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WORKING PAPER 2009:25

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ISSN 1651-1166

# Local polynomial regression with truncated or censored response*

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December 14, 2009

**Abstract**

Truncation or censoring of the response variable in a regression model is a problem in many applications, e.g. when the response is insurance claims or the durations of unemployment spells. We introduce a local polynomial regression estimator which can deal with such truncated or censored responses. For this purpose, we use local versions of the STLS and SCLS estimators of Powell (1986) and the QME estimator of Lee (1993) and Laitila (2001). The asymptotic properties of our estimators, and the conditions under which they are valid, are given. In addition, a simulation study is presented to investigate the finite sample properties of our proposals.

**Keywords:** Non-parametric regression, Truncation, Censoring, Asymptotic properties.
**JEL:** C14

*Acknowledgments: We are grateful to two anonymous referees for helpful comments. We acknowledge the following funding bodies for their support: Institute of Labour Market Policy Evaluation, Jan Wallanders and Tom Hedelius Foundation, and the Swedish Research Council through the Ageing and Living Condition program and the Umeå node of the Swedish Innitiative for Research on Microdata in the Social and Medical Sciences.

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# 1 Introduction

Truncation or censoring of the response variable in a regression model is a problem appearing in many applications. Truncation occur, for instance, when studying the value of insured property damages due to fire, theft or a similar event, because any loss that are below the deductible will not be reported to the insurance company. Censoring often occur when studying durations, e.g., unemployment spells in labour economics, survival times in medical experiments and failure times for components in industrial processes.

We consider the following regression model

$$y_i^* = m(x_i) + \varepsilon_i^*, \qquad i = 1, 2, ..., n^* \tag{1}$$

where $y^*$ is a latent response variable, $x$ is an explanatory variable, $m(x)$ is an unknown $p+1$ ($p \geq 1$) times diffentiable function, and $\varepsilon^*$ is a random error term independently and identically distributed with mean zero and finite variance.

With left (right) truncated, the pairs of observations $(x_i, y_i^*)$ are only observed if $y_i^* > t$ ($y_i^* < t$), where $t$ is a known constant truncation point. For simplicity let $t = 0$. This can be done without loss of generality, by subtracting $t$ from $y_i^*$ and from $m(x_i)$. Let $y$ denote the observed response variable and let $n$ denote the observed sample size. A similar problem to truncation is censoring where data is said to be left (right) censored if $y = \max(y^*, c)$ ( $y = \min(y^*, c)$), where $c$ is a known censoring point. Again, for simplicity and without loss of generality, let $c = 0$. Note that the explanatory variable $x$ is observed under censoring, but not under truncation of the response variable. Moreover, under left (right) censoring it is known that the response is smaller (larger) than $c = 0$, but the exact value is unknown. Such information is not available under truncation, and we do not even know how many observations are truncated.

In this paper we introduce a local polynomial regression estimator of the function $m(x)$, which is able to deal with truncated or censored outcomes. There are few available non-parametric estimators, in particular when it comes to truncated or non-random censored responses; exceptions include Chen et al. (2005) for the censored case and Lewbel and Linton (2002) for both the truncated and censored situations.

Local polynomial regression is a popular nonparametric regression technique due to its attractive asymptotic properties, in particular at the border of the support. For fully observed responses, a local polynomial regression estimate of $m(x_0)$ is obtained by estimating a polynomial in $x$ with weighted ordinary least squares. Each unit in the study is weighted depending of its distance in $x$ to the design point of interest (focal value) $x_0$, thereby making the procedure local. In order to obtain a local polynomial regression estimator for truncated or censored data we propose to replace ordinary least squares with other distribution-free estimators designed for the estimation of a parametrized mean function, e.g. linear in the parameters $m(x_i) = x_i^T \boldsymbol{\beta}$, with truncated or censored responses. Such estimators are reviewed in Lee and Kim (1998), including symmetrically trimmed/censored least squares estimators (STLS and SCLS) suggested first by Powell (1986), and the quadratic mode estimator (QME), suggested by Lee (1993); see also Laitila (2001) and Karlsson (2004). Thus, we present in Section 2 a local-STLS (and local-SCLS) polynomial regression estimator, deriving also asymptotic propeties. A local-QME estimator is described in Section 3. Section 4 presents a Monte Carlo study of the finite sample properties of these estimators. The paper is concluded with a discussion in Section 5.

