $g^e(\cdot)$. Thus, these shocks also leave the wedge unchanged when $\rho\theta = 1$. The intuition behind this result is that, under this parametrization, the marginal effect of the real exchange rate on consumption utility and labour disutility offset each other and no stabilization process is needed.

Figure 4 shows the case of exogenous fluctuations in government expenditure. Because fiscal shocks do not affect g^{flex} and g^e proportionally, their effect on efficiency depends on the steady state level of output. In general, fiscal disturbances create inefficient movements in the wedge between g^{flex} and g^e , as represented in the figure. The only circumstance in which there are no such movements is when the steady state level of output is efficient ($\bar{Y}^{flex} = \bar{Y}^*$). This result is consistent with the findings of Benigno and Woodford (2004) in a closed economy setting.

Therefore, the assumptions needed in order to have an inflation target as the optimal plan are: (1) $\rho\theta = 1$; (2) there should be no mark-up shocks ($\mu_t = 0, \forall t$); and, in the case of fiscal shocks, (3) the steady state level of output ought to be efficient from the small open economy's point of view (i.e. $\mu = 1/(1-\lambda)$). These conditions guarantee that the flexible price equilibrium characterizes the efficient allocation.

Under this specification, the weights on the loss function are:

$$\frac{\Phi_Y}{(1-\lambda)} = (\eta + \rho) \quad (46)$$

$$\Phi_{RS} = 0 \quad (47)$$

$$\frac{\Phi_\pi}{(1-\lambda)} = \frac{\sigma}{k}. \quad (48)$$

And the target for output is:

$$\widehat{Y}_t^T = \widehat{Y}_t^{Flex} = (\eta + \rho)^{-1} \{\eta \varepsilon_{Y,t} + g_t\}. \quad (49)$$

The relative weights specified in equations (46) and (48) are analogous to those in the closed economy and the policy target coincides with the flexible price allocation. The assumption of $\mu = 1/(1-\lambda)$ guarantees that steady state output is efficient from the point of view of the small open economy. In addition, the restriction of $\rho\theta = 1$, ensures that exchange rate movements do not affect welfare since its the marginal effect on consumption utility and labour disutility offset each other. Moreover, the optimal plan does not respond to external shocks. In what follows, under this specification, the optimal monetary policy in a small open economy is isomorphic to a closed economy. This result is consistent with the findings of Gali and Monacelli (2005)¹³.

4.2 Quantitative results

4.2.1 The General Optimal Plan:

In this section we present some numerical analysis of the optimal monetary policy. In our benchmark specification we assume a unitary elasticity of intertemporal substitution, i.e. $\rho = 1$. Following Rotemberg and Woodford (1997) we assume $\eta = 0.47$. Furthermore, the elasticity of substitution between home and foreign goods, θ , is assumed to be 3¹⁴ (Obstfeld and Rogoff (1998) argue that it should be between 3 and 6). The degree of openness, λ , is assumed to be 0.2, implying a 20% import share of the GDP. In addition, the baseline calibration considers the case of a "optimal subsidy" policy, where τ is set such that $\mu = 1/(1-\lambda)$. Moreover, the elasticity of substitution between differentiated goods σ is assumed to be 10 as in Benigno and Benigno (2003). To characterize an average length of

¹³The authors have characterized the loss function for a small open economy in the case in which trade imbalances or steady state monopolistic distortions are absent (i.e. $\rho = \theta = 1$ and $\mu = 1/(1-\lambda)$).

¹⁴This leads to a specification where Home and Foreign goods are substitutes in the utility, given that $\rho\theta > 1$.

price contract of 3 quarters, we assume $\alpha = 0.66$. Finally, we assume $\beta = 0.99$. Starting from this specification, we analyze how optimal monetary policy responds to the different shocks.

Figure 7 shows the impulse response of consumption, output, the real exchange rate and producer price inflation following a productivity shock. Comparing the optimal policy with an inflation target highlights that there are no quantitatively significant differences between the two. Under both regimes a higher productivity at home increases domestic output and consumption. In addition, a larger supply of domestic goods leads to a depreciation in the real exchange rate.

The zero measure specification of the Home economy enables us to study how the monetary authority should respond to fluctuations on external conditions, when there are no feedback effects. Figure 6 presents the impulse response of the various domestic variables to a foreign shock - represented by an innovation in C_t^* . Again, the optimal plan is quantitatively similar to an inflation targeting regime. Domestic consumption increases with the increase in foreign consumption and there is a real exchange rate appreciation. The impact on domestic competitiveness now leads to a fall in home production.

As illustrated in Figure 7, when the economy is subject to mark up shocks, optimal monetary policy departs from price stabilization. The optimal plan reacts to fluctuations in the wedge between marginal utility of consumption and marginal disutility of production. The policy response to a mark up shock implies an exchange rate depreciation and an increase in the domestic consumption of home goods. As a result, domestic output increases. As shown in figure 8, this is not the case when the economy is closed. In this case, inflation stabilization is larger, requiring a contraction in the level of economic activity.

The optimal response to a fiscal shock is presented in figures 9, 10 and 11¹⁵. Figure 9 compares the optimal monetary policy with an inflation targeting regime. It shows that the exchange rate depreciation is smaller in the former. Consequently, crowding out in consumption is smaller under the optimal regime. As a result, whereas output falls under a policy of price stability, domestic production increases under the optimal plan. Conversely, as portrayed in figure 10, the optimal plan in a closed economy is closer to an inflation target and involves a larger fall in consumption.

¹⁵Given that \hat{g}_t is defined as $\frac{(G-\bar{G})}{\bar{Y}}$ an innovation in \hat{g}_t is measured in percentages of the GDP.

These results change significantly when the goods are complementary in utility. As displayed in figure 11, when $\theta = 0.7$, a fiscal shock leads to an exchange rate appreciation and a fall in domestic consumption.

4.2.2 Ranking Standard Policy Rules

Exercises such as the ones shown above demonstrate that the source of the shock affecting the economy is an important determinant of the performance of policy rules. In the optimal targeting rule, this is captured by the composition of the target variables \widehat{Y}_t^T and \widehat{RS}_t^T , which stipulate how optimal policy should respond to different shocks. The quantitative analysis also illustrates the role of the economy's characteristics (that is, variations in the structural parameters such as λ and θ) in the policy prescription. In analytical terms this is captured by the formulation of the weights of the variables in the loss function and in the targeting rule.

In order to verify how robust the results presented in the previous section are (specially regarding the validity of producer price inflation targeting), some sensitivity analysis should be conducted. However, as shown in section (4) of the appendix, not every combination of parameter specification is consistent with the second order condition of the minimization problem. For this reason, the computation of the optimal plan as shown above is not valid for those specifications. Therefore, an alternative robustness check has to be done by evaluating the performance of an inflation targeting regime compared with other standard policy rules for the different parameter values and types of disturbances. This exercise is also interesting *per se*, as it allows the evaluation of policies currently used by international monetary authorities.

We compute a ranking of policy rules (more specifically, domestic inflation targeting, CPI inflation targeting and exchange rate peg) for different values of ρ, θ and λ . We start by varying θ and ρ , while maintaining $\lambda = 0.4$. Alternatively, we can keep the log utility specification and analyze different scenarios for θ and λ . Moreover, we consider the case of 1% standard deviation productivity, fiscal and mark up shocks¹⁶.

¹⁶Kehoe and Perri (2000) find and estimate of 0.7% for the productivity shock standard deviation. Gali et al (2002) finds a standard deviation for price mark ups of 4.3% (implying variance of approximately 0.0016). Perotti (2005) estimates the standard deviation of a government spending shock for various countries. The estimates range from 0.8% to 3.5%.

Tables 1 and 2 show the policy rule that leads to the highest level of welfare, following a productivity shock. Domestic inflation targeting is the preferred policy rule for low levels of θ , ρ and λ . A large elasticity of substitution between domestic and foreign goods increases the sensitivity of home demand to exchange rate movements. As a result, exchange rate fluctuations have a higher impact on the rate of marginal utility of consumption and marginal disutility of production. For this reason, when θ is high, the small open economy benefits from adopting an exchange rate peg. The same happens when the coefficient of risk aversion is large. Moreover, an exchange rate peg becomes superior to PPI or CPI inflation targeting when the economy is relatively open.

