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Abstract

We consider the biases that can arise in bias elicitation when expert assessors make random errors. We illustrate the phenomenon for two sources of bias: that due to omitting important variables in a least squares regression and that which arises in adjusting relative risks for treatment effects using an elicitation scale. Results show that, even when assessors' elicitations of bias have desirable properties (such as unbiasedness and independence), the nonlinear nature of biases can lead to elicitations of bias that are, themselves, biased. We show the corrections which can be made to remove this bias and discuss the implications for the applied literature which employs these methods.

Keywords: Bias reduction; Expert elicitation; Elicitation scales; Omitted variable bias

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1 Introduction

There is growing interest in the use of experts to make elicitations about suspected biases in biomedical research (Thompson et al., 2011; Wilks et al., 2011; Turner et al., 2009). Such work sits within a broader literature which considers the role of expert opinion for dealing with uncertainty in statistical research (Oakley and O'Hagan, 2007; Greenland, 2005; Wolpert and Mengersen, 2004; Spiegelhalter and Best, 2003; Smith et al., 1995; Eddy et al., 1992). According to O'Hagan et al. (2006), expert opinion can add significantly more information to a study than can better data analysis or higher quality data sets.

Yet there is acknowledgement that the use of experts can present its own problems. Turner et al. (2009), in an analysis of a health technology appraisal in antenatal care, note that elicitation can be time-consuming, challenging and requires knowledgeable and motivated assessors who have, ideally, been recruited from a range of disciplines. Ioannidis (2011), commenting on bias adjustment for meta-analyses of observational studies, notes that some biases might be difficult to elicit with any degree of accuracy and that the sheer volume of potential biases can make it difficult for any expert to assess them rigorously. Kynn (2008) argues that much recent statistical research using probability elicitations has lagged behind psychological research. She presents a series of recommendations to improve elicitations, including the need to frame elicitation questions appropriately, to decompose the elicitation process into manageable tasks, to check for coherency in the elicitations and, if possible, to repeat the elicitation process at a later date to check the self-consistency of experts.

This note considers the biases that can arise in bias elicitation when expert assessors make random errors in their elicitations. It shows that, even when the elicitation process is carried out by assessors of high quality - we define these as assessors who make unbiased, independent, elicitations of bias - bias elicitation can, itself, be biased. The result occurs when the bias term to be elicited is a nonlinear function of the random errors made by elicitors. The value of the bias may be approximated in a straightforward manner using a Taylor series polynomial of degree two.

We illustrate the phenomenon using two examples: elicitation for the classical 'omitted variables' problem in least squares regression, and the use of elicitation scales to assess bias in relative risk for studies used in a recent National Institute for Clinical Excellence technology appraisal in antenatal care (Turner et al., 2009). Proofs show the adjustments that must be made to remove the bias in the bias elicitation. We conclude by discussing the broader implications of our results for elicitations for other sources of bias that may be encountered in statistical research.

2 Biased elicitation of bias

2.1 Elicitation for omitted variables bias in least squares regression

Assume that the true data generating process (dgp) for an observation in a regression model is:

$$y = \mathbf{x}'\boldsymbol{\beta} + u, \tag{1}$$

where $\mathbf{x}$ is a $K \times 1$ vector of regressors, $\mathbf{x} = [x_1, \ldots, x_K]'$, $\boldsymbol{\beta}$ is a $K \times 1$ vector of parameters, $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]'$, and $u \sim N(0, \sigma_u^2)$. Least squares regression is used to estimate the relationship:

$$y = \tilde{\mathbf{x}}'\tilde{\boldsymbol{\beta}} + v, \tag{2}$$

where the vector $\tilde{\mathbf{x}} = [x_1, \ldots, x_{K-L}]'$ contains a $(K-L) \times 1$ subset of the regressors in $\mathbf{x}$, such that the variables $x_{K-L+1}, \ldots, x_K$ from $\mathbf{x}$ have been incorrectly omitted. $\tilde{\boldsymbol{\beta}} = [\tilde{\beta}_1, \ldots, \tilde{\beta}_{K-L}]'$ is the corresponding $(K-L) \times 1$ parameter vector. Define $\mathbf{z} = [x_{K-L+1}, \ldots, x_K]'$ as the $L \times 1$ vector containing the regressors from Eq. (1) that are incorrectly omitted in Eq. (2). It is the case that $v = \mathbf{z}'\boldsymbol{\alpha} + u$, where $\boldsymbol{\alpha}$ is an $L \times 1$ vector of parameters.

