Heterogeneous dynamics, aggregation and the persistence of economic shocks.

Laura Mayoral*
Institute for Economic Analysis, (CSIC)

Abstract

It has been recently emphasized that, if individuals have heterogeneous dynamics, estimates of shock persistence based on aggregate data are significatively higher than those derived from its disaggregate counterpart. However, a careful examination of the implications of this statement on the various tools routinely employed to measure persistence is missing in the literature.

This paper formally examines this issue. We consider a disaggregate linear model with heterogeneous dynamics and compare the values of several measures of persistence across aggregation levels. Interestingly, we show that the average persistence of aggregate shocks, as measured by the impulse response function (IRF) of the aggregate model or by the average of the individual IRFs, is identical on all horizons. This result remains true even in situations where the units are (short-memory) stationary but the aggregate process is long-memory or even nonstationary. In contrast, other popular persistence measures, such as the sum of the autoregressive coefficients or the largest autoregressive root, tend to be higher the higher the aggregation level. We argue, however, that this should be seen more as an undesirable property of these measures than as evidence of different average persistence across aggregation levels. The results are illustrated in an application using U.S. inflation data.

Key words: Heterogeneous dynamics, aggregation, persistence, impulse response function, sum of the autoregressive coefficients, U.S. inflation persistence.

*I am very grateful to Manuel Arellano, Lola Gadea, Albert Marcet, Thies van Rens, Enrique Sentana, Paolo Zaffaroni and participants in seminars at the Universities of Harvard, MIT, Glasgow, Southampton, Pompeu Fabra, CEMFI and IAE. Financial support from the Spanish Ministry of Education through grant SEJ2006-00369 and from the Fundación Ramón Areces is gratefully acknowledged. The usual disclaimer applies.

1. INTRODUCTION

In this paper we are interested in the measurement of the persistence of economic shocks across different aggregation levels in a context where the underlying processes are allowed to have heterogeneous dynamics. By persistence, we refer to the speed and pattern of adjustment of the process (or processes) of interest to economic shocks of different natures.

Heterogeneous dynamics at the individual level have been found to be important in a wide variety of contexts, such as in the speed of reversion of real exchange rates (Imbs et al., 2005 and Crucini and Shintani, 2008) and of income shocks (Hu and Ng, 2004), in the dynamics of saving behavior (Haque et al., 2000), in inflation dynamics (Altissimo et al., 2006a and the references therein), in labor demand across firms (Zhang and Small, 2006), etc. In fact, one could argue that the existence of some degree of heterogeneity across individuals is likely to be the rule rather than the exception in most contexts.

A recent strand of literature has pointed out that, if the underlying DGP is a dynamic model with heterogeneous coefficients, estimates of shock persistence based on aggregate data are significatively higher than those derived from averaging the corresponding persistence measures computed with disaggregate data. Two explanations have been provided to account for this phenomenon. On the one hand, building on the results of Pesaran and Smith (1995), it has been argued that estimates computed with aggregate data are biased and that the sign of the bias is positive (see Imbs et al., 2005). On the other, influenced by the results in Robinson (1978) and Granger (1980), some authors have suggested that the aggregation of heterogeneous dynamic processes is not an innocuous operation and that it may tend to increase overall shock response (see Altissimo et al., 2009 among others).

Several empirical studies corroborate these arguments by finding estimates of persistence that vary considerably across aggregation levels and are, in general, higher, the higher the level of aggregation (see Imbs et al., 2005, Altissimo et al., 2009, 2006b, Clark, 2006, Lünnemann and Mathä, 2004, etc). In contrast, some authors have reported very similar persistence values across aggregation levels in highly heterogeneous datasets (Crucini and Shintani, 2008, Gadea and Mayoral, 2009). The conclusions of these articles are typically drawn by comparing averages of individual persistence measures with their corresponding

values computed with aggregate data. However, different papers employ different tools to measure persistence and therefore, it is not clear whether these findings are related to specific properties of the considered data or whether the use of a particular persistence measure can systematically bias the conclusions towards a specific direction.

This paper examines the relationship among measures of persistence of aggregate shocks computed at different levels of aggregation. We consider a model with a large number of individual processes that may have heterogeneous dynamics, the dynamic random coefficients model, and we use the most widely employed measures to establish the comparisons, namely, the impulse response function and other popular scalar measures such as the sum of the autoregressive coefficients, the largest autoregressive root, the half-life, etc.

Our results demonstrate that not all the measures routinely employed in applications to establish persistence comparisons fair equally well. We show that the response to an aggregate shock, as measured by the impulse response function (IRF) of the aggregate model or by the average of the individual IRFs, is the same on all horizons with or without individual dynamic heterogeneity. This implies that, according to the IRF, the aggregation of heterogeneous units does not magnify the *average* response to a shock and that one could use averages of individual IRFs (and its related scalar measures) to calibrate macro phenomena.

This conclusion is not in contradiction with previous studies that have shown that the stochastic properties of the aggregate process can differ from those of the individual units under heterogeneity. Thus, as is well known, the aggregation of heterogeneous (short-memory) stationary processes can give rise to an aggregate variable that presents long memory or is even nonstationary (see Robinson, 1978, Granger, 1980 and Zaffaroni, 2004 for a complete characterization of this phenomenon).[1] However, it is shown that the above-described relationship between the disaggregate and aggregate IRF still holds in these set-ups, implying a similar *average* shock response across aggregation levels even when the aggregate process is long memory or nonstationary.

This apparent contradiction highlights the fact that there is no consensus in the literature as to the definition of persistence. This paper emphasizes that, if by persistence one refers

[1] An stationary process is short (long)-memory if its autocorrelation function is (is not) summable.

to the evolution of the response to shocks as described by the IRF, then the existence of individual heterogeneity does not magnify the *average* response to a shock when more aggregated data is considered. Given the empirical relevance of the IRF as a tool to track the evolution of economic shocks, this distinction can be very important in applications.

In contrast, other popular persistence measures, such as the sum of the autoregressive coefficients (SAC), are not invariant to aggregation when individual heterogeneity is allowed for, being typically larger, the higher the level of aggregation. However, we argue that this should be interpreted more as an undesirable property of these measures, rather than as a sign of different average persistence across aggregation levels. The reason is the following: the SAC was introduced as a summary of the IRF because it has a direct relation with the cumulative impulse response (CIR), namely, SAC=1-CIR^{-1} (Andrews and Chen, 1994). Typically, when applied to disaggregate data, the SAC is computed for each of the micro-units and its average is used as a measure of micro-persistence. Then, this value is compared to the SAC obtained with aggregate data. However, since the relation between the CIR and the SAC is not linear, the average SAC has not a direct connection with the average CIR. We show that, while the average CIR remains constant across aggregation levels, a straightforward application of Jensen's inequality ensures that the average SAC tends to increase systematically with the level of aggregation. Thus, by relying on the latter measure, one could conclude that the average response to a shock increases with the level of aggregation when, in fact, a more thorough analysis of the IRF would suggest the opposite. Similar problems appear when other tools, such as the largest autoregressive root (LAR), are employed for these purposes.

The previous results have some important implications on macroeconomic modelling. Some authors have warned against the practice of using averages of microparameters to calibrate macro models (Altissimo et al., 2009). Our findings qualify this statement by showing that the adequacy of this procedure entirely depends on the type of microparameter employed. Thus, averages of individual IRFs can be used in macro calibration independently of whether there is heterogeneity or not at the individual level or whether the aggregate process is long memory or nonstationary while the individual units are stationary. However, as mentioned above, this conclusion cannot be extended to other popular parameters.

The findings detailed above are true for the population. We have further explored whether good sample counterparts of the relevant aggregate and disaggregate persistence measures can be obtained. Two issues have been analyzed. Firstly, to derive the above-described results, it has been assumed that the aggregate process equals the expected value of the individual models. However, aggregate data is usually computed as a (possibly weighted) average of the individual data. We review the conditions that are needed for the holding of a Law of Large numbers relating the sample mean and the expected value of the individual relations. Secondly, it has been argued that under individual dynamic heterogeneity it can be difficult to obtain good estimates of the aggregate model because it usually has a very complicated dynamic structure (Pesaran and Smith, 1995). We show that standard estimation methods allow one to obtain estimates of the IRF computed with aggregate data with good large and finite sample properties. Interestingly, we show that in situations where persistence is high, the finite sample properties of aggregate estimates can improve, rather than deteriorate, as the degree of individual heterogeneity increases. Another important implication of our theoretical results is that not only aggregate but also disaggregate data can be employed to obtain estimates of the aggregate IRF. Moreover, it is shown that the use of the latter type of data can bring about important efficiency gains.

Finally, to illustrate the theoretical results, U.S. inflation data at different levels of aggregation has been analyzed and it has been shown that the IRF remains fairly constant across them. Furthermore, we find that other measures, such as the SAR, systematically increase with the level of aggregation, as reported in previous studies. Thus, by focusing only on the latter set of measures, one can easily conclude that the effect of aggregate shocks changes across aggregation levels when, in fact, a careful examination of the IRF would suggest the opposite.

The structure of this paper is as follows. Section 2 presents a simple model for the individual data, the dynamic random coefficients model (RCM), and defines some tools for measuring the average impact of an aggregate shock on the economy. Section 3 compares the measures of average micro-persistence derived in Section 2 with those obtained from the corresponding aggregate model. Section 4 considers aspects that are important for the empirical application of the results derived in Section 3, such as the holding of a Law of

Large Numbers that relates the aggregate model (obtained as the expected value of the micro-relations) with real data (defined as a weighted sum of individual variables), as well as a discussion of the asymptotic and finite sample properties of the estimators of the IRF under heterogeneity. Section 5 extends the results obtained in previous sections in several directions. Section 6 presents an empirical illustration that analyzes the persistence of U.S. inflation data at different levels of aggregation and Section 7 concludes.

2. HETEROGENEITY AND PERSISTENCE AT THE MICRO-LEVEL

Consider the problem of assessing the impact of an aggregate shock when micro data is available and the different units can be heterogeneous. This section presents a standard univariate disaggregate model and reviews some measures of shock persistence that can be employed, or have been employed in the past, for analyzing this issue. Section 3 applies similar measures to the aggregate model and compares the two sets of results, while Section 5 extends the analysis to the multivariate setting.

A simple but commonly postulated model for microeconomic behavior that allows for heterogeneous dynamics is the Random Coefficients Model.[2] It describes an economy in which each unit i satisfies a linear regression with a Koyck lag given by:

$$y_{it} = a_i y_{it-1} + b_i' x_{it} + \nu_{it},\ i = 1, ..., N,\ t = 1, ...T, \tag{1}$$

$$\nu_{it} = \rho_i u_t + \varepsilon_{it}, \tag{2}$$

where t denotes time, y_{it} and x_{it} are observable variables and $a_i = \bar{a} + \eta_i^a$, $b_i = \bar{b} + \eta_i^b$, $\rho_i = 1 + \eta_i^\rho$ are unknown coefficients, where η^k, for $k \in \{a, b, \rho\}$, are mutually independent, zero-mean random variables with variance σ_k^2. The innovation ν_{it} is the sum of two orthogonal, zero-mean martingale difference sequences, one common to all agents and one idiosyncratic, with variances $\sigma_u^2 > 0$ and σ_ε^2, respectively. The distribution of a has bounded support in the interval (-1,1] and it is assumed that $E_I\left(a^h\right)$ exists for all h, where $E_I(.)$ denotes expectation across the distribution of individuals.[3] The process x_{it} is assumed to be independent of u_t

[2] See Hsiao and Pesaran (2007) for a recent survey and Pesaran et al. (2002) and Pesaran (2006) for details on the estimation of this model.

