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Time-Varying Beta Estimators in the Mexican Emerging Market

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Abstract

This paper compares the performance of three different time-varying betas that have never previously been compared: the rolling OLS estimator, a nonparametric estimator and an estimator based on GARCH models. The study is conducted using returns from the Mexican stock market grouped into six portfolios for the period 2003-2009. The comparison, based on asset pricing perspective and mean-variance space returns, concludes that GARCH based beta estimators outperform the others when the comparison is in terms of time series while the nonparametric estimator is more appropriate in the cross-sectional context.

Keywords: Time-varying beta, nonparametric estimator, GARCH based beta estimator

JEL: G15, C12, C14

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1 Introduction

Precise estimates for market betas are crucial in many financial applications including asset pricing, corporate finance and risk management. From a pricing perspective, the empirical failure of the unconditional Capital Asset Pricing Model (CAPM) has led to two possible ways of relaxing restrictive assumptions under the model being considered: The first is the use of an intertemporal framework, as in Merton (1973), that implies multiple sources of systematic risk. The ad-hoc three-factor model of Fama and French (1993) and the four-factor model of Carhart (1997) are successful examples of multifactor models. The second is to eliminate the static context in the relationship between expected return and risk by allowing time variation in both factors and loadings. In that sense, Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Petkova and Zhan (2005) find that betas of assets with different characteristics move differently over the business cycle and Campbell and Vuolteenaho (2004), Fama and French (1997) and Ferson and Harvey (1999) show that time-variation in betas helps to explain anomalies such as value, industry or size. However, this conditional time-varying framework does not seem to be enough to improve the weak fit of the CAPM, as shown by Lewellen and Nagel (2006). The main problem in beta dynamics literature is that the investor's set of conditioning information is unobservable and consequently some assumptions have to be made. There are two main alternatives: making assumptions about the dynamics of the betas or making assumptions about the conditional covariance matrix of the returns.

For the first alternative, many different structures have been considered. There are studies that assume standard stochastic processes driving the dynamics of betas, such as random walk, autoregressive, mean reverting and switching models. Some examples can be found in Wells (1994), Moonis and Shah (2003) and Mergner and Bulla (2008). Other studies use parametric approaches based on Shanken (1990), in which betas are modeled as a function of state variables or firm characteristics as in Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Santos and Veronesi (2004). More recently state-varying betas have also been nonparametrically estimated by Ferreira et al. (2011). Betas have also been assumed as a function of time, with both linear and parabolic functional forms, as in Lin et al. (1992) and Lin and Lin (2000). Nonetheless neither empirical estimation nor simulation results can produce a clear conclusion about the best way to model betas. If no parametric functions are specified and no additional conditions are assumed except that betas vary smoothly over time, then the seminal work of Fama and MacBeth (1973) suggests the use of a rolling window ordinary least squares (OLS) estimation of the market model. This data-driven approach has the advantage of no parameterisation but requires the prior selection of the window length. More recently, but based on the same idea, other estimators in the family of recursive least squares have been considered. In this sense, time-varying conditional betas have been nonparametrically estimated by Esteban and Orbe (2010), Li and Yang (2011) and Ang and Kristensen (2011) assuming that betas vary smoothly over time and possibly nonlinearly. The flexibility of this nonparametric setting avoids the problem of misspecification derived from selecting a functional form but it also requires that window length be selected. These studies are based on the nonparametric time-varying estimator proposed first by Robinson (1989) and extended by Orbe et al. (2005) and Orbe et al. (2006).

The second alternative, consisting of making assumptions about the conditional covariance matrix of the returns, relies on the simple parametric approach of ARCH-class models.

In this context the assumptions under multivariate GARCH (MGARCH) models make it possible to estimate time-varying betas. In fact, the transmission of volatility between assets is captured by a time-varying conditional covariance matrix whose elements are used to calculate the beta as a ratio of covariance to variance. As the conditional covariance matrix is time dependent, the beta obtained will also be time dependent. There has been a great proliferation of multivariate models with GARCH structures in the last few decades (see Bauwens et al., 2006 or Silvennoinen and Teräsvirta, 2009 for a survey) and it must be decided which specific structure is to be used in order to estimate the betas. Some examples of the use of MGARCH models to estimate time-varying betas can be found in Bollerslev et al. (1988), Ng (1991), De Santis and Gérard (1998) and more recently in Choudhry (2005) and Choudhry and Wu (2008), among others.

Given the wide variety of time-varying beta estimates, some papers compare different approaches. The most common comparison is between GARCH based estimators and Kalman filter approaches. In general, results indicate that the latter class of estimators performs better in terms of forecasting ability (Faff et al., 2000, and Choudhry and Wu, 2008). However, there is no agreement about the best process assumption for beta dynamics. Moreover, when Kalman filter is compared with estimators in the class of least squares, as in Ebner and Neumann (2005), the latter outperform the former.

In this paper three alternative methodologies for estimating time varying betas are compared: the well known rolling window OLS estimator, the nonparametric time-varying estimator proposed in Esteban and Orbe (2010) and a beta estimator based on a GARCH process for the conditional covariance matrix of returns. These methodologies are selected because they avoid the need to impose assumptions about the specific functional form of beta dynamics. The main theoretical difference between the OLS and nonparametric estimators is that the latter has guaranteed consistency if the bandwidth is optimally chosen. In practice, there is an advantage in using the nonparametric estimator since there are many data-driven window selection criteria while the OLS estimator uses the rule of the thumb. The GARCH based beta estimator does not rely on a smoothness assumption but has the advantage of taking into account the potential conditional heteroscedasticity of the returns. The three estimation methodologies can be compared because they all imply that beta is the ratio of covariance to variance. This is not necessarily true when additional sources of risk or other time-varying estimators are considered. Specifically, the OLS, the nonparametric estimator with a uniform and a Gaussian kernel, and for the GARCH based estimator the bivariate BEKK and the bivariate dynamic conditional correlation (DCC) structure are considered.

The analysis is applied to daily returns for the Mexican stock market over the period between 2003 and 2009. This is a tight market that produces high cross-sectional dispersion in the sensitivity of individual returns to market returns. This is a desirable characteristic for the aim of the paper because it makes it possible to analyse the performance of the estimates in relation to different levels of beta. The sample period also contributes to the aim of the paper because it includes the recent financial and economic crisis, ensuring enough time variation in betas. Finally, the data frequency selection seeks to exploit the benefits of using high-frequency data in measuring systematic risk while avoiding problems of errors in variables that stem from nonsynchronous trading effects.

The accuracy of the alternative estimators is compared in terms of their usefulness for asset pricing or portfolio management purposes. The CAPM fit in both time series and cross-sectional frameworks is analysed and the variance of the minimum variance portfolio that results from the use of the different estimators is also compared. Interesting results are found. On the one hand, GARCH based estimators better reproduce the time series relationship between individual returns and the market return: the market model is better fitted and the quadratic sum of Jensen's alphas is lower. A more detailed analysis reveals that the gain comes from the improvement in measuring the systematic risk for the stocks with the lowest and most unstable betas. In fact, GARCH based estimators are the worst when they are applied to stocks highly correlated with the market. On the other hand, the conditional heteroscedasticity assumption also seems to have benefits for portfolio diversification since it is possible to reduce the overall risk of the portfolio if the estimation of individual risks is based on betas from GARCH models. However, when the aim is to estimate the risk premium, nonparametric estimators produce more accurate results. In fact, results show cross-sectional evidence that the CAPM holds only when betas have been estimated with nonparametric techniques.

