# Platform Ownership*

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First version: November 2002; This version: 6 September 2006

**Abstract**

We develop a theoretical framework of trade on a platform on which buyers and sellers interact. The platform may be owned by a single large, or many small independent or vertically integrated intermediaries. We provide a positive and normative analysis of the impact of platform ownership structure on platform size. The strength of network effects is important in the ranking of ownership structures by induced platform size and welfare. While vertical integration may be welfare-enhancing if network effects are weak, monopoly platform ownership is socially preferred if they are strong.

**Keywords:** Two-Sided Markets, Network Effects, Intermediation, Product Diversity, Entry

**JEL-Classification:** L10, D40

*The authors would like to thank the Editor (Xavier Vives), two anonymous referees, Javier Elizalde, Sven Rady, Jean-Charles Rochet, Yossi Spiegel, Jean Tirole, Christian Wey, and various seminar and conference audiences. They are grateful for financial support by the Deutsche Forschungsgemeinschaft (SFB TR 15). The first author also gratefully acknowledges financial support by the National Science Foundation (grant SES-0422778).

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# 1 Introduction

Most markets do not form spontaneously, but need to be organized. In a market place which may be a physical or virtual location, buyers and sellers interact on what we call a *trading platform*. Examples of market places in physical location are abundant in economic geography. An example of a market place in virtual location is the internet platform. The interesting central feature common to many market places are two-sided network effects: buyers are attracted to market places or platforms that house many sellers; sellers are attracted to market places that draw many buyers. The size of the trading platform reflects the number of sellers it houses: a larger number of sellers need a larger platform.

We observe widely differing institutional arrangements or ownership structures of platforms. The platform may be owned by a monopoly intermediary, by many small intermediaries, or by buyers or sellers active on the platform. Dispersed platform ownership may further be distinguished by contractual arrangements and property rights: incumbent platform owners may or may not have the right to restrict entry onto the platform. Examples of monopoly platform ownership are shopping malls controlled by a single developer, and independently-owned internet and software platforms. Also, large supermarket chains (such as Walmart) and department stores (such as Harrods) act as monopoly platforms to the extent that they delegate pricing to the manufacturers and charge them for the provision of retail space. An example of dispersed ownership with unrestricted access is the retail space supplied by landlords within a downtown shopping district. But zoning laws may restrict the development of new slots, and so the city council may play the role of a public platform holder. If the city council is captured by the interests of incumbent property owners, the shopping district may be viewed as a closed platform in which owners of developed slots can block the development of additional slots for commercial purposes. A virtual location example of dispersed ownership with restricted access is the New York Stock Exchange where the seat holders form a club that controls entry. Here, the companies listing their stocks correspond to the sellers on the platform, and the investors correspond to the buyers.[1] Finally, we observe many variants of vertically-integrated platforms such as B2B electronic marketplaces. For example, Covisint is owned by downstream automotive manufacturers, and Supply-On by their upstream suppliers.

In recent years, antitrust authorities in the U.S. and Europe have become increasingly concerned about potential anticompetitive practices in such market places. Indeed, it is the presence of two-sided network effects that raises doubts about the validity of received antitrust guidelines. For example, the welfare consequences of a dominant platform owned by a single company have been central to recent antitrust investigations in the case of Microsoft's Windows operating system. Another recent example is the U.S. Federal Trade Commission's investigation of Covisint.[2] Similarly, the European Commission recently investigated conditions of access to airport slots in the context of mergers and alliances in the airline industry.[3]

In this paper, we develop a theoretical framework of trade on a platform, in which we avoid

[1] Companies have to pay an initial listing fee and an annual renewal certification fee. Resulting revenues are linearly distributed among seat holders according to the number of seats.

[2] See FTC press release on September 11, 2000.

[3] See, for example, "Competition Policy in the Air Transport Sector", speech by EU Commissioner Karel van Miert, given to the Royal Aeronautic Society in 1998.

making functional form assumptions and allow for both, dispersed, or horizontally or vertically integrated ownership structures. Our framework is sufficiently rich to encompass as special cases a number of existing models of intermediated and non-intermediated trade, with very different micro foundations.

We apply this framework to address two sets of questions. (1) What is the impact of ownership structure on platform size or, equivalently, on the number of varieties offered by sellers? Does monopoly platform ownership lead to more or less product diversity than a platform owned by a large number of small intermediaries? How does the platform's ability to charge not only sellers but also buyers for access affect platform size under different ownership structures? What is the effect of vertical integration on platform size? How do these answers depend on the strength of platform externalities? (2) From a social point of view, does the "market" over- or underprovide product variety? How does this depend on the ownership structure and on the ability of the platform to charge not only sellers but also buyers? Are monopoly intermediation or vertical integration necessarily harmful for welfare? What are the welfare effects of allowing incumbent intermediaries to exclude potential entrants from the platform? More generally, which ownership structure is socially preferred?

In our model, we consider a market in which buyers and sellers interact exclusively on one trading platform. A large number of heterogeneous sellers each rent a platform slot to sell a differentiated product. The platform slots are let by either a large number of intermediaries or by a monopoly platform owner. A large number of heterogeneous buyers decide whether or not to visit the platform to purchase the products offered by the sellers.[4] Network effects are two-sided: the larger the number of sellers (i.e., the greater the product variety), the more attractive is the platform to buyers; and *vice versa*, for a given platform size, the larger the number of buyers, the more attractive it is for sellers to rent a platform slot. The basic platform ownership structures we consider are: two modes of competitive ownership, namely an open access platform and a closed platform or club, where access is restricted by the incumbent intermediaries; and monopoly platform ownership. We extend this model in two directions. First, we analyze vertically integrated platform ownership where each platform slot is owned by one seller, and access to the platform may be either open or closed. Second, we consider two-sided pricing where platform owners charge not only sellers but also buyers for access.

Our model is sufficiently rich to address some major competition policy concerns. One concern is that the downward integration of sellers onto a platform would give rise to undue restrictions on the size of a platform. However, this concern is misplaced in the context of our model where vertical integration of sellers onto the platform is weakly welfare-improving.

Another concern by policy-makers is that incumbent platform owners may impose undue restrictions on new entrants. In the context of our model, this is indeed a valid concern. But allowing incumbents to restrict access may actually increase welfare if platform effects are weak.

Yet another concern is that giving market power to a monopoly platform owner would lead to high rental charges, and thus drive down seller profits. Under free entry of sellers, this would induce a smaller platform size than an open platform ownership. This intuition is correct only if platform effects are weak. If platform effects are strong, on the other hand,

[4] A zero access price applies to consumers if it is not feasible to charge buyers or if the platform would choose to subsidize buyers but can only set non-negative access prices. The latter situation is likely to occur in an extended model with two platforms provided that sellers multi-home and buyers single-home.

monopoly ownership induces a larger platform size and is socially preferable to other ownership structures. But even if platform effects are weak, monopolization can be socially desirable as an open ownership structure can induce a socially excessive platform size.

Our analysis thus indicates that competition policy in two-sided markets should be aware of some important aspects. First, allowing sellers to integrate downwards onto the platform may be socially beneficial, in particular under closed ownership and weak platform effects. Second, allowing incumbent intermediaries to exclude potential entrants has ambiguous welfare effects if platform effects are weak, and has no effects otherwise. Third, monopoly platform ownership is socially preferable to fragmented ownership if platform effects are strong and possibly even if they are weak.

*Related Literature.* Our model is related to several strands of the literature. What sets our paper apart from the existing literature, however, is our focus on the effect of platform ownership structure, and the strength of the platform effects, on trade.

First, our paper is closely related to the recent literature on two-sided markets (e.g., Rochet and Tirole, 2003 and 2006; Armstrong, 2006), which builds on the older literature on network effects in non-intermediated trade (e.g., Katz and Shapiro, 1985) and, in particular, on indirect network effects (e.g., Chou and Shy, 1990; Church, Gandal, and Krause, 2002). As in our model, there are two-sided network effects on the trading platform: buyers care about the number of sellers, and sellers care about the number of buyers. In this literature, it is typically assumed that sellers do not compete with each other for buyers and that buyers and sellers are charged for access and usage of the platform.[5] The objective of much of this literature is to analyze the pricing structure on both sides of the market when the platform is owned by a single owner, while our objective is to analyze platform size and product variety on a platform for different ownership structures.

Second, our paper is closely related to the literature on optimal product diversity (e.g., Spence, 1976; Dixit and Stiglitz, 1977) and on optimal entry (e.g., von Weizsäcker, 1980; Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). At the heart of this literature lies the question whether the market over- or underprovides product variety or the number of sellers. The literature differs from our paper in that there is no intermediated trade, and typically there are no indirect network effects.

Third, our paper is also related to the literature on geographical market places (e.g., Stahl, 1982; Schulz and Stahl, 1996; Legros and Stahl, 2002). While the focus of this literature is different from that of our paper, the models may be interpreted as special cases of our general framework with non-intermediated trade.

Fourth, our paper is loosely connected to the literature on intermediation (e.g., Spulber, 1999), which is concerned with explaining the emergence and role of intermediaries. In contrast, we take the existence of a trading platform as given and focus on the effect of ownership

[5] Baye and Morgan (2001) analyze a model of a monopoly gatekeeper in the internet, with pricing on both sides of the market. Anderson and Coate (2005) analyze pricing of advertisements on a media platform, and allow in most of their analysis for pricing on only one side of the market. They show that there may be over- or underprovision of advertising from a social point of view. Rysman (2004) structurally estimates a related model of the market for yellow pages, where pricing is again only on one side of the market. Few other papers have looked at imperfect competition among sellers; exceptions are Belleflamme and Toulemonde (2004) or Hagiu (2004). (See also footnote 6.) Only Hagiu (2004), following our approach, considers the possibility that the platform may not be under monopoly ownership.

structure of the platform.

*Plan of the Paper.* In the next section, we present our theoretical framework of trade on a platform. In section 3, we analyze equilibrium and welfare under the three basic ownership structures. In section 4, we provide a positive and normative analysis of vertically integrated ownership structures. In section 5, we re-consider the equilibrium under the basic ownership structures when platform owners can charge not only sellers but also buyers for accessing the platform. In section 6, we discuss re-interpretations and extensions of our model. We conclude in section 7.

## 2 A Model of Trading on a Platform

We consider a model of trading on one platform. There are three types of economic agents: sellers, buyers, and intermediaries who own the trading platform. To offer her good to buyers, each seller needs to rent a slot on the platform. The rental charge of any given slot is determined by its owner (intermediary). We allow for various ownership structures of the trading platform, which are described in detail in sections 3 and 4.

*Buyers.* There is a continuum of (atomless) buyers who decide whether or not to visit the platform. A buyer's type is denoted by $\zeta$; the empirical support of $\zeta$ in the population of buyers is $[0, \infty)$. If a buyer of type $\zeta$ takes up the outside option, he obtains a utility of $g(\zeta)$. If instead he decides to visit the market place, he derives a utility $u(m_s) \geq 0$, where $m_s$ is the measure of sellers on the platform. (Alternatively, we may assume that the value of the outside option is zero for all types, and $g(\zeta)$ is the "transport cost" of visiting the platform.) We assume that $u(\cdot)$ is a continuously differentiable function, and $g(\cdot)$ continuously differentiable and strictly increasing with $g(\zeta) \to \infty$ as $\zeta \to \infty$. The properties of $g(\cdot)$ ensure existence of a marginal type $z(m_s) \equiv g^{-1}(u(m_s))$ such that a buyer of type $\zeta$ visits the market place if and only if $\zeta \leq z(m_s)$. Since $g(\cdot)$ can always be chosen appropriately, we assume w.l.o.g. that, for any $x > 0$, the measure of buyers whose types fall into the interval $[0, x]$ is given by $x$.

*Sellers.* There is a continuum of (atomless) sellers who decide whether or not to rent a platform slot.[6] A seller's type is denoted by $\mu_s$; the empirical support of $\mu_s$ in the population of sellers is $[0, \infty)$. If a seller decides not to participate on the platform, she obtains a zero payoff. Otherwise, she needs to rent a platform slot at rental price $r$.[7] A seller of type $\mu_s$ then has to incur a setup cost of $f(\mu_s)$. A seller's gross profit per unit mass of buyers is given by $\pi(m_s)$, where $m_s$ is the measure of entering sellers. Further, since all buyers (who decide to visit the market place) have identical demand, a seller's variable profit is proportional to the mass $z(m_s)$ of buyers visiting the market place. We assume that $\pi(\cdot)$ is a continuously differentiable function, and $f(\cdot)$ continuously differentiable and strictly increasing with $f(\mu_s) \to \infty$ as $\mu_s \to \infty$. Since $f(\cdot)$ can always be chosen appropriately, we assume w.l.o.g. that, for any $x > 0$, the measure of potential sellers whose types fall into the interval $[0, x]$ is $x$.