# 2 Local symmetrically trimmed least squares estimators

Symmetrically trimmed least squares estimators (Powell, 1986) can be used to address either truncation or censoring in the setting of (semi-)parametric regression models, that is when $m(x_i)$ in (1) can be described parametrically, for example with a polynomial $m(x_i) = \beta_0 + \beta_1 x_i + \ldots + \beta_p x_i^p$. Truncation (or censoring) of the response variable introduces an asymmetry in its distribution. The STLS and SCLS estimators symmetrically truncates and censors, respectively, the response variable in order to restore the distribution symmetry about $\beta_0 + \beta_1 x_i + \ldots + \beta_p x_i^p$. In this way, the least squares estimator is consistent and asymptotically normal, under some regularity conditions, including the assumption of symmetrically distributed error terms.

In the case of left truncation (at $t = 0$), and for a polynomial model, the parametric

STLS estimator is defined by

$$\text{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{n} \left( y_i - \max \left( \frac{1}{2} y_i, \boldsymbol{x}_i^T \boldsymbol{\beta} \right) \right)^2, \tag{2}$$

where $\boldsymbol{x}_i = (1, x_i, \ldots, x_i^p)^T$ and $\boldsymbol{\beta} = (\beta_0, \ldots, \beta_p)^T$, or equivalently as the value of $\boldsymbol{\beta}$ solving

$$\sum_{i=1}^{n} 1[y_i < 2\boldsymbol{x}_i^T \boldsymbol{\beta}] \big( y_i - \boldsymbol{x}_i^T \boldsymbol{\beta} \big) \boldsymbol{x}_i = 0, \tag{3}$$

where $1[E]$ is the indicator function of the event $E$, which takes value 1 if $E$ is true and 0 otherwise.

Hence, all observations $y_i$ larger than $2\boldsymbol{x}_i^T \boldsymbol{\beta}$ are trimmed ("truncated") to restore the symmetry of the error distribution. Similarly, for the SCLS estimator, $(1/2y_i)^2 - \big(\max\big(0, \boldsymbol{x}_i^T \boldsymbol{\beta}\big)\big)^2$ is added to the objective function in (2) for all observations larger than $2\boldsymbol{x}_i^T \boldsymbol{\beta}$ so these observations get "censored".

The proposal in this paper is to generalize the STLS and SCLS estimation procedures into local procedures that produce a non-parametric fit by introducing a set of "localizing" weights $K\big((x_i - x_0)/h\big)$, inspired by the local polynomial regression estimator for non-truncated/non-censored data (Fan and Gijbels, 1996). The points $x_0$ are called focal values and are often, but do not have to be, equal to the $x$-points in the sample. The focal values should, however, belong to the space spanned by the observed $x$-values.

Thus, define a local-STLS estimator for $m(x_0)$ in (1) with a left truncated response variable at $t = 0$ by $\hat{m}(x_0) = \boldsymbol{e}^T \hat{\boldsymbol{\theta}}_h(x_0)$, where $\boldsymbol{e} = (1, 0, \ldots, 0)^T$ and

$$\hat{\boldsymbol{\theta}}_h(x_0) = \text{argmin}_{\boldsymbol{\theta}} \sum_{i=1}^{n} K\left( \frac{x_i - x_0}{h} \right) \left( y_i - \max \left( \frac{1}{2} y_i, \boldsymbol{z}_i^T \boldsymbol{\theta} \right) \right)^2, \tag{4}$$

where $\boldsymbol{z}_i = (1, (x_i - x_0), \ldots, (x_i - x_0)^p)^T$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)^T$ and where $K$ is a kernel function of order $r$, that is satisfying $\int K(u)du = 1$, $\int u^k K(u)du = 0$ for $k = 1, \ldots, r-1$ and $\int u^r K(u)du \neq 0$. Typical choices for $K(\cdot)$ are the Gaussian p.d.f. or the tricube function as in (8) below.