The rest of the paper is structured as follows. Section 2 presents the estimation methodologies. Section 3 describes the data. Section 4 compares beta estimates descriptively. Section 5 provides the empirical results regarding the comparison of the beta estimators in two frameworks: asset pricing and mean-variance portfolio analysis. Section 6 concludes and the Appendix contains the data information.

2 Methodology

The Capital Asset Pricing Model due to Sharpe (1964) and Lintner (1965) relates the expected return on an asset to its systematic market risk or beta. This beta is the sensitivity of the asset return to changes in the return of the market portfolio. That is, beta is the slope of the market model:

$$R_{it} = \alpha_i + \beta_i R_{mt} + u_{it} \qquad i = 1, \ldots, N \quad t = 1, \ldots, T, \tag{1}$$

where R_{it} and R_{mt} are the return on asset or portfolio i and on the market portfolio at time t, respectively. Commonly, the unknown coefficients in (1) are estimated by OLS applied to the linear regression for each portfolio.

Under the assumption that these coefficients vary with time, model (1) must be rewritten as:

$$R_{it} = \alpha_{it} + \beta_{it} R_{mt} + u_{it} \qquad i = 1, \ldots, N \quad t = 1, \ldots, T.$$

2.1 The rolling OLS beta estimator

As Fama and MacBeth (1973) proposed, one simple way to obtain time series estimates of betas is by a recursive OLS estimation of the market model. This consists of the minimisation of a local sum of squared residuals for each portfolio i:

$$\min_{(\alpha_{it},\beta_{it})} \sum_{j=t-1}^{t-r} (R_{ij} - \alpha_{it} - \beta_{it} R_{mj})^2, \tag{2}$$

where r indicates the amount of past observations to be considered at each estimation point. From the first order conditions of the optimisation problem (2) the rolling OLS estimator is obtained as:

$$(\hat{\alpha}_{it} \quad \hat{\beta}_{it})'_{ROLL} = \left(\sum_{j=t-1}^{t-r} X_j X_j' \right)^{-1} \sum_{j=t-1}^{t-r} X_j R_{ij} \qquad i = 1, \ldots, N,$$

where $X_j = (1\ R_{mj})'$ is the j-th observation of the data matrix and the subscript $ROLL$ denotes the OLS rolling estimator.

In the empirical application of this estimator, a window of 120 observations for data with daily frequency is used. The sampling frequency is selected based on the findings of Bollerslev and Zhang (2002) or Ghysels and Jacquier (2006), who show that high-frequency data result in a more effective measure of betas than the commonly used monthly returns. Since the Mexican stock market is tight, a lot of stocks are far from being continuously traded with the nonsynchronicity effects on beta estimates, so intraday data are discarded. A window length of 120 days is used. An alternative number of observations was also considered but it did not alter the main conclusions of the paper[2].

2.2 The nonparametric time-varying beta estimator

This estimator is, as before, a recursive least squares estimator. It relies on the assumption that the unknown time-varying coefficients, α_{it} and β_{it}, are smooth functions (linear or nonlinear) of the time index. It is derived from minimising a smoothed sum of squared residuals for a given portfolio i and for a pre-selected smoothness degree h_i:

$$\min_{(\alpha_{it},\ \beta_{it})} \sum_{j=t-1}^{t-Th_i} K_{h_i,tj}(R_{ij} - \alpha_{it} - \beta_{it} R_{mj})^2,$$

where $K_{h_i,tj} = h_i^{-1} K\left((t/T - j/T)/h_i\right)$ is a weight function and $K(\cdot)$ is a symmetric second order kernel. The shape of this kernel determines how past observations are to be weighted. If a uniform kernel is used all selected past observations are equally weighted but if the Epanechnikov or the Gaussian kernels are used, larger weights are given to those observations

[2]Specifically, windows of 90 and 400 days were analysed. Results are available upon request.

closer to the estimation time point and smaller weights to those farther away in time. The parameter h_i is the bandwidth that controls the amount of smoothness imposed on the coefficients associated with the ith portfolio. Solving the first-order conditions, the estimator has the following expression:

$$(\hat{\alpha}_{it} \quad \hat{\beta}_{it})'_{NP} = \left(\sum_{j=t-1}^{t-Th_i} K_{h_i,tj} X_j X_j' \right)^{-1} \sum_{j=t-1}^{t-Th_i} K_{h_i,tj} X_j R_{ij} \qquad i = 1, \ldots, N,$$

where all elements are already defined and the subscript NP indicates the nonparametric estimator.

Once the smoothness degree h_i is fixed, the estimator obtained is consistent with the standard rate of convergence in nonparametric settings and has a closed form, so neither iterative methods nor initial values are needed to calculate the estimations. Since the role of the bandwidth is to determine the amount of smoothness imposed on the betas and therefore the number of relevant past observations to be taken into account when estimating these betas, it is crucial to select it adequately in advance. If the bandwidth is large, the sub-sample of significantly weighted observations is larger, that is, more past observations are considered relevant in each local estimation. This results in a time series of estimated betas with little variability due to the high smoothness degree. But if the bandwidth is small the estimation sub-sample is narrowed and the estimated betas have more dispersion. Different bandwidths (h_i) are allowed for the portfolios in order to capture different possible variations and curvatures of the betas. In consequence, the sub-sample size used at any estimation time point is the same when estimating the betas for a given portfolio but can be different for betas from another portfolio.

In regard to the practical issues of choosing the kernel and the bandwidths, it is well known that all kernels are asymptotically equivalent but that this is not the case for the bandwidth value. An optimal bandwidth is such that it minimises an error criterion in order to reach a trade off between the squared bias and the variance of the beta estimator. Thus a small bandwidth leads to a small bias and a larger variance while a large bandwidth leads to the contrary results. Generally speaking, there are three types of methods for selecting bandwidths in a nonparametric estimation setting: Leave-one-out techniques, penalised sum of squared residuals and plug-in methods. Härdle et al. (1988), Härdle (1990) and Wand and Jones (1995) provide detailed discussions and some practical comparisons of these criteria. In the context of conditional factor models Ang and Kristensen (2011) and Li and Yang (2011) propose a bandwidth selection criteria for two-sided kernels: considering symmetric sub-samples that take into account not only past observations but also future observations. In this paper, only past observations are taken into account for estimating conditional betas and the considered data-driven method for selecting the bandwidths simultaneously is based on the proposal of Esteban and Orbe (2010), where the error is minimised for all regressions together in order to take into account the relationships between the different bandwidths.

Finally, note that this nonparametric estimator generalises the rolling OLS estimator since it can be derived as a particular case. If a uniform kernel that weights past observations equally is considered and $h_i = r/T$ is imposed instead of selecting the smoothness degree using a data-driven method, then the estimations obtained by the two estimators match. Nonetheless, in this case the estimation does not take advantage of weighting the nearest observations more highly, and since the value of the bandwidth is possibly not the optimal, consistency is not guaranteed. In Section 3 the estimation results obtained by a uniform and a Gaussian kernels are presented, each for its corresponding optimal bandwidth according to the selection criteria used. The comparison between the rolling OLS estimator and the nonparametric estimator using a uniform kernel highlights differences from using the optimal bandwidth while the comparison with the nonparametric estimator using a Gaussian kernel is more instructive since weighting past observations is equivalent to using information more efficiently.

2.3 The time-varying beta estimator based on multivariate GARCH models.

The literature on financial econometric volatility has provided evidence of fluctuations and high persistence in conditional variance of asset returns and conditional covariance with the market return (see Andersen et al., 2010 for a survey). Since market betas are ratios of conditional covariances and variances, $\hat{\beta}_{it} = \widehat{cov}_t(R_i, R_m)/\widehat{var}_t(R_m)$, if these second moments are adequately estimated by a multivariate GARCH, then betas are also expected to be accurate estimators.

