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Abstract

In 2003, Swiss Re introduced a mortality-based security designed to hedge excessive mortality changes for its life book of business. The concern was apparently brevity risk, i.e., the risk of premature death. The brevity risk due to a pandemic is similar to the property risk associated with catastrophic events such as earthquakes and hurricanes and the security used to hedge the risk is similar to a CAT bond. This work looks at the incentives associated with insurance-linked securities. It considers the trade-offs an insurer or reinsurer faces in selecting a hedging strategy. We compare index and indemnity-based hedging as alternative design choices and ask which is capable of creating the greater value for shareholders. Additionally, we model an insurer or reinsurer that is subject to insolvency risk, which creates an incentive problem known as the judgment proof problem. The corporate manager is assumed to act in the interests of shareholders and so the judgment proof problem yields a conflict of interest between shareholders and other stakeholders. Given the fact that hedging may improve the situation, the analysis addresses what type of hedging tool would be best to use. We show that an indemnity-based security tends to worsen the situation, as it introduces an additional incentive problem. Index-based hedging, on the other hand, under certain conditions turns out to be beneficial and therefore clearly dominates indemnity-based strategies. This result is further supported by showing that for the same strike prices the current shareholder value is greater with the index-based security than the indemnity-based security.

Keywords: alternative risk transfer, insurance, default risk

JEL classification: G22, G32, D82

Introduction

As populations grow and change, markets integrate and medical science advances, the character and structure of the risks faced by governments, corporations and individuals also change. The threat of SARS in 2003 and avian flu in 2004 have provided reminders that governments and life insurers face correlated risks on a large scale in events such as pandemics.

In December 2003, Swiss Re introduced a mortality-based security designed to hedge excessive mortality changes for its life book of business.[1] The concern was apparently brevity risk, i.e., the risk of premature death. Brevity risk can be managed with the standard tools as long as there are no correlated mortality surprises. Such would be the case with a recurrence of the Spanish flu or more generally with the occurrence of a new avian flu. The potential for pandemics introduces correlated risks on a large scale and so the potential for mortality surprises. The brevity risk due to a pandemic is similar to the property risk associated with catastrophic events such as earthquakes and hurricanes and the security used to hedge the risk is similar to a CAT bond (Dubinsky and Laster 2003).

The model constructed here is designed to analyze the potential usefulness of mortality-based securities in hedging risk. A publicly held and traded corporation with books of life business is constructed. The corporation may be an insurer or reinsurer, although it will be referred to as a reinsurer throughout most of this article. The organization is structured so that it faces brevity risk in addition to other standard risks such as credit and interest rate risk. Under these conditions, a (re)insurer facing a capital constraint may find a mortality-based security to be a natural risk management tool and therefore turn to the capital markets to hedge these risks. It may also traditionally reinsure its book of business. The model employed here is sufficiently general to allow for both types of instruments to be considered. The focus, however, is aimed at highlighting a design choice that is particularly important in catastrophe bond issues; the question is whether an index or indemnity trigger is used as the underlying for such a transaction.

The literature on Alternative Risk Transfer (ART) explains how the securitization of catastrophic exposures can create value. Some articles have identified the trade-offs involved in the design of optimal risk management programs integrating traditional (re)insurance and ART instruments (Doherty 1997; Froot 1997; Croson and Kunreuther 2000). On the one hand, securitization of insurance risk offers advantages over traditional reinsurance arrangements, such as the potential to substantially reduce moral hazard, credit risk and transaction costs. On the other hand, possible improvements typically come at a cost of the basis risk incurred by an index-linked transaction; this is true since an index cannot perfectly represent the individual risk and would therefore only provide an imperfect hedge.

This recent literature focuses on transactions based upon index triggers. This approach seems justified in light of empirical observations in the CAT bond market: While earlier CAT bond issues were mainly based upon indemnity triggers (which have also traditionally been used in insurance and reinsurance coverage), the last few years indicate a greater market share for indexed instruments (McGhee, Faust and Clarke 2005).[2] As index-based solutions create the problem of basis risk, their recent popularity

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[1] A similar mortality-based instrument was introduced by Swiss Re in April 2005. See Artemis – the alternative risk transfer portal.

[2] A recent study by Guy Carpenter & Company (Guy Carpenter 2005), for instance identifies new risk capital in the amount of $915.3 million ($1.47 billion) that was provided through index-linked CAT bonds in 2004 (2003),

naturally raises the question of why the industry prefers index over indemnity triggers. The straightforward answer is that, besides potentially reducing transaction cost, an appropriately constructed index reduces or eliminates moral hazard. The introduction of a catastrophe index in a CAT bond issue or the use of a population's average life expectancy in a mortality-based security solve the moral hazard problem inherent in almost any insurance or reinsurance transaction.

An index trigger is a new device for addressing moral hazard. If compensation from a reinsurance contract or any other hedging instrument is based upon an index beyond the hedging party's control, this party will still reap the entire benefit of loss control in addition to the hedge. The other party, e.g. a reinsurer or the investors in an insurance-linked security, do not need to be concerned anymore about monitoring the cedent's, respectively the issuer's, risk selection or loss-handling practices. A trade-off results between these benefits and the basis risk incurred by the index instrument.

A few papers have addressed the trade-offs analytically: Cummins and Mahul (Cummins and Mahul 2000) consider an insurance product that is subject to credit risk as well as basis risk,[3] as the insurer's payment is tied to an exogenous index. The interaction between these two factors is also analysed by Richter (Richter 2003) albeit with two different instruments: Insurance, on the one hand, is subject to credit risk but can be used to generate a perfect hedge. Risk securitization, on the other hand, comes without credit risk but incurs basis risk. The analysis shows that under these conditions the indexed security is beneficial whenever the credit risk on the reinsurance exists. As a tool that mainly counteracts reinsurance credit risk, securitization primarily replaces reinsurance for high levels of the loss. The latter result is confirmed by Nell and Richter (Nell and Richter 2004) who study the trade-off between the implicit transaction cost incurred by a reinsurer's risk aversion and the basis risk of a CAT bond.

