Multilevel Modeling of Complex Survey Data

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Outline

- Model-based and design based inference
- Multilevel models and pseudolikelihood
- Pseudo maximum likelihood estimation for U.S. PISA 2000 data
- Scaling of level-1 weights
- Simulation study
- Conclusions

Multistage sampling: U.S. PISA 2000 data

- Program for International Student Assessment (PISA): Assess and compare 15 year old students' reading, math, etc.
- Three-stage survey with different probabilities of selection
  - Stage 1: Geographic areas k sampled
  - Stage 2: Schools $j=1,\ldots,n^{(2)}$ sampled with different probabilities π_j (taking into account school non-response)
  - Stage 3: Students $i=1,\ldots,n_j^{(1)}$ sampled from school j, with conditional probabilities $\pi_{i|j}$
- Probability that student i from school j is sampled:

$$\pi_{ij} = \pi_{i|j}\pi_j$$

Model-based and design-based inference

- **Model-based inference**: Target of inference is parameter β in infinite population (parameter of data generating mechanism or statistical model) called **superpopulation** parameter
  - Consistent estimator (assuming simple random sampling) such as maximum likelihood estimator (MLE) yields estimate $\widehat{\beta}$
- **Design-based inference**: Target of inference is statistic in **finite population** (FP), e.g., mean score $\overline{y}^{\mathrm{FP}}$ of all 15-year olds in LA
  - Student who had a $\pi_{ij} = 1/5$ chance of being sampled represents $w_{ij} = 1/\pi_{ij} = 5$ similar students in finite population
  - Estimate of finite population mean (Horvitz-Thompson):

$$\widehat{\overline{y}}^{\mathrm{FP}} = \frac{1}{\sum_{ij} w_{ij}} \sum_{ij} w_{ij} y_{ij}$$

  - Similar for proportions, totals, etc.

Model-based inference for complex surveys

- Target of inference is superpopulation parameter β
- View finite population as simple random sample from superpopulation (or as realization from model)
- MLE $\widehat{\beta}^{\mathrm{FP}}$ using finite population treated as target (consistent for β)
- Design-based estimator of $\widehat{\beta}^{\mathrm{FP}}$ applied to complex survey data
  - Replace usual log likelihood by weighted log likelihood, giving **pseudo maximum likelihood estimator** (PMLE)
- If PMLE is consistent for $\widehat{\beta}^{\mathrm{FP}}$, then it is consistent for β

Multilevel modeling: Levels

- Levels of a multilevel model can correspond to stages of a multistage survey
  - Level-1: Elementary units i (stage 3), here students
  - Level-2: Units j sampled in previous stage (stage 2), here schools
  - Top-level: Units k sampled at stage 1 (primary sampling units), here areas
- However, not all levels used in the survey will be of substantive interest & there could be clustering not due to the survey design
- In PISA data, top level is geographical areas — details are undisclosed, so not represented as level in multilevel model

Two-level linear random intercept model

- Linear random intercept model for continuous y_{ij}:

$$y_{ij} = \beta_0 + \beta_1 x_{1ij} + \cdots + \beta_p x_{pij} + \zeta_j + \epsilon_{ij}$$

  - x_{1ij},. . .,x_{pij} are student-level and/or school-level covariates
  - β_0,. . .,β_p are regression coefficients
  - $\zeta_j \sim$ N$(0, \psi)$ are school-specific random intercepts, uncorrelated across schools and uncorrelated with covariates
  - $\epsilon_{ij} \sim$ N$(0, \theta)$ are student-specific residuals, uncorrelated across students and schools, uncorrelated with ζ_j and with covariates

Two-level logistic random intercept model

- Logistic random intercept model for dichotomous y_{ij}
  - As generalized linear model

$$\text{logit}[\Pr(y_{ij}=1|\mathbf{x}_{ij})] = \beta_0 + \beta_1 x_{1ij} + \cdots + \beta_p x_{pij} + \zeta_j$$