Similarly, define a local-SCLS estimator for $m(x_0)$ in (1) with a left censored response variable (at $c = 0$) by $\tilde{m}(x_0) = \boldsymbol{e}^T \tilde{\boldsymbol{\theta}}_h(x_0)$ and

$$\begin{aligned}
&\tilde{\boldsymbol{\theta}}_h(x_0) \\
&= \operatorname{argmin}_\theta \left[ \sum_{i=1}^{n} K\left(\frac{x_i - x_0}{h}\right) \left(y_i - \max\left(\frac{1}{2} y_i, \boldsymbol{z}_i^T \boldsymbol{\theta}\right)\right)^2 \right. \\
&+ \left. \sum_{i=1}^{n} K\left(\frac{x_i - x_0}{h}\right) 1\left[y_i > 2\boldsymbol{z}_i^T \boldsymbol{\theta}\right] \left( \left(\frac{1}{2} y_i\right)^2 - \left(\max\left(0, \boldsymbol{z}_i^T \boldsymbol{\theta}\right)\right)^2 \right) \right]. \qquad (5)
\end{aligned}$$

## 2.1 Asymptotics

Define $\hat{\boldsymbol{\theta}}_h(x_0)$ as the solution of (4), and $\boldsymbol{\theta}_h(x_0)$ as the solution of its population version

$$\operatorname{argmin}_{\boldsymbol{\theta}} E\left( K\left(\frac{x_i - x_0}{h}\right) \left(y_i - \max(\frac{1}{2} y_i, \boldsymbol{z}_i^T \boldsymbol{\theta})\right)^2 \right), \qquad (6)$$

where the expecation is over the joint distribution of $(y_i, x_i)$. First it is shown that $\hat{\boldsymbol{\theta}}_h(x_0) \to \boldsymbol{\theta}_h(x_0)$ almost surely as $n \to \infty$, and that the estimator is asymptotically normally distributed around $\boldsymbol{\theta}_h(x_0)$. Then it is shown that, under some regularity conditions, $\boldsymbol{\theta}_h(x_0) \to \boldsymbol{\theta}_m$ as $h \to 0$ and $n \to \infty$, where $\boldsymbol{\theta}_m^T = (m(x_0), m^{(1)}(x_0), \ldots, m^{(p)}(x_0))$ and $m^{(i)}(x_0)$ is the $i^{th}$ derivative of $m$ evaluated at $x_0$.

**Assumption 1** *The true parameter vector $\boldsymbol{\theta}_h(x_0)$ is an interior point of a compact parameter space $\Theta$.*

**Assumption 2** *The regressors $\boldsymbol{z}_i$ are independently distributed random vectors with $E(||\boldsymbol{z}_i||^{4+\eta}) < K_0$ for some positive $K_0$ and $\eta$, and $\nu_n$, the minimum characteristic root of the matrix*

$$N_n = \frac{1}{n} \sum_{i=1}^{n} E\left( 1[\boldsymbol{z}_i^T \boldsymbol{\theta} \geq \epsilon_0] K\left(\frac{x_i - x_0}{h}\right) \boldsymbol{z}_i \boldsymbol{z}_i^T \right)$$

*has $\nu_n > \nu_0$ whenever $n > n_0$, some positive $\epsilon_0$, $\nu_0$ and $n_0$.*

**Assumption 3** *The error terms $\epsilon_i^*$ are mutually independent distributed, and, conditionally on $\boldsymbol{z}_i$, are continuously symmetrically distributed about zero, with densities which are bounded above and continuous and positive at zero, uniformly in $i$. That is, if $F(\lambda|\boldsymbol{z}_i, i) \equiv F_i(\lambda)$ is the conditional c.d.f. of $\epsilon_i^*$ given $\boldsymbol{z}_i$, then $dF_i(\lambda) = f_i(\lambda) d\lambda$,* *where* $f_i(\lambda) = f_i(-\lambda)$, $f_i(\lambda) < L$, *and* $f_i(\lambda) > \zeta_0$ *whenever* $|\lambda| < \zeta_0$, *some positive constants* $L_0$ *and* $\zeta_0$. *In addition, the error densities* $f_i(\lambda)$ *are unimodal, with strict unimodality, uniformly in* $i$, *in some neighborhood of zero. That is,* $f_i(\lambda_2) \geq f_i(\lambda_1)$ *if* $|\lambda_1| \geq |\lambda_2|$, *and there exists some* $\alpha_0 > 0$ *and some function* $h(\lambda)$, *strictly decreasing in* $[0, \alpha_0]$, *such that* $f_i(\lambda_2) - f_i(\lambda_1) \geq h(|\lambda_2|) - h(|\lambda_1|)$ *when* $|\lambda_2| \leq |\lambda_1| \leq \alpha_0$.

