Mixed Duopoly, Merger and Multiproduct Firms

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Abstract

The literature on mergers has extensively analyzed the decision to merge by private firms but it has not considered the decision to merge by private and public firms. We assume that when a private firm and a public firm merge (or when one of them acquires the other), they set up a multiproduct firm in which the government owns an exogenous percentage stake. In this framework, we show that the decision to merge by firms depends on the degree to which goods are substitutes and on the percentage of the shares owned by the government in the multiproduct firm.

Keywords: Mixed Duopoly; Merger; Multiproduct Firm

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1. Introduction

The factors that influence the decision to merge by private firms and the effects of that decision on social welfare have been analyzed by the literature on mergers. For example, Farrell and Shapiro (1990) show that, under cost asymmetries, mergers by private firms can increase social welfare. This is the case when the losses caused by the reduction in market competition are outweighed by the cost savings obtained by transferring the output from less efficient plants to more efficient ones. From the point of view of private firms, strategic mergers entail, among other benefits, market power increases and cost reductions derived from management, production, financing and distribution economies (see, for example, Mueller, 1980).

The literature on mergers has extensively analyzed the decision to merge by private firms but it has not considered the decision to merge by private and public firms. The purpose of this paper is to fill this gap in literature.[1] It must be noted that the literature on public firms usually assumes that private firms maximize profits, public firms maximize social welfare and firms with a mixture of private and public ownership maximize the weighted average of the payoff of the government and their own profit.[2] Therefore, when considering the decision to merge by private and public firms it must be taken into account that these firms have different objective functions.

The above issue is important because of the recent debates concerning the privatization of public firms. Public firms are usually acquired by private firms, which reorganize their organizational structure by setting up multiproduct firms after the acquisition. However, the literature on public firms considers that when a public firm is privatized a new uniproduct private firm arises. In this framework, De Fraja and Delbono (1989) show that when firms

[1] In this paper we will consider as a merger those cases in which one firm acquires another firm.

[2] Matsumura (1998) argues that with the exception of the USA, we can observe many firms with a mixture of private and public ownership. He shows that this type of firm is a reasonable choice for the government in the context of a mixed duopoly with uniproduct firms.

produce a homogeneous good and the number of private firms is high enough, it is socially desirable to privatize the public firm.

In this paper we assume that when a private firm and a public firm merge, they set up a multiproduct firm. The new firm will be neither public nor private, and the government will own an exogenous percentage of the shares in the multiproduct firm. We consider that the private firm and the public firm produced heterogeneous products and that, after merging, the multiproduct firm has two divisions, producing one variety in each division.[3] We show that the decision to merge by a private firm and a public firm crucially depends on the degree to which goods are substitutes and on the percentage of the shares that the government owns in the multiproduct firm.

One example of the question that we want to study is given by the acquisition of SEAT, a Spanish publicly owned automobile manufacturer, by Volkswagen in 1986. After the acquisition, Volkswagen continued producing cars with both brands. Volkswagen is partially owned by the government of Lower Saxony, which owns a 20 percent stake in the firm (Esser, 1998).

Similarly, the French firm Renault acquired a 36.8% equity stake in Nissan Motor and a 22.5% stake in Nissan Diesel (*Renault Presse*, 20-10-99) in 1999. Renault was privatized in 1996, and its share capital is owned 55.8% by private shareholders and 44.2% by the French State. Recently, "Renault and the Romanian State Ownership Fund have just carried out the planned transfer of capital and shares, thus sealing the agreement signed on July 2, 1999. Renault has acquired 51% of the equity capital of Dacia ... Dacia, now a private-sector company, thus becomes a second marque for the Renault group" (*Renault Presse*, 30-9-99).

[3] Though product homogeneity is convenient as a first approximation, empirical evidence shows that most real world markets are characterized by product differentiation. Thus, Cremer *et al.* (1991) argue that introducing heterogeneous products is a significant step towards achieving a better understanding of actual mixed oligopolies.

We consider a duopoly in which one of the two firms is public and the other private (a mixed duopoly). We study whether the public firm and the private one want to merge to set up a multiproduct firm in which the government owns an exogenous percentage of the shares. We show that when the percentage of the shares owned by the government in the multiproduct firm takes an intermediate value and the degree to which goods are substitutes is low enough, both the private firm and the public firm will want to merge. When the percentage of the shares owned by the government takes an intermediate value and the degree to which goods are substitutes is high enough, neither the private firm nor the public firm will want to merge. We also show that when the multiproduct firm is able to obtain economies of scope, the private firm and the public firm want to merge for a broader range of the values of the percentage of the shares owned by the government and the degree to which goods are substitutes.

The paper is organized as follows. Section two presents the model. Section three studies the results. Section four analyzes the decision to merge by firms when there are no economies of scope. Section five analyzes the decision to merge by firms when there are economies of scope, and conclusions are drawn in section six.

