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Concentration, Separation, and Dispersion: Economic Geography and the Environment

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by

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Abstract
The paper investigates the spatial patterns of industrial location and environmental pollution in a new-economic-geography model. Factors of production and their owners are mobile, but factor owners are not required to live in the region in which their factors are employed. Under laisser faire, a chase-and-flee cycle of location is possible: people, who prefer a clean environment, are chased by polluting industries, which want to locate geographically close to the market. Locational patterns under optimal environmental regulation include concentration, separation, dispersion and several intermediate patterns. Moreover, it is shown that marginal changes in environmental policy may induce discrete changes in locational patterns.

Keywords: economic geography, migration, trade, pollution, environmental regulation

JEL codes: Q52, Q56, Q58, R30, F12

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1 Introduction

Economic geography deals with the allocation of economic activity in geographical space. With his short monograph "Geography and Trade", Paul Krugman [12] revived the interest in this field of economic research, which had been largely neglected for a long time. Using tools of modern trade theory, in particular Krugman's [10, 11] intra-industry trade models based on Dixit-Stiglitz [4] monopolistic competition, a "new economic geography" (NEG) was established. The NEG is a theory that does not rely on ad-hoc arguments and heuristics to explain spatial patterns of economic activity but instead builds on a consistent micro-founded modelling framework. General features of the NEG are the interaction of centripetal and centrifugal forces (which explain agglomeration and spatial dispersion of economic activities, respectively), bifurcations, and path dependencies. Small parameter changes may drive an economy from agglomeration to dispersion and vice versa and adjustment paths towards long-term equlibria are not unique such that the outcomes of dynamic allocation processes in space are indeterminate and may depend on historical pre-conditions or self-fulfilling expectations. The state of the art is summarized in books and survey papers such as Fujita et al., [6] Neary [15], Baldwin et al. [2], and Brakman et al. [3].

Geographical space is an important category in environmental economics, too. With the exception of greenhouse gases, CFCs, and possibly some other global pollutants, emissions generate more environmental harm in the geographical proximity of their source than far away. Catastrophic events like the London smog disaster of 1953 and the accidents in Seveso (1976), Bhopal (1984), and Chernobyl (1986) tragically testify this relationship. Nevertheless, the environmental-economics literature on the geographical dimension of pollution and, in particular its interaction with the spatial patterns of economic activity is still rather small. An early paper is Siebert's [22] handbook article, which was written before the arrival of the NEG and is by and large based on traditional models of foreign trade. Markusen et al. [14], Rauscher [18], and Hoel [8] look at interjurisdictional competition for mobile polluting firms and find that, depending on the severity of environmental harm, the outcome of this competition is either a "race to the bottom" or "not in my backyard". Similar results are obtained by Pflüger [16], who looks at the issue in a trade model incorporating Dixit-Stiglitz monopolistic competition and "iceberg" transportation costs. Although the model contains the main building blocks of NEG theory, it is still a trade model in that factors of production are immobile and agglomeration is excluded. Kanbur et al. [9] look at interjurisdictional competition, too. Modelling space as a continuous variable, they find that small jurisdictions have incentives to charge lower environmental taxes than larger ones. Agglomeration, which is the central issue of the NEG, is not addressed in any of these papers. Papers explicitly addressing issues of economic geography are Rauscher [19], van Marrewijk [23], and Lange/Quaas [13]. Rauscher [19] looked at a variety of NEG models and derived optimal environmental policies only for a world in which factors are immobile whereas residents can change their locations. The genuine NEG issues that arise from the mobility of factors were, however, only sketched in this paper. Elbers/Withagen [5] and van Marrewijk [23] investigate core-periphery NEG models in which centrifugal and centripetal forces do exist and derive the result that agglomeration forces are mitigated by environmental externalities. Similar results are reported by Lange/Quaas [13] who derive a wide range of economic-geography patterns including full agglomeration, partial agglomeration, and dispersion of economic activities. The underlying reason for less agglomeration is the centrifugal character of environmental pollution. Workers demand compensating wage differentials if a job requires them to live in a polluted industrial agglomeration. This centrifugal force mitigates or even offsets the agglomerative forces that are present in NEG models

Similar issues are discussed in the present paper. As expected environmental pollution constitutes a centrifugal force that works against agglomeration in this paper, too. However, the following analysis differs from the aforementioned papers in one important aspect. I assume that people can escape pollution. They are not tied to the region in which their income is generated. This generates some additional patterns of geographical organization of the economy that have not been addressed in the previous literature.

To start off, I define three stylized patters which are rarely observed in their purest form bat that are useful as benchmark scenarios in a theoretical analysis,

- concentration, i.e. a geographical pattern where the most populated regions are the most polluted ones, e.g. big cities like Shanghai, Mexico City, or Los Angeles,
- the separation of pollution and population, the example being nuclear power stations, which are often located in peripheral regions with low population densities, and
- dispersion, i.e. a pattern where people and pollution are (more or less) evenly distributed in space.

To address these issues, I use an economic-geography model based on the standard ingredients, i.e. Dixit-Stiglitz [4] preferences and "iceberg" transportation costs. Moreover, like in many other NEG models, regions are symmetric. The main difference compared the standard core-periphery model of the NEG literature, which was extented by Elbers/Withagen [5] and others to deal with environmental externalities, is that factor owners do not have to live where their factors are employed. This is related to the footloose-capital model used by Pflüger [17], where the factor is mobile whereas its owners are not. As already mentioned, I go one step further by assuming that factor owners are mobile as well, but that they can choose their location of residence independently of that of their factor. With this assumption, two new patterns that cannot occur in other NEG models are detected. The first one is the "chase-and-flee" phenomenon: residents try to avoid pollution and leave agglomerations, but the industry wants to be where the market is and follows the consumers. The second one is separation: residents and the industry agglomerate in different regions if environmental damage is large.

The paper is organized as follows. The next section presents the model. Section 3 is devoted to the analysis of the spatial patterns of economic activities and pollution under laisser faire. In Section 4, I look at welfare maximization and derive optimal allocations of production and consumption in geographical space. In Section 5, the choice of environmental-policy instrument choice is discussed. Section 6 introduces costly emission abatement and Section 7 summarizes.

2 The Model

Assume a world consisting of two regions, East and West. Both regions are identical as regards preferences and technology. All variables related to the West are indicated by asterisks. To save space, the following paragraphs will perform the steps to derive the market equilibrium only for one region, the East. As regions are symmetric, the same steps can be performed for the West and the results are analogous.

Households

The total population of the two regions is 1, of which $ß$ live in the East and $(1\text{-}ß)$ in the West. All households are identical. There is a single factor of production which is equally distributed across households. Factor supply is inelastic and equals 1, of which a share k is employed in the East and $(1–k)$ in the West. $ß$ and k are endogenous variables in this model: households and factors are mobile but people do not have to live where their factor is employed. Think of mobile capitalists who, for example, prefer to live in pleasant environments such as the Cote d'Azur or the Swiss canton of Ticino whereas their capital is employed in an export processing zone in China. With factor prices w and w^*, the factor income of a representative household is $kw+(1–k)w^*$. Moreover, each household inelastically supplies one unit of a numéraire good such that its budget is $1+ kw+(1–k)w^*$. Let x be the consumption of the numéraire good. The other good is a differentiated good and it is available in many different varieties. Product variety is modelled as a continuum. $c(i)$ and $c^*(j)$ denote consumption of domestic varieties in the East and in the West, $m(j)$ and $m^*(i)$ denote the imports of foreign varieties, and $\theta > 1$ is the "horses fed with grain"[1] transportation cost mark-up. $p(i)$ and $p^*(j)$ are the prices of Eastern

[1] In his seminal work on the "isolated state", von Thünen [24 , p. 16] made the assumption that the horses pulling the carts of grain from the rural region to the city are fed with grain from the carts. Thus, the transportation cost is proportional to the value of the commodity transported. Some 100 years later, this concept was introduced into Heckscher-Ohlin trade theory by Samuelson [21] and termed "melting-iceberg" transportation cost. The advantages of the approach are (i) that a transportation sector does not need to be modelled and that (ii) that price elasticities of demand are not affected by the introduction of transportation costs.

and Western goods, respectively, and n and n^* measure product variety. Thus, the Eastern households' budget constraint is

$$\int_0^n p(i)c(i)di + \int_0^{n^*} \theta\, p^*(j)m(j)dj + x = 1 + kw + (1-k)w^* . \tag{1}$$

Preferences are of the love-of-variety type à la Dixit/Stiglitz [4]. Utility from consumption is quasilinear and it is augmented by subtracting the disutility from environmental pollution.[2] Thus, the utility of a representative resident is

$$u = \frac{1}{\gamma}\left(\int_0^n c(i)^\gamma di + \int_0^{n^*} m(j)^\gamma dj\right) + x - \frac{\delta e^{1+z}}{1+z}, \tag{2}$$

$\gamma \in (0,1)$ is a measure of substitutability. The elasticity of substitution between different varieties is $\sigma=(1-\gamma)^{-1}$. Quasilinearity implies that all income effects are captured by the numéraire good. Finally, e denotes emissions (e^* being the Western equivalent of e) and we assume that there are no transboundary pollution spillovers. δ measures the impact or intensity of environmental pollution. Moreover, the environmental damage is increasing and convex, the curvature parameter, z, being positive. The reason for specifying the damage function in this way is that some results to be derived in this paper depend on the curvature of the marginal-damage function. In this respect, the present model differs from the standard environmental-economics model, where assumptions are made only on the slope, but not on the curvature of the marginal-damage function. If z=1, the marginal damage is linear in emissions, for z<1 it is concave, for z>1 it is convex.