The trade-off between moral hazard and basis risk has been discussed analytically by Doherty and Mahul (Doherty and Mahul 2001) and Doherty and Richter (Doherty and Richter 2002), who investigate the interaction of these two problems, when insurance can be used to cover the basis risk of an index-linked transaction. It is shown that combining the two hedging tools might extend the possibility set and therefore lead to efficiency gains.

This work will consider the trade-offs an insurer or reinsurer faces in selecting a hedging strategy to maximize current shareholder value. Like Doherty and Mahul and Doherty and Richter we consider index and indemnity triggers; the focus here, however, is on a publicly held and traded corporation acting in the interests of shareholders rather than a manager maximizing expected utility. Rather than considering a mix of hedging instruments, we compare index and indemnity-based hedging as alternative design choices and ask which is capable of creating the greater value for shareholders. Additionally, and very importantly, we model an insurer or reinsurer that is subject to insolvency risk; this risk of insolvency creates an additional incentive problem known as the judgment proof problem. The corporate manager is assumed to act in the interests of shareholders and so the judgment proof problem yields a conflict of interest between shareholders and other stakeholders. The judgment proof problem then yields a situation in which management does not have an incentive to select the socially optimal level of care.

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while new indemnity-based transactions only amounted to 227.5 (260) million. Contrasting this, indemnity-based transactions in 1998 (1997) amounted to $846.1 ($431) million while index-based CAT bonds generated risk capital in the amount of $0 ($202 million).

[3] We refer to one of the risks as credit rather than default risk since the organization that is the object of analysis is not subject to default but rather owns a contract that is subject to default. The recently published version of (Cummins and Mahul 2004), however, does not include the basis risk.

A solution for the underinvestment problem suggested in the risk management literature is that potential creditors demand that the corporation hedges insolvency risk, e.g., (Jensen and Meckling 1976; Smith and Stulz 1985; Mayers and Smith 1987; Froot, Scharfstein and Stein 1993; Garven and MacMinn 1993; MacMinn 2005). This requirement can be enforced, for instance, by adding a covenant to the debt that requires the company to hedge. Given the fact that hedging improves the situation, the following analysis will address, in light of the new financial instruments described above, what type of hedging tool would be best to use. We ask whether one of the two types of hedging that were discussed earlier is better than the other as a solution for the incentive distortions created by insolvency risk. Thus, the primary interest here is in the incentives associated with index versus indemnity-based insurance-linked securities (or other forms of hedging) in a framework in which the issuer faces the risk of insolvency. We consider the impact such securities have on the company's actions, e.g., regarding underwriting.

The analysis here extends previous work by incorporating basis risk and moral hazard in modeling an insurer or reinsurer facing insolvency risk on its books of business. In a financial market setting, we show that an indemnity-based security tends to worsen the situation, as it introduces the additional incentive problem. Index-based hedging, on the other hand, under certain conditions turns out to be beneficial and therefore clearly dominates indemnity-based strategies. This result is further supported by showing that for the same strike prices the current shareholder value is greater with the index-based security than the indemnity-based security.

The financial market model with a life reinsurer's brevity risk is introduced in the next section. The socially efficient operating decisions are also derived here. The following section on triggers and incentives introduces the indemnity and index instruments; there the incentive effects of each are analyzed. The penultimate section compares the current shareholder values for the indemnity and index triggers given the same strike prices on those options.

A Financial Market Model with Brevity and Insolvency Risk

Consider an organization in a competitive economy operating between the dates t = 0 and 1 (MacMinn 1987). The dates t = 0 and 1 are subsequently referred to as *now* and *then*, respectively. Decisions are made *now* and payoffs on those decisions are received *then*. The economy is composed of organizations and risk averse investors. Investors make portfolio decisions on personal account to maximize expected utility subject to a budget constraint. The organization will initially take the corporate form and will be assumed to act on behalf of its principals, i.e., the investors who are shareholders.[4]

In a standard reinsurance transaction the profitability of a contract depends on the cedent's loss control effort or equivalently the level of care. A (re)insurer selects a portfolio of risks and negotiates the contract terms with the insured. This includes required safety and loss reduction operations as well as aspects of product design such as deductibles, retention levels or coinsurance arrangements. When claims arise, a primary settles those claims with its policyholders. Each of these activities and considerations is costly, yet each activity can affect the frequency and severity of claims. If a primary is heavily reinsured or if a reinsurer is covered by a significant retrocession, it still bears the cost of loss reduction, but the other contracting party reaps the benefit. To address

[4] The assumption is only for convenience. The corporate objective function can be derived; for example, see MacMinn (2005).

this incentive conflict, reinsurance has contractual controls. Contracts may be experience rated or retrospectively priced. Additionally, long term and brokered relationships are common in reinsurance which provide further incentive to undertake loss control. We abstract from the plethora of methods available to the reinsurer to manage its risks and focus on a single activity we call the level of care and suppose that increasing the level of care reduces risk.