Assumptions 1 and 3 are the same as in Powell (1986, assumption $P$, $E1$ and $E2$). Assumption 2 is a modified version of assumption $R$ in Powell (1986) to take into account the local weighting scheme $K(\cdot)$. The condition on the minimum characteristic root, $\nu_n$, of $N_n$ in Assumption 2 is the essential condition for unique identification of $\boldsymbol{\theta}$, by assuring that $\boldsymbol{z}$ is sufficiently variable.

**Theorem 4** *Under Assumptions 1, 2 and 3*

1. $\hat{\boldsymbol{\theta}}_h(x_0) \rightarrow \boldsymbol{\theta}_h(x_0)$ *almost surely when* $n \rightarrow \infty$.
2. $Z_n^{-1/2}(W_n - V_n)\sqrt{n}(\hat{\boldsymbol{\theta}}_h(x_0) - \boldsymbol{\theta}_h(x_0)) \rightarrow \mathcal{N}(\mathbf{0}, I)$ *in distribution when* $n \rightarrow \infty$, *where*

$$
\begin{aligned}
W_n &= \frac{1}{n}\sum_{i=1}^{n} E\Big(1[-\boldsymbol{z}_i^T\boldsymbol{\theta} < \varepsilon_i < \boldsymbol{z}_i^T\boldsymbol{\theta}]K\big(\frac{x_i - x_0}{h}\big)\boldsymbol{z}_i\boldsymbol{z}_i^T\Big)\\
V_n &= \frac{1}{n}\sum_{i=1}^{n} E\Big(1[\boldsymbol{z}_i^T\boldsymbol{\theta} > 0]K\big(\frac{x_i - x_0}{h}\big)\frac{2\boldsymbol{z}_i^T\boldsymbol{\theta} f_i(\boldsymbol{z}_i^T\boldsymbol{\theta})}{F_i(\boldsymbol{z}_i^T\boldsymbol{\theta})}\boldsymbol{z}_i\boldsymbol{z}_i^T\Big)\\
Z_n &= \frac{1}{n}\sum_{i=1}^{n} E\Big(1[-\boldsymbol{z}_i^T\boldsymbol{\theta} < \varepsilon_i < \boldsymbol{z}_i^T\boldsymbol{\theta}]K\big(\frac{x_i - x_0}{h}\big)\varepsilon_i^2\boldsymbol{z}_i\boldsymbol{z}_i^T\Big)
\end{aligned}
$$

   *and* $Z_n^{-1/2}$ *is any square root of the inverse of* $Z_n$.

The proof follows exactly the same lines as the proof of Theorem 2 in Powell (1986) and is therefore omitted. For the censoring case, a similar proof can be obtained for the estimator, $\tilde{\boldsymbol{\theta}}(x_0)$ defined by (5) by adapting the assumptions $P$, $R$ and $E1$ and following the lines of Theorem 1 in Powell (1986).

Theorem 4 establishes consistency and asymptotic normality of $\hat{\boldsymbol{\theta}}_h(x_0)$ around $\boldsymbol{\theta}_h(x_0)$. This is of interest because $\boldsymbol{\theta}_h(x_0) \rightarrow \boldsymbol{\theta}_m$ as $h \rightarrow 0$ under regularity conditions. This may be shown by taking the derivative of the expectation in (6), equating it to zero, and using a Taylor expansion:

$$
\begin{aligned}
& \frac{\partial}{\partial \boldsymbol{\theta}} E\left[K(\frac{x_i - x_0}{h})\{y_i - \max(\frac{1}{2}y_i, \boldsymbol{z}_i^T\boldsymbol{\theta})\}^2\right] \\
= \; & E_x\left\{E\left(K(\frac{x_i - x_0}{h})\frac{\partial}{\partial \boldsymbol{\theta}}\left[\{y_i - \max(\frac{1}{2}y_i, \boldsymbol{z}_i^T\boldsymbol{\theta})\}^2\right]\Big| x\right)\right\} \\
= \; & -2E_x\left\{E\left(K(\frac{x_i - x_0}{h})1[y_i < 2\boldsymbol{z}_i^T\boldsymbol{\theta}](y_i - \boldsymbol{z}_i^T\boldsymbol{\theta})\boldsymbol{z}_i\Big| x\right)\right\} \\
= \; & -2E_x\Bigg[K(\frac{x_i - x_0}{h}) \\
\times \; & \left(\boldsymbol{z}_i\int_0^{2\boldsymbol{z}_i^T\boldsymbol{\theta}}(\boldsymbol{z}_i^T\boldsymbol{\theta}_m - \boldsymbol{z}_i^T\boldsymbol{\theta} + (y_i - m(x_i)) + R(x_i))f(y_i|x_i)dy_i\right)\Bigg] \\
= \; & -2E_x\left[K(\frac{x_i - x_0}{h})(\boldsymbol{z}_i^T\boldsymbol{\theta}_m - \boldsymbol{z}_i^T\boldsymbol{\theta})\boldsymbol{z}_i\int_0^{2\boldsymbol{z}_i^T\boldsymbol{\theta}} f(y_i|x_i)dy_i\right] \\
& -2E_x\left[K(\frac{x_i - x_0}{h})\boldsymbol{z}_i\int_0^{2\boldsymbol{z}_i^T\boldsymbol{\theta}}(y_i - m(x_i))f(y_i|x_i)dy_i\right] \\
& -2E_x\left[K(\frac{x_i - x_0}{h})R(x_i)\boldsymbol{z}_i\int_0^{2\boldsymbol{z}_i^T\boldsymbol{\theta}} f(y_i|x_i)dy_i\right] = \mathbf{0},
\end{aligned}
$$

where we have used an order $p$ Taylor expansion around $x_0$ of $m(x_i)$: $m(x_i) = \boldsymbol{z}_i^T\boldsymbol{\theta}_m + R(x_i)$.

Assuming regularity conditions, we have that the last term in the sum above tends to zero when $h \to 0$, because then $K\left(\frac{x_i - x_0}{h}\right)$ set all weight on $x_i = x_0$ and $R(x_0) = 0$. The same hold for the second term when setting $\boldsymbol{\theta} = \boldsymbol{\theta}_m$, because $K\left(\frac{x_i - x_0}{h}\right)$ set all weight on $x_i = x_0$ and $\int_0^{2m(x_i)}(y_i - m(x_i))\boldsymbol{z}_i f(y_i|x_i)dy_i = 0$ by construction. Finally, because the first term is set to zero for $\boldsymbol{\theta} = \boldsymbol{\theta}_m$ for any $h$, we can say that $\boldsymbol{\theta}_h(x_0)$ tends to $\boldsymbol{\theta}_m$ as $h \to 0$.

Note that the above behaviour is in line with classical results in non-parametric regression where asymptotic results are obtained by letting $h$ tend towards zero (see the above) when $n$ tends towards infinity (see Theorem 4). The above developments do not, however, say how fast $h$ has to tend to zero with respect to $n$. It is to be expected that $h$ has to tend to zero at a slower rate than $n$ tends to infinity as it is generally the case in non-parametric regression. The convergence rate of $h$ has not much practical relevance however. Instead it is more interesting to have a data-driven method that chooses $h$ optimally in some respect. Thus, an important question for further research.

# 3 Alternatives/related estimators

As an alternative to the "localization" of the STLS and SCLS estimators to obtain a non-parametric fit for regression models when the response variable is truncated or censored it is possible to build on a different available proposal for parametric truncated models, for example the quadratic mode estimator (QME) of Lee (1993).

Define the local QME estimator of $m(x_0)$ in (1) by $\breve{m}(x_0) = \boldsymbol{e}^T \breve{\boldsymbol{\theta}}_h(x_0)$ with

$$\breve{\boldsymbol{\theta}}_h(x_0) = \text{argmin}_{\boldsymbol{\theta}} \sum_{i=1}^{n} K\left(\frac{x_i - x_0}{h}\right)\left(\left(y_i - \max\left(\boldsymbol{z}_i^T\boldsymbol{\theta}, \delta\right)\right)^2 - \delta^2\right), \tag{7}$$

where $\delta$ is a trimming threshold parameter determined by the researcher.

A particular appealing feature of the QME estimator over the STLS estimator is that with the former the assumption of symmetrically distributed errors can be relaxed, see Laitila (2001), when the slope parameters can be consistently estimated under asymmetrically distributed errors. Nevertheless, for consistent estimation of the intercept symmetrically distributed errors still are required. This means that if the interest is to estimate the derivatives of $m(\cdot)$, as is the case in many economic applications, the local-QME is suitable for this purpose even when asymmetry is suspected. A drawback of QME is the need to choose the extra tuning parameter, $\delta$.

