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REAL INTEREST RATES AND INDEX LINKED GILTS

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ABSTRACT

This paper derives the *ex ante* paths of the future expected short (one period) real interest rates at quarterly frequency from observations on the prices of a set of UK index linked bonds. These rates are used to investigate the impact of monetary policy and the nature of expectations formation in the bond market.

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Introduction

The *ex ante* real interest rate is the central variable in most conventional neo-classical macroeconomics. If one had a time series of such rates, many controversies could be resolved. The problem is, of course, that the *ex ante* real interest rate is not usually observed. One observes nominal interest rates, but not price expectations. Price expectations can be obtained from survey data, about which economists are notoriously sceptical; or by regression forecasts. Woodward (1990) notes that such forecasts "basically amount to conjecture".

Index linked bonds (that is, coupon and principal payments rise in line with a price index) offer a direct measure of the *ex ante* real interest rate. A number of countries have issued, and continue to issue, index linked debt. The British market is by far the largest, with the most comprehensive range of maturities. For governments, one advantage of index linked bonds occurs when private sector expectations of inflation exceed those of the government, perhaps because markets are wary of government intentions. In this case index linked bonds enable funding at lower cost (provided the government does indeed deliver lower inflation), as well as the opportunity for the government to influence expectations. This argument led the recently elected British government to introduce index linked bonds (gilts) in 1981. For much of the 70s funding had been at negative ex post real interest rates, leading to market scepticism about anti-inflationary resolve. Mrs Thatcher's administration had, as it turned out, a strong commitment to reduced inflation. More generally, governments may feel that catering for a wider range of tastes will reduce the cost of financing the deficit.

The major worry about index linked bonds, certainly in the US, is how to tax the indexed component. In 1985, Francis X. Cavanaugh of the US Treasury argued before the Joint Economic Committee of Congress that tax privileges would have to be awarded to index linked bonds to make them attractive, thus raising the cost to the Treasury. However, in principle, given any tax regime, index linked bonds would trade at some price, and a low issue price presumably reflects the present value of future tax payments remitted to the Treasury. Thus, apart from timing, the Treasury's position is unaffected. This issue is sidestepped in the UK, where capital gains on all gilts are tax-exempt. Another worry in the US, also expressed by the Bank of England at the first issue, is that increased use of index linked contracts could weaken anti-inflationary resolve. Even so, without a detailed model of the costs of inflation, it is not clear that this would entail a welfare loss.

Between 1981 and mid-1992 the British government issued 14 index linked gilts[1], with maturities up to about 30 years, constituting, by the early 90s, about a fifth of the market value of all gilts. This paper uses the prices of these securities to calculate the expected path of short *ex ante* real interest rates at each date from 1984q1 to 1992q2, up to the 30 year horizon.

The paper is organised as follows. Section 1 uses a present value formula for bond prices to derive the path of expected future short real interest rates, given observations on the prices of a set of perfectly indexed bonds. Such observations enable the calculation of the expected *average* future real interest rate *between* the maturities of the available bonds, i.e. they give the future expected path as a step function. These can then be smoothed in a variety of ways. This technique uses only the information in the prices. It does not assume a specific model of the term structure, nor, in particular, does it make assumptions about expectations.

Unfortunately the indexation is not perfect: coupons and principal are adjusted in line with the retail price index *lagged* eight months.[2] Thus the index linked bond price includes elements of the nominal term structure. Section 3 describes how to deal with this without making *ad hoc* assumptions about expectations. Essentially we use the observable nominal term structure, calculated from a set of conventional bonds, to discount the eight month lag in indexation. Woodward (*op. cit.*) has introduced this technique, but calculates only yields to maturity. Our aim, however, is to calculate the full expected future path of the short real interest rate at each date.

Section 4 uses these real interest rates to test some macroeconomic propositions. So far, British Index Linked gilts have not been used extensively for this purpose. Barro (1987), in his macroeconomic text, lists some yields to maturity as evidence of the relative stability of the *ex ante* real interest rate compared to the nominal rate. Arak and Kreicher (1985) use them to generate a path of expected UK inflation. Robertson and Symons (1994) study the behaviour of real interest rates and inflation expectations, over the period of sterling's exit from the ERM. Most other discussion has taken place in the finance literature. Section 5 concludes.

1. The Path of Expected Future Short Real Interest Rates with Perfectly Indexed Bonds

We commence our analysis by assuming the existence of a riskless bond which pays a coupon c, perfectly indexed to an appropriate price index, with an indexed maturity payment of £100 in m periods' time. If a single risk-neutral zero-tax-paying investor willingly holds this security as well as nominal securities of all intervening maturities arbitrage implies

(1) $g_t=(1/p_b)[cp_{t+1}/(1+r_t)+cp_{t+2}/(1+r_t)(1+r_{t+1})+..(c+100)p_{t+m}/(1+r_t)..(1+r_{t+m-1})]$

where g_t is the price of the bond

r_t is the nominal interest rate in period t

p_t is the price level in period t,

b is the base period for indexation

Price levels dated later than t are assumed to be expected values formed at t. If we define the short real interest rate ρ_t as

$$1+\rho_t=(1+r_t)/(p_{t+1}/p_t)$$

the arbitrage condition takes the form

(2) $g_t=(p_t/p_b)[c/(1+\rho_t)+c/(1+\rho_t)(1+\rho_{t+1})+....+(c+100)/(1+\rho_t)...(1+\rho_{t+m-1})]$

Write (2) more succinctly as

(3) $$g_t=g_t(\rho)$$

where $\boldsymbol{\rho}$ is the vector of real interest rates from t to maturity. The published yield to maturity of this security would be the solution y to

(4) $$g_t=g_t(y\mathbf{1})$$

where $\mathbf{1}$ is a vector of 1s. We can expand (3) as a Taylor series about $y\mathbf{1}$:

(5) $$g_t=g_t(y\mathbf{1})+(\partial g_t/\partial\rho)'(\rho-y\mathbf{1})$$

where the derivative is evaluated at y**1** and we have neglected higher order terms. Making use of (4) we obtain

(6) $$y=(1/(\partial g_t/\partial \rho)'\mathbf{1})((\partial g_t/\partial \rho)'\rho)$$

The derivative $\partial g_t/\partial \rho$ can be readily evaluated from (2). For example, the first entry is

(7) $$\partial g_t/\partial \rho_t=(p_t/p_b)[-c/(1+y)^2-c/(1+y)^3-....-(c+100)/(1+y)^{m+1}]$$

The next entry is obtained by deleting $c/(1+y)^2$; and, in general, successive entries are obtained in like manner by deleting terms from the left in (7). Thus (6) expresses the yield on a security as a weighted average of future short rates where the weights decline for more distant rates. It is easy to see that the weights decline quite slowly: approximately at the rate $1/(1+y)$.