[6] Our reduced-form approach in principle allows for large oligopoly sellers. In this case, the number of sellers is discrete, and so a formal analysis would require taking into account integer constraints (and potential multiplicity of equilibria). The qualitative features of our results carry over to the discrete setting.

[7] The (uniform) rental price $r$ is thus the membership fee that each seller has to pay to obtain access to the platform. Once access is granted, there are no further fees. See Rochet and Tirole (2006) for a discussion of the reasons for the exclusive use of membership fees.

We will refer to $z(m_s)\pi(m_s) - f(\mu_s)$ as the profit of a seller of type $\mu_s$, and to $z(m_s)\pi(m_s) - f(\mu_s) - r$ as her net profit. Optimal entry decisions imply existence of a marginal type $m_s$ such that a seller rents a platform slot if and only if $\mu_s \leq m_s$. Hence, the measure $m_s$ of entering sellers is implicitly defined by $z(m_s)\pi(m_s) - f(m_s) - r = 0$.

*The Platform.* There is a continuum of (atomless) slots on the platform which can be developed by their owners to host sellers. A slot type is denoted by $\mu_p$; the empirical support of $\mu_p$ is $[0, \infty)$. If its owner decides to develop a slot of type $\mu_p$, he has to incur a fixed cost $c(\mu_p)$. Under each ownership structure, platform owners will first develop the slots with the lowest development costs. We assume that $c(\cdot)$ is a continuous and weakly increasing function: it becomes increasingly costly to offer the same services or convenience to buyers as more retailers are active on the platform. Since $c(\cdot)$ can always be chosen appropriately, we assume w.l.o.g. that, for any $x > 0$, the measure of platform slots with types in $[0, x]$ is $x$. For simplicity, each platform slot hosts one seller, and so sellers of measure $m_s$ need retail space of measure $m_p = m_s$. Retail space of measure $m_p$ is provided at a minimum development cost of $C(m_p) \equiv \int_0^{m_p} c(\mu_p) d\mu_p$. Since $c(\cdot)$ is weakly increasing, $C(\cdot)$ is weakly convex. Note that platform slots differ only in their development costs; from the sellers' point of view, all developed slots are homogeneous. Consequently, the equilibrium rental price is independent of the type of the slot.

*Market Clearing.* Under any ownership structure, the rental market for platform slots will clear in equilibrium. Since a single slot accommodates a single seller, in equilibrium, the measure of entering sellers must be equal to the measure of developed platform slots, i.e., $m_s = m_p = m$. We will refer to $m$ as the *platform size.*

*Payoffs.* To summarize, we can write the equilibrium payoff of a seller of type $\mu_s$ as $z(m)\pi(m) - f(\mu_s) - r$, where $m$ is the equilibrium platform size and $r$ the (uniform) equilibrium rental price. Below we will often refer to the marginal seller's profit (gross of the rental price $r$) which is denoted by $\Phi(m) \equiv z(m)\pi(m) - f(m)$. The equilibrium surplus of a buyer of type $\zeta$ from visiting the platform (rather than taking up the outside option) is $u(m) - g(\zeta)$. Aggregate profits for intermediaries are the revenues collected through rental charges, $mr$, minus the accumulated platform development costs $C(m)$.

*Timing.* We consider the following sequence of decisions, involving first the intermediaries, then the sellers, and finally the buyers:[8]

**Stage 1a** The measure $m_p$ of platform slots is developed by their owner(s).

**Stage 1b** The rental charge $r$ is set by the platform owners (intermediaries).

**Stage 2** Sellers decide whether or not to rent a slot on the trading platform.

**Stage 3** Buyers decide whether or not to visit the platform.

*Underlying Micro Structure.* While not necessary for our formal analysis, it is helpful to consider the micro structure we have in mind. Each seller offers a unique variety of a differentiated good, and hence faces a downward-sloping residual demand curve. Varieties are

[8] Throughout the paper, we focus on the equilibrium with the largest number of active sellers in each stage-2 subgame.

symmetric, and so the platform size $m$ is the measure of varieties offered by sellers. We expect a seller's gross profit (per unit mass of buyers), $\pi(m)$, to (eventually) decrease with $m$ for two reasons. First, for given prices, buyers purchase less from each seller as the number of sellers increases.[9] This may be dubbed the *market share effect*. Second, as the number of sellers increases, competition becomes more intense and prices fall (assuming that the goods offered by sellers are substitutes). This may be dubbed the *price effect*.[10] The *competition effect* is then defined as the sum of the market share and price effects. It constitutes a negative direct network effect in the reduced form.

Likewise, buyers' utility $u(m)$ and, hence, the mass $z(m)$ of buyers visiting the platform, should be increasing in variety $m$ for two reasons. First, buyers have a taste for variety. Second, as the number of sellers increases, prices tend to fall (provided varieties are substitutes). The effect of an increase in $m$ on the mass $z(m)$ of buyers may be dubbed the *market expansion effect*. It constitutes a positive indirect network effect in the reduced form.

If varieties are substitutes, an increase in platform size thus exercises two countervailing forces on a seller's variable profit $z(m)\pi(m)$: a positive market expansion effect and a negative competition effect.[11] The following example provides one particular micro-foundation for our general model. We will return to it at various points in the paper.

**Example 1 (CES)** *Suppose consumers (buyers) make a discrete choice between visiting the market place and an outside option. Consumer type $\zeta$'s value of the outside option or, equivalently, the transport cost associated with visiting the platform, is given by $g(\zeta) = t\zeta$. Conditional on visiting the market place, consumers have CES preferences over the variants offered by the retailers (sellers). Demand for variant $j$ is*

$$x(j) = \frac{p(j)^{-\frac{1}{1-\rho}} E}{\int_0^m p(i)^{-\frac{\rho}{1-\rho}} di},$$

*where $p(j)$ is the price of variant $j$, $E$ is income spent on the differentiated goods industry, and $\rho \in (0,1)$ is inversely related to the degree of product differentiation. Each retailer $j$ maximizes her gross profit (per unit mass of buyers) $\pi = (p-c)p^{-\frac{1}{1-\rho}} A$, where $c$ is the marginal cost of production, and $A = E/\int_0^m p(j)^{-\frac{\rho}{1-\rho}} dj$. Using symmetry, the first-order conditions of profit maximization yield the equilibrium price $p = c/\rho$. In this example, an increase in product*

[9] There may be a countervailing effect as variety increases, however, namely that variety-seeking consumers may optimally decide to spend a larger fraction of their income on the goods produced in the differentiated goods industry.

[10] If the goods offered were complements, or alternatively, search or participation externalities would lead to demand complementarities, the price effect would be positive, at least initially.

[11] In much of the literature on optimal product variety (e.g., Dixit and Stiglitz, 1977), which is concerned with non-intermediated trade, it is assumed that the number of active buyers is exogenous, and so $z$ is a constant. Yet there is competition between firms, and so $\pi$ is decreasing in $m$.

In contrast, throughout much of the recent literature on two-sided markets (e.g., Rochet and Tirole, 2003; Armstrong, 2006), it is assumed that there is no competition effect, and so $\pi$ is a constant, while buyer surplus depends on the number of sellers, and so $z$ is not constant.

An exception is the recent work on the credit card payment industry, which provides a formal analysis of the functioning of Visa and Mastercard; see Rochet and Tirole (2002) and Schmalensee (2002). Other exceptions include Ambrus and Argenziano (2004), Belleflamme and Toulemonde (2004), and Hagiu (2004).

*variety has no price effect, so $\partial p/\partial m = 0$. Equilibrium output per seller is given by $(E/m)(\rho/c)$, so there is a market share effect, and $\pi(m) = (1-\rho)E/m$ is decreasing in $m$. In equilibrium, utility $u(m) = E(\rho/c)m^{\frac{1-\rho}{\rho}}$ is increasing in $m$. The marginal consumer $z$ who is indifferent between visiting the market place and taking up the outside option is defined by $u(m) - tz = 0$, and hence $z(m) = u(m)/t$. A retailer's gross profit is then given by*

$$z(m)\pi(m) = \frac{E^2}{t}\frac{\rho(1-\rho)}{c}m^{\frac{1-2\rho}{\rho}}.$$

*Here, $z(\cdot)\pi(\cdot)$ is monotone in $m$: it is decreasing if the variants are sufficiently good substitutes, $\rho > 1/2$, and increasing if the reverse inequality holds. Hence, a necessary condition for an interior maximum in the mass of entering sellers is that $\rho < 1/2$.*

*As regards costs, we assume that sellers' fixed costs take the form $f(\mu_s) = a\mu_s^b$, where $a \geq 0, b > 0$; and that the development cost function takes the form $c(\mu_p) = \alpha\mu_p^\beta$, where $\alpha \geq 0, \beta > 0$.*

*Reduced-Form Assumptions.* Our discussion of the underlying micro-structure motivates the following assumptions on the reduced-form payoff functions.

**Assumption 1** *The marginal seller's profit $\Phi(m) \equiv z(m)\pi(m) - f(m)$ is single-peaked. Specifically, there exists a unique $\widehat{m} \in [0,\infty) \cup \{\infty\}$ such that $\Phi(\cdot)$ is strictly increasing on $[0,\widehat{m})$ and strictly decreasing on $(\widehat{m},\infty)$.*

If $0 < \widehat{m} < \infty$, the maximizer is implicitly defined by the first-order condition

$$\Phi'(\widehat{m}) = 0. \tag{1}$$

In this case, the seller's gross profit $z(m)\pi(m)$ initially rises faster with platform size than the fixed cost $f(m)$, which can occur only if the market expansion effect initially outweighs the competition effect. As $m$ increases, one would expect the market expansion and competition effects to become smaller while less and less efficient sellers enter the platform so that eventually the marginal seller's profit $\Phi(m)$ declines. But we also allow for the possibility that $\Phi(m)$ is monotonic: if $\widehat{m} = \infty$, $\Phi(m)$ is monotonically increasing in $m$; if $\widehat{m} = 0$, $\Phi(m)$ is monotonically decreasing in $m$.

**Assumption 2** *There exists a unique $m^* > 0$ such that marginal seller's profit is equal to the development cost of his slot:*

$$\Phi(m^*) - c(m^*) = 0. \tag{2}$$

*For smaller platform sizes, $m < m^*$, the profit of the marginal seller exceeds the development cost of the marginal slot, $\Phi(m) > c(m)$, while the opposite is true for larger platform sizes, $\Phi(m) < c(m)$ if $m > m^*$.*

**Example 2 (CES)** *In the CES-Example, assumptions 1 and 2 hold under the restriction that $\rho > \max\left\{\frac{1}{\beta+2}, \frac{1}{b+2}\right\}$, which requires a minimum degree of convexity of the two aggregate cost functions, assuming that $a > 0$ and $\alpha > 0$. (See the appendix for details.) We will return to three special/limiting cases:*

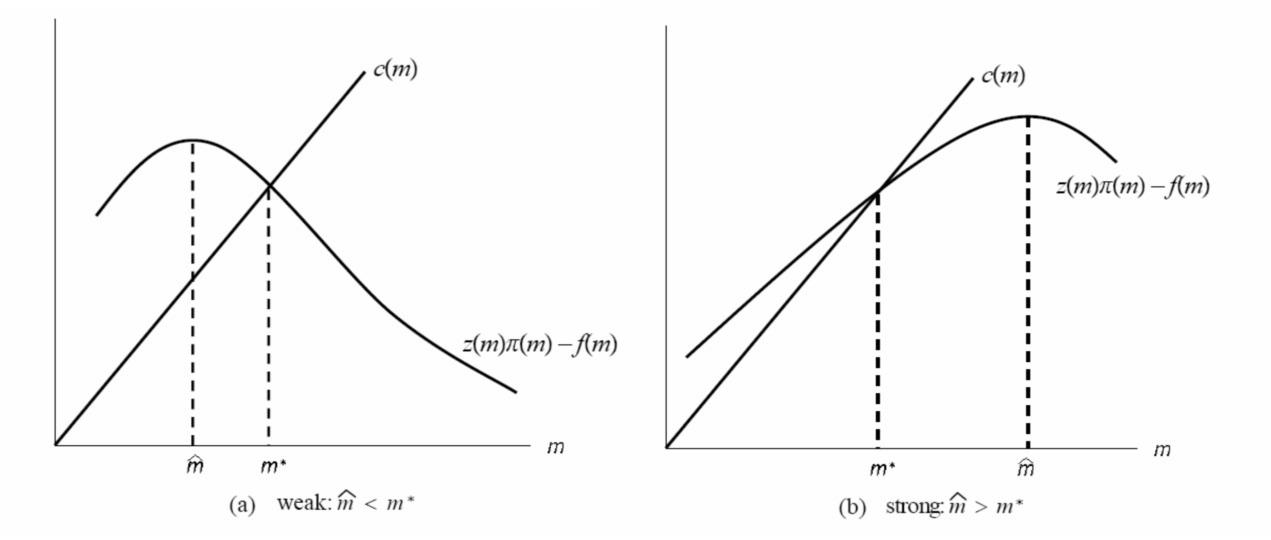


Figure 1: Weak and Strong Platform Effects

**Case I** *The cost functions $f(\cdot)$ and $c(\cdot)$ coincide, $a = \alpha$ and $b = \beta$. In this case, our assumptions hold if $\rho > \frac{1}{\beta+2} = \frac{1}{b+2}$.*

