An agent-based retail location model on a supply chain network

Arthur Huang* and David Levinson†

Abstract

Clusters of business locations, which considerably impact daily activities, have been prominent phenomena. Yet the question of how and why firms cluster in certain areas has not been sufficiently studied. This paper investigates the emergence of clusters of business locations on a supply chain network comprised of suppliers, retailers, and, consumers. Krugman (1996) argued that urban concentration involved a tension between the *centripetal* and the *centrifugal* forces. Based on that notion, this research proposes an agent-based model of retailers' location choice in a market of homogeneous products. In this game, retailers endeavor to maximize their profits by changing locations. Retailers' distribution patterns are measured by entropy and cluster density. Simulation results reveal that as more retailers engage in the game, clusters autonomously emerge and the entropy of clusters increases. Once retailers exceed a certain number, mean density of clusters begins to decline; all discrete clusters gradually merge to a large cluster, spreading out uniformly. This research thus finds that the centripetal force attracts retailers to supplier locations; with even more retailers entering the market, the centrifugal force disperses them. The sensitivity results on model parameters and consumers' demand elasticity are also discussed.

Keywords: clustering, supply chain network, location choice, distribution pattern

1 Introduction

In economic geography, clusters are geographical agglomerations of firms with similar or complementary capabilities (Richardson, 1972). Geographical clusters of business locations have been prominent phenomena in almost all countries and regions. Global integration increasingly contributes to regional specialization, with decreasing transportation costs and trade barriers enabling firms to closely interact with other firms to benefit from local economies of scale (Brackman et al., 2001; Baldwin et al., 2003).

*University of Minnesota, Department of Civil Engineering, 500 Pillsbury Drive SE, Minneapolis, MN 55455 USA, Email: huang284@umn.edu

†RP Braun-CTS Chair of Transportation Engineering, University of Minnesota, Department of Civil Engineering, Email: dlevinson@umn.edu, web: http://nexus.umn.edu

An early investigation of clustering was performed by Marshall (Marshall, 1890), who argued that while firms were directly connected through business exchange, they were also indirectly linked through competition for labor and production factors, and that clustering of locations represented the distribution of economic activities. The Marshallian model considered the clustering of labor markets, production factors, and technological spillovers as economic externalities. Weber (1957) [1]proposed a theory of industrial location where industrial organizations locate to minimize transportation costs of raw materials and final product. He summarized two categories of products: weight-gaining and weight-losing. If the weight of the raw materials is more than the weight of the final product, it is the *weight losing* case; the optimal location to process the good is where the raw material is located. If the weight of the raw materials is less than the weight of the final product, it is the *weight gaining* case; the optimal location to process the good is at the market.

Krugman (1996) argued that the geographical distribution of firms is balanced by "centripetal" and "centrifugal forces". The centripetal forces that encourage firms to cluster include: (1) natural advantages of certain sites, such as labor availability and convenient transportation; (2) market-size external economies, for instance, access to markets (backward linkages) and access to products (forward linkages); and (3) pure external economies, such as knowledge spillovers. The centrifugal forces mainly refer to the factors that cause firms to scatter, such as: the need for reducing cost and the negative externalities of clustering, competition for the market, bidding-up prices for land, labor, and services, and other negative spillovers (Maskell and Kebir, 2006).

Clusters have gained increasing attention. Porter (1990, 1994) formulated a *diamond model* to identify the mechanism of fostering industrial dynamism and long-term development. The model consisted of four mutually-related factors: factor input conditions, demand conditions, related and supporting industries, and firm structure, strategy, and rivalry. Levinson and Krizek (2008) proposed four factors impacting a firm's decision of where to locate on a spatially-structured supply chain networks: complementors, competitors, connectors, and customers, which comprised the *diamond of exchange*.

Other research considered organization structure, knowledge creation, and entrepreneurship in clusters. Malmberg and Power (2005), for instance, hypothesized that knowledge in clusters was created through various local inter-organizational collaborative interaction, increased competition, and/or knowledge spillover. Evaluation of the pharmaceutical industry in the mid-Atlantic region and the automobile cluster in Detroit disclosed that they originated from a series of serendipitous individual entrepreneurial activities (Feldman and Schreuder, 1996; Kepper, 2002). Additionally, Markusen (1996); Gordon and McCann (2000); Paniccia (2002) evaluated cluster topologies according to different classification criteria, such as size of industry, degree of specialization, and type of interdependence. Johnston et al. (2003) analyzed different forms of urban spatial concentration based on the features and densities of clusters. In sociology, some studies have broached the issue of inter- and intra-organizational ties in clusters and their externalities on innovation and learning (Guilati, 1995; Gilsing, 2005). Topics like trust between firms, strength of ties, cognitive distance, exploration and exploitation have been tackled (March, 1991; McAllister, 1995; Krackhardt, 1999; Nooteboom, 2004).

[1]Weber's theory of industrial location was originally published in his book *ber den Standort der Industrie* in 1909, written in German.

There also have been empirical evaluations on land-use and regional development models and factors that impact clusters (van Eck and Koomen, 2007; Agarwal et al., 2002); statistical analyses are the major methodology. For example, as early as the 1970s, a classification of economic features were summazried based on geographical patterns of clusters (Bergeman et al., 1975). Sohn (2004) examined the spatial association of manufacturing activities, and argued that the spatial distribution pattern of business locations was significantly positively related with inter-industry economic linkage. Bade and Nerlinger (2000) studied the spatial distribution of new technology-based firms (NTBFs) and disclosed that the locations of NTBFs were associated with their proximity to Research and Development facilities. In addition, Based on the data of Japanese manufacturing firms in the US, the level of agglomeration economies was also found to be highly correlated with the physical proximity of firms (Head et al., 1995, 1999).

Yet studies of clustering are incomplete. Most theories on clustering are descriptive or qualitative. Statistical analyses, which study association from a macroscopic perspective, cannot persuasively reveal causality. A constellation of factors and notions attached on the concept of clusters are often highly correlated; casual relationships between such factors and clustering have not been sufficiently untangled. Quantitative theoretical models that can properly answer the question of how and why clusters emerge and prosper need to be formulated. Microscopically, clusters may form by experiencing a complex and self-organized process in developmental stages (Feldman and Francis, 2006). Maskell and Malmberg (1999) considered clusters as "ecologies of mutually dependent firms and institutions", which are constantly learning and adapting. The relationship between individual firms' decision-making and the emergence of clusters is still unclear. Since clusters result from individual decision-making, it would be of interest to examine how individual agents' seemingly random and chaotic decisions and interactions as a whole lead to firms' clustering. This paper posits a model of how firms cluster with the existence of centripetal and centrifugal forces.

We model retailers' location choice in a simplified supply chain network of suppliers, retailers, and consumers. A Supply chain network, as a multi-agent system for moving products or services from suppliers to consumers, can provide a comprehensive view about multilevel interactions between sellers and buyers. Decision-making and dynamics in supply chains have been a topic of extensive research (Poirier and Reiter, 1996; Simchi-Levi et al., 2004; Miller, 2002). There are two main methodologies. One is analytical analysis on supply chain equilibrium. Nagurney (1999); Nagurney et al. (2002), for example, proposed finite-dimensional variational inequality formulas to study dynamics in supply chains from a network economics perspective. The other is the agent-based model for simulating individual agents' decision-making in supply chains (Swaminathan et al., 1997; Julka et al., 2002; Kaihara, 2003). Each method provides specific insights that the other cannot offer.

This research builds an agent-based model to examine retailers' location choice. In this model, retailers directly purchase goods from suppliers and sell them to consumers. We assume that all players locate in a circular city comprised of discrete locations. In contrast with previous research, this study analyzes a pure model of agglomeration of firms without forms of co-operation and other inter-organizational linkages. We endeavor to explore the basic elements that drive the formation of cluster. The basic assumptions of this model are: (1) Only homogeneous products exist in the market; alternative choices of the single good are not considered. (2) Players have perfect information. (3) Retailers are profit-maximizing. (4) Suppliers all have the same unit sales price and keep their price and locations fixed at all times. (5) Retailers also have the same fixed unit sales prices, which indicates that retailers maximize profits by changing locations. (6) No cost is associated with moving. (7) Consumers share the same utility function, suggesting that they have the same taste of patronizing retailers.