[3] A sufficient, although not necessary, condition for this assumption to hold for all h is that the support of a is strictly contained in the interval [-1,1].

and the expectation $E_I(b'x_t)$ is assumed to exist. If x_{it} is just a constant, then (1) is simply the first-order autoregressive model. The distribution of agents can be discrete or continuous and the number of agents is assumed to be countably or uncountably infinite.

Suppose now that, at time t, a unitary aggregate shock occurs and one is interested in measuring its *average* impact over time on a system like (1), populated by heterogeneous individuals. For each unit i, the impact of this shock can be evaluated through the IRF, defined as the difference between two forecasts (see Koops et al., 1996)

$$IRF^i(t,h) = E(y_{it+h}|u_t = 1; z_{it-1}) - E(y_{it+h}|u_t = 0; z_{it-1}) \tag{3}$$

where the operator E(.|.) denotes the best mean squared error predictor and $z_{it-1} = \left(\begin{array}{cccccc} y_{it-1}, & y_{it-2}, & ... & x_{it-1}, & x_{it-2}... \end{array} \right)'$. Application of this definition to (1) yields

$$IRF^i(t,h) = \rho_i a_i^h, \text{ for } h \geq 0. \tag{4}$$

The average response to this aggregate shock can be computed as the expected value of (4) over the distribution of units, that we denote as *disaggregate IRF* (IRF_{dis}), given by

$$IRF_{dis}(t,h) = E_I(IRF(t,h)) = E_I\left(a^h\right), \text{ for } h \geq 0, \tag{5}$$

since $E_I(\rho)$ has been normalized to 1. For the simple DGP considered in this section, IRF_{dis} is given by the h-th moment of the distribution of a.

Since the IRF is an infinite vector of numbers, it is a rather unwieldy measure of persistence. For this reason, scalar measures are frequently preferred, such as the Cumulated Impulse Response (CIR), which evaluates the total cumulative effect of a shock over time, and the half life (HL), defined as the number of periods it takes until half the effect of a shock dissipates. Using expression (5), it is straightforward to define the corresponding versions of these measures at the micro level. The average cumulative response (denoted as CIR_{dis}) can be computed as

$$CIR_{dis} = \sum_{h=0}^{\infty} IRF_{dis}(t,h), \tag{6}$$

which, for the simple model considered in this section, yields $\text{CIR}_{dis} = E_I\left(\frac{1}{1-a}\right)$. As for

the *disaggregate* HL (HL_{dis}), it can be defined as the value of h that verifies[4]

$$IRF_{dis}(t,h)|_{h=\mathrm{HL}_{dis}} = 0.5. \tag{7}$$

In many applications average shock persistence is evaluated using the *sum of the autoregressive coefficients* (SAC) and the *largest autoregressive root*. Typically, they are computed for each individual time series and, then, averages (or distribution quantiles) are reported as measures of average *micro* persistence; See Altissimo et al. (2006a), Bilke (2005), Clark (2006), Lünnemann and Mathä (2004), etc.

Finally, other authors have employed an alternative definition of 'disaggregate' IRF to the one presented in this section to draw their conclusions about persistence. This is the case of Imbs et al. (2005), who first construct an 'artificial' representative agent model, y_t^*, that has the same autoregressive structure as the units y_{it} but whose AR coefficients are given by the average of the individual-specific coefficients. Then, they compute the 'disaggregate' IRF by applying the standard definition of the IRF in (3) on y_t^*.

In the following section we assess the performance of the above-defined measures of persistence when used to compare persistence across aggregation levels.

3. INDIVIDUAL HETEROGENEITY AND PERSISTENCE AT THE MACRO LEVEL

This section compares the response to the same aggregate shock over time when it is computed from the micro model (1) or from its corresponding aggregate process. To establish the relation, we first focus on the IRF and, then, the behavior of other popular persistence measures is also examined. For the sake of clarity, this section deals with a very simple model. Section 5 will extend the present analysis by relaxing some of the assumptions that will be imposed below.

How is the IRF computed from aggregate data related to the average of the individual impulse responses defined in Section 2? In a representative agent economy, the answer is trivial. This is because the individual and the aggregate models share the same dynamics

[4] Notice that, since the HL is a nonlinear function of the IRF, the average of the individual HLs does not coincide, in general, with the HL associated with IRF_{dis} defined in (7).

so the IRFs derived from each model are also the same. When the individuals are heterogeneous, however, the dynamics of the aggregate process are generally different from those of the individual units. If all the micro-units are stationary and there is a finite number of them, it is still easy to derive the relationship between the two sets of IRFs.

Consider the simplest case: let y_{1t} and y_{2t} be defined as in (1) and (2), and let $Y_t = (y_{1t} + y_{2t})/2$ be the aggregate process. If y_{it}, for $i = \{1, 2\}$, is weakly stationary (assuming that $b_i = 0$ for simplicity), then

$$y_{1t} = \rho_1 u_t + a_1 \rho_1 u_{t-1} + a_1^2 \rho_1 u_{t-2} ... + \epsilon_{1t}, \tag{8}$$

$$y_{2t} = \rho_2 u_t + a_2 \rho_2 u_{t-1} + a_2^2 \rho_2 u_{t-1} ... + \epsilon_{2t}, \tag{9}$$

and

$$Y_t = \frac{\left((\rho_1 + \rho_2) u_t + (a_1 \rho_1 + a_2 \rho_2) u_{t-1} + (a_1^2 \rho_1 + a_2^2 \rho_2) u_{t-2} ... + \epsilon_{1t} + \epsilon_{2t}\right)}{2}, \tag{10}$$

where $\epsilon_{it} = \sum_{j=0}^{\infty} a_i^j \varepsilon_{it-j}$, $i = \{1, 2\}$. Since the IRF to a unitary change of u_t is given by the coefficients of u in the expressions above, it is clear that the IRF of Y_t is given by the average of the individual impulse responses. Thus, if consistent estimators are employed, the limit of the IRF estimated with aggregate data and that of the average of the micro IRFs should be the same since they are estimates of the same population quantity.

The former derivation has important limitations since it relies heavily on the existence of the MA(∞) representation of the micro-units and of the aggregate process. Thus, if some of the processes at the disaggregate level contain a unit root, it could not be applied. Furthermore, if the number of individual processes is allowed to go to infinity, the existence of the MA(∞) representation of the aggregate process is not guaranteed, even when all the micro units are weakly stationary (see Granger, 1980). For the sake of generality, in the following we consider the AR representation instead, since it does not present these problems. The calculations, however, are not so straight forward as when the MA one is employed.

We now analyze the relation between the aggregate and individual IRFs in the more general case where there is an infinite number of individual processes that can be stationary or not. Before computing the aggregate IRF, it is first necessary to consider the aggregation of model (1) in these circumstances. This problem has been addressed by Lewbel (1994),

who followed the"stochastic" approach to aggregation introduced by Kelejian (1980) and formalized by Stoker (1984). The latter author defines an aggregate function as the expected value over the distribution of agents of the micro relations (see Stoker, 1984, Definition 2)

$$Y_t = E_I\left(y_t\right) = \int y_t P(y_t|\theta_t) dy_t, \tag{11}$$

where $\theta\left(t\right)$ is a parameter vector that could vary over time but not across individuals and $E_I\left(.\right)$ gives the time path of the dependent variable mean.

To apply the results in Lewbel (1994), we further assume that $B = E_I\left(b\right)$, $X_t = E_I\left(x_t\right)$, and $E_I\left(\nu_t\right) = u_t$ exist. For simplicity, we also consider that the variables b and x are uncorrelated for all t, and that a is independent of the distribution of $(b'x + \nu)$. Some of these conditions can be easily relaxed without any change in the main result, as will be shown in Section 5. So,

$$Y_t = E_I\left(ay_{t-1}\right) + B'X + u_t, \tag{12}$$

where $Y_t = E_I(y_t)$. Lewbel (1994) showed that, under the above-mentioned assumptions, expression (12) can be written as

$$Y_t = \sum_{s=1}^{\infty} A_s Y_{t-s} + B'X_t + u_t, \tag{13}$$

for constants A_1, A_2, ..., defined as $A_s = E\left(\alpha_s\right)$, where $\alpha_1 = a$ and $\alpha_s = \left(\alpha_{s-1} - A_{s-1}\right)a$ for $s > 1$.

These constants can be easily shown to satisfy the equation

$$A_s = m_s - \sum_{r=1}^{s-1} m_{s-r} A_r. \tag{14}$$

Thus, model (13) can be employed to evaluate how the aggregate variable Y_t responds to changes in the aggregate shock, u_t.

3.1 Comparing aggregate versus disaggregate response

As in Section 2, we consider a unitary common shock occurring at time t and now we are interested in its effect on the aggregate variable Y_{t+h}, for $h \geq 1$. The (aggregate) IRF (denoted henceforth as IRF_{AG}) can be easily computed as

$$IRF_{AG}(t,h) = E\left(Y_{t+h}|u_t = 1; Z_{t-1}\right) - E\left(Y_{t+h}|u_t = 0; Z_{t-1}\right), \tag{15}$$

where $Z_{t-1} = (Y_{t-1},\, Y_{t-2}, ...\, ,\, X_{t-1},\ X_{t-2}, ...)$. Application of this definition to (13) yields

$$IRF_{AG}(t,h) = \begin{cases} IRF_{AG}(t,0) = 1, \\ \sum_{j=1}^{h} A_j IRF_{AG}(h-j), \text{ if } h \geq 1. \end{cases} \tag{16}$$

Although this expression seems complex, it can, in fact, be notably simplified. Notice that equation (14) can be rewritten as $m_s = \sum_{r=0}^{s-1} m_r A_{s-r}$ and, iterating the latter expression, it is straightforward to show that[5]

$$\begin{aligned} IRF_{AG}(t,\ 1) &= m_1 = E_I(a) = IRF_{dis}(t,\ 1) \\ IRF_{AG}(t,\ 2) &= m_2 = E_I\left(a^2\right) = IRF_{dis}(t,\ 2) \\ &\ldots \\ IRF_{AG}(t,\ h) &= m_h = E_I\left(a^h\right) = IRF_{dis}(t,\ h). \end{aligned} \tag{17}$$

Thus, there is a direct link between the micro and the aggregate response to a common shock: the IRF computed in the aggregate model is just the expected value of the individual IRFs and, as will be shown in the next section, this is also true under less stringent assumptions than the ones imposed above. This result is very interesting because it implies that the aggregation of heterogeneous processes does not amplify the *average* response to a given shock. It also implies that, if the aggregate process is highly persistent, it is not so only because the micro-units are heterogeneous but because, on average, they are highly persistent too. Another interesting consequence is that in order to obtain estimates of the standard (aggregate) IRF not only aggregate but also disaggregate data can be employed and, as will be shown in Section 4, the use of the latter type of data can bring about important efficiency gains.

The result above does not imply that other key properties of the micro processes are invariant to aggregation. As is well known, the aggregation of short-memory stationary processes may induce long memory or nonstationary behavior in the aggregate variable (see Section 4 for details). Similarly, if some of the individual processes are I(1) while the others are strictly stationary, the aggregate will also be an I(1) process. Nevertheless, even in this

[5] A related and independently developed result has recently been put forward by Caballero and Engel (2007). They consider a model of infrequent price adjustment at the disaggregate level and show that the response to a monetary shock is the same at the micro and at the macro level.

case, the above-described relationship among IRFs across different aggregation levels still applies.