The gains or losses of adopting different policy regimes are represented by the following measure:

$$W_d^{a,b} = \frac{W^a - W^b}{U_c(\bar{C})} = \frac{2(1 - \beta)(U_0^a - U_0^b)}{U(C)},$$

where U_0 is the expected life time utility of the representative agent. $W_d^{a,b}$ measures the percentage difference in steady state consumption under regime a and b . Table 3 illustrates the welfare gains or losses of adopting an inflation targeting rather than an exchange rate peg when the economy is subject to productivity shocks. Although an exchange rate peg is superior to an inflation targeting regime when θ , ρ and λ are large, the quantitative welfare loss is not very significant: it ranges from 0.001% to 0.004% of steady state consumption. As shown in section 5.1 when $\rho = \theta = 1$ and the economy is subject to productivity shocks, a domestic inflation target coincides with the optimal policy rule. In this case, the welfare losses of a fixed exchange rate regime is 0.010% of steady state consumption. Table 9 shows that these costs increase to 0.013% when the economy is relatively closed ($\lambda = 1/5$).

In the case of foreign shocks, figures for the preferred policy are identical to the case of domestic productivity shocks. Pegging the exchange rate outperform an inflation targeting regime when the economy is relatively open, and demand is sensitive to exchange rate movements (i.e., θ is large) and the intertemporal elasticity of substitution is small (high levels of ρ). This is illustrated in tables 5 and 6.

Turning to fiscal shocks, for intermediate levels of ρ, θ and λ , CPI targeting is the best of the

However, in the present paper we consider equally variable shocks, with $\sigma^2 = 0.0001$.

three standard policy forms evaluated. That is, under these specifications, the central bank improves welfare by targeting a weighted average of domestic inflation and exchange rate depreciation. This is illustrated in tables 7 and 8. However, the cost of imposing an inflation targeting regime under this parametrization is insignificant: at most 0.001% loss in steady state consumption (see highlighted statistics in table 9 and 10). Moreover, as in the case of foreign and productivity shocks, when λ , θ and ρ are large, fixing the exchange rate is the best alternative

In the case of mark up shocks, an inflation target is the preferred standard policy only under the knife-edge specification where $\rho = \theta = 1$ (see table 11). With unitary elasticity of substitution and $\rho > 1$, CPI targeting is the preferred policy rule. In addition, whenever $\theta > 2$ pegging the exchange rate leads to higher welfare than PPI inflation targeting. The steady state consumption losses associated with strict domestic price stabilization compared with a fixed exchange rate regime are shown in table 12. When mark up fluctuations are the source of disturbance affecting the small open economy, $W_d^{IT,PEG}$ may reach 0.043%, when $\rho = \theta = 6$.

The costs of adopting a welfare inferior policy rule presented in the above tables are small in magnitude. The shift in steady state consumption is never larger than 0.05%. We should note however that these costs are of the same order of magnitude as the costs of business cycles reported by Lucas (1987) (the author estimates a 0.1% shift in steady state consumption).

5 Conclusion

This paper formalizes a small open economy model as a limiting case of a two country general equilibrium framework and characterizes its utility-based loss function. It also derives the optimal monetary plan, represented by a targeting rule, for a small open economy. The setup developed in this work encompasses, as special cases, the closed economy framework and the small open economy case with efficient levels of steady state output. As a result, the examination of monetary policy in such environments is nested in our analysis.

The utility-based loss function for a small open economy is a quadratic expression in domestic inflation, the output gap and the real exchange rate. This paper demonstrates that a small open

economy, completely integrated with the rest of the world, should be concerned about exchange rate variability. Therefore, the optimal policy in a small open economy is not isomorphic to that in a closed economy and also, it does not prescribe a pure floating exchange rate regime. Price stability (or domestic inflation targeting) is optimal only under a specific parameterization of the model. In the cases where the economy experiences productivity and foreign shocks exclusively, domestic inflation targeting is only optimal under a particular specification for preferences. If fiscal disturbances are also present, price stability as the optimal plan further requires a production subsidy. When these restrictions on the steady state level output and preferences are relaxed, deviations from inward looking policies arise in the optimal plan.

Nevertheless, under our benchmark calibration, the optimal monetary policy mimics closely an inflation targeting regime when the economy experiences domestic productivity shocks and external disturbances. In the case of fiscal and mark up shocks the optimal plan departs from price stability. Moreover, the openness of the economy modifies the optimal responses to the referred shocks significantly.

The sensitivity analysis exercise demonstrates that inflation targeting, when comparing CPI and exchange rate targeting, is the preferred policy if the economy is relatively closed and its demand is not sensitive to exchange rate movements. Conversely, if λ , θ and ρ are large, the small open economy may improve welfare by adopting a fixed exchange rate regime.

The tools developed in this paper can be applied to different economic environments. It is important to notice that the model presented here assumes that there are complete asset markets. Relaxing such assumption would lead to a more realistic representation of the model. Moreover, the introduction of asset market imperfections and their welfare consequences would enrich the optimal monetary policy analysis.

Another interesting extension would involve analyzing fiscal policy by allowing proportional taxation to be an endogenous variable. This would enable the investigation of the interaction between fiscal and monetary authorities and the optimal policy mix. The small open economy representation allows for the assessment of interesting issues such as the implication of different government bond denominations

for fiscal policy. Moreover, one could analyze optimal fiscal arrangements under a fixed exchange rate regime.

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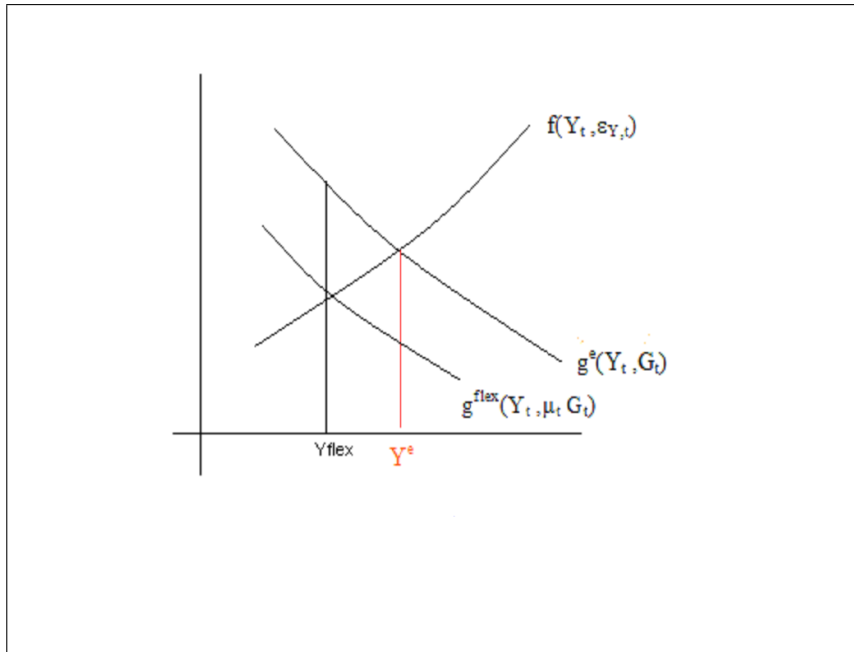