In this model, retailers' revenues come from selling products to consumers; retailers' costs include purchasing cost and shipping cost of products. Retailers' location choice depends not only on their distance to the market, but also on their distance to suppliers. In each round of the game, a retailer selects a locality that is expected to provide the highest profits of all locations. Retailers' geographical distribution patterns are assessed when stable patterns emerge. The patterns are measured via entropy and cluster density. Our simulation results show that clustering is a self-organizational process that can exhibit different patterns with the balance of centripetal and centrifugal forces. Policy implications are also derived from from our sensitivity tests.

The rest of the paper is organized as follows. Section 2 introduces the theory of clustering. Section 3 describes autonomous players in this game and describes the structure of the supply chain network. Section 4 defines the concept of entropy and cluster density to measure retailers' distribution patterns. Section 5 depicts the design of the experiments and analyzes simulation results. Section 6 shows the result of sensitivity tests on some parameters and demand elasticity. Finally, Section 7 concludes the paper and discusses the implications of clustering.

2 Theory of clustering

Hotelling (1929) modeled retailers' location choice with consumers uniformly distributed along a line segment, who patronized retailers with the minimum cost. In the case of duopoly, in equilibrium, both retailers tend to stay in the middle of the line and offer the same price. If a third retailer appears, he will prefer to stay close to the other two players, but not between them. As more retailers join this game, their tendency is to "cluster unduly" (Hotelling, 1929).

The market is more complicated than Hotelling's scenario. For example, multiple commodities, rather than one, usually exist in the market where retailers can decide which kind to sell. When selecting locations, retailers also need to consider costs, which include but not limited to, shipping costs of products from suppliers, land price, and cost of moving. Consumers' tastes of choice are diverse as well; multiple factors impact their choice of retailers. Hotelling's model leaves a lot of room to be extended.

This research extends Hotelling's model to a game of location choice among multiple retailers. We explore this problem from the supply chain perspective. We structure the model to illustrate Krugman's centripetal and centrifugal forces shaping the distribution of retailers. Table 1 summarizes some representative centripetal and centrifugal forces in retailers' clustering.

We build a simple agent-based model to exemplify this theory. In this model, products (here we assume homogeneous products) flow from suppliers, via retailers, to consumers, while cash proceeds in the opposite direction. By changing locations, retailers aim to maximize their individual profits. Two factors affect a retailer's decision-making. One is the market; the other is cost. On the one hand, retailers would like to cover a larger market by avoiding direct competition of their peers, which is the centrifugal force to scatter them. On the other hand, they hope to stay close to suppliers in order to reduce shipping cost—the centripetal force of clustering. Location choice balances these forces.

3 The model

This study uses a multi-agent paradigm for modeling retailers' location choice. This sections introduces agents in this model: suppliers, retailers, and consumers.

3.1 Notation

Variables and constants use in this research are listed in Table 2.

3.2 Consumers

This model assumes that consumers have the same fixed demand in the beginning of each round. A consumer selects a retailer to patronize based on the retailer's attractiveness index, which depends on the observable distance between the consumer and the retailer, and other unobservable factors. Consumers' utility function takes the form of gravity model, which has been widely employed to model consumers' choice (Reilly, 1929; Bucklin, 1971; Levinson and Kumar, 1994). Retailer j's attractiveness index for consumer i is represented as:

$$A_{ij} = k_1 \cdot d_{ij}^{\beta} \tag{1}$$

β is expected to be negative because longer travel distance would generally diminish consumers' willingness of patronizing. Although many other factors such as price, brand, product quality, and considerations about scheduling and trip chaining can impact consumers' choice of retailers, our simplified model only considers the effect of distance. To account for preferences associated with factors other than traffic cost (and thus avoid a deterministic model, which is a special case where travel cost dominates), we use a logit model in which the probability of a consumer patronizing a retailer depends on travel cost but has a large random component. After calculating all retailers' attractiveness indexes, a consumer probabilistically selects a retailer to patronize. The probability for consumer i to patronize retailer j, ρ_{ij}, is calculated based on the following logit model:

$$\rho_{ij} = \frac{e^{A_{ij}}}{\sum_{j \in J} e_j^{A_{ij}}} \tag{2}$$

The *Roulette Wheel Selection* method is adopted for a consumer to select a retailer in each round. The approach, which can be found in Gen and Cheng (1997), basically means that retailer j with higher ρ_{ij} for consumer i has a higher chance to be selected by this consumer. A consumer's probabilities of patronizing all retailers comprise his *wheel of selection*, which is updated in every round. A spin of the wheel selects a retailer; once a retailer is selected, a consumer buys needed products from this retailer. The sequence for consumers to patronize retailers is randomly decided for each round.

3.3 Retailers

Retailers connect suppliers and consumers in the supply chain. In the beginning of each round, a retailer evaluates expected profits of all localities. A retailer's expected profit at locality m, Ω_m, is calculated as:

$$\Omega_m = (\sum_{i=1}^{N} \lambda \cdot \rho_{im}) \cdot [\theta - \sum_{k=1}^{K} (\delta + u \cdot \sigma_{mk}) l_{mk}] \tag{3}$$

Where $\sum_{i=1}^{N} \lambda \cdot \rho_{im}$ stands for total expected sales of products in locality m, given other retailers are geographically fixed at that time. The following part in brackets refers to expected profit per product, equaling sales price minus cost. The cost of a product for a retailer includes the purchasing cost from a supplier and the shipping cost. Here we assume that a retailer patronizes the closest supplier. After evaluating profitability of all localities, a retailer moves to the locality that has the highest expected profit (revenue - cost). Each retailer can only move once per round, and its sequence to move is randomly determined.

As all retailers select their locations, consumers begin to patronize retailers; the method has been introduced in the previous section. Retailers' *actual* profits are calculated at the end of one round. A typical formulation of retailer j's actual profit, Π_j, is as follows:

$$\Pi_j = (\sum_{i=1}^{N} \lambda \cdot x_{ij}) \cdot [\theta - \sum_{k=1}^{K} (\delta + u \cdot m_{jk}) l_{jk}] \tag{4}$$

Compared with function (3), the main difference is the way total sales amount is calculated, which in this function equals retailer j's sales price times actual sales amount at the end of one round. The actual profit and expected profit may be different for two reasons: First, actual profit is calculated when all retailers have had an opportunity to relocate, that is at the end of a round, whereas expected profit maybe estimated before other retailers find new locations. Second, consumers patronize retailers based on probability.

3.4 Suppliers

The model assumes that all suppliers keep the same unit sales price. In addition, they are evenly distributed in the circle and are fixed in all rounds. We also presume that suppliers can always produce enough goods to meet market demand.

4 Measuring spatial distribution

4.1 Entropy

The concept of entropy was first introduced by Shannon (1948) to measure information uncertainty. Entropy has been used to investigate the heterogeneity of complex social, biological, and transportation networks. (Balch, 2000; Ben-Naim et al., 2004; Xie and Levinson, 2006). We employ a similar logic to measure retailers' clustering patterns. A cluster here is defined as an agglomeration of retailers that are geographically adjacent or in the same locality of the circle.

The entropy of n retailers' distribution pattern, γ_n, is represented as:

$$\gamma_n = -\sum_{i=1}^{M} \frac{\varepsilon_i}{n} log(\frac{\varepsilon_i}{n}) \tag{5}$$

Where M is the total number of clusters; n is the total number of retailers. ε_i is the number of retailers in cluster i.

4.2 Cluster density

We also measure cluster density for distribution patterns. A cluster density is calculated as the number of retailers in a cluster divided by the number of locations in the cluster. The mean average cluster density of n retailers, φ_n, is formulated as:

$$\varphi_n = \frac{1}{M} \sum_{i=1}^{M} \frac{\varepsilon_i}{\tau_i} \tag{6}$$

where τ_i is the number of locations in cluster i; M is the number of emergent clusters.

Figure 1 gives three examples of 5 retailers' distribution patterns. In Figure 1-a, retailers are widely dispersed, and each isolated retailer is a special case of cluster. The entropy of this pattern equals 0.70, and mean cluster density equals 1. Figure 1-b shows that two clusters emerge and one retailer is geographically discrete. Its entropy equals 0.46, and mean cluster density equals 1.33. Similarly, the entropy for Figure 1-c equals 0.22, and average cluster density equals 1.17.