To illustrate this argument, consider again model (1) under the assumptions of Sections 2 and 3, with a having support in the interval $(0,1)$. Clearly, since $a_i < 1$ for all i, all individual processes are (short-memory) stationary. Thus, they admit a Wold representation (for simplicity, we assume that $b_i = 0$ for all i)

$$y_{it} = \nu_{it} + a_i \nu_{it-1} + a_i^2 \nu_{it-2} + ..., \ i = 1, ..., N, \ t = 1, ...T. \tag{18}$$

The aggregate process $Y_t = E_I(y_t)$ could be computed by taking expectations in (18), provided the expected value of the right-hand side yields a well-defined MA representation. This will happen as long as the resulting MA coefficients are square-summable, that is, if $\sum_{j=0}^{\infty} E_I\left(a^j\right)^2 < \infty$. As shown by Zaffaroni (2004), the holding of this condition crucially depends on the behavior of the distribution of a in a neighborhood of 1, as will be explained in more detail in Section 4. Consider, for instance, that a follows a Beta(p, q) distribution in the interval (0,1). The moments of a are given by

$$m_j = E\left(a^j\right) = \frac{B(p + j/2, q)}{B(p, q)},$$

where p, $q > 0$ are the parameters of the distribution and $B(p, q)$ is the Beta function. It holds, for large j, that

$$m_j \approx (p + j/2)^{-q}. \tag{19}$$

Clearly, for values of q smaller than 0.5, the sequence $\{m_j\}_{j=0}^{\infty}$ is not square-summable, implying that the MA representation of Y_t is not mean-square convergent and, so, Y_t is not covariance-stationary. Notice, however, that the AR expansion described in (12) exists and is well defined, since $A(L) Y_t$ is a mean-square convergent random variable. If $0.5 < q < 1$, Y_t is stationary and the k-th autocorrelation can be approximated by $\rho_k \approx ck^{1-2q}$. Then, if $q \in (0.5, 1)$ the correlations are not summable and Y_t is a long memory process.[6] Notice, however, that the aggregate IRF derived from Y_t would equal the average of the micro IRFs derived from the stationary processes, since all the conditions needed for obtaining (17) are fulfilled.

[6] More specifically, Y_t could be approximated by a fractionally-integrated proces with order of integration $d = (1 - q)$. See Granger (1980) and Zaffaroni (2004) for details.

3.2 Other measures of persistence

We now examine how the remaining measures of persistence mentioned in Section 2 perform in this framework.

The aggregate versions of the CIR and the HL are defined as

$$CIR_{AG} = \sum_{h=0}^{\infty} IRF_{AG}\left(t,h\right),$$

and

$$IRF_{AG}\left(t,h\right)|_{h=\mathrm{HL}_{AG}} = 0.5,$$

respectively. Clearly, the equality between the IRF_{AG} and the IRF_{dis}, established in (17), implies that the CIR and the HL also present the same population values across aggregation levels.

Many empirical papers focus on the sum of autoregressive coefficients for comparing shock response across aggregation levels. In these articles, the SAC computed with aggregate data (denoted henceforth as SAC_{AG}) is compared with the average of the individual SACs (SAC_{dis}). This measure was originally introduced by Andrews and Chen (1994) because it has a direct relation with the CIR through the expression

$$SAC = 1 - CIR^{-1}, \tag{20}$$

and so, *"different values of the SAC can be interpreted easily in terms of persistence because they correspond straightforwardly to different values of the CIR"* (Andrews and Chen, 1994).

However, although this relation holds when a single series is employed, it is clear that relation (20) no longer holds when the SAC and the CIR are replaced by the corresponding averages. In fact, although the CIR remains constant across aggregation levels, the SAC increases systematically with the level of aggregation. To see this, notice that, for the simple model described in (1), CIR_{dis} is given by $CIR_{dis} = 1 + E_I\left(a\right) + E_I\left(a^2\right) + ... = E_I\left(\frac{1}{1-a}\right)$ and recall that $CIR_{dis} = CIR_{AG}$. Since $\frac{1}{1-a}$ is a convex function, Jensen's inequality implies that

$$SAC_{AG} = 1 - \left(E_I\left(\frac{1}{1-a}\right)\right)^{-1} > 1 - \left(\frac{1}{1-E_I\left(a\right)}\right)^{-1} = E_I\left(a\right) = SAC_{dis}, \tag{21}$$

so the aggregate SAC is bigger than the average of the individual SACs unless there is no heterogeneity, in which case both measures are equal. Notice, however, that this result does not imply that average persistence increases with the level of aggregation: it only implies that under individual heterogeneity, SAC_{dis} is a poor summary of CIR_{dis}, and therefore, is not a suitable tool for establishing persistence comparisons.

The inequality in (21) can be illustrated with a simple example. Consider, again, a collection of individuals behaving as in (1) (with $b_i = 0$, for all i for simplicity) with a heterogenous autoregressive parameter, a, that follows a U(0,1) distribution. Thus, the (population) value of the average of the individual SACs, SAC_{dis}, is equal to 0.5. With respect to CIR_{dis}, the moments of the uniform distribution, given by $E\left(a^h\right) = (h+1)^{-1}$, are not summable and so $\text{CIR}_{dis} = \sum_{h=0}^{\infty} E\left(a^h\right) = \infty$.

As for the corresponding aggregate values, notice that, in this case, Y_t can be obtained by taking expectations in (18) since the aggregate MA(∞) representation is well defined.[7] Bearing in mind that $A(z) = M(z)^{-1}$ for all z, where $\text{M}(z) = \sum_{j=0}^{\infty} E\left(a^j\right) z^j$, it follows that, in particular, $A(1) = M(1)^{-1} = \left(\sum_{h=0}^{\infty} E\left(a^h\right)\right)^{-1} = 0$. Thus, $\text{SAC}_{AG} = 1 - A(1) = 1$. Finally, it is easy to check that $CIR_{AG} = CIR_{dis} = \infty$. So, although the CIR remains constant across aggregation levels, the SAC increases considerably (from 0.5 to 1). This could clearly lead to the wrong conclusion that shocks are more persistent at the aggregate than at the micro level.

The LAR suffers from similar problems to the SAC. For instance, for the example above, it is easy to see that $\text{LAR}_{dis} = SAR_{dis} = 0.5$ and $\text{LAR}_{AG} = SAR_{AG} = 1$, where LAR_{dis} and LAR_{AG} are the average of the individual LARs and the LAR associated with Y_t, respectively. Thus, the LAR should not be employed to compare persistence across aggregation levels either.

Finally, in some applications, an alternative definition of disaggregate IRF to the one presented in this paper has been employed (see Imbs et al., 2005). As mentioned in Section 2, the strategy consists of fitting autoregressive models to each of the individual processes and to compute the average of the AR coefficient estimates. Next, an "averaged" model is constructed, having the same AR structure as the individuals but whose coefficients are

[7] Y_t is stationary with variance $\sigma_u^2 \sum_{j=0}^{\infty} (j+1)^{-2} < \infty$.

given by these averaged estimates. Then, the IRF of this artificially generated process is computed as in a model with homogeneous coefficients. Thus, estimates of the function

$$\overline{IRF}\,(t,h) = E_I\,(a)^h\,,\ \text{for } h \geq 0, \tag{22}$$

are provided as measures of micro-persistence. But clearly, since the IRF is not a linear function, (22) does not capture the average response of the micro units. Furthermore, whenever the support of a is positive, a^h is strictly convex and the application of Jensen's inequality yields

$$E_I(IRF(t,h)) = E_I\left(a^h\right) > \ \overline{IRF}\,(t,h) = E_I\,(a)^h\,, \text{for all } h > 1. \tag{23}$$

It follows that, if estimates of (22) are compared with those of IRF_{AG}, differences might arise but mainly due to the underestimation of the average shock response at the micro level rather than to an overestimation of the response when aggregate data is employed.[8]

The discussion above suggests that one of the reasons why empirical papers have found different degrees of shock persistence across aggregation levels is related to the type of tools employed to measure it. The following section reviews further issues that are also important for understanding why such differences might arise in practise.

4. AGGREGATION, THE LAW OF LARGE NUMBERS AND ESTIMATION

The discussion contained in Section 3 is based on results that are true for the population. But, in order to be of interest to practitioners, it must be possible to obtain good approximations to the population values when the corresponding sample counterparts are employed, at least when N and T are sufficiently large. Thus, two issues deserve consideration here. Firstly, the population aggregate model is defined as the expected value of the individual processes. However, aggregate data, denoted as $\bar{Y}_{Nt}$ henceforth, is usually constructed as (a possibly weighted) average of the individual data. Hence, $\bar{Y}_{Nt}$ would approximately follow

[8] Gadea and Mayoral (2008) have shown that the reduction in real exchange rate persistence found by Imbs et al. (2005) using sectoral real exchange rates is due to their definition of average micro-persistence. When the disaggregate IRF defined in (5) is employed, standard estimates of persistence are recovered.

the aggregate model in (13) if a LLN relating Y_t and $\bar{Y}_{Nt}$ holds. Secondly, it has been argued that estimation of the aggregate model can be problematic when the individuals are heterogeneous such that the resulting persistence estimates can be biased upwards (Pesaran and Smith, 1995, Imbs et al. 2005). In this section we briefly examine these two issues.

4.1. Aggregation and the LLN

For simplicity, we assume that the aggregate data $\bar{Y}_{Nt}$ is constructed as a simple average of a large number of individual processes

$$\bar{Y}_{Nt} = \frac{\sum_{i=1}^{N} y_{it}}{N},$$

where y_{it} is defined as in (1) with $b_i = 0$ for all i. $\bar{Y}_{Nt}$ can be written as the sum of two terms

$$\bar{Y}_{Nt} = \frac{1}{N}\left(\sum_{i=1}^{N} \frac{\varepsilon_{it}}{(1-a_i L)} + \sum_{i=1}^{N} \frac{\rho_i u_t}{(1-a_i L)}\right), \tag{24}$$

that will be referred to as the idiosyncratic and common components, respectively. Then, $\bar{Y}_{Nt}$ would be a good approximation of the aggregate model derived in Section 3 provided that a LLN applies, such that $\bar{Y}_{Nt}$ and $Y_t = E_I(y_t)$ are close for large N. However, it has been argued that, for the disaggregate model considered in this paper, such a LLN might not hold (see, for instance, Forni and Lippi, 1997, p. 17). Since the applicability of the results obtained in Section 3 relies on this convergence, it is worth considering this issue in more detail.

The holding of a LLN relating $\bar{Y}_{Nt}$ and Y_t hinges on whether the limit of $\bar{Y}_{Nt}$ when $N \to \infty$ is stationary or not. So, before considering the convergence of $\bar{Y}_{Nt}$ and Y_t, the asymptotic properties of $\bar{Y}_{Nt}$ as N increases should be reviewed.