**Case II** *Platform development costs are constant (and zero), $\alpha = 0$. In this case, our assumptions are satisfied if $\rho > \frac{1}{b+2}$.*

**Case III** *All sellers have the same fixed cost (zero), $a = 0$. In this case, assumption 1 is always satisfied since $z(m)\pi(m)$ is a monotone function, while assumption 2 holds if $\rho > \frac{1}{\beta+2}$.*

*A Taxonomy of Platform Effects.* The two numbers, $m^*$ and $\widehat{m}$, allow us to provide a simple but powerful taxonomy of the network or platform effects at work. Intuitively, if the market expansion effect (indirect network effect) is weak (relative to the competition effect and the rising fixed cost $f(\cdot)$), then the marginal seller's profit $\Phi(m)$ will be increasing only for small $m$, or even be monotonically decreasing. In this case, $\widehat{m} < m^*$, and the platform effects are said to be weak. This case is illustrated in panel (a) of figure 1. If, on the other hand, the market expansion effect outweighs the competition effect and rising fixed cost for a large range of platform sizes, then $\widehat{m} > m^*$, and the platform effects are said to be strong. This case is illustrated in panel (b) of figure 1.

**Definition 1** *Platform effects are* weak *if $\widehat{m} < m^*$, and so $\partial\Phi(m^*)/\partial m < 0$. Platform effects are* strong *if $\widehat{m} > m^*$, and so $\partial\Phi(m^*)/\partial m > 0$.*

In the following remarks, we investigate under what conditions platform effects are weak versus strong.

**Remark 1** *The distinction between weak and strong platform effects can be linked to a "stand-alone utility" $\theta$ provided by the platform. Suppose buyers' utility function takes the form*

| | weak | strong |
|---|---|---|
| $a = \alpha$ | $\rho > \frac{1}{(b/2)+2}$ | $\frac{1}{(b/2)+2} > \rho > \frac{1}{b+2}$ |
| $\alpha = 0$ | $\rho > \frac{1}{b+2}$ | $\emptyset$ |
| $a = 0$ | $\rho > \frac{1}{3}$ | $\frac{1}{3} > \rho > \frac{1}{b+2}$ |

Table 1: Weak and Strong Platform Effects in the CES-Example (with $b = \beta$)

*$u(m;\theta) = v(m) + \theta$, and assume that $v'(m) \geq 0$ for all $m \geq 0$ and $g''(z) \geq 0$ for all $z \geq 0$. This implies that $\partial z(m;\theta)/\partial\theta > 0$ and $\partial^2 z(m;\theta)/\partial\theta\partial m \leq 0$. Then, $\widehat{m}(\theta)$ is weakly decreasing in $\theta$ (and strictly decreasing if $\widehat{m}(\theta) > 0$), while $m^*(\theta)$ is strictly increasing in $\theta$. Hence, if there exists a $\widehat{\theta}$ such that $\widehat{m}(\widehat{\theta}) = m^*(\widehat{\theta})$, then the platform effect is strong for $\theta < \widehat{\theta}$ and weak for $\theta > \widehat{\theta}$. That is, if the platform provides a high stand-alone utility (e.g., because of good services that are provided independently of platform size), then the platform effects purely generated by buyer-seller interaction are weak, while the opposite is true if the platform provides a low stand-alone utility.*

**Remark 2** *The distinction between weak and strong platform effects is closely linked to the strength of indirect network effects. This can be seen in a simple parametric example. To focus on indirect network effects, suppose that sellers offer independent products so that $\pi$ is constant. Each buyer consumes one unit of each product and receives a constant utility per product, i.e. $u(m) = \alpha m$. Furthermore, suppose that $g(\zeta) = \zeta$. Then, $z(m) = \alpha m$. Finally, let $c(m) = c_0 m^3/2$ and $f(m) = f_0 m^2/2$. A seller's revenue is then $\pi z(m)$ and a buyer's utility $\alpha m$ so that the parameters $\pi$ and $\alpha$ measure the strength of indirect network effects. It turns out that platform effects are strong, $\widehat{m} > m^*$, if and only if $\alpha\pi > f_0^2/c_o$. That is, platform effects are strong if and only if indirect network effects are above a certain threshold, and platform effects are weak otherwise.*

**Example 3 (CES)** *In the CES-example, the distinction between weak and strong platform effects can be linked to the degree of product differentiation between sellers' offerings. As table 1 illustrates for the three special cases, platform effects are weak if the products offered by sellers are sufficiently good substitutes ($\rho$ large), while they are strong if products are sufficiently differentiated ($\rho$ small). Intuitively, the more differentiated are products, the faster increases buyers' utility with the number of products offered on the platform (in the CES-example, this is due to consumers' love of variety), and so the stronger are the indirect network effects.*

# 3 Independently Owned Platforms

In this section, we derive the equilibrium under three ownership structures that are all vertically disintegrated: open platform ownership ($O$), closed platform ownership ($C$), and monopoly platform ownership ($M$).

- *Open Platform Ownership ($O$).* A population of ex ante identical (potential) intermediaries sequentially enter the market for intermediation on the platform and decide which platform slots to develop. Each intermediary can develop at most one slot $\mu_p$ at cost $c(\mu_p)$, and offer it to one seller at the competitive rental price.

- *Closed Platform Ownership (C).* A population of ex ante identical (potential) intermediaries sequentially enter the market for intermediation on the platform, and reserve platform slots for development. However, early entrants (incumbent club members) can deny access to the platform to later entrants (prospective members).[12] After being admitted, each intermediary develops his reserved slot $\mu_p$ at cost $c(\mu_p)$, and offers it to house one seller at the market-clearing rental price.

- *Monopoly Platform Ownership (M).* All platform slots are owned by a monopoly intermediary. The monopolist decides which platform slots to develop, and sets a uniform rental price for all (developed) platform slots.[13]

Ownership is fragmented under open and closed platform ownerships and concentrated under monopoly ownership. Under open platform ownership, incumbent intermediaries cannot deny access to other intermediaries – in contrast to the cases of closed and monopoly platform ownerships. As regards "clubs" or "closed platform ownership", there are two polar cases to consider. To the extent that frictionless Coasian bargaining between incumbent club members and prospective members is feasible, the equilibrium outcome would replicate the one under monopoly platform ownership ($M$). The other polar case is where side payments between intermediaries are infeasible. This is the case we analyze under the label "closed platform ownership" ($C$). One example relates to retailing with fragmented ownership. Here, the modifications of zoning restrictions typically require the approval of the city council. A closed platform then corresponds to the situation in which the city council acts according to the interests of incumbent platform slot owners.

### 3.1 Positive Analysis

*Open Platform Ownership.* Under open platform ownership, free entry of rental-price taking intermediaries implies that the cost of developing the marginal platform slot of type $m_p$ is equal to the rental price $r$, i.e., $r = c(m_p)$. Profit-maximizing participation decisions of sellers imply that the marginal seller's profit net of the rental price is equal to zero, $z(m_s)\pi(m_s) - f(m_s) = r$. The competitive rental price $r$ clears the rental market, $m_p = m_s$. The equilibrium platform size $m^O$ is thus determined by

$$\Phi(m^O) \equiv z(m^O)\pi(m^O) - f(m^O) = c(m^O). \tag{3}$$

(From assumption 2, it follows that $m^O$ is unique.) Comparing (2) and (3), we obtain the following lemma.

[12] As we will later show, the interests of all of the admitted members are perfectly aligned. Hence, we can remain agnostic about the club's admission procedure (e.g., giving individual veto power to each club member yields the same outcome as requiring unanimity).

[13] While it is easy to look at this possibility, we do not consider his potential ability to discriminate between different types of sellers.

**Lemma 1** *Under open platform ownership, there is a unique equilibrium platform size given by* $m^O = m^*$.

Graphically, equilibrium under open platform ownership is at the intersection of the $\Phi(\cdot)$ and $c(\cdot)$ curves in figure 1.

*Closed Platform Ownership.* Under closed platform ownership, incumbent intermediaries can deny prospective intermediaries access to the platform. Since the most efficient platform slots will be reserved first, the optimization problem for incumbent intermediaries consists in maximizing each active intermediary's profit with respect to the number of active intermediaries, subject to: the participation constraint for sellers, the market clearing condition for platform slots, and the condition that each intermediary makes nonnegative profits. Formally, $\max_{m_p} r - c(\mu_p)$ subject to $z(m_s)\pi(m_s) - f(m_s) = r$, $m_s = m_p$, and $r \geq c(m_p)$. Conditional on being granted access to the platform, all intermediaries on the platform agree on the optimal number of active intermediaries. Market clearing implies a platform size $m = m_s = m_p$. The maximization problem can thus be rewritten as

$$\max_{m} z(m)\pi(m) - f(m) \quad (4)$$
$$\text{s.t.} \qquad z(m)\pi(m) - f(m) \geq c(m).$$

The constraint reflects that prospective intermediaries cannot be forced to join the platform. It follows trivially that the equilibrium platform size $m^C$ cannot be larger than under open platform ownership: $m^C \leq m^O = m^*$. Solving the maximization problem, we obtain the following result.

**Lemma 2** *Under closed platform ownership, there is a unique equilibrium platform size given by* $m^C = \min\{\widehat{m}, m^*\}$.

**Proof.** See appendix. ■

Graphically, the equilibrium under closed platform ownership is at the peak of the $\Phi(\cdot)$ curve when platform effects are weak; see figure 1(a). When platform effects are strong, the equilibrium is at the intersection of the $\Phi(\cdot)$ and $c(\cdot)$ curves in figure 1(b).

*Monopoly Platform Ownership.* Suppose now that all platform slots are owned by a monopolist. The monopolist's problem consists in maximizing his profit with respect to the total number of platform slots $m_p$, subject to the free-entry condition for sellers and the rental market clearing condition. Formally, $\max_{m_p} m_p r - C(m_p)$, subject to $z(m_s)\pi(m_s) - f(m_s) = r$ and $m_s = m_p$. Market clearing implies a platform size $m = m_s = m_p$. The maximization problem can thus be rewritten as $\max_m m\,[z(m)\pi(m) - f(m)] - C(m)$. Solving this problem, we obtain the following result.

**Lemma 3** *Under monopoly platform ownership, the equilibrium platform size* $m^M$ *satisfies* $m^M \in (\widehat{m}, m^*)$ *if platform effects are weak, and* $m^M \in (m^*, \widehat{m})$ *if platform effects are strong. Otherwise, if* $\widehat{m} = m^*$*, we have* $m^M = \widehat{m} = m^*$.

**Proof.** See appendix. ■

Suppose first that platform effects are weak, $\widehat{m} < m^*$. In this case, we have $m^M \in (\widehat{m}, m^*)$. Consider figure 1(a). To see that $m^M > \widehat{m}$, note that at any platform size $m \leq \widehat{m}$, a marginal increase in platform size has a non-negative effect on profits for *inframarginal* platform slots as $\Phi'(m) \geq 0$. Further, at any $m < m^*$, the marginal slot itself makes a positive contribution to total profits as $\Phi(m) > c(m)$. To see that $m^M < m^*$, note that at any $m > \widehat{m}$, a marginal increase in platform size reduces the profits for inframarginal platform slots as $\Phi'(m) < 0$. Moreover, at any $m \geq m^*$, the marginal slot makes a non-positive contribution to total profits as $\Phi(m) \leq c(m)$.