The estimation procedure for MGARCH models involves maximising the following log-likelihood function for each portfolio i:

$$lnL(\theta) = -\frac{1}{2}\sum_{t=1}^{T} ln|H_{it}| - \frac{1}{2}\sum_{t=1}^{T} y_{it}' H_{it}^{-1} y_{it},$$

where $y_{it} = (R_{it}\ R_{mt})'$ is the vector of dependent variables containing a bivariate vector of constants, θ is the vector of parameters to be estimated and the specification of the conditional covariance matrix (H_{it}) depends on the MGARCH structure considered.

This analysis considers two different MGARCH structures often used in financial literature: BEKK and DCC. The former is the bivariate BEKK (1,1,1) due to Engle and Kroner

(1995) and has the advantage that the positive-definite constraint of the conditional covariance matrix is guaranteed by construction. The latter is the bivariate dynamic conditional correlation specification proposed by Engle (2002), where the conditional covariance matrix is decomposed into time-varying correlations and conditional standard deviations estimated using univariate GARCH models.

Once the conditional covariance matrix is estimated, the time-varying GARCH based beta for portfolio i is calculated as:

$$\hat{\beta}^l_{it} = \frac{\hat{H}^l_{i12t}}{\hat{H}^l_{i22t}} \quad l = BEKK, DCC,$$

where $\hat{H}^l_{i12t}$ is the estimated conditional covariance between the ith portfolio returns and the market returns and $\hat{H}^l_{i22t}$ is the estimated conditional variance of the market return for $l = BEKK, DCC$ conditional covariance matrix structures.

3 Data

This analysis uses daily logarithms of returns on 42 stocks traded on the Mexican Stock Exchange between January 2, 2003 and December 31, 2009. The data series have been computed from daily prices taking into account dividends and splits. The sample is selected on the basis of representative criteria in terms of both market capitalisation and trading volume. The sample basically coincides with the 35 firms included in the "Índice de Precios y Cotizaciones" (IPC, hereafter). As the composition of this market index is revised annually, this gives a total number of 42 firms in the sample period. The proxy for the risk-free asset is the 28-day maturity Treasury Certificate and data for this proxy are collected from the Banco de Mexico.

To show the representativeness of the selected sample, the table in the Appendix provides the names of the firms selected, their industrial classifications and the percentage of the total trading volume in pesos on the Mexican Stock Exchange at the end of 2009 accounted for by each stock. At that time the market comprised stocks issued by 85 firms, with five of them being non domestic companies. Although the sample only contains half of the firms extant, it accounts for 95% of the market in terms of trading volume in pesos in 2009, as can be seen by adding the weights in the last column of the table in the Appendix.[3] Moreover, the firms selected represent all the different industrial categories.

The individual stocks are sorted and grouped into portfolios. Since one of the aims is to analyse the appropriateness of the estimators in relation to the level and the volatility of beta, it is important for the sorting criteria to be able to produce sufficiently different portfolio betas. In that sense, individual betas could be used for sorting and locating stocks in portfolios. However, this would imply, on the one hand, selecting a beta estimation methodology first to conduct the analysis of the appropriateness of each estimator. On the other hand, in subsequent sections asset pricing tests are used for comparing beta estimators and the results would be subject to the concerns raised by Lewellen et al. (2010). This is why stocks have been sorted by individual money trading volume. The composition of the portfolios is updated monthly by using the volume in pesos of the total trades for each stock during the month and the return of the portfolio is computed daily as the equally weighted average of the returns on stocks in the portfolio. Thus, Portfolio 1 contains the less liquid

[3]The same calculation using trading volumes for other years in the sample period gives similar percentages of representativeness.

stocks while the most frequently traded stocks are in Portfolio 6.[4]

Table 1 reports the summary statistics for the return on the six portfolios, on the market index and on the risk free asset covering the whole sample period. The mean and the standard deviation are expressed on an annual basis. The beta estimator for each portfolio and its standard error come from the OLS estimation of the market model using the full sample period. Finally, the last row reports the average in time and across stocks within each portfolio of the monthly trading volume in millions of pesos. As can be seen, major differences in trading volume are observed; Portfolio 6 concentrates a large part of the market trading and their stocks have 70 times more trading volume than Portfolio 1. These liquidity differences do not imply differences in portfolio return volatilities, since standard deviation is similar for all six portfolios, but curiously they produce increasing mean returns ranging from 14% for Portfolio 1 to 29% for Portfolio 6. Thus, it seems that this market does not show an illiquidity premium. More importantly, betas are monotonously increasing from Portfolio 1 to Portfolio 6 and also have different levels of standard errors. Therefore, the portfolio formation criterion produces the desirable dispersion in portfolio betas. The distribution of the returns is negatively skewed for the risk-free asset and all portfolios except the fifth and the market index, for which the return's distributions are symmetric at the 5% significance level. Regarding the kurtosis coefficient, there is a significant positive excess of kurtosis for all cases except for the risk-free asset, for which the coefficient is negative. Therefore, the returns are not normally distributed. This is confirmed by the Jarque Bera test.

[4]The classification has also been drawn up using trading volume in terms of number of shares and the characteristics of the resulting portfolios are very similar.

4 Conditional beta estimates

In this section descriptive statistics regarding the five time series beta estimators obtained by the three considered methodologies are presented and compared. Rolling window OLS is obtained with subsamples of 120 previous observations for all portfolios and denoted by ROLL. The nonparametric estimator uses two alternative kernels: the uniform (NP-U) and the Gaussian (NP-G). The selected bandwidth is 0.1279 for Portfolios 1, 2, 3 and 6 and 0.0896 for Portfolios 4 and 5 when the uniform kernel is used, while for the Gaussian kernel the selected bandwidth is 0.0591 for all portfolios except the fifth, for which is 0.0398. Therefore, although bandwidths are allowed to vary with portfolios, the data-driven selected values indicate that betas have the same smoothness degree for most portfolios and hence the number of relevant past observations is the same. Finally, the two alternative GARCH specifications produce time series of beta estimates that are denoted as BEKK and DCC. In the GARCH context the total sample information is used (producing series of 1764 daily betas) and the estimation method does not weight the observations according to their temporal neighbourhood but according to the conditional heteroscedasticity structure. In order to provide a homogeneous comparable context, the sample of beta estimates is restricted to the period between 17th October, 2003 and 31st December, 2009, with a total of 1564 daily beta estimates for each estimator.

Table 2 presents the mean and the standard deviation of the time series of estimated betas for each portfolio and for all alternatives considered. The general conclusion is that all estimation methods produce conditional betas series that move around a very similar mean value, smaller than the point beta estimate from the market model (see Table 1), and differences between estimates are observed in standard deviations. Comparing ROLL and the NP estimates, it can be seen that the former one has, in general, a smaller mean but a larger standard deviation, which are similar when different kernels are used. The comparison between ROLL and GARCH estimates shows that GARCH beta estimates are more volatile, with BEKK estimates having larger standard deviations and smaller means than DCC estimates. These results are confirmed in Figure 1 which shows the time series beta estimates for the two extreme portfolios. Subfigures 1(a) and 1(b) compare ROLL and NP estimates while Subfigures 1(c) and 1(d) compare ROLL and GARCH based estimates. All betas move around the same long term mean, the NP methods produce smoother betas than ROLL and changes in the short term are much more pronounced in estimates from GARCH structures. In addition, independently of the estimation methodology, mean betas increase and standard deviations of betas decrease, almost monotonously, from the portfolio containing the least liquid stocks to the portfolio containing the most liquid stocks.