2. The model

To study the decision to merge by a public firm and a private one, we consider an economy made up of a monopolistic sector and a competitive numeraire one. The monopolistic sector comprises two firms producing a differentiated good: one firm is public and the other firm is private, denoted by 0 and 1, respectively.[4] On the consumption side, there is a continuum of consumers of the same type whose utility function is linear and separable in the numeraire

[4] Bös (1986, p.231) points out that when public and private firms exist in the economy, "Partial analysis must be centered on duopolistic or oligopolistic structures where one of the participants is a public enterprise. The great advantage of such an analysis is that it reveals the whole range of possible outcomes as depending on the different possible reactions of the economic agents considered".

good. The representative consumer maximizes $U(q_0,\ q_1)$ - p_0q_0 - p_1q_1, where $q_i \geq 0$ is the amount of the good i and p_i is its price (i = 0, 1). The function $U(q_0,\ q_1)$ is assumed to be quadratic, strictly concave and symmetric in q_0 and q_1:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}(q_1^2 + 2bq_0q_1 + q_0^2),\ 1>b\geq 0,^{5}$$

where parameter b measures the degree to which goods are substitutes. Then, inverse demand functions are given by:

$$p_i = a - q_i - b\,q_j,\quad i \neq j;\ \ i, j = 0, 1.$$

As is usually assumed (see, for example, De Fraja and Delbono, 1989), firms have identical technologies, represented by the quadratic cost function:

$$C(q_i) = F + \frac{1}{2}kq_i^2,\ k\geq 1.5,\ i = 0, 1,^{6} \qquad (1)$$

where F=0 with no loss of generality, since entry decisions are not considered. The profit function of firm i is given by:

$$\pi_i = q_i\,(a - q_i - bq_j) - \frac{1}{2}kq_i^2,\ i \neq j;\ i,\ j = 0, 1. \qquad (2)$$

[5] We consider a simplified version of the model used by Vives (1984). Following Vives, we assume that b<1 to assure that the function $U(q_0, q_1)$ is strictly concave.

[6] To eliminate non relevant cases we consider that parameter k is high enough, that is, $k\geq 1.5$.

The private firm chooses the output level, q_1, that maximizes its profit given by (2) for i=1 and j=0. The public firm chooses the output level, q_0, that maximizes social welfare. We measure social welfare as the sum of consumer surplus (denoted by CS) and producers' surplus. Therefore, social welfare is given by:

$$W = CS + \pi_0 + \pi_1, \tag{3}$$

where consumer surplus is given by:

$$CS = \frac{1}{2}(q_0^2 + q_1^2) + bq_0q_1. \tag{4}$$

The public and the private firms can decide to merge and set up a multiproduct firm with two divisions, 0 and 1.[7] If they merge, we assume that the government owns s percent of the shares in the multiproduct firm. Following Matsumura (1998) we consider that a firm which is jointly owned by the public and private sectors maximizes the weighted average of the payoff of the government and its own profit. Therefore, in this case, the multiproduct firm chooses the output level of its two divisions, q_0 and q_1, that maximizes:

$$V = s\,W + (1\text{-}s)\,(\pi_0 + \pi_1), \tag{5}$$

where π_0 and π_1 are given by (2) and W is given by (3).

We propose a two stage game with the following timing. First, the private and public firms decide whether to merge and set up a multiproduct firm whose ownership is shared by the

[7] We do not consider that two independent private firms arise in case of privatization. De Fraja and Delbono (1989) show that the government prefers a mixed duopoly rather than two private firms when goods are homogenous. This result holds when goods are not perfect substitutes.

private and the public sectors. Second, the uniproduct firms or the multiproduct firm make production decisions. To obtain a subgame perfect equilibrium, the game is solved backwards.

3. Results

Given that firms have to decide whether to merge or not, we have two cases. First, the public firm and the private one do not merge; in this case there is a mixed duopoly denoted by the superscript *G*. Second, firms merge and set up a multiproduct firm in which the government owns *s* percent of the shares; we denote this case by the superscript *S*.

We solve first the second stage of the game when there is a mixed duopoly. The private firm chooses the value of q_1 that maximizes (2), for i=1 and j=0. The public firm chooses the value of q_0 that maximizes (3). Solving these two problems simultaneously, we get:

$$q_0^G = \frac{a\,(2-b+k)}{2-b^2+3k+k^2},\ q_1^G = \frac{a\,(1-b+k)}{2-b^2+3k+k^2},\ p_0^G = \frac{a\,k(2-b+k)}{2-b^2+3k+k^2},\ p_1^G = \frac{a\,(1+k)\,(1-b+k)}{2-b^2+3k+k^2},$$

$$\pi_0^G = \frac{a^2\,k\,(2-b+k)^2}{2\,(2-b^2+3k+k^2)^2},\ \pi_1^G = \frac{a^2\,(2+k)\,(1-b+k)^2}{2\,(2-b^2+3k+k^2)^2},$$

$$W^G = \frac{a^2\,(7+2\,b^3+15k+10k^2+2k^3-2b^2(1+k)-2b\,(3+4k+k^2))}{2\,(2-b^2+3k+k^2)^2}.$$

It is easy to see that the output of the public firm is greater than that of the private one independently of the degree to which goods are substitutes. Thus, in equilibrium, the public firm has higher marginal and total costs than the private one. Social welfare strictly decreases

with parameter *b*, since the industry's output (and thus the consumer surplus) and profit decrease also with this parameter.[8]

When the two firms merge, they set up a multiproduct firm that chooses the values of q_0 and q_1 that maximize (5). Thus, we get:

$$q_0^S = q_1^S = \frac{a}{(2-s)(1+b)+k}, \; p_0^S = p_1^S = \frac{a((1-s)(1+b)+k)}{(2-s)(1+b)+k},$$

$$\pi^S = \frac{a^2(2(1-s)(1+b)+k)}{((2-s)(1+b)+k)^2}, \; W^S = \frac{a^2((3-2s)(1+b)+k)}{((2-s)(1+b)+k)^2},$$

where $\pi^S$is the total profit of the multiproduct firm. The social welfare and the output of the industry increase with parameter *s* while the profit of the multiproduct firm decreases with this parameter. The reason is that the greater the value of parameter *s* is, the greater the weight of social welfare is in the objective function of the multiproduct firm. As a result, when parameter *s* increases, the output level of the two divisions and the consumer surplus increase, which outweighs the decrease in the profit of the multiproduct firm.

