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# Pricing Interrelated Goods in Oligopoly

***Sandro Shelegia***

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*International School of Economics at Tbilisi State University*
*16 Zandukeli Street, Tbilisi 0108, Georgia*
*www.iset.ge e-mail: publications@iset.ge*

# Pricing Interrelated Goods In Oligopoly

Sandro Shelegia*
Department of Economics and Business
Universitat Pompeu Fabra

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**Abstract**

In this paper we propose a two-good model of price competition in an oligopoly where the two goods can be complements or substitutes and each retailer has a captive consumer base *à la* Burdett and Judd (1983). We find that the symmetric Nash Equilibrium of this model features atomless pricing strategies for both goods. When the two goods are complements the prices charged by any retailer are, at least locally, negatively correlated so if one of the goods is priced high the other one is on a discount. This finding is supported by an empirical observation that simultaneous discounts of complements are infrequent. In contrast, if the goods are substitutes or independently valued the prices will be randomized independently unless the less valuable substitute is not sold at all. In the case of complements the retailers earn higher profit relative to the case of selling both goods only as a bundle. The ability to "discriminate" between the captives and the shoppers through keeping the sum of the two prices high while setting one of the prices low drives the result. Such discrimination is impossible when the goods are substitutes as consumers switch to buying the lower priced substitute. Additionally, we provide some insights on bundling in the price dispersion setting.

## 1 Introduction

In this paper we present a stylized model of price competition between retailers selling two interrelated but homogeneous products. To avoid Bertrand outcome we assign each of the competing retailers a captive group of consumers *à la* the second stage of competition in Burdett and Judd (1983). We show that, in simultaneous move Nash

*Email: sandro.shelegia@upf.edu; Website: `http://www.econ.upf.edu/~shelegia`

Equilibrium, the retailers use purely mixed strategies for both prices leading to a price dispersion. If the two goods are complements the prices of the two goods within every shop will be, at least locally, negatively correlated while if they are substitutes or have independent valuations the two prices will be randomized independently.

Theoretical literature on multiproduct price competition is relatively scarce. The inherent difficulty lies in an ability of consumers to mix and match goods offered by different retailers leading to a complicated pattern of choices. The modeling exercise is further complicated if the demand for goods is interdependent. Nevertheless, several authors have tackled the issue of a multiproduct price competition. Armstrong and Vickers (2001) have shown that if the consumers are required to buy all the goods at one shop (a practice usually referred to as "pure bundling") the competition can be modeled in terms of utility offers made by retailers and thus solved relatively easily. The aim of this study is distinctly different: we want to analyze pricing strategies of multiproduct retailers when, at least some, consumers are able to buy products at different shops. This phenomena seems to be widespread (e.g. savvy shoppers who buy a TV set at one shop and a DVD player at another) and Armstrong and Vickers (2001) do not address it in their approach.

Some authors have modeled the multiproduct price competition in differentiated goods where, as in our model, consumers can engage in mixing purchases between shops. Lal and Matutes (1989) have studied price competition in a duopoly selling two independent goods. They find that in equilibrium retailers will charge different but deterministic prices and may even capture the entire consumer surplus of the less mobile consumers. As opposed to our model in Lal and Matutes (1989) goods are differentiated (for some consumers), the consumers differ in their willingness to pay and there is a economies of scale in shopping. [1]. In contrast, we assume that the two goods are homogeneous across retailers and all the consumers have identical preferences. Such setting allows us to analyze the interaction between the demand dependency between goods and the information asymmetry between the consumers.[2] Demand interrelation between goods is not, in itself, sufficient to avoid pricing at the marginal cost so we introduce captive consumers to soften price competition.

We will argue that the multidimensionality of product offering will allow retailers to jointly discriminate between the captive consumers and the shoppers (consumers who

[1] Consumer heterogeneity in preferences is a standard assumption in the multiproduct literature (see Whinston (1990), Chen (1997), Choi and Stefanadis (2001), Hosken and Reiffen (2007) and Denicolò (2000) among others)

[2] In fact the information asymmetry can be made endogenous while keeping all the consumers *ex ante* identical

are able to compare offers of several retailers and buy each good at the lowest price) when the two goods are complements. In the Burdett and Judd (1983) model retailers are selling only one good and thus setting only one price. As a result, while lowering it to attract the shoppers they also lose sure profit they earn from the captives. In contrast, if retailers are selling two complements, they can keep the sum of the two prices constant at the joint reservation value of the two goods (thus ensuring that the profit earned from the captives is unchanged) and lower one of the prices, engaging aggressively in a price competition for the shoppers.

Joint discrimination is impossible if the two goods are substitutes. Unlike complements, substitutes that a retailer sells compete not only with the same goods sold by other retailers but also with each other. As a result, the retailer cannot keep the sum of prices equal to their joint reservation value and start decreasing one of them to compete for the captives as by doing so she induces the captive consumers to buy only the cheaper good. Not only the retailers are not able to earn additional profits through discriminating the two groups of consumers but also they will earn lower profits than they would if all of them were selling only the bundle. When forcing consumers to buy both goods together retailers disallow an implicit competition between the two goods within their store. This phenomenon is unrelated to the competition between retailers and is also true for the monopolist selling two *imperfect* substitutes.

There are numerous empirical studies of the single good price dispersion (see for example Lach (2002)) but recently Hosken and Reiffen (2004) have argued that most of the these are incapable of fully rationalizing price distributions of individual prices. They argue that the multiproduct approach to modeling retail pricing is the key to understanding some aspects of retail behavior (Hosken and Reiffen (2004), p. 144).

There have been some attempts in the marketing literature to do just that (see Mulhern and Leone (1991)). There is a consensus that when one of the complementary goods is on sale the other one is unlikely to be discounted as well. Even thought most of the literature does not directly document this negative relationship there are several attempts to prove that discounting both complementary goods at the same time could be a profitable strategy. Theoretical justification for holding sale on only one of two complements is grounded in the monopoly paradigm: if a shop lowers a price for one of the goods the optimal price for the other rises as the demand for it increases thus having both goods on sale should not be optimal. It is not obvious why, in the first place, the monopolist has to lower one of the prices from its optimal monopoly level so the empirical literature studying price competition grounded on such theoretical bases seems to be vulnerable to criticism. Here we attempt to provide a theoretical model that justifies such reasoning in the oligopoly setting where retailers, in fact, have incentive to lower one of the prices to attract additional shoppers and also realize that the monopoly price for the other good increases subsequently.

Relatively recently, after the availability of scanned data from supermarkets some authors started studying the choices of individual consumers in response to price discounts rather then focusing on the overall store sales. Such approach allows to separate the effect price reduction has on the number of items individual consumer buys from the number of consumers price reduction attracts to the shop. Moreover, it became possible to see which goods consumers purchase together and how this behavior changes in response to price reductions. Van den Poel et al. (2004) have studied consumer decisions based on their basket of purchases and found that: "[s]imultaneous large discounts on both main and complementary products occur rarely..." This finding is strongly in line with the prediction of our model that when one of the products is on the deepest discount the other one is priced high. Van den Poel et al. (2004) also find that: "[t]he situation in which a complementary product is in promotion when the main product is not in promotion does not occur frequently." The "main" product is defined as the highest sales and profit generator. In our model we find that If one of the goods is valued much higher than the other (the case of "Intermediate Complements" in what follows) the interval where the price of the less valuable complement is randomized can be substantially smaller thus leading to the empirical regularity described above.

We will show that in the two-good nonsequential search model prices of the two goods will be randomized and depending on the nature of demand interrelation we expect to find no correlation between them if the goods are substitutes or independently valued and negative correlation if they are complements. The latter result derives from an inability to charge the highest price for each good simultaneously as at such pair of prices none of the complements will be purchased at all. When deciding to carry both substitutes the retailers are not able to charge excessive price for each of them and hence neither of the prices is restricting the other.

The paper is organized as follows: in Section 2 we specify our model and provide some insights on the behavior of the consumers and a monopolist, in Section 3 we solve the oligopoly model for all the cases, in Section 4 we provide discussion of bundling and complementarity in shopping and Section 5 we conclude.

# 2 The Model

## 2.1 Assumptions

What follows is similar to Burdett and Judd (1983). Consider a market with $N$ retailers selling two homogeneous goods labeled $A$ and $B$. Marginal cost of production $c_i$ $(i = a, b)$ is assumed to be constant, independent of a production level of the other good and equal among the retailers. Cost of producing a bundle of one unit of each good will be denoted by $c_{ab}$ and is equal to $c_a + c_b$. All consumers have identical tastes and their mass per retailer is normalized to one. The consumers demand exactly one unit of each good and the triplet $(v_a, v_b, v_{ab})$ describes their reservation prices for one unit of $A$, one unit of $B$ and a bundle composed of one unit of $A$ and $B$. We will assume that the consumers can freely dispose the goods thus $v_{ab} \geq \max\{v_a, v_b\}$ and they get utility of zero if they do not consume anything. While identical in tastes the consumers differ in their shopping behavior and come in two types: proportion $\theta$ of the consumers visits only one retailer at random (we refer to these consumers as "captives") while the rest of the consumers (proportion $1 - \theta$) visit two retailers at random (to be called "shoppers").[3] The shoppers can buy each good at the lowest price they observe without paying an extra transportation cost if they choose to buy goods at different shops. If the retailers charge the same price for a good the shoppers will be allocated equally among them. We allow the two goods to be asymmetric and without loss of generality we assume that

**Assumption 1.** $v_b - c_b \geq v_a - c_a$.

If Assumption 1 holds with the strict inequality we refer to $B$ as the "more valuable" or the "main" good.