Suppose second that platform effects are strong, $m^* < \widehat{m}$. We then have $m^M \in (m^*, \widehat{m})$. Consider figure 1(b). To see that $m^M > m^*$, note that at any platform size $m < \widehat{m}$, a marginal increase in platform size increases the profits for inframarginal platform slots as $\Phi'(m) > 0$. Further, at any $m \leq m^*$, the marginal slot makes a non-negative contribution to total profits as $\Phi(m) \geq c(m)$. To see that $m^M < \widehat{m}$, note that at any platform size $m \geq \widehat{m}$, a marginal increase in platform size has a non-positive effect on profits for inframarginal platform slots as $\Phi'(m) \leq 0$. Moreover, at any $m > m^*$, the marginal slot makes a negative contribution to total profits as $\Phi(m) < c(m)$. Hence, when platform effects are strong, the monopoly platform owner optimally "subsidizes" the marginal seller by setting a rental price below marginal development cost.[14]

*A Comparison of Platform Sizes under the Three (Vertically Non-Integrated) Ownership Structures.* From our analysis, it is immediate to see that monopoly ownership unambiguously leads to a larger platform size than closed platform ownership. Compared to the open platform, the monopoly platform has a larger platform size if and only if platform effects are strong. This result is summarized in the following proposition.

**Proposition 1** *The equilibrium platform size under the three vertically non-integrated ownership structures can be ranked as follows. If platform effects are weak, $\widehat{m} < m^*$, then $m^C < m^M < m^O$. If platform effects are strong, $m^* < \widehat{m}$, then $m^O = m^C < m^M$.*

**Proof.** This follows immediately from lemmas 1 to 3. ■

The corresponding rental charges satisfy $r^C > r^M > r^O$ if platform effects are weak and $r^O = r^C < r^M$ if platform effects are strong. Hence, a monopoly platform owner always charges a higher rental fee than an open platform – and yet platform size is larger under monopoly ownership when platform effects are strong. To develop a better intuition for the result, interpret platform size $m$ as the "output" of the intermediaries, the "price" of which is the rental charge $r$. The intermediaries as producers of platform slots face the industry production cost function $C(m)$. Under a fragmented ownership structure, each potential producer has a capacity of one unit of output. The "inverse demand function" for platform slots is then given by the marginal seller's willingness to pay for a slot, $\Phi(m)$. By assumption 1, the inverse demand function is single-peaked and obtains a unique maximum at $\widehat{m} \geq 0$. When comparing open and monopoly platform ownerships, we are thus comparing a competitive market with a monopoly.

[14] Such cross-subsidization between platform slots on the same market side is distinct from the cross-subsidization between market sides that has been analyzed in the two-sided markets literature (e.g., Rochet and Tirole, 2003; Armstrong, 2006).

Suppose first that platform effects are weak, $\widehat{m} < m^*$. In the special case $\widehat{m} = 0$, inverse demand is monotonically downward-sloping. As is well known from intermediate microeconomics, equilibrium output will then be lower under monopoly than in a competitive market, since the monopolist has an incentive to restrict output so as to be able to charge a higher price. Under closed platform ownership, members have an incentive to deny access as further entry will reduce the price members can charge. Since each (atomless) member produces a single unit of output (which is a slot of measure zero), closed platform ownership leads to a measure zero of output. Equilibrium output is thus lower than under monopoly. With $\widehat{m} > 0$, the comparison between the three ownership structures is similar to the case where $\widehat{m} = 0$, the main difference being that the equilibrium platform size under closed platform ownership is of positive measure. Note that if the marginal development cost function $c(m)$ were constant (as typically assumed in the literature on network effects; see Economides, 1996), then platform effects would necessarily be weak.

Suppose now that platform effects are strong, $m^* < \widehat{m}$. In this case, the industry's marginal (production) cost function $c(m)$ intersects the inverse demand function in the latter's increasing part (compare figure 1b). Consequently, the equilibrium output levels of the open and closed competitive markets coincide: under closed platform ownership, incumbent producers would like to increase industry output above and beyond the level provided under open platform ownership, but cannot force production of additional producers at a price below cost. In contrast, the monopolist can internalize network effects (which cause the inverse demand function to be locally upward-sloping) and subsidize production at the margin. Hence, the ranking of output levels between competitive markets and monopoly is reversed if the relevant part of the inverse demand function is upward-sloping.

## 3.2 Welfare Analysis

In this subsection, we investigate the welfare properties of equilibria under the different ownership structures. Henceforth, we will assume that buyers' utility is strictly increasing in platform size $m$. As discussed in section 2, this can be for two reasons: consumers value variety, and prices may decrease with the number of sellers.

**Assumption 3** *The utility function $u(\cdot)$ is continuously differentiable and strictly increasing.*

An immediate implication of this assumption is that the mass $z(m)$ of buyers visiting the market place is strictly increasing in the mass $m$ of sellers on the platform: $z'(m) = u'(m)/g'(z(m)) > 0$.

*Consumer Surplus Standard.* In many countries, antitrust authorities seem to have adopted a consumer (rather than a total) surplus standard. Since at given $m$, buyers enter until $g(z(m)) = u(m)$, the aggregate surplus of all buyers visiting a platform of size $m$ is given by $S(m) = \int_0^{z(m)} [g(z(m)) - g(\xi)]\, d\xi$. Assumption 3 implies that one ownership structure induces a larger buyer surplus than another if and only if it induces a larger platform size. As the following result shows, the socially preferred ownership structure under the consumer surplus standard is the open platform if platform effects are weak, and the monopoly platform if platform effects are strong.

**Proposition 2** *The ranking of ownership structures by consumer (buyer) surplus is:*

$$\begin{aligned} S(m^C) &< S(m^M) < S(m^O) \text{ if platform effects are weak;} \\ S(m^C) &= S(m^O) < S(m^M) \text{ if platform effects are strong.} \end{aligned}$$

**Proof.** This follows from proposition 1 and assumption 3. ■

The social ranking of ownership structures is less straightforward if total surplus is used as the welfare criterion. It is to this issue that we now turn.

*Total Surplus Standard.* We now consider total surplus of buyers, sellers, and intermediaries as the welfare criterion. When platform size is $m$, total surplus is given by

$$W(m) = \int_0^{z(m)} [g(z(m)) - g(\xi)]\, d\xi + \int_0^m [f(m) - f(\mu_s)]\, d\mu_s + \{m\Phi(m) - C(m)\}, \quad (5)$$

where the first term is the aggregate surplus of buyers as before and the second term is the aggregate surplus of sellers, while the last term in curly brackets is the aggregate surplus of platform owners.

Let $m^W = \arg\max_m W(m)$ denote the platform size chosen by a benevolent social planner.[15][16] The first-order condition for $m^W$ is given by

$$z(m^W)u'(m^W) + m^W f'(m^W) + \left\{m^W \Phi'(m^W) + \Phi(m^W) - c(m^W)\right\} = 0. \quad (6)$$

We now compare the platform sizes induced by the different ownership structures with the socially optimal platform size. First, we claim that the open platform is too small from a social point of view, $m^O < m^W$, if and only if

$$z(m^W)u'(m^W) + m^W f'(m^W) + m^W \Phi'(m^W) > 0. \quad (7)$$

To see this, note that $\Phi(m^W) - c(m^W) < 0$ if and only if (7) holds. The claim then follows from $\Phi(m^O) - c(m^O) = 0$ and assumption 2. Note that the l.h.s. of condition (7) represents the sum of the externalities on active buyers, sellers, and intermediaries induced by a small increase in platform size when $m = m^W$: the first term is the increase in aggregate buyer surplus, the second term the increase in aggregate seller surplus, while the third term represents the aggregate change in rental payments received by the existing $m^W$ platform owners. Since $u'(m) > 0$ and $f'(m) > 0$ for all $m$, condition (7) can be violated only if $\Phi'(m^W) < 0$. The single-peakedness of $\Phi(\cdot)$ (assumption 1) then implies that $\Phi'(m^O) < 0$ is a necessary condition for $m^O > m^W$. That is, the open platform induces a socially excessive platform size *only if* platform effects are weak.

[15] We follow the I.O. literature on socially optimal entry (e.g., Spence, 1976; von Weizsäcker, 1980; Mankiw and Whinston, 1986) in assuming that the planner chooses the mass of entering sellers (and thus the mass of developed slots), taking consumer behavior and sellers' pricing as given. This has been called the "structural second-best" as it corresponds to structural regulation (Vives, 1999).

[16] In the context of our retailing example, the social planner may be a total surplus maximizing city council that designs zoning laws and provides subsidies to developers of retail space, thereby in effect determining platform size $m$.

**Example 4 (CES)** *Consider the CES-example and assume that there is no heterogeneity on the seller side, $a = 0$ (case III). Holding fixed all other parameters, there exists a critical value $\widehat{\rho}$ such that product diversity offered by an open platform is excessively large, $m^O > m^W$, if and only if the products offered are sufficiently good substitutes, $\rho > \widehat{\rho}$.*

Second, we claim that monopoly ownership always induces a platform that is too small from a social point of view, $m^M < m^W$. To see this, note that $m^M$ maximizes the joint profits of intermediaries, the third term in equation (5). The claim then follows from the observation that the first two terms (the aggregate surplus of buyers and sellers) are monotonically increasing in platform size $m$. In conjunction with proposition 1, we thus obtain that $m^C < m^M < m^W$ if platform effects are weak, while $m^C = m^O < m^M < m^W$ if platform effects are strong.

We do *not* assume that total surplus $W(m)$ is single-peaked in $m$. Nevertheless, we obtain the following result on welfare.

**Proposition 3** *The ranking of ownership structures by total surplus is:*

$$\begin{aligned} W(m^C) &< W(m^M) < W(m^W) \text{ if platform effects are weak;} \\ W(m^C) &= W(m^O) < W(m^M) < W(m^W) \text{ if platform effects are strong.} \end{aligned}$$

**Proof.** To see that $W(m^C) < W(m^M)$, independently of whether platform effects are weak or strong, note first that $m^M$ maximizes the third term in equation (5), while $m^C$ does not. Furthermore, the first two terms in (5) are strictly larger when evaluated at $m^M$ rather than $m^C < m^M$ as both terms are monotonically increasing in $m$. To see that $W(m^M) < W(m^W)$, recall that $m^W = \arg\max_m W(m)$, while $m^M$ maximizes the third term in (5), but not the first two terms. Finally, if platform effects are strong, $m^C = m^O$, and so $W(m^C) = W(m^O)$. ■

*Implications for Competition Policy.* For competition policy, two important sets of questions arise. First, given that the platform is owned by independent intermediaries, should the incumbent owners be allowed to exclude potential entrants? As we have shown, exclusion of potential entrants has welfare effects only if platform effects are weak, and these welfare effects are ambiguous since an open platform may lead to a socially excessive or insufficient platform size. Note that if the antitrust authority were to use consumer surplus as its welfare criterion, then exclusion is never socially beneficial as it always weakly reduces platform size.

Second, given that the seller side is fragmented, is horizontal integration (monopolization) necessarily harmful from a social point of view? As proposition 3 indicates, the answer is "no": horizontal integration is beneficial to welfare if platform effects are strong. Even if platform effects are weak, monopolization may increase total surplus (and necessarily does so in the case of monopolization of a closed platform).[17] Importantly, this efficiency defense for monopoly intermediation is not based on cost efficiency but on the internalization of externalities.

More specifically (and related to the literature on optimal product variety), with respect to the degree of product differentiation, our analysis suggests that in markets in which products are close substitutes, the open platform may lead to socially excessive product variety due to the

[17] In the CES-example, there exist parameters such that $W(m^O) < W(m^M)$ even when platform effects are weak.

competition (or "business-stealing") effect among sellers.[18] A monopoly owner of a platform takes this competition effect into account. However, due to its market power, it sets high rental charges which leads to a socially insufficient number of sellers and too little product variety. Here, an open platform leads to more product variety than a monopoly platform. In markets in which products are bad substitutes (we may think of market segments for luxury goods) the competition effect is less pronounced. Here, the monopoly platform provides actually more (but socially insufficient) variety than an open platform because it takes into account that it may achieve inframarginal gains by subsidizing platform slots at the margin. Such a subsidization at the margin is not possible on the open platform.

# 4 Vertically Integrated Platforms

In this section, we turn to the analysis of vertically integrated platforms, where each active seller owns her own platform slot, and so trade can be thought of as being non-intermediated. We consider two vertically integrated platform ownership structures: an open integrated ($OI$) and a closed integrated ($CI$) ownership structure. Since in our reduced-form approach we do not explicitly consider sellers' pricing decisions, we do not analyze the case of a vertically integrated monopoly supplier-intermediary.