The multiproduct firm internalizes that divisions 0 and 1 produce substitute goods. Thus, as competition between the divisions increases with the degree to which goods are substitutes, the greater parameter *b* is, the lower the output level of the two divisions and the consumer surplus are. Therefore, the output of industry, the profit of the multiproduct firm and social welfare decrease with parameter *b.*

[8] It can be shown that the output of both firms decreases with parameter *b*. The inverse demand function of good *i* is given by $p_i = a - q_i - b\,q_j$; therefore, for a given q_j, if *b* increases $a\text{-}bq_j$ decreases and, thus, the market size of good *i* also decreases. As a result, q_i decreases if *b* increases.

4. Analysis of the decision by firms to merge

In order to analyze the decision by firms to merge it must be pointed out that the objective functions of the firms (see (2), (3) and (5)) are influenced, in a different way, by parameters *b* and *s*. Thus, we can identify two effects, the s-effect and the b-effect, which we describe below.

The s-effect only affects the multiproduct firm. When parameter *s* increases, for a given value of parameter *b*, the weight of social welfare in the objective function of the multiproduct firm is greater and, thus, output level, consumer surplus and social welfare increase, while profit decreases.

The second effect is the b-effect: for a given value of parameter *s*, the greater the degree to which goods are substitutes is, the lower the firms' output level is. This effect is made up of two further effects. On the one hand, when parameter *b* increases, the market size of each good, and thus the output level, decreases. On the other hand, when parameter *b* increases, the multiproduct firm and the public one internalize that goods are substitutes (thus reducing their output level). Both effects influence output level in the same way and thus, for a given value of parameter *s*, when parameter *b* increases the private firm reduces its output level less than the multiproduct firm and the public one.

We shall now analyze whether firms want to merge and set up a multiproduct firm. First we consider whether the private firm wants to merge or not. In case of merging the private firm will own 1-*s* percent of the shares in the multiproduct firm. Therefore, the owners of the private firm will accept the merger if the profit that they will obtain in the multiproduct firm, $(1-s)\pi^S$, is greater than the profit obtained by the private firm in the mixed duopoly, π_1^G. Let s_P denote the value of parameter *s* such as $(1-s)\pi^S=\pi_1^G$; this value, shown in the appendix, increases with parameter *b*.

Proposition 1. $(1\text{-}s)\pi^S > \pi_1^G$ *if and only if* $s < s_P$, where $0<s_P<1$.

This proposition shows that the private firm will want to merge with the public firm if, after the merger, the shareholders of the private firm own a high enough percentage of the shares in the multiproduct firm (i. e., if $s<s_P$).[9] When s is high enough (i. e., if $s \geq s_P$) the private firm would prefer not to merge. This result is shown in figure 1 for a given value of parameter k. This figure show how the decision on merging by the private firm depends on parameters b and s.

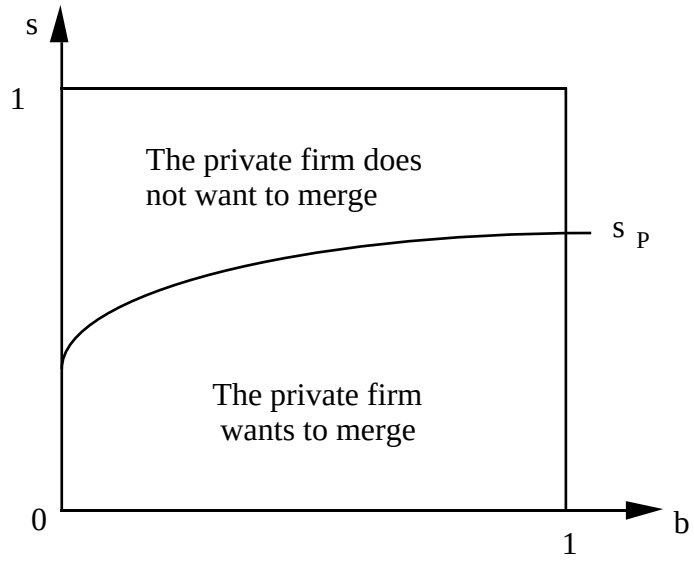

Figure 1. Illustration of proposition 1 for a given value of parameter k.

When parameter s is low or high enough, only the s-effect is important in explaining the result. When parameter s is low enough, the shareholders of the private firm will always want the merger since they will have a high enough percentage of the shares in the multiproduct firm.

[9] It is easy to see that, for the values of parameter s at which the private firm wants to merge with the public firm, the total profit of the industry is greater with a multiproduct firm than with a mixed duopoly.

When parameter *s* is high enough, the result is reversed since they will have a low enough percentage of the shares.

When parameter *s* takes an intermediate value, parameter *b* is important in explaining the decision on merging by the private firm. If parameter *b* is low (high) enough, given an intermediate value of parameter *s*, the private firm does not want (wants) to merge. If *b*=0, goods are independent in demand; as the private firm is a monopolist and does not take social welfare into account, we have that $(1\text{-}s)\pi^S<\pi_1^G$. If *b* tends to 1, the degree to which goods are substitutes is high. Thus, the profit of the private firm is very low since the public firm is very aggressive and competition in the product market is strong. On the other hand, the multiproduct firm internalizes that the degree to which goods are substitutes is high, reducing the output of the two goods to decrease market competition. As a result, $(1\text{-}s)\pi^S>\pi_1^G$. Up till now we have only considered two extreme cases (*b*=0 and *b* tends to 1); if we consider other values of parameter *b*, it must be noted that when *b* increases, for a given value of parameter *s*, π_1^G decreases more than $(1\text{-}s)\pi^S$. Thus, when *b* is low (high) enough, for an intermediate value of parameter *s*, π_1^G is greater (lower) than $(1\text{-}s)\pi^S$.