The insurance company considered will face the standard capital market risks such as interest rate and insolvency risks but will also face life risk. The premium income will be generated *now* and invested in a capital market portfolio. The composition of the organization's portfolio will be determined endogenously and will be shown to depend on the risk management choices made. The losses on the books of business occur *then* and depend on the state of nature revealed. The following partially summarizes the notation used in the development of the model:

ω	state of nature
$\Omega = [0,\zeta]$	set of states
$p(\omega)$	basis stock price *now*
$P(\omega)$	sum of basis stock prices $\varepsilon \le \omega$; $P(\omega) = \int_0^\omega p(\varepsilon)\,d\varepsilon$
$\Gamma(\omega)$	premium income *then* on the book of business; $\Gamma' > 0$ [5]
a	effort spent in composing the book of business measured in dollars
$L(a,\omega)$	loss on book of business
$\Pi(a,\omega)$	payoff on book of business, i.e., $\Pi(a,\omega) = \Gamma(\omega) - L(a,\omega) - a$
$I(\omega)$	mortality index
i	exercise price for mortality security
S	stock value

[5] As the economy improves in state so does the premium income *then* since the premium income is invested.

Suppose the financial markets are competitive. In the absence of any insurance-linked security and any insolvency risk, the stock market value of the (re)insurer may be expressed as the value of its books of business as follows:

$$S(a) = \int_{\Omega} \max\{0, \Pi\}\, dP = \int_{0}^{\zeta} \Pi\, dP \qquad (1)$$

Consider the reinsurer. The reinsurer has the payoff max{0, Π} in the absence of the mortality-based security. The reinsurer may create a mortality-based security for its life book by forming a special purpose entity (SPE) similar to that for a CAT bond; the essence of the security from the perspective of the insurer, however, is the creation of an option that yields a payoff of L(a, ω) dollars in state ω for losses on its life book in excess of a trigger amount i; equivalently, the security pays $\max\{0, L(a,\omega) - i\}$. This is the case if the reinsurer uses an indemnity trigger. Alternatively, if the reinsurer uses an index trigger then the essence of the SPE is the creation of a security that yields an indexed payoff of I(ω) in state ω for losses on its life book in excess of a trigger amount i; equivalently, the index security pays $\max\{0, I(\omega) - i\}$

In an economy such as this the 1958 Modigliani-Miller theorem (Modigliani and Miller 1958) will hold and so mortality-based securities will not, *ceteris paribus*, increase the stock value of the (re)insurer. Once the *ceteris paribus* assumption is relaxed we will investigate the incentive effects of the mortality-based securities and ultimately the impact on value. Additionally, by adding longevity risk and a similar mortality-based security for life annuity books of business, the natural hedge between the (re)insurer's life and annuity books of business might be investigated.

From the reinsurer's perspective the life book of business exposes the corporation to the risk that an insured's life is briefer than expected and so we refer to it as brevity risk. The reinsurer may also be exposed to insolvency risk and credit risk in the financial markets; the insolvency risk introduces the judgment proof problem with the associated incentive problems. The ability to securitize some risk in capital markets introduces a moral hazard problem in the underwriting operation. In this section we concentrate on the life book of business and the associated risks.

First, consider the value of the reinsurer with and without the mortality-based security; we will refer to the mortality-based security more generally as an insurance-linked security (ILS). The unhedged reinsurer has a stock value S^u. If there is insolvency risk in the event of a pandemic then let the state δ be implicitly defined by $\Pi(a,\delta) = 0$. The unhedged stock value may then be expressed as

$$\begin{aligned} S^u(a) &= \int_{\Omega} \max\{0, \Pi(a,\omega)\}\, dP(\omega) \\ &= \int_{\delta}^{\zeta} \Pi(a,\omega)\, dP(\omega) \end{aligned} \qquad (2)$$

Observe that the reinsurer selects the underwriting effort to maximize the current shareholder value. The first order condition is

$$\frac{dS^u}{da} = \int_\delta^\zeta D_1\Pi\left(a^u,\omega\right)dP(\omega)$$

$$= \int_\delta^\zeta \left(-1 - D_1L(a,\omega)\right)dP(\omega)\ ^{6} \tag{3}$$

$$= 0$$

Equation (3) implicitly defines the optimal underwriting activity a^u. The underwriting effort by the reinsurer is assumed to reduce the brevity risk. This assumption is formalized in the following:

Assumption One. The reinsurer's payoff $L(a,\omega)$ satisfies the principle of decreasing uncertainty (PDU) and the PDU is defined by the following derivative properties: $D_2L < 0$ and $D_{12}L > 0$.[7]

After compensating for the change in the mean, the PDU provides a decrease in the risk of the payoff in the Rothschild-Stiglitz sense (Rothschild and Stiglitz 1970; MacMinn and Holtmann 1983).

Next, consider the second order condition. Observe that

$$\frac{d^2S^u}{da^2} = \int_\delta^\zeta D_{11}\Pi\left(a^u,\omega\right)dP(\omega) - D_1\Pi\left(a^u,\delta\right)p(\delta)\frac{d\delta}{da} < 0 \tag{4}$$

The concavity of Π or equivalently the convexity of L suffices to make the first term on the right hand side of (4) negative. The PDU suffices to show that $D_1\Pi(a^u,\delta) > 0$ and

$$\frac{d\delta}{da} = -\frac{D_1\Pi(a^u,\delta)}{D_2\Pi(a^u,\delta)} < 0 \tag{5}$$

Hence, the second order condition holds if the second term on the RHS of (4) is less than the first. It may also be noted that the second order condition reduces to just the first term in the absence of insolvency risk and so the concavity of the payoff suffices to show that the condition holds. We will assume that the second order condition is satisfied in the remaining analysis.

It is useful to compare the underwriting decisions of the firms with and without insolvency risk. Recall that Shavell (Shavell 1986) has described the situation in which an insurer does not possess the resources to cover all losses with certainty as the judgment proof problem. The insurer facing the judgment proof problem does not have the incentive to select the socially efficient level of care as noted in the following claim and proof. Let a^e denote the level of care or equivalently the underwriting choice made by the reinsurer with no insolvency risk.[8]

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[6] $D_1\Pi$ is standard notation for the partial derivative of the function Π with respect to its first argument. Similarly $D_{12}\Pi$ is standard notation for the partial derivative of the function $D_2\Pi$ with respect to its first argument.