Figure 1: Efficiency Analysis

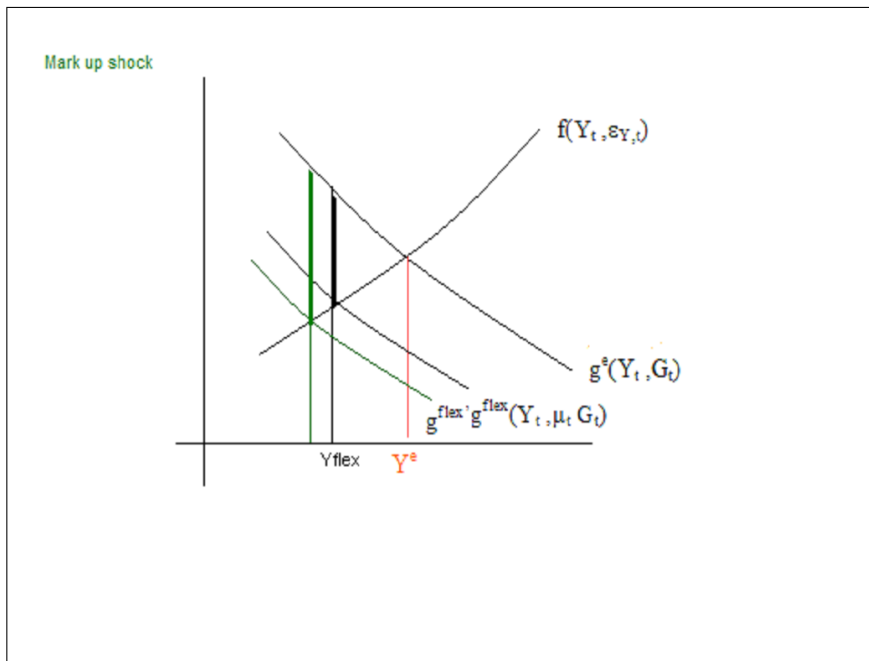


Figure 2: Efficiency Analysis - the case of Mark up shocks

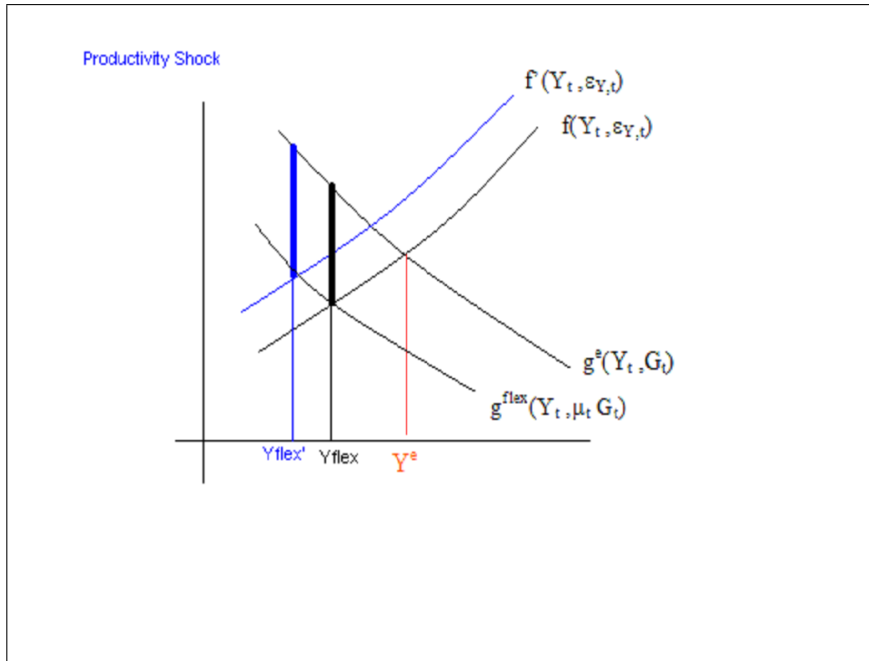


Figure 3: Efficiency Analysis - the case of Productivity shocks

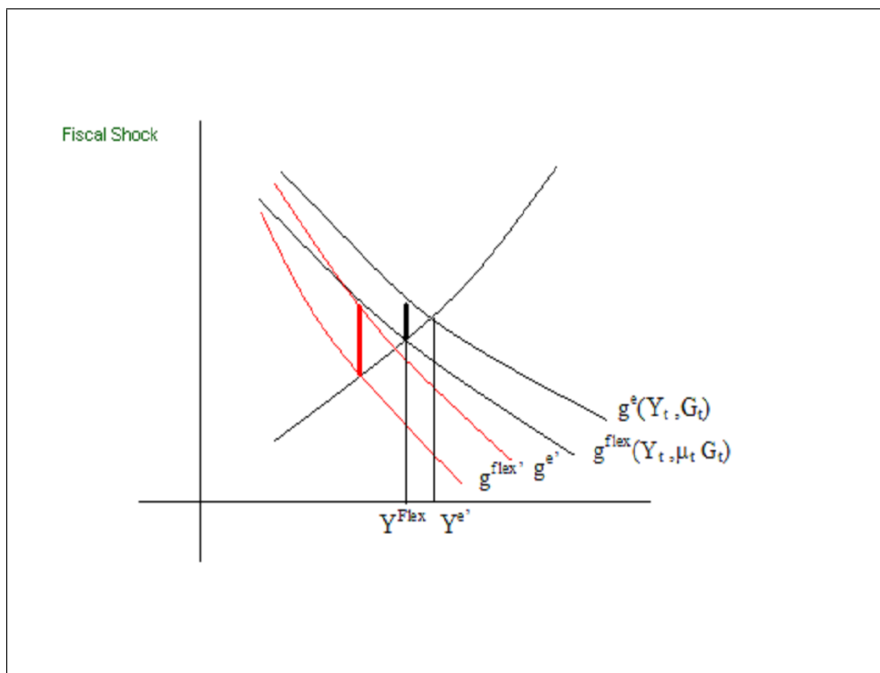


Figure 4: Efficiency Analysis - the case of Fiscal shocks

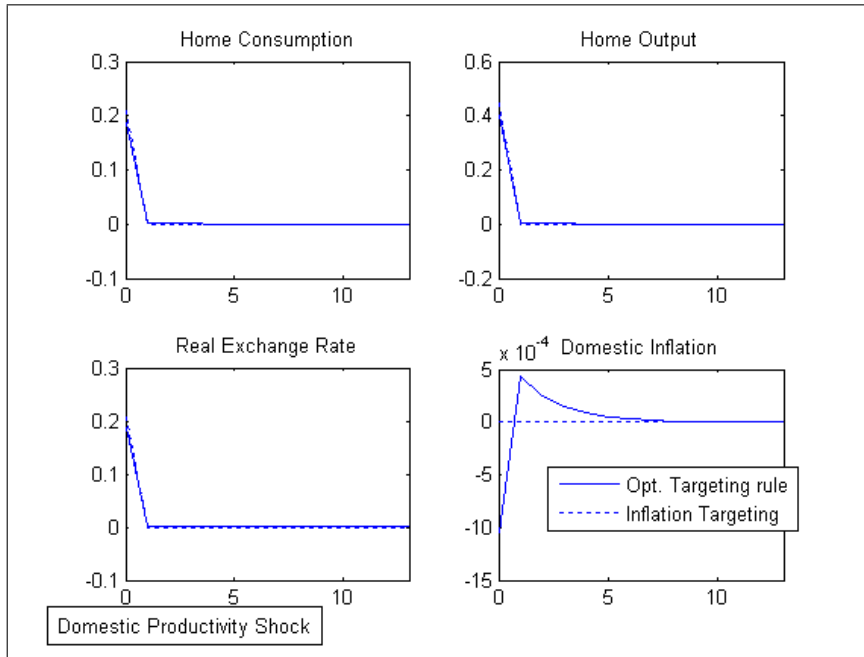


Figure 5: Impulse responses following a Productivity Shock

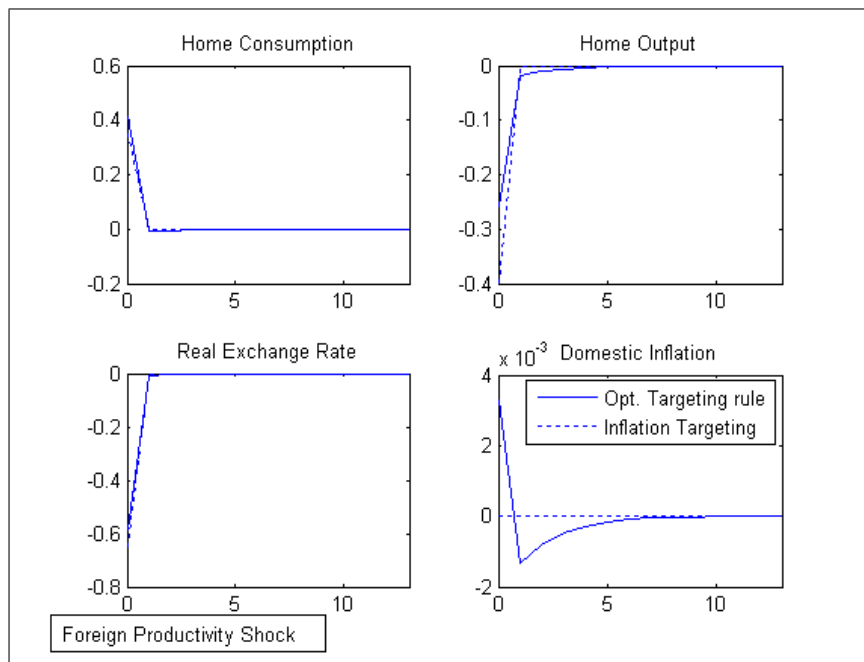


Figure 6: Impulse responses following a Foreign Shock

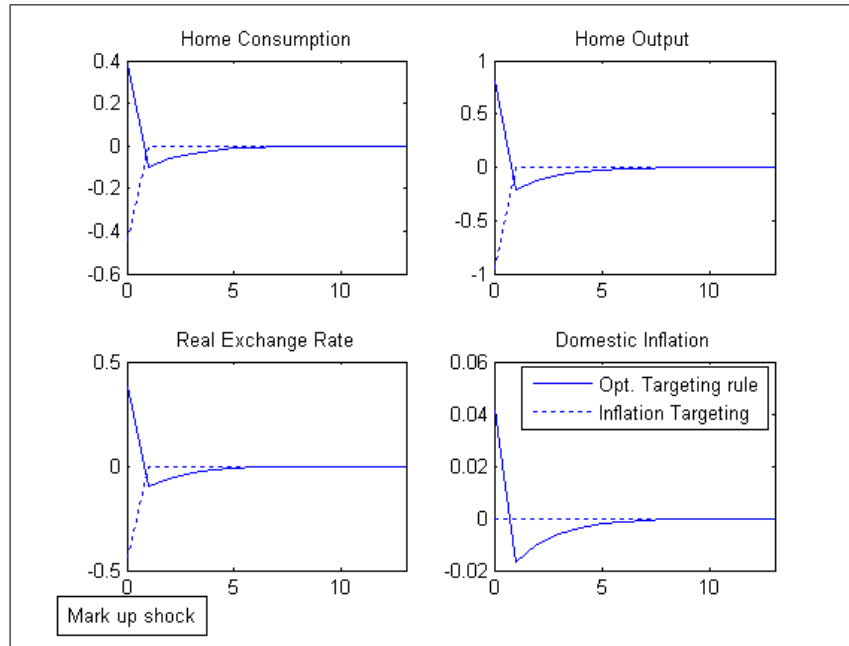


Figure 7: Impulse responses following a Mark up Shock

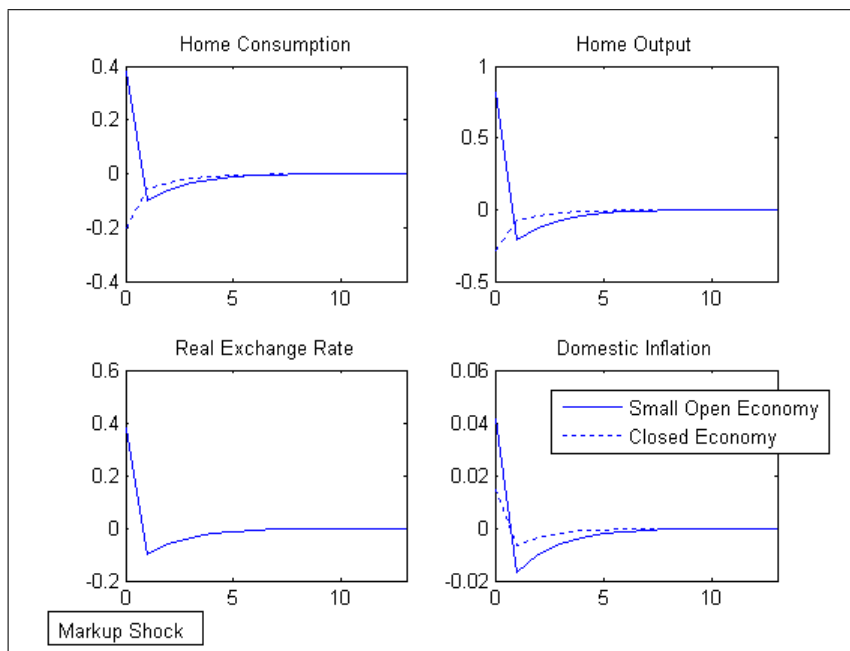


Figure 8: Impulse responses following a Markup Shock - Open vs Closed Economy

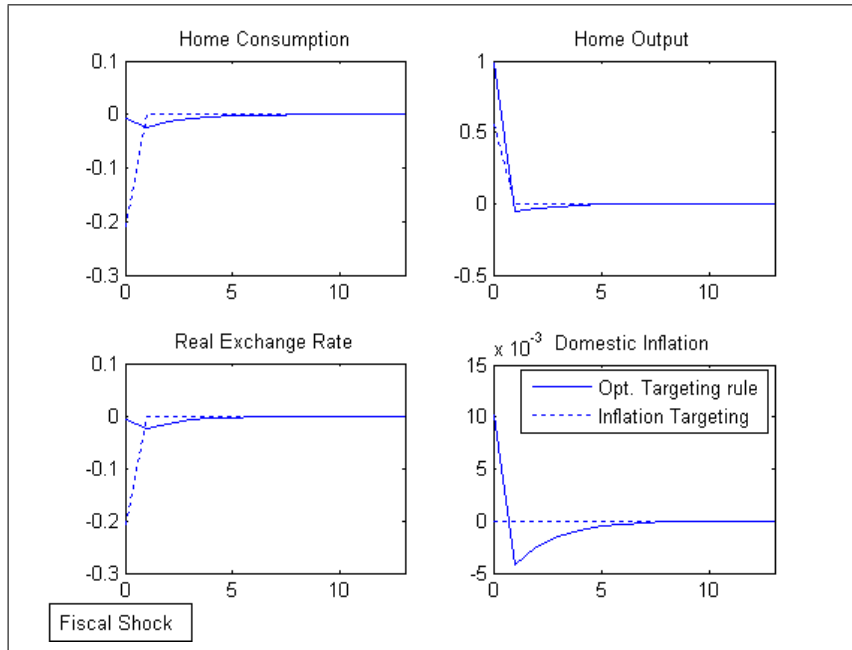


Figure 9: Impulse responses following a Fiscal Shock

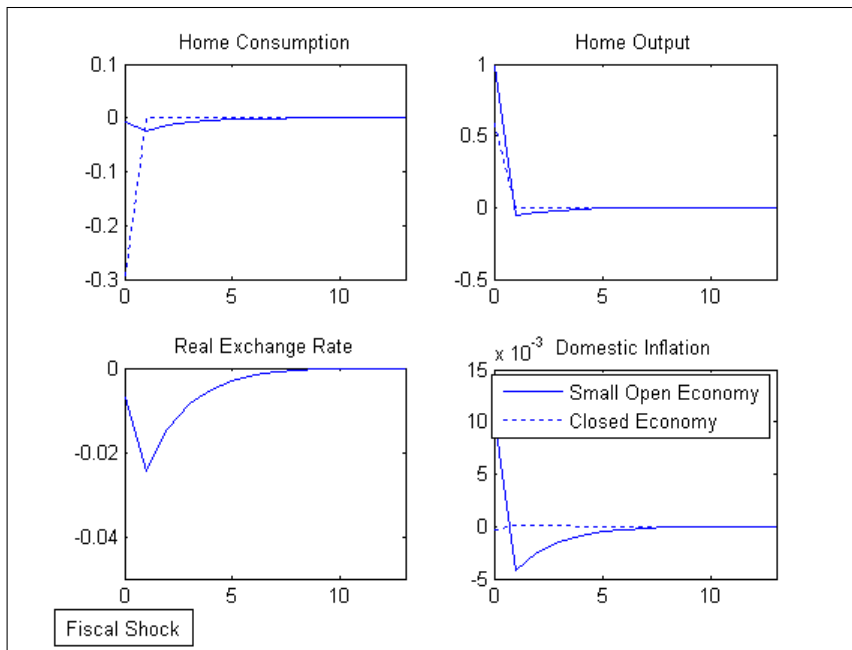


Figure 10: Impulse responses following a Fiscal Shock - Open vs Closed Economy

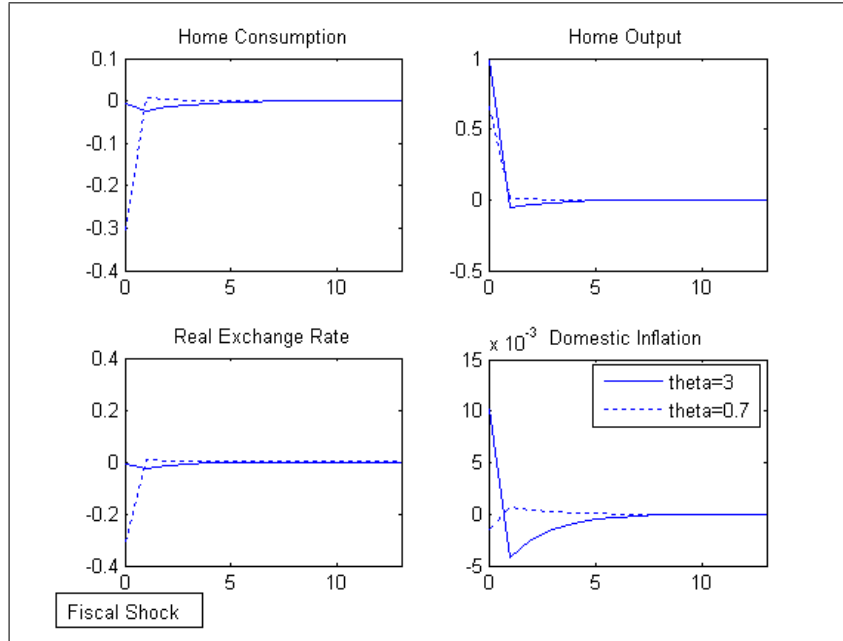


Figure 11: Impulse responses following a Fiscal Shock - varying the elasticity of substitution between domestic and foreign goods

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	IT	PEG	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 1: Preferred policy rule following an productivity shock - varying the degree of openness and the intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Table 2: Preferred policy rule following a productivity shock- varying the intertemporal and intratemporal elasticity of substitution

Table 3: Welfare costs following a productivity shock- varying the intertemporal and intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	0.010%	0.004%	0.001%	-0.001%	-0.002%	-0.002%