Utility maximization results in inverse demand functions for Eastern and Western commodities

$$p(i) = c(i)^{\gamma-1} \qquad \text{for} \quad i \in (0,n), \tag{3}$$

$$p^*(j) = \theta^{-1} m(j)^{\gamma-1} \qquad \text{for} \quad j \in (0,n^*) \tag{3*}$$

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2 This quasilinear utility function is probably the simplest way of specifying the love-of-variety model. It is even simpler than the model suggested by Pflüger [17], who also uses a quasilinear specification. The disadvantage of quasilinear utility is that model is only a partial-equlibrium model in that it neglects interactions between the market for differentiated goods and other goods markets.

Compared to many other economic-geography models, the demand functions are very simple (e.g., see Baldwin et al. [2]; Fujita et al. [6] and Neary [15]). They do not contain a CES price index. This is a direct result from specifying the utility function (2) as quasi-linear.

Moreover, it is assumed that all varieties are produced with the same technologies and the same factor requirements. Then the prices for these varieties are identical and the arguments *i* can be dropped. Using (1), (3), and (3*) in (2) gives the utility function of an individual living in the East:

$$\tilde{u} = 1 + kw + (1-k)w^* + \frac{1-\gamma}{\gamma}\left(np^{\frac{\gamma}{\gamma-1}} + n^*(\theta p^*)^{\frac{\gamma}{\gamma-1}} \right) - \frac{\delta e^{1+z}}{1+z}. \quad (4)$$

The producers

The supply side of the market for the non-numéraire good is characterized by Dixit-Stiglitz monopolistic competition. There is only one factor of production and its remuneration, which is exogenous to the firm, is w. All producers use the same technology characterized by increasing returns to scale. In particular, I assume constant marginal cost vw and fixed cost Fw, F and v being technological unit input requirements. Let q denote the output of a representative firm. Its profits are

$$\pi = pq - (F + vq)w \quad (5)$$

Profit maximization yields

$$\gamma p = vw,$$

i.e. marginal revenue equals marginal cost. To simplify notation, choose units of the input such that $\gamma = v$. Thus,

$$w = p. \quad (6)$$

Using this in the zero-profit condition yields:

$$q = q^* = \frac{F}{1-\gamma}. \quad (7)$$

This is a standard result of the Dixit-Stiglitz model with constant marginal cost. The output of a single variety is determined by the price elasticity and the parameters of the cost function, but it does not depend on any other variables of the model.

Since all firms produce identical quantities, the number of firms can be inferred from the factor market equilibrium. As k is factor supply in the East and factor demand is $n(F+vq)$, the factor-market equilibrium is

$$n = (1-\gamma)\frac{k}{F}. \tag{8}$$

Using (7) again, we have

$$Q=nq=k. \tag{9}$$

Emissions and environmental damage

Emissions are linearly related to production. For simplicity choose units such that the factor of proportionality is unity. Thus, $e=Q$ and $e^*=Q^*$. From (9), it then follows that

$$e=k. \tag{10}$$

In the other region, $e^*=1-k$. Thus environmental damages in the East and in the West are $D=\delta k^{1+z}/(1+z)$ and $D^*=\delta(1-k)^{1+z}/(1+z)$, respectively. In the remainder of the paper, the differential in environmental damage, $D - D^*$, will be of major importance. Its derivative with respect to k is

$$\frac{d(D-D^*)}{dk} = \delta\left(k^z + ((1-k)^z\right) \quad > \quad 0,$$

which is hump-shaped for $z<1$, u-shaped for $z>1$ and horizontal for $z=1$. It follows that $D-D^*$ is S shaped for $z<1$ and inversely S-shaped for $z>1$. See Figure 1. These curvature properties imply that, starting from a symmetric equilibrium, $k=0.5$, the first unit of factor movement causes the largest (smallest) change in the difference in environmental damage across regions if $z<1$ ($z>1$).

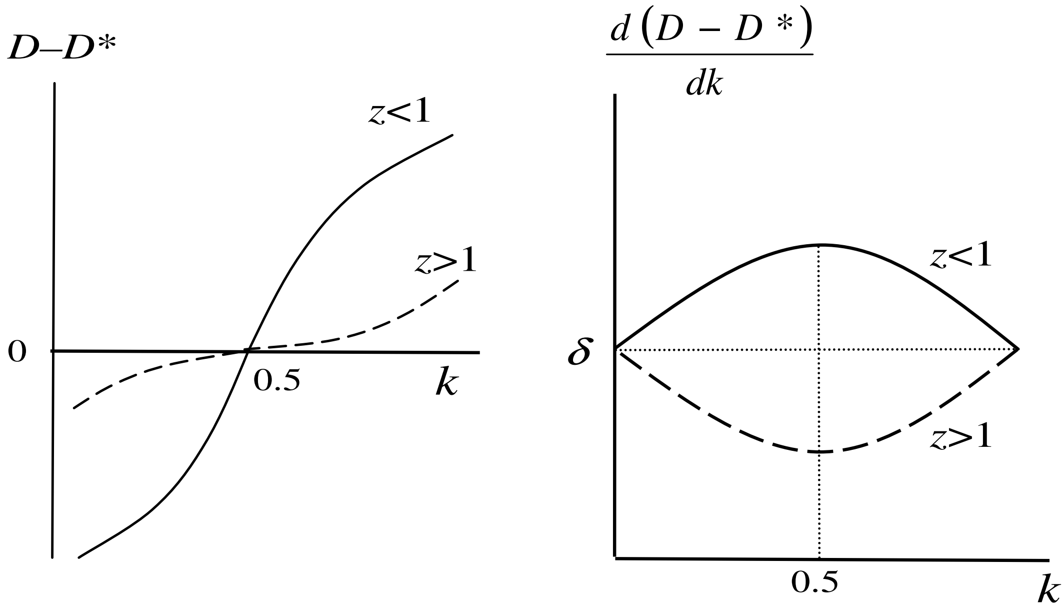

Figure 1: The differential in environmental damages across regions and its change

Goods market equilibrium

In the goods-market equilibrium, supply and demand are equal for each variety, i.e.

$$q = \beta c + (1-\beta)\theta m^*,$$

$$q^* = \beta \theta m + (1-\beta) c^*,$$

Inserting the Eastern inverse demand function, (3), and its Western analogue, and then using (7) to substitute for q and q^*, we obtain

$$p = \left[\frac{1-\gamma}{F}\left(\Theta + \beta(1-\Theta)\right)\right]^{1-\gamma}, \qquad (11)$$

$$p^* = \left[\frac{1-\gamma}{F}\left(1 - \beta(1-\Theta)\right)\right]^{1-\gamma}, \qquad (11^*)$$

with $\Theta = \theta^{\frac{\gamma}{\gamma-1}} < 1$ as a measure of trade freeness.

Indirect utility

Finally, the indirect utility can now be represented as functions of the exogenous parameters of the model and of the patterns of location of factors and factor owners, *k* and *ß*, respectively. In order to keep things a bit clearer, we do not eliminate prices. Thus, using (6) to substitute for *w*, (8) to substitute for *n*, and (9) to substitute for *e* in (4) yields the indirect utility of a representative Eastern resident:

$$\tilde{u} = 1 + kp + (1-k)p^* + \frac{(1-\gamma)^2}{\gamma F}\left(kp^{\frac{\gamma}{\gamma-1}} + \Theta(1-k)p^{*\frac{\gamma}{\gamma-1}}\right) - \frac{\delta k^{1+z}}{1+z}. \qquad (12)$$

Performing the same steps for the West yields

$$\tilde{u}^* = 1 + kp + (1-k)p^* + \frac{(1-\gamma)^2}{\gamma F}\left(\Theta kp^{\frac{\gamma}{\gamma-1}} + (1-k)p^{*\frac{\gamma}{\gamma-1}}\right) - \frac{\delta (1-k)^{1+z}}{1+z}, \qquad (12^*)$$

with *p* and *p** being determined by (11) and (11*), respectively.