Assume that the statistician observes data with $i = 1, \ldots, N$ observations on y and $\tilde{\mathbf{x}}$, and stack these by row so that:

$$\mathbf{y} = \tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}} + \mathbf{v}. \tag{3}$$

$\mathbf{y}$ is a $N \times 1$ vector of observations on the dependent variable, $\tilde{\mathbf{X}}$ a $N \times (K-L)$ matrix of observations on the regressors. As is well known, as long as the variables in $\mathbf{z}$ are correlated with those in $\tilde{\mathbf{x}}$ and have non-zero correlation with y in Eq. (1), the least squares estimator $\hat{\tilde{\boldsymbol{\beta}}}$ from Eq. (3) will be biased, but more efficient, than that from Eq. (1), as follows:

$$E\left[\hat{\tilde{\boldsymbol{\beta}}}\right] = \boldsymbol{\beta}^{*} + \mathbf{b}\boldsymbol{\beta}^{\dagger} \tag{4}$$

$$\operatorname{var}\left(\hat{\tilde{\boldsymbol{\beta}}}\right) = \sigma_u^2(\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1}. \tag{5}$$

In (4), $\boldsymbol{\beta}^{*} = [\beta_1, \ldots, \beta_{K-L}]'$ (the true parameters for the first $K - L$ variables in Eq. (1)), $\mathbf{b}$ is a $(K - L) \times L$ matrix containing the appropriate regression coefficients from the auxiliary regressions, the regressions of the excluded variables on all of the included variables[1], and $\boldsymbol{\beta}^{\dagger} = [\beta_{K-L}, \ldots, \beta_K]'$, the true parameters for the final L variables in Eq. (1):

$$y = \tilde{\mathbf{X}}'\boldsymbol{\beta}^{*} + \mathbf{Z}'\boldsymbol{\beta}^{\dagger} + u, \tag{6}$$

where $\mathbf{Z}$ is the stacked matrix of the omitted variables.

The bias term for the parameter vector is therefore:

$$E\left[\hat{\tilde{\boldsymbol{\beta}}}\right] - E\left[\hat{\boldsymbol{\beta}}^{*}\right] = \mathbf{b}\boldsymbol{\beta}^{\dagger}. \tag{7}$$

And the difference in efficiency of the two estimators is (Greene, 2003):

$$\operatorname{var}\left(\hat{\tilde{\boldsymbol{\beta}}}\right) - \operatorname{var}\left(\hat{\boldsymbol{\beta}}^{*}\right) = \sigma_u^2(\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1} - \sigma_u^2\left[\tilde{\mathbf{X}}'\tilde{\mathbf{X}} - \tilde{\mathbf{X}}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\tilde{\mathbf{X}}\right]^{-1}. \tag{8}$$

2.1.1 Bias elicitation

A group of M expert assessors, indexed $l = 1, \ldots, M$ and operating independently of each other, is presented with the results of the estimation of Eq. (2) and is asked to make elicitations about potential omitted variable biases in the point

[1] The auxiliary regressions are:

$$\begin{aligned} x_{K-L+1} &= \alpha_{K-L+1} + b_{K-L+1,1}x_1 + \ldots + b_{K,K-L}x_{K-L} + w_{K-L+1} \\ \ldots &= \ldots \\ x_K &= \alpha_K + b_{K,1}x_1 + \ldots + b_{K,K-L}x_{K-L} + w_K. \end{aligned}$$

where the ws are assumed to be zero mean and constant variance random variables. The matrix of coefficients is therefore:

$$\mathbf{b} = \begin{bmatrix} b_{K-L+1,1} & \ldots & b_{K,1} \\ \ldots & \ldots & \ldots \\ b_{K-L+1,K-L} & \ldots & b_{K,K-L} \end{bmatrix}.$$

estimates and their variances. Predicate the analysis on the following assumptions: 1. that each expert correctly judges that the true data generating process is Eq. (1) and therefore that the variables in $\mathbf{z}$ have been incorrectly omitted from Eq. (2); 2. following the advice of Kynn, the omitted variable bias is broken down into its constituent parts and each expert makes an elicitation about the parameters that appear in Eqs. (7) and (8); 3. assessors' elicitations are unbiased (that is, their elicitations are random variables with expected values equal to the true values of the omitted parameters); 4. (for simplicity) all elicitations have common variance (both within and between elicitors); 5. the pooled elicitations of bias are obtained by averaging the assessors' elicitations.