  - As latent response model

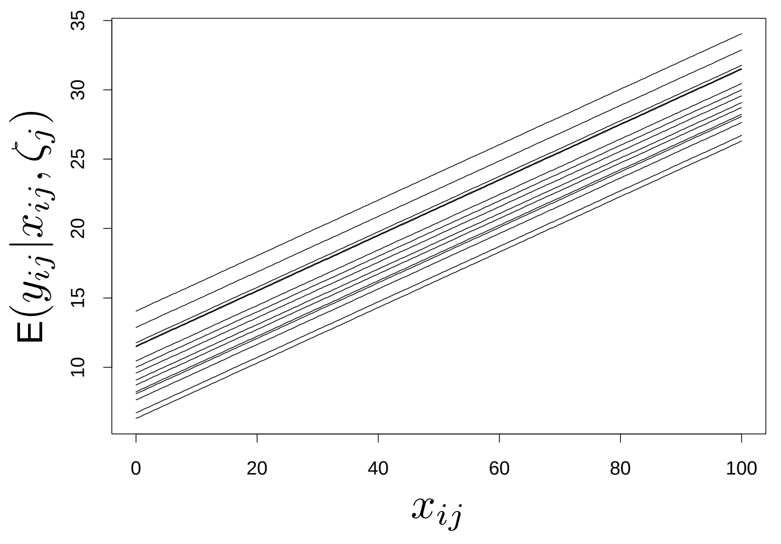
$$y^*_{ij} = \beta_0 + \beta_1 x_{1ij} + \cdots + \beta_p x_{pij} + \zeta_j + \epsilon_{ij}$$

$$y_{ij} = 1 \text{ if } y^*_{ij} > 0, \; y_{ij} = 0 \text{ if } y^*_{ij} \leq 0$$

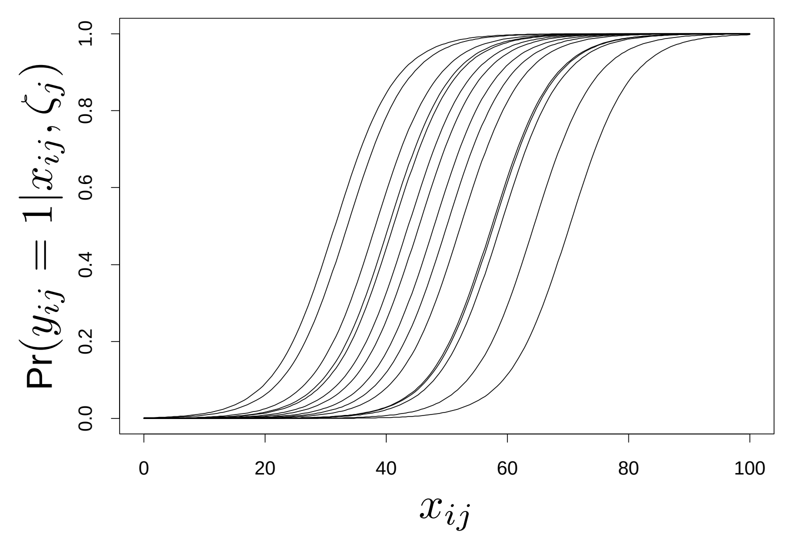
  - $\zeta_j \sim \text{N}(0, \psi)$ are school-specific random intercepts, uncorrelated across schools and uncorrelated with covariates
  - $\epsilon_{ij} \sim$ Logistic are student-specific residuals, uncorrelated across students and schools, uncorrelated with ζ_j and with covariates

Illustration of two-level linear and logistic random intercept model

$$\mathsf{E}(y_{ij}|x_{ij},\zeta_j) = \beta_0 + \beta_1 x_{ij} + \zeta_j$$



$$\mathsf{Pr}(y_{ij} = 1|x_{ij},\zeta_j) = \frac{\exp(\beta_0+\beta_1 x_{ij}+\zeta_j)}{1+\exp(\beta_0+\beta_1 x_{ij}+\zeta_j)}$$