5 Experiments and Results

In this model, all players sit along a circle subdivided into 100 discrete units, where 5000 consumers and 5 suppliers are evenly distributed. A consumer's demand on the product is 20. Retailers, whose original locations are randomly assigned, move to their profit-maximizing locations. We examine different scenarios with the number of retailers ranging from 2 to 100. For each scenario, 10 rounds are tested.

The values of parameters used in this experiment are shown in Table 3. We examine the pattern of retailers' location distribution when stable patterns become visible. Typically, a stable pattern emerges after round 2 or 3, which is in all likelihood an equilibrium—an individual retailer cannot improve their profits by unilaterally changing its location, given the same is true of other retailers. Figure 2 and Figure 3 respectively depict entropies and cluster densities for different scenarios. Simulations results are described as follows.

When the number of retailers rises from 2 to 5, they are widely dispersed; each stays at a supplier location. Figure 2 displays a sharp increase of entropy. As the number of retailers increases to 10, retailers double up at supplier locations, while evenly sharing the market. The entropy curve basically turns flat. The mean cluster density curve, as shown in Figure 3, exhibits a steep increase from 1 to 2. As the number of retailers increases in this game, retailers nevertheless no longer continue to accumulate on top of suppliers. As Figure 3 illustrates, when the number of retailers gets larger than 12, mean cluster density almost all becomes 1, which means retailers begin to be more dispersed around the circle. Yet they keep centering around suppliers. Such patterns reveal a trade-off between the centripetal force—approximating suppliers, and the centrifugal force—approximating the market. It is interesting to note that as the number of retailers increases from 96 to 100, entropy declines drastically. This is because when retailers reach a certain number, clusters begin to merge. While 100 retailers partake in the game, they cover every discrete location of the circle. In this situation, all retailers constitute a big cluster; the value of entropy then becomes zero. Some examples of retailers' distribution patterns are illustrated in Figure 4. Additionally, Figure 5 illustrates individual retailers' expected profits and actual profits in equilibrium for the scenarios of 10 retailers and 21 retailers. For the case of 10 retailers, retailers' expected profits in equilibrium are the same, as they are evenly distributed and double up at supplier locations. Yet due to some randomness in consumers' choice model in each round, we can notice small variations among retailers' actual profits. In the case of 21 retailers, an odd number, we expect retailers to have different expected profits in equilibrium; actual profits of retailers are also impacted by some randomness in consumers' choice model. Figure 10 displays retailers' average profit in different scenarios (marked by green dots), wherein we can clearly notice a decline of average profit with the increase of participants in this game.

Our above analysis indicates that in this model, when the number of retailers is no more than 10, the centripetal force induces them to double up at supplier locations, while the centrifugal force keeps the pattern symmetric. As the number retailers continues to grow, retailers tend to disperse themselves in the circle; the existence of centripetal force, however, keeps them stay close to suppliers. Overall, different players in the game lead to different sizes and densities of clusters.

6 Sensitivity tests

6.1 Sensitivity of parameters

To further explore the effects of the "centripetal" and "centrifugal" forces on retailers' distribution patterns, this research examines different values for β and u. We test the number of retailers from 2 to 30. Figure 6 and Figure 7 show results of sensitivity test on β; Figure 8 and Figure 9 display the results of sensitivity tests on u. It should be noted that when we examine different values of β or u, other parameters are kept the same as in Table 3.

First, we examine the value of β from -2.0 to 0, with step size 0.25, given other parameters fixed. Figure 6 and Figure 7 present the results of entropies and cluster densities. It can be observed that retailers' distribution patterns are similar when β is larger than -0.5; the features of patterns have been described in the prior section. When β equals -0.25 or 0, retailers' distribution patterns significantly differ from other cases. When β equals -0.25, retailers only locate in supplier localities, whatever the number of retailers in the game. As can be seen in Figure 7, cluster density has a linear positive relationship with the number of retailers. Particularly in the case of 30 retailers, every supplier-centered cluster is home to 6 retailers. When β equals 0, consumers are indifferent to travel. It's interesting to see that retailers amass at one supplier's location; therefore the entropy equals 0, and cluster density equals the number of retailers in the game. This is an artifact of the simulation model that retailers choose the first most profitable locations and don't consider alternative location of exactly equal profits. They might just as easily cluster uniformly or non-uniformly on any supplier location.

The second sensitivity test varies the value of u from 0 and 0.16, with step size 0.02. As shown in Figure 8, the trends of entropy are consistent when $u \leq 0.04$. The entropy curve for $u = 0.02$ climbs over others when number of retailers is greater than 15. The curve for the case of $u = 0$—when shipping costs no longer matter—shows a smooth uprising trend with the increasing of retailers. In this case, retailers' distribution patterns suggest that retailers no long stay at supplier locations, but tend to evenly disperse themselves along the circle to share the market; cluster density of this scenario always equals 1. Figure 9 further shows cluster densities for different values of u. Our studies find that when u is larger than 0.08, retailers tend to double up on suppliers, as their number booms from 2 to 10. However, as the number of retailers rises to 15, only the case of $u = 0.16$ shows continuing accumulation of retailers at supplier locations. In particular, in the case of 15 retailers, every three retailers stay at a supplier location. When u equals 0.16, although the cluster density curve gradually falls as the number of retailers continues to increase, a rising trend of cluster density can still be noticed when the number of retailers ascends from 20 to 25.

6.2 Demand elasticity

This research also examines the situation when consumer demand fluctuates, aiming to disclose the impact of the number of retailers on consumers' demand and individual retailer's profitability.

Since retailers' sales price is fixed, we relate consumer demand with retailers' attractiveness. Consumer i's demand for product is measured as:

$$D_i = \alpha_0 + \alpha_1 \cdot \sum_{j \in J} e_j^{A_{ij}} + \varepsilon \tag{7}$$

Where $\sum_{j \in J} e_j^{A_{ij}}$ is a function of accessibility to retailers, as introduced in function (2); the error ε term is an i.i.d with zero mean value. This expression originated from studies on evaluating accessibility of location in transportation research (Ben-Akiva and Lerman, 1985). It indicates that the maximum utility of all alternatives in a choice set is a measure of a consumer's expected demand associated with this situation. Despite α_0 and α_1 being common to all consumers, this function reflects the variation of demands of consumers in different locations and in different rounds and for different scenarios.

α_0 is set to be 10, representing a consumers' basic demand on this product; α_1 equals 1.75, which is a consumer's demand elasticity with respect to accessibility to retailers. Values of other parameters are the same as those in Table 3. Undoubtedly, the increase of the number of retailers leads to the rising of the value of accessibility function, which ultimately induces the growing of consumers' demand. Here an upper limit, 60, is set on consumer i's demand, D_i. We test the scenarios wherein the number of retailers increases from 2 to 100, and calculate consumers' average individual demand and retailers' average profits at equilibrium for each case.

Figure 11 shows consumers' average demand for the scenarios of different retailers. As can be seen in Figure 11, in the very beginning, consumers' average demand gradually rises with the increasing of retailers, yet it stops growing when more than 26 retailers enter the game. It implies that as more retailers partake in the game, the implicit product and price differentiation as well as the explicit lowered transportation costs increase demand in the first stage of the survival curve of the product; however, in our case, there is a limit on the maximum size of a market. Figure 10 demonstrates retailers' average profits for the cases of different players; a decreasing trend of profits can be clearly observed. This makes sense in that more retailers mean a lesser market share on average for each. Figure 10 also shows retailers' average profits when consumers' demand are constant (marked by red dots). Comparing with the case of elastic demand (marked by blue dots), we can see a steeper declining curve of average profits, which is a direct consequence of competition.

Moreover, our findings disclose that retailers' distribution pattern when customers have elastic demand is similar to the the case wherein customers' demand is inelastic. Figure 12 compares the entropies of retailers' distribution patterns of these two cases. As can be seen, the trends of both curves are very similar—while entropy firstly increases as more retailers participate in the game, after staying around between 0.7 and 0.8 for a while, the value of entropy sharply descends to zero. In addition, Figure 13 compares cluster densities of retailers' distribution patterns. The results are similar with and without elastic demand, there are minor differences due to the stochastic nature of the models.