3 Patterns of Location and Agglomeration

To analyse the spatial allocation of economic activity, we first look at the locations of the factors and then at the locations of their owners. Factors move if there is a difference in factor remunerations. From *w*=*p*, *w**=*p**, (11) and (11*), we have

$$w - w^* = \left(\frac{1-\gamma}{F}\right)^{1-\gamma}\left((\Theta + \beta(1-\Theta))^{1-\gamma} - (1-\beta(1-\Theta))^{1-\gamma}\right) \qquad (13)$$

Factors are indifferent where to locate if *ß*=0.5. In Figure 2.1, this is represented by the vertical indifference line at *ß*=0.5. The horizontal parts of the line are explained by the fact that factor shares cannot be larger than 1 or less than 0. In a next step, let us consider the behaviour of the factor owners. The difference in utility between East and West is

$$\tilde{u} - \tilde{u}^* = \frac{(1-\Theta)(1-\gamma)^2}{\gamma F}\left(kp^{\frac{\gamma}{\gamma-1}} - (1-k)p^{*\frac{\gamma}{\gamma-1}}\right) - \frac{\delta}{1+z}\left(k^{1+z} - (1-k)^{1+z}\right), \qquad (14)$$

where the last term on the right-hand side is the environmental-damage differential depicted in the left-hand part of Figure 1. Assume for a moment that environmental considerations do not matter such that this term vanishes, δ=0. Then the $\tilde{u} = \tilde{u}^*$ line, along which households are indifferent where to locate, can be derived from (14) by using (11) and (11*):

$$k = \frac{(\Theta + \beta(1-\Theta))^{\gamma}}{(1-\beta(1-\Theta))^{\gamma} + (\Theta + \beta(1-\Theta))^{\gamma}}. \qquad (15)$$

Its slope is

$$\left.\frac{dk}{d\beta}\right|_{\tilde{u}=\tilde{u}^*,\delta=0} = \gamma(1-\Theta^2)\frac{(\Theta+\beta(1-\Theta))^{\gamma-1}(1-\beta(1-\Theta))^{\gamma-1}}{\left[(1-\beta(1-\Theta))^{\gamma} + (\Theta+\beta(1-\Theta))^{\gamma}\right]^2} > 0.$$

From (15), we have that k=0.5 for β=0.5 and that k>0 for β=0 and k<1 for β=1. Moreover, taking the second derivative, one can establish that the slope is minimized in the symmetric equilibrium (β=k=0.5) and that it increases towards the boundaries of the (0,1) interval. The resulting $\tilde{u} = \tilde{u}^*$ line is depicted in 2.1 together with the w=w* line. There are three equilibria: full agglomeration in one of the two regions or an equal distribution of population and factors across the regions. Let us introduce simple adjustment dynamics, such that factors move to the region offering higher wages and people move to the region offering a larger indirect utility

$$\dot{k} = \lambda_k (w - w^*),$$

$$\dot{\beta} = \lambda_\beta (\tilde{u} - \tilde{u}^*).$$

λ_k and λ_β are positive adjustment-speed parameters. The Jacobian of the linearized dynamic system is

$$J = \begin{pmatrix} 0 & \lambda_k \frac{\partial(w-w^*)}{\partial\beta} \\ \lambda_\beta \frac{\partial(\tilde{u}-\tilde{u}^*)}{\partial k} & \lambda_\beta \frac{\partial(\tilde{u}-\tilde{u}^*)}{\partial\beta} \end{pmatrix} \sim \begin{pmatrix} 0 & + \\ + & - \end{pmatrix}, \qquad (16)$$

where the signs of the elements of J are displayed in (16) as well. The adjustment dynamics are indicated by horizontal and vertical arrows in Figure 2.1. There are two centripetal forces and one centrifugal force in this model. Factor movements always foster agglomeration. Factors move to where the majority of the consumers live because the region with larger demand generates higher facor remuneration. In the case of housholds, there are two forces, one centripetal, the other centrifugal. Consumers are attracted by producers offering a large degree of product variety without incurring transportation costs. In this respect, they like agglomeration, too. This is what Fujita et al. [6, p. 346] call "thick markets". On the other hand, they do not like to live where many other households live because the high demand raises local prices. This congestion effect is the centrifugal force of the model. The two centripetal forces, however, dominate and the two agglomeration equilibria are stable whereas the symmetric dispersion equilibrium is unstable – unless a trajectory starts on the saddle path leading to this point. Mathematically, this follows from the fact that the Jacobian, *J*, has a negative determinant. However, as initial conditions are historically given, the probability of starting exactly on this saddle is infinitesimally small. Thus, the corresponding equilibrium is irrelevant and laisser faire implies full agglomeration, i.e. the industry and the households locate in the same region.[3]

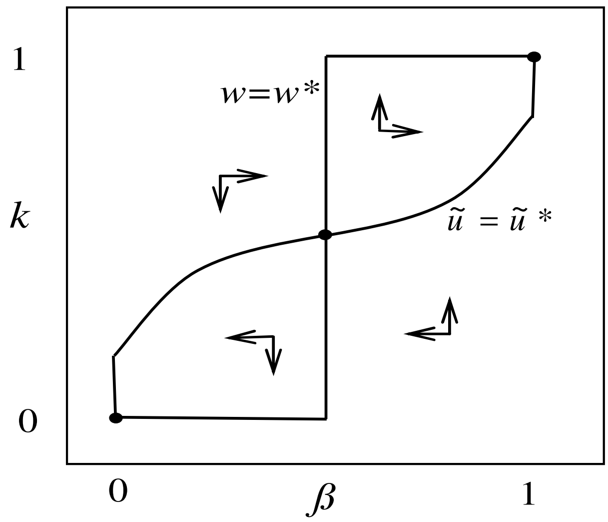

Figure 2.1: Patterns of location in the absence of environmental concerns

[3] Note that unlike in other geography models, dispersion is not a stable equilibrium. This means that a variant of the black-hole condition discussed in Fujita et al. [6, Ch. 4] is violated here due to the quasilinearity of the utility function.

Having determined agglomeration patterns in the absence of environmental problems, let us now consider environmental pollution. In this case, the second-row, first-column element of the Jacobian in (16) may change its sign. The slope of $\tilde{u} = \tilde{u}^*$ line can be determined via total differentiation of (14):

$$\frac{d\beta}{dk} = \frac{\dfrac{1}{\gamma(\Theta+\beta(1-\Theta))^{\gamma}} + \dfrac{1}{\gamma(1-\beta(1-\Theta))^{\gamma}} - \dfrac{F^{1-\gamma}\delta\left(k^{z}+(1-k)^{z}\right)}{(1-\Theta)(1-\gamma)^{2-\gamma}}}{(1-\Theta)\left(\dfrac{k}{(\Theta+\beta(1-\Theta))^{1+\gamma}} + \dfrac{1-k}{(1-\beta(1-\Theta))^{1+\gamma}}\right)} \tag{17}$$

The sign of the slope $\tilde{u} = \tilde{u}^*$ line is ambiguous since the first two terms in the numerator on the right-hand side are positive, whereas the second term is negative. Note that the numerator of the third term contains the marginal environmental-damage differential depicted in Figure 1. Four cases can be distinguished.

- In the first case, the marginal damage is very small such that the change in location of the $\tilde{u} = \tilde{u}^*$ line is so small that the qualitative behaviour of the system is the same is in Figure 2.1. The $\tilde{u} = \tilde{u}^*$, however, line is steeper than the one depicted in Figure 2.1.
- In the second case the marginal damage is so large that the second term in the numerator on the left-hand side always dominates the first term. This implies that slope of the $\tilde{u} = \tilde{u}^*$ line changes its sign.
- The third case is characterized by highly convex marginal environmental damage, i.e. z substantially larger than 1. In this case, the marginal damage differential is small for k close to 0.5, but large for k close to 0 or 1. The $\tilde{u} = \tilde{u}^*$ line is inversely S-shaped
- The fourth case is characterized by highly concave marginal environmental damage, i.e. z close to 0. In this case, the sum marginal damage differential is large for k close to 0.5, but small for k close to 0 or 1. The $\tilde{u} = \tilde{u}^*$ line is S shaped.

In what follows, I will investigate the latter three of the four cases graphically.