Assume there exists at t a family of n indexed-linked securities, differing by maturity dates, and perhaps also by coupon value. Then each security satisfies a version of (6). We can capture the whole content of the observed yields in a matrix equation

(8) $$A_t\boldsymbol{\rho}_t=\mathbf{y}_t$$

where $\mathbf{y}_t$ is the vector of published yields, $_t\rho$ is the vector of short rates out to the maximum duration security in the family and A_t is a matrix whose rows consist of positive declining weights. There is one row for each bond. For convenience, we assume these rows are ranked according to maturity date, shortest to longest. Usually we shall drop the date subscripts in (8).

Equation (8) gives all the information concerning the path of the interest rate implicit in the current set of yields (or, equivalently, the current set of bond prices). It is clear that one cannot, in practice, recover ρ from (8) because there will be many more entries in ρ than different securities, i.e. there are more unknowns in (8) than equations. To identify ρ some further assumptions are required. We now consider the procedure we have found most fruitful.

Define the *mesh* of a family of n securities of different maturities as the decomposition of the total duration of the longest running security $[t, t+m_n]$ into segments

$$[t,t+m_1],\ [t+m_1+1,t+m_2],\ldots..,\ [t+m_{n-1}+1,t+m_n]$$

where $m_1<m_2<\ldots<m_n$ are the periods to maturity of each security. Let $\bar{\rho}_i$ be the mean of ρ on the i'th mesh segment and write $\bar{\rho}=(\bar{\rho}_1, \ldots, \bar{\rho}_n)$. If ρ were constant on mesh segments then

(9) $$\rho=W\bar{\rho}$$

where W is the appropriate $m_n$xn matrix of 1s and Os. In this case the arbitrage condition (8) becomes

(10) $$AW\bar{\rho}=\mathbf{y}$$

It is easy to see that AW is nxn of full rank so

(11) $$\rho=W(AW)^{-1}\mathbf{y}$$

If ρ is not constant over mesh segments then one can still compute the right hand side of (11): call it ρ^s. Now ρ^s is a step function constant over mesh segments, and

$$\begin{aligned}
\rho^s &= W(AW)^{-1}\mathbf{y} \\
&= W(AW)^{-1}A\rho \\
&= W(AW)^{-1}A\,(W\bar{\rho} + \rho-W\bar{\rho}) \\
&= W\bar{\rho} + W(AW)^{-1}A(\rho-W\bar{\rho})
\end{aligned}$$

For smooth-changing ρ and narrow mesh segments, $A(\rho-W\bar{\rho})$ will be approximately zero (the discrepancy arising because the rows of A are not quite constant over mesh segments). It thus follows that

(12) $$\rho^{s} \cong W\bar{\rho}$$

i.e. the values of the steps in ρ^{s} computed according to (11) are approximately the arithmetic averages of ρ over the mesh segments.

The measure ρ^{s} has its attractions as it uses only the arbitrage condition to produce an estimate of the path of ρ. While an estimate of ρ_{t+i} based on ρ^{s} will be the average of values of ρ around ρ_{t+i}, if the mesh size is small, this may be satisfactory.

One problem with ρ^{s} is that it will jump discontinuously at the mesh boundaries. One might think one could get closer estimates of ρ if some sort of smoothness were imposed. Smoothness, however defined, is quite a plausible prior for the path of future interest rates for at least two reasons. A good neoclassical reason is that capital cannot jump instantaneously - only investment - and hence the same applies to its marginal product. A different justification of this prior is that typically, even if one believes quite strongly that some event will take place in the future, one is often unsure exactly when this will be. This uncertainty tends to smooth the expected paths of exogenous variables.

We have studied several procedures to construct smooth paths of interest rates satisfying the arbitrage condition. Consider the solution to:

(13) minimise over ρ : $\rho' D\rho$ subject to $A\rho=\mathbf{y}$

where D is some specified matrix. For example, we might choose D so that

(14) $$\rho' D\rho = \Sigma_{t}(\rho_{t}-\rho_{t-1})^{2}$$

whereupon the above procedure amounts to choosing the interest rate path satisfying the arbitrage condition that minimises the sum of squared changes. This procedure is analogous to the Hodrick-Prescott (1980) filter for constructing the trend of a time series.

A different, more standard, method of imposing smoothness is to assume the interest rate path lies in some specified function space:

(15) $$\rho = H\beta$$

where the columns of the matrix H are smooth functions of time (e.g. polynomials) and β parameters to be fitted. Substituting (15) into (8) we have

(16) $$AH\beta = \mathbf{y}$$

from which β can be recovered by regression for any number of parameters up to the number of securities in the family.