2	0.007%	0.001%	-0.001%	-0.003%	-0.003%	-0.003%
3	0.006%	0.000%	-0.002%	-0.003%	-0.004%	-0.004%
4	0.005%	0.000%	-0.002%	-0.003%	-0.004%	-0.004%
5	0.005%	0.000%	-0.003%	-0.003%	-0.004%	-0.004%
6	0.004%	-0.001%	-0.003%	-0.004%	-0.004%	-0.004%

$$W_d^{IT, PEG}$$

Table 4: Welfare costs following a productivity shock- varying the degree of openess and intratemporal elasticity of substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	0.008%	0.004%	0.002%	0.001%	0.001%	0.001%
1/3	0.011%	0.004%	0.002%	0.001%	0.001%	0.001%
1/4	0.012%	0.003%	0.002%	0.001%	0.001%	0.001%
1/5	0.013%	0.003%	0.002%	0.001%	0.001%	0.001%

$$W_d^{IT, PEG}$$

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	IT	PEG	PEG	PEG	PEG
1/3	IT	IT	IT	PEG	PEG	PEG
1/4	IT	IT	IT	CPI	PEG	PEG
1/5	IT	IT	IT	IT	PEG	PEG

Table 5: Preferred policy rule following an external shock - varying the degree of openess and the intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	IT	IT	PEG	PEG	PEG
2	IT	IT	PEG	PEG	PEG	PEG
3	IT	IT	PEG	PEG	PEG	PEG
4	IT	PEG	PEG	PEG	PEG	PEG
5	IT	PEG	PEG	PEG	PEG	PEG
6	IT	PEG	PEG	PEG	PEG	PEG

Table 6: Preferred policy rule following a foreign shock- varying the intertemporal and intratemporal elasticity of substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	IT	CPI	PEG	PEG	PEG	PEG
1/3	IT	CPI	PEG	PEG	PEG	PEG
1/4	IT	IT	CPI	PEG	PEG	PEG
1/5	IT	IT	CPI	PEG	PEG	PEG

Table 7: Preferred policy rule following a fiscal shock.- varying the degree of openness and the intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	CPI	PEG	PEG	PEG	PEG
2	IT	PEG	PEG	PEG	PEG	PEG
3	CPI	PEG	PEG	PEG	PEG	PEG
4	CPI	PEG	PEG	PEG	PEG	PEG
5	CPI	PEG	PEG	PEG	PEG	PEG
6	CPI	PEG	PEG	PEG	PEG	PEG