In order to gain insight into the similarities of different time-varying betas estimates the average correlations between pairs of conditional beta estimates are computed. Table 3 reports the correlations calculated for each portfolio and then averaged over all of them. Results indicate that the pattern is very similar for beta estimates based on minimising some kind of least squares, on the one hand, and for beta estimates from GARCH specifications, on the other hand. However, the correlation between any of the estimated betas of each group is much smaller, with the largest value being 0.56795 for the correlation between DCC and NP-G estimated betas and the lowest 0.40049, for the correlation between BEKK and NP-U estimated betas. This finding evidences the different consequences that the assumption of the conditional covariance of returns has on the resulting beta estimate.[5]

Regarding the pattern of the estimated betas for all portfolios, Figure 2 shows the rolling OLS, the NP with the Gaussian kernel and the BEKK based beta estimators for all six portfolios. As mentioned above, the volatility of beta estimates decreases and the mean increases from Portfolios 1 to 6 for all estimation methodologies. Independently of the portfolio, the most volatile beta estimates are those obtained using the GARCH specification, while the pattern of beta estimates is smoother and similar among the rolling and nonparametric estimators.

5 Beta estimator comparison

In this section the accuracy of the different estimators is compared. Since true betas are not observable, it is not possible to conduct traditional analyses such as the in-sample bias or the out-of-sample forecasting power for beta estimates. Instead, the comparison is made in terms of the utility of time-varying beta estimates for two important actual applications: Asset pricing and portfolio management.

5.1 The asset pricing perspective

This subsection analyses how systematic risk may be assessed more accurately through the use of one beta estimation methodology or another. For this purpose the simplest asset pricing framework is considered: the CAPM. It must be pointed out that this exercise

[5] Similar results are obtained in Faff et al. (2000) when comparing Kalman filter and GARCH based beta estimators.

does not set out to test the CAPM -the point is not whether the model is misspecified or not- and that the analysis presented here could easily be extended to a multi-factor asset pricing model. However, this model offers a simple way of looking at the expected positive relationship between returns and systematic risk that any underlying investor's preferences would imply. In that sense, a beta estimate is more accurate if it is able to improve this relationship.

Next, two different settings for the comparison are considered. The first is based on time series analysis and the second on cross-section analysis.

5.1.1 Time series analysis

The first comparison between beta estimates relies on the appropriateness of the factor model representation. That is, for each portfolio, the different beta estimates are compared in terms of fit for the market model. Since time-varying coefficients are estimated, R-squared statistics are not necessarily bounded and they cannot be comparable. Instead, the return variance explained by the market model, $VR1 = var(\hat{R}_i)/var(R_i)$, is used as a measure of goodness of fit, and the return variance that the model fails to explain, $VR2 = var(\hat{u}_i)/var(R_i)$, as a measure of the estimation error.[6] It must be pointed out that the computation of $\hat{R}_{it}$ and $\hat{u}_{it}$ requires estimates for parameter α_{it} and BEKK and DCC models do not provide them. In these cases, an estimation of α_{it} is obtained from the average of the market model where the time variation comes from each daily beta estimate:

$$\hat{\alpha}^l_{it} = \bar{R}_i - \hat{\beta}^l_{it}\bar{R}_m \quad i = 1, \ldots, N \quad t = 1, \ldots, T \quad l = \text{BEKK},\ \text{DCC},$$

where $\bar{R}_i$ and $\bar{R}_m$ are the mean returns on portfolio i and the market portfolio, respectively.

[6] These measures are used in Ferson and Harvey (1991) and Harvey et al. (2002), among others.

Table 4 shows the values of $VR1$ and $VR2$ criteria for each portfolio and each estimator. Results for the two measures are very similar when the ROLL and NP estimators are compared, since both estimators are based on the use of recursive least squares. In general, ROLL estimates show a larger fit (larger $VR1$) but also a larger estimation error (larger $VR2$). This finding could be due to the bandwidth size. Since the number of past relevant observations that the method uses is smaller for the rolling OLS than for the NP, the smoothness degree imposed is lesser and in consequence the estimated betas have a smaller bias but a larger variance. Looking at the results overall, it can be concluded that the market model is better explained when beta estimates come from GARCH models. The only exception is Portfolio 2, where NP-G produces the lowest $VR2$. Finally, comparing the criteria when the two GARCH specifications are used the conclusions are not clear, since results differ depending on the portfolio and the fit measure.

The second comparison within this time-series framework employs Jensen's alphas as a measure of the error adjustment of the model. The Jensen's alpha associated with each beta estimator is computed for each portfolio and period as the difference between the observed return and the estimated return:

$$\widehat{\alpha}_{it}^{J} = (R_{it} - R_{ft}) - \widehat{\beta}_{it} (R_{mt} - R_{ft}) \quad i = 1, \ldots, N \quad t = 1, \ldots, T,$$

where R_{ft} represents the risk-free rate.

The quadratic sum of these alphas is calculated as a measure of the model misspecification which allows a comparison to be made between different estimation methods. A large value of the quadratic sum of alphas indicates a poor specification of the model since the estimated returns differ greatly from the observed returns. Table 5 reports this measure. The bottom row shows the total sum for all portfolios. It can be seen that beta estimates from GARCH models produce the lowest alphas for four out of the six portfolios, which implies the lowest values for the quadratic sum of alphas aggregating all portfolios. Specifically, the DCC estimator presents the lowest errors for Portfolios 1 and 5 while BEKK is the best at reducing errors for Portfolios 3 and 4. However, the two GARCH estimators are the worst when pricing Portfolios 2 and 6. Finally, in accordance with the results provided in Table 4, for Portfolio 2 the lowest errors are associated with the NP-G estimator.

In order to learn whether the differences observed in Table 5 are relevant for each portfolio, a pairwise comparison of Jensen's alphas, in absolute values, associated with two beta estimators is conducted by the Wilcoxon signed rank test. Table 6 reports the median difference between the two series of alphas expressed on an annual basis. For example, a comparison of ROLL and NP-U in Portfolio 1, -0.00319 indicates that if the ROLL beta estimate is used the pricing error is 0.319% lower, in terms of annual returns, than if the NP-U estimate is used. The number in parenthesis is the p-value for the test of the null that this median difference is zero. Observation of results for all portfolios reveals a small number of significant differences in absolute alphas. The most notable case occurs for Portfolio 1. Results for this portfolio indicate that DCC performs better than any of the other estimators. This beta estimator produces a pricing error of approximately 1% lower than any estimator based on least squares. As expected, the median pricing difference, although statistically significant, is lower than with the BEKK estimator. In contrast, the DCC estimator produces significant, higher errors than ROLL, NP-G or BEKK beta estimators when pricing Portfolio 6. For the rest of the portfolios, the only notable conclusion is that NP-G performs better than ROLL.

Therefore, the analysis of Jensen's alphas gives a general conclusion: It seems that the conditional heteroscedasticity structure helps in estimating time-varying betas for assets with market betas highly volatile; however this structure is not useful when the relationship between the return on the asset and the market return is relatively stable.

5.1.2 Cross-sectional analysis

In this subsection the estimators are compared in terms of the market risk premium implied by the different estimated betas. Under rational expectations there should be a positive relationship between expected returns and systematic risk cross-sectionally. For this purpose, the simple CAPM framework is used. The model may of course be misspecified but this is not a limitation since the positive relationship between market betas and expected returns could be justified under any other investors preference assumption.