Bias elicitation for $\hat{\hat{\boldsymbol{\beta}}}$

Proposition 1 *Under the dgp given by Eq. (1) and the assumptions made about the assessors' elicitations, the overall elicitation of the bias term in Eq. (7) will, itself, be biased, unless there exists zero correlation between the errors in the assessors' elicitations*

PROOF. Let an expert's elicitation of the bias associated with the parameters in $\mathbf{b}$ and $\boldsymbol{\beta}^{\dagger}$ in Eq. (7) be denoted by the superscript e and the bias-elicited matrices for the expert be $\mathbf{b}^e$ and $\boldsymbol{\beta}^{\dagger e}$, as follows:

$$\mathbf{b}^e = \begin{bmatrix} b^e_{K-L+1,1} & b^e_{K-L+2,1} & \cdots & b^e_{K,1} \\ b^e_{K-L+1,2} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ b^e_{K-L+1,K-L} & b^e_{K-L+2,K-L} & \cdots & b^e_{K,K-L} \end{bmatrix} \tag{9}$$

and

$$\boldsymbol{\beta}^{\dagger e} = \begin{bmatrix} \beta^e_{K-L+1} \\ \beta^e_{K-L+2} \\ \cdots \\ \beta^e_{K} \end{bmatrix}. \tag{10}$$

Given the assumptions about the elicitations, $b^e_{j,k} = b_{j,k} + \epsilon_{j,k}$ and $\beta^e_j = \beta_j + \epsilon_j$, where $j = K-L+1, \ldots, K$ and $k = 1, \ldots, K-L$, where all ϵs are zero-mean random variables with common variance σ^2_ϵ. Substituting these expressions into Eqs. (9) and (10), obtaining the product $\mathbf{b}^e\boldsymbol{\beta}^{\dagger e}$ and applying the expectation operator gives:

$$E\left[\mathbf{b}^e\boldsymbol{\beta}^{\dagger e}\right] = \mathbf{b}\boldsymbol{\beta}^{\dagger} + \begin{bmatrix} \sum_{j=K-L+1}^{K} \mathrm{cov}(\epsilon_{j,1}, \epsilon_j) \\ \sum_{j=K-L+1}^{K} \mathrm{cov}(\epsilon_{j,2}, \epsilon_j) \\ \cdots \\ \sum_{j=K-L+1}^{K} \mathrm{cov}(\epsilon_{j,K-L}, \epsilon_j) \end{bmatrix}. \tag{11}$$

The expectation of the matrix product of the individual assessors' elicitations therefore equals the true bias in Eq. (7), plus a bias term, which will be non-zero unless there exists zero correlation between the errors in the assessors' elicitations. □

Bias elicitation for var $\left(\hat{\hat{\beta}}\right)$

For ease of exposition, we consider a version of Eq. (1) which contains an intercept term and only two regressors, x and w, one of which is incorrectly omitted. The true dgp is therefore:

$$y = \alpha + \beta_x x + \beta_w w + u, \tag{12}$$

and we assume that the incorrectly specified regression omits w. For simplicity, we assume that σ_x^2, σ_w^2 and σ_u^2 are known.

Proposition 2 *Under the dgp given by Eq. (12), the elicitation of the bias associated with the variance of $\hat{\beta}_x$ in a regression which omits w: (a) cannot be separated from elicitation of the bias for the point estimate and (b) will, itself, be biased, even if there exists zero correlation between the errors in the assessors' elicitations.*

PROOF. For the two variable case, Eq. (8) simplifies to:

$$\mathrm{var}\left(\hat{\boldsymbol{\beta}}^*\right) = \mathrm{var}\left(\hat{\hat{\boldsymbol{\beta}}}\right)\phi,$$

where:

$$\phi = \frac{1}{\left(1 - \left(b_{wx}\frac{\sigma_w}{\sigma_x}\right)^2\right)} > 1 \tag{13}$$

Parameter	σ_{ϵ_w}	$\sigma_{\epsilon_{wx}}$	β_w	b_{wx}	σ_u	σ_x	σ_w	$\sigma_{\epsilon_w \epsilon_{wx}}$
Value	1	0.7	10	1	2	1	5	0.16667

Table 1: Simulation parameter values

is the multiplicative bias term. Since we have assumed that σ_x and σ_w are known, the assessors may use their elicitations b_{wx}^e that would have been used for the elicitation of bias for the point estimator (defined in section 2.1.1) and substitute them into (13).