Pseudolikelihood

- Usual marginal log likelihood (without weights)

$$\log \prod_{j=1}^{n^{(2)}} \int \underbrace{\left\{ \prod_{i=1}^{n_j^{(1)}} f(y_{ij}|\zeta_j) \right\}}_{\Pr(\mathbf{y}_j|\zeta_j)} g(\zeta_j)\, \mathrm{d}\zeta_j = \sum_{j=1}^{n^{(2)}} \log \int \exp \left\{ \sum_{i=1}^{n_j^{(1)}} \log f(y_{ij}|\zeta_j) \right\} g(\zeta_j)\, \mathrm{d}\zeta_j$$

- Log pseudolikelihood (with weights)

$$\sum_{j=1}^{n^{(2)}} w_j \log \int \exp \left\{ \sum_{i=1}^{n_j^{(1)}} w_{i|j} \log f(y_{ij}|\zeta_j) \right\} g(\zeta_j)\, \mathrm{d}\zeta_j$$

- Note: need $w_j = 1/\pi_j, \quad w_{i|j} = 1/\pi_{i|j}$; cannot use $w_{ij} = w_{i|j} w_j$
- Evaluate using adaptive quadrature, maximize using Newton-Raphson [Rabe-Hesketh *et al.*, 2005] in `gllamm`

Standard errors, taking into account survey design

- Conventional “model-based” standard errors not appropriate with sampling weights
- **Sandwich estimator** of standard errors (Taylor linearization)

$$\text{Cov}(\widehat{\boldsymbol{\vartheta}}) \; = \; \mathcal{I}^{-1}\mathcal{J}\mathcal{I}^{-1}$$

  - $\mathcal{J}$: Expectation of outer product of gradients, approximated using PSU contributions to gradients
  - $\mathcal{I}$: Expected information, approximated by observed information (‘model-based’ standard errors obtained from $\mathcal{I}^{-1}$)
- Sandwich estimator accounts for
  - Stratification at stage 1
  - Clustering at levels ‘above’ highest level of multilevel model
- Implemented in `gllamm` with `cluster()` and `robust` options

Analysis of U.S. PISA 2000 data

- Two-level (students nested in schools) logistic random intercept model for reading proficiency (dichotomous)
- PSUs are areas, sampling weights $w_{i|j}$ for students and w_j for schools provided
- Predictors:
  - [Female]: Student is female (dummy)
  - [ISEI]: International socioeconomic index
  - [MnISEI]: School mean ISEI
  - [Highschool]/ [College]: Highest education level by either parent is highschool/college (dummies)
  - [English]: Test language (English) spoken at home (dummy)
  - [Oneforeign]: One parent is foreign born (dummy)
  - [Bothforeign]: Both parents are foreign born (dummy)

Data structure and gllamm syntax in Stata

- Data strucure

```
. list id_school wt2 wt1 mn_isei isei in 28/37, clean noobs

   id_school       wt2         wt1     mn_isei    isei
           2    105.82    .9855073    47.76471      30
           2    105.82    .9855073    47.76471      57
           2    105.82    .9855073    47.76471      50
           2    105.82    1.108695    47.76471      71
           2    105.82    .9855073    47.76471      29
           2    105.82    .9855073    47.76471      29
           3    296.95    .9677663          42      56
           3    296.95    .9677663          42      67
           3    296.95    .9677663          42      38
           3    296.95    .9677663          42      40
```

- gllamm syntax

```
gllamm pass_read female isei mn_isei high_school college
   english one_for both_for, i(id_school) cluster(wvarstr)
   link(logit) family(binom) pweight(wt) adapt
```

PISA 2000 estimates for multilevel regression model

	Unweighted Maximum likelihood		Weighted Pseudo maximum likelihood		
Parameter	Est	(SE)	Est	(SE_R)	(SE_R^{PSU})
β_0: [Constant]	$-$6.034	(0.539)	$-$5.878	(0.955)	(0.738)
β_1: [Female]	0.555	(0.103)	0.622	(0.154)	(0.161)
β_2: [ISEI]	0.014	(0.003)	0.018	(0.005)	(0.004)
β_3: [MnISEI]	0.069	(0.001)	0.068	(0.016)	(0.018)
β_4: [Highschool]	0.400	(0.256)	0.103	(0.477)	(0.429)
β_5: [College]	0.721	(0.255)	0.453	(0.505)	(0.543)
β_6: [English]	0.695	(0.283)	0.625	(0.382)	(0.391)
β_7: [Oneforeign]	$-$0.020	(0.224)	$-$0.109	(0.274)	(0.225)
β_8: [Bothforeign]	0.099	(0.236)	$-$0.280	(0.326)	(0.292)
ψ	0.272	(0.086)	0.296	(0.124)	(0.115)