- *Open Integrated Platform Ownership ($OI$).* A population of (potential) sellers sequentially decide whether and which platform slots to develop. Each seller can develop at most one slot $\mu_p$ at cost $c(\mu_p)$ to sell her own good.
- *Closed Integrated Platform Ownership ($CI$).* A population of (potential) sellers sequentially decide whether and which platform slots to reserve for development. However, early entrants (incumbent club members) can deny access to the platform to later entrants (prospective members).[19] After being admitted, each seller develops her reserved slot $\mu_p$ at cost $c(\mu_p)$ to sell her own good.

Since sellers and platform slots are heterogeneous, equilibrium under both ownership structures depends on the sequence in which sellers decide whether and which platform slots to develop. We assume that sellers' participation decisions are taken in increasing order of fixed costs: more efficient sellers move before less efficient sellers.[20] Since each entrant will want to develop the best available platform slot, this sequencing results in a perfectly positive correlation between seller type and slot type across seller-slot pairs.

*Open Integrated Platform Ownership.* Consider first the case of an open integrated platform. Since seller and slot types will be perfectly correlated across active seller-slot pairs, the marginal

[18] While this result on excessive product diversity appears familiar from the existing literature, it is derived in a more general framework that encompasses network externalities.

[19] As in the case of the (non-integrated) closed ownership structure, the interests of all of the admitted members are perfectly aligned.

[20] This assumption may be justified as follows. In a natural dynamic extension of our model, suppose that the sequential entry of sellers onto the platform takes time and sellers discount profits. In this case, more efficient sellers have a higher willingness-to-pay for early (rather than late) entry than less efficient sellers.

seller $m$ with fixed cost $f(m)$ faces development cost $c(m)$. The equilibrium platform size $m^{OI}$ is thus determined by the free-entry condition

$$\Phi(m^{OI}) \equiv z(m^{OI})\pi(m^{OI}) - f(m^{OI}) = c(m^{OI}). \quad (8)$$

Hence, the outcome is identical to that of an open non-integrated platform, $m^O = m^{OI} = m^*$, as summarized in the following lemma.

**Lemma 4** *Under open integrated platform ownership, the unique equilibrium platform size $m^{OI}$ is given by $m^{OI} = m^*$.*

*Closed Integrated Platform Ownership.* Consider now the case of a closed integrated platform. Each incumbent seller-intermediary of type $\mu_s = \mu_p = \mu$ wants to choose platform size $m$ so as to maximize its net profit $z(m)\pi(m) - f(\mu) - c(\mu)$ subject to the constraint that each active seller-intermediary obtains a non-negative net profit, $z(m)\pi(m) - f(m) - c(m) \geq 0$. The solution to this problem, $m^{CI}$, either satisfies the constraint with equality, and so $m^{CI} = m^*$, or else $z'(m^{CI})\pi(m^{CI}) + z(m^{CI})\pi'(m^{CI}) = 0$. We therefore obtain the following result.

**Lemma 5** *Under closed integrated platform ownership, the equilibrium platform size $m^{CI}$ satisfies $\widehat{m} \leq m^{CI} \leq m^*$ if platform effects are weak (with the first inequality being strict if, in addition, $\widehat{m} > 0$), and $m^{CI} = m^*$ if platform effects are strong.*

**Proof.** See appendix. ■

*Vertical Integration and Platform Size.* The effects of vertical integration on equilibrium platform sizes are summarized in the following proposition.

**Proposition 4** *Under an open ownership structure, vertical integration has no effect on equilibrium platform size: $m^{OI} = m^O$. Under a closed ownership structure, vertical integration weakly increases the equilibrium platform size: $m^{CI} \geq m^C$, where the inequality is strict if platform effects are weak and $\widehat{m} > 0$.*

To see why $m^{CI} \geq m^C$, note that, under a closed non-integrated ownership structure, the intermediaries do not capture all of the inframarginal rents of sellers (due to the uniform rental charge). The incumbent intermediaries thus have a smaller incentive to admit additional members than under vertical integration, where they capture all of the inframarginal rents of sellers. Observe that if all sellers were homogeneous (i.e., if $f$ were constant), all intermediaries would make zero profit in equilibrium under vertical separation. In this case, vertical integration would not affect the allocation under closed ownership.

**Example 5 (CES)** *Consider the CES-example, and suppose that $a = \alpha$ and $b = \beta$ (case I). Then vertical integration of a closed platform leads to an increase in platform size from $\widehat{m}$ to $m^*$ if platform effects are weak and $\widehat{m} > 0$, which holds for $\rho \in (2/(4+\beta), 1/2)$.*

*Welfare Effects of Vertical Integration.* What are the welfare effects of vertical integration? Proposition 4 says that vertical integration of an open platform has no effects, while vertical integration of a closed platform weakly increases platform size. As the following proposition shows, vertical integration of a closed platform is weakly welfare increasing. This is due to the internalization of vertical externalities.

**Proposition 5** *Vertical integration of an open platform has no welfare effects:* $S(m^{OI}) = S(m^O)$ *and* $W(m^{OI}) = W(m^O)$*. Vertical integration of a closed platform weakly increases buyer surplus,* $S(m^{CI}) \geq S(m^C)$ *and (assuming that a seller's gross profit* $z(m)\pi(m)$ *is single-peaked in platform size* $m$*) total surplus,* $W(m^{CI}) \geq W(m^C)$*, where the inequalities are strict if platform effects are weak and* $\widehat{m} > 0$*.*[21]

**Proof.** As regards vertical integration of an open platform, the assertion follows immediately from proposition 4. As regards vertical integration of a closed platform, the result on consumer surplus follows from proposition 4 and assumption 3. To see the result on total surplus, note that

$$W'(m) = z(m)u'(m) + \{z(m)\pi(m) - f(m) - c(m)\} + m\left[z'(m)\pi(m) + z(m)\pi'(m)\right].$$

By assumption 3, the first term is strictly positive for all $m$. By assumption 2, the second term is strictly positive for all $m < m^*$. The single-peakedness of $z(m)\pi(m)$ implies that $z'(m^{CI})\pi(m^{CI}) + z(m^{CI})\pi'(m^{CI}) = 0$ if $z'(m^*)\pi(m^*) + z(m^*)\pi'(m^*) \leq 0$, and $z'(m^{CI})\pi(m^{CI}) + z(m^{CI})\pi'(m^{CI}) > 0$ otherwise. It follows that the third term is weakly positive for all $m \leq m^{CI}$. Hence, $W'(m) > 0$ for all $m \leq m^{CI}$, and so $W(m^{CI}) \geq W(m^C)$, where the inequality is strict if $m^{CI} > m^C$. The result then follows by applying proposition 4. ■

# 5 Two-Sided Pricing

In the preceding analysis, buyers had free access to the platform. Indeed, it is often impracticable to charge consumers, especially in brick-and-mortar retail markets. In this section, we consider two-sided pricing: as before, each seller needs to pay $r$ to rent a platform slot, but now each buyer has to pay a "subscription fee" $s$ (which can be positive or negative) to participate on the platform. We confine attention to the three basic ownership structures, $O$, $C$, and $M$. As we will show, the notions of weak and strong platform effects can be extended to the case of two-sided pricing. Most importantly, the ranking of ownership structures by induced platform size (proposition 1) carries over to this setting.

Since each buyer has to pay the subscription fee $s$ to visit the platform, the mass of active buyers is now given by $z(m, s) \equiv g^{-1}(u(m) - s)$. Denote the sum of the profit of the marginal seller (gross of the rental price $r$) and the subscription revenue (from buyers) per seller/developer by

$$\Psi(m, s) \equiv z(m, s)\pi(m) - f(m) + sz(m, s)/m.$$

In analogy to our assumption on $\Phi$, we assume now that $\Psi$ is single-peaked in $m$, holding $s$ fixed. In addition, $\Psi$ is assumed to have a unique interior maximizer with respect to $s$ for any given $m$.[22] For any given $m$, $\widehat{s}(m)$ denotes the unique solution to $\Psi_s(m, s) = 0$.

[21] In the CES-example, the single-peakedness of $z(m)\pi(m)$ holds for all $\rho$. The result also holds if, instead, we assume that the aggregate profit of sellers and intermediaries, $\int_0^m [z(m)\pi(m) - f(\mu) - c(\mu)]\, d\mu$, is single-peaked in $m$.

In the CES-example, we can provide conditions on parameters such that vertical integration strictly increases total surplus. A particular case is the specification from above ($a = \alpha$, $b = \beta$, and $\rho \in (2/(4+\beta), 1/2)$).

[22] We have verified that, for some parameter constellations, all assumptions made in this section are satisfied in our CES-example.

**Remark 3** *Assume $g''(z) \geq 0$. Then, for each $m$, $\Psi(m,s)$ is concave in $s$ and has an interior maximizer. (This is shown in the appendix).*

Correspondingly, let for any given $s$,

$$\widehat{m}(s) = \arg\max_{m \geq 0} \Psi(m,s).$$

If $\Psi(m,s)$ is strictly increasing for all $m \geq 0$, then $\widehat{m}(s) = \infty$. Otherwise, $\widehat{m}(s)$ is implicitly defined by $\Psi_m(\widehat{m}(s), s) = 0$. Suppose that $\Psi(m,s) < c(m)$ for $m$ sufficiently large, holding $s$ fixed, and define $m^*(s)$ as the largest solution to $\Psi(m^*(s), s) - c(m^*(s)) = 0$. Hence, (generically) $\frac{\partial}{\partial m}\{\Psi(m^*(s), s) - c(m^*(s))\} < 0$. We now extend our previous definition of weak and strong platform effects.

**Definition 2** *Platform effects are weak at $s$ if $\widehat{m}(s) < m^*(s)$, and so $\partial\Psi(m^*(s), s)/\partial m < 0$. Platform effects are strong at $s$ if $\widehat{m}(s) > m^*(s)$, and so $\partial\Psi(m^*(s), s)/\partial m > 0$.*

In line with standard results from oligopoly theory (e.g., Vives, 1999), we further assume that sellers' aggregate gross profits (per unit mass of buyers) are weakly decreasing in $m$, i.e., $d[m\pi(m)]/dm \leq 0$. Together with $g''(z) \geq 0$, this implies the following result.

**Lemma 6** *Suppose $d[m\pi(m)]/dm \leq 0$ and $g''(z) \geq 0$. Then, $\Psi_{ms}(m, \widehat{s}(m)) > 0$, and so $\widehat{s}'(m) > 0$.*

We are now equipped to discuss equilibrium allocations under open, closed, and monopoly platform ownership.

*Open Platform Ownership.* Suppose that if a subscription fee is charged to buyers, each active platform owner (intermediary) receives its fair share of the proceeds: $sz(m,s)/m$. In this case, any equilibrium is characterized by (i) free entry of platform developers, i.e., $r + sz(m,s)/m = c(m)$, and (ii) free entry of sellers, i.e, $r = z(m,s)\pi(m) - f(m)$. (Free entry of buyers is implicit in the definition of $z(m,s) = g^{-1}(u(m) - s)$.) In equilibrium, $m$ and $s$ must satisfy

$$m\left[z(m,s)\pi(m) - f(m) - c(m)\right] + sz(m,s) = 0,$$

or $\Psi(m,s) = c(m)$. Note that any tuple $(m,s)$ satisfying this equation can be sustained as a competitive equilibrium: indeed, there is a continuum of equilibria in which all developed slots are rented out, $\mu_p = \mu_s = m$. Since there is free entry of platform developers, any equilibrium $(m,s)$ on the open platform satisfies $m = m^*(s)$.

One natural equilibrium arises when platform developers charge a price equal to marginal cost on each side of the market, and so $s = 0$, and $m = m^O = m^*(0)$. This is the equilibrium that we analyzed before under one-sided pricing. Henceforth, we will focus on another natural equilibrium, namely the one that maximizes developers' joint profits $mr + sz - C(m)$, or $m\Psi(m,s) - C(m)$, subject to the constraint that each developer covers his cost. Hence, we can write the program as $\max_{m,s} m\Psi(m,s) - C(m)$, subject to $\Psi(m,s) = c(m)$. This program can be rewritten as $\max_{m,s} mc(m) - C(m)$ subject to $\Psi(m,s) = c(m)$. Since $d\{mc(m) - C(m)\}/dm = mc'(m) \geq 0$, the program is equivalent to maximizing platform size $m$ subject to the free entry condition for developers, $\Psi(m,s) = c(m)$. The associated Lagrangian is given by $L = m + \lambda\{\Psi(m,s) - c(m)\}$. The first-order conditions are:

$$\frac{\partial L}{\partial m} = 1 + \lambda\{\Psi_m(m,s) - c'(m)\} = 0, \tag{9}$$

$$\frac{\partial L}{\partial s} = \lambda \Psi_s(m,s) = 0, \tag{10}$$

$$\frac{\partial L}{\partial \lambda} = \Psi(m,s) - c(m) = 0. \tag{11}$$

Let $s^{O2}$ and $m^{O2}$ denote the equilibrium subscription fee and platform size, respectively. From (10) and (11), it follows that, $s^{O2} = \widehat{s}(m^{O2})$ and $m^{O2} = m^*(s^{O2})$, which we assumed to be unique. The intersection of these functions is necessarily stable since

$$m^{*\prime}(s^{O2}) = -\frac{\Psi_s(m^{O2}, s^{O2})}{\Psi_m(m^{O2}, s^{O2}) - c'(m^{M2})} = 0,$$

where the second equality obtains since $\Psi_s(m^{O2}, s^{O2}) = 0$ and $\Psi_m(m^{O2}, s^{O2}) < c'(m^{M2})$. Note that the equilibrium subscription fee is given by the elasticity formula:

$$s^{O2} = -\frac{m^{O2}\pi(m^{O2})}{1 + \frac{1}{\varepsilon_{z,s}(m^{O2}, s^{O2})}}.$$

where $\varepsilon_{z,s}(m,s) \equiv z_s(m,s)s/z(m,s)$ is the elasticity of the number of active buyers with respect to the subscription fee $s$.