The asymptotic properties (consistency and asymptotic normality) of the estimator defined by (7) can be obtained by adapting the results in Laitila (2001).

# 4 Simulation study

A simulation study is performed in order to study the finite sample properties of the proposed estimators, local-STLS and local-QME, defined in (4) and (7) under truncation. For comparison, the local polynomial regression estimator, LPR, (Fan and Gijbels, 1996) defined as $\bar{m}(x_0) = \boldsymbol{e}^T \bar{\boldsymbol{\theta}}_h(x_0)$ with

$$\bar{\boldsymbol{\theta}}_{h(x_0)} = \text{argmin}_{\boldsymbol{\theta}} \sum_{i=1}^{n} K\left(\frac{x_i - x_0}{h}\right)\left(y_i - \boldsymbol{z}_i^T\boldsymbol{\theta}\right)^2,$$

ignoring the truncation is also used to estimate $m(x_0)$ in (1).

In the simulation study polynomials of order 1 and 2 are used. The kernel function, $K(\cdot)$ used for all the proposed estimators is the tricube weight function

$$K(u) = (1 - |u|^3)^3. \tag{8}$$

In the simulation three different bandwidths, $h$, are used, viz: 2, 3, and 5. For the local-QME the threshold parameter $\delta$ also has to be chosen. Here $\delta = 0.5\sigma$, $\delta = 1.0\sigma$, and $\delta = 2.0\sigma$ are utilised, where $\sigma$ is the standard deviation of the error term.

Samples of size $n^* = 500$ and $n^* = 1000$ are generated from the following three models:

Model 1: $y^* = 2.5x + \varepsilon^*$, where $x \sim Uniform(2, 8)$, $\varepsilon^* \sim Normal(0, \sigma)$ and $\sigma = 0.3$. The response is truncated if $y^*$ is smaller than the truncation point, $t = 6$.

Model 2: $y^* = \sin(x) + \varepsilon^*$, where $x \sim Uniform(2, 8)$, $\varepsilon^* \sim Normal(0, \sigma)$ and $\sigma = 0.3$. Data is truncated if $y^*$ is smaller than the truncation point, $t = -1.5$.

Model 3: The same model as Model 2, but with a higher truncation point, viz: $t = -1.0$

The truncation points in the first two models yield an overall truncation grade of approximately 5% and 1% respectively. However, at some values of the explanatory variables the truncation grade is much higher. Model 3 is included as an extreme situation where truncation grade is almost 50% at some focal values (10% overall). Figure 1 displays data simulated from the models described above. The estimators are evaluated at 13 equally spaced focal values, $\{2.0, 2.5, 3.0, ..., 8.0\}$ in terms of the average bias and the average mean squared error (MSE) over 1000 replicates. The truncation grade at each focal value is reported in the same tables as the simulation results on bias and MSE.

Model 1 is also estimated with the ordinary ("parametric") STLS, SCLS and QME estimators to provide benchmarks to the preformance of the local-STLS, local-SCLS, and local-QME respectively. For Model 2 and 3, both polynomials of order 1 and of order 2 are used for the local-STLS.

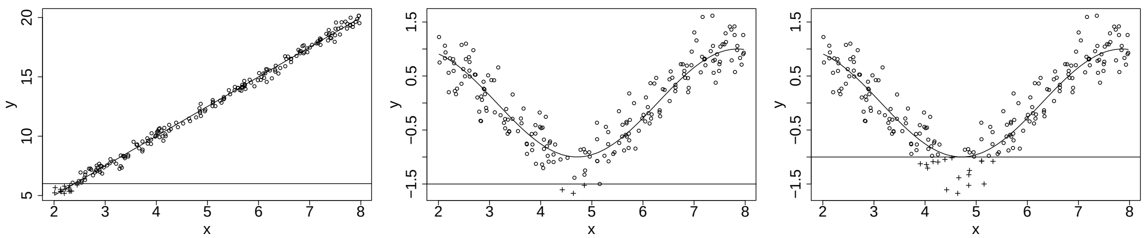

Figure 1: Simulated data ($n^* = 200$) from Models 1-3 (from left to right). The horizontal lines indicate the truncation points for each model.