We studied the performance of these methods by formulating two possible synthetic paths for short term interest rates: first, exponential decline from a current high level to a future lower level; and second, an interest rate which increases from a current low level to a peak in some years time, and then declines to some asymptotic value[3]. Yields to maturity were calculated using these interest rate paths for a family of securities defined by the coupons and maturities of the index-linked gilts existing in the summer of 1992. Using these yields we attempted to recover the original synthetic interest rate path. Two classes of functions were used in (16): polynomials in time $\{t^n\}$ and the Hermite polynomials $\{t^n \exp(-t^2/2)\}$. There are about a dozen index-linked gilts in operation, thus enabling up to twelve polynomials to be fitted. Once the Hermite polynomials were appropriately scaled (i.e. units of time t chosen) they were much superior to time polynomials in terms of maximum and minimum error and variance of error. Naturally, the fit improved with the degree of polynomial used. However, once this methodology was applied to real data, an important problem emerged. High-order polynomials tended to produce interest rate paths with a large number of interior maxima and minima. Moreover the interest rate path tended to be moving about substantially even 30 years into the future. The first of these problems affected also the estimates calculated by the smoothing procedure (13) and (14). We found we obtained plausible results for Hermite polynomials up to a maximum degree of two (i.e. $t^2 \exp(-t^2/2)$). This allows, at most, one interior maximum and one interior minimum.

In practice we did not calculate the fitted path from (16). Instead we smoothed the step function ρ^s. Our reasoning was that it is better to stick as close as possible to what one knows to be true (given arbitrage) rather than to leap into the dark with econometric estimates which might have unpleasant properties given stochastic errors in (8) and (15).

2. Tax and Risk Issues

We note that (1) is not adjusted for taxation. In principle the stream of coupon payments should be adjusted for expected future tax rates. There are two related methods in the literature for dealing with this. One can nominate a tax rate and calculate the corresponding term structure using some method to choose the set of tax efficient bonds for that tax rate (e.g. Schaeffer (1981)), or one can estimate an average tax rate effective in the market by some best fit criterion (e.g. Woodward (*op. cit.*), Levin and Copeland (1992)). Invariably these methods impose a constant tax rate to the maturity horizon of the security; which is plainly questionable.

However there is one large player in the market for whom a future tax rate can be nominated with some certainty: the tax exempt pension funds. There has never been any prospect of change in the tax status of these institutions. At the end of 1990 the pension funds had total assets of £302.7 billion, roughly 60% of UK GDP. Table 1 shows they hold nearly half of the total index linked issue as well as significant holdings of conventional gilts of all maturities, though skewed to longer dated stock. The central contention of this paper is that such a large untaxed collective institution, holding a full range of securities, is sufficient to enforce the arbitrage condition (1).

Levin and Copeland (*op. cit.*) assume the expected path of the real interest rate is constant (but unknown) at any point in time and that inflationary expectations decay exponentially from a current (unknown) level to some long run level (also unknown). With these assumptions they are able to solve for the real interest rate, the path of inflationary expectations and a marginal tax rate effective in the market. They find that this rate declines from an average of about 15% in the 80s to about 5% in the 90s. Woodward (*op. cit.*), estimating over the 80s, finds an average closer to 25%; an OLS regression of his estimates on those of Levin and Copeland gives:

$$\text{(Woodward's tax rate)} = \underset{(4.3)}{16.7} + \underset{(1.8)}{0.42}\ \text{(L\&C's tax rate)}$$

Estimation period 84q1-89q1, quarterly data, R^2=0.15, t-statistics in parentheses

Given the strong assumptions required for either of these estimates, in particular the constancy of marginal tax rates over the horizon of the set of bonds, the lack of agreement between them suggests that the issue is not resolved by this strategy. A better procedure is to select, as we do below, securities that are especially attractive to tax exempt institutions, and assume a zero tax rate in (1). Even if the marginal bond owner does pay tax, in the UK this tax is only ever on coupon payments. For long dated stocks, where the coupon component of the price is large in the present value formula, Table 1 shows that the tax exempt institutions hold a large part of the stock. So taxation is not an issue here. At the short end of the market, the low coupon values mean that the (taxable) coupon stream only comprises a small part of the price. Here, the major part of the return is the (tax free) principal repayment. So tax effects are likely to be minor here as well. We calculate that for a 2% bond with a ten year horizon and a marginal tax rate of 10%, the assumption of a zero tax rate would give an error of at most about a fifth of one percent in the real interest rate calculation if the real interest rate is two per cent.

The presence of risk however would break the arbitrage condition. There are three possible sources of risk for holders of a gilt: the risk of default, the risk of changes in inflation, and the risk of changes in the spot interest rate. The UK has never defaulted and one can safely assume this risk away. We wish to argue that the remaining two categories are small compared to the real interest rate.

Levin and Copeland (*op. cit.*) are also able to calculate an inflation risk parameter at each period. They find that the risk premium is at most about ten percent (and usually considerably less) of the real interest rate. To the extent that their assumptions are some sort of approximation to reality, the consistency of their estimates of the risk premium indicates that it is probably a small effect. Woodward (*op. cit.*) argues similarly, by different methods.

3. Empirical Implementation

British index-linked gilts are not perfectly indexed to the price level. Coupons and the principal repayment are adjusted by the retail price index (RPI) eight months before the payment; and the base price itself is the RPI eight months before the issue of the bond. Thus, letting t refer to months,

$$(17)\quad g_t=(1/p_b)[c_{t+1}p_{t-7}/(1+r_t)+c_{t+2}p_{t-6}/(1+r_t)(1+r_{t+1})+\ldots c_{t+8}p_t/(1+r_t)\ldots(1+r_{t+7})$$

$$+c_{t+9}p_{t+1}/(1+r_t)\ldots(1+r_{t+8})+\ \ldots\ +c_{t+m}p_{t+m-8}/(1+r_t)\ldots(1+r_{t+m-1})]$$

where c_{t+i} is the monthly unadjusted payment, zero if there is no payment in month t+i, and including the principal at maturity. As before prices dated later than t are assumed known only in expectation. In (17) the first eight terms are known at t.[4] Call this component O_t and the remainder F_t (O and F standing for observed and forecast). Then

$$(18)\qquad g_t=O_t(\mathbf{r})+F_t(\mathbf{r},\rho)$$

where

$$(19)\qquad F_t=(p_t/p_b)[c_{t+9}/(1+\rho_t)(1+r_{t+1})..(1+r_{t+8})+\ \ldots$$

$$\ldots+c_{t+m}/(1+\rho_t)..(1+\rho_{t+m-9})(1+r_{t+m-8})\ldots(1+r_{t+m-1})]$$

Equation (18) makes clear that the price of index-linked gilts is the sum of the price O_t of a hypothetical pure nominal bond, and the price F_t of a bond that depends on both the nominal and the real term structure.