To measure the demand interrelation between the two goods we define $\phi$ as the number that solves $v_{ab} = (1 + \phi)(v_a + v_b)$. The goods will be referred to as substitutes if $\phi < 0$, as independently valued if $\phi = 0$ and as complements if $\phi > 0$. Note that the marginal contribution of each good to the value of the bundle is asymmetric when $v_a \neq v_b$. Namely, the consumer gains $v_{ab} - v_a = (1 + \phi)(v_a + v_b) - v_a = \phi v_a + (1 + \phi)v_b$ if she adds $B$ to the previously purchased $A$ and gains $(1 + \phi)v_a + \phi v_b$ in the opposite case. The marginal contribution of each good defines how much can be charged for it when bought on top of the other good and is an important consideration when the retailer wants to induce consumer to buy both goods.

Finally, in order to simplify the treatment of border cases we assume that in the event of indifference the consumers respect the following order: buy both goods, buy

[3] For now $\theta$ will be exogenously given but it can be made endogenous as in Burdett and Judd (1983).

only B, buy only $A$ and do not buy anything.

Firms compete by setting prices for the two goods simultaneously and we look for a symmetric Nash Equilibrium of this model. We will assume that retailers will not, in addition, set a separate price for the bundle. It is equivalent to assuming that the shoppers are able to resale the goods among each other (???). The implications for the model when the retailers are allowed to bundle the two goods are discussed in Subsection 4.1.

## 2.2 Consumer Behavior and The Monopolist

Since each retailer has a monopoly power through the captive consumers, the strategies employed in the oligopolistic equilibrium depend on the pricing behavior of a hyphotetical monopolist facing the captive consumers. Before proceeding to solving the oligopoly model we will illustrate the optimal behavior of the consumers facing any price pair and, subsequently, profit-maximizing strategy of the monopolist. This section will demonstrate that the pricing by the monopolist is fundamentally different depending on whether the two goods are substitutes or complements so we shall solve the oligopoly model for these two cases separately.

Assume a consumer can buy the goods at a price pair $\{p_a, p_b\}$. For the captive consumers these are the two prices charged by the only retailer they visit while for the shoppers each price is the minimum between the prices of each good from the two retailers they have encountered. The consumer has a choice of buying both goods, only A, only $B$ and non at all and gets the surplus of $v_{ab} - p_a - p_b$, $v_a - p_a$, $v_b - p_b$ and 0, respectively.

The consumer will buy both goods iff

$$v_{ab} \geq p_a + p_b \tag{1}$$

$$v_{ab} \geq p_a + v_b \tag{2}$$

$$v_{ab} \geq p_b + v_a. \tag{3}$$

She will buy only good $I$ if $v_i \geq p_i$, $v_i - v_j \geq p_i - p_j$ and $v_{ab} - v_i \leq p_j$ hold at the same time (subscript $J$ denotes the other good). Figure 1 illustrates consumer choices depending on the prices and the sign of $\phi$.

If the two goods are complements the most monopolist can earn when selling both $A$ and $B$ is $v_{ab} - c_{ab}$. In this case Inequality 1 is binding so she can charge any point on

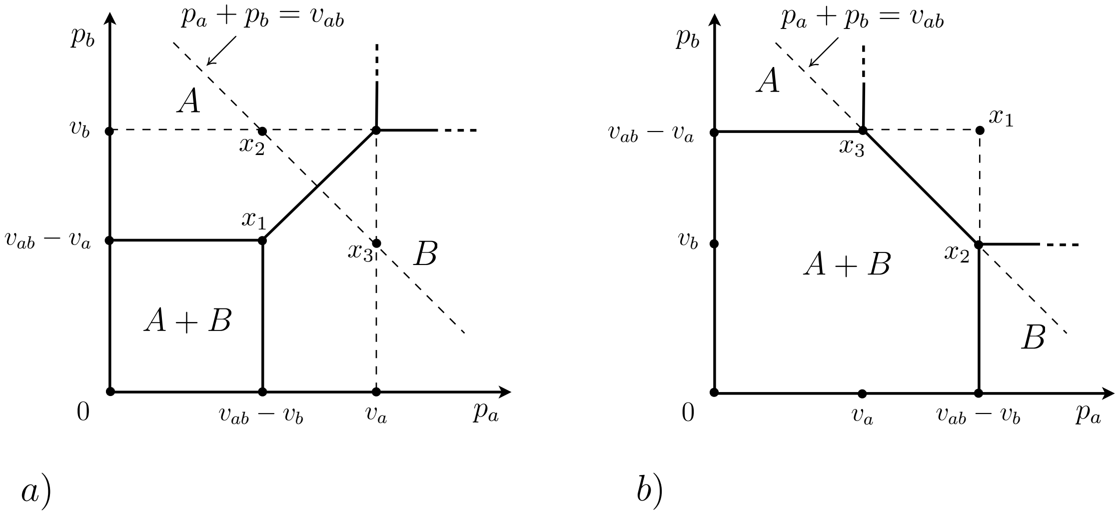


Figure 1: Consumer choice when the goods are *a*) substitutes and *b*) complements. Labels $A$, $B$ and $A + B$ indicate price pairs at which the consumers buy only $A$, only $B$ and both goods, respectively. These areas are delimited with solid lines.

the line connecting $x_2$ and $x_3$ in Figure 1 *b*). It is easy to see that this pricing is only feasible when the goods are complements. Figure 1 *a*) shows that If $v_{ab} = p_a + p_b$ then consumers will not buy the two substitutes together hence, the monopolist is unable to earn $v_{ab} - c_{ab}$. When the monopolist aims to sell both substitutes Inequalities 2 and 3 bind and she earns $(2v_{ab} - v_a - v_b) - c_{ab} < v_{ab} - c_{ab}$ by charging the price pair $x_1$. The inability to earn $v_{ab} - c_{ab}$ is a result of the substitutability between the goods. When the goods are substitutes they effectively compete with each other not allowing the monopolist to extract their joint value from the consumer.[4]

If the monopolist sells only one of the substitutes then she will sell $B$ (recall Assumption 1). She will charge $p_b = v_b$ and any $p_a \geq v_a$ to earn $v_b - c_b$. Hence, if $(2v_{ab} - v_a - v_b) - c_{ab} < v_b - c_b \iff v_{ab} < v_b + \frac{1}{2}(v_a + c_a)$ the monopolist will choose to sell only B. In the opposite case she will sell both goods.

Having verified the pricing by the monopolist we turn to our oligopolistic model. We will consider the complements and substitutes separately as suggested by the monopolist pricing.

[4]In the Subsection 4.1 we discuss implications for the behavior of the monopolist if she can bundle the substittues. It turns out that the monopolist can sell both goods and still earn $v_{ab} - c_{ab}$ if she refuses to sell the goods separately.

# 3 Equilibrium

## 3.1 Complements

In this section we assume that $v_{ab} > v_a + v_b$ ($\phi > 0$). Previously, we have demonstrated that in this case the monopolist will charge such a pair of prices that their sum is $v_{ab}$ (e.g. $p_a = v_a$ and $p_b = v_{ab} - v_a$). Incentive to undercut will make deterministic pricing of this sort impossible in the presence of competing retailers. This pressure on prices is downwards so any retailers will still sell both goods to the captives in equilibrium.

**Proposition 1.** *In any symmetric Nash Equilibrium the retailers will charge only such pairs of prices that the captive consumers buy both goods, that is Inequalities 1-3 hold.*

Imagine the opposite. For simplicity assume that $A$ is the one the captives do not buy. It should be clear that the shoppers will not buy any good that captive do not buy so the retailer will not sell $A$ at all. But one can always lower the price of $A$ to such level that it is still above the marginal cost and the captives buy both goods, a strategy that increases profit. In terms of Figure 1 $B$ is the only good sold if the price point is in region $B$. For any point in this region the retailer can fix the price of $B$ and lower the price of $A$ before the price pair is in the region $A + B$. By doing so the retailer will sell $A$ at a price $v_{ab} - v_b$ that is more than $c_a$ thus increasing profit. More formally:

*Proof.* $B$ is the only good sold to the captives iff all of the following are true

$$\begin{aligned} v_{ab} - v_b &< p_a \\ v_b - v_a + p_a &\geq p_b \\ v_b &\geq p_b. \end{aligned}$$

Let the retailer, instead of $p_a$, charge $\hat{p}_a = v_{ab} - v_b$. It is easy to verify that at $\{\hat{p}_a, p_b\}$ the captive consumers will buy both goods and $\hat{p}_a + p_b - c_{ab} > p_b - c_b$ so the profit earned from the captives will increase. The shoppers were not buying $A$ before and by lowering its price the profit earned from them can only increase. □

In the symmetric equilibrium the cumulative price distributions will be atomless for both prices. If some price was charged with a strictly positive probability, there would be a positive probability of a tie at that price and all the retailers would have an incentive to charge a slightly lower price with the same probability as the old one and serve all the shopper in case of a tie. Not only will the price distributions for each good be atomless, there will be no gaps in the distribution thus each marginal price distribution will be continuous with a closed and connected support. The reason why the distribution function will be gapless is intuitive: charging a price at the lower bound of the gap attracts the shoppers with the same probability as the price at the upper edge of the gap but the latter price gives higher profit. We will denote the interval where the price of $I$ is randomized by $[\underline{p}_i, \overline{p}_i]$.