## 4.1 Results

For Model 1 under truncation, the average MSE and average bias at the 13 focal values are found in Tables 1 and 2, respectively. All tables are found in the Appendix. The average MSE of the local-STLS and local-QME at all focal values decreases when $n^*$ increases from 500 to 1000. The absolute value of the average bias of the local-STLS and local-QME also decreases at most, but not all, focal values when $n^*$ increases. The local-STLS and local-QME with $\delta = 2\sigma$ perform best in terms of bias and MSE for all the bandwidths, $h$, considered. Compared to the ordinary STLS and QME estimators for linear truncated regression models the bias and MSE of the local-STLS and local-QME are higher, as expected, since Model 1 is a linear model.

Figure 2 illustrates results from Model 2 (average over 1000 samples of size $n^* = 1000$) on the estimated functions (left panel) and their corresponding bias (right panel) using the local-STLS estimator, the local-QME estimator, and LPR, with bandwidth $h = 2$. For the local-QME $\delta = 1\sigma$ is used. In the upper panel a first order polynomial is used to approximate $m(x)$ locally for all three estimators and in the lower panel a second order polynomial is used. The estimators using a second order polynomial are denoted local-STLS2, local-QME2 and LPR2. We can note that the local-STLS and local-QME perform better than LPR. When using a second order polynomial the differences between the estimated curves are smaller, but the LPR2 still overestimates the true curve $m(x)$. This is most obvious at focal values where truncation occur more often. All of the estimated curves using second order polynomials are closer to the true curve (less bias) than those using first order, especially where the function $m(x)$

changes direction.

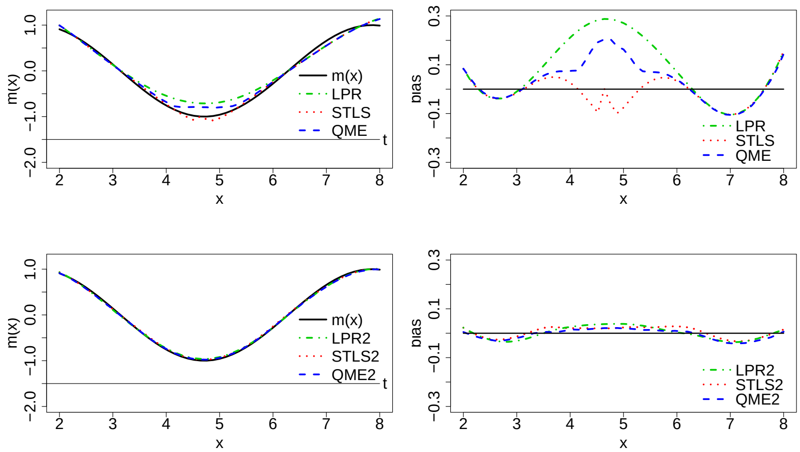


Figure 2: Estimates of $m(x)$ (left panel) and bias (right panel) using local-STLS (dotted), local-QME (dashed), and LPR (dashed and dotted) with 1st order polynomials (upper panel) and 2nd order polynomials (lower panel), $h = 2$, and $\delta = 1\sigma$. Average over 1000 samples of size $n^* = 1000$ from Model 2 under truncation.

Table 3 displays the average MSE of the estimators LPR, local-STLS, local-STLS2, and local-QME for Model 2. The MSE of local-STLS, local-STLS2 and local-QME decreases (or is more or less unchanged) when $n^*$ increases. For focal values between 3.5 and 6.0, MSE is large for LPR due to the truncation, while the other estimators perform well. The results for Model 3 (presented in Table 4) are similar to those for Model 2, with larger MSEs where truncation is more important.

Finally, in simulation experiments (not presented here), local-SCLS behaved similarly under censoring than local-STLS under truncation.

# 5 Discussion

Local polynomial regression is a popular non-parametric regression technique due to its nice properties (local linear regression is for instance more performant at the edges than simple kernel regression) and to its conceptual simplicity which makes it easy to communicate to non-specialists. We have introduced in this paper two versions (local-STLS and local-QME) of a local polynomial regression estimator which are able to deal with truncated or censored outcomes. The asymptotic normal distribution together with an explicit expression for the variance are given, thereby allowing for the construction of confidence bands.

The small sample properties of the estimators are studied in a simulation study. As usual with non-parametric estimators the choice of the bandwidth balances bias and variability. Our results indicate that one should consider adaptive bandwidths, i.e., using smaller bandwidth when the observations fall near the trunction (censoring) point. In situations where an automatic bandwith selection is needed one may consider cross-validation, although more work is needed to adapt the latter out-of-sample validation method to truncated outcomes.