Published yields are the solution y to

$$(20)\qquad g_t=O_t(\bar{r}\mathbf{1})+F_t(\bar{r}\mathbf{1},y\mathbf{1})$$

where $\bar{r}=(1+y)(1+\pi)-1$ and π is some assumed inflation rate, either 5 or 10 per cent in the *Financial Times*. Assume the nominal term structure is known and expand F from (19) as before around the vector y**1**. One obtains

$$g_t=O_t+F_t(\mathbf{r},y\mathbf{1})+(\partial F/\partial\rho)'(\rho-y\mathbf{1})$$

whence

(21) $y+(1/(\partial F/\partial\rho)'\mathbf{1})(g_t-O_t-F_t(\mathbf{r},y\mathbf{1}))=(\partial F/\partial\rho)'\mathbf{1})(\partial F/\partial\rho)'\rho)$

Comparing (21) with (6) one can see that there is an extra term on the left. In practice this term is always quite small. Thus we have reestablished (8) provided a small correction is made to $\mathbf{y}_t$. Given a family of index-linked bonds we have therefore

(22) $$A_t\rho = \mathbf{y}^*_t$$

where now the rows of A_t are $\partial F/\partial\rho$ for each bond.

This analysis is predicated on knowing the nominal term structure since $\partial F/\partial\rho$ depends on $\mathbf{r}$. We amend the methods discussed above. The arbitrage condition for nominal bonds leads to an analogue of (8).

$$A^N_t\mathbf{r}_t=\mathbf{y}^N_t$$

where A^N_t is a readily-calculable weighting matrix, $\mathbf{r}_t$ is the vector of short-term nominal interest rates and $\mathbf{y}^N_t$ is a vector of published yields. We compute the nominal term structure $\mathbf{r}^s_t$ that is constant on mesh segments (for the chosen family of nominal bonds) according to (11). Where $\mathbf{r}^s_t$ stops short of the maturity of the longest dated index-linked, we carry it forward at its computed average over the final two years of its duration; we then smooth $\mathbf{r}^s_t$ onto a constant and the first three Hermite polynomials.

In order to work at quarterly frequency, nominal yields were collected on the last working day of each quarter for the family of nominal gilts recorded in Table 2. This family was selected where possible to be congruent in maturity structure to the index-linked family (listed in Table 3), and, importantly, to have higher coupons than contiguous nominal gilts. Our reasoning here was that such gilts would be especially attractive to zero tax payers, as discussed above. Thus they approximate the appropriate discount factor for the nominal component of index linked gilts held by zero tax payers. This then allows calculation of a gross nominal term structure. Note that the longest dated nominal gilt matures in 2017, seven years before the longest dated index linked, so that we shall use the procedure outlined above to extend the nominal term structure to the full horizon.

Given the nominal term structure we may now compute the rows of A, $\partial F/\partial\rho$, in (22). To convert the existing family of index-linked gilts into a form suitable for analysis at quarterly frequency we replace them by a synthetic family whose coupon drops occur on the last day of the appropriate quarter. For example, if an index-linked pays on February 17 and August 17 we calculate by the present value formula the price of an index-linked of the same yield (as published in the *Financial Times*, taking the 5 per cent inflation variant) and quarters to maturity, but paying at the end of March and September; and replace the original gilt price by this new hypothetical gilt price as g_t in (21). Finally, to convert (19) into quarterly form, we assume nominal and real interest rates are constant within quarters so that, for example, the term $(1+\rho_t)(1+r_{t+1}) \dots (1+r_{t+8})$ is replaced by $(1+\rho)^{1/3}(1+r_{t+2})(1+r_{t+3})^{2/3}$. Note that t in the second expression indexes quarters (and r_t refers to a quarterly interest rate) whereas t indexes months in the first expression.

To summarize, to obtain the term structure of real interest rates:

(a) We evaluate the nominal term structure according to the method described above

(b) This enables $\partial F/\partial\rho$ to be calculated from (19) and evaluated at the published yields.

(c) Thus A_t in (22) is known; and $\mathbf{y}_t^*$ can be computed from (21).

(d) Formula (11) then gives ρ^s, which can be smoothed if desired.

4. UK *ex ante* real interest rates

The results of applying this procedure over the period 84q1 to 92q2 are displayed in Figures 1-4. Figure 1 displays the surface of the underlying step functions for the nominal term structure and Figure 2 the result of smoothing this surface onto the Hermite polynomials. Similarly Figures 3 and 4 graph the surfaces of the underlying step function and the corresponding smoothed surface for the real term structure. A different perspective on the behaviour of the real interest rate is obtained by graphing sections through the surfaces. Figure 5 graphs, and Table 4 lists, the average expected real interest rate over the next eight quarters, and the estimate of the real interest rate in 2024, quarterly from 84q1 to 92q2, taken from the smoothed surface. What is immediately striking is the volatility of the short real interest rate and the stability of the long real interest rate. The long real interest rate is approximately constant at about 2.7% from 1984 to 1989, then rises sharply to about 4.2%. The rise coincides with the emergence of a genuine possibility of market economies in Eastern Europe. The associated capital requirements will raise the demand for world savings and hence imply an increase in the world real interest rate. If this is the correct explanation for the rise in the long rate in Figure 5, it is interesting that the markets take the view that equipping the former communist states will take more than 30 years. The behaviour of the short real interest rate is more problematical from a neo-classical perspective.