**Proposition 2.** *In the symmetric equilibrium price for good $i = a, b$ will be randomized according to a continuous distribution function $F_i(p_i)$ defined over an interval $[\underline{p}_i, \overline{p}_i]$.*

*Proof.* See proofs of Propositions 3 and 8 from Varian (1980). □

Next we will argue that the expected profit earned on $I$ from charging any $p_i \in [\underline{p}_i, \overline{p}_i]$ is equal and independent from the price charged for the other good. Imagine a retailer charging a price pair $\{p_i, p_j\}$. The expected profit from charging $p_i$ for $I$ is equal to

$$\pi_i(p_i) = [\theta + 2(1 - \theta)(1 - F_i(p_i))](p_i - c_i) \tag{4}$$

First we argue that either the profit from selling $I$ is constant for all $p_i \in [\underline{p}_i, \overline{p}_i]$ or it is strictly increasing for all $p_i$. The expected profit cannot be decreasing with $p_i$ because if it were then all the retailers would lower prices, a strategy that increases profit earned on $I$ and does not affect the profit earned on $J$.

Next we argue that if $\pi_i(p_i)$ is constant for some $p_i$ then it has to be constant for all $p_i$. The reason is rather simple. If $\pi_i(p_i)$ is constant for some subinterval of $[\underline{p}_i, \overline{p}_i]$ any retailer that chooses $p_i$ from this subinterval can charge any price for $J$ from a narrow enough interval $[\underline{p}_j, \underline{p}_j + \epsilon]$. Expected profit earned on $J$ in that interval has to be constant or otherwise there will be a point mass on the most profitable price in the interval. If the profit is constant in the interval $[\underline{p}_j, \underline{p}_j + \epsilon]$ then retailers that charge such $p_j$ have to be indifferent between charging any price for $I$ or otherwise there will be a point mass in the equilibrium distribution of $p_i$. So if there is a subinterval in $p_i \in [\underline{p}_i, \overline{p}_i]$ where $\pi_i(p_i)$ is constant so it is everywhere else in this interval.

We have established that $\pi_i(p_i)$ is either constant for all $p_i$ or strictly increasing for all of them. If it is constant then so will be $\pi_j(p_j)$ for all $p_j$ thus establishing what we want to prove.

If $\partial\pi_i(p_i)/\partial p_i \neq 0$ for some $p_i$ then $\partial\pi_i(p_i)/\partial p_i > 0$. Under this condition any retailer would find it optimal to increase $p_i$ so either $p_i \leq v_{ab} - p_j$ or $p_i \leq v_{ab} - v_j$ should bind. We will consider each case in turn.

Take any equilibrium price pair $\{p_i, p_j\}$. Assume $p_i \leq v_{ab} - v_j$ binds. This means that $v_j > p_j$. For all such $p_j$ it is optimal to charge $p_i$ and thus there is a point mass at

$p_i$, a contradiction.

Now assume $p_i \leq v_{ab} - p_j$ binds so $p_i + p_j = v_{ab}$. The last condition along with the strict monotonicity of $\partial\pi_i(p_i)/\partial p_i$ implies that $F_i(p_i) = 1 - F_j(v_{ab} - p_i)$. Writing the derivative of the expected profit from selling both goods with respect to the price of $I$ we get

$$\begin{aligned} \frac{\partial\pi}{\partial p_i} = \frac{\partial\left[\theta(v_{ab}-c_{ab}) + 2(1-\theta)\left[(1-F_i(p_i))(p_i-c_i) + F_i(p_i)(v_{ab}-p_i-c_j)\right]\right]}{\partial p_i} &= 0 \Rightarrow \\ \Rightarrow 1 - 2F_i(p_i) + \frac{\partial F_i(p_i)}{\partial p_i}(v_{ab} - 2p_i - c_j + c_i) &= 0 \end{aligned}$$

Recalling $\partial F_i(p_i)/\partial p_i > 0$ we conclude that if $p_i$ is less (more) than $(v_{ab} - c_j + c_i)/2$ then $F_i(p_i)$ is more (less) than $1/2$. The last condition cannot hold for all $p_i$ as it implies that $F_i(p_i)$ is more than $1/2$ for $p_i$ below $(v_{ab} - c_j + c_i)/2$ and less than $1/2$ for $p_i$ above $(v_{ab} - c_j + c_i)/2$ which contradicts the monotonicity of $F_i(p_i)$. With this we show that expected profit from selling $I$ cannot be increasing for all $p_i$.

**Proposition 3.** *The expected profit earned on $I$ from charging any $p_i \in [\underline{p}_i, \overline{p}_i]$ is constant and independent from the price charged for $J$.*

If any retailer is charging $\overline{p}_i$ for $I$ then she will not sell $I$ to the shoppers (other retailers will charge lower price for $I$ with probability one). Hence, she should increase $\overline{p}_i$ until the captive consumers are indifferent between buying the two goods and either buying only $J$ or not buying anything at all

$$\overline{p}_i = \max p_i : \left\{(v_{ab} - v_j \geq p_i) \cap (v_{ab} - p_j \geq p_i)\right\}. \tag{5}$$

It is intuitive that the highest price for $I$ will be charged along with the lowest price for $J$ when $\underline{p}_j \geq v_j$. Assume the opposite so that a pair $\{\overline{p}_i, \hat{p}_j\}$ is charged such that $\hat{p}_j > \underline{p}_j$ while $\underline{p}_j \geq v_j$. The last two inequalities combined imply $\hat{p}_j > v_j$. In the maximization problem in Equation 5 the second restriction will bind so $\overline{p}_i = v_{ab} - \hat{p}_j$. But then, the retailer can charge the pair $\{v_{ab} - \underline{p}_j, \underline{p}_j\}$ and earn higher profit on $I$ without changing the profit earned on $J$ (Proposition 3). From the last argument it follows that if $\underline{p}_j \geq v_j$ then $\overline{p}_i = v_{ab} - \underline{p}_j$. If the highest price for $I$ is restricted by the price of $J$ (in the sense of Equation 5) a retailer should always choose the lowest price for $J$ in order to increase the highest price for $I$ as much as possible.

**Proposition 4.** *If the lowest price for $I$ is higher than the unit valuation for $I$ ($v_i < \underline{p}_i$) then the highest price for $J$ will be charged only with the lowest price for $I$.*

Now consider the case when $(v_j \geq \underline{p}_j)$. It should be clear that if the retailer wants to increase $\overline{p}_i$ as much as possible she should always charge $\overline{p}_i$ with some $p_j \leq v_j$ thus making $\overline{p}_i = v_{ab} - v_j$. If it were to charge $\overline{p}_i$ with some $\hat{p}_j > v_j$ there would be a possibility to increase profits by charging $\{v_{ab} - v_j, v_j\}$ instead. We have established that when $\underline{p}_i \leq v_i$ then $\overline{p}_j = v_{ab} - v_i$.

The previous discussion implies that there are four possible cases when $A$ and $B$ are complements:

1. $\underline{p}_a \geq v_a$ and $\underline{p}_b \geq v_b \Longrightarrow \overline{p}_a = v_{ab} - \underline{p}_b$ and $\overline{p}_b = v_{ab} - \underline{p}_a$. We will call this the case of Strong Complements.

2. $\underline{p}_a > v_a$ and $\underline{p}_b < v_b \Longrightarrow \overline{p}_a = v_{ab} - v_b$ and $\overline{p}_b = v_{ab} - \underline{p}_a$. Intermediate Complements I.

3. $\underline{p}_a < v_a$ and $\underline{p}_b > v_b \Longrightarrow \overline{p}_a = v_{ab} - \underline{p}_b$ and $\overline{p}_b = v_{ab} - v_a$. Intermediate Complements II, impossible due to Assumption 1.

4. $\underline{p}_a \leq v_a$ and $\underline{p}_b \leq v_b \Longrightarrow \overline{p}_a = v_{ab} - v_b$ and $\overline{p}_b = v_{ab} - v_a$. Weak Complements.

Next we will consider each case separately.

### 3.1.1 Strong Complements

In this case $\underline{p}_b > v_b$ and $\underline{p}_a > v_a$. We will demonstrate that the last inequalities hold only if the complementarity is strong enough ($v_{ab}$ is large enough with respect to $v_a + v_b$), hence the name for the case. When $v_{ab}$ is large enough retailers increase $\overline{p}_i$ up to the point when consumers are indifferent between buying both products or not buying anything at all ($v_{ab} - p_a - p_b = 0$). The retailers never have to be concerned that by increasing $\overline{p}_i$ they may induce consumers to switch to buying only $J$ because the prices for both of the goods are above their individual valuations.