This issue has been analyzed by Zaffaroni (2004) and we briefly summarize the results that are relevant to the problem considered here. The asymptotic behavior of $\bar{Y}_{Nt}$ critically depends on the properties of the distribution of a around 1. As shown by Granger (1980), if the support of a is given by $[\delta_1, \delta_2]$ with $\delta_2 < 1$, the corresponding aggregate process is I(0), for any shape of the distribution of a. On the other hand, if $\delta_2 = 1$ and the distribution of a is such that $P(a = 1) > 0$, then $\bar{Y}_{Nt}$ converges to an I(1) random variable. An interesting intermediate case arises whenever $\delta_2 = 1$ and a belongs to a family of absolutely

continuous distributions such that $P(a = 1) = 0$. To characterize the convergence in this case, Zaffaroni (2004) considers the following semiparametric specification of the density of $a \in (0, 1)$ around unity,[9]

$$f(a) \sim c_b (1 - a)^{-b}, \text{ as } a \to 1, \ 0 < c_b < \infty, \ b \in [0, 1)$$

In this case, $\bar{Y}_{Nt}$ converges to a stationary random variable provided $b < 0.5$ and to a nonstationary one otherwise. Interestingly, if $0 \leq b < 0.5$, the limit of $\bar{Y}_{Nt}$ is a long-memory process and if $b > 0$, the limit process can be characterized as a fractionally integrated process with order of integration $d = b$.

Under similar assumptions as the ones adopted in this paper, Zaffaroni (2004) shows that provided the limit of $\bar{Y}_{Nt}$ is stationary, a strong LLN holds and $\bar{Y}_{Nt} \xrightarrow{L_2} E_t(y_t) = Y_t$. In this case, the idiosyncratic component converges almost surely to zero while the common component converges in L_2 to the corresponding expectation[10]

$$N^{-1} \sum_{i=1}^{N} \frac{\rho_i u_t}{(1 - a_i L)} \xrightarrow{L_2} E_I \left(\frac{\rho u_t}{1 - aL} \right) = \sum_{j=0}^{\infty} E_I \left(a^j \right) u_{t-j}.$$

However, whenever the limit of $\bar{Y}_{Nt}$ is a nonstationary random variable, the convergence above fails: the idiosyncratic component no longer vanishes because the variance of $N^{-1} \sum_{i=1}^{N} \frac{\varepsilon_{it}}{(1 - a_i L)}$ tends to infinity. On the other hand, the common component does not converge to its expected value because neither the Bochner nor the Pettis integral of this component exist.

In principle, this could be a major drawback for the results established in Section 3. If nonstationary variables are observed, $\bar{Y}_{Nt}$ might not be a good proxy for Y_t. Then, one should not expect the persistence estimates obtained with aggregate data, $\bar{Y}_{Nt}$, to be close to those obtained with the corresponding disaggregate variables. Notice, however, that this

[9] This condition is semiparametric because the behavior of the density for any given interval $[0, \gamma_2]$ with $\gamma_2 < 1$ is unspecified. Standard distributions, such as the Uniform or the Beta, are contained in this specification by setting $d = 0$ and $d \geq 0$, respectively.

[10] The expectation of the idiosyncratic component is taken with respect to the Pettis integral (see Uhlig, 1986). This is because the Bochner integral (which extends the definition of the Lebesgue integral to functions taking values in a Banach space) of that component may not exist. This is the well known *measurability problem* (Judd, 1985).

problem has an easy solution. Taking first differences from the original aggregate data, $\bar{Y}_{Nt}$, we obtain

$$(1-L)\,\bar{Y}_{Nt} = \frac{1}{N}\left(\sum_{i=1}^{N}\frac{(1-L)\,\varepsilon_{it}}{(1-a_iL)} + \sum_{i=1}^{N}\frac{(1-L)\,\rho_i u_t}{(1-a_iL)}\right), \tag{25}$$

and, in this case, the same results as in the case where the limit of $\bar{Y}_{Nt}$ is stationary are recovered, that is, the idiosyncratic component in (25) converges to zero while the common one converges to the corresponding expectation. Thus, it holds that

$$(1-L)\,\bar{Y}_{Nt} \overset{L_2}{\to} (1-L)\,Y_t, \tag{26}$$

where $(1-L)\,Y_t$ is the first difference of $Y_t = E_I(y_t)$ and is a stationary process. Thus, whenever nonstationarity is detected, the usual procedure of first differentiating the data would be sufficient in order to guarantee the convergence of $(1-L)\,\bar{Y}_{Nt}$ to $(1-L)\,Y_t$. The IRF of Y_t can be estimated by first estimating the IRF associated with $(1-L)\,\bar{Y}_{Nt}$, and, then, cumulating the corresponding values. That is,

$$\widehat{IRF_{AG}}(t,h) = \sum_{j=1}^{h}\widehat{IRF_{(1-L)\bar{Y}_N}}(j,t),$$

where $\widehat{IRF_{AG}}(t,h)$ and $\widehat{IRF_{(1-L)\bar{Y}}}(j,t)$ denote the estimates of the IRFs associated with Y_t and with $(1-L)\,Y_t$, respectively.

4.2. Aggregation and Inference

As regards the estimation of the aggregate process, difficulties arise because Y_t contains an infinite number of parameters. Several authors have emphasized that this may bias the estimates of the aggregate model in such a way that aggregate persistence measures present an upward bias (Pesaran and Smith, 1995, Imbs et al., 2005).

However, Berk (1974) and Lewis and Reinsel (1985) have shown that, if Y_t is short-memory, $\sqrt{T}$-consistent and asymptotically normally distributed, estimates can be obtained by approximating the AR(∞) process by an AR(k) model, where k does not increase too quickly or too slowly. More specifically, k should verify an upper bound condition, $k^3/T \to 0$, and a lower bound one, $T^{1/2}\sum_{j=k+1}^{\infty}|A_j| \to 0$ as $k,\, T \to \infty$.[11]

[11] Related results for the case where Y_t is a long memory case have been recently presented by Godet

The lag length, k, is the key parameter in implementing procedures that approximate AR(∞) models in applications. However, the above-described theoretical restrictions provide little practical guidance for choosing an appropriate value of k. Ng and Perron (1995) show that standard selection criteria (the AIC and the BIC) choose values of k, $\hat{k}$, that are proportional to $\log T$ and, so, do not verify the lower bound condition stated above. In fact, bias terms arising as a consequence of the asymptotic misspecification of the model when these criteria are employed are of order $T^{-1/2}$, considerably more severe than the usual finite sample biases that are typically of order T^{-1}.

Kuersteiner (2005) has shown that the general-to-specific (GTS) approach introduced by Ng and Perron (1995) can be used to produce a data-dependent selection rule such that the parameters obtained in the AR(k) (VAR(k)) model are consistent and asymptotically normal for the parameters of the underlying VAR(∞) model. Then, the consistency and the asymptotic normality of aggregate IRF estimates in AR(∞) (VAR(∞)) models follow from an application of the delta method.[12]

In the following subsection we present the results of a small Monte Carlo experiment designed to evaluate the finite-sample performance of the above-described estimation strategy when applied to estimate the aggregate IRF under individual heterogeneity. We also analyze the finite sample behavior of IRF estimates based on disaggregate information. A more detailed assessment of the finite-sample performance of these estimation techniques can be found in Mayoral (2009).

4.3. Finite sample properties of IRF estimators under individual heterogeneity

In this Monte Carlo experiment we are interested in evaluating the finite-sample properties of IRFs estimators computed with aggregate and disaggregate data as the degree of heterogeneity of the underlying processes increases. In order to isolate the impact of an increase in heterogeneity from an increase in average persistence, different degrees of het-

(2008).

[12] Bootstrap methods have also been employed for inference in univariate AR(∞) models, see Kreiss (1997), Choi and Hall (2000) and Gonçalves and Kilian (2004) among others. Extensions to the multivariate case have been proposed by Paparoditis (1996) and Inoue and Kilian (2002).

erogeneity have been considered while keeping the level of persistence constant. In order to achieve this, the following approach was adopted. The data has been generated according to the model:

$$y_{it} = a_i y_{it-1} + \rho_i u_t + \varepsilon_{it},\ i = 1, ..., N,\ t = 1, ...T, \tag{27}$$

where $T = \{100, 400\}$, $N = \{200\}$, u_t and ε_{it} are $i.N(0,1)$ and $i.N(0, \sigma_i^{\varepsilon})$ random variables, respectively and σ_i^{ε} and ρ_i are draws from two independent uniform distributions in the interval (0.5, 1.5). The autoregressive parameter a is distributed as a $U(\delta_1, \delta_2)$, for different values of (δ_1, δ_2). These values determine both the degree of heterogeneity of the individual units and the level of persistence of the aggregate. In this exercise, the degree of heterogeneity will be determined by the standard deviation of a, σ_a, while the level of persistence will be measured by the cumulated impulse response up to lag 100, CIR(100).

To select (δ_1, δ_2), the following approach has been pursued. Firstly, we have set three levels of persistence, denoted as LP I, II and III, corresponding to the case where there is no heterogeneity and a takes the values $a = \{0.85, 0.90, 0.95\}$, respectively. The persistence implied by a model like (27) in these cases, as measured by the CIR(100), yields

$$LP\text{ I:} \qquad CIR(100) = \sum_{j=0}^{100} 0.85^j = 6.67.$$

$$LP\text{ II:} \qquad CIR(100) = \sum_{j=0}^{100} 0.9^j = 10.$$

$$LP\text{ III:} \qquad CIR(100) = \sum_{j=0}^{100} 0.95^j = 19.89.$$

Secondly, four values of σ_a have been considered, namely $\sigma_a = \{0,\ 0.01,\ 0.05,\ 0.1\}$. Next, the values (δ_1, δ_2) are chosen in such a way that the desired σ_a is obtained and the resulting set of heterogeneous processes imply a similar value of the CIR(100) as LP I to LP III. Under heterogeneity, CIR(100) $= \sum_{j=0}^{100} E\left(a^j\right) = \left(\frac{1}{\delta_2 - \delta_1}\right) \sum_{j=0}^{100} \left(\delta_2^{j+1} - \delta_1^{j+1}\right)/j + 1$. Then, the restriction on the standard deviation implies that $(\delta_2 - \delta_1) = \sqrt{12}\sigma_a$ while the condition on the CIR allows us to identify unique values for (δ_1, δ_2). Table I reports these values for each of the levels of persistence and heterogeneity considered in this experiment.

TABLE I. DISTRIBUTION OF a

$a \sim U(\delta_1, \delta_2)$									
		σ_a= 0.0		σ_a= 0.01		σ_a= 0.05		σ_a= 0.1	
	CIR(100)	δ_1	δ_2	δ_1	δ_2	δ_1	δ_2	δ_1	δ_2
LP I	6.67	0.85	0.85	0.83	0.866	0.7470	0.9202	0.6155	0.9619
LP II	10	0.90	0.90	0.882	0.9166	0.7900	0.9632	0.6455	0.9919
LP III	19.89	0.95	0.95	0.9300	0.9646	0.826	0.992	0.6700	1.00

The aggregate process, Y_t, has been computed as the simple average of the $y'_{it}s$

$$Y_t = \frac{\sum_{i=i}^{N} y_{it}}{N}.$$

To estimate the resulting data, an AR(k) process has been fitted to Y_t, where k was chosen according to different approaches, namely, the AIC and the GTS. Following Ng and Perron (1995), the maximum value of k, $k\max$, was set according to the rule $k\max(Y_T) \approx 10 * (T/100)^{0.25}$, which for the values of T considered in this experiment yields $kmax = \{10, 14\}$. Disaggregate data has also been employed to estimate the IRF following a similar approach. This time, $k\max$ was set to $k\max(Y_T)/3$. For each replication r, the estimated disaggregate and aggregate models have been employed to compute the corresponding IRFs, denoted as $\widehat{IRF_{dis,r}}$ and $\widehat{IRF_{AG,r}}$, respectively. $\widehat{IRF_{dis,r}}$ has been obtained as the mean of the individual IRFs while $\widehat{IRF_{AG,r}}$ is the sample analog of (16) .