If we assume the major instrument of monetary policy rate in the UK in this period to be the treasury bill rate, and that changes in this rate correspond to monetary shocks, we can investigate the correlations between monetary policy and the real interest rate. Defining the short real interest rate as the average expected rate over the next two years, the medium rate as the average expected rate between horizons of four and five years, and the long rate as that expected at the horizon of the longest dated index linked bond (i.e. 2024), we obtain the following OLS estimates from 84q1 to 92q2 (t-statistics in parentheses):

$$\underset{}{(\text{short RIR})} = \underset{(2.5)}{1.3} + \underset{(5.5)}{0.69(\text{short RIR})_{-1}} + \underset{(2.1)}{0.17\ \Delta TBR} \qquad R^2=0.49$$

$$(\text{medium RIR}) = \underset{(1.8)}{1.3} + \underset{(3.7)}{0.68(\text{medium RIR})_{-1}} + \underset{(0.8)}{0.04\ \Delta TBR} \qquad R^2=0.34$$

$$(\text{long RIR}) = \underset{(0.3)}{0.08} + \underset{(13.1)}{1.0(\text{long RIR})_{-1}} + \underset{(0.1)}{0.003\ \Delta TBR} \qquad R^2=0.85$$

where ΔTBR is the one quarter change in the UK treasury bill rate. There is little evidence of residual misspecification in these regressions. The level of the treasury bill was insignificant in all regressions.

These regressions suggest that monetary policy, as captured by movements in the treasury bill rate, does have an important effect on the short real interest rate, but only on the short rate. The impact effect of a 1 percentage point reduction in the treasury bill rate is a 0.17 percentage point reduction in the expected short real interest rate. After one year, this reduction implies a fall of about 0.03 percentage points in the expected short real interest rate. To obtain a more persistent fall in the short real interest rate will require continual cuts in the treasury bill rate. Thus from early 1986 to mid 1988, the treasury bill rate fell from 12.5% to 7.3% in a sequence of downward and upward movements around a downwards trend, accompanied by the long fall in short real interest rates evident in Figure 5. The 1988-1989 monetary squeeze ran these events in reverse. Levin and Copeland (*op. cit.*) detect similar movements with their version of the real interest rate. The estimates imply that only 20% of the effects of a monetary shock remain after four quarters. This seems to imply that, at least via this channel, the classical dichotomy is approximated within one year.

The calculated paths of expected interest rates also offer the opportunity to study expectations formation and, in particular, a direct test of the rationality of expectations. By the law of iterated expectations we have

$$E(\rho_H|\Omega_t)=E(E(\rho_H|\Omega_{t+1})|\Omega_t)$$

where H>t is a (fixed) horizon date and Ω_t the information set available at t. This implies that revisions to the mathematical expectation are orthogonal to t-dated information. If the market expectation coincides with the mathematical expectation then our calculated expected rates should possess this property.

We take as the information set current and lagged growth rates of GDP, the GDP deflator and the retail price index, and current and lagged absolute changes in the treasury bill rate. For the four horizons: 1995q1, 2001q1, 2005q1 and 2010q1, regressions of the change in the expectation of the real interest rate at these horizons upon current and first lagged values of the information set between 1984q1 and 1992q2 gave F-statistics that were insignificant in all four regressions at the 10% level. Univariate regressions of the change in the expectation on the current and first lagged values of each of the variables in the information set, thirty-two regressions in all, yielded only two significant t-ratios at the 5% level (2001q1 on the lagged change in the treasury bill rate, t=-2.3; and 2005q1 on the lagged change in the treasury bill rate, t=-2.0).

These results are consistent with the rational expectations hypothesis. Whilst they may be consistent with other expectation formation mechanisms, note that the absence of significant correlations implies there are no forecastable arbitrage gains in this market, a weak form of efficiency.

In Robertson and Symons (*op. cit.*) we applied the above techniques to daily data covering the period of sterling's exit from the ERM. We found that this monetary action brought about an immediate fall in the *ex ante* short real interest rate, leaving more distant rates unchanged. Inflation expectations, which are then readily calculated (with certain assumptions about risk premia) were found to decline in the short run but to increase in the long run. Thus useful insights can be obtained from the study of these securities.

5. Conclusion

This paper has described a method to obtain estimates of the *ex ante* path of future real interest rates from British Government index linked debt. The term structure of expected future real interest rates at quarterly frequency was obtained for the period 84q1 to 92q2. These *are* the implied real one period returns at future dates available in the market. With the assumption of free capital markets and purchasing power parity, they are then estimates of the world real interest rate. They allow a consideration of the impact of monetary policy, and an assessment of the expectations formation mechanism in the market.

Endnotes

* Centre for Economic Performance and Centre for Economic Forecasting, London Business School; and Centre for Economic Performance and Department of Economics, University College London; respectively. The Centre for Economic Performance is financed by the Economic and Social Research Council.

1. A further three were issued in September 1992.

2. We believe this is to permit the calculation of accrued interest before the six-monthly coupon drops, allowing for a two month lag in the publication of the retail price index. The Bank of England publishes bond prices net of accrued interest (the so called "clean price") so that prices do not jump at the coupon drop. It is a pity that this minor convenience has been allowed to contaminate the macroeconomic signals implicit in the bond prices. Even the publication lag is escapable, as all payments could be rolled over at short nominal rates until all information has appeared. It does seem, however, that imperfect indexation is widespread: see Bootle (1991), Appendix 4.

3. The economic interpretation of these paths could be, firstly, tight current monetary policy; and, secondly, future increased investment demand such as might follow a growth take-off in Eastern Europe. We felt that, as a minimum requirement, an estimation procedure should be able to cope with these paths.

4. Even though the RPI is published with a lag of one month, we assume agents in the market know the current price level.

Table 1 Market & Pension Fund Holdings of Gilt Edged Stock, end 1990

	Market Value £billion	(%)	Pension Fund Holdings £billion	%ge of Pension Fund Gilt portfolio (%)	% of market
Shorts (0-5years)	31.2	(27.1)	1.2	(4.3)	3.8
Mediums (5-15years)	52.0	(45.2)	12.7	(45.3)	24.4
Longs (>15years)	8.3	(7.2)	4.3	(15.4)	51.8
Index Linked	23.5	(20.5)	9.8	(35.0)	41.7
Total Stock	115.0	(100.0)	28.0	(100.0)	

Sources: Pension Fund Holdings: *CSO Financial Statistics*, February 1993.
Market Holding of Gilts: *Bank of England Quarterly Bulletin*, Feb. 1992, p56.