Problem with using weights in linear models

- Linear variance components model, constant cluster size $n_j^{(1)} = n^{(1)}$

$$y_{ij} \;=\; \beta_0 + \zeta_j + \epsilon_{ij}, \qquad \text{Var}(\zeta_j) = \psi, \qquad \text{Var}(\epsilon_{ij}) = \theta$$

- Assume sampling independent of ϵ_{ij}, $w_{i|j} = a > 1$ for all i, j
- Get biased estimate of ψ:
  - Weighted sum of squares due to clusters

$$\text{SSC}^{\text{w}} \;=\; \sum_j (\overline{y}_{.j} - \overline{y}_{..})^2 = \sum_j (\zeta_j - \overline{\zeta}_{.})^2 + \sum_j (\overline{\epsilon}^{\text{w}}_{.j} - \overline{\epsilon}^{\text{w}}_{..})^2 \;=\; \text{SSC}$$

  - Expectation of SSC^{w}, same as expectation of unweighted SSC

$$\text{E}(\text{SSC}^{\text{w}}) \;=\; (n^{(2)} - 1)\left[\psi + \frac{\theta}{n^{(1)}}\right]$$

  - Pseudo maximum likelihood estimator

$$\widehat{\psi}^{\text{PML}} = \frac{\text{SSC}^{\text{w}}}{n^{(2)}} - \frac{\widehat{\theta}^{\text{w}}}{an^{(1)}} \;>\; \widehat{\psi}^{\text{ML}} = \frac{\text{SSC}}{n^{(2)}} - \frac{\widehat{\theta}^{\text{ML}}}{n^{(1)}}$$

Explanation for bias and anticipated results for logit/probit models

- Clusters appear bigger than they are (a times as big)
  - Between-cluster variability in $\bar{\epsilon}^{\mathrm{w}}_{\cdot j}$ greater than for clusters of size $an^{(1)}$
  - This extra between-cluster variability in $\bar{\epsilon}^{\mathrm{w}}_{\cdot j}$ is attributed to ψ
  - However, if sampling at level 1 stratified according to ϵ_{ij}, e.g.

$$\pi_{i|j} \approx \begin{cases} 0.25 & \text{if } \epsilon_{ij} > 0 \\ 0.75 & \text{if } \epsilon_{ij} \leq 0 \end{cases}$$

    variance of $\bar{\epsilon}^{\mathrm{w}}_{\cdot j}$ decreases, and upward bias of $\widehat{\psi}^{\mathrm{PML}}$ decreases
- Bias decreases as $n^{(1)}$ increases
- In logit/probit models, anticipate that $|\widehat{\beta}^{\mathrm{PML}}|$ increases when $\widehat{\psi}^{\mathrm{PML}}$ increases; therefore biased estimates of β

Solution: Scaling of weights?

- Scaling method 1 [Longford,1995, 1996; Pfeffermann *et al.*, 1998]

$$w^*_{i|j} = \frac{\sum_i w_{i|j}}{\sum_i w^2_{i|j}} w_{i|j} \quad \text{so that} \quad \sum_i w^*_{i|j} = \sum_i w^{*2}_{i|j}$$

  - In linear model example with sampling independent of ϵ_{ij}, no bias

```
egen sum_w = sum(w), by(id_school)
egen sum_wsq = sum(wˆ2), by(id_school)
generate wt1 = w*sum_w/sum_wsq
```

- Scaling method 2 [Pfeffermann *et al.*, 1998]

$$w^*_{i|j} = \frac{n_j^{(1)}}{\sum_i w_{i|j}} w_{i|j} \quad \text{so that} \quad \sum_i w^*_{i|j} = n_j^{(1)}$$