Table II reports the average mean squared error (MSE) of $\widehat{IRF_{dis}}$ and $\widehat{IRF_{AG}}$ which is defined as

$$\overline{MSE}_{dis} = R^{-1} \sum_{r=1}^{R} \left(h^{-1} \sum_{j=0}^{h} \left(\widehat{IRF_{dis,r}}(j) - IRF_{true,r}(j) \right)^2 \right),$$

and

$$\overline{MSE}_{AG} = R^{-1} \sum_{r=1}^{R} \left(h^{-1} \sum_{j=0}^{h} \left(\widehat{IRF_{AG,r}}(j) - IRF_{true,r}(j) \right)^2 \right),$$

where $R = 1000$ is the number of replications, $h = 50$ is the horizon of the IRF and $IRF_{true,r}$

is the true IRF in replication r, given by

$$IRF_{true,r}(j) = N^{-1}\sum_{i=1}^{N} a_{i,r}^{j},\ j = 1, ..., h.$$

Only values obtained by selecting k using the GTS approach are reported. Those obtained by using the AIC were very similar so they are omitted for the sake of brevity.

TABLE II. MEAN SQUARED ERROR, GTS SELECTION METHOD

	T=100							
	σ_a= 0.0		σ_a= 0.01		σ_a= 0.05		σ_a= 0.1	
	$\overline{MSE}_{dis}$	$\overline{MSE}_{AG}$	$\overline{MSE}_{dis}$	$\overline{MSE}_{AG}$	$\overline{MSE}_{dis}$	$\overline{MSE}_{AG}$	$\overline{MSE}_{dis}$	$\overline{MSE}_{AG}$
LP I	0.0016	0.0067	0.0015	0.0068	0.0017	0.0074	0.0021	0.0078
LP II	0.0060	0.0229	0.0060	0.0208	0.0062	0.0192	0.0069	0.0183
LP III	0.04828	0.1010	0.0402	0.0876	0.0236	0.0528	0.0111	0.0270
				T=400				
LP I	0.0003	0.0021	0.0003	0.0021	0.0004	0.0022	0.0004	0.0026
LP II	0.0012	0.0058	0.0013	0.0060	0.0012	0.0062	0.0012	0.0065
LP III	0.0072	0.0246	0.0073	0.0241	0.0037	0.0153	0.0018	0.0101

Figures I to III present the average over the number of replications of IRF$_{true}$, $\widehat{IRF_{dis}}$ and $\widehat{IRF_{AG}}$ for LP I-III, respectively. For each sample size, $T = \{100, 400\}$, four graphs are presented in each figure, corresponding to the different degrees of heterogeneity considered. Graphs in the upper left corner correspond to the case where there is no heterogeneity ($\sigma_a = 0$), while the remaining graphs display estimated IRFs under positive $\sigma_a's$.

Several interesting conclusions can be drawn from inspecting Table II and Figures I to III. Firstly, good estimators of the IRF can be obtained with either aggregate or disaggregate data using standard estimation methods. Thus, dynamic heterogeneity at the individual level does not seem to be an obstacle to obtain estimators of IRF$_{AG}$ with good finite-sample properties, even when AR(k) models with moderate values of k are used to fit the aggregate model. Figures I to III show that the average bias is in general small and fairly similar for the $\sigma_a = 0$ and the $\sigma_a > 0$ cases. However, the bias is usually smaller for

$\widehat{IRF_{dis}}$ than for $\widehat{IRF_{AG}}$. Similar conclusions are drawn by analyzing the average MSE: $\overline{MSE}_{dis}$ is always smaller than $\overline{MSE}_{AG}$, even in the absence of heterogeneity. This result is very interesting since it suggests that the use of disaggregate information can bring about important efficiency gains in the estimation of aggregate quantities.

Secondly, increasing heterogeneity for a given level of persistence does not affect much $\overline{MSE}_{dis}$ unless persistence is high (LP III), in which case $\overline{MSE}_{dis}$ tends to decrease as heterogeneity increases. A similar pattern is observed for $\overline{MSE}_{AG}$: for moderate levels of persistence (LP I), increasing σ_a deteriorates $\overline{MSE}_{AG}$. However the opposite effect is found in more persistent situations. For instance, for LP III with $T = 100$, the $\overline{MSE}_{AG}$ for $\sigma_a = 0.1$ is only 1/4 the $\overline{MSE}_{AG}$ obtained for the case of no heterogeneity ($\sigma_a = 0$). This evidence is surprising and at odds with the common belief that individual heterogeneity hurts the properties of aggregate estimators.

Thirdly, increasing the level of persistence always increases the MSE. This increase may be due to the well-known downward bias affecting the OLS estimator of AR coefficients when persistence is high. Interestingly, the increase tends to be smaller in the presence of heterogeneity. For instance, for $\sigma_a = 0$ and $T = 100$, the MSE of $\widehat{IRF_{dis}}$ $\left(\widehat{IRF_{AG}}\right)$ increases by a factor of 8 (4.5) when moving from LP II to LP III. However, for $\sigma_a = 0.1$, the MSE of $\widehat{IRF_{dis}}$ $\left(\widehat{IRF_{AG}}\right)$ rises only by a factor of 1.5 (1.5) from LP II to LP III. Finally, both the bias and the MSE drop considerably when larger sample sizes are considered.

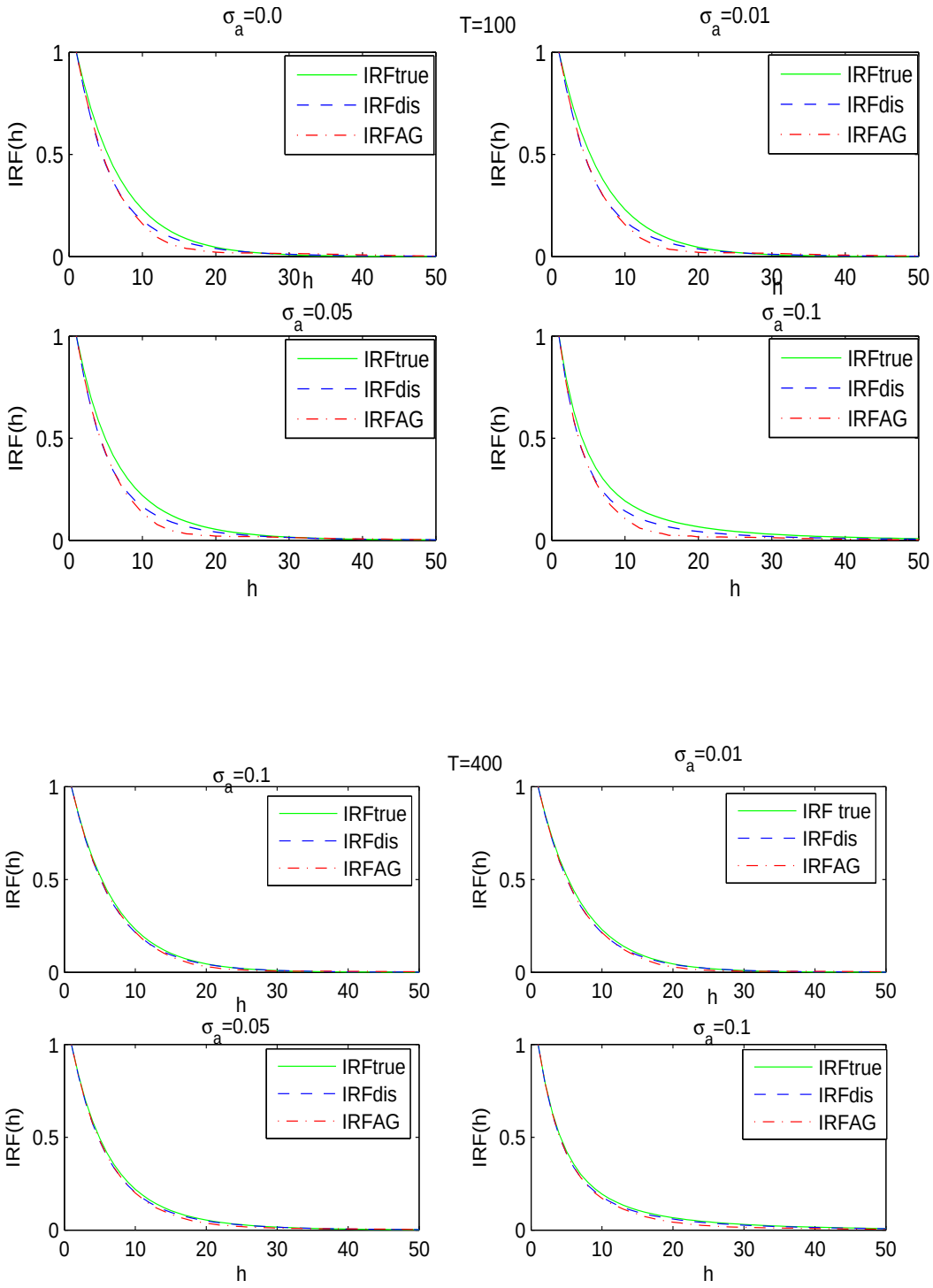

FIG 1. Estimated IRFs, LP I. T={100, 400}.

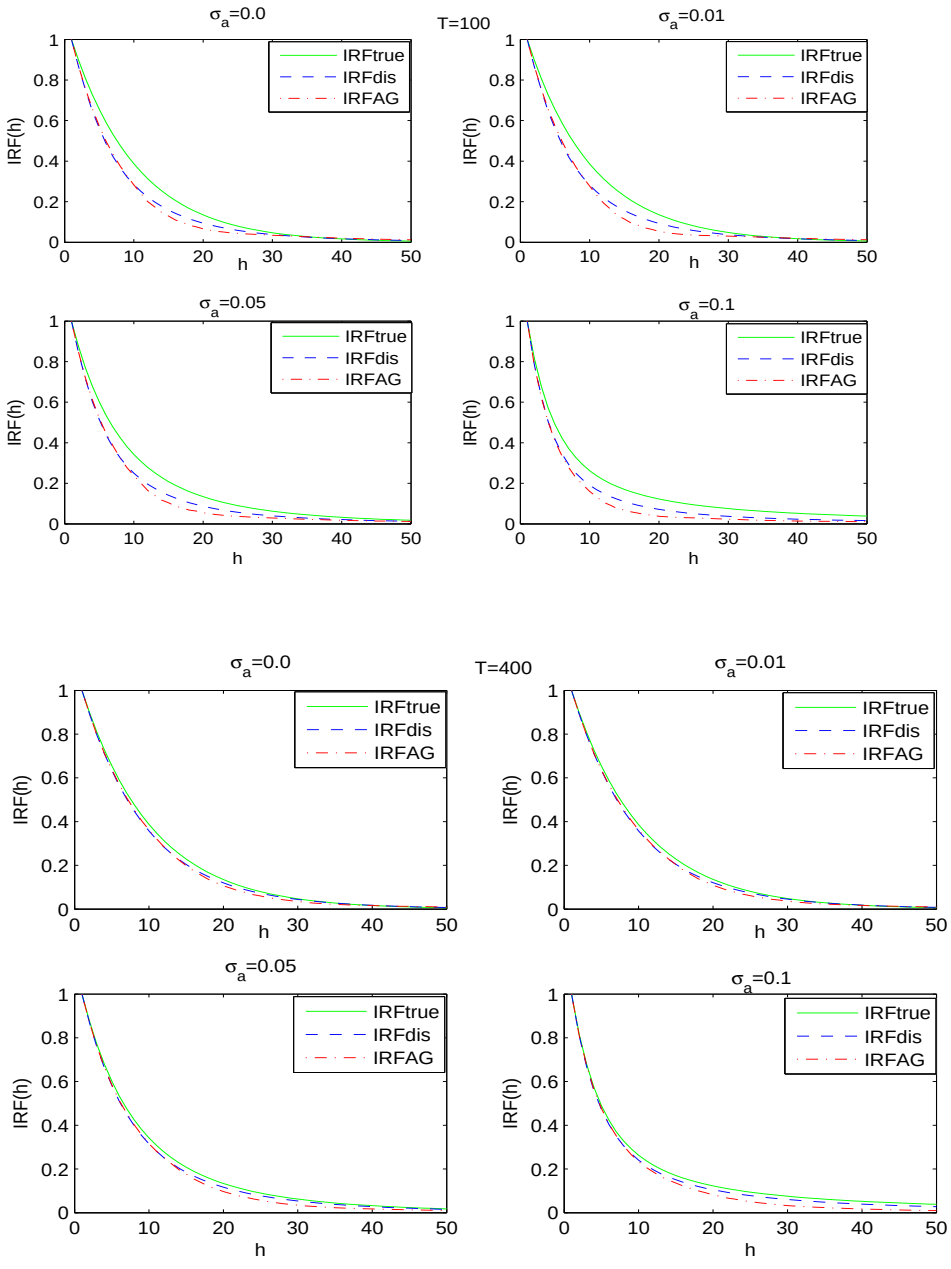

FIG 2. Estimated IRFs, LP II. T={100, 400}.