Substitute $b_{wx}^e = b_{wx} + \epsilon_{wx}$ into (13), rearrange and apply the expectation operator to obtain:

$$E[\phi^e] = E\left[\frac{\sigma_w^2}{\sigma_w^2 - (b_{wx} + \epsilon_{wx})^2 \sigma_x^2}\right].$$

Use a Taylor series polynomial of degree two to approximate the term on the right hand side:

$$E[\phi^e] \approx \phi + \sigma_{\epsilon_{wx}}^2 \left(\frac{\sigma_w^2 \sigma_x^2 \left(3\sigma_x^2 (b_{wx})^2 + \sigma_w^2\right)}{\left(\sigma_w^2 - \sigma_x^2 (b_{wx})^2\right)^3} \right). \tag{14}$$

Hence: (a) the appearance of the term b_{wx} from the elicitation for the parameter estimator means that elicitation for the variance cannot be separated from elicitation for the parameter estimator itself; (b) the expectation of the elicited adjustment required to the variance of the estimator in Eq. (12) is, itself, biased. □

2.1.2 Simulation

To illustrate the results, consider a scenario in which ten assessors are asked to make elicitations for omitted variable bias for a parameter estimate and its variance in the two-regressor scenario of Eq. (12), using the methods and assumptions described above. The parameter values we choose for the simulation are summarised in Table 1. Note that the non-zero covariance implies that the assessors' elicitations are not independent. We run the elicitation exercise 100000 times and calculate the eliciation biases that are given in Eqs. (11) and (14).

- Elicitation for bias in $\hat{\tilde{\boldsymbol{\beta}}}_{\boldsymbol{x}}$. Given the result in Eq. (11), we would expect the bias in the elicitations to equal the covariance between ϵ_w and ϵ_{wx}.

From Table 1, the covariance equals 0.16667. The average of the bias in the elicitations across the 100000 simulations is 0.16861, a difference of 1.2%.

- Elicitation for bias in var($\hat{\beta}_x$). The true bias associated with the elicitations for the adjustment required to the variance in (14) is 0.02501. The average of the bias across the simulations is 0.02521, a difference of 0.8%.

2.2 Elicitation for bias in log relative risks using elicitation scales

Turner et al. (2009) consider bias elicitation for a range of biases in a series of studies in antenatal care. To illustrate biased elicitation of bias, we consider elicitation for one possible source of bias in log relative risk from one published study only, using the elicitation scale approach proposed by Turner et al.. In this context, the study parameter estimate of log relative risk, $\hat{\theta}$, is adjusted by adding a value μ^e, a pooled estimate of bias, derived from assessors' elicitations μ_i^e, $i = 1, \ldots, M$, for M assessors (where, once again, we use the superscript 'e' to denote 'elicitation'). The standard error of $\hat{\theta}$, s^2, is adjusted by adding the pooled estimate of the bias for the standard error, $(\sigma^e)^2$.

Assessor i's elicitations for the two bias parameters, μ_i^e and σ_i^e, are given by:

$$\mu_i^e = \frac{\log(a_i^e) + \log(1/b_i^e)}{2} = \frac{1}{2}\log\left(\frac{a_i^e}{b_i^e}\right) \quad \text{and} \tag{15}$$

$$\sigma_i^e = \frac{\log(1/b_i^e) - \log a_i^e}{2} = \frac{1}{2}\log\left(\frac{1}{a_i^e b_i^e}\right), \tag{16}$$

where a_i^e and b_i^e are elicitor i's chosen upper and lower ranges on an elicitation scale for the degree of bias in the intervention group (left hand part of the scale, running from 0.1 (risk much lower in the intervention group) to 1 (no bias)) and the control group (right had part of the scale, running from 1 (no bias) to 0.1 (risk much lower in the control group)).[2] Assume that the true values of the lower and upper ranges that should be selected are $\bar{a}$ and $\bar{b}$ and that assessors, on average, get their elicitations of these two values correct, but with random error with constant variance. Then the elicitations of the range end-points in Eqs. (15) and (16) are given by the random variables $a_i^e = \bar{a} + \epsilon_a$ and $b_i^e = \bar{b} + \epsilon_b$, where