Using $\underline{p}_i > v_i$ $(i = a, b)$ along with Proposition 4 gives

$$\overline{p}_a = v - \underline{p}_b, \tag{6}$$

$$\overline{p}_b = v - \underline{p}_a. \tag{7}$$

Since $\overline{p}_i$ never attracts the shoppers and $\underline{p}_i$ attracts them with probability one the expected profit from charging any of this two has to be equal so

$$2(1-\theta)(\underline{p}_i - c_i) = \theta(\overline{p}_i - c_i). \tag{8}$$

Using the latter along with Equations 6 and 7 we get

$$\overline{p}_a = \frac{1}{2}\left[\theta c_a + (2-\theta)\left(v_{ab} - c_b\right)\right] \tag{9}$$

$$\underline{p}_a = \frac{1}{2}\left[(2-\theta)c_a + \theta\left(v_{ab} - c_b\right)\right] \tag{10}$$

$$\overline{p}_b = \frac{1}{2}\left[\theta c_b + (2-\theta)\left(v_{ab} - c_a\right)\right] \tag{11}$$

$$\underline{p}_b = \frac{1}{2}\left[(2-\theta)c_b + \theta\left(v_{ab} - c_a\right)\right] \tag{12}$$

Recall that $\underline{p}_a \geq v_a$ and $\underline{p}_b \geq v_b$ should hold in this case. After some algebra we are left with

$$v_{ab} \geq \frac{2\left(v_a - c_a\right)}{\theta} + c_{ab}$$

$$v_{ab} \geq \frac{2\left(v_b - c_b\right)}{\theta} + c_{ab}.$$

Assumption 1 implies that $\frac{2(v_a-c_a)}{\theta} + c_{ab} \leq \frac{2(v_b-c_b)}{\theta} + c_{ab}$ so the restriction on unit valuations for the case of Strong Complements is given by

$$\frac{2(v_b - c_b)}{\theta} + c_{ab} \leq v_{ab}. \tag{13}$$

At this point we need to verify that the captive consumers buy both goods at all the price pairs charged in equilibrium.

$$\overline{p}_a = \frac{1}{2}\left[\theta c_a + (2-\theta)\left(v_{ab} - c_b\right)\right] \leq v_{ab} - v_b \tag{14}$$

$$\overline{p}_b = \frac{1}{2}\left[\theta c_b + (2-\theta)\left(v_{ab} - c_a\right)\right] \leq v_{ab} - v_a. \tag{15}$$

These reduce to

$$v_{ab} \geq \frac{2\left(v_a - c_a\right)}{\theta} + c_{ab} \tag{16}$$

$$v_{ab} \geq \frac{2\left(v_b - c_b\right)}{\theta} + c_{ab}, \tag{17}$$

the two conditions for the Strong Complements.

As touched upon above, we refer to this case as Strong Complements because if $v_{ab}$ is large enough the price for both goods will always exceed their individual reservation values ($\underline{p}_i > v_i$) and the price range and equilibrium strategies are independent of $v_a$ and $v_b$. The expected profit for a retailer charging price pair $\{p_a, p_b\}$ such that $p_a + p_b \leq v_{ab}$

is given by $\pi_{ab} = \pi_a + \pi_b$ where $\pi_i$ $(i = a, b)$ is given by

$$\pi_i = [\theta + 2(1-\theta)(1 - F_i(p_i))] \, (p_i - c_i). \tag{18}$$

Given that $p_a + p_b \leq v_{ab}$ the expected profit from selling $I$ is constant for all $p_i \in [\underline{p}_i, \overline{p}_i]$ and is equal to $\pi_i = \theta(\overline{p}_i - c_i)$. As a result, the marginal price distribution for $A$ and $B$ in the symmetric equilibrium will be

$$F_a(p_a) = \frac{(2-\theta)\,(2p_a - v_{ab}\theta - (2-\theta)c_a + \theta c_b)}{4(1-\theta)\,(p_a - c_a)} \tag{19}$$

$$F_b(p_b) = \frac{(2-\theta)\,(2p_b - v_{ab}\theta - (2-\theta)c_b + \theta c_a)}{4(1-\theta)\,(p_b - c_b)}, \tag{20}$$

respectively. Any joint distribution $F(p_a, p_b)$ such that $p_a + p_b \leq v_{ab}$ for all price pairs and the derived marginal distributions are $F_a(p_a)$ and $F_b(p_b)$ will form a symmetric equilibrium. It is easy to see that number of such joint distribution functions is infinite. Here we present the simple randomization rule available to any retailer: to randomize price of $A$ according to the marginal distribution function in Equation 19 and set $p_b$ according to some monotonically decreasing function $b(p_a)$ such that the resulting marginal distribution of the price of $B$ is exactly as in Equation 20. Such function exists and is implicitly defined by an equation $F_a(p_a) = 1 - F_b(b(p_a))$. After some manipulation one can check that the function $p_a + b(p_a)$ is decreasing at $\overline{p}_a$, increasing at $\underline{p}_a$ (at both points it is equal to $v_{ab}$) and the derivative $\partial(p_a + b(p_a))/\partial p_a$ changes sign only once on the interval $[\underline{p}_a, \overline{p}_a]$ thus $p_a + b(p_a) \leq v_{ab}$ for all $p_a \in [\underline{p}_a, \overline{p}_a]$. One can introduce slight noise to the function $b(p_a)$ and obtain some other joint distribution function which has the necessary marginals, hence the multiplicity of such functions. Equilibrium marginal densities and the function $b(p_a)$ are illustrated in Figure 2.

The pricing strategies for the two goods are highly dependent. There is no such range of $p_a$ and $p_b$ that the two prices can be randomized independently in that range. Alternatively, for any $p_i \in [\underline{p}_i, \overline{p}_i]$ the restriction $p_i + p_j \leq v_{ab}$ is binding for some $p_j$.

When the goods are Strong Complements retailers earn expected profit of $\pi_{ab} = \theta(2-\theta)\,(v_{ab} - c_{ab})$ that is larger than the profit they would obtain if the two goods were sold only together ($\pi = \theta\,(v_{ab} - c_{ab})$). Intuition for this profit bump is the following: when setting the sum of prices to $v_{ab}$ the retaielrs can surely sell one of the goods to the shoppers by setting its price low enough thus earning $\theta\,(v_{ab} - c_{ab})$ from the captives and $\theta(1-\theta)\,(v_{ab} - c_{ab})$ from the shoppers. If instead, they sold only the composite good, $v_{ab}$

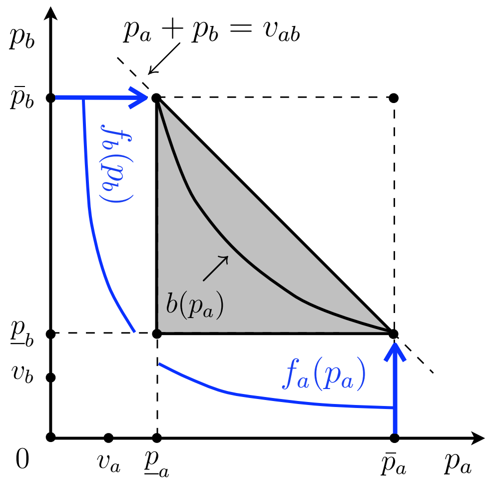


Figure 2: Strong Complements. The shaded area indicates price pairs that can be charged in equilibrium ($p_a + p_b \leq v_{ab}$). The blue axis are the marginal densities for the price of each good.

would be the highest price ever charged and at that price only the captives would buy the bundle giving the retailer the profit of $\theta\,(v_{ab} - c_{ab})$. As this case demonstrates when the two goods are complements the retailers can jointly discriminate between the captives and the shoppers and earn higher profit than when selling the two goods together.

### 3.1.2 Weak Complements

Here we assume that $\underline{p}_i \leq v_i$ holds for $i = a, b$. The goods in this section are called Weak Complements because their individual valuation are large enough (relative to $v_{ab}$) to be higher than at least some prices charged for them. Unlike the case of Strong Complements, here the process of increasing $\overline{p_i}$ stops when consumers are ready to switch to buying only good $J$ and this is possible because $\underline{p}_j \leq v_j$. As $v_{ab}$ will be shown to have to be small enough the independent valuations ($v_{ab} = v_a + v_b$) is included here as a border case.

Recall that $(2-\theta)(\underline{p}_i - c_i) = \theta(\overline{p}_i - c_i)$ so the boundaries for prices will be given by

$$\overline{p}_b = v - v_a \tag{21}$$
$$\overline{p}_a = v - v_b \tag{22}$$
$$\underline{p}_a = \frac{2(1-\theta)c_a + \theta\,(v_{ab} - v_b)}{2-\theta} \tag{23}$$
$$\underline{p}_b = \frac{2(1-\theta)c_b + \theta\,(v_{ab} - v_a)}{2-\theta} \tag{24}$$

We impose $\underline{p}_a \leq v_b$ and $\underline{p}_b \leq v_b$ to get

$$v_a \geq \frac{2(1-\theta)c_a + \theta\,(v_{ab} - v_b)}{2-\theta}$$
$$v_b \geq \frac{2(1-\theta)c_b + \theta\,(v_{ab} - v_a)}{2-\theta}.$$

Rewriting in terms of $v_{ab}$ and remembering Assumption 1 the last two restrictions reduce to

$$v_{ab} \leq \frac{(2-\theta)\,(v_b - c_b)}{\theta} + v_a + c_b \tag{25}$$

The marginal price distributions for $A$ and $B$ in the symmetric equilibrium will be derived as in Subsection 3.1.1 using Equation 18 and we get

$$F_a(p_a) = \frac{p_a - v_{ab}\theta - c_a + \theta c_a + \theta v_b}{(1-\theta)\,(p_a - c_a)} \tag{26}$$
$$F_b(p_b) = \frac{p_b - v_{ab}\theta - c_b + \theta c_b + \theta v_a}{(1-\theta)\,(p_b - c_b)}. \tag{27}$$

The equilibrium joint distribution function should satisfy these conditions: the derived marginal distributions should coincide with the two we have obtained and for all price pairs $\{p_a, p_b\}$ their sum should be no more than $v_{ab}$ $(p_a + p_b \leq v_{ab})$.

Note that when $v_{ab} = v_a + v_b$, that is when goods are independent, $\overline{p}_a = v_a$ and $\overline{p}_b = v_b$ so $p_a + p_b \leq v_{ab}$ for all $\{p_a, p_b\}$. In this case there will be no restriction linking the marginal pricing strategies for the two goods so in the symmetric equilibrium the strategies can be independent and the joint distribution function is written simply as a product of marginal distributions: $F(p_a, p_b) = F_a(p_a)F_b(p_b)$. Figure 4 $c$) illustrates the set of price pairs over which the joint distribution function is be defined along with the marginal density functions for each good.