*Closed Platform Ownership.* Under closed ownership structure, $r$ and $s$ are chosen so as to maximize the profit of each active developer, $r + sz/m$. The program is thus given by $\max_{m,s} \Psi(m,s)$ subject to $\Psi(m,s) \geq c(m)$. If the constraint is non-binding, the first-order conditions are given by $\Psi_s(m,s) = 0$ and $\Psi_m(m,s) = 0$. Their solution is assumed to be unique, and it is given by $s^{C2} = \widehat{s}(m^{C2})$ and $m^{C2} = \widehat{m}(s^{C2})$. We assume that the intersection of $\widehat{s}(\cdot)$ and $\widehat{m}(\cdot)$ in $(m,s)$-space is "stable", i.e.,

$$\widehat{s}'(m^{C2}) < \frac{1}{\widehat{m}'(s^{C2})}. \tag{12}$$

Even if the constraint $\Psi(m,s) \geq c(m)$ is binding, it is optimal for platform owners to set $s$ so as to maximize $\Psi(m,s)$ for any given $m$, and so $s^{C2} = \widehat{s}(m^{C2})$. However, in this case, the program becomes the same as that of the open platform, and so $m^{C2} = m^{O2}$ and $s^{C2} = s^{O2}$. Independently of whether or not the constraint is binding, the equilibrium subscription fee is given by the same elasticity formula as under the open platform:

$$s^{C2} = -\frac{m^{C2}\pi(m^{C2})}{1 + \frac{1}{\varepsilon_{z,s}(m^{C2}, s^{C2})}}.$$

*Monopoly Platform Ownership.* The problem for a monopoly platform owner is to maximize $m\Psi(m,s) - C(m)$ with respect to $m$ and $s$. For any given $m$, the profit-maximizing subscription fee $\widehat{s}(m)$ satisfies $\Psi_s(m, \widehat{s}(m)) = 0$. For any given $s$, the profit-maximizing platform size $\widetilde{m}(s)$ satisfies $\widetilde{m}(s)\Psi_m(\widetilde{m}(s), s) + \Psi(\widetilde{m}(s), s) - c(\widetilde{m}(s)) = 0$.

We assume that in $(m, s)$-space, the curves $\widetilde{m}(s)$ and $\widehat{s}(m)$ have a unique intersection at $(m^{M2}, s^{M2})$ with $m^{M2} = \widetilde{m}(s^{M2})$ and $s^{M2} = \widehat{s}(m^{M2})$. Again, the intersection of these curves is assumed to be "stable", i.e.,

$$\widehat{s}'(m^{M2}) < \frac{1}{\widetilde{m}'(s^{M2})}. \tag{13}$$

From the first-order condition, the profit-maximizing subscription fee is given by the same elasticity formula as under the open platform

$$s^{M2} = -\frac{m^{M2}\pi(m^{M2})}{1 + \frac{1}{\varepsilon_{z,s}(m^{M2}, s^{M2})}}.$$

*Comparison of Ownership Structures.* As in section 3, we compare the equilibrium outcomes under the different ownership structures. In the remainder, we assume that sellers' aggregate gross profits (per unit mass of buyers) are weakly decreasing in platform size, whereas total gross surplus of buyers and sellers per unit mass of buyers are increasing in platform size, i.e., $d[m\pi(m)]/dm \leq 0 < d[m\pi(m) + u(m)]/dm$. In addition, we assume $g''(z) \geq 0$. We then obtain the following ranking of allocations with respect to ownership structures.

**Proposition 6** *If platform effects are weak at $s^{O2}$, then $m^{C2} < m^{M2} < m^{O2}$, $z^{C2} < z^{M2} < z^{O2}$, and $s^{C2} < s^{M2} < s^{O2}$. If platform effects are strong at $s^{O2}$, then $m^{C2} = m^{O2} < m^{M2}$, $z^{C2} = z^{O2} < z^{M2}$, and $s^{C2} = s^{O2} < s^{M2}$.*

**Proof.** See appendix. ■

The proposition shows that the ordering of ownership structures by induced platform size carries over to our setting with two-sided pricing. In spite of the fact that, in equilibrium, the subscription fee is larger for an ownership structure with a larger platform size, it continues to hold that an ownership structure that leads to a larger platform size also attracts more buyers. With respect to the prevalence of weak versus strong platform effects, there is an important difference compared to our previous analysis. Here, $\partial\Psi(m^*(s), s)/\partial m$ is evaluated at $s = s^{O2}$, whereas under one-sided pricing it is evaluated at $s = 0$. The case of weak platform effects is illustrated in figure 2.

*Welfare Properties.* As regards buyer surplus, an ownership structure that attracts more consumers than another generates a larger buyer surplus. Since there is a positive monotone relationship between $m$ and $z$ across ownership structures (see proposition 6), buyer surplus is higher under one ownership structure than under another if and only if its induced size is larger. The ranking of ownership structures by consumer surplus then follows immediately from proposition 6.

Total surplus under two-sided pricing is given by

$$W(m, s) \equiv z(m, s)u(m) - \int_0^{z(m,s)} g(\xi)d\xi + mz(m, s)\pi(m) - \int_0^m f(\mu_s)\mu_s - C(m).$$

Let $m^{W2}$ denote the welfare-maximizing platform size under two-sided pricing. In analogy to our welfare result under one-sided pricing, we state the following result.

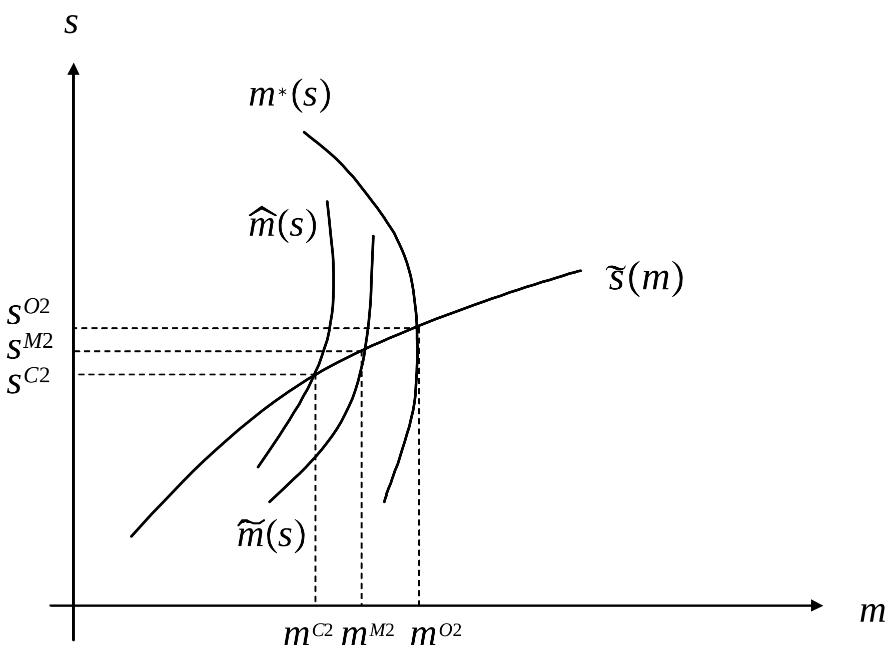


Figure 2: Weak Platform Effects at $s^{O2}$.

**Proposition 7** *The ranking of ownership structures by total surplus is:*

$$W(m^{C2}) < W(m^{M2}) < W(m^{W2}) \text{ if platform effects are weak at } s^{O2};$$

$$W(m^{C2}) = W(m^{O2}) < W(m^{M2}) < W(m^{W2}) \text{ if platform effects are strong at } s^{O2}.$$

As under one-sided pricing, a monopoly platform ownership may generate a larger total surplus than an open ownership structure, and necessarily does so if platform effects are strong at $s^{O2}$.[23]

# 6 Discussion

In this section, we briefly discuss possible re-interpretations and extensions of our model.

*Economics of Zoning Laws.* Our analysis can be used to study the implications of different zoning restrictions on the number of slots developed for retailing. It suggests that if products are good substitutes (so that the competition effect is sufficiently strong), an unregulated market for retailing may lead to a socially excessive number of shops. This provides a rationale for zoning laws.[24] However, if zoning laws are made on behalf of existing property owners

[23] In the CES-example, there exist parameters such that there is excessive product diversity under open platform ownership, $m^{O2} > m^{W2}$.

[24] Similarly, the government may become a regulator by issuing licenses for certain types of business, where the allocation mechanism must be chosen such that the most efficient firms enter.

(who restrict the number of slots) there is always social underprovision of shops. This means that a laissez-faire policy is often performing better than a policy in which the legislator is captured by the interests of existing property owners. However, the latter allocation can also be improved by horizontal or vertical integration. In particular, if the legislator is captured by the interests of established owners, horizontal integration is necessarily welfare improving. For the case in point this implies that (national) antitrust policy optimally depends on whether the (local) legislator is captured by a particular interest group.

*Brick-and-Mortar Retailing Reconsidered.* In brick-and-mortar retailing, different ownership structures coexist. On the one hand, many cities' downtown retailing districts are characterized by open and dispersed ownership. In a great number of instances, landlords occupy single lots and offer space to one retailer. This corresponds to open platform ownership. In some instances, the sellers are also the owners of their retail space, which corresponds to an open integrated platform.[25] On the other hand, most suburban shopping malls are operated by a shopping center developer, and are thus characterized by monopoly platform ownership.

Does this suggest that one of these ownership structures is socially inferior? The answer is, "no, not necessarily," as we will argue now. The crucial difference between a downtown retailing district and a suburban shopping mall is that many consumers work (and sometimes live) downtown, while suburban shopping malls are located in areas with a relatively low population density, and so consumers travel to them only for shopping.

This may be formalized by reconsidering the parameter $\theta$ in the market size function $z$ to capture location-specific demand. As in remark 1, let $z(m;\theta)$ denote the number of consumers visiting a market place of size $m$ and location characteristic $\theta$. A higher value of $\theta$ will be associated with a larger location-specific "stand-alone" demand and, therefore, less responsiveness of demand to the number of sellers: $z(m;\theta)$ is increasing in $\theta$, while $\partial z(m;\eta)/\partial m$ is decreasing in $\theta$. Since a downtown location ($d$) has arguably a larger stand-alone demand than a suburban location ($s$), $\theta^d > \theta^s$.

As discussed in remark 1, this implies that $\widehat{m}(\theta^d) \leq \widehat{m}(\theta^s)$, where the inequality is strict if $\widehat{m}^s > 0$, and $m^*(\theta^d) > m^*(\theta^s)$. Hence, everything else equal, platform effects are "more likely" to be strong in a suburban location than in a downtown location. Since monopoly ownership is the welfare-maximizing ownership structure under strong platform effects while open ownership can be welfare-maximizing under weak platform effects, a shopping mall may be the socially optimal ownership structure in a suburban location, while open ownership may be socially optimal in a downtown location. (Note that this does *not* imply that product diversity is larger in a suburban shopping mall than in a downtown retailing district.)

To the extent that a shopping center developer actively selects retailers and adjusts rental contracts to internalize externalities, there is an additional reason for the platform effect to be strong in a shopping mall.[26]

[25] In both cases, the assumption that the costs of providing such space are non-decreasing in the number of slots can be justified by invoking a constant quality standard of slot provision: the worse the location of a retail slot, the larger the costs that have to be incurred to make the slot as attractive to consumers as a better located slot. If the cost is incurred by the seller, the rental price paid to the landlord has to be adjusted accordingly.