[2] A full description may be found in Turner et al. (2009, pages 29-30). If $a_i^e = b_i^e$, it implies that the elicitor believes that the biases 'cancel'; they are as likely to favour the intervention group as they are the control.

$\epsilon_a \sim f(0, \sigma^2_{\epsilon_a})$ and $\epsilon_b \sim g(0, \sigma^2_{\epsilon_b})$, $\sigma^2_{\epsilon_a} > 0, \sigma^2_{\epsilon_b} > 0$, f and g being two density functions. We assume that the pooled elicitations are obtained by averaging the μ_is and σ_is, that is, $\mu^e = \sum_{i=1}^M \mu_i^e/M$ and $\sigma^e = \sum_{i=1}^M \sigma_i^e/M$.[3]

Proposition 3 *Under the bias elicitation process and assumptions described above, the pooled elicitations of the bias terms μ^e and σ^e will, themselves, be biased.*

Proof. Substitute $a_i^e = \bar{a} + \epsilon_a$ and $b_i^e = \bar{b} + \epsilon_b$ into Eqs. (15) and (16) and calculate approximations to the expected values $E[\mu^e]$ and $E[\sigma^e]$, using a Talyor series polynomial of degree two:

$$E[\mu^e] \approx \frac{1}{2}\log\left(\frac{\bar{a}}{\bar{b}}\right) - \frac{\sigma^2_{\epsilon_a}}{4\bar{a}^2} + \frac{\sigma^2_{\epsilon_b}}{4\bar{b}^2}, \tag{17}$$

$$E[\sigma^e] \approx \frac{1}{2}\log\left(\frac{1}{\bar{a}\bar{b}}\right) + \frac{\sigma^2_{\epsilon_a}}{4\bar{a}^2} + \frac{\sigma^2_{\epsilon_b}}{4\bar{b}^2}. \tag{18}$$

Hence the pooled bias elicitations μ^e and σ^e are themselves biased. □

Eq. (17) shows that the bias in the bias elicitation - the final two terms in the equation - can be positive, zero, or negative. Eq. (16) shows that the bias is strictly positive. Figure 1 plots the bias in bias elicitation term from Eq. (17) using level curves. It shows how this bias in bias elicitation changes as $\bar{a}$ and $\bar{b}$ - the true lower and upper limits on the elicitation scale - change (for the purposes of illustration, we assume that $\sigma^2_{\epsilon_a} = \sigma^2_{\epsilon_a} = 1$). The figure shows that, when $\bar{a} = \bar{b}$, the bias in the bias elicitation equals zero, because our assumptions on $\sigma^2_{\epsilon_a}$ and $\sigma^2_{\epsilon_b}$ mean that the two bias terms in Eq. (17) cancel. Further, we know from Eq. (15) that the true bias equals zero in this situation. Hence, if $\bar{a} = \bar{b}$, which corresponds to there being no difference between intervention and control in terms of: 1. the degree of bias favouring each and 2. the amount of bias adjustment which is required to the elicited bias term, the overall adjustment of $\hat{\theta}$ for elicited bias and its bias equals zero. However, as Figure 1 shows, the more $\bar{a}$ and $\bar{b}$ differ, the greater the bias in the bias elicitation: to the left of the line $\bar{a} = \bar{b}$, the bias term for the elicitation of bias is negative and decreasing; to the

[3]Other pooling methods - for example, using the median, as in Turner et al. (2009), could be considered. We concentrate on averaging to illustrate the general point that the expected value of a nonlinear function of a random variable is not the same as the function of its expected value.