Table 8: Preferred policy rule following a fiscal shock- varying the intertemporal and intratemporal elasticity of substitution

$\lambda \setminus \theta$	1	2	3	4	5	6
1/2	0.003%	0.000%	-0.002%	-0.002%	-0.003%	-0.003%
1/3	0.003%	0.000%	-0.001%	-0.002%	-0.002%	-0.002%
1/4	0.002%	0.000%	-0.001%	-0.002%	-0.002%	-0.002%
1/5	0.002%	0.000%	-0.001%	-0.001%	-0.002%	-0.002%

$$W_d^{IT,CPI}$$

Table 9: Welfare costs following a fiscal shock - varying the degree of openness and intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	0.003%	0.000%	-0.001%	-0.002%	-0.002%	-0.002%
2	0.001%	-0.002%	-0.003%	-0.003%	-0.003%	-0.003%
3	0.000%	-0.002%	-0.003%	-0.003%	-0.003%	-0.003%
4	0.000%	-0.003%	-0.003%	-0.004%	-0.004%	-0.003%
5	-0.001%	-0.003%	-0.003%	-0.004%	-0.004%	-0.003%
6	-0.001%	-0.003%	-0.004%	-0.004%	-0.004%	-0.003%

$$W_d^{IT,CPI}$$

Table 10: Welfare costs following a fiscal shock - varying the intertemporal and intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	IT	CPI	PEG	PEG	PEG	PEG
2	CPI	PEG	PEG	PEG	PEG	PEG
3	CPI	PEG	PEG	PEG	PEG	PEG
4	CPI	PEG	PEG	PEG	PEG	PEG
5	CPI	PEG	PEG	PEG	PEG	PEG
6	CPI	PEG	PEG	PEG	PEG	PEG

Table 11: Preferred policy rule following an mark up shock- varying the intertemporal and intratemporal elasticity of substitution

$\rho \setminus \theta$	1	2	3	4	5	6
1	0.029%	-0.004%	-0.020%	-0.029%	-0.033%	-0.035%
2	0.017%	-0.014%	-0.028%	-0.035%	-0.038%	-0.040%
3	0.012%	-0.017%	-0.031%	-0.037%	-0.040%	-0.041%
4	0.009%	-0.019%	-0.032%	-0.038%	-0.041%	-0.042%
5	0.008%	-0.021%	-0.033%	-0.039%	-0.041%	-0.042%
6	0.007%	-0.021%	-0.034%	-0.039%	-0.042%	-0.043%

$$W_d^{IT,PEG}$$

Table 12: Welfare costs following a mark up shock - varying the degree of openness and intratemporal elasticity of substitution