The private firm is indifferent to merging for $s=s_P$, and this value increases with parameter *b*. To explain this result, both the s-effect and the b-effect must be taken into account. The s-effect implies that $(1\text{-}s)\pi^S$ decreases with parameter *s* while π_1^G does not change with this parameter. The b-effect implies that if parameter *b* decreases, for a given value of parameter *s*, π_1^G increases more than $(1\text{-}s)\pi^S$. As a result, if *b* decreases, it is necessary that the shareholders of the private firm receive a higher percentage of the shares in the multiproduct firm (i. e., parameter *s* must decrease) to be indifferent to merging.

Now we are going to analyze whether the government prefers a multiproduct firm, in which it owns *s* percent of the shares, to a mixed duopoly. The government prefers a multiproduct firm to a mixed duopoly if $W^S > W^G$. Let s_W denote the value of parameter *s* such that $W^S = W^G$; this value, shown in the appendix, increases with parameter *b*.

Proposition 2. $W^S > W^G$ *if and only if* $s > s_W$, where $0 < s_W < 1$.

This proposition shows that the government prefers a multiproduct firm to a mixed duopoly only when it owns a high enough percentage of the shares in the multiproduct firm (i. e., if $s > s_W$). Proposition 2 is shown in figure 2 for a given value of parameter *k*. This figure shows how the decision on merging by the public firm depends on parameters *b* and *s*.

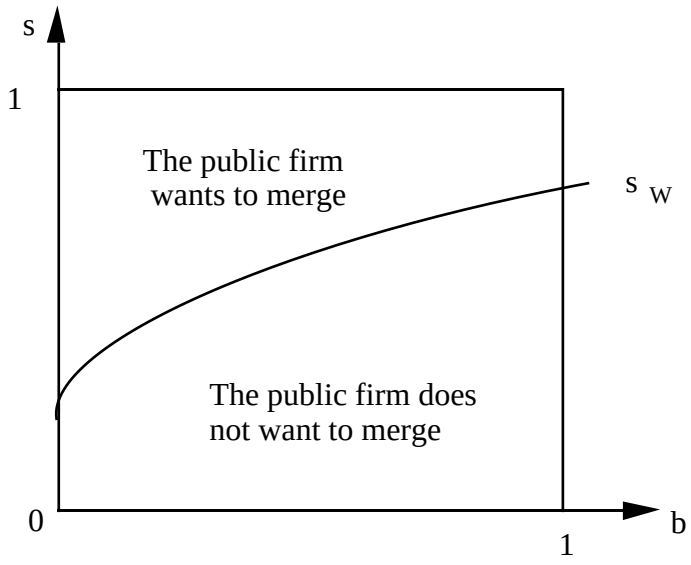

Figure 2. Illustration of proposition 2 for a given value of parameter *k*.

As in the preceding case, when parameter *s* is low or high enough, only the s-effect is important in explaining the decision by the government on merging. When parameter *s* is high enough, the public firm will always want to merge, since the government will have a high enough percentage of the shares in the multiproduct firm. When parameter *s* is low enough, the result is reversed, since the percentage of the shares owned by the government is low enough.

When parameter *s* takes an intermediate value, parameter *b* is important in explaining the decision by the government on merging. If parameter *b* is high (low) enough, given an intermediate value of parameter *s*, the public firm does not want (wants) to merge. If b=0, goods are independent in demand. The private firm is a monopolist and does not take social welfare into account. As the public firm cannot influence the market served by the private firm, the government prefers a multiproduct firm to a mixed duopoly. If *b* tends to 1, the degree to which goods are substitutes is high. Thus, social welfare is high in the case of a mixed duopoly since the public firm is very aggressive and competition in product market is strong; as a result, the government prefers a mixed duopoly to a multiproduct firm. If we consider other values of parameter *b*, it must be noted that when *b* increases, for a given value of parameter *s*, W^S decreases more than W^G. Thus, when *b* is high (low) enough, for an intermediate value of parameter *s*, the public firm does not want (wants) to merge.

The public firm is indifferent to merging for $s=s_W$, and this value increases with parameter *b*. To explain this result, both the s-effect and the b-effect must be taken into account. The s-effect implies that W^S increases with parameter *s* while W^G does not change with this parameter. The b-effect implies that if parameter *b* increases, for a given value of parameter *s*, W^S decreases more than W^G. As a result, if *b* increases, the government must obtain a higher percentage of the shares in the multiproduct firm to be indifferent to allowing the merger.

In order to determine the equilibrium of the game (i. e., whether the private and public firms merge or not) we have to compare the values s_P and s_W. Given that this comparison is very complex, we shall compare s_P and s_W for a given value of parameter *k*.

Lemma 1. *For* $k=2$, $s_W \geq s_P$ *if and only if* $b \geq b^*=0.4432$.

This lemma shows that s_W is lower than s_P for low enough values of parameter b, while the result is reversed for high enough values of parameter b. The result obtained in lemma 1 holds for all k, as shown in figure 3; this figure shows that b^* decreases with parameter k.