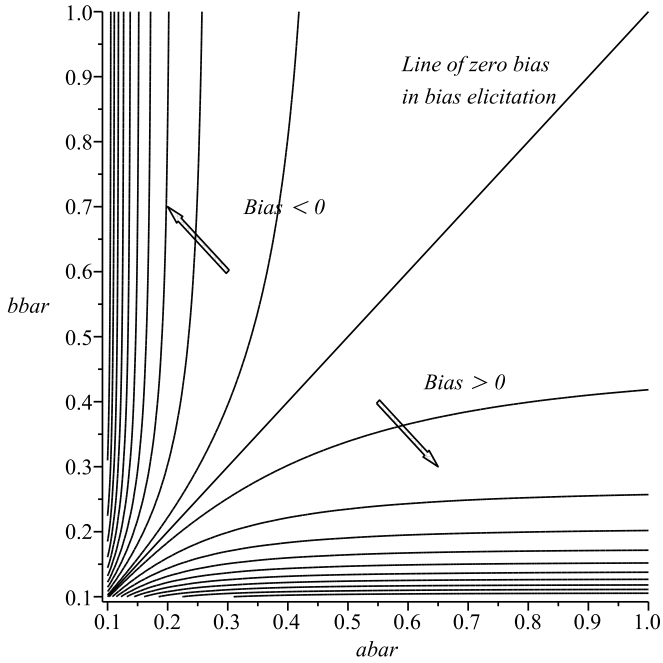

Figure 1: Level curves for the bias in bias elicitation term for μ in Eq. (17): $-\sigma^2_{\epsilon_a}/(4\bar{a}^2) + \sigma^2_{\epsilon_b}/(4\bar{b}^2)$, assuming $\sigma^2_{\epsilon_a} = \sigma^2_{\epsilon_b} = 1$

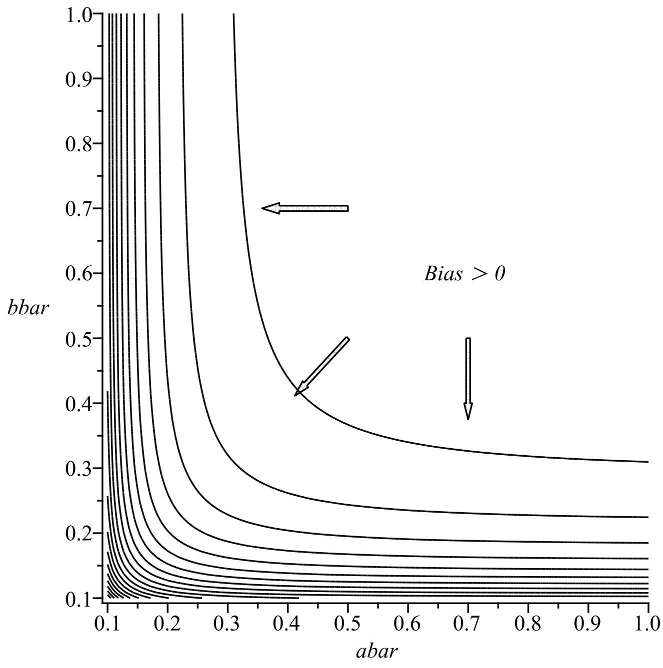

Figure 2: Level curves for the bias in bias elicitation term for σ in Eq. (18): $\sigma^2_{\epsilon_a}/(4\bar{a}^2) + \sigma^2_{\epsilon_b}/(4\bar{b}^2)$, assuming $\sigma^2_{\epsilon_a} = \sigma^2_{\epsilon_b} = 1$

right of the line it is positive and increasing. Hence, deviations of either $\bar{a}$ or $\bar{b}$ from the line $\bar{a} = \bar{b}$ will lead to the bias adjustment being biased. The level curves in Figure 1 show that the severity of the impact of a marginal change in either $\bar{a}$ or $\bar{b}$ is higher the closer one moves to the point (1/10,1/10); for studies with large biases, a relatively small difference between $\bar{a}$ and $\bar{b}$ will lead to a greater absolute bias in the bias adjustment term. This is due to the strictly concave nature of the bias function for a^e and the strictly convex nature of the bias function for b^e. Figure 2 plots the level curves for the bias term for σ. The appendix gives more on the intuition behind these results.