For any retailer the expected profit in equilibrium will be equal to $\pi_{ab} = \theta(2v_{ab} - v_a - v_b - c_{ab}) \geq \theta(v_{ab} - c_{ab})$ and is at least as large as the profit that obtains when the two goods are sold as a pure bundle.

### 3.1.3 Intermediate Complements

Here we assume that $\underline{p}_b < v_b$ and $\underline{p}_a > v_a$. The two goods are evidently asymmetric here and we will prove that Intermediate Complements exist only when Assumption 1 holds with strict inequality. the latter implies that $B$ is more profitable than $A$ fixing the surplus obtained by consumers. This case is a mixture of the previous two in a sense that $\overline{p}_a$ is constrained by $v_{ab} - v_b$ while $\overline{p}_b$ is constrained by $\underline{p}_a$. For this range of $v_{ab}$ the prices charged for $A$ will always exceed its individual reservation price while for $B$ that will not be true. One can think of a laptop and a laptop bag as an example of Intermediate Complements. Clearly in this case Assumption 1 should hold with a strict inequality and as a result laptop bags will always be overpriced relative to their intrinsic value (i.e. bags of similar quality and shape without a dedicated laptop functionality, a phenomenon widely observed in practice)

We know that $\overline{p}_b = v_{ab} - \underline{p}_a$ and $\overline{p}_a = v_{ab} - v_b$. Remembering that $\overline{p}_a$ attracts only the captives and $\underline{p}_a$ attracts shoppers with probability one we write

$$\underline{p}_a = \frac{\theta(v_{ab} - v_b) + 2(1-\theta)c_a}{2-\theta}. \tag{28}$$

From the previous equation we get

$$\overline{p}_b = v_{ab} - \underline{p}_a = \frac{2(1-\theta)\left(v_{ab} - c_a\right) + \theta v_b}{2-\theta} \tag{29}$$

$$\underline{p}_b = \frac{2(1-\theta)\left(v_{ab}\theta + 2c_b - \theta\left(c_a + c_b\right)\right) + \theta^2 v_b}{(2-\theta)^2}. \tag{30}$$

We impose the restrictions for this case

$$v_a < \frac{\theta(v_{ab} - v_b) + 2(1-\theta)c_a}{2-\theta}$$
$$v_b > \frac{2(1-\theta)\left(\theta v_{ab} + 2c_b - \theta\left(c_a + c_b\right)\right) + \theta^2 v_b}{(2-\theta)^2}$$

to get

$$\frac{2}{\theta}(v_b - c_b) + c_a + c_b > v_{ab} > \frac{(2-\theta)}{\theta}(v_b - c_b) + v_a + c_b. \tag{31}$$

Note that the goods are intermediate complements only if $v_b - c_b > v_a - c_a$. I the two goods are equally profitable then it is impossible that only one of the goods is always sold at a price above its individual reservation price.

The price ranges for Intermediate Complements are illustrated in Figure 4 *b*). We have already exhausted all the possible values of $v_{ab}$. It is trivial to show that the case of Intermediate Complements II cannot occur. Using the same methodology as for Intermediate Complements I conditions on $v_{ab}$ can be derived and they are impossible to fulfill given Assumption 1.

The marginal distribution functions for the price of $A$ and $B$ in the symmetric equilibrium will be

$$F_a(p_a) = \frac{p_a - v_{ab}\theta - c_a + \theta c_a + \theta v_b}{(1-\theta)(p_a - c_a)} \tag{32}$$

$$F_b(p_b) = \frac{p_b - v_{ab}(1-\theta)\theta - c_b + \theta\left((1-\theta)c_a + c_b - \theta v_b\right)}{(1-\theta)(p_b - c_b)} \tag{33}$$

The joint distribution function should have derived marginal distributions as in the previous two equations and for all pairs $\{p_a, p_b\}$ $p_a + p_b \leq v_{ab}$ should hold.

Naturally we verify that the expected profit in this case are larger than when the retailers use pure bundling.

## 3.2 Substitutes

In this section we assume that $v_{ab} \leq v_a + v_b$. In Subsection 2.2 we have demonstrated that the monopolist compares $2v_{ab} - v_a - v_b - c_{ab}$ and $v_b - c_b$ and prices accordingly. If $v_{ab} \geq \frac{1}{2}(v_a - c_a) + v_b$ the prices charged will be $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$, a price pair at which the captive consumers buy both goods. Instead, if $v_{ab} < \frac{1}{2}(v_a - c_a) + v_b$ the prices charged will be $p_b = v_b$ and $p_a \geq v_a$ and captive consumers buy only $B$. It turns out that these two ranges for $v_{ab}$ are important even when the competitors are present. We will call the two goods Weak Substitutes when all the retailers sell both goods which is the case when $v_{ab} \geq \frac{1}{2}(v_a - c_a) + v_b$. When the goods are close enough to being independently valued, all the retailers still choose to sell them both. As the goods become better substitutes the retailers will find it less and less profitable to sell both as this requires lowering both prices and at some point they switch to selling only $B$.

When $v_{ab} < \frac{1}{2}(v_a - c_a) + v_b$ the monopolist would sell $B$ only. We will show that in our model only for some part of this range $B$ is the only good sold while for the rest of the range retailers sometimes sell both goods and sometimes only $B$. The former case will be referred to as Strong Substitutes while the latter as Intermediate Substitutes.

### 3.2.1 Weak Substitutes

Here we assume that in the symmetric equilibrium retailers sell both goods to the captives with probability one. We will prove that this is the case if and only if $v_a + v_b > v_{ab} \geq \frac{1}{2}(v_a + c_a) + v_b$.

If all the retailers sell both goods the distribution function $F_i(p_i)$ will be atomless and defined over a closed and connected support so price of $I$ will be randomized over the interval $[\underline{p}_i, \overline{p}_i]$. In order for the retailers to sell both goods it has to be true that $\overline{p}_a \leq v_{ab} - v_b$ and $\overline{p}_b \leq v_{ab} - v_a$. Note that for each price the condition of selling both goods only depends on the price of that good so provided these are true expected profit earned on each good will be independent of the price of the other. Since the distribution functions are atomless the shoppers will not buy the good priced at $\overline{p}_i$. So any retailer will increase this price up to the maximum possible provided that both goods are sold, that is:

$$\overline{p}_a = v_{ab} - v_b \tag{34}$$

$$\overline{p}_b = v_{ab} - v_a. \tag{35}$$

The expected profit earned in equilibrium will be $\pi = \theta(2v_{ab} - v_a - v_b - c_{ab})$. We have to make sure no retailer wants to deviate and sell only one of the goods. It is obvious that if $I$ is the only good sold then $p_i > \overline{p}_i$, otherwise the retailer can decrease the price of the other good and sell both which leads to higher profit. But if only $I$ is sold to the captives when $p_i > \overline{p}_i$ it will never be sold to the shoppers so $p_i = v_i$. If this is true the retailer will earn $\theta(v_b - c_b)$. Because we want both goods to be sold in equilibrium it has to be the case that $\theta(2v_{ab} - v_a - v_b - c_{ab}) \geq \theta(v_b - c_b) \iff v_{ab} \geq \frac{1}{2}(v_a + c_a) + v_b$.

Now imagine that $v_{ab} \geq \frac{1}{2}(v_a + c_a) + v_b$. We will argue that in this case all the retailers will choose to sell both goods. Assume the opposite so retailers in the symmetric equilibrium sell only $B$ with a positive probability. There will be the highest price that is ever charged for $B$ and it is sold to the captives while the shoppers do not buy anything. Such highest price should be equal to $v_b$ and the profit earned will be $\theta(v_b - c_b)$. Charging $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$ will give more than $\theta(2v_{ab} - v_a - v_b - c_{ab})$ which given $v_{ab} \geq \frac{1}{2}(v_a + c_a) + v_b$ is larger than $\theta(v_b - c_b)$ so selling only $B$ brings strictly less profit than selling both at a price pair $\{v_{ab} - v_b, v_{ab} - v_a\}$.

We conclude that in equilibrium both goods are sold to the captives with the probability one iff $v_{ab} \geq \frac{1}{2}(v_a + c_a) + v_b$

The lowest prices any retailer will charge for $A$ and $B$ are the ones that attract shoppers with probability one and give the same profit as charging the highest price for the good and attracting no shoppers so

$$\underline{p}_a = \frac{2(1-\theta)c_a + \theta\,(v_{ab} - v_b)}{2-\theta} \tag{36}$$

$$\underline{p}_b = \frac{2(1-\theta)c_b + \theta\,(v_{ab} - v_a)}{2-\theta} \tag{37}$$

In equilibrium price for $I$ ($I = A, B$) will be randomized in the interval $[\underline{p}_i, \overline{p}_i]$ according to the marginal distribution

$$F_a(p_a) = \frac{p_a - \theta v_{ab} - c_a + \theta c_a + \theta v_b}{(1-\theta)\,(p_a - c_a)} \tag{38}$$

$$F_b(p_b) = \frac{p_b - \theta v_{ab} - c_b + \theta c_b + \theta v_a}{(1-\theta)\,(p_b - c_b)}, \tag{39}$$

for $A$ and $B$, respectively. The price ranges for Weak Substitutes are illustrated in Figure 5 $a$).

The two prices will be randomized independently as in the case of the independent valuations. Note that the marginal price distributions are identical to those from Weak Complements but the joint distribution function in the latter case can never be independent. Because the goods are substitutes there is no opportunity to "discriminate" between the captives and the shoppers. Monopoly profit from the captives obtains only for one price pair and subsequently the retailers do not have opportunity to keep the monopoly sum constant while lowering one of the prices.