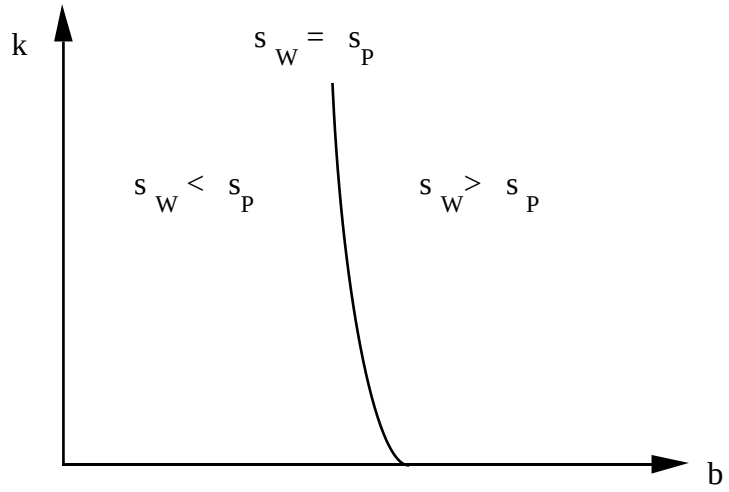

Figure 3. Comparison of s_P and s_W.

Taking into account lemma 1 and propositions 1 and 2, we have the following result.

Proposition 3. *For k=2, the private firm and the public firm will merge when* $b<b^*$ *and* $s_W<s<s_P$.[10]

This proposition shows that the private firm and the public one want to merge, setting up a multiproduct firm, for $s_W<s<s_P$. This is the case when the degree to which goods are substitutes is low enough ($b<b^*$). From lemma 1 and propositions 1 and 2 we can illustrate proposition 3 in figure 4.

[10] The result obtained in proposition 3 holds for values of parameter k other than 2.

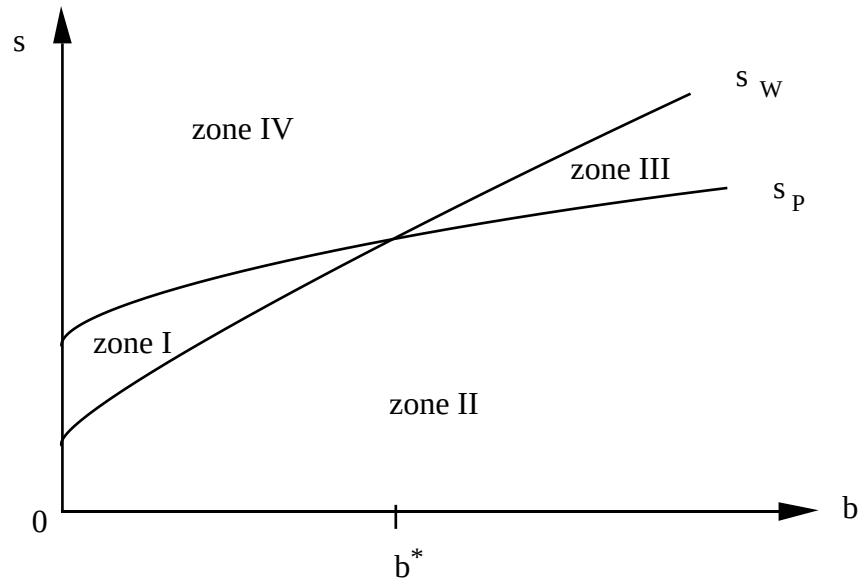

Figure 4. Illustration of proposition 3.

In figure 4 we can distinguish four zones. In zone I, both the private firm and the public one prefer to merge. In zone II, the private firm wants to merge but the government prefers a mixed duopoly. In zone III, neither the private firm nor the public firm want to merge. Finally, in zone IV the government prefers a multiproduct firm but the private firm does not want to merge. The results shown in zones I to IV in figure 4 are explained by the interaction of the s-effect and the b-effect. In order to explain these results we will assume first that parameter *b* is given and only parameter *s* can change. Subsequently we will also change the value of parameter *b*.

We shall assume firstly that b=0 and only parameter s can change (see table 1); thus, the results obtained are due to the s-effect only. In this case, we will analyze whether the public firm and the private one want to merge or not. As we have seen, the government allows the merger if $W^S > W^G$, whereas the private firm wants to merge if $(1-s)\pi^S > \pi_1^G$.

When s=0, $W^G > W^S$, since the multiproduct firm is private whereas there is a public firm in the mixed duopoly. When parameter s increases, the weight of social welfare in the objective

function of the multiproduct firm is higher and, as a result, W^S increases. Given that W^G does not depend on parameter *s*, if this parameter is high enough (i. e., if $s>0.3557=s_W(b=0)$), W^S will be greater than W^G and the government will prefer a multiproduct firm. However, when $s<s_W(b=0)$ the government will prefer a mixed duopoly.

		$s=0$	$s=0.2$	$s=0.4$	$s=0.6$	$s=0.8$
W^S	$b=0$	$0.3125\ a^2$	$0.3185\ a^2$	$0.3240\ a^2$	$0.3287\ a^2$	$0.3320\ a^2$
	$b=0.2$	$0.2892\ a^2$	$0.2958\ a^2$	$0.3019\ a^2$	$0.3071\ a^2$	$0.3109\ a^2$
W^G	$b=0$	$0.3229\ a^2$	$0.3229\ a^2$	$0.3229\ a^2$	$0.3229\ a^2$	$0.3229\ a^2$
	$b=0.2$	$0.\ 3033\ a^2$	$0.\ 3033\ a^2$	$0.\ 3033\ a^2$	$0.\ 3033\ a^2$	$0.\ 3033\ a^2$
π_1^G	$b=0$	$0.125\ a^2$	$0.\ 125\ a^2$	$0.\ 125\ a^2$	$0.\ 125\ a^2$	$0.\ 125\ a^2$
	$b=0.2$	$0.1096\ a^2$	$0.1096\ a^2$	$0.1096\ a^2$	$0.1096\ a^2$	$0.1096\ a^2$
$(1-s)\pi^S$	$b=0$	$0.25\ a^2$	$0.1994\ a^2$	$0.1481\ a^2$	$0.0968\ a^2$	$0.0468\ a^2$
	$b=0.2$	$0.2272\ a^2$	$0.1812\ a^2$	$0.1343\ a^2$	$0.0874\ a^2$	$0.0419\ a^2$

Table 1. Values of the objective functions of the government and the private firm for $k=2$.