Case 1

Since the $\tilde{u} = \tilde{u}^*$ line is steeper than the one depicted in Figure 2.1, equilibria with incomplete agglomeration of households are possible. The underlying reason is that living in an agglomeration is not that beneficial anymore if producers generate environmental pollution. Thus, if all people lived in one region, the sum of congestion and pollution effects would dominate the agglomeration effect. A fraction of the inhabitants would have an incentive to emigrate, thereby mitigating the congestion effect for the non-migrants until inhabitants of both regions are equally well off. It turns out that pollution is a centrifugal force. See Elbers/Withagen [5] and van Marrewijk [23] for the same conclusion in a similar modelling framework.

Case 2

Case 2 is depicted in Figure 2.2. Compared to Figure 2.1, $\tilde{u}=\tilde{u}^*$ line is rotated in a counter-clockwise fashion. If the resulting indifference line is rather flat like the one depicted in Figure 2.2, there will be vertical segments for β=0 and β=1. The point of intersection with the $w=w^*$ line is either a stable node or a stable spiral. This follows from the Jacobian, (16), which now has a positive determinant. However, as the Jacobian is a linear approximation of the true dynamics which is reasonably accurate only close to the equilibrium, unstable paths farther from the equilibrium cannot be excluded. Figure 2.2 shows an example starting from agglomeration. Assume that all firms and housholds are located in the East, i.e. $\beta=k=1$. Households suffer from pollution and start moving to the West. At β=0.5, firms start to relocate, too, since factor incomes are higher in regions where demand is larger. At some point, all households have moved, but firms are still relocating to the West. If the number of firms is getting too large, people flee environmental pollution and start relocating to the East again. When β=0.5 again, firms start to follow until at some time housholds relocate to the West. There are two effects generating this cycle. On the one hand consumers like to live in a clean environment, i.e. they want to locate far away from the producers. The producers,on the other hand, like big markets and want to locate close to the consumers. Thus, the producers chase the consumers and the consumers try to flee. Since there are only two regions in the model, the result is a cycle. In the real world, there are, of course, more than two regions, but analogous patterns are observable in the process of de-urbanization. People move from the city to the country side, but shopping centres, petrol stations etc follow and tend to disturb the idyll. As a reaction people move even farther.

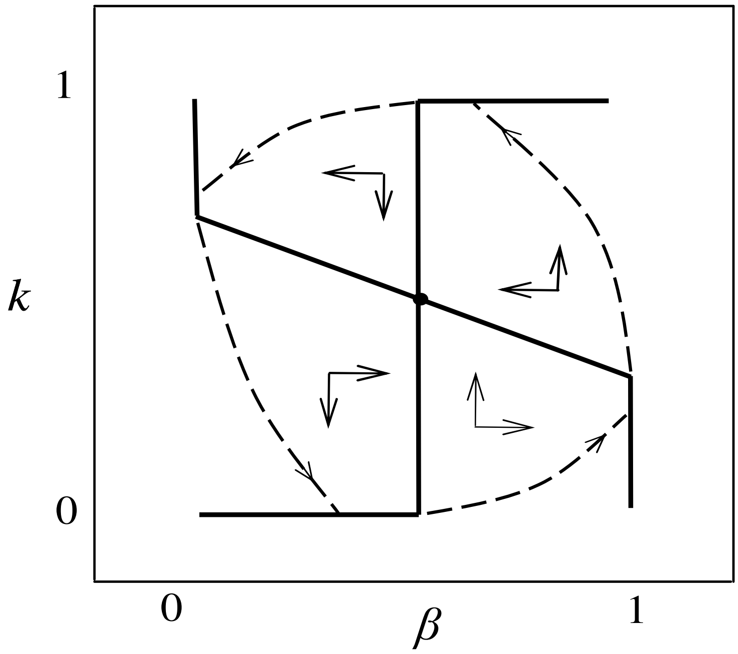

Figure 2.2: Patterns of location for large values of δ: The chase case

Two additonal remarks shall be made here. As already mentioned, the dispersion equilibrium, β=k=0.5, is locally asymptotically stable. Thus, instead of the chase-and-flee circle, the long-run laisser-faire solution of the model might be one in which firms and housholds are equally distibuted across the two regions. This is likely if the $\tilde{u} = \tilde{u}^*$ line is not too flat, i.e. if pollution is substantial but not extreme. The larger the impact of pollution, δ, flatter the $\tilde{u} = \tilde{u}^*$ line and the more likely is the chase-and-flee case. The second remark is related to the question as to which parameters induce the change from Scenario 1 to Scenario 2. Inspection of ion (17) shows that besides δ, which is obvious, the fixed cost, F, is a critical parameter, too. F occurs in the last term in the numerator on the right-hand-side of this equation and, thus, has the same effect, at least qualitatively as the pollution parameter δ. The reason is that a large fixed cost reduces utility derived from consumption. See equations (12) and (12*). Thus, the compensation consumers get in terms of consumer surplus when they locate close to dirty producers is small and environmental concerns tend to dominate material wants.

Case 3

If z is large, the differential in environmental damage between the East and the West is increased more than proportionally in relation to the number of firms if firms relocate. Close to the dispersion equilibrium, the system has the same properties as in Case 1. It is a saddle and, thus, unstable. When $d\beta/dk$=0 in equation (18), the $\tilde{u}=\tilde{u}^*$ line bends backwards and additional equilibria with fully dispersed households and incomplete industrial agglomeration are feasible. See Figure 2.3. The possibility of such an inverse S shape is confirmed by numerical simulations (see the diagrams in the Appendix). The partial-agglomeration equilibria are stable nodes or spirals and the initial conditions determine which equilibrium is approached. A chase-and-flee cycle like depicted in Figure 2.2 cannot be excluded, however. For a large degree of industrial agglomeration, the marginal and absolute damages in the industrialized region are very large such that households living have large incentives to relocate and like in Case 2 factors tend to follow.

Case 4

If z is small, the curve may be S-shaped like the one depicted in Figure 2.4. Close to the dispersion equilibrium, the system exhibits the same qualitative behaviour as the one discussed in Case 2. The dispersion equilibrium is locally stable. Far from this equilibrium, agglomeration forces dominate and the properties of the system resemble those of the one discussed in Case 1. There are two equilibria with dispersed households and partially agglomerated factors and they are unstable. Finally, there are two stable agglomeration equilibria with full agglomeration of the industry in one region and a majority (which may be 100%) of the households residing in the same region. Thus, depending on historical conditions, factor movements and household migration may either result in agglomeration or in dispersion.