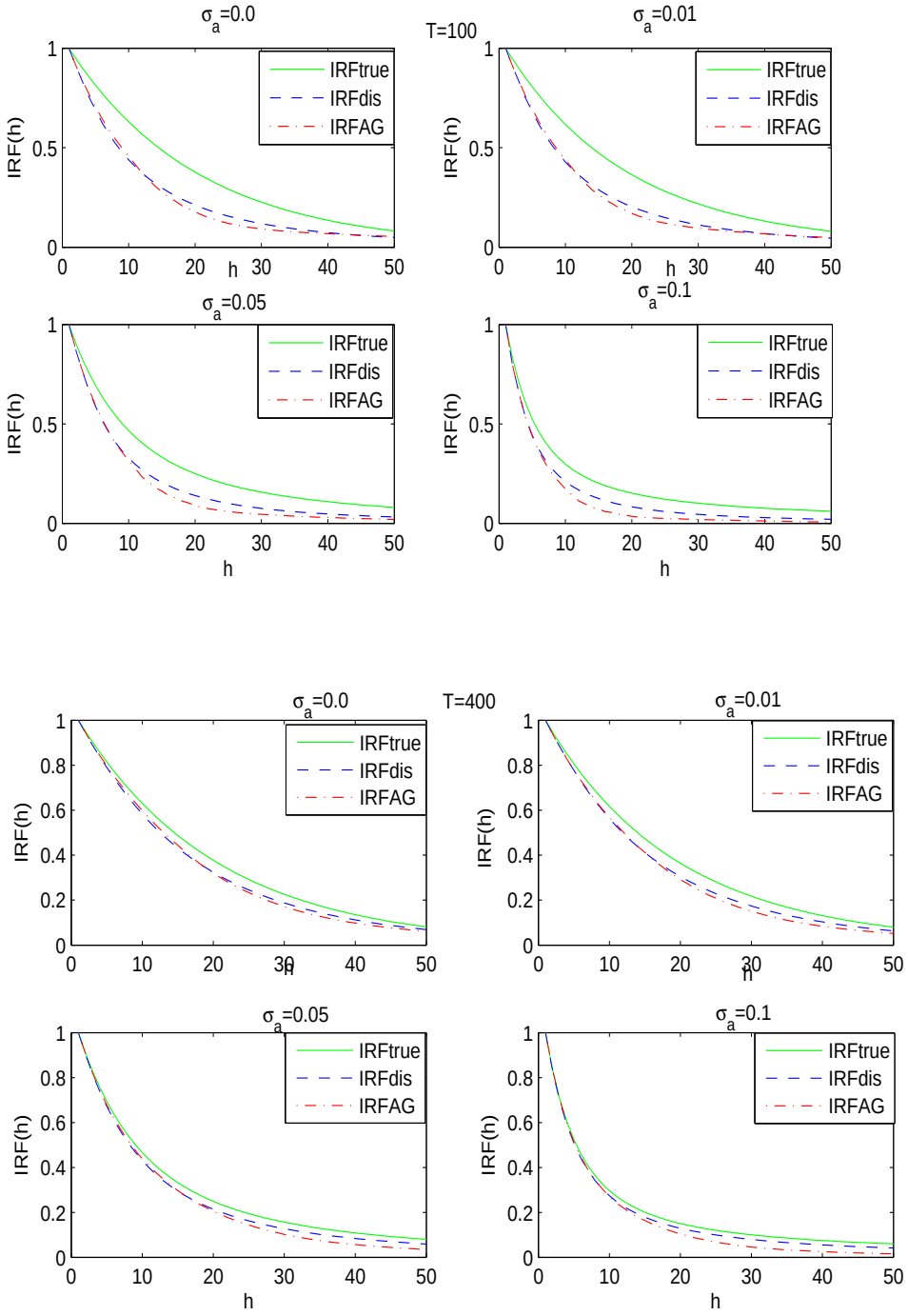

FIG 3. Estimated IRFs, LP III. T={100, 400}.

5. EXTENSIONS

A more realistic approach than the one analyzed in Section 3 would entail considering vector autoregressive processes, fewer assumptions of independence and uncorrelatedness between the variables of the model as well as allowing for more general individual dynamics. This section extends the results of Section 3 in these directions.

We now consider vector autoregressive (VAR) systems at the disaggregate level. To do so, equation (1) is reinterpreted so that y_{it} and $\nu_{it} = \rho_i u_t + \varepsilon_{it}$ are now J vectors of random variables, a_i is a $J \times J$ matrix of random coefficients, ρ_i is a diagonal $J \times J$ matrix verifying that $E_I(\rho_i) = I_J$, where I_J is the identity matrix of order J, and for simplicity, b_i is set to 0 for all i.

The individual IRF associated with this model can be obtained by

$$IRF^i(t,h) = E(y_{it+h}|u_t = \delta; z_{it-1}) - E(y_{it+h}|u_t = \bar{0}; z_{it-1}) \tag{28}$$

where $z_{it-1} = \begin{pmatrix} y_{it-1}, & y_{it-2}, & \dots \end{pmatrix}'$, $\bar{0}$ is a J vector of zeroes and δ are the relevant experimental shocks. Under the assumptions of Section 3, it is easy to check that, in the vector case, the individual IRF is given by

$$IRF^i(t,h) = a_i^h \rho_i \delta, \text{ for } h \geq 0. \tag{29}$$

and, so, the average response can be defined as

$$IRF_{dis}(t,h) = E_I(IRF(t,h)) = E_I\left(a^h\right)\delta, \text{ for } h \geq 0. \tag{30}$$

The aggregate VAR can be obtained using a similar strategy as in the scalar case. Taking expectations in (1) and adding and subtracting the matrix $A_1 = E(a_1)$, we get

$$Y_t = E_I(y_t) = A_1 Y_{t-1} + E_I\left((A_1 - E(A_1))(a y_{t-2} + \nu_{t-1})\right) + u_t.$$

Defining the $J \times J$ matrix $A_s = E(\alpha_s)$, where $\alpha_s = (\alpha_{s-1} - A_{s-1})a$ and $\alpha_1 = a$, one can iterate the procedure above to obtain the aggregate model

$$Y_t = \sum_{s=1}^{\infty} A_s Y_{t-s} + u_t.$$

The aggregate IRF can be obtained as,

$$IRF_{AG}(t,h) = E\left(Y_{t+h}|u_t = \delta; Z_{t-1}\right) - E\left(Y_{t+h}|u_t = \bar{0}; Z_{t-1}\right)$$

where $Z_{t-1} = \left(Y_{t-1}, Y_{t-2}, ...\right)$, which yields the expression $IRF_{AG}\left(t,0\right) = \delta$ and

$$IRF_{AG}\left(t,h\right) = \sum_{j=1}^{h} A_j IRF_{AG}\left(h-j\right), \text{ for } h \geq 1. \tag{31}$$

As in the scalar case, it is easy to show that the relations $A_s = m_s - \sum_{r=1}^{s-1} m_{s-r}A_r$ and $m_s = \sum_{r=0}^{s-1} m_r A_{s-r}$ also hold in the matrix case, where m_s, defined as $m_s = E_I\left(a^s\right)$, is of order $J \times J$. Thus, it is straightforward to check that

$$IRF_{AG}\left(t,h\right) = E_I\left(a^h\right)\delta. \tag{32}$$

So, comparing (32) and (30), one can conclude that in the vector case, the average response to an economic shock is the same, regardless of the level of aggregation at which it is considered.

When correlation between some of the random variables entering the micro model is allowed, in general, additional terms enter the aggregate equation. For instance, the assumption most likely to be violated is that of independence between a and b. Whenever this assumption is violated, it is obtained that (see Lewbel, 1994)

$$Y_t = \sum_{s=1}^{\infty} A_s Y_{t-s} + B'X_t + \sum_{s=1}^{\infty} Cov\left(\alpha_s, b\right) X_{t-s} + u_t, \tag{33}$$

where $\alpha_1 = a$ and $\alpha_{s+1} = \left(\alpha_s - E\left(\alpha_s\right)\right)a$, for $s \geq 1$. However, it is straightforward to check that the expression of the aggregate IRF in (16) is not altered by the addition of this new term and, hence, the relation between the micro and macro IRFs established in Section 3 is preserved. If X_t is a simply a vector of ones, then, provided $\sum_{s=1}^{\infty} Cov\left(\alpha_s, b\right) < \infty$, the new term is just an addition to the aggregate intercept. Otherwise, in order to have consistent estimates of the aggregate responses, an increasing number of lags of X_t should be included in the aggregate equation.

If the distributions of b and x_t are not independent, the aggregate model becomes,

$$Y_t = \sum_{s=1}^{\infty} A_s Y_{t-s} + B'X_t + Cov\left(b, x_t\right) + u_t, \tag{34}$$

and, as in the previous case, the expression of the aggregate IRF is not affected by this term. If the covariance between b and x_t is finite constant over time, the $Cov(b, x_t)$ is also an addition to the aggregate constant term.