3 Discussion

Our results show that, even when high quality assessors are tasked with making elicitations for bias, the nonlinearities in biases can lead to biased elicitations of bias. For the case of omitted variables in least squares regression, Proposition 1 shows that the bias associated with the assessors' elicitations for point estimators will equal zero only if the elicitations have zero covariance. Proposition 2 shows that bias in bias elicitations for the variance of the point estimators is present even if the errors in the elicitations have zero covariance: it is sufficient that assessors' elicitations are random, that is, that they have non-zero variance (the term σ^2_ϵ in Eq. (14)). In the case of bias elicitation for log relative risk in Turner et al., if it is assumed that assessors make unbiased elicitations of the lower and upper ranges on the elicitation scale, elicitations are biased.

According to Chavalarias (2010), there are 235 potential biases in biomedical research. Since biases and elicitation methods are likely to differ across studies, such analysis could proceed on a case-by-case basis, or a group of researchers could catalogue the main approaches to bias elicitation and the biases therein. We believe that the technique of Taylor series polynomials presented here offers an accessible and elegant approach to approximating and interpreting these biases in bias elicitations.

A Bias for μ using an elicitation scale for log relative risk

To aid exposition, assume that $\bar{b}$ is known to be equal to 1. Then the elicitations of bias for assessor i become, from Eq. (15):

$$\mu_i^e = \frac{1}{2}\log(a_i^e). \tag{19}$$

for $i = 1, \ldots, M$. We assume that the elicitations are random with expectation equal to $\bar{a}$, which equals the true value of the lower range of the elicitation scale. Again, to aid exposition, we assume that the density function for a^e has a clear upper and lower-bound (a^l and a^u). The mapping from a^e to μ^e via the nonlinear function h is shown in Figure 3.

The true bias adjustment is $h(E[a^e]) = h(\bar{a})$. Under the transformation given by h, the expectation of the μ_i^es will be biased downward, that is, $E[h(a^e)] < h(E[a^e]) = h(\bar{a})$, as shown. Other things equal, the further to the left lies the centre of mass of the distribution of a^e, that is, the greater the true bias $\bar{a}$, the greater the divergence between $E[h(a^e)]$ and $h(E[a^e])$, meaning the greater is the bias in the bias elicitation. This is due to the increasing, strictly concave nature of h and explains the shape of the level curves in Figure 1.

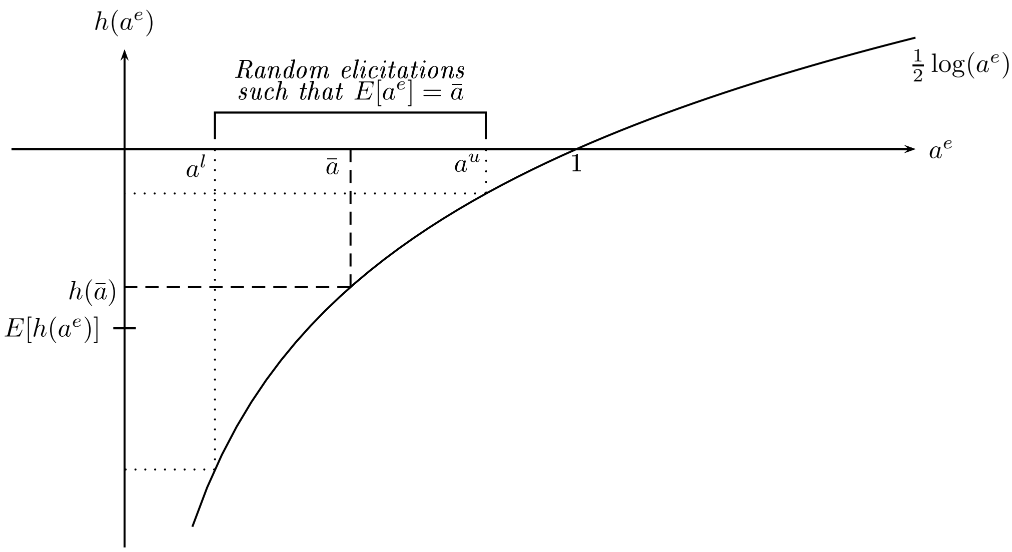

Figure 3: Difference between $\mu = E[h(a^e)]$ and $h(E[a^e]) = h(\bar{a})$ for Eq. (17) and the function $h = (1/2)\log(a^e)$, where we assume $\bar{b}$ is known to equal 1

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