Table 2 Nominal Gilts used to evaluate Term Structures

Bond		Maturity Date	quarters to maturity from 84q1
14%	1986	29th Oct	11
13.25%	1987	22nd Jan	12
12.5%	1990	22nd Mar	24
13.5%	1992	22nd Sep	34
13.5%	1994	27th Apr	41
14%	1996	22nd Jan	48
15%	1997	27th Oct	55
13%	2000	14th Jul	66
10%	2003	8th Sep	78
10.5%	2005	20th Sep	86
8.5%	2007	11th Jul	94
9%	2008	13th Oct	99
9%	2012	6th Aug	114
8.75%	2017	25th Aug	134

Table 3 Index Linked Gilts used to evaluate Term Structures

Bond		Maturity Date	quarters to maturity from 84q1
2%	1988	20th Mar	16
2%	1990	25th Jan	24
2%	1992	23rd Mar	32
2%	1994	16th May	41
2%	1996	16th Sep	50
2.5%	2001	24th Sep	70
2.5%	2003	20th May	77
2%	2006	19th Jul	90
2.5%	2009	20th May	101
2.5%	2011	23rd Aug	110
2.5%	2013	16th Aug	118
2.5%	2016	26th Jul	130
2.5%	2020	16th Apr	145
2.5%	2024	17th Jul	162

Table 4 Short and Long Expected Real Interest Rates 84q1-92q2

	Short	Long
84q1	5.097	2.617
84q2	5.752	2.912
84q3	5.713	2.828
84q4	5.418	2.615
85q1	4.905	2.644
85q2	4.482	2.685
85q3	4.342	2.638
85q4	5.121	2.786
86q1	4.317	2.806
86q2	3.964	2.738
86q3	4.587	2.536
86q4	4.007	2.866
87q1	3.076	2.663
87q2	2.969	2.650
87q3	3.807	2.792
87q4	3.416	2.973
88q1	2.827	2.994
88q2	3.298	3.145
88q3	3.869	3.209
88q4	4.435	2.951
89q1	4.058	2.908
89q2	4.498	2.864
89q3	3.989	2.838
89q4	4.318	2.724
90q1	5.030	3.018
90q2	4.900	2.957
90q3	4.678	3.264
90q4	4.546	3.421
91q1	4.202	3.379
91q2	4.855	3.570
91q3	3.509	3.999
91q4	4.527	4.020
92q1	4.138	4.280
92q2	4.661	4.058

Note:
(i) "short" is the average expected rate over the following eight quarters.
(ii)"long" is the expected rate in 2024.
(iii) These data are graphed in Figure 5.