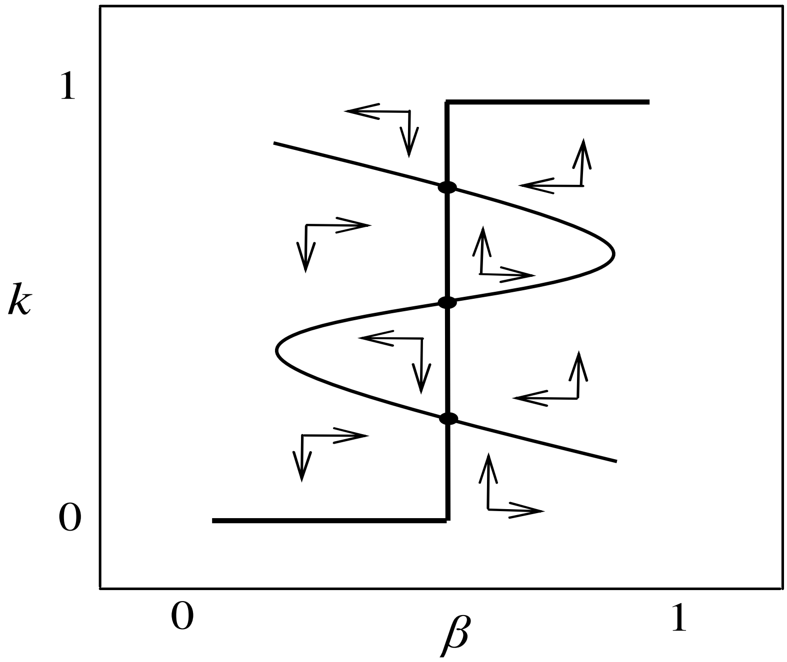

Figure 2.3: Patterns of location for large values of z

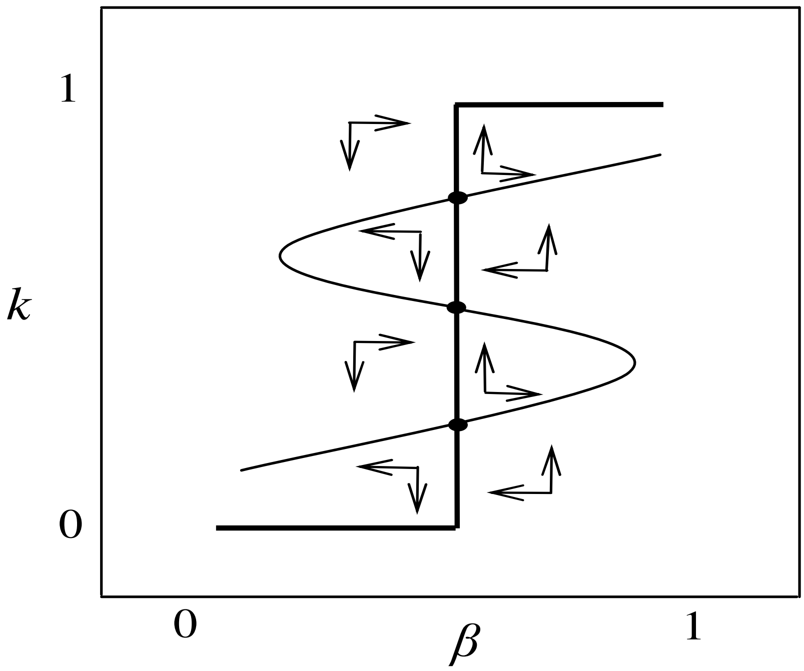

Figure 2.4: Patterns of location for small values of z

4. Optimal Environmental Policy

Let W denote welfare. Welfare is the population-weighted sum of utilities in the East and in the West, $\beta\tilde{u}+(1-\beta)\tilde{u}^*$. From (12) and (12*), rearranging terms and using (11) and (11*) to replace (Θ+β(1–Θ)) and (1–β(1–Θ)) , we have

$$W=1+\frac{1}{\gamma}(kp+(1-k)p^*)-\frac{\beta\delta}{1+z}k^{1+z}-\frac{(1-\beta)\delta}{1+z}(1-k)^{1+z}, \tag{18}$$

with p and p^* given by (11) and (11*), respectively

This is to be maximized with respect to respect to β and k. The derivatives with respect to k and β are:

$$\frac{\partial W}{\partial k}=\frac{w-w^*}{\gamma}-\delta\beta k^z+\delta(1-\beta)(1-k)^z, \tag{19}$$

$$\frac{\partial W}{\partial \beta}=\tilde{u}-\tilde{u}^*, \tag{20}$$

where the wage differential and the indirect-utility differential are determined by eqs. (13) and (14), respectively. It is not surprising that the indifference condition $\tilde{u}=\tilde{u}^*$ is the first-order condition with respect to household location. Households do not generate externalities (the congestion effect being internalized the market). Thus, individual behaviour is compatible with welfare maximization. If households do not want to move, i.e. if $\tilde{u}=\tilde{u}^*$, they have located optimally, individually as well as socially, and the social planner's first-order condition is met. Firms, in contrast, do cause external effects and, therefore, the planner's first order condition with respect to firm location differs from the corresponding private-sector indifference condition.

The social optimum can be derived from the first-order conditions (19) and (20). Before this is done, we look two special cases that are quite simple.

- Absence of environmental damage. If δ=0, full agglomeration is optimal. Equations (19) and (20) imply that welfare is increasing (decreasing) in β and k if the corresponding wage or utility differential is positive (negative). This implies that the gradient pointing to the maximum corresponds to the arrows depicting the adjustment dynamics in Figure 2.1. The agglomeration equilibria are maxima and the symmetric dispersion equilibrium is a saddle.

- Dominant environmental damage. The case in which environmental damage is so large that it dominates everything else can be considered by letting δ go to infinity in eq. (18) or by setting the openness parameter to unity, $\Theta=1$. The only welfare and component to be affected by location choices then is the environmental-harm term at the end of the right-hand side of (18), which is always negative. It can be minimized by setting $\beta=0$ and $k=1$ or vice versa, i.e. by perfectly separating households and factors from each other.

Having discussed the two extreme cases of full concentration and perfect separation, we proceed by looking at the intermediate case. Matters are more difficult if environmental damages are neither negligible nor dominating everything else. Solutions cannot be determined explicitly anymore since the first-order conditions, (19) and (20), are highly nonlinear and, moreover, optima are not always interior. Thus, a numerical approach was employed as follows.

1. In a first step select parameters to calibrate the Dixit-Stiglitz model. In what follows, $F=\Theta=\gamma=0.5$. Then, choose values z such that the marginal environmental damage is either convex or concave.

2. In a second step, vary the pollution-impact parameter δ from zero to infinity (in practice a very large value) and determine the corresponding welfare maxima numerically taking into account that boundary welfare maxima are feasible.

3. Finally, the welfare-maximizing allocations of firms and households corresponding to changes in the environmental-impact parameter δ are depicted in the (β,k) space.

The second step of the procedure, which is a bit cumbersome, is described in detail in the appendix. In what follows, we present only the final result, obtained from performing step 3.

Figures 3.1, and 3.2 show results for convex and concave marginal environmental damage corresponding to the parameter values z=4 and z=0.2, respectively. The parameter δ is increased from zero to a large value and the arrows depict the movement of the optimum allocation of production and habitation as δ increases. Solid lines represent continuous movements of the welfare-maximizing spatial pattern and dotted lines indicate jumps from one maximum to another one. The figures are stylized insofar as they depict the qualitative results. In particular, the solid lines may be bended rather than linear as drawn in the diagrams. The figures show that the optimum is full agglomeration if δ=0 and perfect separation if δ is very large. For some intermediate levels of δ, the welfare maximum is attained by dispersion, i.e. by β=k=0.5. Other possible welfare maxima include the possibilities of imperfect agglomeration and imperfect separation. In particular, we have the following results:[4]

- z=4. For small values of δ, there is full agglomeration. As δ is increased, the optimum moves to partial agglomeration and then to total dispersion, i.e. 50% of factors and 50% of factor owners locating in the East and the remaining 50% in the West. As δ is further increased beyond a particular critical value, the equilibrium jumps to partial separation: all households should be located in one region and the majority of the production in the other region. If δ becomes very large, perfect separation becomes optimal.
- z=0.2. Full agglomeration is again optimal for small values of δ. As δ is increased beyond some critical value of δ, a jump occurs and dispersion is the optimum. However, unlike in the above case, the movement of the optimum from dispersion to perfect separation is smooth.

Of course the arrows in Figures 3.1 and 3.2 do not represent dynamics, but comparative statics. The important result is that besides full agglomeration and perfect separation almost all other geographical patterns can be optimal: partial agglomeration, partial separation, and dispersion.[5]

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4 The appendix discusses these scenarios in much more detail, including cases in which the first-order conditions are not sufficient for a welfare maximum, but rather give rise to a saddle point.

5 Some spatial patterns can be exluded, for example a partial-agglomeration equilibrium which is possible under laisser faire in Scenario 3 (Figure 2.3) and wich is characerized by an equal number of housholds in the two regions and an uneven distribution of producers.