7 Conclusions

Although clusters have been a topic of extensive research, both theoretically and empirically, we still have a limited understanding of how clusters emerge and why they occur in certain areas and not others. This research proposes an agent-based framework to model retailers' location choice in a supply chain of suppliers, retailers, and consumers. In this model, retailers aim to maximize profits by changing locations. Two categories of factors impact retailers' decision-making. One is the distance to suppliers, which can be seen as the centripetal force that encourages retailers to cluster. The other is the distance to markets, as the centrifugal force that tends to scatter retailers to avoid peer competition and be near consumers. Our agent-based model reveals that clusters emerge, and retailers double up on supplier locations as the number of retailers ascends to a certain number. When more partake in this game, the size of each cluster increases, and cluster density decreases (which means clusters are *flattened*). As number of retailers approximates the number of locations in the circle, clusters begin to merge with each other. Our sensitivity tests also find different patterns when the shipping cost or consumers' willingness to travel changes. The development of a market does not always lead to condensed agglomeration of business locations on a simple buy-and-sell supply chain network. Different features of markets, such as transportation cost and market size, can result in different location distribution patterns. Our findings demonstrate that the balance between the centripetal and centrifugal forces greatly impacts the size and density of clusters.

By assessing demand elasticity, this research replicates the phenomenon that market demand boosts in the early period as more product providers enter the market. In this stage, although retailers' average profit decreases as more competitors join the game, each retailer can still be fairly profitable due to the increasing market demand. Yet as market demand saturates, the increasing number of players quickly drags average profit down. While this research does not cover this topic (we would need to extend the model to incorporate other fixed and variable costs of doing business), we can reasonably speculate that as profit keeps declining, many retailers will begin to quit the game and leave the market. It can be further deduced that to strike a balance between consumer welfare and retailer profit call for a limited number of retailers participating in the game, which the market may be able to achieve if entry and exit to the market are enabled.

Undoubtedly, there are more factors that can influence retail location choice. This model provides a basic framework that allows for many interesting extensions. One extension, for example, could be introducing complementary products in the supply chain network. It is also of interest to investigate consumers' trip-chaining behavior when patronizing different stores. Another extension is to examine retailers' distribution patterns when suppliers also change their locations in this game. Future work can also consider the variation of input prices or land in different locations, as well as the impact of labor market. Finally, a natural extension would give retailers powers of pricing and product differentiation.

References

Agarwal, C., Green, G, M., Evans, T. and Schweik, C. (2002), "A review and assessment of land-use change models: Dynamics of space, time, and human choice", *Gen Tech Rep NE-297, U.S. Department of Agriculture, Forest Service, Newtown Square, PA* .

Bade, F. and Nerlinger, E. (2000), "The spatial distribution of new technology-based firms: Empirical results for West-Germany", *Papers in Regional Science* , Vol. 79, Springer, pp. 155–176.

Balch, T. (2000), "Hierarchic social entropy: An information theoretic measure of robot group diversity", *Autonomous Robots* , Vol. 8, Springer, pp. 209–238.

Baldwin, R., Forslid, R., Martin, P., Ottaviano, G. and Robert-Nicoud, F. (2003), *Economic Geography and Public Policy*, Priceton University Press.

Ben-Akiva, M. E. and Lerman, S. R. (1985), *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press.

Ben-Naim, E., Frauenfelder, H. and Toroczkai, Z. (2004), *Complex Networks*, Springer.

Bergeman, J., Greenston, P. and Healy, R. (1975), "A classification of economic activities based on location patterns", *Journal of Urban Economics* , Vol. 2, pp. 1–28.

Brackman, S., Garrestsen, H. and van Marrewijk, C. (2001), *An Introduction to Geographical Economics*, Cambridge University Press.

Bucklin, L. (1971), "Retail Gravity Models and Consumer Choice: A Theoretical and Empirical Critique", *Economic Geography* , Vol. 47, JSTOR, pp. 489–497.

Feldman, M. P. and Francis, J. L. (2006), "Entrepreneurs as agents in the formation of industrial clusters", *Clusters and Regional Development: Critial Reflections and Explorations* , pp. 116–136.

Feldman, M. P. and Schreuder, Y. (1996), "Initial advantage: The origins of the geographic concentration of the pharmaceutical industry in the mid-Atlantic region", *Industrial and Corporate Change* , Vol. 5, pp. 839–62.

Gen, M. and Cheng, R. (1997), *Genetic Algorithms and Engineering Design*, John Wiley & Sons, Inc.

Gilsing, V. A. (2005), *The Dynamics of Interfirm Networks and Strategic Alliances, Exploration, Exploitation and Co-evolution*, Cheltenhan: Edward Elgar.

Gordon, I. R. and McCann, P. (2000), "Industrial clusters: Complexes, agglomeration and/or social networks", *Urban Studies* , Vol. 37, pp. 513–32.

Guilati, R. (1995), "Familiarity breeds trust? the implications of repeated ties on contractual choice in alliances", *Academy of Management Journal* , Vol. 38, pp. 85–112.

Head, C. K., Ries, J. C. and Swenson, D. L. (1995), "Agglomeration benefits and location choices: Evidence from Japanese manufacturing investments in the United States", *Journal of International Economics* , Vol. 38, pp. 223–247.

Head, C. K., Ries, J. C. and Swenson, D. L. (1999), "Attracting foreign manufacturing: Investment promotion and agglomeration", *Regional Science and Urban Economics* , Vol. 29, Elsevier, pp. 197–218.

Hotelling, H. (1929), "Stability in competition", *Economic Journal* , Vol. 39, JSTOR, pp. 41–57.

Johnston, R., Voas, D. and Poulsen, M. (2003), "Measuring spatial concentation: The use of threshold profiles", *Environmental Planning B* , Vol. 30, pp. 3–14.

Julka, N., Srinivasan, R. and Karimi, I. (2002), "Agent-based supply chain management1: framework", *Computers and Chemical Engineering* , Vol. 26, Elsevier, pp. 1755–1769.

Kaihara, T. (2003), "Multi-agent based supply chain modelling with dynamic environment", *International Journal of Production Economics* , Vol. 85, Elsevier, pp. 263–269.

Kepper, S. (2002), "The capabilities of new firms and the evolution of the US automobile industry", *Industrial and Corporate Change* , Vol. 11, pp. 645–66.

Krackhardt, D. (1999), "The ties that torture: Simmelian tie analysis in organizations", *Research in Sociology of Organizations* , Vol. 16, pp. 183–210.

Krugman, P. (1996), “Urban concentration: The role of increasing returns and transport costs”, *International Regional Science Review* , Vol. 19, pp. 5–30.

Levinson, D. and Krizek, K. (2008), *Planning for Place and Plexus : Metropolitan Land Use and Transport*, New York, NY: Routledge.

Levinson, D. and Kumar, A. (1994), “A Multi-Modal Trip Distribution Model”, *Transportation Research Record* , Vol. 1466, pp. 124–131.

Malmberg, A. and Power, D. (2005), “(how) do (firms in) clusters create knowledge”, *Industry and Innovation* , Vol. 12, pp. 305–321.

March, J. (1991), “Exploration and exploitation in organizational learning”, *Organization Science* , Vol. 2, pp. 101–23.

Markusen, A. (1996), “Sticky places in slippery space: A typology of industrial districts”, *Economic Geography* , Vol. 72, pp. 293–313.

Marshall, A. (1890), *Principles of Economics*, London: Macmillan.

Maskell, P. and Kebir, L. (2006), “What qualifies as a cluster theory?”, *Clusters and Regional Development: Critial Reflections and Explorations* .

Maskell, P. and Malmberg, A. (1999), “Localised learning and industrial competeitiveness”, *Cambridge Journal of Economics* , Vol. 23, pp. 167–185.

McAllister, D. J. (1995), “Affect- and cognition based trust as foundations for interpersonal cooperation in organizations”, *Academy of Management Journal* , Vol. 38, pp. 24–59.

Miller, T. (2002), *Hierarchical Operations and Supply Chain Planning*, Springer.

Nagurney, A. (1999), *Network Economics: A Variational Inequality Approach*, Kluwer Academic Publishers.

Nagurney, A., Dong, J. and Zhang, D. (2002), “A supply chain network equilibrium model”, *Transportation Research Part E* , Vol. 38, Elsevier, pp. 281–303.

Nooteboom, B. (2004), Density and strength of ties in innovation networks: A competence and governance view, Technical report, ERIM Report Series Reference No. ERS-2004-005-ORG.