Using the Fama and MacBeth (1973) methodology, the following cross-sectional regression is estimated for each day in the sample period:

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{mt}\beta_{it} + e_{it} \quad i = 1, \ldots, N, \tag{3}$$

where the beta is approximated by each of the five beta estimators considered. A reasonable beta estimator should produce a positive, significant market risk premium and the more precise the above cross-sectional relationship is, the more accurate the beta estimator is. Additionally, since excess returns are used as dependent variable, an intercept statistically equal to zero indicates a good model fit.

The results from the Fama-MacBeth estimation of the model are presented in Table 7. This table reports the estimates of the intercept and the market risk premium (x10^2), their t-statistics for individual significance and the corresponding Shanken (1992) adjusted t-statistics. The left panel of the table shows the results when daily portfolio returns and betas are used in the estimation of (3) and running one regression each day. The right panel provides the results when monthly returns and the beta estimator corresponding to the last day of the previous month are used to reduce the excessive noise that daily observations could introduce into this cross-sectional analysis. In this case, the number of regressions is 75 corresponding to the number of months in the period analysed.

Intercepts are non-statistically different from zero and market risk premia are positive for all beta estimates and the two data frequencies. However, differences in the value and significance of the risk premia are observed for different beta estimators. In both panels, the best results are obtained for the two non-parametric estimators. At daily frequency, these are the only cases in which risk premia are significantly positive at the 10% level. The results for the monthly frequency are still more conclusive. Risk premia associated with NP beta estimators are positively significant at the 5% level while risk premium values and the standard errors for ROLL and GARCH structures are similar and not significant.

Thus, the results of this analysis indicate that nonparametric time-varying beta estimates are better at capturing the cross-sectional dispersion in mean returns. This suggests, on the one hand, that the size of the window and the right weighting matters since these are the only differences between NP-U and ROLL estimators. Therefore, an optimal mechanism for choosing the bandwidth is important. On the other hand, the high volatility in the GARCH based betas seems to have a negative effect on the stability of the relationship between systematic risk and mean returns. And then, for estimating the price of risk, methodologies based on smoothness mechanisms are more appropriate for the prior estimation of systematic risk.

5.2 Portfolio management analysis

An important application of betas is their use in portfolio management. Since individual betas are part of the variance of a portfolio, the power of prediction of the different beta estimators can be studied by analysing whether the purpose indicated in the portfolio construction criterion is achieved in the next period.

For each of the estimation methodologies considered, betas for all six portfolios are taken in order to obtain an estimation of the next period covariance matrix, which can then be used to obtain the composition of the overall minimum variance portfolio. Thus, the beta estimators are compared by analysing the variance of the resulting portfolio.

Specifically, according to the market model, for a given month s the covariance matrix of a set of N asset returns is:

$$\Sigma_s = \sigma^2_{ms} B_s B'_s + D_s,$$

where σ^2_{ms} is the variance of the market return, B_s is an N-vector of individual betas and D_s is an $N \times N$ matrix of the idiosyncratic variance-covariances, all them measured in month s. The variance of the market return is estimated using daily returns within month s; beta estimates on the last day of month $s-1$ are used as predictors of elements of B_s; and D_s is estimated as the residual covariance matrix from the market model consistent with these beta estimates employing daily returns within month s:

$$\widehat{D}_s = \frac{1}{T_d} \widehat{U}'_s \widehat{U}_s,$$

where $\widehat{U}_s$ is a $T_d \times N$ matrix containing the residuals $\widehat{u}_{isd} = R_{isd} - \widehat{\alpha}_{is-1} - \widehat{\beta}_{is-1} R_{msd}$ for $i = 1, \ldots, N$, $d = 1, ...T_d$, where T_d is the number of days in the month s and $s = 1, \ldots, S$ with S being the number of months in the sample.

The portfolio formation criterion consists of investing in the minimum variance portfolio which implies choosing the portfolio weights (ω_s) that solve the following problem:

$$Min\ \omega_s' \Sigma_s \omega_s$$

$$s.t. \quad \omega_s' \mathbf{1} = 1$$

This optimisation problem is solved for each month and each beta estimate, then the daily return of the minimum variance portfolio is computed for all the days in the month and its variance is recorded. The most successful beta estimator should lead to portfolios with the lowest variance.

Table 8 provides the results for the comparisons of pairs of series of the variance of the minimum variance portfolio conducted via the Wilcoxon median test. For each pair of estimates, the median difference between the two variance series and the p-value for the null that this difference is zero are reported.

The results are quite conclusive: in between 70% and 90% of the months the GARCH based beta estimators produce lower variances than estimators based on least squares, and this difference in variance has a high value on the median and is clearly significant. Thus, for the purpose of risk hedging in portfolio decisions, beta estimates from autoregressive conditional heteroscedasticity assumptions are superior to methods that do not assume structure on variance-covariance returns. Finally, when ROLL and NP estimators are compared the differences in the resulting variance portfolio are not so big but NP-G is significantly better than rolling OLS with both the standard selection of the window size and the optimal window size.

6 Conclusions

This paper compares the performances of three time-varying beta estimators for the market model that have never previously been compared with one another homogenously: the rolling window OLS estimator, a nonparametric estimator and the time-varying beta estimator from a GARCH structure for the conditional variance of the errors of the market model. These three methodologies were selected out of all the different estimation possibilities because they maintain the beta definition: the ratio of the covariance between the return on an asset and the market return and the variance of the market return. It is important to note that a multivariate model and other methodologies that require parametric assumptions about beta dynamics may disturb this definition. In this sense, any potential advantages of each estimator over the others can be easily identified. The nonparametric estimator allows the optimal window length to be chosen while the GARCH based estimator has the advantage of taking into account the return's conditional heteroscedasticity.

Specifically, both uniform and Gaussian kernels are used for the nonparametric estimator and DCC and BEKK models are considered in the GARCH specifications. Therefore, five beta estimates are obtained for each of the six portfolios of daily returns for the Mexican stock market in the period 2003-2009.

The analysis is conducted under two frameworks: an asset pricing perspective that assumes the CAPM and the mean-variance space for returns. In the first case the accuracy of beta estimates is analysed using different measures of the time-series fit of the model and looking at the cross-sectional relationship between mean returns and market betas. In the mean-variance context, the forecasting power of different beta estimates is obtained by comparing the results of the minimum variance portfolio.

The comparison of the different estimators gives a clear message. GARCH based beta estimators are more volatile, which improves the fit of the market model for all portfolios. In fact, the conditional heteroscedasticity structure on the return errors especially improves the estimation of time-varying betas for those assets with highly volatile market betas. As a result, for the purpose of risk hedging, beta estimates from GARCH assumptions are superior, as the minimum variance portfolio analysis shows.

Nonetheless, nonparametric time-varying beta estimates are better at capturing the cross-sectional dispersion in mean returns while the high volatility in GARCH based betas has a negative effect on the stability of the relationship between systematic risk and mean returns. Consequently, in estimating the price of risk, methodologies based on smoothness mechanisms are more appropriate for the prior estimation of systematic risk.

Given the different conclusions are obtained depending on whether returns are analysed in a time-series or in a cross-sectional setting, one possible improvement could be to propose a new estimator that combines the advantages of both these estimators, i.e. an estimator that imposes smoothness on the betas and simultaneously takes into account the conditional heteroscedasticity structure on the return errors. Future research will be based on this idea.