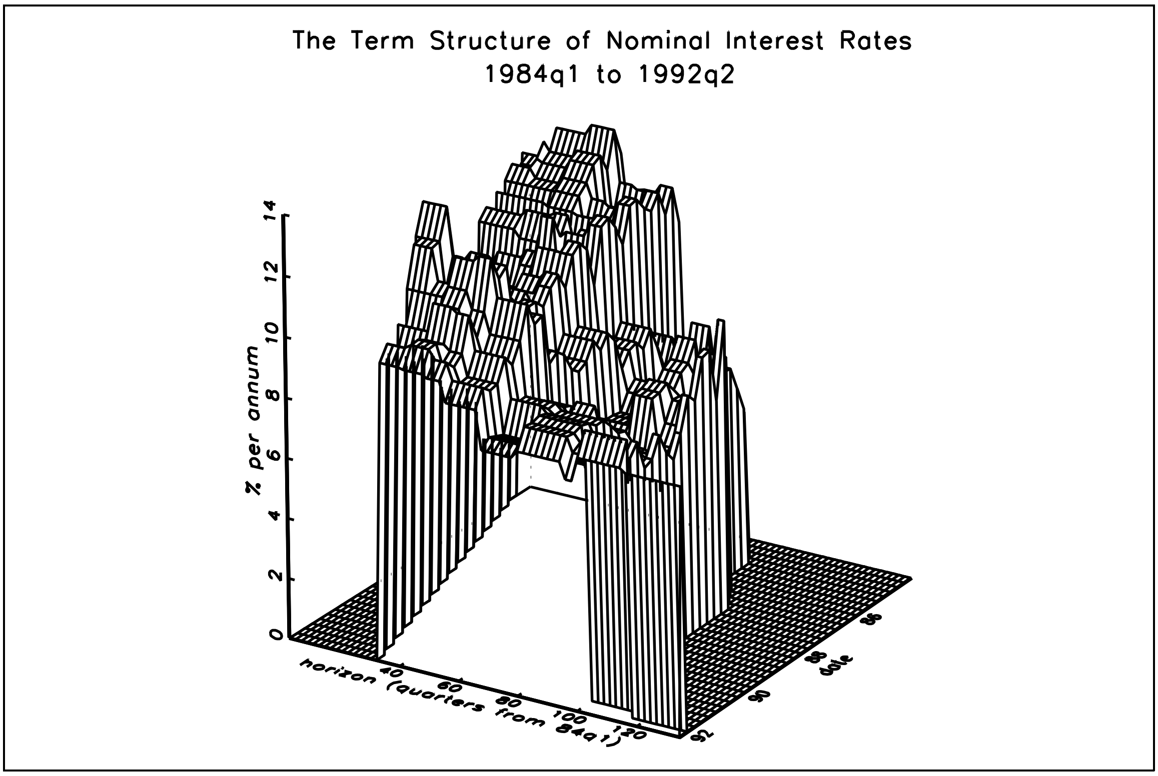

Figure 1

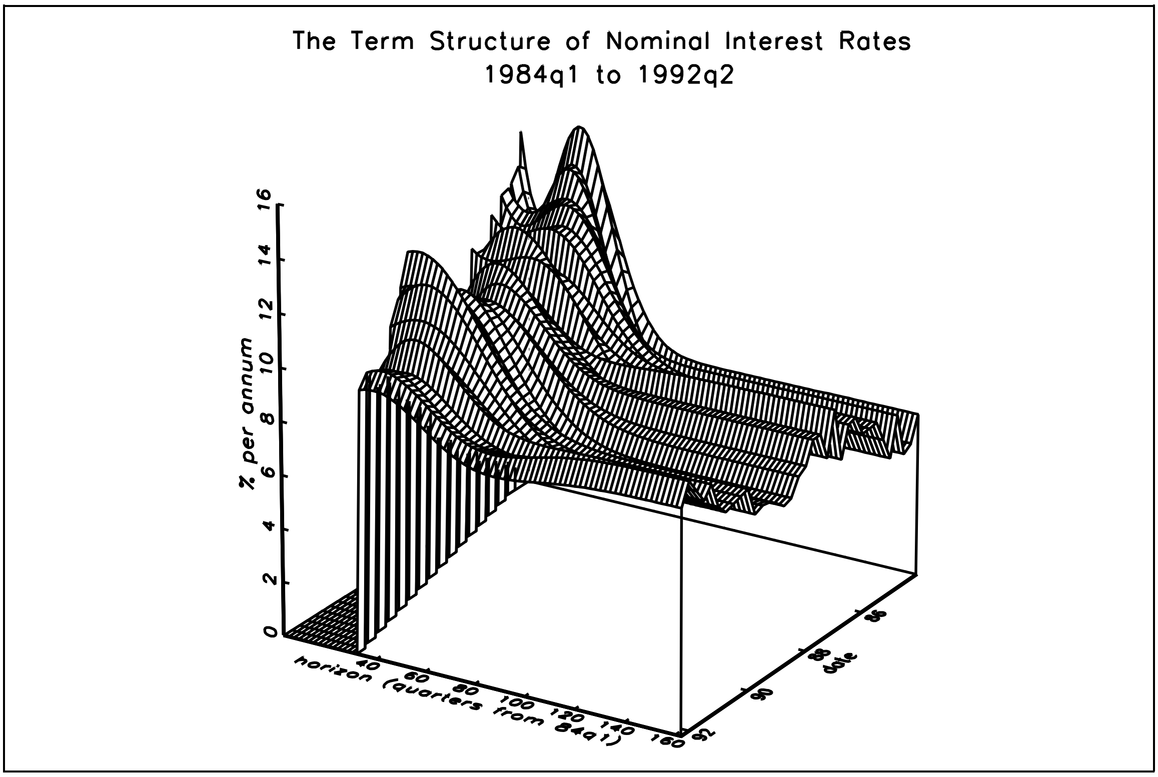

Figure 2

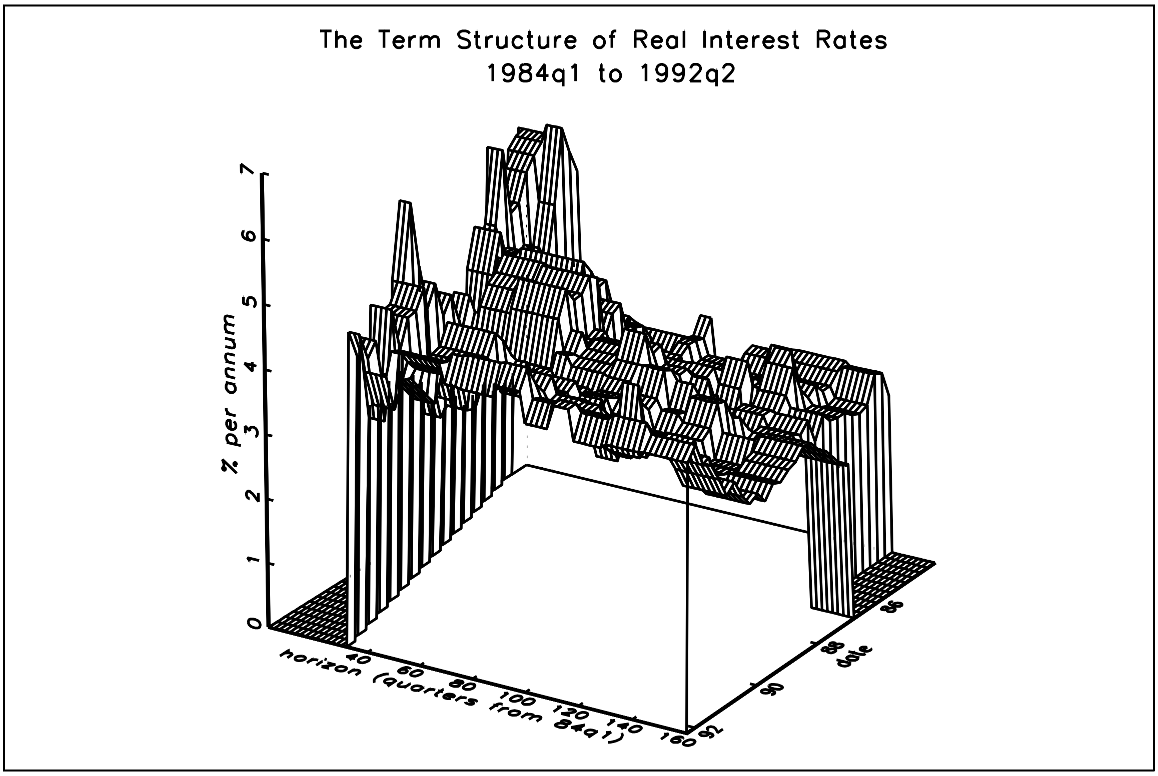

Figure 3

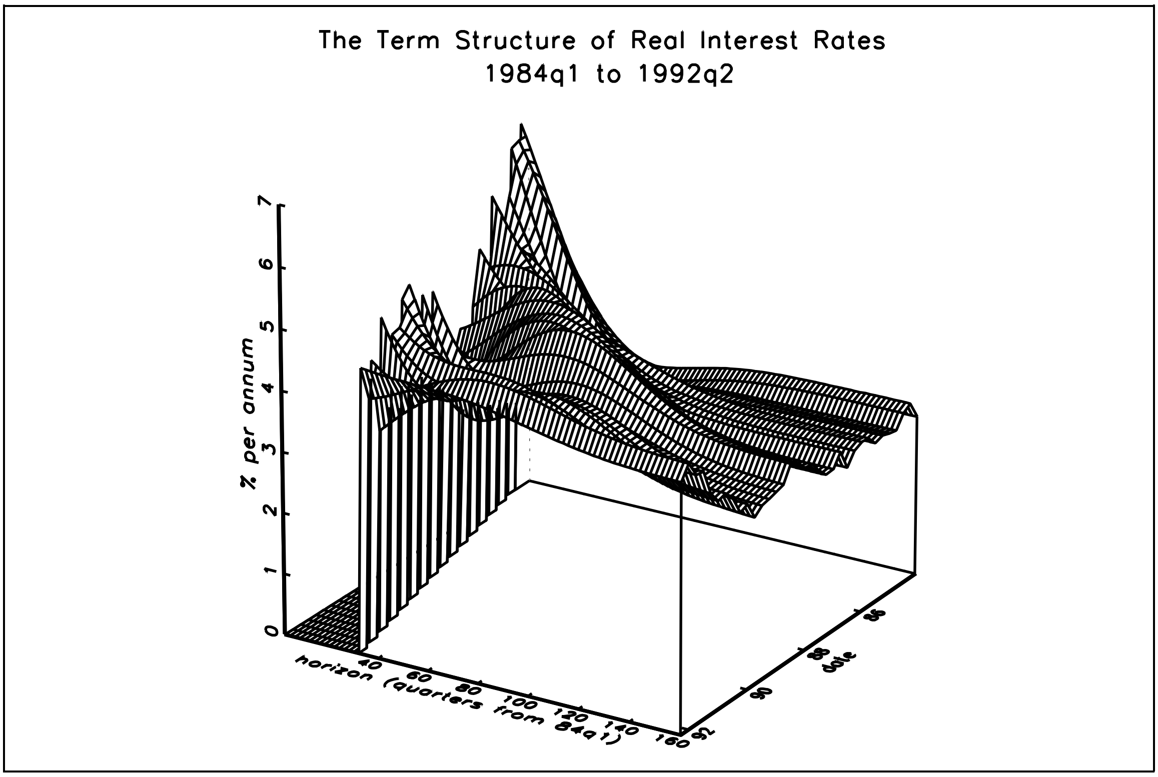

Figure 4