[7] See MacMinn and Holtmann 1983 for a description of this principle. It is a mirror image of the principle of increasing uncertainty introduced by Leland, i.e., see Leland, H. (1972). "Theory of the Firm Facing Uncertain Demand." American Economic Review **62**: 278-91.

[8] The socially efficient care is that level that maximizes the value for all stakeholders in the enterprise and so can also be described in situations with insolvency risk as well. Equation (6) would still apply.

Claim: The level of care selected by the reinsurer is greater in the absence of insolvency risk, i.e., $a^e > a^u$

Proof. In the absence of insolvency risk the value is S^e where

$$S^e(a) = \int_0^{\zeta} \Pi(a,\omega)\, dP(\omega) \tag{6}$$

and the first order condition for a socially optimal level of care is

$$\begin{aligned} \frac{dS^e}{da} &= \int_0^{\zeta} D_1\Pi(a^e,\omega)\, dP(\omega) \\ &= \int_0^{\zeta} \left(\Gamma(\omega) - 1 - D_1L(a^e,\omega)\right) dP(\omega) \\ &= 0 \end{aligned} \tag{7}$$

Hence, the claim follows by noting that

$$\begin{aligned} \left.\frac{dS^e}{da} - \frac{dS^u}{da}\right|_{a=a^u} &= \int_0^{\zeta} D_1\Pi(a^u,\omega)\, dP(\omega) - \int_{\delta}^{\zeta} D_1\Pi(a^u,\omega)\, dP(\omega) \\ &= \int_0^{\delta} D_1\Pi(a^u,\omega)\, dP(\omega) \\ &> 0 \end{aligned} \tag{8}$$

follows by the PDU since $D_1\Pi(a^u,\delta)$ is positive and $D_{12}\Pi < 0$ yields $D_1\Pi(a^u,\omega) > 0$ for all $\omega \le \delta$. QED

It may also be noted that the social optimum noted in equation (7) is, with appropriate discounting, equivalent to the optimum noted in the literature by (Shavell 1986; Kahan 1989; MacMinn 2002). The social optimum is the level of care such that the present value of the marginal benefit equals that of the marginal cost, as seen in the following rewrite of equation (7)

$$\begin{aligned} \frac{dS^e}{da} &= \int_0^{\zeta} \left(-D_1L(a^e,\omega) - 1\right) dP(\omega) \\ &= -\int_0^{\zeta} D_1L(a^e,\omega)\, dP(\omega) - \int_0^{\zeta} dP(\omega) \\ &= 0 \end{aligned} \tag{9}$$

The first term on the RHS of the second equality is the marginal benefit or equivalently the present value of the marginal loss reduction and the second term is the marginal cost or equivalently the present value of the last dollar spent on care.

Triggers and Incentives

Next, consider the introduction of insurance-linked securities. The first case considered here is that of a mortality-based bond issued by an SPE. The instrument is designed to pay the reinsurer in the event of mortality surprise, e.g., if the mortality rate is 130% or more of what had been projected. Such an instrument may be constructed with an indemnity, index or parametric trigger. In the indemnity case the payoff from the perspective of the reinsurer would be an option payoff like max{0, L(a, ω) – i} where i is the strike price. In the second case of an index trigger the payoff from the perspective of the reinsurer would be max{0, I(ω) - i} where I(ω) is the loss index.

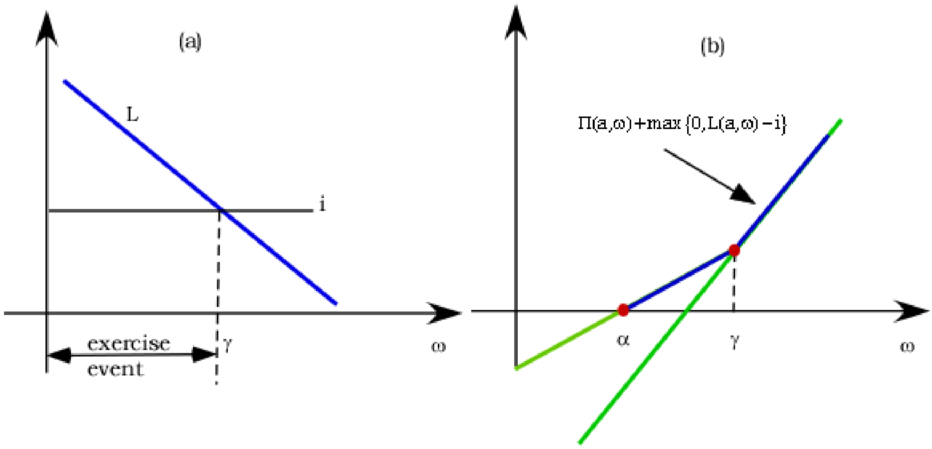
Indemnity trigger

The indemnity trigger case of an insurance-linked security costs C^m dollars now where C^m is the call option price for the coverage. Then

$$\begin{aligned} C^m(a,i) &= \int_\Omega \max\{0, L(a,\omega) - i\}\, dP \\ &= \int_0^\gamma (L(a,\omega) - i)\, dP \end{aligned} \qquad (10)$$

where γ is the boundary of the option's in the money event as shown in figure one.[9]

Figure 1



The stock value *now* of the corporation with this ILS is

$$\begin{aligned} S^m(a,i) &= \int_\Omega \max\{0, \Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\}\, dP \\ &= \int_\alpha^\zeta \left(\Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\right) dP \end{aligned} \qquad (11)$$

where α is the boundary of the insolvency event as shown in figure 1(b). Finally the current shareholder value is S^{mo}