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Appendix

Description of Individual Stocks

Ticker	Firm Name	Sector	Trading Volume
AMX-L	América Móvil	Telecomunications/Services	23.22
TELMEX-L	Teléfonos de Mexico	Telecomunications/Services	3.49
TELINT-L	Telmex Internacional	Telecomunications/Services	2.09
TELECOM-A1	Carso Global Telecom	Telecomunications/Services	1.89
AXTEL-CPO	Axtel	Telecomunications/Services	1.84
TLEVISA-CPO	Grupo Televisa	Telecomunications/Radio and Television	3.33
TVAZTCA-CPO	TV Azteca	Telecomunications/Radio and Television	1.07
ICH-B	Industrias CH	Materials/Steel	0.21
SIMEC-B	Grupo Simec	Materials/Steel	0.17
GMEXICO-B	Grupo Mexico	Materials/Metals and Mining	7.65
AUTLAND-B	Compañía minera Autland	Materials/Metals and Mining	0.12
CEMEX-CPO	Cemex	Materials/Construction	4.63
MEXCHEM	Mexichem	Materials/Chemical Products	0.93
ASUR-B	Grupo Aeroportuario del Sureste	Industrials/Transportation	0.87
GAP-B	Grupo Aeroportuario del Pacífico	Industrials/Transportation	0.50
OMA-B	Grupo Aeroportuario del Centro Norte	Industrials/Transportation	0.15
GEO-B	Corporación Geo	Industrials/Construction	1.73
URBI	Urbi Desarrollos Urbanos	Industrials/Construction	1.40
HOMEX	Desarrolladora Homex	Industrials/Construction	1.39
ICA	Empresas ICA	Industrials/Construction	1.33
IDEAL-B1	Impulsora del Desarrollo y el Empleo en América Latina	Industrials/Construction Construction	1.11
ARA	Consorcio Ara	Industrials/Construction	1.10
SARE-B	Sare Holding	Industrials/Construction	0.06
ALFA-A	Alfa	Industrials/Capital Goods	1.43
GCARSO-A1	Grupo Carso	Industrials/Capital Goods	1.02
LAB-B	Genomma Lab Internacional	Health/Medicine Distrib.	1.50
BOLSA-A	Bolsa Mexicana de Valores	Financial Services/Financial Markets	0.24
GFNORTE-O	Grupo Financiero Banorte	Financial Services/Financial Groups	2.04

GFINBUR-O	Grupo Financiero Inbursa	Financial Services/Financial Groups	1.07
COMPART-O	Banco Compartamos	Financial Services/Commercial Banks	0.79
WALMEX-V	Wal-Mart de Mexico	Consumer Staples/Hypermarkets	13.22
COMERCI-UBC	Controladora Comercial Mexicana	Consumer Staples/Hypermarkets	0.07
KIMBER-A	Kimberly-Clark Mexico	Consumer Staples/Household Products	1.06
BIMBO-A	Grupo Bimbo	Consumer Staples/Food	1.00
GRUMA-B	Gruma Sab de C.V.	Consumer Staples/Food	0.51
FEMSA-UBD	Fomento Económico Mexicano	Consumer Staples/Beverages	5.82
GMODELO-C	Grupo Modelo	Consumer Staples/Beverages	1.70
ARCA	Embotelladoras Arcas	Consumer Staples/Beverages	0.54
KOF-L	Coca-cola Femsa	Consumer Staples/Beverages	0.07
ELEKTRA	Grupo Elektra	Consumer Discret./Retails	1.28
GFAMSA-A	Grupo Famsa	Consumer Discret./Retails	0.50
SORIANA-B	Organización Soriana	Consumer Staples/Hypermarkets	1.01

Notes:

For each stock this table reports the name of the ticker, the corresponding firm, its industrial classification and the proportion for which each stock accounts in the total volume in Mexican pesos traded on the stock market at the end of 2009.

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Table 1: Summary Statistics of Returns and Beta Estimates

	Port. 1	Port. 2	Port. 3	Port. 4	Port. 5	Port. 6	IPC	TC
Mean	0.1401	0.2062	0.3263	0.2053	0.2404	0.2914	0.2366	0.0496
Std. Dv.	0.2634	0.2182	0.2370	0.2662	0.2579	0.2719	0.2310	0.0006
Skewness	-0.5418	-0.5357	-0.1317	-1.4388	-0.0451	-0.1263	0.1023	-0.2892
Excess Kurtosis	5.2145	3.6191	7.5339	29.5635	5.6639	5.0603	5.3426	-0.4960
JB ($\times 10^3$)	2.0848	1.0470	4.1769	64.8480	2.3585	1.8867	2.1010	0.0426
Beta	0.6523	0.6949	0.7958	0.9027	0.9667	1.1059		
(std. err.)	(0.022)	(0.015)	(0.015)	(0.017)	(0.013)	(0.009)		
Volume (million)	119.874	298.962	497.886	817.236	1682.89	8280.50		

Notes:
This table presents the summary statistics for the daily returns on 6 portfolios where stocks are sorted by trading volume, the market index (IPC) and the risk-free asset (TC) returns for the period between January 2, 2003 and December 31, 2009. Means and standard deviations are on an annual basis. Beta is the OLS estimate from the market model and its standard error is shown bellow in parentheses. Volume indicates the average over time and across stocks in each portfolio of the monthly trading volume expressed in millions of Mexican pesos.

Table 2: Summary Statistics of Beta Estimates

Port.	Statistic	ROLL	NP-U	NP-G	BEKK	DCC
1	Mean	0.64306	0.64418	0.64356	0.63208	0.63859
	Std. Dv.	0.21544	0.15340	0.17320	0.29100	0.22748
2	Mean	0.69788	0.68721	0.69150	0.71146	0.73322
	Std. Dv.	0.13793	0.09058	0.10527	0.15658	0.13397
3	Mean	0.75433	0.75913	0.75943	0.74916	0.78374
	Std. Dv.	0.11777	0.08998	0.10166	0.13661	0.12069
4	Mean	0.82728	0.82880	0.82879	0.80581	0.84891
	Std. Dv.	0.15181	0.14398	0.13860	0.16433	0.15550
5	Mean	0.91797	0.91952	0.91949	0.91076	0.92989
	Std. Dv.	0.11253	0.10464	0.11301	0.14380	0.12329
6	Mean	1.07423	1.07515	1.07530	1.07224	1.07493
	Std. Dv.	0.06711	0.05676	0.06013	0.06877	0.08035

Notes:
This table presents summary statistics for time series of portfolio beta estimates that are daily in frequency and cover the period between October 17, 2003 and December 31, 2009. Portfolios are constructed by sorting stocks by trading volume in pesos such that Portfolios 1 and 6 contain the least and the most liquid stocks, respectively. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models.

Table 3: Average Correlations of Alternative Beta Estimates

	ROLL	NP-U	NP-G	BEKK	DCC
ROLL	1	0.87181	0.95698	0.48085	0.51768
NP-U		1	0.92973	0.40049	0.44023
NP-G			1	0.52510	0.56795
BEKK				1	0.83994
DCC					1

Notes:
This table presents average correlations between pairs of conditional beta estimates for the period between October 17, 2003 and December 31, 2009. Correlations are calculated for each portfolio and averaged over all of them. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models.

Table 4: Model Fit Criteria

Port.	Crit.	ROLL	NP-U	NP-G	BEKK	DCC
1	VR1	0.35305	0.32071	0.33601	0.41067	0.39818
	VR2	0.64917	0.65419	0.64650	0.63800	0.63676
2	VR1	0.56155	0.51100	0.53289	0.61597	0.58320
	VR2	0.44636	0.44984	0.44371	0.46298	0.45917
3	VR1	0.60847	0.57303	0.59854	0.63702	0.64128
	VR2	0.38480	0.38834	0.38279	0.37247	0.37605
4	VR1	0.63827	0.63492	0.62377	0.66514	0.72413
	VR2	0.37584	0.37499	0.37398	0.35524	0.36889
5	VR1	0.78174	0.77337	0.78270	0.80586	0.79655
	VR2	0.24562	0.24510	0.24412	0.24012	0.23905
6	VR1	0.88501	0.87285	0.88357	0.88993	0.89473
	VR2	0.11092	0.11007	0.11026	0.10964	0.11106

Notes:
This table presents the return variance explained by the market model, VR1, and the return variance that the model fails to explain, VR2, for each portfolio in the period between October 17, 2003 and December 31, 2009. Portfolios are constructed by sorting stocks by trading volume in pesos such that Portfolios 1 and 6 contain the least and the most liquid stocks, respectively. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models.