Paniccia, I. (2002), *Industrial Districts: Evolution and Competitiveness in Italian Firms*, Cheltenham and Northampton, MA: Edward Elgar Publishing.

Poirier, C. and Reiter, S. (1996), *Supply Chain Optimization: Building the Strongest Total Business Network*, Berrett-Koehler Publishers.

Porter, M. (1990), *The Competitive Advantage of Nations*, London: Macmillan.

Porter, M. (1994), "The role of location in competition", *International Journal of the Economics of Business* , Vol. 1, pp. 35–40.

Reilly, W. (1929), *Methods for the study of retail relationships*, University of Texas, Austin, Tex.

Richardson, G. (1972), "The organization of industry", *Economic Journal* , Vol. 82, pp. 883–896.

Shannon, C. E. (1948), "A mathematical theory of communication", *Bell System Technical Journal* , Vol. 27, pp. 379–423, 623–656.

Simchi-Levi, D., Wu, S. and Shen, Z. (2004), *Handbook of quantitative supply chain analysis: modeling in the e-business era*, Boston: Kluwer Academic Press.

Sohn, J. (2004), "Do birds of a feather flock together? Economic linkage and geographic proximity", *The Annuals of Regional Science* , Vol. 38, pp. 47–73.

Swaminathan, J., Smith, S. and Sadeh-Koniecpol, N. (1997), "Modeling supply chain dynamics: A multiagent approach", *Decision Sciences* .

van Eck, J. R. and Koomen, E. (2007), "Characterising urban concentration and land-use diversity in simulations of future land use", *Annals of Regional Science* , Vol. 42, pp. 123–140.

Weber, A. (1957), *Theory of the Location of Industries*, University of Chicago Press.

Xie, F. and Levinson, D. (2006), "Measuring the structure of road networks", *Geographical Analysis* , Vol. 39, pp. 336–356.

Table 1: Representative centripetal and centrifugal forces in retailers' clustering

Centripetal forces	Centrifugal forces
Locating close to suppliers	Staying close to a larger market
Providing customer with ability to comparison shops	less competition
Providing customers with ability to complementary products	Low rent/land price
Availability of cheap/skilled labor	

Table 2: List of Variables

Variables	Description
d_{ij}	distance between consumer i and retailer j
β	exponent for distance decay
A_{ij}	retailer j's attractiveness index for consumer j
x_{ij}	binary variable, which equals 1 if consumer i patronizes retailer j
ρ_{ij}	probability for consumer i to patronize retailer j
m_{kj}	shortest distance between supplier k and retailer j
Ω_m	expected profit at location m
Π_j	retailer j's actual profit
σ_{mk}	shortest distance between supplier k and location m
l_{jk}	binary variable, which equals 1 if retailer j patronizes supplier k
l_{mk}	binary variable, which equals 1 if a retailer in location m patronizes supplier k
ε_i	number of retailers in cluster i
γ_n	entropy of the distribution pattern of n retailers
τ_i	number of locations in cluster i
φ_n	mean cluster density of the distribution pattern of n retailers
ρ_{im}	probability for consumer i to patronize the retailer at location m
D_i	consumer i's demand of products
Constants	**Description**
k_1	constant value in the function of retailers' attractiveness
θ	retailers' sales price
λ	individual customer's product demand
u	retailers' unit shipping cost per product
δ	suppliers' sales price
N	total number of consumers
K	total number of suppliers
M	total number of clusters

Table 3: Parameters used in the experiment

Basic parameter values	
# of localities	100
K	5
N	5000
λ	20
u	0.08 ($)
δ	1.5 ($)
θ	3.5 ($)
β	- 1.0
k_1	1
Demand elasticity	
α_0	10
α_1	1.75

Note: every locality is home to 50 consumers.

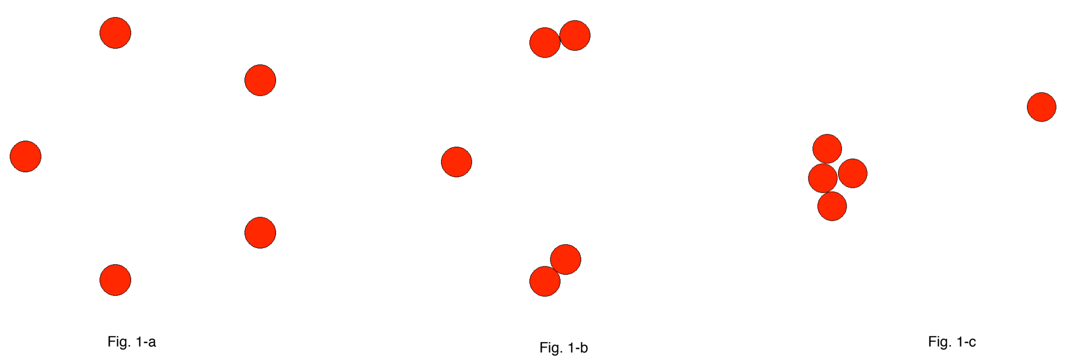

Figure 1: Examples of entropies and cluster densities for 5 retailers. Fig.1-a: retailers are dispersed and isolated; $\gamma = 0.70$, $\varphi = 1$. Fig.1-b: three clusters emerge, one of which is a special case of cluster with only one isolated retailer; $\gamma = 0.46$, $\varphi = 1.33$. Fig.1-c: $\gamma = 0.22$, $\varphi = 1.17$.

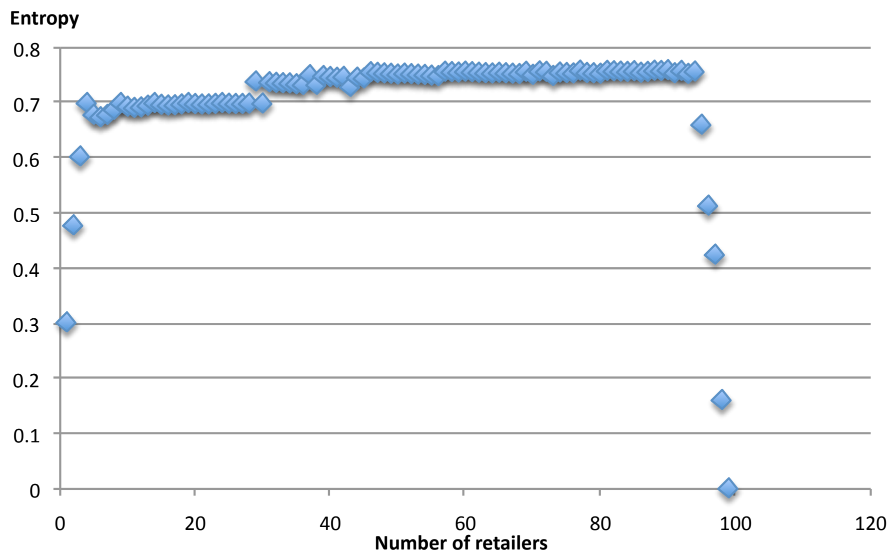

Figure 2: Entropies of retailers' distribution patterns for the number of retailers from 2 to 100. As the number rises from 2 to 10, retailers evenly disperse themselves on supplier locations; entropy rises with the increase of retailers. As the number of retailers continues to grow, retailers begin to be more dispersed around suppliers. Cluster density decreases, whereas the the number of clusters remains the same. When the number of retailers approximates the number of locations in the circle, clusters gradually merge with each other, and finally constitute a large cluster.

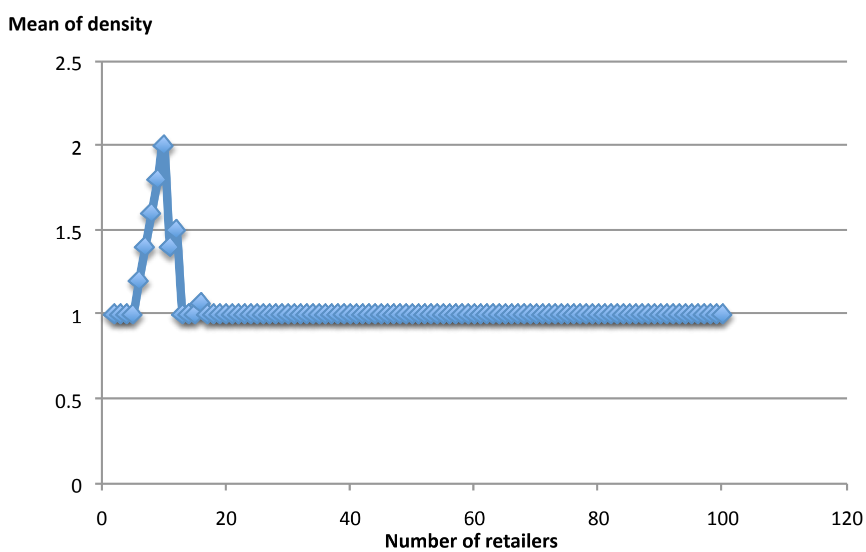

Figure 3: Mean cluster density for the number of retailers from 2 to 100. As the number of retailers increases to 10, mean cluster density increases. As the number of retailers keeps ascending, mean cluster density descends.