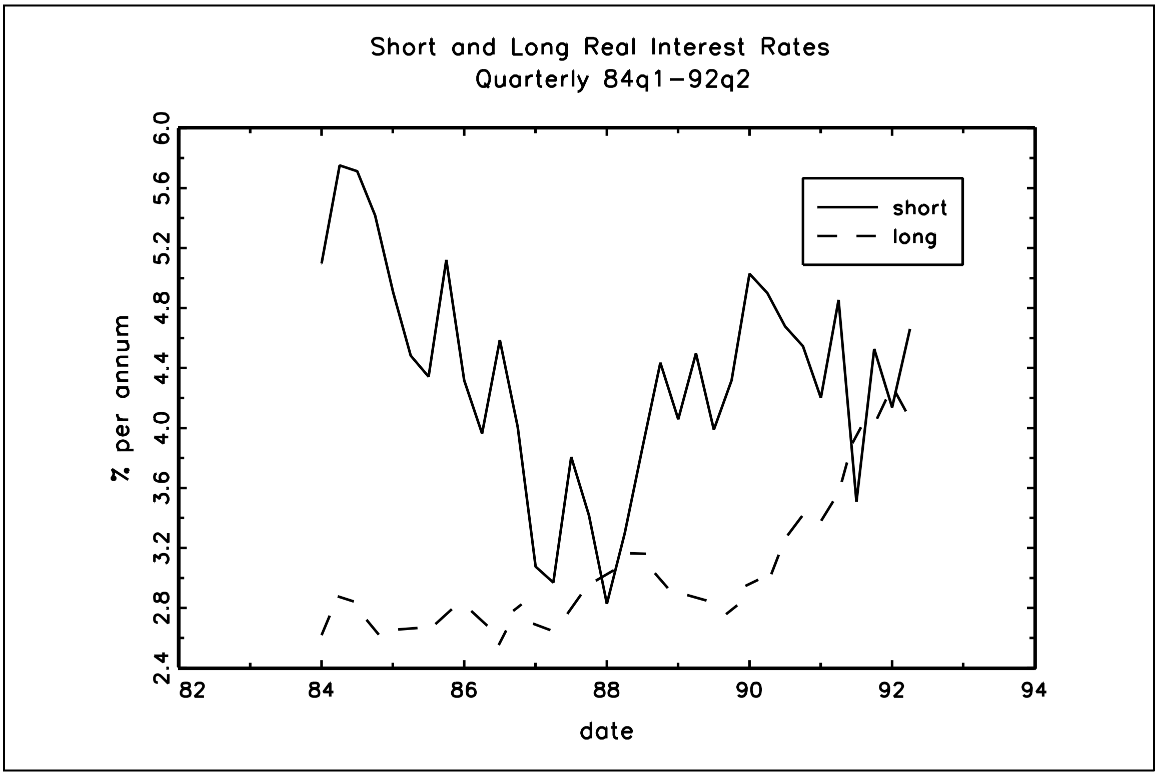

Figure 5

Note
(i) "short" is the average expected rate over the following eight quarters.
(ii) "long" is the expected rate in 2024.

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