Table 5: Quadratic Sum of Jensen's Alphas

Portfolio	ROLL	NP-U	NP-G	BEKK	DCC
1	0.2787	0.2801	0.2777	0.2740	0.2734
2	0.1381	0.1391	0.1375	0.1442	0.1428
3	0.1406	0.1415	0.1400	0.1370	0.1382
4	0.1775	0.1771	0.1774	0.1696	0.1763
5	0.1074	0.1074	0.1068	0.1063	0.1058
6	0.0542	0.0541	0.0540	0.0542	0.0549
Sum	0.8964	0.8993	0.8934	0.8854	0.8913

Notes:
This table reports the quadratic sum of daily Jensen's alphas from October 17, 2003 to December 31, 2009 from the CAPM for each portfolio and each alternative estimator. The sum for all portfolios appears in the last row. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models.

Table 6: Comparison of Jensen's Alphas in Absolute Values. Median Test

Portfolio 1					Portfolio 2				
	NP-U	NP-G	BEKK	DCC		NP-U	NP-G	BEKK	DCC
ROLL	-0.00319 (0.14)	-0.00010 (0.44)	0.00753 (0.16)	0.01149 (0.01)	ROLL	-0.00126 (0.18)	0.00074 (0.18)	0.00551 (0.27)	0.00320 (0.30)
NP-U		0.00460 (0.00)	0.00754 (0.03)	0.01134 (0.00)	NP-U		0.00233 (0.00)	0.00744 (0.05)	0.00306 (0.09)
NP-G			0.00447 (0.24)	0.00718 (0.02)	NP-G			0.00482 (0.38)	0.00373 (0.29)
BEKK				0.00329 (0.02)	BEKK				-0.00197 (0.79)
Portfolio 3					Portfolio 4				
	NP-U	NP-G	BEKK	DCC		NP-U	NP-G	BEKK	DCC
ROLL	0.00299 (0.29)	0.00141 (0.02)	0.00421 (0.05)	0.00201 (0.17)	ROLL	0.00090 (0.02)	0.00119 (0.02)	-0.00093 (0.69)	-0.00162 (0.92)
NP-U		0.00022 (0.52)	0.00362 (0.08)	0.00220 (0.18)	NP-U		0.00053 (0.12)	-0.00046 (0.75)	0.00079 (0.97)
NP-G			0.00210 (0.15)	0.00068 (0.31)	NP-G			-0.00062 (0.90)	-0.00306 (0.56)
BEKK				0.00048 (0.62)	BEKK				0.00214 (0.99)
Portfolio 5					Portfolio 6				
	NP-U	NP-G	BEKK	DCC		NP-U	NP-G	BEKK	DCC
ROLL	0.00028 (0.41)	0.00118 (0.04)	0.00298 (0.43)	0.00735 (0.10)	ROLL	0.00063 (0.78)	0.00001 (0.35)	-0.00371 (0.09)	-0.00259 (0.04)
NP-U		0.00023 (0.19)	-0.00004 (0.49)	0.00455 (0.15)	NP-U		0.00018 (0.47)	-0.00135 (0.30)	-0.00255 (0.07)
NP-G			-0.00001 (0.64)	0.00486 (0.16)	NP-G			-0.00182 (0.11)	-0.00231 (0.02)
BEKK				0.00105 (0.79)	BEKK				-0.00187 (0.00)

Notes:
This table shows the results of the Wilcoxon signed rank test comparing a pair of Jensen's alpha series obtained from the CAPM using alternative estimators. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models. Alphas are daily in frequency, from October 17, 2003 to December 31, 2009, with absolute values on an annual basis. For each pair, the median difference and in parentheses the corresponding p-value for the null of equal median are reported.

Table 7: Cross-Sectional Estimation

		Daily frequency		Monthly frequency	
		γ_0	γ_1	γ_0	γ_1
ROLL	Estimate	0.0101	0.0821	0.4088	1.6655
	t-stat.	0.181	1.251	0.320	1.502
	Adj. t-stat.	0.181	1.250	0.311	1.459
NP-U	Estimate	-0.0045	0.1012	-0.2203	2.3191
	t-stat.	-0.081	1.568	-0.181	2.210
	Adj. t-stat.	-0.081	1.566	-0.176	2.147
NP-G	Estimate	-0.0054	0.1045	-0.0386	2.1628
	t-stat.	-0.098	1.601	-0.031	1.955
	Adj. t-stat.	-0.098	1.599	-0.030	1.898
BEKK	Estimate	0.0329	0.0566	0.9704	1.1540
	t-stat.	0.615	0.891	0.847	1.196
	Adj. t-stat.	0.614	0.890	0.823	1.161
DCC	Estimate	0.0462	0.0436	0.4642	1.6817
	t-stat.	0.852	0.671	0.410	1.682
	Adj. t-stat.	0.851	0.670	0.398	1.634

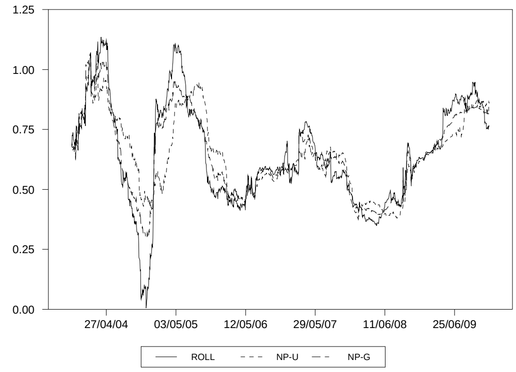
Notes:
This table shows the results of the cross-sectional estimation of the CAPM for all alternative estimators. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models. The Fama and MacBeth methodology is applied to the period from October 17, 2003 to December 31, 2009. The left panel shows daily data (1564 cross-sectional regressions) and the right panel shows monthly returns and the beta estimation for the last day of the previous corresponding month (75 cross-sectional regressions). For each risk premium, the estimate ($\times 10^2$), its t-statistic and the corresponding Shanken-adjusted t-statistic are reported.

Table 8: Out of Sample Variance Comparison for the Global Minimum Variance Portfolio

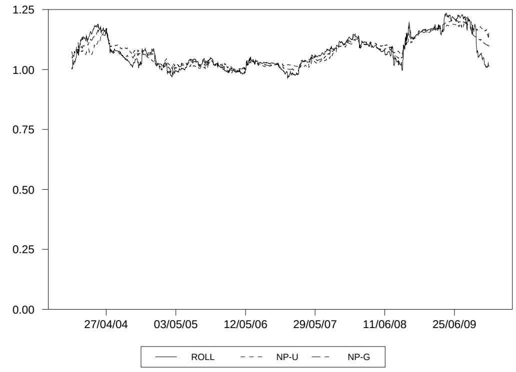
x/y		NP-U	NP-G	BEKK	DCC
	% $x > y$	48.00	70.67	76.00	77.33
ROLL	Median ($\times 10^4$)	-0.3820	0.8594	5.4509	4.7913
	p-value	0.2767	0.0009	0.0000	0.0000
	% $x > y$		73.33	82.67	90.67
NP-U	Median ($\times 10^4$)		1.0738	7.2178	3.9241
	p-value		0.0001	0.0000	0.0000
	% $x > y$			78.67	78.67
NP-G	Median ($\times 10^4$)			4.3094	3.3413
	p-value			0.0000	0.0000
	% $x > y$				42.67
BEKK	Median ($\times 10^4$)				-0.5491
	p-value				0.7077

Notes:
This table shows the results of the Wilcoxon signed rank test when comparing a pair of series of estimated variances of the minimum variance portfolio obtained by alternative estimators. Specifically, the percentage of cases for which x produces higher variance than y, the median differences between the variances and the p-value for the null that this difference is zero are reported. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models.