The next step would be to allow higher order lags at the disaggregate level. To simplify the exposition, we next analyze the AR(2) case. The AR(p) one, though notationally cumbersome, can be analyzed in the same manner. Suppose the individual agents i behave according to the model,

$$y_{it} = a_{1i}y_{it-1} + a_{2i}y_{it-2} + u_{it},\ i = 1, ..., N,\ t = 1, ...T. \tag{35}$$

In addition to the assumptions stated in Section 3, we now require the existence of all the moments of a_2 and all the cross-moments between a_1 and a_2. Notice that we do not need to assume independence between a_1 and a_2. The aggregate model is obtained by taking expectations in (35),

$$Y_t = E_I(a_1y_{t-1}) + E_I(a_2y_{t-2}) + u_t,\ t = 1, ...T.$$

which can be written as an AR(∞) process

$$Y_t = \sum_{j=0}^{\infty} C_jY_{t-j} + u_t. \tag{36}$$

In order to obtain the coefficients C_j, define $\alpha_{j1} = a_j$, $A_{jk} = E(\alpha_{jk})$ and $\alpha_{jk+1} = (\alpha_{jk} - E(\alpha_{jk}))\, a_j$, for $j = 1, 2$ and $k = 1, 2$ ..., and notice that

$$\begin{aligned} E_I(a_1y_{t-1}) &= E_I(a_1 + E_I(a_1) - E_I(a_1)\, y_{t-1}) = \\ &\quad E_I(a_1)\, Y_{t-1} + E_I(\alpha_{12}y_{t-2}) + E_I((a_1 - E(a_1))\, a_2y_{t-3}), \end{aligned}$$

and

$$\begin{aligned} E_I(a_2y_{t-2}) &= E_I(a_2 + E_I(a_2) - E_I(a_2)\, y_{t-2}) = \\ &\quad E_I(a_2)\, Y_{t-2} + E_I((a_2 - E_I(a_2))\, a_1y_{t-3}) + E_I(\alpha_{22}y_{t-4}),\ \text{etc.} \end{aligned}$$

Iterating the procedure above, after some algebra, one can obtain that[13]

$$\begin{aligned} C_1 &= E_I(\alpha_{11}) = E_I(a_1) \\ C_2 &= E_I(\alpha_{12}) + E_I(\alpha_{21}) = Var(a_1) + E_I(a_2) \\ C_3 &= E_I(\alpha_{13}) + 2cov(a_1, a_2) = Cov(a_1^2, a_1) + 2cov(a_1, a_2) \\ C_4 &= E_I(\alpha_{14}) + E_I(\alpha_{22}) + 3cov(a_1 a_2, a_1), \text{ etc.} \end{aligned} \tag{37}$$

Next, we check that the disaggregate and aggregate IRFs are the same. The IRF for each of the micro-units can be easily computed from definition (3) and, taking expectations, it is obtained that

$$\begin{aligned} IRF_{dis}(t,1) &= E_I(a_1);\ IRF_{dis}(t,2) = E_I\left(a_1^2\right) + E_I(a_2); \\ IRF_{dis}(t,3) &= E_I\left(a_1^3\right) + 2E_I(a_1 a_2);\ IRF_{dis}(t,4) = E_I\left(a_1^4\right) + 3E_I\left(a_1^2 a_2\right) + E_I\left(a_2^2\right), \end{aligned}$$

and, in general,

$$IRF_{dis}(t,h) = E_I(IRF(t,h)) = E_I(a_1 IRF(t,h-1) + a_2 IRF(t,h-2))). \tag{38}$$

It is not difficult, although algebraically tedious, to check that the aggregate IRF associated with the aggregate process (35) coincides with the one obtained from averaging the individual IRFs. For instance, consider the first values of the aggregate IRF that are given by

$$\begin{aligned} IRF_{AG}(t,1) &= C_1 = E_I(a_1) = IRF_{dis}(t,1) \\ IRF_{AG}(t,2) &= C_1^2 + C_2 = E_I\left(a_1^2\right) + E_I(a_2) = IRF_{dis}(t,2) \\ IRF_{AG}(t,3) &= C_1^3 + 2C_1 C_2 + C_3 = E_I\left(a_1^3\right) + 2E_I(a_1 a_2) = IRF_{dis}(t,3) \\ IRF_{AG}(t,4) &= C_1^4 + 3C_1^2 C_2 + 2C_3 C_1 + C_2^2 + C_4 \\ &= E_I\left(a_1^4\right) + 3E_I\left(a_1^2 a_2\right) + E_I\left(a_2^2\right) = IRF_{dis}(t,4), \text{ etc.} \end{aligned} \tag{39}$$

Hence, the aggregate IRF is just the expected value of the individual IRFs, as shown in Section 3 for the simple AR(1) case. The same result also holds when more general AR(p) dynamics are considered.

[13] Notice that these coefficients are different from the ones provided by Lewbel (1994) (Equations 6' and 16) since those expressions contain some typos.

6. EMPIRICAL ILLUSTRATION

Monetary authorities and central banks are very interested in knowing how sluggishly inflation returns to its long-run equilibrium level after the arrival of a shock. Recently, it has been argued that the persistence of shocks to inflation tends to increase with the level of aggregation. For instance, one of the conclusions of the Inflation Persistence Network, created by the European Central Bank with the aim of analyzing the patterns of European inflation persistence, was that there is clear evidence of large differences across sectors and *"that measures of the degree of inflation persistence increase with the level of aggregation. Individual or highly disaggregate price series are, on average, much less persistent than aggregate ones"* (see Angeloni et al., 2007, and the references therein).[14] Similar findings have been reported for U.S. inflation (Clark, 2006).

Most of the empirical studies that have compared the persistence of inflation shocks across different aggregation levels have relied on averages of the SAC (or other scalar measures such as the LAR) as measures of disaggregate persistence (see Clark, 2006, Altissimo et al., 2006*a*, and the references therein). However, this approach can be problematic for the reasons discussed in Section 3.

This section compares estimates of U.S. inflation persistence computed at different levels of aggregation. In addition to the SAC, cross-sectional averages of the IRFs are considered as measures of shock response.

We use a similar data set to Clark (2006). Price indexes and nominal expenditures for all components of consumption, as measured in the NIPA accounts, have been obtained from the webpage of the Bureau of Economic Analysis (BEA). This dataset permits breakdowns at various levels of aggregation. We focus on core inflation, which excludes food and energy prices. Then, the aggregate variable, denoted as Level 1, is core inflation. We also report results for data broken into several levels of disaggregation, each spanning all the core inflation. The most disaggregate level (that we will refer to as Level 4) contains 109 disaggregate prices. Level 3 and Level 2 aggregate these 109 series into 46 and 11 categories, respectively. See Clark (2006) for details on the construction of these variables. The data

[14] These conclusions are based on a number of studies that can be found in the network's webpage at http://www.ecb.int/events/conferences/html/inflationpersistence.en.html.

is quarterly and covers the period 1976 to 2002.[15]

To illustrate the existence of heterogeneity in inflation data, the mean, the standard deviation and the sum of the autoregressive coefficients (with and without small-sample bias correction) have been computed for each of the 109 series in Level 4. Table III summarizes the results by presenting some descriptive statistics of the quantities obtained. More specifically, it reports the average, the weighted average, the three quantiles and the minimum and maximum values of the individual statistics detailed above. The weights employed to construct the weighted averages represent the listed components shares of core PCE nominal spending in 2001, in percentage terms. To compute the SAC, AR(k) processes have been fitted, where k has been chosen according to the GTS. Bias-corrected estimates (denoted as $\text{SAC}_{\text{B-C}}$ in the table) have been calculated following Kilian's (1998) method.

TABLE III. DESCRIPTIVE STATISTICS (LEVEL 4)

	Mean inflation	Std inflation	SAC	$\text{SAC}_{\text{B-C}}$
Average	3.70	5.53	0.660	0.771
W. average	3.95	5.69	0.741	0.832
S. deviation	1.50	4.09	0.218	0.185
First quantile	2.88	3.45	0.538	0.678
Second quantile	4.04	4.36	0.729	0.804
Third quantile	4.75	5.89	0.810	0.911
Min	-1.91	2.25	-0.240	-0.141
Max	7.64	29.4	0.955	1.00

Inspection of Table III reveals that disaggregate inflation data is highly heterogeneous. The mean inflation values over the period considered range from -1.91 to 7.46 with a mean (median) of 3.70 (4.04) and a standard deviation of 1.5. Individual dispersion also shows important disparities across series, as shown by the minimum and maximum values (2.25 and 29, respectively). More importantly, the dynamics of inflation series, as measured by the SAC, also seem to be very heterogeneous. Values of the SAC (bias-corrected SAC)

[15] Clark (2006) analyzed the period 1959-2002. Nevertheless, in order to avoid problems derived from the existence of structural breaks around the 1973 crisis, which would have a great impact on persistence estimates (see Perron, 1989), we have preferred to avoid this period by considering only post-crisis data.

range from -0.24 (-0.14) to 0.95 (1.00), with a mean of 0.66 (0.77) and a standard deviation of 0.21 (0.18).

Table IV reports several measures of persistence computed at different levels of aggregation (Levels 1 to 4). To compute impulse response functions, AR(k) processes have been estimated where the order k has been chosen according to the GTS and to the AIC.[16] The first four rows of Table IV present the sum of the first h values of the IRFs relative to aggregation levels 1 to 4, for $h = \{4, 8, 12, 16 \text{ and } 20\}$, that is, the cumulated response of inflation from 1 to 5 years after the shock occurs. The bottom row of Table IV reports the average value of the SAC for the four aggregation levels. Figure IV, in turn, depicts the IRFs associated to aggregation levels 1 to 4. Confidence intervals have been computed using bootstrap methods.

In agreement with the theoretical results, Table IV shows that impulse responses computed from aggregate and sectoral data are very close at all the considered horizons. As can be seen from Figure IV, the four IRFs present approximately the same values and the same pattern of decay, so they imply a very similar degree of shock persistence, in line with the results presented in Table IV.

The values of the SAC, however, vary considerably across aggregation levels, reproducing what has been found in previous studies. Values of the SAC range from the 0.66 corresponding to aggregation level 4 (with a confidence interval of (0.64, 0.68)) to the 0.86 for level 1 (with a C.I. of (0.82, 0.92)). Therefore, from only looking at these figures, one would conclude that the response to a shock is higher, the higher the level of aggregation at which it is measured. However, a more detailed analysis of the evolution of the shock as described by the IRF suggest the opposite conclusions.

[16] The maximum number of lags was set to 20, 16, 12 and 8, for aggregation levels 1 to 4, respectively. The significance level for applying the general-to-specific criterion was 10%. Small sample-bias corrected estimates have also been computed but they are not reported since results are qualitatively identical.