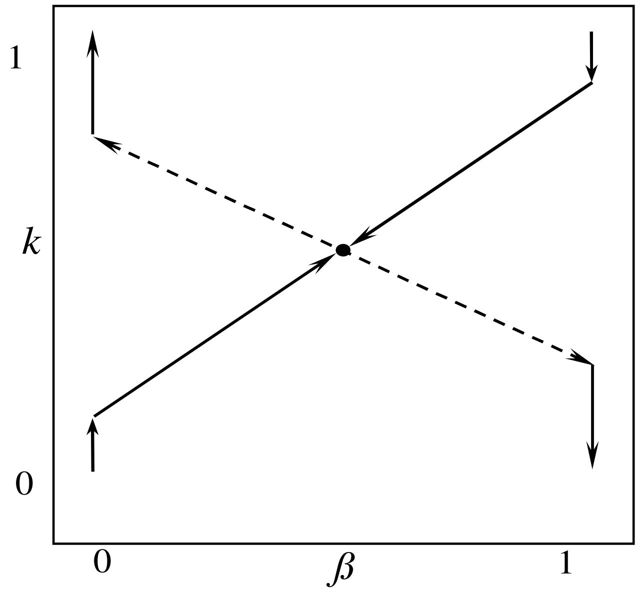

Figure 3.1: Effects of increasing δ for z=4

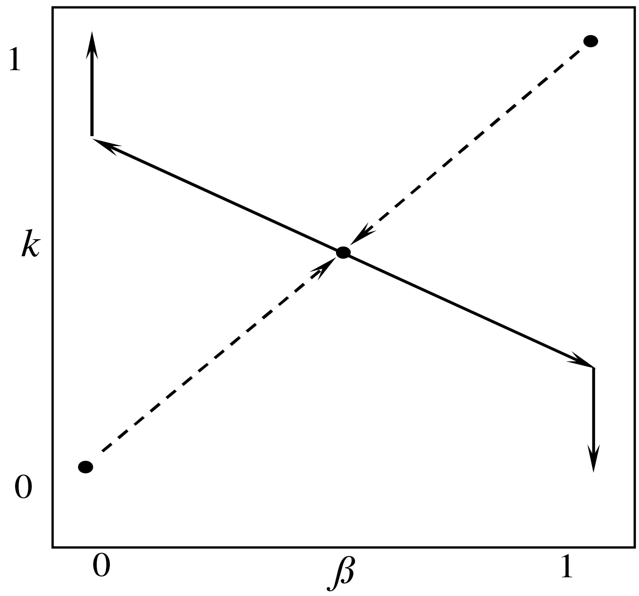

Figure 3.2: Effects of increasing δ for z=0.2

5. A Note on Instruments of Environmental Policy

How can the optimum be achieved? For small environmental damages, the policy is laisser faire. If environmental damage matters, however, full agglomeration is no longer optimal: incomplete agglomeration of firms is warranted and for extreme environmental damages even perfect separation of firms and residents is desirable. The standard instrument in environmental economics to implement the welfare-maximizing allocation is an environmental tax on the emission of pollutants. In our model, the emissions are proportional (and with the normalizations made in Section 2 even identical) to the use of factors. Thus, one could employ factor taxation instead. However, such taxes do not always do the job. The impact of a tax on factor utilization in the East is shown in Figure 4. It s seen that the laisser-faire indifference locus for producers known from Figure 2.1 is shifted to the right. The disadvantage of the tax can only be compensated for by a larger market, i.e. $\beta>0.5$. As partial-agglomeration optima lie on the upward-sloping parts of the $\tilde{u}=\tilde{u}^*$ line (see Figure A.1 in the appendix), the equilibria resulting after the introduction of a tax are saddles and, thus, unstable. Although the industry's indifference line is moved, centripetal forces continue to dominate the factor allocation. Factors move to where the consumers are. It is not possible to induce factors to agglomerate only partially in a region where consumers agglomerate. The underlying reason is that there are no decreasing returns to factor utilization in this model like they are known from standard models of international factor movements. See Ruffin [20] for a survey. Factor incomes depend only on the size of the product market, but not on the factor supply in a region. Equation (13) confirms this.[6]

If taxes do not inhibit industrial concentration in a region, the policy must be command and control. The optimum allocation of factors across regions must be enforced by law as economic incentives are ineffective. Once the optimum allocation of production has been established, consumers choose their locations optimally without further government intervention.

[6] In the case of partial separation, matters are different. The $\tilde{u}=\tilde{u}^*$ schedule is downward-sloping and the equilibrium is a stable node or a stable spiral. However, the chase-and-flee dynamics cannot be excluded generally.

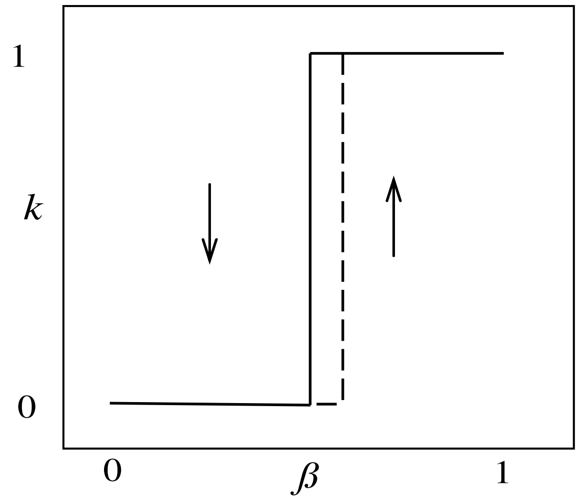

Figure 4: The impact of factor taxation in the East

6 Costly Emission Abatement

Up to here, the only way of reducing environmental damage was to relocate producers to the other region – of course at the expense of increasing emissions there. This section introduces proper emission abatement. In the absence of abatement efforts, baseline emissions are k and $1-k$ in the East and in the West, respectively. See equation (10). Emissions after abatement then are $k-a$ and $1-k-a^*$, where a and a^* denote the abatement levels. Abatement is costly and we model this by an increase in the fixed cost. Think of a large investment in cleaner technology that renders production cleaner once and for all. Let the abatement cost be increasing and convex, i.e. $F'(a)>0$, $F''(a)>0$.. From (7), (8), and (9) we then have that

- the output per firm is increasing in a,
- the number of firms is decreasing in a, and
- the region's total output (and thus its baseline emissions, too) is unaffected by a change in a.

The underlying logic is that the increase in fixed cost makes large-scale production more profitable. The number of firms declines. Without the change in abatement, this would result in an overall cost reduction due to increasing returns to scale. This cost saving is, however, eaten up by the higher cost of abatement. That the effect on total output (and, thus, on baseline emissions as well) is zero, is an artefact of this model. Nevertheless, the following results will go through for more complex models of monopolistic completion as well.

In what follows, we will look at the impact of a change in environmental regulation on the patterns economic geography. Let us assume that environmental regulation is of the command-and-control type and that the Eastern government unilaterally tightens environmental standards whereas the West does business as usual, i.e. $da>0$ and $da^*=0$. With different environmental standards in the two regions, the factor-income differential, (13), and the utility differential, (14), have to be rewritten such that

$$w - w^* = \left(\left(\frac{1-\gamma}{F(a)} (\Theta + \beta(1-\Theta)) \right)^{1-\gamma} - \left(\frac{1-\gamma}{F(a^*)} (1-\beta(1-\Theta)) \right)^{1-\gamma} \right), \tag{21}$$

$$\tilde{u} - \tilde{u}^* = (1-\Theta)(1-\gamma)^2 \left(\frac{kp^{\frac{\gamma}{\gamma-1}}}{\gamma F(a)} - \frac{(1-k)p^{*\frac{\gamma}{\gamma-1}}}{\gamma F(a^*)} \right) - \frac{\delta}{1+z} \left((k-a)^{1+z} - (1-k-a^*)^{1+z} \right). \tag{22}$$

Note that p and p^* in (22) depend on the abatement levels via (11) and (11').

Let us investigate the impact of the parameter change in the (β,k) diagram by looking at the indifference lines, $k=k^*$ and $\tilde{u}=\tilde{u}^*$. Without loss of generality assume that the initial situation is characterized by $a=a^*=0$. Moreover, we consider a scenario with a relatively small environmental-damage parameter such that the consumers' indifference line still is upward-sloping. This is like in Figure 2.1, however with a slightly steeper $\tilde{u}=\tilde{u}^*$ line. Totally differentiating the left-hand sides of (21) and (22), keeping k constant, we have

$$\left.\frac{d\beta}{da}\right|_{w=w^*,a=0} = \frac{(1-\Theta^2)F'}{(1-\beta(1-\Theta))^2 F} > 0, \tag{23}$$

$$\left.\frac{d\beta}{da}\right|_{\tilde{u}-\tilde{u}^*,a=0} = \frac{(1-\Theta)(1-\gamma)^{3-\gamma} kF^{\gamma-2} F' - \delta k^z}{(\Theta+\beta(1-\Theta))^{\gamma} \partial(\tilde{u}-\tilde{u}^*)/\partial\beta}, \tag{24}$$

where the arguments of F and F' have been omitted for notational brevity. From (23), it is clear that the ($w=w^*$) indifference line is shifted to the right, the more the higher the marginal abatement cost, F'. Since costs are increased by tighter environmental standards and this has a negative impact on factor income in the region affected by these standards, factor owners can only be indifferent where to locate their factors if the regulated economy is larger than the unregulated economy, i.e. if $\beta>0.5$ in our case. As can be seen from (24), there are two effects that shift the $\tilde{u}=\tilde{u}^*$ indifference line. The denominator on the right-hand side is always negative. The first term in the numerator is positive. This means that material well-being is reduced by relatively more for residents in the region with tightened environmental standards. Of course, residents in both regions are negatively affected by the higher production cost in the East, but those living in the West are to some extent insulated from the adverse effects of tighter standards by the transportation cost. The other effect is the environmental-quality effect. People living in the region with stricter environmental standards benefit from lower pollution. Given that we assumed that a=0 initially, the environmental quality effect is likely to dominate unless the marginal abatement cost is prohibitively high. Then, $\tilde{u}=\tilde{u}^*$ line is shifted to the right as well. Possible implications for economic geography are depicted in Figure 5.