A Appendix: The Steady State

In this appendix we derive the steady state conditions. All variables in steady state are denoted with a bar. We assume that in steady state $1 + i_t = 1 + i_t^* = 1/\beta$ and $P_t^H/P_{t-1}^H = P_t^F/P_{t-1}^F = 1$. Moreover we normalize the price indexed such that $\bar{P}_H = \bar{P}_F$. This implies that $\frac{\bar{P}_H}{\bar{P}} = \bar{R}\bar{S} = 1$. From the demand equation at Home, we have:

$$\bar{Y} = v\bar{C} + \frac{v^*(1-n)}{n}\bar{C}^* + \bar{G} \quad (\text{A.1})$$

$$\bar{Y}^* = \frac{(1-v)n}{1-n}\bar{C} + (1-v^*)\bar{C}^* + \bar{G}^* \quad (\text{A.2})$$

If we specify the proportion of foreign-produced goods in home consumption as $1 - v = (1 - n)\lambda$ and the proportion of home-produced goods in foreign consumption is $v^* = n\lambda$, and take the limiting case where $n = 0$, we have.

$$\bar{Y} = (1 - \lambda)\bar{C} + \lambda\bar{C}^* + \bar{G} \quad (\text{A.3})$$

And from the Foreign demand we have

$$\bar{Y}^* = \bar{C}^* + \bar{G}^* \quad (\text{A.4})$$

Moreover, applying our normalization to the price setting equations we have:

$$U_C(\bar{C}) = \mu V_y (\lambda\bar{C}^* + (1 - \lambda)\bar{C} + \bar{G}) \quad (\text{A.5})$$

$$U_C(\bar{C}^*) = \mu^* V_y (\bar{C}^* + \bar{G}^*) \quad (\text{A.6})$$

Where

$$\mu = \frac{\sigma}{(1 - \bar{\tau})(\sigma - 1)}$$

We are also imply the following notation throughout the appendix

$$(1 - \phi) = \frac{1}{\mu}$$

$$\phi = 1 - \frac{(1 - \bar{\tau})(\sigma - 1)}{\sigma}$$

$$0 \leq \phi < 1; \mu > 1$$

The Symmetric Steady State:

Iterating the complete asset market assumption we have:

$$RS_t = \kappa_0 \left(\frac{C_t}{C_t^*} \right)^\rho \quad (\text{A.7})$$

where

$$\kappa_0 = RS_0 \left(\frac{C_0}{C_0^*} \right)^\rho \quad (\text{A.8})$$

So if we assume an initial level of wealth such that $\kappa_0 = 1$, the steady state version of (A.7) imply $\bar{C} = \bar{C}^*$. Moreover, throughout the appendix we assume $\bar{G}^* = \bar{G} = 0$. Under this condition, equations (A.5) and (A.6) imply: $\mu = \mu^*$.

B Appendix: A Second Order Approximation to the Utility Function

In this appendix, we derive the first and second order approximation to the equilibrium conditions of the model under the assumptions that $\bar{C} = \bar{C}^*$ and $\bar{G}^* = \bar{G} = 0$. Moreover, we obtain the second order approximation to the utility function in order to address welfare analysis. To simplify and clarify the algebra, we use the following isoelastic functional forms:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \quad (\text{B.9})$$

$$V(y_t(h), \varepsilon_{Y,T}) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1} \quad (\text{B.10})$$

B.1 Demand

As shown in the text, home demand equation is:

$$Y_t = \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{RS_t} \right)^{-\theta} C_t^* \right] + g_t \quad (\text{B.11})$$

The first order approximation to demand in the small open economy is therefore:

$$\hat{Y}_t = -\theta \hat{p}_{H,t} + (1-\lambda)\hat{C}_t + \lambda \hat{C}_t^* + \theta \lambda \widehat{RS}_t + \hat{g}_t \quad (\text{B.12})$$

Note that fiscal shock \hat{g}_t is defined as $\frac{G_t - \bar{G}}{\bar{Y}}$, allowing for the analysis of this shock even when the zero steady state government consumption is zero. And the second order approximation to the demand function is:

$$\sum \beta^t \left[d'_y y_t + \frac{1}{2} y'_t D_y y_t + y'_t D_e e_t \right] + s.o.t.i.p = 0 \quad (\text{B.13})$$

where

$$y_t = \begin{bmatrix} \hat{Y}_t & \hat{C}_t & \hat{p}_{H,t} & \widehat{RS}_t \end{bmatrix}$$

$$e_t = \begin{bmatrix} \hat{\varepsilon}_{y,t} & \hat{\mu}_t & \hat{g}_t & \hat{C}_t^* \end{bmatrix}$$

$$d'_y = \begin{bmatrix} -1 & 1-\lambda & -\theta & \theta\lambda \end{bmatrix}$$

$$d'_e = \begin{bmatrix} 0 & 0 & 1 & \lambda \end{bmatrix}$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda(1-\lambda) & 0 & -\theta\lambda(1-\lambda) \\ 0 & 0 & 0 & 0 \\ 0 & -\theta\lambda(1-\lambda) & 0 & \theta^2\lambda(1-\lambda) \end{bmatrix}$$

$$D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\lambda) & -\lambda(1-\lambda) \\ 0 & 0 & \theta & 0 \\ 0 & 0 & -\theta\lambda & \theta\lambda(1-\lambda) \end{bmatrix}$$

B.2 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}} \quad (\text{B.14})$$

Assuming the symmetric steady state equilibrium, the log linear approximation to the above condition is:

$$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho} \widehat{RS}_t \quad (\text{B.15})$$

Given our utility function specification, equation (B.14) gives rise to a exact log linear expression and therefore the first and second order approximation are identical.

In matrix notation, we have:

$$\sum E_t \beta^t \left[c'_y y_t + \frac{1}{2} y'_t C_y y_t + y'_t C_e e_t \right] + s.o.t.i.p. = 0 \quad (\text{B.16})$$

$$c'_y = \begin{bmatrix} 0 & -1 & 0 & \frac{1}{\rho} \end{bmatrix}$$

$$c'_e = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C'_y = 0$$

$$C'_e = 0$$

B.3 The Real Exchange Rate

Given that in the rest of the world $P_F = SP^*$, equation (23) can be expressed as:

$$\left(\frac{P_t}{P_{H,t}} \right)^{1-\theta} = (1-\lambda) + \lambda \left(RS_t \frac{P_t}{P_{H,t}} \right)^{1-\theta} \quad (\text{B.17})$$

Therefore, the first order approximation to the above expression is:

$$\tilde{p}_{H,t} = -\frac{\lambda \widehat{RS}_t}{1-\lambda} \quad (\text{B.18})$$

Moreover, the second order approximation to equation (B.17) is:

$$\sum E_t \beta^t \left[f'_y y_t + \frac{1}{2} y'_t F_y y_t + y'_t F_e e_t \right] + s.o.t.i.p. = 0 \quad (\text{B.19})$$

$$f'_y = \begin{bmatrix} 0 & 0 & -(1-\lambda) & -\lambda \end{bmatrix}$$

$$f'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F'_y = \lambda(\theta - 1) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1-\lambda/(1-\lambda)) \end{bmatrix}$$

$$F'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B.4 Price Setting

The first and second-order approximation to the price setting equation follow Benigno and Benigno (2001) and Benigno and Benigno (2003)¹⁷. These conditions are derived from the following first order condition of sellers that can reset their prices:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_T \left[\frac{\tilde{p}_t(h)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \mu_T \frac{V_y(\tilde{y}_{t,T}(h), \varepsilon_{Y,t})}{U_c(C_T)} \right] \right\} = 0 \quad (\text{B.20})$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_t \quad (\text{B.21})$$

and

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) (\tilde{p}_t(h))^{1-\sigma} \quad (\text{B.22})$$