When $s=0$, the private firm wants to merge, since it will own all the shares in the multiproduct firm. When parameter *s* increases, the s-effect implies that $(1-s)\pi^S$ decreases while π_1^G does not change. Then, if parameter *s* is high enough (i. e., if $s\geq0.4900=s_P(b=0)$), the private firm will not want to merge.

The reason why $s_P(b=0)$ is greater than $s_W(b=0)$ is the following. When $s=0$, the difference between π_1^G and $(1-s)\pi^S$ is great, and therefore parameter *s* has to increase a great deal in order for π_1^G to be equal to $(1-s)\pi^S$. However, the difference between W^G and W^S is small, and therefore only a small increase in parameter *s* is necessary for W^G to be equal to W^S. As a result, when $b=0$ the public and private firms want to merge for the values of parameter *s* such

that $0.3557<s<0.4900$ (zone I). When $s\geq0.4900$ only the public firm wants to merge (zone IV), while when $s\leq0.3557$ only the private firm wants to merge (zone II).

Next we shall consider that $b=0.2$ (see table 1) to introduce the b-effect into the explanation of the results. Given that parameter b is still small, the b-effect has a small weight in the explanation of the results; they are explained by the s-effect. When $b=0.2$ it is obtained that $W^S>W^G$ for $s>0.4065=s_W(b=0.2)$ and $(1-s)\pi^S>\pi_1^G$ for $s<0.4973=s_P(b=0.2)$. Then, $s_W(b=0.2)$ is still smaller than $s_P(b=0.2)$, but the difference between the two values is less than when $b=0$. The reason is that when parameter b increases W^S decreases more than W^G, for a given value of parameter s. As a result, parameter s must increase strongly in order for the government to accept the merger. However, for a given value of parameter s, when parameter b increases, π_1^G decreases more than $(1-s)\pi^S$; therefore, the private firm will accept the merger even with a small increase in parameter s. Then, when $b=0.2$ both the private and public firms want to merge for the values of parameter s such that $0.4065<s<0.4973$ (zone I). When $s\geq0.4065$ only the public firm wants to merge (zone IV) while when $s\leq0.4973$ only the private firm wants to merge (zone II).

We have seen in the above cases that when the value of parameter b is low, the results are due to the s-effect. However, when parameter b is high enough, the b-effect dominates the s-effect. This is the case, for example, when $b=0.6$ (see table 2); the value of parameter b is now so high that $s_W(b=0.6)$ is greater than $s_P(b=0.6)$. As parameter b is high enough, the value of parameter s necessary for the public firm to want to merge is greater than that allowing the private firm to want to merge. In this case, $W^G < W^S$ for $s>0.5618$ and $\pi_1^G \geq (1-s)\pi^S$ for $s\geq0.5396$. As a result, neither the public firm nor the private one want to merge for

0.5396<s<0.5618 (zone III). When s≥0.5618 only the public firm wants to merge (zone IV), while when s≤0.5396 only the private firm wants to merge (zone II).

		s=0	s=0.2	s=0.4	s=0.6	s=0.8
W^S	b=0.6	0.2514 a^2	0.2586 a^2	0.2654 a^2	0.2714 a^2	0.2759 a^2
	b=0.9	0.2288 a^2	0.2362 a^2	0.2432 a^2	0.2495 a^2	0.2543 a^2
W^G	b=0.6	0. 2703 a^2	0. 2703 a^2	0. 2703 a^2	0. 2703 a^2	0. 2703 a^2
	b=0.9	0. 2499 a^2	0. 2499 a^2	0. 2499 a^2	0. 2499 a^2	0. 2499 a^2
π_1^G	b=0.6	0.0850 $a2$	0.0850 a^2	0.0850 a^2	0.0850 a^2	0.0850 a^2
	b=0.9	0.0704 a^2	0.0704 a^2	0.0704 a^2	0.0704 a^2	0.0704 a^2
$(1\text{-}s)\pi^S$	b=0.6	0.1923 a^2	0.1531 a^2	0.1131 a^2	0.0729 a^2	0.0343 a^2
	b=0.9	0.1724 a^2	0.1372 a^2	0.1010 a^2	0.0648 a^2	0.0301 a^2

Table 2. Values of the objective functions of the government and the private firm for k=2.

When we consider a value of parameter b greater than 0.6, for example b=0.9 (see table 2), the b-effect becomes stronger. Thus, the interval of values of parameter s in which the public firm and the private firm do not want to merge, 0.5688<s<0.6130 (zone III), is greater than when we considered b=0.6. When s≥0.6130 only the public firm wants to merge (zone IV), while when s≤0.5688 only the private firm wants to merge (zone II).

5. The decision to merge by firms when there are economies of scope

One advantage of multiproduct firms is that they are able to obtain economies of scope. The term economies of scope (see Panzar and Willig, 1981) is used to describe a basic property of production: cost saving which results from scope rather than scale. Chandler (1990, p. 41) argues that "the most common stimulus to diversification was the potential for economies of scope existing in an enterprise's major functional units-production, distribution, and research. At most enterprises the first step toward such product diversification was the development of a full line that exploited the firm's facilities and capabilities in all the three major activities".