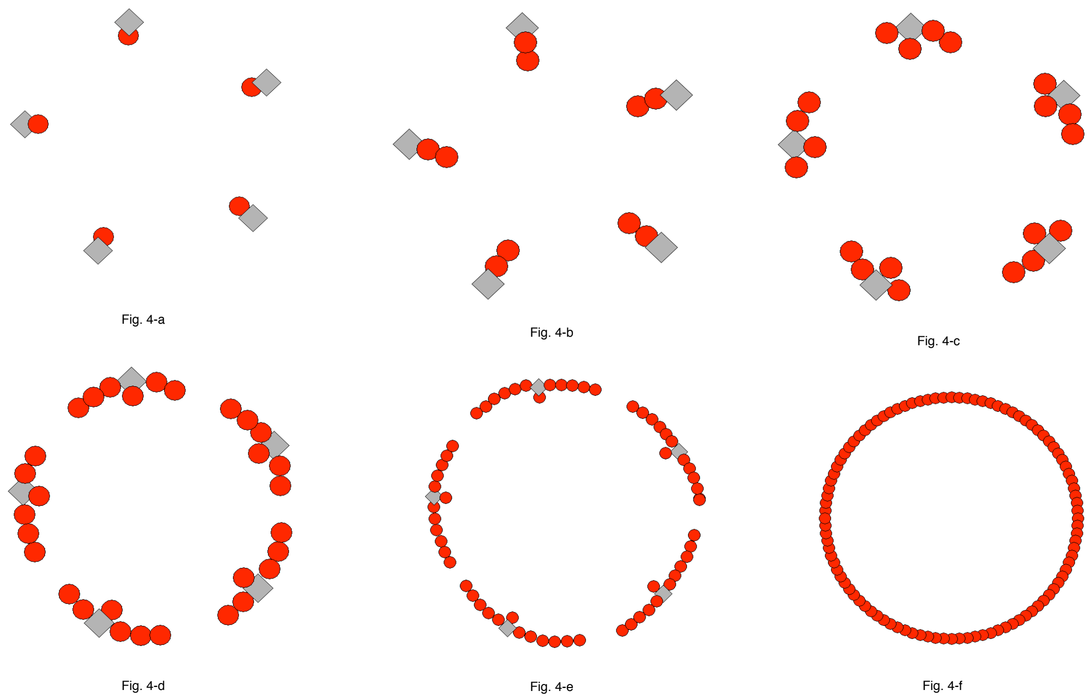

Figure 4: Examples of retailers' distribution patterns. (Red circles stand for retailers, and gray diamonds stand for suppliers. Objects' sitting on top of each each means they share the same location; objects in adjacency indicate that they are geographically adjacent.). Fig.4-a represents the distribution pattern of 5 retailers, where retailers sit on supplier localities. Fig.4-b shows the pattern of 10 retailers. Fig. 4-c shows the distribution pattern of 20 retailers. Fig.4-d depicts the case of 30 retailers. Fig.4-e and Fig.4-f respectively illustrate the scenario of 60 retailers and 100 retailers.

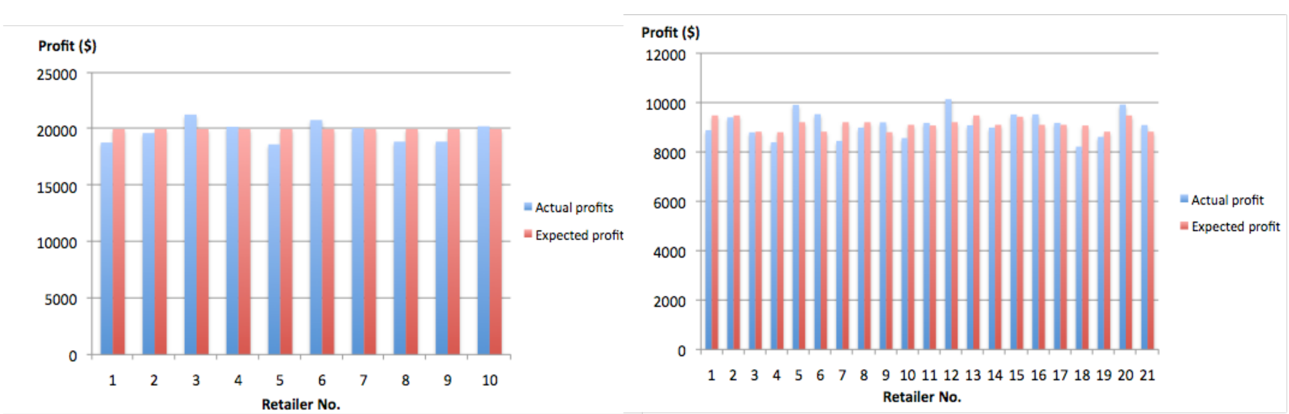

Fig. 5-a

Fig. 5-b

Figure 5: Expected profits and actual profits for scenarios of 10 retailers and 21 retailers when stable patterns merge. Fig. 5-a: Expected profits and actual profits for 10 retailers. As can be seen, all retailers are expected to have the same profits because they are evenly distributed and double up at supplier localities. The differences between their actual profits are due to consumers' choice of probability. Fig 5-b: Expected profits and actual profits for 21 retailers. Retailers expected profits, though different, are close. Retailers' actual profits are also impacted by consumers' randomness in choice to some extent.

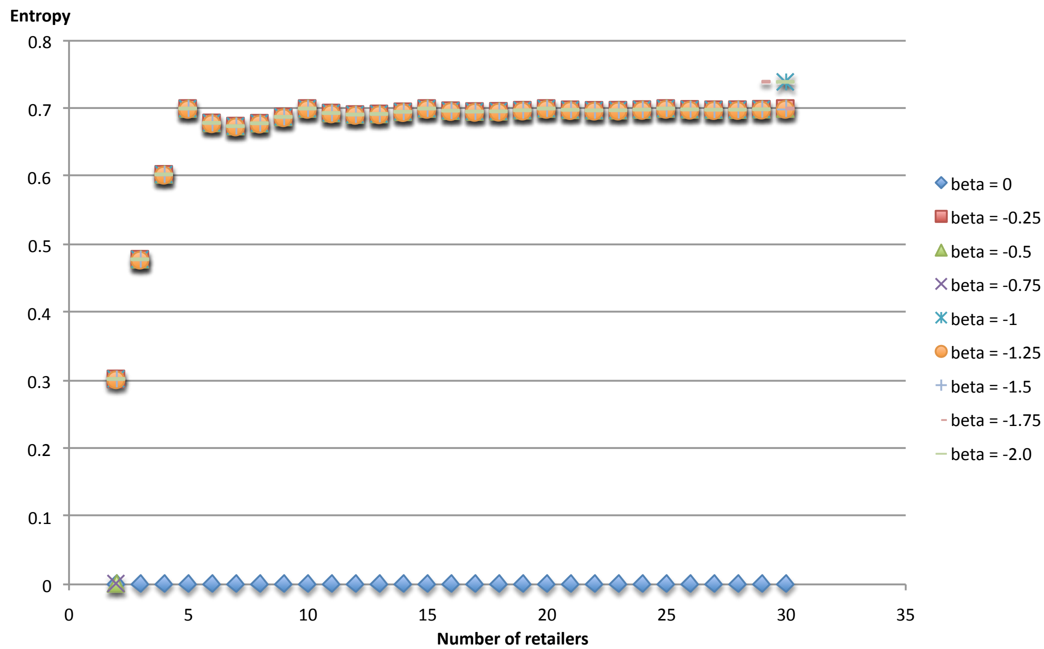

Figure 6: Entropies of retailers' distribution patterns: results of sensitivity test on the distance decay parameter, β.