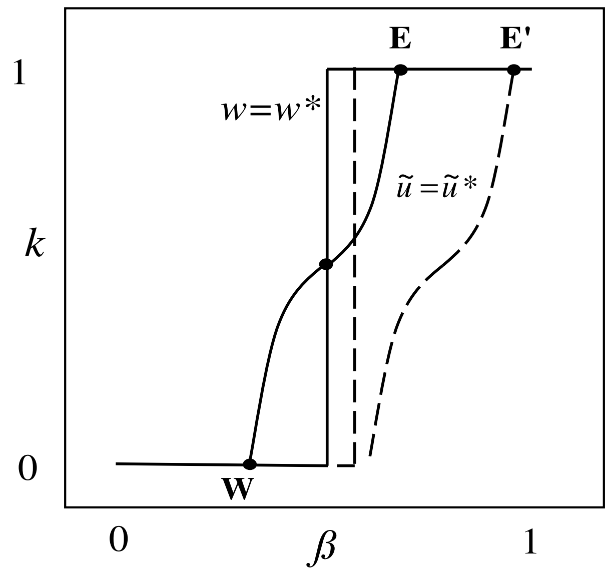

Figure 5: Effects of tighter environmental standards in the East

Figure 5 shows a scenario in which F' is small compared to δ such that the environmental-quality effect dominates the other two effects contained in (23) and (24). The shift of the $w=w^*$

line is minuscule compared to the shift of the $\tilde{u}=\tilde{u}^*$ curve. It is seen that one of the stable equilibria, *W*, indicating agglomeration of firms in the West, vanishes, whereas the Eastern equilibrium, *E*, only changes its location. What does this mean? Stricter standards in the East are bad for factors, but good for citizens residing in this region. As the East becomes more attractive as a place to live, ecologically sensitive people emigrate from the West and move to the East. As firms benefit from the larger market, they relocate as well if the market-size effect is stronger than the abatement-cost effect. The result is a catastrophic change in the spatial patterns of habitation and production, which occurs if the abatement parameter a is increased beyond a certain threshold value. A marginal change in environmental regulation can induce a landslide of the spatial organization of the economy.

7 Summary and Conclusions

This paper has looked at a world in which factors of production and their owners are mobile, but in which factor owners do not have to live where their factors are employed. The set-up was a two-region Dixit-Stiglitz model with trade costs. Due to the assumption of quasilinear preferences the baseline version of the model without environmental externalities unambiguously generates agglomerations. The centripetal always dominate the centrifugal forces. Environmental externalities induce an additional centrifugal force such that agglomeration tendencies are mitigated. However, this only affects mobile households, but not mobile production. Thus, if environmental harm is large, a chase-and-flee pattern is possible. People want to live in a clean environment and they avoid industrial agglomerations. However, their role as factor owners induces an investment pattern that contradicts their self-interest as consumers of environmental quality. They tend to locate their factors where the demand is, i.e. close to the consumers. Since each individual's contribution to the environmental harm is marginal, the environmental concern is not taken into account and the industry chases the consumers.

Optimal spatial patterns in the presence of environmental concerns range from complete separation over dispersion to complete agglomeration. Complete separation is optimal if environmental harm is large: no one wants to live near a hot spot and no one should live near a hot spot. It has then been shown that partial-agglomeration optima are sometimes difficult to be implemented. As there are no decreasing returns to industrial concentration, environmental taxes can be ineffective when an optimal pattern of production and habitation is strived for. A way out is to implement the optimal allocation of factors across regions by command and control. Finally, it was shown that the unilateral introduction of environmental standards via abatement requirements may induce catastrophic changes in the geographical patterns of habitation and production.

An interesting feature of this model is that many results depend on the curvature of the *marginal* environmental damage curve. In standard environmental-economics models, it is assumed that marginal environmental damage is increasing, but its second derivative does not matter. Here it does matter. The underlying reason is that locational decisions involve a comparison of environmental damages in the two regions. In other words, locational decisions are based not on the absolute level of environmental damage but on its East-West differential. This implies that third derivatives of the damage function play a role in situations where only second derivatives would occur in standard models.

Some extensions of the model come into mind. Firstly, all pollution was treated as a purely local public bad in this paper. Transboundary pollution spillovers were neglected. They can be introduced easily and the results would be quite straightforward. The pollution differential between regions would be mitigated and the chase-and-flee scenario as well as the optimality of separation would be more unlikely. A second extension would pertain to the adjustment dynamics in the laisser-faire case. Like Krugman [12, Appendix B] and Baldwin [1], one could consider forward-looking expectations instead of the static-expectations used in this paper to model adjustment processes. The conjecture is that history-vs.-expectations outcomes will be possible if environmental damage is small. Whether the introduction of forward-looking expectations could induce more desirable patterns of location in situations in which this paper found a chase-and-flee cycle, remains to be investigated. Finally, the case of costly abatement deserves deeper analysis, in particular regarding the design of optimal environmental policies in a geographical context.

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Appendix: Derivation of Optimal Solutions

Optima will be derived via the first order conditions, $\partial W/\partial \beta=0$, (19), and $\partial W/\partial k=0$, (20), and they will be depicted in the (β,k) space. Total differentiation (19 and (20) yields the Hessian,

$$H=\begin{pmatrix} -z\delta\left(\beta k^{z-1}+(1-\beta)(1-k)^{z-1}\right) & \dfrac{\partial(\tilde{u}-\tilde{u}^*)}{\partial k} \\ \dfrac{\partial(\tilde{u}-\tilde{u}^*)}{\partial k} & \dfrac{\partial(\tilde{u}-\tilde{u}^*)}{\partial \beta} \end{pmatrix}, \tag{A1}$$

It has negative diagonal elements and the signs of the off-diagonal elements are ambiguous. The sign of the first diagonal element is obvious for $\delta>0$, the sign of the second one is determined by

$$\frac{\partial(\tilde{u}-\tilde{u}^*)}{\partial \beta}=-\frac{(1-\Theta)^2(1-\gamma)^{2-\gamma}}{F^{1-\gamma}}\left(k(\Theta+\beta(1-\Theta))^{1-\gamma}+(1-k)(1-\beta(1-\Theta))^{1-\gamma}\right), \tag{A2}$$

and for the off-diagonal elements we have

$$\frac{\partial(\tilde{u}-\tilde{u}^*)}{\partial k}=\frac{(1-\Theta)(1-\gamma)^{2-\gamma}}{\gamma F^{1-\gamma}}\left(\frac{1}{(\Theta+\beta(1-\Theta))^{\gamma}}+\frac{1}{(1-\beta(1-\Theta))^{\gamma}}\right)-\delta k^{z}-\delta(1-k)^{z}. \tag{A3}$$

They are negative (positive) if the environmental damage parameter δ is large (small). The elements of the Hessian determine the slopes of the $\partial W/\partial \beta=0$ and the $\partial W/\partial k=0$ curves in the (β,k) space. That the signs of the off-diagonal elements are indeterminate indicates that both curves may be positively or negatively sloped, depending on δ, and as will be seen, that they can even be non-monotonous. The determinant of the Hessian may be negative and this implies that a point in which both first-order conditions are satisfied is not always a maximum. What is decisive for the following investigation, however, is that the signs of the diagonal elements are negative. Thus, for given values of the one variable, the first-order condition with respect to the other variable maximizes the welfare functional. This has the following implications:

- In all points in the (β,k) space located above the $\partial W/\partial k=0$ curve, k is too large and should be reduced. The opposite applies to points below this locus.
- In all points located to the left of the $\partial W/\partial \beta=0$ curve, β is too small and should be increased. The opposite applies to all points to the right of this locus.

To calibrate the model, we use the following numerical values of the parameters of the geography module:

$$F = \gamma = \Theta = 0.5 \,.$$

For the environmental-damage function, we use the values z=4 and z=0.2 to indicate convex and convave environmental marginal damages respectively. As regards the δ parameter of the model we start with δ=0 and increase it to very large values. Figures A1 and A2 depict the loci of the first-order conditions in the (β,k) space for z=4 and z=0.2, respectively, and selected values of δ. Figures have been drawn using MATHEMATICA®. $\partial W/\partial \beta$=0 is depicted as a solid line, $\partial W/\partial k$=0 as a dotted line. For δ=0, the phase diagram is the same as in Figure 2.1: the dispersion equilibrium is a saddle and the agglomeration equilibria are the welfare optima. With increasing values of δ, the isoclines rotate and change their shapes. It is seen that the $\partial W/\partial \beta$=0 line is rotated in a counter-clockwise fashion whereas the $\partial W/\partial k$=0 line is rotated clockwise. Figures A.1 and A.2 reveal the following:

- For z=4, we start with agglomeration at small values of δ. As δ gets larger, the number of firms in the agglomeration declines whereas households are still fully agglomerated. For even larger values of δ, the $\partial W/\partial \beta$=0 line has an inverse S shape. The resulting optima involve partial-agglomeration. E.g., for δ=3.7, the optimum is characterized by some 90% of the households and 60% of the firms locating in the same region. The partial-agglomeration equilibria converge to total dispersion. The dispersion optimum remains optimal for a large parameter range, e.g. for δ=5.5 and for δ=9.5 as shown in the diagram. For even larger values of δ, the two lines converge and intersect, the intersection point being a saddle. For very large values of the damage parameter, not shown in the diagram, both lines have negative slopes, the $\partial W/\partial \beta$=0 line being flatter than the $\partial W/\partial k$=0 line, and the optimum is perfect separation with firms and households located in different regions.
- For z=0.2, we start with agglomeration at small values of δ again. As δ gets larger, depicted here for δ=0.58, there are multiple intersection points, but the partial-agglomeration points are saddles and, therefore, not optimal. At certain critical value of δ, the optimum jumps to the centre, i.e. to dispersion, and stays there for some range of parameters, depicted for δ

=0.6 and δ=0.7. In the latter case, the S shape of the $\partial W/\partial\beta$=0 line becomes obvious. With further increases in the damage parameter we move to partial separation, e.g. some 80% of the households and some 20% of the industry locating in the same region for δ=0.7, and finally to perfect separation.