With mark up shocks μ_t defined as $\frac{\sigma}{(\sigma-1)(1-\tau_t^H)}$, the the first order approximation to the price setting equation can be written in the following way:

$$\widehat{\pi}_t^H = k \left(\rho \widehat{C}_t + \eta \widehat{Y}_t - \widehat{p}_{H,t} + \widehat{\mu}_t - \eta \widehat{\varepsilon}_{Y,t} \right) + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{B.23})$$

where $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

The second order approximation to equation (B.20) can be written as follows:

$$Q_{to} = \phi \sum E_t \beta^t \left[a'_y y_t + \frac{1}{2} y'_t A_y y_t + y'_t A_e e_t + \frac{1}{2} a_\pi \pi_t^2 \right] + s.o.t.i.p. \quad (\text{B.24})$$

$$a'_y = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix}$$

$$a'_e = \begin{bmatrix} -\eta & 1 & 0 & 0 \end{bmatrix}$$

$$A'_y = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & 0 \\ \rho & -\rho^2 & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$a_\pi = (\eta+1) \frac{\sigma}{k}$$

B.5 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function can be written as:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j) - V(y_s^j, \varepsilon_{Y,s}) \right] \quad (\text{B.25})$$

¹⁷For a detail derivation of the first-order approximation to the price setting see technical appendix in Benigno and Benigno (2001). Benigno and Benigno (2003) have the details on the second-order approximation.

$$W_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w_\pi \pi_t^2 \right] + s.o.t.i.p \quad (\text{B.26})$$

$$w'_\pi = \frac{\sigma}{\mu k}$$

$$w'_y = [-1/\mu \quad 1 \quad 0 \quad 0]$$

$$W'_y = \begin{bmatrix} \frac{(1+\eta)}{\mu} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W'_e = \begin{bmatrix} -\frac{\eta}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In addition, using the second order approximation to the equilibrium condition derived in section 8.1 to 8.4, we can eliminate the term $w'_y y_t$ from equation (B.26). In order to do so, we derive the vector Lx , such that:

$$[a_y \quad d_y \quad f_y \quad c_y] Lx = w_y$$

Where a_y, d_y, f_y, c_y were previously defined in this appendix. Therefore, we have:

$$Lx_1 = \frac{1}{(\rho + \eta) + l\eta} [l\mu^{-1} + (1 - \lambda) - \mu^{-1}] \quad (\text{B.27})$$

$$Lx_2 = \frac{1}{(\rho + \eta) + l\eta} [\rho(\mu^{-1} - (1 - \lambda)) + (1 - \lambda)(\eta + \rho)] \quad (\text{B.28})$$

$$Lx_3 = \frac{1}{(\rho + \eta) + l\eta} [(\rho\theta - 1)(1 - \lambda)\mu^{-1} - (\eta\theta + 1)] \quad (\text{B.29})$$

where $l = (\rho\theta - 1)\lambda(2 - \lambda)$

And the loss function L_{t_0} will have the following form:

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y'_t L_y y_t + y'_t L_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + s.o.t.i.p \quad (\text{B.30})$$

where:

$$L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y$$

$$L_e = W_e + Lx_1 A_e + Lx_2 D_e$$

$$L_\pi = w_\pi + Lx_1 a_\pi$$

To write the model just in terms of the output, real exchange rate and inflation, we define the matrixes N and N_e mapping all endogenous variables into $[Y_t, T_t]$ and the errors in the following way:

$$y'_t = N [Y_t, T_t] + N_e e_t \quad (\text{B.31})$$

$$N = \begin{bmatrix} 1 & 0 \\ 1 & -\frac{l+\lambda}{\rho(1-\lambda)} \\ 0 & -\frac{\lambda}{(1-\lambda)} \\ 0 & 1 \end{bmatrix}$$

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (B.30) can therefore be expressed as:

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} [\widehat{Y}_t, \widehat{RS}_t]' L'_y [\widehat{Y}_t, \widehat{RS}_t] + [\widehat{Y}_t, \widehat{RS}_t]' L'_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + t.i.p \quad (\text{B.32})$$

where:

$$L'_y = N' L_y N$$

$$L'_e = N' L_y N_e + N' L_e$$

Finally, we rewrite the previous equation with variables expressed as deviations from their targets:

$$L_{t_0}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] + s.o.t.i.p \quad (\text{B.33})$$

where:

$$\begin{aligned} \Phi_Y &= (\eta + \rho)(1 - \phi) + \frac{(\rho - 1)[-l(1 - \phi) - (\lambda - \phi)]}{(1 + l)} \\ &+ Lx_1 \left[(\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l)} \right] \\ &- \frac{Lx_2(1 - \lambda)^2 \lambda(\rho\theta - 1)}{(1 + l)} \end{aligned}$$

$$\begin{aligned} \Phi_{RS} &= -\frac{(\lambda + l)(\rho - 1)}{(1 - \lambda)\rho^2} \\ &+ \frac{Lx_1 l(\rho - 1 - l)}{(1 - \lambda)^2 \rho} \\ &+ \frac{Lx_2 \lambda(\rho\theta - 1)[\rho\theta(1 - \lambda) + \lambda + l]}{\rho^2} \\ &+ \frac{Lx_3 \lambda(\theta - 1)}{1 - \lambda} \end{aligned}$$

$$\Phi_\pi = \frac{\sigma}{\mu k} + (1 + \eta) \frac{\sigma}{k} Lx_1$$

and

$$\widehat{Y}_t^T = q_y^e e_t, \text{ and } \widehat{RS}_t^T = q_{rs}^e e_t$$

with

$$q_y^e = \frac{1}{\Phi_Y} \begin{bmatrix} \frac{\eta}{\mu} + Lx_1(1+\eta)\eta & -Lx_1(1+\eta) & \frac{(\rho-1)(1-\lambda)+Lx_2}{1+l} & 0 \end{bmatrix}$$

$$q_{RS}^e = \frac{1}{\Phi_{RS}} \begin{bmatrix} 0 & 0 & \frac{(\rho-1)Lx_1}{(1-\lambda)} + \frac{Lx_2\lambda(\lambda(1-\lambda)+1)(\rho\theta-1)}{\rho(1-\lambda)} & \frac{-Lx_2\lambda(1-\lambda)(\rho\theta-1)}{\rho} \end{bmatrix}$$

Moreover, we can write the constraints of the maximization problem as:

$$\widehat{\pi}_t^H = k \left(\eta(\widehat{Y}_t - \widehat{Y}_t^T) + (1-\lambda)^{-1}(\widehat{RS}_t - \widehat{RS}_t^T) + u_t \right) + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{B.34})$$

$$(\widehat{Y}_t - \widehat{Y}_t^T) = (\widehat{RS}_t - \widehat{RS}_t^T) \frac{(1+l)}{\rho(1-\lambda)} + \chi u_t \quad (\text{B.35})$$

where

$$u_t = \left[\eta, \frac{1}{1-\lambda} \right] \left[(\widehat{Y}_t^T - \widehat{Y}_t^{Flex}), (\widehat{RS}_t^T - \widehat{RS}_t^{Flex}) \right]'$$

$$\chi = \left[\frac{1}{\eta}, \frac{(1+l)}{\rho} \right]$$

and \widehat{Y}_t^{Flex} and \widehat{Y}_t^{Flex} are the flexible price allocation for output and the real exchange rate:

$$\widehat{Y}_t^{Flex} = [(\eta + \rho) + \eta l]^{-1} \left\{ \eta(1+l)\widehat{\varepsilon}_{Y,t} - (1+l)\widehat{\mu}_t + \rho\widehat{g}_t - \rho l \widehat{C}_t^* \right\} \quad (\text{B.36})$$

$$\frac{\widehat{RS}_t^{Flex}}{(1-\lambda)} = [(\eta + \rho) + \eta l]^{-1} \rho \left\{ \eta\widehat{\varepsilon}_{y,t} - \widehat{\mu}_t - \eta\widehat{g}_t - (\eta + \rho)\widehat{C}_t^* \right\} \quad (\text{B.37})$$

B.6 Special Cases

In this section we present the special cases described in the main text.