[9] The payoffs in the figures will be represented as linear only due to the authors' limited drawing ability; the analysis does not depend on the linear functions represented in the figures.

$$
\begin{aligned}
S^{mo}(a,i) &= -C^{m}(a,i) + S^{m}(a,i) \\
&= -\int_0^{\gamma}\big(L(a,\omega)-i\big)\,dP + \int_{\alpha}^{\zeta}\big(\Pi(a,\omega)+\max\{0,L(a,\omega)-i\}\big)\,dP \qquad (12) \\
&= -\int_0^{\alpha}\big(L(a,\omega)-i\big)\,dP + \int_{\alpha}^{\zeta}\big(\Gamma(\omega)-a-L(a,\omega)\big)\,dP
\end{aligned}
$$

Let S^{uo} denote the current shareholder value in the unhedged case with no insolvency risk. It follows that if the probability of insolvency is zero so that α is zero then (12) becomes

$$
\begin{aligned}
S^{mo}(a,i) &= -C^{m}(a,i) + S^{m}(a,i) \\
&= -\int_0^{\gamma}\big(L(a,\omega)-i\big)\,dP + \int_{0}^{\zeta}\big(\Pi(a,\omega)+\max\{0,L(a,\omega)-i\}\big)\,dP \\
&= \int_0^{\zeta}\Pi(a,\omega)\,dP \qquad (13) \\
&= S^{uo}(a)
\end{aligned}
$$

and (13) demonstrates that no value is added by the ILS. This is confirmation of the generalized version of the 1958 Modigliani-Miller theorem. It is achieved with the usual *ceteris paribus* assumption; in this case no change in underwriting care due to the ILS is allowed in making the comparison and that assumption will be changed in the subsequent analysis. It may also be noted that the current shareholder value S^{mo} diminishes relative to the unhedged current shareholder value S^{uo} given insolvency risk; this is not a real diminution of value but rather a redistribution of value from shareholders to policyholders or other stakeholders.

$$
\begin{aligned}
S^{mo} - S^{uo} &= -\int_0^{\alpha}\big(L(a,\omega)-i\big)\,dP + \int_{\alpha}^{\zeta}\Pi(a,\omega)\,dP - \int_{\delta}^{\zeta}\Pi(a,\omega)\,dP(\omega) \\
&= -\int_0^{\alpha}\big(L(a,\omega)-i\big)\,dP + \int_{\alpha}^{\delta}\Pi(a,\omega)\,dP \qquad (14) \\
&< 0
\end{aligned}
$$

Incentive effects of the indemnity trigger

The ILS with an indemnity trigger will have an impact on the incentive to take care in underwriting. The indemnity trigger has the effect of full loss coverage in some states and that in turn impacts the underwriting choice; equivalently, a well known moral hazard problem (Shavell 1979) occurs with this form of the ILS. The relationship between the option coverage and the underwriting care will be specified in the function $a^{m}(i)$ where i is the exercise price of the option. The function a^{m} is implicitly defined by the first order condition for the stock value expressed in equation (11).

$$\frac{\partial S^m}{\partial a} = \int_\alpha^\zeta \frac{\partial \Pi}{\partial a} dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP$$

$$= \int_\alpha^\zeta \left(-\frac{\partial L}{\partial a} - 1 \right) dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP \tag{15}$$

$$= 0$$

Suppose the second order condition for a maximum holds. Then the function $a^m(i)$ exists and its derivative is

$$\frac{da^m}{di} = -\frac{\dfrac{\partial^2 S^m}{\partial a\,\partial i}}{\dfrac{\partial^2 S^m}{\partial^2 a}} \geq 0 \tag{16}$$

if the numerator is non-negative. To see that the numerator in (16) is non-negative observe that

$$\frac{\partial^2 S^m}{\partial a\,\partial i} = \frac{\partial}{\partial i}\left(\int_\alpha^\zeta \left(-\frac{\partial L}{\partial a} - 1 \right) dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP \right)$$

$$= -\left(-\frac{\partial L}{\partial a} - 1 \right) p(\alpha)\frac{\partial \alpha}{\partial i} - \frac{\partial L}{\partial a} p(\alpha)\frac{\partial \alpha}{\partial i} + \frac{\partial L}{\partial a} p(\gamma)\frac{\partial \gamma}{\partial i} \tag{17}$$

$$= p(\alpha)\frac{\partial \alpha}{\partial i} + \frac{\partial L}{\partial a} p(\gamma)\frac{\partial \gamma}{\partial i}$$

$$> 0$$

The inequality in (17) follows because α is increasing in i, γ is decreasing in i and L is decreasing in a.[10] Therefore, the inequality in (16) is a strict inequality and $a^m(i)$ is increasing in the strike price i.

Figure one and equation (11) are appropriate for some exercise prices but there are two more generic cases that must also be included. First, there is an exercise price $\hat{i}$ sufficiently large that α equals δ; for larger i the boundary of the insolvency event remains at δ. For $i \geq \hat{i}$ the stock value is that provided in equation (7) and so $a^m(i)$ is a