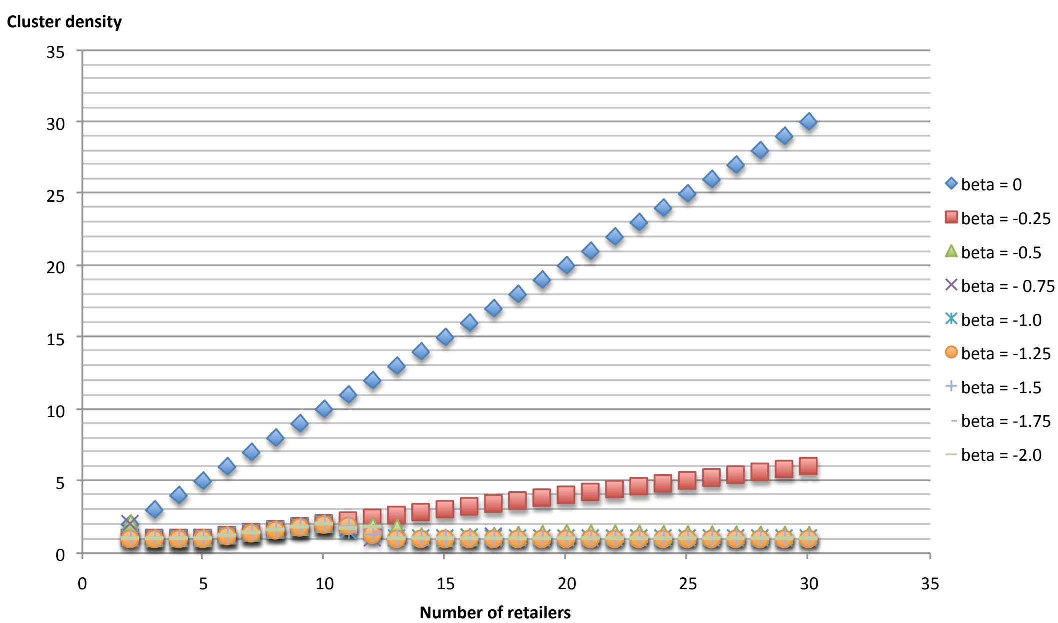

Figure 7: Mean cluster density of retailers' distribution patterns: results of sensitivity test on distance decay parameter, β.

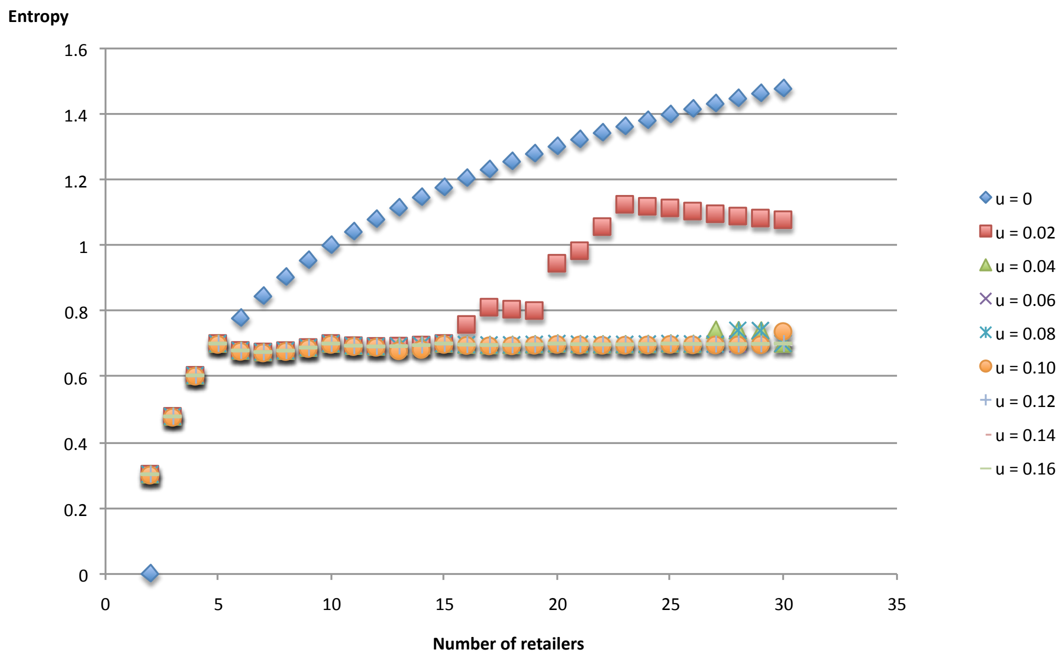

Figure 8: Entropies of retailers' distribution patterns: results of sensitivity test on unit shipping cost, u.

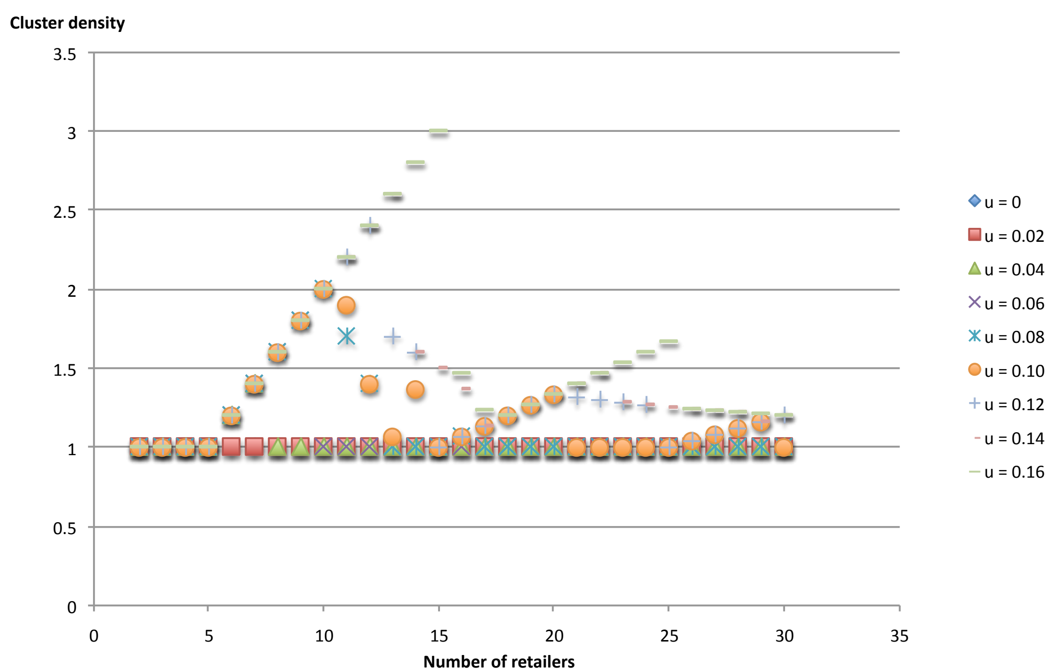

Figure 9: Mean cluster density of retailers' distribution patterns: results of sensitivity test on unit shipping cost, u.