Special Case 1:

In this case we assume:

1. $\rho\theta = 1$
2. No mark-up or fiscal shocks.

$$\Phi_Y = (\eta + \rho)(1-\lambda) + ((1-\lambda) - \mu^{-1})(1-\rho)$$

$$\Phi_{RS} = 0$$

$$\Phi_\pi = \frac{\sigma}{k}(1-\lambda) + ((1-\lambda) - \mu^{-1})(1-\rho) \frac{\sigma}{k(\rho + \eta)}$$

Moreover:

$$\widehat{Y}_t^T = q_y^e e_t = \widehat{Y}_t^{Flex} = [(\eta + \rho)]^{-1} \{ \eta\widehat{\varepsilon}_{y,t} \}$$

Special Case 2:

In this case we assume:

1. $\rho\theta = 1$
2. $\mu = 1/(1-\lambda)$
3. No mark-up shocks.

$$\Phi_Y = (\eta + \rho)(1 - \lambda)$$

$$\Phi_{RS} = 0$$

$$\Phi_\pi = \frac{\sigma}{k}(1 - \lambda)$$

Moreover:

$$\widehat{Y}_t^T = q_y^e e_t = \widehat{Y}_t^{Flex} = [(\eta + \rho)]^{-1} \{ \eta \widehat{\varepsilon}_{y,t} + g_t \}$$

Special Case 3: The closed economy

$$\frac{\Phi_\pi^c}{\Phi_Y^c} = \frac{\sigma}{k(\eta + \rho)}$$

and

$$\widehat{Y}_t^{T,c} = \frac{\eta \varepsilon_{Y,t}}{(\eta + \rho)} - \frac{(\mu - 1)(\eta + 1)\mu_t}{(\eta + \rho)(\mu\eta + \rho + (\mu - 1))} + \frac{\rho(\eta\mu + \rho)g_t}{(\eta + \rho)(\mu\eta + \rho + (\mu - 1))}$$

$$\widehat{Y}_t^{Flex,c} = \frac{\eta \varepsilon_{Y,t} - \mu_t + \rho g_t}{(\eta + \rho)}$$

C Appendix: Proof of determinacy

In this section we show that the optimal targeting rule, together with the policy constraints and the initial condition for inflation delivers a determinate equilibrium. The equilibrium conditions given by equations (34), (35) and (39) can be rewritten as:

$$\widehat{\pi}_t^H = \gamma_1(\widehat{Y}_t - \widehat{Y}_t^T) + k\delta_t + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{C.38})$$

$$\gamma_2 \Delta(\widehat{Y}_t - \widehat{Y}_t^T) - \gamma_3 \Delta\delta_t + \gamma_4 \widehat{\pi}_t^H = 0 \quad (\text{C.39})$$

where δ_t is a linear combination of shocks and follows a AR(1) process

$$\delta_t = \omega\delta_{t-1} + e_t \quad (\text{C.40})$$

and

$$\gamma_1 = k(\eta + \rho(1 + l)^{-1})$$

$$\gamma_2 = (1 + l)\Phi_Y + \frac{\rho^2(1 - \lambda)^2}{(1 + l)}\Phi_{RS}$$

$$\gamma_3 = \chi \frac{(1 + l)}{\rho^2(1 - \lambda)^2}$$

$$\gamma_4 = (\rho + \eta(1 + l))k\Phi_\pi$$

Moreover, we can reduce the system given by conditions (C.38) and (C.39) to the following equation:

$$\beta E_t \widehat{\pi}_{t+1}^H - (1 + \beta + \gamma_1\gamma_4/\gamma_2)\widehat{\pi}_t^H + \widehat{\pi}_{t-1}^H = (\gamma_1\gamma_3/\gamma_2 + k)\xi_t \quad (\text{C.41})$$

where ξ_t is a stationary shock¹⁸. The characteristic polynomial associated with this equation is:

$$P(a) = \beta a^2 - (1 + \beta + \gamma_1\gamma_4/\gamma_2)a + 1 \quad (\text{C.42})$$

¹⁸More specifically $\xi_t = e_t - (1 - \omega)\delta_{t-1}$

Equations (C.38) and (C.39) form a system with one predetermined variable and one endogenous variable. Therefore, determinacy is guaranteed if the above polynomial has one root inside the unit circle and one outside. This is true if $\gamma_2/\gamma_4\gamma_1 > -1/2(1 + \beta)$. More specifically:

$$\frac{(1+l)^2\Phi_Y + \rho^2(1-\lambda)^2\Phi_{RS}}{(1+l)^2(\rho + \eta(1+l))^2\Phi_\pi} > -\frac{k^2}{2(1+\beta)} \quad (\text{C.43})$$

D Appendix: Randomization problem

In order to ensure that the policy obtained from the minimization of the loss function is indeed the best available policy; we should certify that no other random policy plan can be welfare improving. Equation (34) combined with (35) leads to the following expression:

$$\widehat{\pi}_t^H = k \left((\eta + \rho^{-1}(1+l))(\widehat{Y}_t - \widehat{Y}_t^T) + u_t \right) + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{D.44})$$

or alternatively

$$\widehat{\pi}_t^H = k \left(\frac{\eta(1+l) + \rho}{\rho(1-\lambda)} (\widehat{RS}_t - \widehat{RS}_t^T) + u_t \right) + \beta E_t \widehat{\pi}_{t+1}^H \quad (\text{D.45})$$

Therefore a random realization that adds $\varphi_j v_j$ to π_{t+j} , also increases \widehat{Y}_t by $\alpha_y k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ and \widehat{RS}_t by $\alpha_{rs} k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$. Where

$$\alpha_{rs} = \left(\frac{\eta(1+l) + \rho}{\rho(1-\lambda)} \right) \quad (\text{D.46})$$

and

$$\alpha_y = (\eta + \rho^{-1}(1+l)) \quad (\text{D.47})$$

Consequently, the total contribution to the loss function is

$$U_c \bar{C} \beta^t \sigma_v^2 E_{t_0} \sum \beta^t [\Phi k^{-2}(\varphi_j - \beta\varphi_{j+1})^2 + \Phi_\pi(\varphi_j)^2] \quad (\text{D.48})$$

where

$$\Phi = \Phi_Y \alpha_y^2 + \Phi_{RS} \alpha_{rs}^2$$

It follows that policy randomization cannot improve welfare if the expression given by equation (D.48) is positive definite. Hence, the first order conditions to the minimization problem are indeed a policy optimal if Φ and Φ_π are not both equal to zero and either: (a) $\Phi \geq 0$ and $\Phi + (1 - \beta^{1/2})^2 k^{-2} \Phi_\pi \geq 0$ or (b) $\Phi \leq 0$ and $\Phi + (1 - \beta^{1/2})^2 k^{-2} \Phi_\pi \geq 0$ holds. This analysis follow closely Woodford and Benigno (2003). The authors also demonstrate that these conditions coincide with the second order condition for the linear quadratic optimization problem.

In the case of our small open economy conditions (a) and (b) involve complicated linear combinations of the structural parameters. Even though they are satisfied under our benchmark calibration, for many parameter combinations this is not the case. The following tables illustrate when a randomization is never welfare improving.

$\rho \setminus \theta$	1	2	3	4	5	6
1	Yes	Yes	Yes	No	No	No
2	No	No	No	No	No	No
3	No	Yes	No	No	No	No
4	No	No	No	No	No	No
5	No	No	No	No	No	No
6	No	No	No	No	No	No

Table 13: Parameterization under which the second order condition to the minimization problem is satisfied (1)

$\theta \setminus \lambda$	1/2	1/3	1/4	1/5
1	No	No	Yes	Yes
2	No	No	No	Yes
3	No	No	No	Yes
4	No	No	No	No

Table 14: Parameterization under which the second order condition to the minimization problem is satisfied (2)

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