TABLE IV. PERSISTENCE MEASURES

IRFS AT DIFFERENT AGGREGATION LEVELS

	Level 1		Level 2		Level 3		Level 4	
	$\sum_{i=1}^{h} IRF_{Level\ 1}(h)$		$\sum_{i=1}^{h} IRF_{Level\ 2}(h)$		$\sum_{i=1}^{h} IRF_{Level\ 3}(h)$		$\sum_{i=1}^{h} IRF_{Level\ 4}(h)$	
	GTS	AIC	GTS	AIC	GTS	AIC	GTS	AIC
$h = 4$	1.77	1.92	2.05	1.97	1.88	1.86	1.81	1.82
$h = 8$	3.06	3.77	3.28	3.05	3.04	2.98	2.91	2.91
$h = 12$	4.01	4.27	4.08	3.69	3.90	3.77	3.72	3.76
$h = 16$	4.58	4.86	4.67	4.17	4.59	4.41	4.33	4.43
$h = 20$	4.86	5.30	5.20	4.60	5.13	4.93	4.81	4.99
SAC AT DIFFERENT AGGREGATION LEVELS								
	0.86 (0.82,0.92)	0.86 (0.82,0.92)	0.81 (0.74,0.83)	0.77 (0.73,0.81)	0.69 (0.65,0.71)	0.69 (0.65,0.71)	0.67 (0.64,0.69)	0.66 (0.64,0.68)

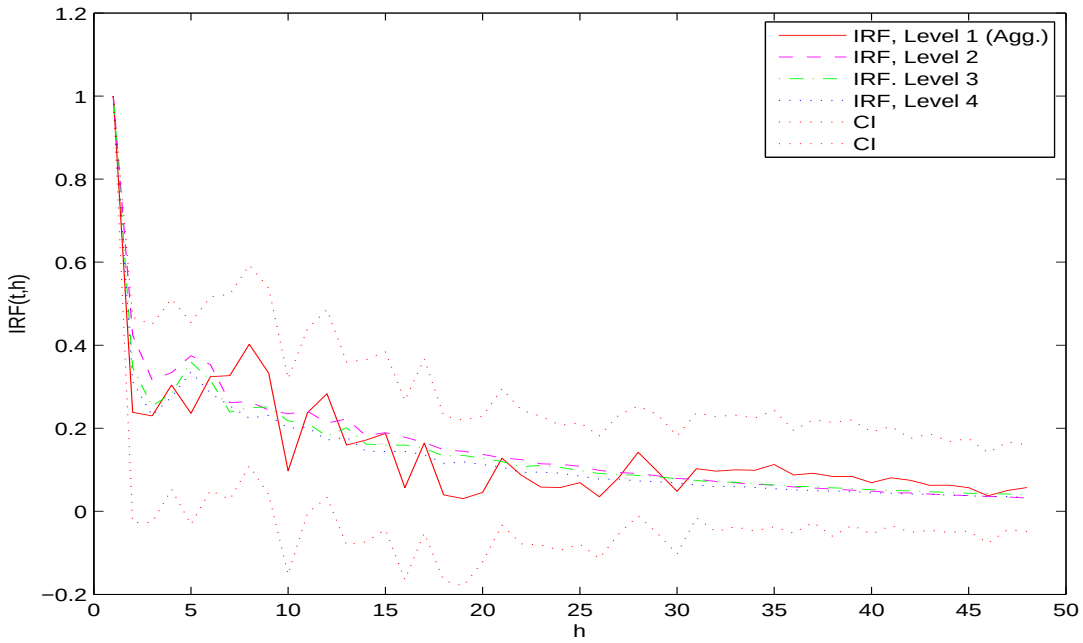

FIG IV. Impulse responses of U.S. inflation computed at different aggregation levels

7. CONCLUSION

This paper examines the relations among shock persistence measures computed at different levels of aggregation in a context where the units may exhibit heterogeneous linear dynamics. It is shown that the average of the individual IRFs equals the aggregate IRF at all horizons, implying that the *average* effect of aggregate shocks is not amplified by the aggregation of heterogeneous processes. A similar relationship also holds for some scalar measures of persistence such as the CIR. However, other tools as the SAR or the LAR have poor properties in this framework. In particular, they tend to yield higher values the higher the level of aggregation considered. Since these measures are the most employed in applications, it is not surprising that different average shock behavior has been reported in many empirical papers. An empirical application using U.S. inflation data that illustrates the theoretical results has also been provided.

Our results have some important implications on macroeconomic modelling. Some authors have warned against the practice of using averages of microparameters to calibrate macro models (Altissimo et al., 2009). However, our results qualify this statement by showing that the adequacy of this procedure depends on the type of microparameter employed. Thus, averages of individual IRFs can be used in macro calibration independently of whether there is heterogeneity or not at the individual level or whether the aggregate process is long memory or nonstationary while the individual units are stationary.

This paper opens several avenues for future research. Our results imply that under linearity both macro (aggregate) or micro (disaggregate) data can be used to estimate the impulse response function associated to an aggregate shock. Furthermore, they suggest that important efficiency gains can be obtained by using micro information. Thus, a thorough examination of the large and finite sample properties of micro and macro estimators is needed. A first step in this direction can be found in Mayoral (2009).

The relation between the micro and the aggregate IRFs can also be exploited in other ways. If disaggregate data is available, it is possible to completely determine the dynamics of the aggregate process. If the latter is stationary, the disaggregate IRF defined in (5) equals the coefficients of the Wold representation of the aggregate process. Then, the macro

dynamics can be identified very simply from the (average) of the micro impulse responses (and vice versa). If the aggregate process is not stationary, its Wold representation does not exist. However, the coefficients of the polynomial $A(z) = M(z)^{-1}$, where $\mathrm{M}(z)=\sum_{j=0}^{\infty} m_j z^j$ are m_j are the coefficients of the disaggregate IRF, are those corresponding to the AR(∞) representation of the aggregate process. These relationships could potentially be used to improve the accuracy of the estimation combining micro and macro data, in the spirit of Imbens and Lancaster (1994).

It would also be possible to recover micro information when only aggregate data is available. Since the aggregate IRF identifies all the moments of the distribution of the autoregressive parameter, a, it is possible to infer some interesting information about the distribution of this parameter only from the aggregate data. For instance, probability bounds for a can be easily computed from the aggregate data by applying Markov and Chevichev inequalities. Furthermore, under certain conditions, the whole distribution of the autoregressive parameter a can be recovered using only aggregate data. This is the well known "moment problem", that consists of inverting the mapping that takes a probability measure to the sequences of moments. This problem already has a long tradition, begun by Stieljes in 1894 and developed by subsequent authors (see, for instance, Lin, 1997 for a description of the conditions needed to recover the density from the moments).

REFERENCES

Altissimo, F., B. Mojon and P. Zaffaroni (2009), "Can aggregation explain the persistence of inflation?," *Journal of Monetary Economics* 56, 231-241.

Altissimo, F., L. Bilke, A. Levin, T. and B. Mojon (2006a), "Sectoral and Aggregate Inflation Dynamics in the Euro," *Journal of the European Economic Association*, Vol. 4, pp. 585-593.

Altissimo, F., M. Ehrmann and F. Smets (2006b), "Inflation persistence and price setting behaviour in the euro area", Occassional paper series, N^o 46, ECB.

Andrews, D. and H. Chen (1994), "Approximately median-unbiased estimation of autoregressive models," *Journal of Business and Economic Statistics*, 12, pp. 187-204.

Angeloni, I., L. Aucremanne, M. Ehrmann, J. Galí, A. Levin and Frank Smets, (2005), "New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modelling," Economics Working Papers 910, Department of Economics and Business, Universitat Pompeu Fabra.

Berk, K.N. (1974), "Consistent autoregressive spectral estimates," *The Annals of Statistics* 2, pp. 489-502.

Bilke, L. (2005), "Break in the mean and persistence of inflation: a sectoral analysis of French CPI," ECB Working Paper 463.

Caballero, R.J. and E. M.R.A. Engel (2007), "Price Stickiness in Ss models: New interpretations and new results," *Journal of Monetary Economics* 54, pp. 100–121.

Choi, E. and P. Hall (2000), "Boostrap confidence regions commputed from autoregressions of arbitrary order," *Journal of the Royal Statistical Society, Series B, 62,* pp. 461–477.

Clark, T. (2006), "Disaggregate evidence on the persistence of consumer price inflation", *Journal of Applied Econometrics,* Vol. 21, 5, pp. 563-587.

Crucini, M. J. and M. Shintani (2008), "Persistence in Law-of-One-Price Deviations: Evidence from Micro-data," Journal of Monetary Economics, 55, pp. 629-644.

Forni M. and M. Lippi (1997), Aggregation and the microfoundations of dynamic macroeconomics, Oxford University Press Inc., New York.

Gadea, L. and L. Mayoral (2007), "Aggregation is not the solution: the PPP puzzle

strikes back," forthcoming *Journal of Applied Econometrics.*

Godet, F. (2007), "Linear prediction of long-memory processes: asymptotic results on mean-squared errors," Mimeo.

Goncalves, S. and L. Kilian (2004), "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form," *Journal of Econometrics* 123, pp. 89-120.

Granger, C.W.J. (1980), "Long memory relationships and the aggregation of dynamic models", *Journal of Econometrics* 14, pp. 227-238.

Haque, N., M.H. Pesaran and S. Sharma (2000), "Neglected heterogeneity and dynamics in cross-country savings regressions," in (eds.) J. Krishnakumar and E. Ronchetii, *Panel Data Econometrics- Future Directions: papers in honor of Pietro Baleestra*, Elsevier, Amsterdam.

Hsiao, C. and M. H. Pesaran (2007), "Random coefficient panel data models," in L. Matyas and P. Sevestre (eds.), The Econometrics of Panel Data, Kluwer Academic Publisers.

Hu. X. and S. Ng (2004), "Estimation covariance structures of dynamic heterogeneous panels," Mimeo.

Imbens, G. W. and T. Lancaster (1994), "Combining micro and macro data in microeconometric models," *Review of Economic Studies* 61, pp. 631-653.

Imbs, J, A. Muntaz, M.O. Ravn and H. Rey (2005), "PPP Strikes Back: Aggregation and the Real Exchange Rate," *Quarterly Journal of Economic*s 120, pp. 1-43.

Kreiss, J.P. (1997) , "Asymptotic properties of residual bootstrap for autoregressions," mimeo. Institute for Mathematical Stochastics, Technical University of Braunschweig.

Kuersteiner, G. M. (2005), "Automatic Inference for Infinite Order Vector Autoregressions", *Econometric Theory*, 21, pp. 85-115.

Lewbel, A. (1994), "Aggregation and Simple Dynamics," *American Economic Review* 84, pp. 905-918.

Lewis, R. and G. Reinsel (1985), "Prediction of multivariate time series by autoregressive model fitting," *Journal of Multivariate Analysis* 64, pp. 393-411.

Lin, G. D. (1997) "On the moment problems", *Statist. Probab. Lett.* 35 85-90.

Lünnemann, P. and T. Matha (2004), "How persistent is disaggregate inflation? An analysis across EU Countries and HIPC Sub-indices," ECB Working paper 415.

Kelejian, H. J. (1980), "Aggregation and disaggregation of nonlinear equations," in: J. Kmenta and J. B. Ramsey (eds.): Evaluation of Econometric Models, New York: Academic Press, 135-153.

Koops, G., M. Pesaran and S.M. Potter (1996), "Impulse response analysis in nonlinear multivariate models," *Journal of Econometrics* 74 (1), pp. 199-47.

Judd, K. L. (1985), "The law of large number with a continuum of IID random variables," *Journal of Economic Theory* 35, 19-25.

Mayoral, L. (2009), "On the estimation of aggregate impulse response functions under dynamic heterogeneity with macro and micro data," mimeo.

Ng, S. and P. Perron, (1995), "Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag," *Journal of the American Statistical Association* 90, pp. 268-281.

Paparoditis, E. (1996) , "Bootstrapping autoregressive and moving average parameter estimates of infinite order vector autoregressive processes" *Journal of Multivariate Analysis* 57, pp. 277-296.

Pesaran, M. H. (2006), "Estimation and inference in large heterogeneous panels with a multifactor error structure." *Econometrica* 74.

Pesaran, M. H. and R. Smith (1995), "Estimating long-run relationships from dynamic heterogeneous panels", *Journal of Econometrics* 68, pp. 79-113.

Robinson, P.M. (1978), "Statistical inference for a random coefficient autoregressive model", *Scandinavian Journal of Statistics* 5, pp. 163-168.

Stoker, T. M. (1984), "Completeness, distribution restrictions and the form of the aggregate function," *Econometrica*, Vol. 52, N ^{o} 4, *pp.*887-907.

Uhlig, H. (1996), "A law of large numbers for large economies," *Economic Theory* 8, 41-50.

Zaffaroni, P. (2004), "Contemporaneous aggregation of linear dynamic models in large economies," *Journal of Econometrics* 120, pp. 75-102.

Zhang, P. and D. Small (2006), "Bayesian Inference for random coefficient dynamic panel data models," Mimeo.