[10] Note that α is implicitly defined by the condition $\Pi(a,\alpha) + \left(L(a,\alpha) - i\right) = 0$ or equivalently by $\Gamma(\alpha) - a - i = 0$ and so

$$\frac{\partial \alpha}{\partial i} = \frac{1}{\Gamma'(\alpha)} > 0\,.$$

Similarly, γ is implicitly defined by the condition $L(a,\gamma) - i = 0$ and so

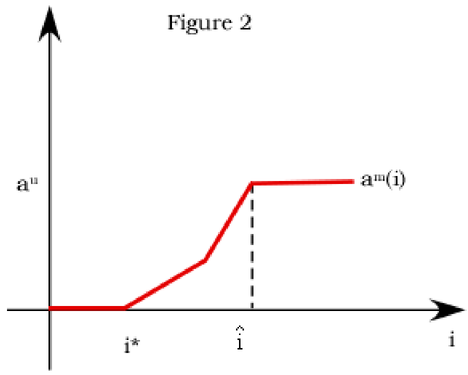
$$\frac{\partial \gamma}{\partial i} = \frac{1}{\dfrac{\partial L}{\partial \gamma}} < 0\,.$$

constant on the interval $i \geq \hat{i}$. Second, there is an exercise price i^* such that α equals zero and so i^* eliminates insolvency risk; for $i \leq i^*$ the stock value is

$$\begin{aligned} S^m &= \int_0^{\gamma}(\Pi + L - i)\,dP + \int_{\gamma}^{\zeta}\Pi\,dP \\ &= \int_0^{\gamma}(\Gamma - a - i)\,dP + \int_{\gamma}^{\zeta}(\Gamma - L - a)\,dP \end{aligned} \tag{18}$$

and again it is apparent that $a^m(i)$ is a constant on the interval $i \leq i^*$; i^* may but need not be as small as zero[11] but at i = 0 the care decision clearly goes to zero as shown in figure two. The results are collected in figure two. Note that $a^m(i)$ is non-decreasing and this is confirmation of a moral hazard problem since an increase in the strike price i is equivalent to less loss coverage and that generates more care in underwriting but more loss coverage generates less care. The figure also suggests that the indemnity trigger ILS cannot provide the incentive for adequate care in underwriting since the maximum care is that for the unhedged case. Clearly, rather than improving or solving the incentive problem the introduction of the indemnity hedge aggravates the problem.

Figure 2



Index trigger

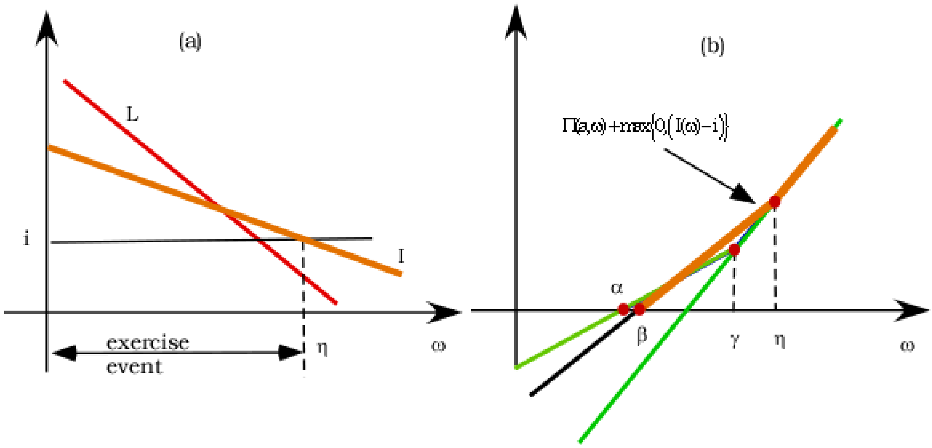
The index trigger case of an insurance-linked security costs C^b dollars now where C^b is the call option price for the coverage. Then

$$\begin{aligned} C^b(i) &= \int_{\Omega}\max\{0, I(\omega) - i\}\,dP \\ &= \int_0^{\eta}(I(\omega) - i)\,dP \end{aligned} \tag{19}$$

where η is the boundary of the in the money or equivalently the exercise event for this option as shown in figure three.

[11] This claim may be justified by direct calculation since γ increases without limit.

Figure 3



The stock value of the corporation with this ILS is

$$\begin{aligned} S^b(a,i) &= \int_\Omega \max\left\{0, \Pi(a,\omega) + \max\left\{0, I(\omega) - i\right\}\right\} dP \\ &= \int_\beta^\zeta \left(\Pi(a,\omega) + \max\left\{0, I(\omega) - i\right\}\right) dP \end{aligned} \tag{20}$$

where β is the boundary of the insolvency event as shown in figure 3(b). The current shareholder value in this case is

$$\begin{aligned} S^{bo}(a,i) &= -C^b(i) + S^b(a,i) \\ &= -\int_0^\eta \left(I(\omega) - i\right) dP + \int_\beta^\zeta \left(\Pi(a,\omega) + \max\left\{0, I(\omega) - i\right\}\right) dP \\ &= -\int_0^\eta \left(I(\omega) - i\right) dP + \int_\beta^\eta \left(\Gamma(\omega) - a + \left(I(\omega) - L(a,\omega)\right) - i\right) dP \\ &\quad + \int_\eta^\zeta \left(\Gamma(\omega) - a - L(a,\omega)\right) dP \end{aligned} \tag{21}$$

Note that the second term on the RHS of the third equality includes the basis risk (I – L). The connection with the generalized 1958 Modigliani-Miller theorem can be made here as in the last case when the insolvency risk is zero.