### 3.2.2 Intermediate Substitutes

In the previous section we demonstrated that both goods are always sold iff $v_{ab} \geq \frac{1}{2}(v_a - c_a) + v_b$. So if $v_{ab} < \frac{1}{2}(v_a - c_a) + v_b$ with some probability only one good will be bought by the captives. In this section we consider the case when probability of selling both goods is still more than zero, albeit less than one. We will argue that in the presence of the price competition $A$ will never be the only good sold to the captives.

**Proposition 5.** *When $v_{ab} < \frac{1}{2}(v_a - c_a) + v_b$ good $A$ will never be the only good sold to the captives.*

*Proof.* Assume the opposite so that for some $\{p_a, p_b\}$ $A$ is the only good sold. There are two possibilities: either $v_a \geq p_a > v_{ab} - v_b$ and then it has to be true that $p_b > v_b - v_a + p_a$, or $v_{ab} - v_b \leq p_a$ and then $p_b > v_{ab} - v_a$. Let us consider the latter case first. As before, $B$ will not be bought by the shoppers so the retailer can decrease her price to $v_{ab} - v_a$ and earn strictly higher profit by selling both goods instead of selling only $A$. If $v_a \geq p_a > v_{ab} - v_b$ then decreasing the price of $B$ can only induce the captives and the shoppers to switch to buying $B$ but will never lead to selling both goods. Assume that the shoppers in this case were buying $A$ with a probability $\lambda_A$. Since $p_a > v_{ab} - v_b$ in the case shoppers buy $A$ they do not buy anything else from the other retailers and they get overall surplus of $v_a - p_a$. Now consider setting the price of $B$ at $v_b - v_a + p_a$. Then with the probability $\lambda_A$ the shoppers will buy $B$ instead of $A$. The expected profit will be at least $(v_b - v_a + p_a - c_b)(\theta + 2(1-\theta)\lambda_A)$ which is larger than the previous profit of $(p_a - c_a)(\theta + 2(1-\theta)\lambda_A)$, a contradiction.

□

We have established that either both goods are bought or only $B$ is bought by the captives. It should be clear that when retailers sell only $B$ ($p_b > v_{ab} - v_a$ and $v_b - p_b \geq v_a - p_a$) they will randomize $p_b$ in some interval $[\tilde{p}_b, v_b]$ where $\tilde{p}_b > v_{ab} - v_a$. Charging $\tilde{p}_b$ the retailer will sell $B$ to the captives with the same probability they would sell both goods to them if they were to charge $p_a = v_{ab} - v_b$ and $p_b = v_{ab} - v_a$ so $\tilde{p}_b - c_b = 2v_{ab} - v_a - v_b - c_{ab}$. Since $\tilde{p}_b > v_{ab} - v_a$ we can derive the first condition for intermediate substitutes

$$v_{ab} > v_b + c_a. \tag{40}$$

If the retailer charges the highest price for $B$ she will only sell $B$ and only to the captives so $\overline{p}_b = v_b$ and the quilibrium profit of all the retailers is $\theta(v_b - c_b)$. So when $p_b > v_{ab} - v_a$ the distribution function of $p_b$ is

$$F_b(p_b) = \frac{2p_b - 2c_b(1-\theta) - (p_b + v_b)\theta}{2(p_b - c_b)(1-\theta)} \tag{41}$$

while $p_a$ can be chosen arbitrarily provided that $p_a \geq v_b - v_a + p_b$. Note that the distribution function for $p_b$ coincides with the one from a single good model.

Now we will require that $\tilde{p}_b \geq \frac{\theta v_b + 2(1-\theta)c_b}{2-\theta}$ because no retailer aiming to sell only $B$ would ever charge a price below $\frac{\theta v_b + 2(1-\theta)c_b}{2-\theta}$. Hence, the second condition for Intermediate Substitutes is

$$2v_{ab} - v_a - v_b - c_{ab} \geq \frac{\theta v_b + 2(1-\theta)c_b}{2-\theta} \Rightarrow v_{ab} \geq \frac{v_a + c_a + 3c_b}{2} + \frac{v_b - c_b}{2-\theta}. \tag{42}$$

If $v_{ab}$ is more than the maximum between $v_b + c_a$ and $\frac{v_a+c_a+3c_b}{2} + \frac{v_b-c_b}{2-\theta}$ then both goods will be sold in equilibrium with a positive probability.

Now lets turn to price pairs such that the captives buy both goods. This is the case when $p_a \leq v_{ab} - v_b$ and $p_b \leq v_{ab} - v_a$. In this case, if the price of $A$ is such that it never attracts the shoppers then it will be set to the maximum so $\overline{p}_a = v_{ab} - v_b$. Now assume the retailer is charging the highest price for $B$ of those below $v_{ab} - v_a$. This price will attract shoppers only when other retailers charge $p_b$ above $v_{ab} - v_a$ so the retailer will get the highest profit only when this price is equal to $v_{ab} - v_a$. The expected profit from charging any price pair such that $p_a \leq v_{ab} - v_b$ and $p_b \leq v_{ab} - v_a$ should be equal so for such prices

$$(p_a - c_a)\left[\theta + 2(1-\theta)(1 - F_a(p_a))\right] + (p_b - c_b)\left[\theta + 2(1-\theta)(1 - F_b(p_b))\right] = \theta(v_b - c_b). \quad (43)$$

The expected profit from charging $\{v_{ab} - v_b, v_{ab} - v_a\}$ should be such that

$$\theta(v_b - c_b) = \left[\theta + 2(1-\theta)(1 - F_b(v_{ab} - v_a))\right](2v_{ab} - v_a - v_b - c_{ab}) \quad (44)$$

which defines $F_b(v_{ab} - v_a)$. We know that $F_b(v_{ab} - v_a) = F_b(\tilde{p}_b)$ and we verify that $\tilde{p}_b = 2v_{ab} - v_a - v_b - c_a$ as derived before.

If a retailer charges $p_b = v_{ab} - v_a$ along with some $p_a \leq v_{ab} - v_b$ the profit earned from selling $B$ is equal to

$$\frac{\theta(v_{ab} - v_b - c_a)(v_b - c_b)}{2v_{ab} - v_a - v_b - c_{ab}}. \quad (45)$$

The lowest price charged for $B$ attracts shoppers with probability one and should give the same expected profit so

$$\underline{p}_b = \frac{\theta(v_{ab} - v_b - c_a)(v_b - c_b)}{(2-\theta)(2v_{ab} - v_a - v_b - c_{ab})} + c_b. \quad (46)$$

The distribution function $F_b(p_b)$ for $p_b \in [\underline{p}_b, v_{ab} - v_a]$ is defined by

$$(p_b - c_b)\left[\theta + 2(1-\theta)(1 - F_b(p_b))\right] = \frac{\theta(v_{ab} - v_b - c_a)(v_b - c_b)}{2v_{ab} - v_a - v_b - c_{ab}}. \quad (47)$$

If a retailer charges $p_a = v_{ab} - v_b$ along with some $p_b \leq v_{ab} - v_a$ the profit earned from selling $A$ is equal to

$$(p_a - c_a)\left[\theta + 2(1-\theta)(1 - F_b(v_{ab} - v_a))\right] = \frac{\theta(v_{ab} - v_a - c_b)(v_a - c_a)}{2v_{ab} - v_a - v_b - c_{ab}}. \quad (48)$$

The lowest price charged for $A$ attracts shoppers with probability one and earns the same profit so

$$\underline{p}_a = \frac{\theta(v_{ab} - v_a - c_b)(v_a - c_a)}{(2-\theta)(2v_{ab} - v_a - v_b - c_{ab})} + c_b. \tag{49}$$

The distribution function $F_a(p_a)$ for $p_a \in [\underline{p}_a, v_{ab} - v_b]$ is defined by

$$(p_b - c_b)\left[\theta + 2(1-\theta)(1 - F_a(p_a))\right] = \frac{\theta(v_{ab} - v_b - c_a)(v_b - c_b)}{2v_{ab} - v_a - v_b - c_{ab}}. \tag{50}$$

The price ranges for Intermediate Substitutes are illustrated in Figure 5 *b*). Note that for the range of parameters discussed in this subsection the oligopolistic industry provides both goods to the captives with some probability while the monopolist would only sell $B$. In this range the competition leads to a larger variety offered to consumers.

### 3.2.3 Strong Substitutes

In this section we consider the case when the only good sold to the captives is $B$ (We have established in the previous section that $A$ cannot be the only good sold). We have shown so far that if $v_{ab} \geq \max\left(v_b + c_a, \frac{v_a+c_a+3c_b}{2} + \frac{v_b-c_b}{2-\theta}\right)$ with nonzero probability both goods are sold. As a result we consider the case when

$$v_{ab} \in \left[\max(v_a, v_b), \max\left(v_b + c_a, \frac{v_a + c_a + 3c_b}{2} + \frac{v_b - c_b}{2-\theta}\right)\right].$$

We have shown that for such $v_{ab}$ the retailers will choose to sell only $B$ in the symmetric equilibrium. The most retailer can charge for $A$ if she aims to sell both goods to the captives is $v_{ab} - v_b$ which is either less than the marginal cost $c_a$ or gives less profit than charging $v_b$ and selling only $B$ would. Hence, she will sell only $B$ and earn expected profit of $\theta(v_b - c_b)$ in the equilibrium. The equilibrium distribution of $p_b$ will be atomless and defined over a closed interval. The highest price ever charged will never attract shoppers so it will be equated to $v_b$. As a result, the price of $B$ will be randomized over the interval $[\underline{p}_b, v_b]$ where $\underline{p}_b = \frac{\theta v_b - 2(1-\theta)c_b}{2-\theta}$ according to a distribution function

$$F_b(p_b) = \frac{2p_b - 2c_b(1-\theta) - (p_b + v_b)\theta}{2(p_b - c_b)(1-\theta)}. \tag{51}$$

The retailers randomize $p_b$ as if $B$ is the only good available and charge $p_a$ such that consumers never choose to buy $A$ (for example $p_a = v_a$). The price ranges for Strong Substitutes are illustrated in Figure 5 *c*).