In some other instances, the downtown retailing district is constrained by zoning restrictions. If these restrictions coincide with those preferred by incumbent owners, this corresponds to a closed platform.

[26] In our framework, consumers care only about the number of sellers on the platform, and all sellers impose the same externality on all other sellers. In the real world, however, some sellers may impose greater (positive)

*Cost Sharing of Development Costs and Congestion Externalities.* In our model, we have assumed that each intermediary bears the development costs of his slot. However, if platform owners form a cooperative, development costs may be shared among them. In the case of equal cost sharing, if incumbent owners have the right to exclude potential entrants, incumbent owners are more restrictive in admitting additional intermediaries than without cost redistribution because joining owners inflict a negative externality on incumbent owners by increasing the average fixed cost born by the typical incumbent. If exclusion is not possible but equal cost sharing is maintained, more intermediaries enter the platform than in the absence of equal cost sharing, and independent intermediaries make zero profit in equilibrium since cost sharing makes them homogeneous ex post. Hence, any overprovision of product diversity under open platform ownership becomes more pronounced when development costs are shared equally.

Alternatively, the positive indirect network effect present in our model could be counteracted by negative congestion externalities arising from too many buyers on the platform. Such externalities often arise in physical markets. But they also come about in virtual markets whenever there are capacity constraints. These externalities can easily be incorporated into our model. Suppose, in particular, that a platform slot is developed at zero cost but that there exists a congestion cost $K(m)$ so that the congestion cost per intermediary is $K(m)/m$. This interpretation is then formally equivalent to the model with cost redistribution.

*Platforms without Slot Structure.* Software platforms do not have a slot structure, as assumed in our paper. Nevertheless, our analysis still applies to integrated and monopoly ownership structures. Even in the absence of a slot structure, platform size can be interpreted as the number of applications that can be run on a platform as long as platform development costs are weakly increasing and weakly convex in the number of potential applications. This includes zero marginal costs as a special case.

Consider, for instance, software platforms for mobile phones. There, an example of monopoly ownership is the platform offered by Microsoft, while an example of vertically integrated ownership is Symbian, a platform jointly developed by several handset makers (for details, see Evans, Hagiu, and Schmalensee, 2006).

*Transaction Fees.* In a number of markets, intermediaries do not charge a fixed access fee or rental price but a price which depends on transaction volume. Examples include internet travel agents (e.g., Expedia, Opodo) and card payment systems (e.g., Visa, Mastercard, American Express).[27] Is it possible to extend our model to allow for a fee per transaction (rather than a

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externalities on rival sellers than some other sellers (e.g., by offering complementary rather than substitute products). A monopoly platform owner is then in a better position than fragmented owners are to select the sellers that impose the greatest externalities on the other sellers. Indeed, there is empirical evidence that shopping mall developers use the rental pricing scheme to internalize some of these externalities. See, for example, Pashigian and Gould (1998), and Gould, Pashigian, and Prendergast (2002). It is in principle possible to extend our framework to accommodate such differences between ownership structures, namely by parameterizing the variable profit function by the ownership structure's ability to internalize externalities between sellers. However, we feel that such an extension would be more fruitful in the context of a particular example that imposes additional structure on the problem.

[27] Expedia and American Express correspond to monopoly ownership in the sense that control rights are concentrated in the publicly traded firm's board. Opodo, an internet travel agency owned and operated by nine leading European airlines (and a travel industry technology provider) corresponds to a vertically integrated ownership structure. Visa and Mastercard correspond to open platform ownership: the banks which own Visa and Mastercard act as financial intermediaries between buyers and sellers.

fixed access charge)? If we introduce a transaction fee in our CES example, a seller's profit is of the form $h(y)z(m)\pi(m) - f(\mu_s)$, where $h$ is a decreasing function of the transaction fee (in percent of revenues) $y$. The impact of different ownership structures on platform size can also be analyzed in this case. In the presence of a transaction charge, we would expect that platform size under open ownership depends on whether or not ownership is vertically integrated. We believe this to be an interesting avenue for further research. However, since competition in the product market is directly affected by the level of the transaction fee, a reduced-form approach seems less useful and we expect results to be less general.

*Stability of Ownership Structures.* In our analysis, the ownership structure was taken as given. But which ownership structure is likely to emerge? To address this question, we provide a "stability analysis" in an earlier version of this paper, Nocke, Peitz, and Stahl (2004). There, we call an ownership structure "stable" (with respect to horizontal integration) if no outside player can make a positive profit by acquiring all property rights on the platform in an appropriately-defined acquisition game. We show that monopoly is the only stable ownership structure when platform effects are strong, while the vertically-integrated ownership structures $OI$ and $CI$ can be stable when platform effects are weak; see Nocke, Peitz, and Stahl (2004) for details.

*Competition between Market Places.* An important extension of our framework is to consider competition not only within, but also between platforms. We will analyze this issue in a separate paper (Nocke, Peitz, and Stahl, 2006; see also Smith and Hay, 2005).

# 7 Conclusion

Intermediaries figure very importantly in the formation of two-sided markets. In this paper, we have developed a general theoretical framework of a trading platform with two-sided network effects, which allows for intermediated and non-intermediated trade. We have provided a useful binary index of the strength of the platform effects involved ("weak" versus "strong"). Indeed, as we have shown, the ranking of ownership structures by induced platform size (or product diversity) and welfare crucially depends on this index of platform effects. If platform effects are strong, equilibrium platform size under monopoly ownership is larger than under dispersed ownership. In this case, all ownership structures induce a socially insufficient product diversity, and monopoly is the socially preferred ownership structure. In contrast, if platform effects are weak, monopoly ownership induces a smaller platform than an open ownership structure. In this case, only an open ownership structure may induce a socially excessive product variety. As we have shown, these insights do not depend upon whether platform owners can charge only sellers for platform access, or both sellers and buyers.

As discussed in the introduction, antitrust authorities in the U.S. and Europe have recently been concerned about anticompetitive practices in markets where buyers and sellers need access to a trading platform (e.g., airports, ports, and B2B platforms). Our theoretical framework allows us to address three important policy issues: (1) monopolization, (2) vertical integration, and (3) exclusion.

(1) Assuming the seller side is fragmented, monopolization of the platform is welfare-enhancing if platform effects are strong. If platform effects are weak, the welfare effect of monopolization of an open platform is ambiguous. However, the monopolization of a closed ownership platform is always socially beneficial.

(2) Assuming that ownership of the platform is dispersed, vertical integration weakly increases welfare if platform effects are weak, and has no welfare effects otherwise.

(3) The effects on total surplus of allowing incumbent platform owners to exclude new entrants are ambiguous. Exclusion may actually increase total surplus if platform effects are weak. However, if consumer surplus is used as a welfare criterion, exclusion is always socially harmful if exercised.

Interestingly, a report by the U.S. Federal Trade Commission staff on B2B electronic platforms[28] does not express concerns about vertical integration *per se* but only about potential exclusion of competitors on the vertically integrated side of the market. This is in line with our welfare results.

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# Appendix: Proofs

**The CES-example.** Here, we discuss assumptions 1 and 2 in the context of our CES-example. Assumption 1 concerns the marginal seller's profit. If $\rho > 1/2$, then each seller's gross profit $z(m)\pi(m)$ is monotonically decreasing in $m$. Since the marginal seller's fixed cost $f(m)$ is increasing in $m$, the marginal seller's profit $\Phi(m)$ must be decreasing in $m$, and so $\widehat{m} = 0$, and the assumption is satisfied. If $\rho < 1/2$, then each seller's gross profit $z(m)\pi(m)$ is monotonically increasing in $m$ while the marginal seller's fixed cost is still weakly increasing in $m$. Then, an interior solution for $\widehat{m}$ may exist. However, if $\Phi'(m) > 0$ for $m$ sufficiently large then $\widehat{m} = \infty$. This condition is equivalent to $\rho < \frac{1}{b+2}$. Hence, there exists a unique interior maximum $\widehat{m}$ only if $\frac{1}{b+2} < \rho < 1/2$.[29]

Assumption 2 says that $\Phi(m) - c(m)$ is strictly positive for $m < m^*$, and strictly negative for $m > m^*$. the sum of marginal net profits of sellers and platform owners. If $\rho > 1/2$, $\Phi(m)$ is monotonically decreasing, while $c(m)$ is increasing in $m$, and so the assumption holds trivially. Using Descartes' sign rule, one can show that the assumption also holds if $\max\left\{\frac{1}{\beta+2}, \frac{1}{b+2}\right\} < \rho < 1/2$.

**Proof of lemma 2.** Suppose platform effects are weak, $m^* > \widehat{m}$. Then, the marginal development cost function $c(m)$ intersects the marginal seller's profit function to the right of the latter's peak; see figure 1. Since incumbent club members' interests with respect to platform size are perfectly aligned (provided each member makes nonnegative profits), each club member's profit is maximized at the platform size $m^C = \widehat{m}$, where the marginal seller's profit $\Phi(m)$ is maximized. The marginal club member at platform size $\widehat{m}$ obtains a strictly positive profit, $r - c(\widehat{m}) > 0$, where $r = \Phi(\widehat{m})$. In the special case $\widehat{m} = 0$, the equilibrium platform size is of measure zero since, in this case, an incumbent club member's profit is decreasing in the number of club members.

Suppose now that platform effects are strong, $m^* < \widehat{m}$. Then, the marginal club member would make a loss at platform size $\widehat{m}$. In this case, incumbent club members would like to admit more members than the number of entering intermediaries under free entry, and so the equilibrium platform size is equal to that under open platform membership, $m^C = m^O = m^*$.

Finally, note that it follows from assumptions 1 and 2 that $m^C$ is unique. ■

[29] A sufficient condition for the marginal seller's profit to be single-peaked is the logconcavity of this function. Logconcavity is implied by $\rho \geq \frac{1}{b+1}$. This can be seen as follows: $z(m)\pi(m) - f(m)$ is of the form $d_1(d_2 m^x - m^y) = d_1 m^x (d_2 - m^{y-x})$. This expression is logconcave in $m$ if $m^{y-x}$ is convex in $m$, i.e., $y - x \geq 1$. Using the parameters of our model, this inequality becomes $b - \frac{1-2\rho}{\rho} \geq 1$, which is equivalent to $\rho \geq \frac{1}{b+1}$.

**Proof of lemma 3.** The equilibrium platform size under monopoly ownership, $m^M$, must satisfy the first-order condition

$$\Phi(m^M) + m^M \Phi'(m^M) = c(m^M) \tag{14}$$

Note that there may be more than one platform size which satisfies the first-order and second-order conditions of profit maximization. In general, our assumptions do not imply that the monopolist's objective function is single-peaked.[30] Clearly, the monopolist selects the solution to the first-order condition that maximizes his profit. Hence, $m^M$ is generically unique.

We can rewrite the first-order condition as $\varphi(m^M) + m^M\Phi'(m^M) = 0$, where $\varphi(m) \equiv \Phi(m) - c(m)$. By assumption 1, $\Phi'(m) > 0$ if $m < \widehat{m}$, and $\Phi'(m) < 0$ if the reverse inequality holds. Similarly, by assumption 2, $\varphi(m) > 0$ if $m < m^*$, and $\varphi(m) < 0$ if the reverse inequality holds. Consequently, if $\widehat{m} \neq m^*$, we have $\varphi(m) + m\Phi'(m) > 0$ for $m \leq \min\{\widehat{m}, m^*\}$ and $\varphi(m) + m\Phi'(m) < 0$ for $m \geq \max\{\widehat{m}, m^*\}$. If $\widehat{m} = m^*$, then $\varphi(m) + m\Phi'(m) > 0$ for $m < \widehat{m} = m^*$ and $\varphi(m) + m\Phi'(m) < 0$ for $m > \widehat{m} = m^*$. ■

**Proof of lemma 5.** The constraint in the maximization problem implies that $m^{CI} \leq m^*$. The first-order condition of the unconstrained problem may be written as

$$d[z(m^{CI})\pi(m^{CI})]/dm = 0. \tag{15}$$

If platform effects are very weak, $\widehat{m} = 0$, $m^{CI} \geq \widehat{m}$ is trivially satisfied. If platform effects are weak but not very weak, we have $\widehat{m} > 0$, and the l.h.s. of (15) is strictly positive if evaluated at any $m \leq \widehat{m}$. Hence, $m^{CI} > \widehat{m}$. If platform effects are strong, $\widehat{m} > m^*$, the constraint in the maximization problem is necessarily binding, and so $m^{CI} = m^*$. ■

**Proof of remark 3.** $\widehat{s}(m)$ satisfies

$$\Psi_s(m, \widehat{s}(m)) = z_s(m, \widehat{s}(m))\pi(m) + \frac{z(m, \widehat{s}(m)) + \widehat{s}(m)z_s(m, \widehat{s}(m))}{m} = 0. \tag{16}$$

We show that $\Psi_{ss}(m, \widehat{s}(m)) < 0$. That is, holding $m$ fixed, $\Psi(m, s)$ has at most one extremum, and this extremum is necessarily a maximum. We have

$$\Psi_{ss}(m, \widehat{s}(m)) = z_{ss}(m, \widehat{s}(m)) \left\{ \frac{m\pi(m) + \widehat{s}(m)}{m} \right\} + \frac{2z_s(m, \widehat{s}(m))}{m}.$$

Note that

$$z_s(m, s) = -\frac{1}{g'(z(m, s))} < 0,$$

and

$$z_{ss}(m, s) = \frac{g''(z(m, s))z_s(m, s)}{[g'(z(m, s))]^2} \leq 0,$$

where the last inequality follows from our assumption that $g''(z) \geq 0$. From the first-order condition (16),

$$m\pi(m) + \widehat{s}(m) = -\frac{z(m, \widehat{s}(m))}{z_s(m, \widehat{s}(m))} > 0.$$

[30] In the CES example, there is a unique solution to the first-order condition, provided parameters are such that assumptions 1 and 2 holds.