Incentive effects of the index trigger

The ILS with an index trigger will have an impact on incentive to take care in underwriting. Unlike the indemnity trigger, this instrument does not generate a moral hazard problem but it does generate basis risk. The relationship between the option coverage and the underwriting care will be specified in the function $a^b(i)$ where i is the exercise price of the option. The function a^b is implicitly defined by the first order condition for equation (20)

$$
\begin{aligned}
\frac{\partial S^b}{\partial a} &= \int_\beta^\zeta \frac{\partial \Pi}{\partial a} dP \\
&= \int_\beta^\zeta \left(-\frac{\partial L}{\partial a} - 1 \right) dP \qquad (22) \\
&= 0
\end{aligned}
$$

Suppose the second order condition for a maximum holds. Then the function $a^b(i)$ exists and its derivative is

$$
\frac{da^b}{di} = -\frac{\dfrac{\partial^2 S^b}{\partial a\, \partial i}}{\dfrac{\partial^2 S^b}{\partial^2 a}} \leq 0 \qquad (23)
$$

if the numerator non-positive. Suppose that $\left(\partial I/\partial \omega - \partial L/\partial \omega\right) > 0$ so that the corporate loss distribution has more weight in its tails. Then observe that

$$
\begin{aligned}
\frac{\partial^2 S^b}{\partial a\, \partial i} &= \frac{\partial}{\partial i}\left(\int_\beta^\zeta \left(-\frac{\partial L}{\partial a} - 1 \right) dP \right) \\
&= -\left(-\frac{\partial L}{\partial a} - 1 \right) p(\beta) \frac{\partial \beta}{\partial i} \qquad (24) \\
&< 0
\end{aligned}
$$

The inequality follows because β is an increasing function of i and by the PDU the term in parentheses is positive at β.[12] The inequality in (24) yields a care function $a^b(i)$ that is decreasing in i.

Figure three and equation (20) are appropriate for some exercise prices but again there are two more generic cases that must also be included. First, there is an exercise price $\overline{i}$ sufficiently large that β equals δ; for larger i the boundary of the insolvency event remains at δ. For $i \geq \overline{i}$ the stock value is that provided in equation (2) and so $a^b(i)$ is a constant on the interval $i \geq \overline{i}$. Second, if there is an exercise price $i^\bullet$ such that β equals zero then $i^\bullet$ eliminates insolvency risk; for $i \leq i^\bullet$ the stock value is

[12]Note that α is implicitly defined by the condition $\Pi(a,\beta) + \left(I(\beta) - i\right) = 0$ or equivalently by $\Gamma(\beta) - a - \left(L(a,\beta) - I(\beta)\right) - i = 0$ and so

$$
\frac{\partial \beta}{\partial i} = \frac{1}{\dfrac{\partial I}{\partial \beta} - \dfrac{\partial L}{\partial \beta}} > 0 .
$$

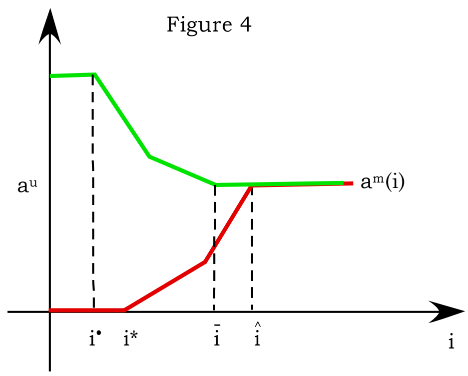
$$S^b = \int_0^\gamma (\Pi + I - i)\,dP + \int_\gamma^\zeta \Pi\,dP$$
$$= \int_0^\gamma (\Gamma - a + (I - L) - i)\,dP + \int_\gamma^\zeta (\Gamma - L - a)\,dP \tag{25}$$

and again it is apparent that $a^b(i)$ is a constant on the interval $i \le i^\bullet$ if an $i^\bullet$ exists which makes $\beta = 0$. If such an $i^\bullet$ exists then below that threshold the care becomes the socially efficient choice as the following derivative shows

$$\frac{\partial S^b}{\partial a} = \int_0^\gamma \left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_\gamma^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP$$
$$= \int_0^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP \tag{26}$$
$$= 0$$

since (26) is equivalent to (7). The results are collected in figure four. Note that $a^b(i)$ is non-increasing and this is confirmation that the moral hazard problem can be eliminated and the incentive distortions due to insolvency risk can be mitigated or eliminated.

Figure 4



The analysis shows that the index trigger dominates the indemnity trigger in the sense that it reduces the insolvency without creating an incentive problem; the indemnity trigger, on the other hand, reduces the insolvency risk but engenders an incentive that tends to increase the insolvency risk. The dominance is investigated in the next section by comparing current shareholder values.

Comparison of Current Shareholder Values

The analysis shows that the insurance-linked security with an index trigger can under certain conditions provide the corporate manager, *ceteris paribus*, with the incentive to take additional care as the level of protection is increased. The security with an indemnity trigger, however, does not align incentives and in fact an additional protection provides the corporate manager, *ceteris paribus*, with an incentive to reduce rather than increase care. Indeed, in the case of the indemnity trigger the care level taken by an unhedged firm provides an upper bound on the care that the manager with this instrument will take. Still, the two triggers provide different current shareholder values first because of the difference in the cost of the protection and second because of the difference in incentive effects.

The goal of this section, therefore, is to compare the current shareholder values with the index versus the indemnity trigger, taking into account the incentives associated with these instruments. Thus we will compare S^{mo} evaluated at $\left(a^{m}(i),i\right)$ and S^{bo} evaluated at $\left(a^{b}(i),i\right)$.

First note that from (21), the current shareholder value in the case of index trigger for a given (a, i) may be rewritten as follows:

$$S^{bo}(a,i) = -\int_0^{\beta}\left(I(\omega)-i\right)dP + \int_{\beta}^{\zeta}\left(\Gamma(\omega)-a-L(a,\omega)\right)dP \tag{27}$$

Second from (12), the current shareholder value in the case of the indemnity trigger for a given (a, i) may be expressed as follows:

$$S^{mo}(a,i) = -\int_0^{\alpha}\left(L(a,\omega)-i\right)dP + \int_{\alpha}^{\zeta}\left(\Gamma(\omega)-a-L\left(a,\omega\right)\right)dP \tag{28}$$

Consider the difference in current shareholder value at the same care and protection level for the standard case in which $\beta > \alpha$.