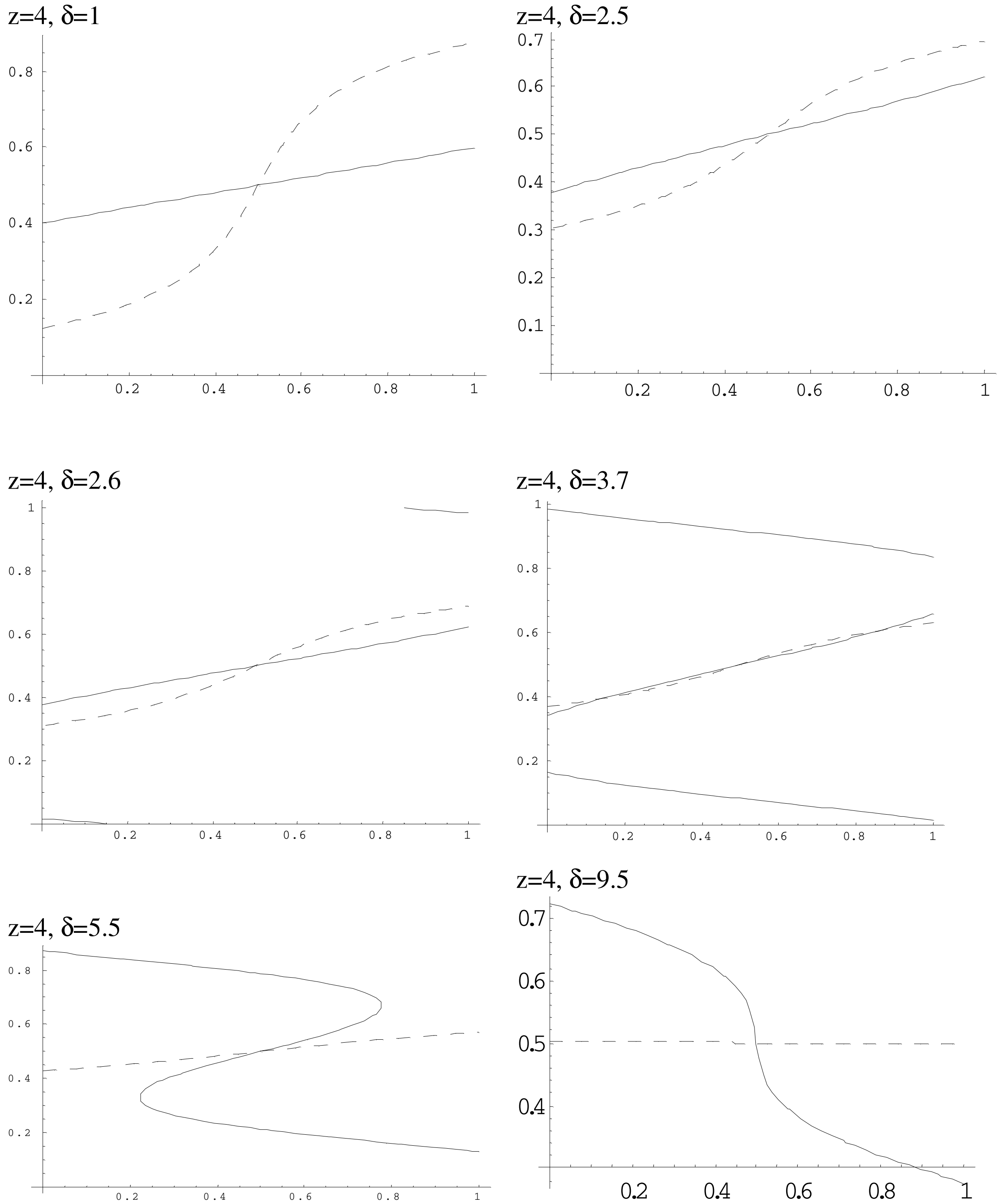

Figure A1 The first-order conditions for z=4

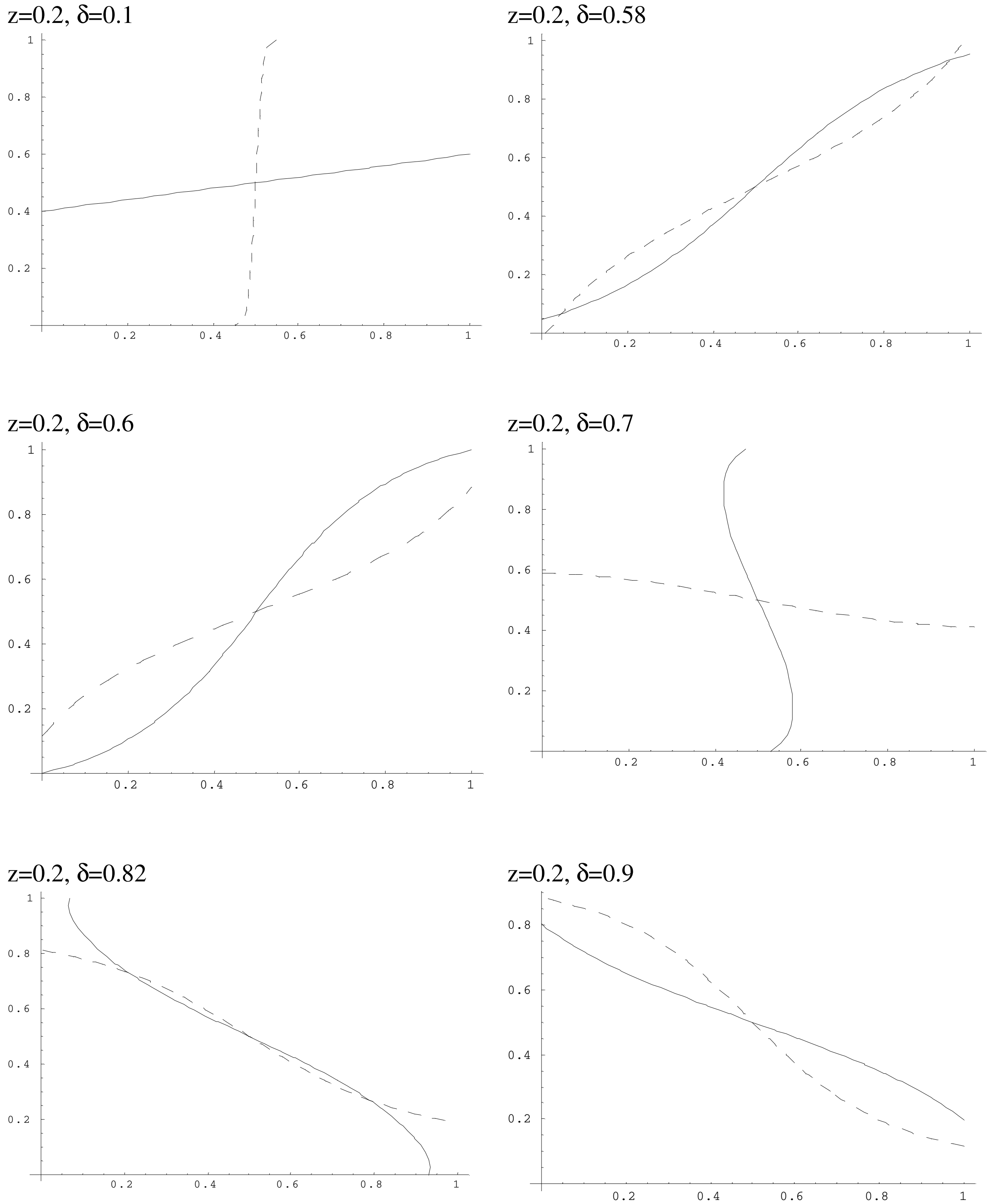

Figure A2 The first-order conditions for z=0.2