Hence, $\Psi_{ss}(m, \widehat{s}(m)) < 0$. It follows that, for any $m$, $\Psi(m, s)$ is single-peaked in $s$. If $\widehat{s}(m)$ exists, it is the unique maximizer of $\Psi(m, s)$.

Existence is guaranteed since, for any $m$, $\Psi$ is increasing in $s$ for $s$ sufficiently small and decreasing in $s$ for $s$ sufficiently large. We first show that $\Psi$ is decreasing in $s$ for $s$ sufficiently large. This can be seen as follows. Note that $z$ is strictly decreasing and concave in $s$. Hence, for any $m$ there exists some $\overline{s}(m)$ such that $z(m, \overline{s}(m)) = 0$. Recall that

$$\Psi_s(m, s) = z_s(m, s)\pi(m) + \frac{sz_s(m, s)}{m} + \frac{z(m, s)}{m}$$

Hence, $\Psi_s(m, \overline{s}(m)) < 0$ for any given $m$. We now show that $\Psi$ is increasing in $s$ for $s$ sufficiently small. Fix any $m$. Then the sum of the first two terms on the r.h.s. of the above equation becomes positive for $s < 0$ sufficiently small (i.e. large in absolute value) because $z_s$ is negative and $\pi(m) + s/m$ is also negative. Concerning the third term, for $s$ sufficiently small $z$ is positive. Hence, $\Psi_s$ is positive for $s$ sufficiently small. ■

**Proof of lemma 6.** Recall that $z_s(m, s) = -1/g'(z(m, s)) < 0$. We assumed that $u'(m) > 0$, and so $z_m(m, s) = u'(m)/g'(z(m, s)) > 0$. We also assume that $g''(z) \geq 0$. This implies that $z_{ms}(m, s) \geq 0$ since

$$z_{ms}(m, s) = \frac{g''(z(m, s))z_m(m, s)}{[g'(z(m, s))]^2}.$$

We have

$$\begin{aligned}
\Psi_{ms}(m, \widehat{s}(m)) &= z_{ms}(m, \widehat{s}(m))\pi(m) + z_s(m, \widehat{s}(m))\pi'(m) \\
&\quad + \frac{[z_m(m, \widehat{s}(m)) + \widehat{s}(m)z_{ms}(m, \widehat{s}(m))] - \frac{[z(m,\widehat{s}(m)) + \widehat{s}(m)z_s(m,\widehat{s}(m))]}{m}}{m} \\
&= z_{ms}(m, \widehat{s}(m))\pi(m) + z_s(m, \widehat{s}(m))\pi'(m) \\
&\quad + \frac{[z_m(m, \widehat{s}(m)) + \widehat{s}(m)z_{ms}(m, \widehat{s}(m))] + z_s(m, \widehat{s}(m))\pi(m)}{m},
\end{aligned}$$

where the second equality follows from (16). We have $\Psi_{ms}(m, \widehat{s}(m)) > 0$ if

$$\begin{aligned}
& z_m(m, \widehat{s}(m)) + z_{ms}(m, \widehat{s}(m)) \left\{m\pi(m) + \widehat{s}(m)\right\} \\
& + z_s(m, \widehat{s}(m)) \left[\pi(m) + m\pi'(m)\right] \\
> \ & 0.
\end{aligned}$$

But this inequality is satisfied since $z_m(m, s) > 0$, $z_s(m, s) < 0$, $z_{ms}(m, s) \geq 0$, $\pi(m) + m\pi'(m) \leq 0$ by assumption, and since from (16),

$$m\pi(m) + \widehat{s}(m) = -z(m, \widehat{s}(m))/z_s(m, \widehat{s}(m)) > 0.$$

Applying the implicit function theorem,

$$\widehat{s}'(m) = -\frac{\Psi_{ms}(m, \widehat{s}(m))}{\Psi_{ss}(m, \widehat{s}(m))} > 0,$$

where the inequality follows since $\widehat{s}(m)$ maximizes $\Psi(m, s)$ and $\Psi_{ms}(m, \widehat{s}(m)) > 0$. ■

**Proof of proposition 6.** Recall that the monopolist's profit-maximizing platform size $\widetilde{m}(s)$ satisfies

$$[\Psi(\widetilde{m}(s), s) - c(\widetilde{m}(s))] + \widetilde{m}(s)\Psi_m(\widetilde{m}(s), s) = 0. \quad (17)$$

(1) Suppose that platform effects are weak at $s^{O2}$. Then, $\widehat{m}(s^{O2}) < m^*(s^{O2}) = m^{O2}$. From the stability condition (12), and since $\widehat{s}'(m) > 0$, it follows that, for any $s \geq s^{O2}$, $\widehat{m}(s) < \widehat{s}^{-1}(s)$. Hence, the unique intersection of $\widehat{s}(m)$ and $\widehat{m}(s)$, $(m^{C2}, s^{C2})$, is such that $s^{C2} < s^{O2}$ and $m^{C2} < m^{O2}$. Since the intersection of $m^*(s)$ and $\widehat{s}(m^{O2})$ at $(m^{O2}, s^{O2})$ is stable, it also follows that $m^{C2} = \widehat{m}(s^{C2}) < m^*(s^{C2})$, i.e., the constraint $\Psi(m^{C2}, s^{C2}) \leq c(m^{C2})$ is nonbinding.

We will now show that $\widehat{m}(s^{O2}) < \widetilde{m}(s^{O2}) < m^*(s^{O2})$. (i) Note that $\widetilde{m}(s^{O2}) < \widehat{m}(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) < c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution. To see this, let $m'$ denote the smallest platform size larger than $\widetilde{m}(s^{O2})$ such that $\Psi(m', s^{O2}) = c(m')$. Since $\Psi(\widehat{m}(s^{O2}), s^{O2}) > c(\widehat{m}(s^{O2}))$, $m'$ exists. But then, the monopolist's profit is larger when choosing platform size $m'$ rather than $\widetilde{m}(s^{O2})$:

$$\begin{aligned} m'\Psi(m', s^{O2}) - C(m') &= m'c(m') - C(m') \\ &> \widetilde{m}(s^{O2})c(\widetilde{m}(s^{O2})) - C(\widetilde{m}(s^{O2})) \\ &> \widetilde{m}(s^{O2})\Psi(\widetilde{m}(s^{O2}), s^{O2}) - C(\widetilde{m}(s^{O2})), \end{aligned}$$

where the first inequality follows from the fact that $mc(m) - C(m)$ is increasing in $m$ since $c'(m) \geq 0$. (ii) Next, note that $\widetilde{m}(s^{O2}) < \widehat{m}(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) \geq c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution either since, in this case, the l.h.s. of (17) is strictly positive. (iii) For the same reason, $\widetilde{m}(s^{O2}) \neq \widehat{m}(s^{O2})$. (iv) Finally, we cannot have $\widetilde{m}(s^{O2}) \geq m^*(s^{O2})$ since, in this case, the l.h.s. of (17) is strictly negative. Hence, $\widehat{m}(s^{O2}) < \widetilde{m}(s^{O2}) < m^*(s^{O2})$. Using the same steps, one can show that $\widehat{m}(s^{C2}) < \widetilde{m}(s^{C2}) < m^*(s^{C2})$. It follows that $\widetilde{m}(s^{O2}) < \widehat{s}^{-1}(s^{O2})$ and $\widetilde{m}(s^{C2}) > \widehat{s}^{-1}(s^{C2})$. Hence, there exists a (unique) $s^{M2} \in (s^{C2}, s^{O2})$ such that $\widetilde{m}(s^{M2}) = \widehat{s}^{-1}(s^{M2})$. Since $\widehat{s}'(m) > 0$, it follows that the unique intersection of $\widetilde{m}(s)$ and $\widehat{s}(m)$, $(m^{M2}, s^{M2})$, is such that $\widehat{m}(s^{O2}) < m^{M2} = \widetilde{m}(m^{M2}) < m^*(s^{O2})$ and $s^{C2} < s^{M2} < s^{O2}$.

(2) Suppose that platform effects are strong at $s^{O2}$. Then, $\widehat{m}(s^{O2}) > m^*(s^{O2}) = m^{O2}$. From our stability condition (12), and since $\widehat{s}'(m) > 0$, it follows that, for any $s \leq s^{O2}$, $\widehat{m}(s) > \widehat{s}^{-1}(s)$. Hence, the unique intersection of $\widehat{s}(m)$ and $\widehat{m}(s)$, $(m', s')$, is such that $s' > s^{O2}$ and $m' > m^{O2}$. Since the intersection of $m^*(s)$ and $\widehat{s}(m^{O2})$ at $(m^{O2}, s^{O2})$ is stable, it also follows that $m' = \widehat{m}(s') > m^*(s')$, i.e., the constraint $\Psi(m', s') \leq c(m')$ is violated. Hence, at the solution $(m^{C2}, s^{C2})$, the constraint must be binding, and so $m^{C2} = m^*(s^{C2}) = m^{O2}$ and $s^{C2} = s^{O2}$.

We will now show that $m^*(s^{O2}) < \widetilde{m}(s^{O2})$. (i) Note first that $\widetilde{m}(s^{O2}) < m^*(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) < c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution. The argument is the same as under (1), (i), above: the monopolist could increase its profit by choosing the smallest platform size $m' > \widetilde{m}(s^{O2})$ such that $\Psi(m', s^{O2}) = c(m')$. (ii) Next, note that $\widetilde{m}(s^{O2}) \leq m^*(s^{O2})$ with $\Psi(\widetilde{m}(s^{O2}), s^{O2}) \geq c(\widetilde{m}(s^{O2}))$ cannot be a profit-maximizing solution either since, in this case, the l.h.s. of (17) is strictly positive. Hence, $m^*(s^{O2}) < \widetilde{m}(s^{O2})$, and so $\widetilde{m}(s^{O2}) > \widehat{s}^{-1}(s^{O2})$. It follows that the unique intersection of $\widetilde{m}(s)$ and $\widehat{s}(m)$, $(m^{M2}, s^{M2})$, is such that $m^{M2} = \widetilde{m}(m^{M2}) > m^*(s^{O2})$ and $s^{M2} = \widehat{s}(m^{M2}) > s^{O2} = s^{C2}$.

It remains to be shown that there is a positive monotone relationship between $m$ and $z$. Note that $\Psi_s(m, s) = z_s(m, s)\pi(m) + [z(m, s) + sz_s(m, s)]/m$.Since $s = u(m) - g(z(m, s))$ and

$z_s(m,s) = -1/g'(z(m,s))$, we can rewrite the first-order condition $\Psi_s(m,s) = 0$ as $m\pi(m) + u(m) - zg'(z) - g(z) = 0$, which determines $z$ as a function of $m$; denote it by $\widetilde{z}(m)$. We have

$$\widetilde{z}'(m) = -\frac{\pi(m) + m\pi'(m) + u'(m)}{-2g'(\widetilde{z}(m)) - \widetilde{z}(m)g''(\widetilde{z}(m))} > 0$$

since $\pi(m) + m\pi'(m) + u'(m) > 0$ and $g''(z) \geq 0$ by assumption. Hence, $z^{\omega} > z^{\omega'}$ if and only if $m^{\omega} > m^{\omega'}$, where $\omega, \omega' \in \{M2, C2, O2\}$. ■