$$\begin{aligned} S^{bo}(a,i)-S^{mo}(a,i) &= \left[\int_0^{\alpha}\left(L(a,\omega)-i\right)dP - \int_0^{\beta}\left(I(\omega)-i\right)dP\right] \\ &\quad + \left[\int_{\beta}^{\zeta}\left(\Gamma(\omega)-a-L\left(a,\omega\right)\right)dP - \int_{\alpha}^{\zeta}\left(\Gamma(\omega)-a-L\left(a,\omega\right)\right)dP\right] \\ &= \int_0^{\alpha}\left(L-I\right)dP - \int_{\alpha}^{\beta}\left(I-i\right)dP - \int_{\alpha}^{\beta}\left(\Gamma-a-L\right)dP \\ &= \int_0^{\alpha}\left(L-I\right)dP - \int_{\alpha}^{\beta}\left(\Gamma-a-\left(L-I\right)-i\right)dP \\ &> 0 \end{aligned} \tag{29}$$

The inequality in (29) follows since L > I for states in the insolvency event $\omega \le \alpha$ and because the second term on the right hand side of the third equality is negative for all $\omega \in \left(\alpha,\beta\right)$; to see that the second term on the right hand side of the third equality is negative recall that the boundary β of the insolvency event is implicitly defined by the condition $\Gamma(\beta)-a-\left(L(a,\beta)-I(\beta)\right)-i=0$. Hence, $\Gamma(\omega)-a-\left(L(a,\omega)-I(\omega)\right)-i$ increasing in ω and zero at β suffices to show that this integral is negative.

Next, observe that from the inequality in (29) it follows that $S^{bo}\left(a^{b}(i),i\right) > S^{mo}\left(a^{m}(i),i\right)$ if S^{mo} decreases as a is reduced from a^{b} to a^{m}; equivalently, this is the case if S^{mo} is increasing in a. Hence, observe that

$$\left.\frac{\partial S^{mo}}{\partial a}\right|_{a=a^m} = -\int_0^{\alpha} D_1 L\, dP - \big(L(a,\alpha) - i\big)\, p(\alpha)\frac{\partial \alpha}{\partial a} + \int_{\alpha}^{\zeta} \big(-1 - D_1 L\big)\, dP$$

$$- \big(\Gamma(\alpha) - a - L(a,\alpha)\big)\, p(\alpha)\frac{\partial \alpha}{\partial a}$$

$$= -\int_0^{\alpha} D_1 L\, dP + \int_{\alpha}^{\zeta} \big(-1 - D_1 L\big)\, dP - \big(\Gamma(\alpha) - a - i\big)\, p(\alpha)\frac{\partial \alpha}{\partial a} \tag{30}$$

$$= \int_0^{\alpha} dP + \int_0^{\zeta} \big(-1 - D_1 L\big)\, dP$$

$$> 0$$

The third equality follows by noting that α is implicitly defined by the condition $\Gamma(\alpha) - a - i = 0$. The inequality follows by noting that $a^m < a^e$ and so the second term on the right hand side of the third equality is positive. Therefore, we have the current shareholder value larger for the index trigger than the indemnity trigger, i.e., $S^{bo}\left(a^b(i), i\right) > S^{mo}\left(a^m(i), i\right)$.

Concluding Remarks

The analysis begins by noting that insolvency risk in conjunction with limited liability creates an incentive problem known as the judgment proof problem. The manager of a publicly held and traded (re)insurance corporate represents the stockholders interest and the judgment proof problem puts those interests in conflict with those of other stakeholders. Not surprisingly we show that such a manager selects a level of care less than the socially efficient level. The conflict of interest described here also generates an underinvestment problem; while not our focus, that problem does motivate the analysis of some new capital market instruments that have been designed to manage (re)insurer insolvency risk. The new capital market instruments considered here are similar to the CAT bonds discussed in the literature in the sense that they may be designed with triggers that are either indemnity or index based so that the options attached to the bonds are in the money if the reinsurer suffers a sufficiently large loss or if the index of losses, i.e., mortality, is sufficiently large. We study the incentive effects associated with each instrument and show that the index based instrument dominates the indemnity based instrument in the sense that it reduces insolvency risk and provides the corporate manager with the incentive to take more rather than less care. We go on to show that, given the same strike price, the current shareholder value of the index based instrument exceeds that of the indemnity based instrument.

There is a growing literature that is concerned with hedging longevity risk, i.e., the risk of out living ones wealth, e.g., see (MacMinn, Brockett and Blake 2006). For further research we note that to date there has been no similar comparison of mortality-based securities to hedge excessive mortality changes for annuity books of business; such mortality-based securities would, of course, be designed to cover excessive mortality changes in the opposite direction. The concern here would be longevity risk, i.e., the risk of living too long. Mortality improvements are being reported; there has been acceleration in the mortality improvements at older ages in Sweden (Wilmoth, Deegan, Lundstrom and Horiuchi 2000). There has also been some evidence that genetics plays a major role in the ability to survive to extremely old ages and hence that genetic research may yield insights into how to slow the aging process (Strauss 2001). The mortality improvements do yield correlated risks for insurers with life annuity books. To the extent that the improvements can be predicted accurately over the horizon of the life annuities, the longevity risk can be managed by insurers with the standard tools. Life annuities, however, have tails that are quite long and so although the mortality improvements may seem less surprising, the correlated risks are just as problematic.

Survivor bonds (Blake and Burrows 2001) have been suggested as an effective means of managing longevity risk. The survivor bond is essentially a reverse tontine; the bond pays a coupon that is proportional to the number of survivors in a cohort. A basis risk problem might remain depending on how the cohort is structured. An instrument similar to that issued by Swiss Re for brevity risk could also be structured for longevity risk. The Swiss Re instrument is in the money if the mortality rate becomes too large but one could also write a security that would be in the money if the mortality rate became too small.

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