When a private and a public firm merge, setting up a multiproduct firm, they are able to obtain economies of scope. An example is given by the partial acquisition of Nissan by Renault. They will implement synergies covering the whole scope of their activities, particularly in the areas of purchasing, product strategy and research. They plan to develop a common line of platforms and powertrains. These synergies will also apply to the strong complementary nature of their geographical locations. *Renault Presse* (20-10-99) points out that "Work already done by the Cross Company Teams confirms the transaction rationale and feasibility of the objective of overall savings of $ 3.3 billion (3 billion euros) for the 2000-2002 period alone. In the longer term, these synergies would reach $ 3 billion each year from 2005 onwards".

We can introduce economies of scope into our model by assuming that the cost function of the multiproduct firm is:

$$C(q_0, q_1) = F + \frac{1}{2} kq_0^2 + \frac{1}{2} kq_1^2 - dq_0q_1,\ 0 < d < k.$$

We denote by $s_P(d)$ and $s_W(d)$ the values of parameter s such that the private firm and the public firm, respectively, are indifferent to merging when there are economies of scope. The values of $s_P(d)$ and $s_W(d)$ are set down in the appendix. The government prefers a multiproduct firm to a mixed duopoly if $s>s_W(d)$ and the private firm wants to merge if $s<s_P(d)$. In order to determine the equilibrium of the game we have to compare the values $s_P(d)$ and $s_W(d)$. Given that this comparison is very complex, we shall compare $s_P(d)$ and $s_W(d)$ for k=2. It can be shown that $s_W(d)$ strictly decreases with d for all b and that $s_P(d)$ strictly increases with d for all b.

Proposition 4. *For k=2, the private firm and the public firm will merge if and only if* $s_W(d)<s<s_P(d)$, *where* $s_W(d)< s_W$ *and* $s_P< s_P(d)$.[11]

This proposition shows that when there are economies of scope the private firm and the public one want to merge and set up a multiproduct firm only when $s_W(d)<s<s_P(d)$. Thus, for a given value of parameter k, the public firm and the private firm want to merge for a broader range of values of parameters b and s than when there are no economies of scope. The comparison of propositions 3 and 4 is shown in figure 5 for k=2, d=0 and d=0.2.

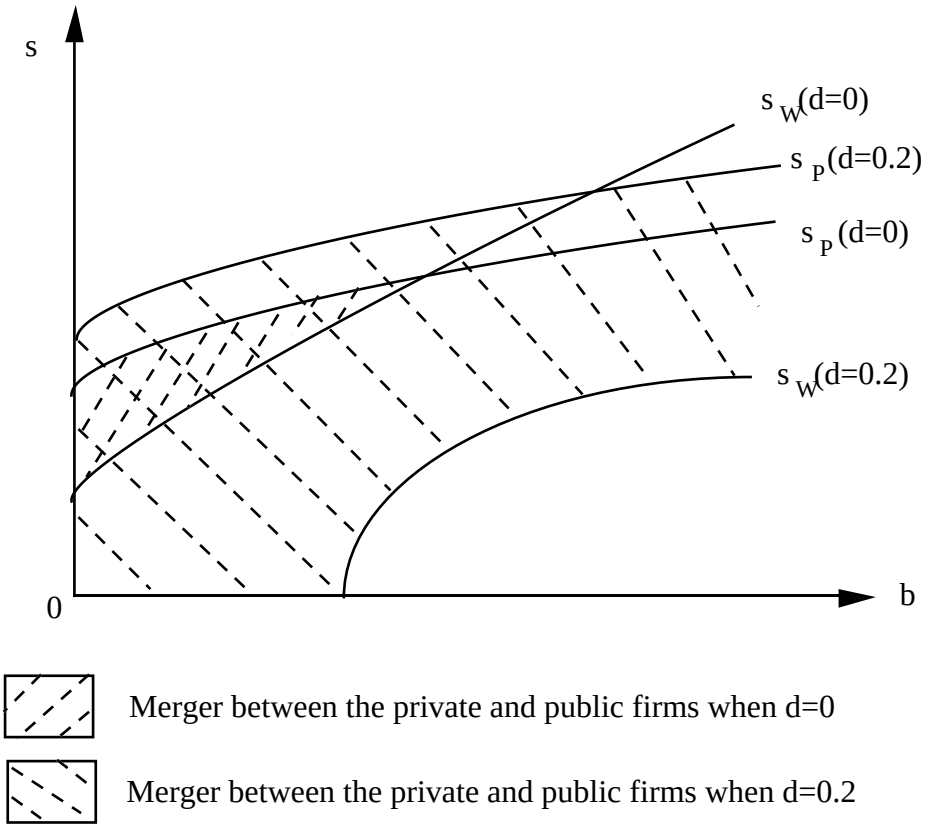

Figure 5. Illustration of proposition 4.

The reason of this result is that the existence of economies of scope implies that, for a given output level, the production cost of the multiproduct firm is lower than that of the public and private firms. As a result, the profit of the multiproduct firm increases with parameter d,

[11] The result obtained in proposition 4 holds for values of parameter k other than 2.

for a given value of parameters *b* and *s*, implying that $s_P < s_P(d)$; i. e. the existence of economies of scope implies that the shareholders of the private firm will accept the merger for a lower percentage of the shares in the multiproduct firm. Similarly, for a given output level, W^S increases with parameter *d*, since the production cost of the multiproduct firm decreases with this parameter. Therefore, $s_W(d) < s_W$; i. e. the existence of economies of scope implies that the government will accept the merger for a lower percentage of the shares in the multiproduct firm. Thus, economies of scope make the merger attractive for even lower values of *s* than before: values that would not be conducive to mergers without the economies of scope (see figure 5).

6. Conclusions

Literature on mergers has extensively analyzed the decision to merge by private firms but it has not considered the decision to merge by private and public firms. The purpose of this paper is to study this type of merger, which is an important issue because of the recent debates concerning the privatization of public firms. These firms are usually acquired by private firms, with multiproduct firms being set up after the acquisition. However, the literature on public firms considers that when a public firm is privatized a new uniproduct private firm arises.