# 4 Discussion

## 4.1 Bundling

Bundling refers to a practice of selling several goods together at a joint price. Pure bundling occurs when the goods are not sold separately but only as a bundle while mixed bundling is, as the name indicates, a strategy when the goods are offered for sale both as separate items and as a bundle. The practice of selling individual goods separately without bundling them is referred to as pure components.

Pure and mixed bundling is a relatively common practice in the retail industry but by no mean is widespread for many types of goods. Its importance in preventing entry or making it less profitable has been stressed by many authors (see Nalebuff (2004)). Nevertheless, theoretical links between bundling and mixed strategies have not been developed. The reason for such negligence is relatively straightforward: when retailers use mixed or pure bundling their expected profit depends not only on the distribution of individual prices but also on the distribution of their sum. If one adds demand interrelation the analysis becomes almost intractable. Venkatesh and Kamakura (2003) have analyzed bundling by a monopolist when consumers have uniformly distributed valuations. When monopolist offers a mixed bundling scheme against these uniform valuations the expected profit she gets is similar to an oligopolist in our setting pricing against a competitor that randomizes prices. Unfortunately, in the equilibrium when goods are complements marginal distributions are not uniform and moreover the distributions are dependent so the expected profit from mixed or pure bundling is hard to compute (Venkatesh and Kamakura (2003) use computer simulation to compute the profits even when valuations are uniform).

Nevertheless, some insights about bundling in our setting can be illustrated. First, let us consider the case of independent valuations ($\phi = 0$). Assume that one of the retailers randomizes the two prices independently according to the distribution functions given in equations 38 and 39. When $\phi = 0$ these densities are identical to the Burdett and Judd (1983) solution for individual goods. It turns out that charging only one price for a bundle $p_{ab}$ above but close enough to $\underline{p}_a + \underline{p}_b$ outperforms any pure components strategy $\{p_a, p_b\} : p_i \in [\underline{p_i}, \overline{p_i}]$ $(i = a, b)$. We know that charging any such price pair will give equal expected profit against equilibrium strategy of the other retailers. Now let one of the retailers instead of randomizing separately charge $p_{ab}$ for a bundle. Figure 3 illustrates the difference between the two pricing strategies. The retailer could also choose any of the price pairs in the grey area, namely she could choose $\{\underline{p}_a, p_{ab} - \underline{p}_a\}$, a price pair that gives the revenue of $p_{ab}$ if both goods are sold. These two strategies lead to the same profit earned on the captives with the only difference between them being the profit earned on the shoppers. When the prices set by the other retailer fall into the area denoted by $X$ both pure bundling and pure components strategies lead to selling the two goods so the profits in this area will be identical. In the area denoted by $Y$ pure bundling still leads to selling both goods while pure components leads to selling only $A$, the difference in profits is in favor of the pure bundling strategy and is equal to $p_{ab} - p_a - c_b$. In the area denoted by $Z$ pure bundling would lead to no sales while pure components would lead to selling $A$. The difference in profits is in favor of the pure components strategy and is equal to $p_a - c_a$. Choosing $p_{ab}$ sufficiently close to $\underline{p}_a + \underline{p}_b$ makes the losses from pure bundling second order while the gains remain first order so pure bundling increases profits.[5]

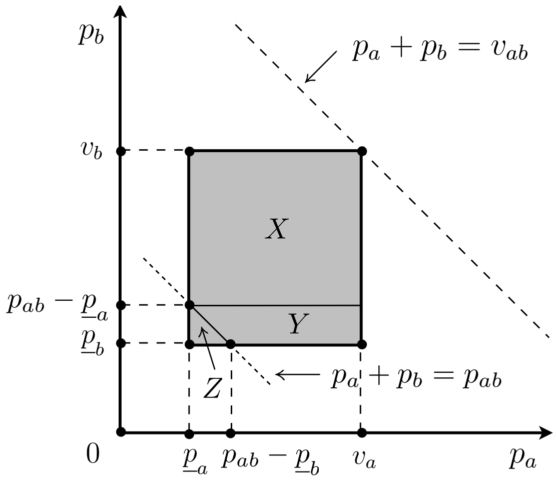


Figure 3: Pure bundling vs. pure components. In region $X$ both strategies give the same profit, in region $Y$ pure bundling gives higher profit and in region $Z$ pure components gives higher profit.

Correlation between prices can change this analysis. If the other retailer positively correlates the two prices then the pure components strategy might be a preferred one as prices would rarely fall in the region $Y$. For example, if the other retailer correlates the two prices with a monotonically increasing function then both pure bundling and pure components give the same expected profit. Negative correlation has an opposite effect as it makes pure bundling more attractive by making region $Z$ unlikely. For example, in the case of Strong Complements one of the solutions involves randomizing $p_a$ and then using

[5]The probability of a price pair chosen by the opponent falling into the area $Z$ is going to zero with a quadratic speed.

a strictly decreasing function $b(p_a)$ to charge $p_b$. If one examines Figure 2 it becomes clear that any pure bundling strategy such that $p_{ab}$ is below the lowest sum of prices of the opponent leads to selling both goods with probability one ($p_{ab} \leq \min_{p_a}\{p_a + b(p_a)\}$). But since $p_{ab} > \underline{p}_a + \underline{p}_b$ this strategy strictly increases profits thus making the pure components equilibrium unsustainable.

When the goods are substitutes we have established that the monopolist would bundle the goods as long as $v_{ab} - v_i > c_j$ for both goods. In the oligopolistic competition this considerations is still there. consider the case of Weak Substitutes. We showed that both goods are sold in equilibrium and the highest price charged for $I$ is $v_{ab} - v_j$. Since all the price pairs give the same profit so does $\{v_{ab} - v_b, v_{ab} - v_a\}$. At this price point none of the goods are sold to the shoppers. If the retailer, instead, bundles the two goods and charges $v_{ab}$ for the bundle she will sell it only to the captives but will earn a strictly higher profit. Surprisingly, bundling is more powerful when the goods are substitutes since the deviation gain can be shown to be deterministic and independent of the joint distribution function characteristics. The kind of bundling deviation that occurs with complements or independent goods can be resolved in the same fashion as it was with the latter: by correlating the prices positively.

There are several way so sustain the pure components equilibrium we have solved. Most importantly, the two goods in question can have different life-cycles leading to existence of consumers who already own one of the goods and are shopping only for the other. If number of such consumers is substantial the small gains that bundling brings can be offset with losses from these consumers who will be less likely to buy a bundle. Clearly we have not dealt with such consumers in our paper but we strongly believe that such an extension will not change the main findings of our model while eliminating bundling as a profitable deviation. This argument seems to be especially appealing when the goods are substitutes, incidentally, the very case when pure bundling is known to strictly increase profits regardless of the opponents strategy.

Furthermore, when the two goods are complements pure components equilibrium features higher profits for all the retailers relative to the pure bundling equilibrium. If the retail industry shares profits with the producers of these two goods in some proportional way, none of the producers have incentive to allow bundling as that would make the competition between retailers harsher and will lead to less profit for the industry.

Lastly, assembling goods into a bundle is a costly exercise for retailers. These costs can outdo the small benefits the bundling brings against randomizing competitors and sustain the pure components equilibrium. On top of the physical costs of bundling there are costs associated with dealing with wholesalers and producers of the goods. Usually these entities run special promotion programs for their goods and these programs would be hard to carry out in tandem with the other suppliers who's good happens to be bundled with supplier's own good.

The considerations listed above makes us think that in many instance bundling is not feasible or desirable strategy for a retailer and can be omitted from our analysis. We do not wish to downplay its importance as incorporating bundling into price dispersion literature could be an important step forward for both lines of research.

### 4.2 Complementarity in Shopping

The crucial assumption that allows as to solve the two-good model relatively simply is the ability of the shoppers to combine purchases from two stores without incurring additional cost. This assumption can be justified for internet shopping or the cases when as Stahl, II (1989) puts it "Casual empiricism suggests that there is a non-negligible measure of consumers who seem to derive enjoyment from shopping itself". It is equivalent to an assumption of costless recall in the sequential search literature as essentially the captives find out prices at two stores and are able to "go back" if one or both prices are lower at the first shop (see Reinganum (1979), Stahl, II (1989), Stahl, II (1996) and Robert and Stahl, II (1993) for the models with sequential search and costless recall).

The immediate problem one faces when introducing the additional cost of buying at two different shops is the inability to separate the pricing strategies for the two goods in any way. The complementary in shopping does not eliminate the possibility to discriminate the shoppers and the captives when the two goods are complements. Rather, it gives an additional ability to do such discrimination without lowering prices too much as the shoppers have a stronger incentive to shop at one place. We were unable to solve our model when these costs are present as this task requires finding the marginal distributions as well as the distribution of the sum of the two prices. It is our opinion that solving the model presented in this paper with an additional cost $t$ incurred by the shoppers when they buy the two goods at different shops will add important insights to a theoretically unfounded argument on economies of scale in shopping being a major consideration when co-pricing goods.