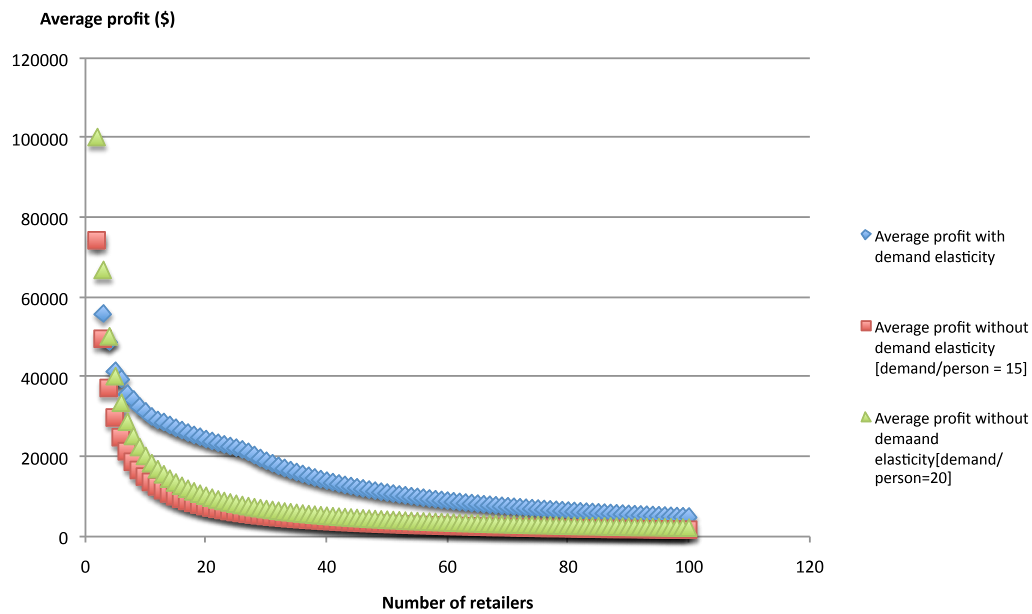

Figure 10: Average profit of total retailers from 2 to 100. (Blue dot illustrates retailer's average profit when consumers' demand burgeons with the increase of their accessibility to retailers; Red dot represents retailers' average profit when consumers' individual demand equal 15 and is inelastic; Green dot represents retailers' average profit when consumers' individual demand equal 20 and is inelastic.

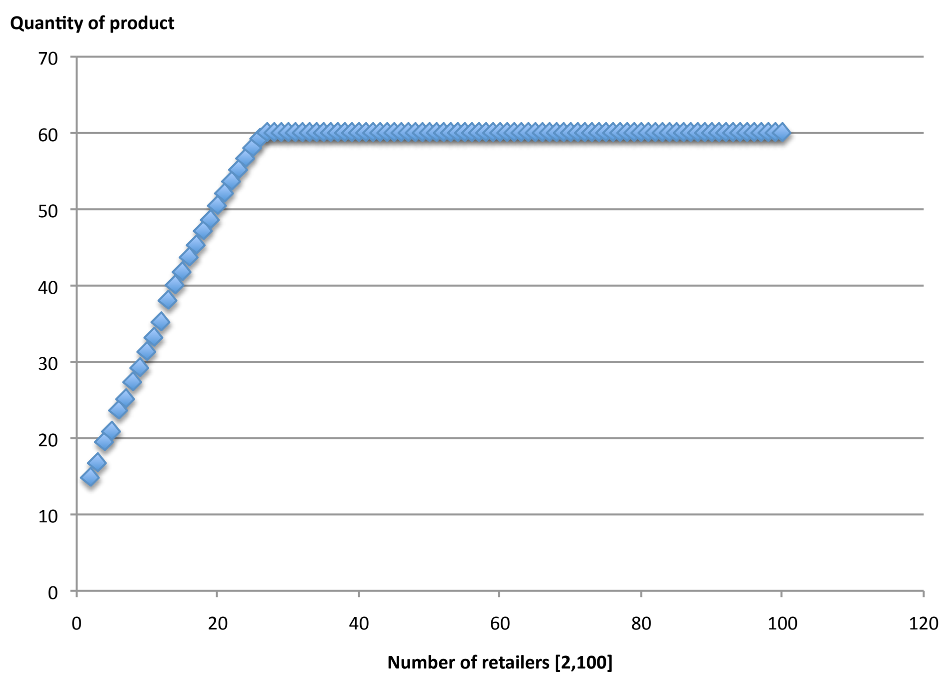

Figure 11: Consumers' average individual demand on the product increases with the rising of accessibility, yet does not exceed 60. As the number of retailers increases, accessibility ascends, enabling individual demand to grow. When the number of retailers exceeds 26, consumers' average individual demand reaches 60 and does not increase any more.

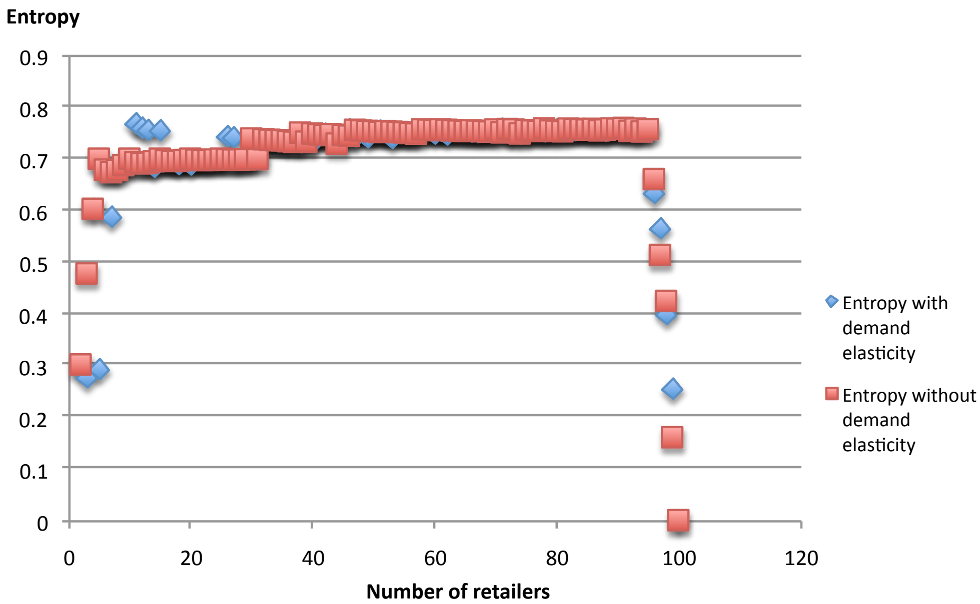

Figure 12: Entropies of retailers' distribution patterns with elastic demand and with inelastic demand. (Blue line: entropies of retailers' distribution patterns when consumers' demand is elastic; Red line: entropies of retailers' distribution patterns without demand elasticity.)

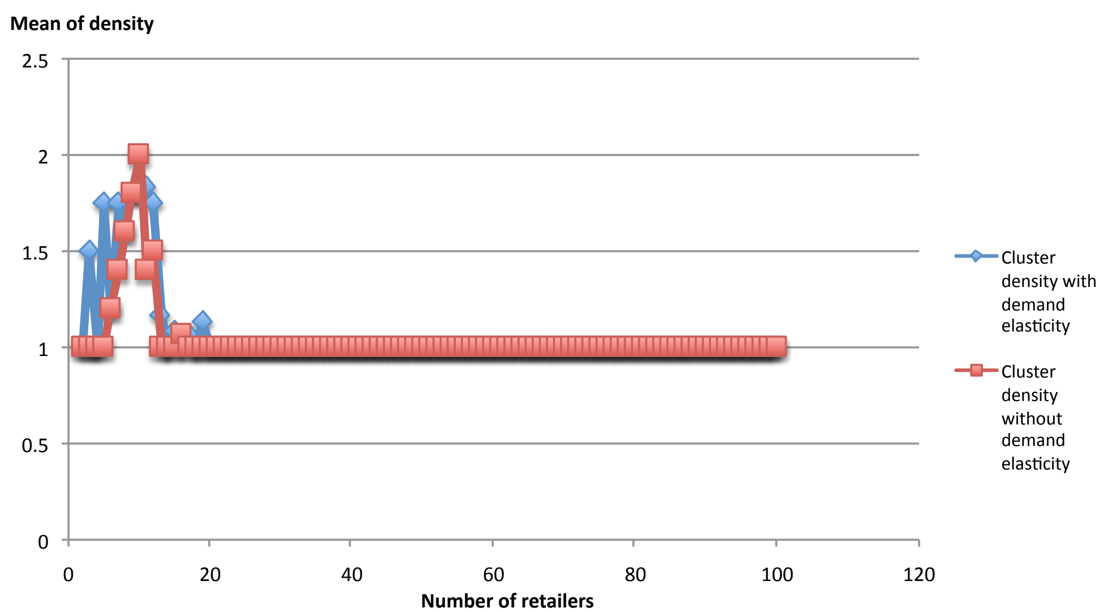

Figure 13: Cluster density of retailers' distribution patterns with elastic demand and with inelastic demand. (Blue line: cluster densities of retailers' distribution patterns when consumers' demand is elastic; Red line: cluster densities of retailers' distribution patterns without demand elasticity.)