In this paper we assume that when a private firm and a public one merge, they set up a multiproduct firm in which the government owns an exogenous percentage stake. We show that the decision to merge by firms depends crucially on the percentage of shares owned by the government in the multiproduct firm and on the degree to which goods are substitutes. When the percentage of the shares owned by the government takes an intermediate value and the degree to which goods are substitutes is low enough, both the private firm and the public firm will want to merge. When the percentage of the shares owned by the government takes an intermediate value and the degree to which goods are substitutes is high enough, neither the private nor the public firm will want to merge.

Multiproduct firms are usually able to obtain economies of scope. If we introduce economies of this type into the model we obtain that, for a given value of parameter *k*, the public and the private firm want to merge for a broader range of values of parameters *b* and *s* than when there are no economies of scope.

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Appendix

Let s_P denote the value of s such that $(1 - s)\pi^S = \pi_1^G$.

$$s_P = \Big(12 + 16b - 8b^2 - 16b^3 + 4b^5 + (40 + 42b - 14b^2 - 18b^3 - b^4)\, k + (51 + 41b - 9b^2 - 5b^3)\, k^2 + (31 + 18b - 2b^2)\, k^3 + (9 + 3b)\, k^4 + k^5 - (2 - b^2 + 3k + k^2)\,(4b^5\,(2 + k) - 4b^2\,(1 + k)\,(2 + k)^2 - 2b^3\,(2 + k)^2\,(2 + 3k) + b^4\,(8 - k^2) + (2 + 2k + k^2)\,(2 + 3k + k^2)^2 + 2b\,(1 + k)^2\,(4 + 10k + 6k^2 + k^3))^{1/2}\Big) \Big/ \Big((1 + b)\,(4b^4 - b^3\,(2 + k) + (1 + k)^2\,(14 + 15k + 4k^2) - b^2\,(14 + 19k + 6k^2) + b\,(2 + k - 2k^2 - k^3))\Big).$$

Let s_W denote the value of s such that $W^S = W^G$:

$$s_W = \Big(2\,(3 + b - 4b^2 + b^4) + (13 + 8b - 10b^2 - 2b^3)\, k + (9 + 8b - 2b^2)\, k^2 + 2\,(1 + b)\, k^3 - (2 - b^2 + 3k + k^2)\,(2\,(1 + k)\,(1 + k - b))^{1/2}\Big) \Big/ \Big((1 + b)\,(7 + 2b^3 + 15k + 10k^2 + 2k^3 - 2b^2\,(1 + k) - 2\mathrm{b}\,(3 + 4k + k^2))\Big).$$

When there are economies of scope the profit of the multiproduct firm and social welfare are:

$$\pi^S = \frac{a^2\,((1 - s)\,(1 + b) - d + k)^2}{((2 - s)\,(1 + b) - d + k)^2},$$

$$W^S = \frac{a^2\,((3 - 2s)\,(1 + b) - d + k)}{((2 - s)\,(1 + b) - d + k)^2}.$$

Let $s_W(d)$ denote, when there are economies of scope, the value of parameter s such that $W^S = W^G$:

$s_W(d)$= -(6 - $2b^5$ - 13k - $9k^2$- $2k^3$ - $2b^4$ (d -1 + k) + $2b^3$ (4 + (6 - d)k + k^2) +b (1 + k) (d - 8+ $2k^2$ (d - 2) + k (6d - 13)) + d (7 + 15k + $10k^2$ + $2k^3$) - $2b^2$ (1 + k) (4d - 3 + k (d + 2)+ k^2) + (2 (($1 + b$)^2 (2 - b^2+ 3k + k^2)^2 ($2b^3d$ - $2b^2d$ (1 + k) - b (1 + k) (1 + 2d(3 + k)) - (1 + k) (1 + k + d (7 + 8k + 2 k^2)))))$^{1/2}$) / ((1 + b)^2 (7 + $2b^3$ + 15k + $10k^2$+ $2k^3$- $2b^2$ (1 + k) - 2b(3 + 4k + k^2))).

Let $s_P(d)$ denote, when there are economies of scope, the value of parameter s such that $(1 - s)\pi^S = \pi_1^G$:

$s_P(d)$= (12 + $4b^5$ + 40k + $51k^2$ + $31k^3$ + $9k^4$ + k^5 + b^3 (d - 8 - 5k) (2 + k) - d (1 + k)^3 (2 + k) -b^4 (d + k) + b^2 (2 + k) (d - 4 - 5k - $2k^2$) + b (2 + 3k + k^2) (8 + d (k - 1) + 9k + $3k^2$) - ((2 - b^2+ 3k + k^2)^2 (4b^5 (2 + k) + $2b^3$ (2 + k) (d^2 - 4 - 8k + 4d k - $3k^2$) + 2b (2 + 3k + k^2) (2 + d^2(k - 1) + 6k + $5k^2$ + k^3 - 2d k (2 + k)) + b^4 (8 + d^2 - k^2 - 4d (3 + 2k)) + (1 + k)^2 (2 + k) (4 + 6k + $4k^2$ + k^3 + d^2 (4 + k) - 2d (3 + 4k + k^2)) - $2b^2$ (2 + k) (d^2 (2 + 3k) + 2 (2 + 3k + k^2) - d (6 + 10k + $3k^2$)))))$^{1/2}$) / ((1 + b) (4b^4 - b^3 (2 + k) + (1 + k)^2 (14 + 15k + $4k^2$) -b^2 (14 + 19k + $6k^2$) + b (2 + k - $2k^2$ - k^3))).

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