  - In line with intuition (clusters do not appear bigger than they are)

```
egen nj = count(w), by(id_school)
generate wt1 = w*nj/sum_w
```

Simulations

- Dichotomous random intercept logistic regression (500 clusters, N_j units per cluster in FP), with

$$y_{ij}^* = \underbrace{1}_{\beta_0} + \underbrace{1}_{\beta_1} x_{1j} + \underbrace{1}_{\beta_2} x_{2ij} + \zeta_j + \epsilon_{ij}, \quad \psi = 1$$

- Stage 1: Sample clusters with probabilities

$$\pi_j \approx \begin{cases} 0.25 & \text{if } |\zeta_j| > 1 \\ 0.75 & \text{if } |\zeta_j| \leq 1 \end{cases}$$

- Stage 2: Sample units with probabilities

$$\pi_{i|j} \approx \begin{cases} 0.25 & \text{if } \epsilon_{ij} > 0 \\ 0.75 & \text{if } \epsilon_{ij} \leq 0 \end{cases}$$

- Vary N_j from 5 to 100, 100 datasets per condition, 12-point adaptive quadrature

Results for $N_j = 5$

Parameter	True value	Unweighted ML	Weighted Pseudo maximum likelihood		
			Raw	Method 1	Method 2
		Model parameters: Conditional effects			
β_0	1	0.40	1.03	0.68	0.75
		(0.11)	(0.19)	(0.16)	(0.15)
β_1	1	1.08	1.19	0.96	0.98
		(0.18)	(0.32)	(0.26)	(0.26)
β_2	1	1.06	1.22	0.94	0.96
		(0.22)	(0.35)	(0.25)	(0.26)
$\sqrt{\psi}$	1	0.39	1.47	0.58	0.70
		(0.37)	(0.21)	(0.31)	(0.30)

Effect of level-1 stratification method ($N_j = 10$)

- (1) Strata based on sign of ϵ_{ij}
- (2) Strata based on sign of ξ_{ij}, Cor$(\epsilon_{ij}, \xi_{ij}) = 0.5$
- (3) Strata based on sign of ξ_{ij}, Cor$(\epsilon_{ij}, \xi_{ij}) = 0$

	True	Raw			Method 1		
Parameter	value	(1)	(2)	(3)	(1)	(2)	(3)
β_0	1	1.04	1.10	1.29	0.83	0.88	1.01
		(0.16)	(0.16)	(0.21)	(0.14)	(0.13)	(0.16)
β_1	1	1.06	1.11	1.26	0.91	0.92	0.99
		(0.23)	(0.26)	(0.30)	(0.20)	(0.23)	(0.25)
β_2	1	1.11	1.12	1.17	0.91	0.91	0.96
		(0.20)	(0.21)	(0.25)	(0.16)	(0.17)	(0.19)
$\sqrt{\psi}$	1	1.19	1.33	1.77	0.40	0.61	0.98
		(0.13)	(0.15)	(0.15)	(0.34)	(0.24)	(0.16)

Simulation results for pseudo maximum likelihood estimation

- Little bias for $\sqrt{\psi}$ when $N_j \geq 50$ (cluster sizes in sample $n_j^{(1)} \geq 25$)
- For smaller cluster sizes:
  - Raw level-1 weights produce positive bias for $\sqrt{\psi}$
  - Scaling methods 1 and 2 overcorrect positive bias for $\sqrt{\psi}$ – apparently due to stratification based on sign of ϵ_{ij}
  - Inflation of $\boldsymbol{\beta}$ estimates whenever positive bias for $\sqrt{\psi}$
  - Good coverage using sandwich estimator (1000 simulations) for $N_j = 50$

Conclusions

- Pseudo maximum likelihood estimation allows for stratification, clustering, and weighting
- Three common methods for scaling level-1 weights: no scaling, scaling method 1, scaling method 2
- Inappropriate scaling can lead to biased estimates
  - If clusters are sufficiently large, little bias — similar results with all three scaling methods
  - If level-1 weights based on variables strongly associated with outcome, use no scaling
  - If level-1 weights based on variables not associated with outcome, use method 1
  - For intermediate situations, use method 2?

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