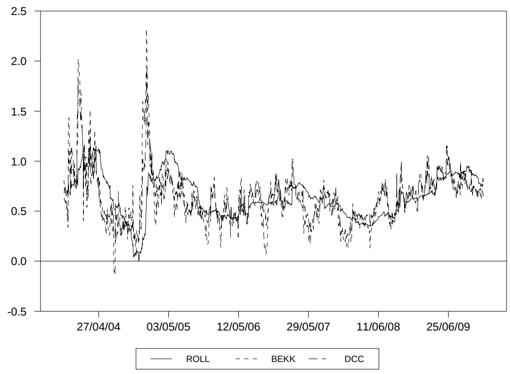
Figure 1: Rolling Beta Estimates vs Alternatives



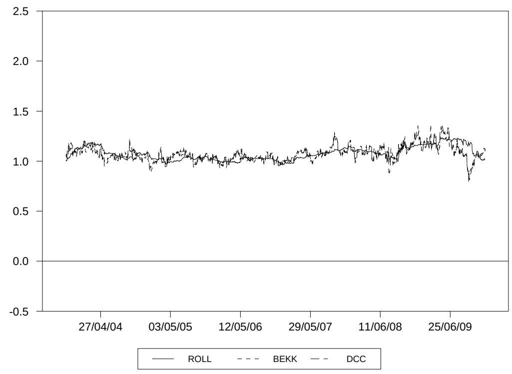
(a) ROLL vs NP (Portfolio 1)



(b) ROLL vs NP (Portfolio 6)



(c) ROLL vs GARCH (Portfolio 1)

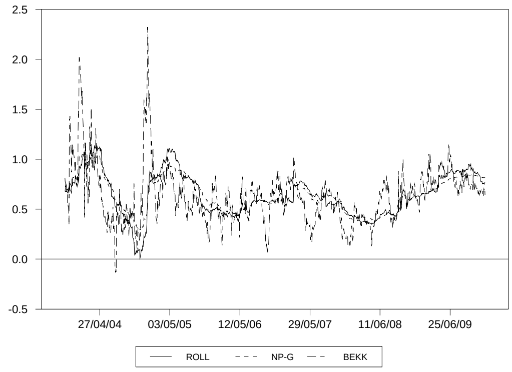


(d) ROLL vs GARCH (Portfolio 6)

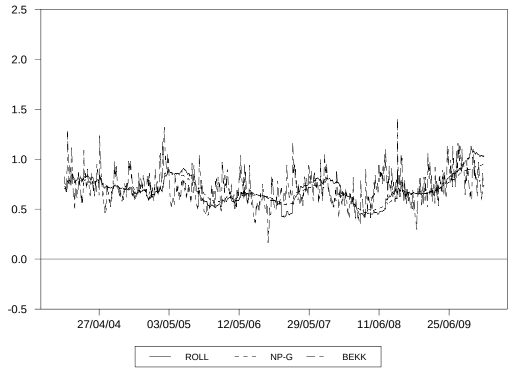
Notes:

This figure shows the rolling beta estimates versus the estimates obtained with the other two estimation methods for Portfolios 1 and 6. ROLL indicates the rolling 120-day window OLS, NP-U and NP-G refer to the nonparametric beta estimates using uniform and Gaussian kernels, respectively, and BEKK and DCC refer to beta estimates computed using covariances and variances from BEKK or DCC GARCH models. Portfolios are constructed by sorting stocks by trading volume in pesos such that Portfolios 1 and 6 contain the least and the most liquid stocks, respectively.

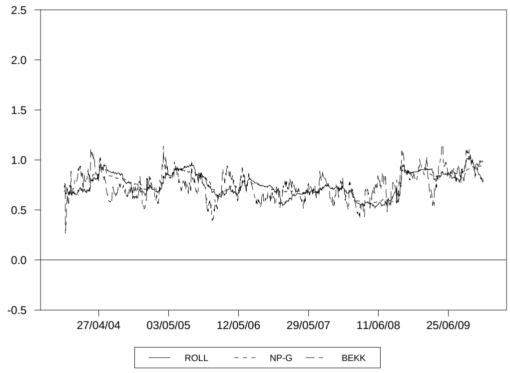
Figure 2: Alternative Beta Estimates for all Portfolios



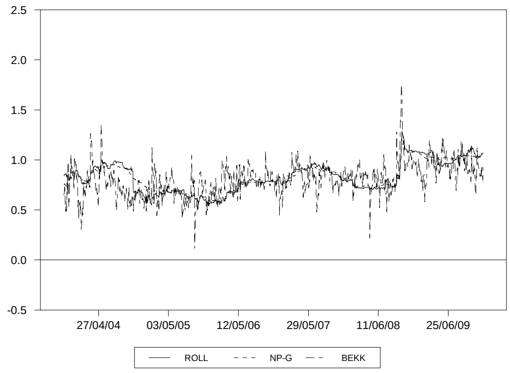
(a) Portfolio 1



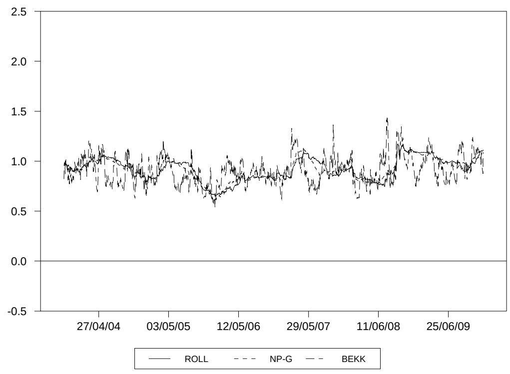
(b) Portfolio 2



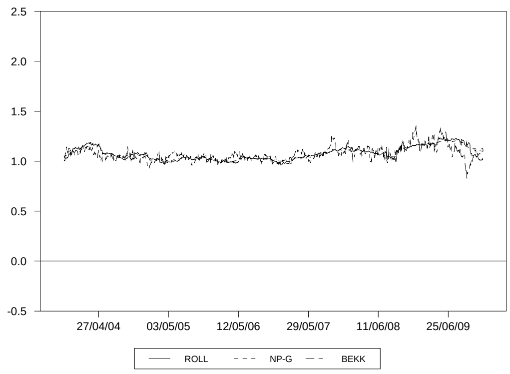
(c) Portfolio 3



(d) Portfolio 4



(e) Portfolio 5



(f) Portfolio 6

Notes:
This figure shows the beta estimates using alternative estimators for Portfolios 1 to 6. ROLL indicates the rolling 120-day window OLS, NP-G refers to the nonparametric beta estimates using a Gaussian kernel and BEKK refers to beta estimates computed using covariances and variances from a BEKK model. Portfolios are constructed by sorting stocks by trading volume in pesos such that Portfolios 1 and 6 contain the least and the most liquid stocks, respectively.