## 5 Conclusion

We have presented a two-good price competition model where the goods are either complements or substitutes. The model exposed substantial difference between demand interrelation. Namely, it was shown that if the two goods are complements retailers are able to discriminate between the more informed consumers (the shoppers) and the less informed ones (the captives) by enticing the former with one of the goods on a deep discount while taxing the latter by keeping the overall price tag high. This practice requires that the retailers are able to charge different combinations of prices that have a fixed sum and yet induce consumers to buy both goods. Through this discrimination the retailers are able to improve their profitability relative to selling the two goods as a bundle. It turns out that even when the two goods are somewhat substitutable retailers lack the above-mentioned space for maneuver and thus are incapable to discriminate between the two groups.

In a symmetric Nash Equilibrium the prices of both goods are randomized in an atomless fashion for the most part of the parameter space apart from the case when the two goods are strongly substitutable in which case the less valuable good is not sold at all. Only when the goods are either independently valued or are substitutes is it feasible that the two prices are randomized independently. When the goods are complements and one of the goods is priced high the other can not be priced in the upper part of its support implying *local* negative correlation between the two prices. The stronger is the complementarity between the goods the more restrictive is the price of one of them for the other. These results are supported by some empirical studies on the pricing of related goods.

We have also demonstrated that bundling is a particularly effective pricing strategy against the opponent's pure components pricing when the goods are complements hence the opponent negatively correlates the prices. Bundling in this case gives ability to sell both goods for sure without charging the lowest price for each of them. This effect is grounded in the nature of the competition and is unrelated to monopoly pricing because a monopolist can achieve maximal profit without bundling the goods. In contrast, when the goods happen to be substitutes the opponent does not correlate prices and the appeal of bundling is purely monopoly based as a retailer not facing any competition would still bundle the two goods to achieve maximal profits. Surprisingly, even when the two goods have independent demand and price of each of them is randomized independently according to a single-good model Nash Equilibrium strategy bundling is a profitable deviation. Unlike the other two cases correlating the two prices positively eliminates this deviation.

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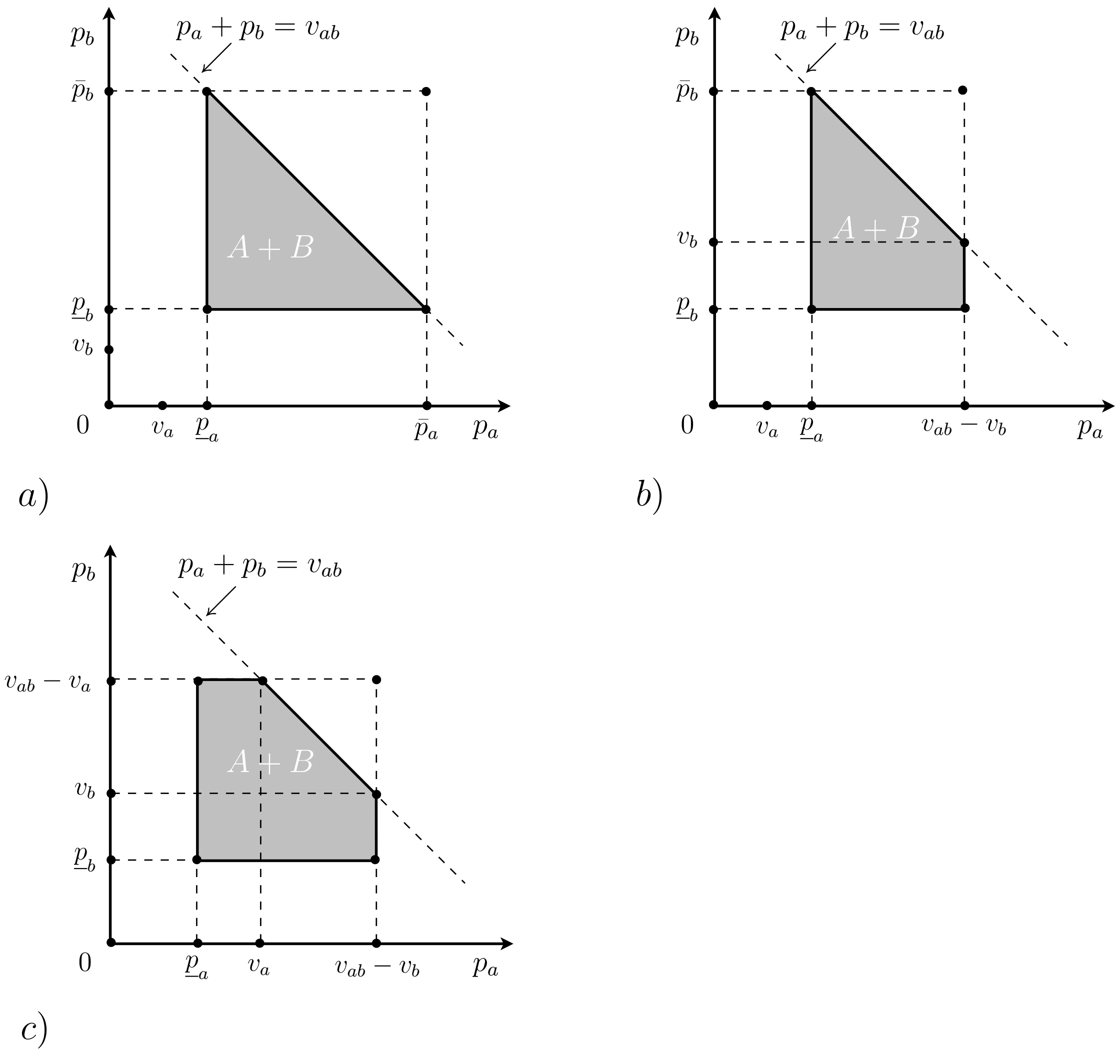


Figure 4: Permitted price pairs ($p_a + p_b \leq v_{ab}$) in the case of strong, intermediate and week complements are illustrated with a shaded area in $a$), $b$) and $c$), respectively.

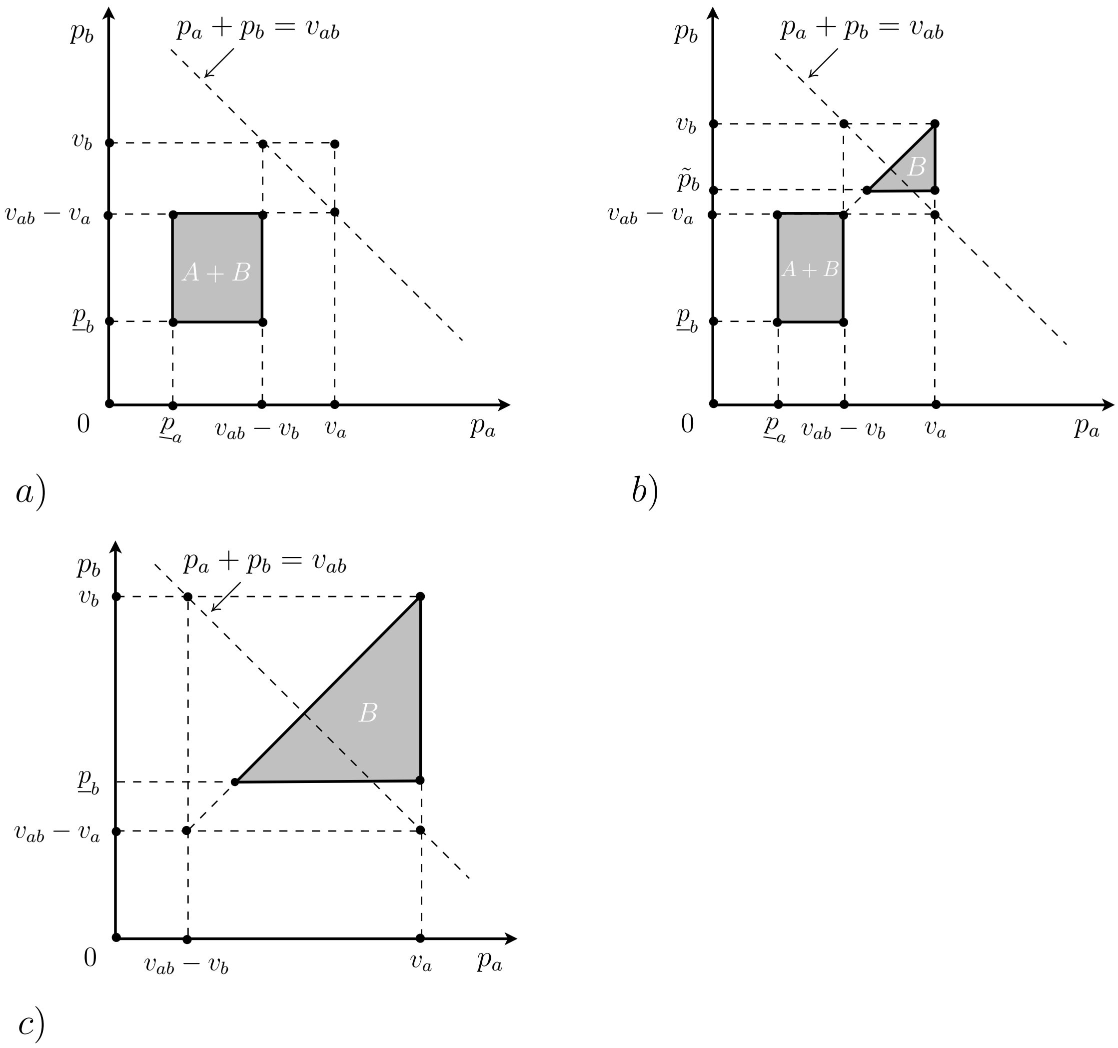


Figure 5: Permitted price pairs in the case of strong and week substitutes are illustrated with a shaded area in $a)$ and $b)$, respectively. Labels $A + B$ and $B$ indicate areas where both goods and only $B$ are sold, respectively.