Local-QME has an extra tunning parameter. By varying the latter, local-QME could not outperform local-STLS in the simulated situations. On the other hand, local-QME does not require symmetrically distributed errors when estimating the derivative of the objective function, and has therefore wider applicability.

# References

Chen, S., Dahl, G. B., Kahn, S. (2005), Nonparametric identification and estimation of a censored location-scale regression model, *Journal of the American Statistical Association*, **100**, 212–221.

Fan, J., Gijbels, I. (1996), *Local polynomial modelling and its applications.* Chapman & Hall.

Karlsson, M. (2004), Finite Sample Properties of the QME, *Communications in Statistics - Simulation and Computation*, **33**, 567–583.

Laitila, T. (2001), Properties of the QME under asymmetrically distributed disturbances, *Statistics & Probability Letters* **52**, 347–352.

Lee, M. J. (1993), Quadratic mode regression, *Journal of Econometrics* **57**, 1–19.

Lee, M. J., Kim, H. (1998), Semiparametric econometric estimation for a truncated regression model: A review with an extension, *Statistica Neerlandica* **52**, 200–225.

Lewbel, A., Linton, O. (2002), Nonparametric censored and truncated regression, *Econometrica* **70**, 765–779.

Powell, J. L. (1986), Symmetrically trimmed least squares estimation for tobit models, *Econometrica* **54**, 1435–1460.

## Appendix: Tables

Table 1: Average MSE at focal values in 1000 replicates of Model 1 under truncation.

| focal values, $x_0$ <br> $P(y^* \leq t \| x = x_0)$ | 2.0 <br> (0.99) | 2.5 <br> (0.20) | 3.0 <br> (0.00) | 3.5 <br> (0.0) | 4.0 <br> (0.00) | 4.5 <br> (0.00) | 5.0 <br> (0.00) | 5.5 <br> (0.00) | 6.0 <br> (0.00) | 6.5 <br> (0.00) | 7.0 <br> (0.00) | 7.5 <br> (0.00) | 8.0 <br> (0.00) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $n^* = 500$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0596 | 0.0123 | 0.0028 | 0.0008 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0012 | 0.0035 |
| local-STLS | 0.0174 | 0.0041 | 0.0013 | 0.0006 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0012 | 0.0035 |
| local-QME, $\delta = 0.5\sigma$ | 0.0635 | 0.0205 | 0.0092 | 0.0067 | 0.0064 | 0.0058 | 0.0061 | 0.0060 | 0.0061 | 0.0064 | 0.0075 | 0.0104 | 0.0193 |
| local-QME, $\delta = 1.0\sigma$ | 0.0512 | 0.0131 | 0.0046 | 0.0024 | 0.0020 | 0.0019 | 0.0019 | 0.0017 | 0.0018 | 0.0019 | 0.0026 | 0.0047 | 0.0123 |
| local-QME, $\delta = 2.0\sigma$ | 0.0265 | 0.0065 | 0.0019 | 0.0009 | 0.0006 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0007 | 0.0015 | 0.0046 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0222 | 0.0073 | 0.0026 | 0.0010 | 0.0005 | 0.0003 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0005 | 0.0010 | 0.0023 |
| local-STLS | 0.0064 | 0.0025 | 0.0011 | 0.0006 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0005 | 0.0010 | 0.0023 |
| local-QME, $\delta = 0.5\sigma$ | 0.0377 | 0.0175 | 0.0096 | 0.0069 | 0.0051 | 0.0047 | 0.0045 | 0.0046 | 0.0051 | 0.0058 | 0.0072 | 0.0104 | 0.0162 |
| local-QME, $\delta = 1.0\sigma$ | 0.0233 | 0.0094 | 0.0044 | 0.0025 | 0.0017 | 0.0013 | 0.0012 | 0.0012 | 0.0013 | 0.0016 | 0.0024 | 0.0044 | 0.0088 |
| local-QME, $\delta = 2.0\sigma$ | 0.0099 | 0.0037 | 0.0016 | 0.0008 | 0.0005 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0004 | 0.0007 | 0.0013 | 0.0030 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0064 | 0.0033 | 0.0018 | 0.0010 | 0.0006 | 0.0004 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0005 | 0.0008 | 0.0013 |
| local-STLS | 0.0023 | 0.0013 | 0.0008 | 0.0005 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0005 | 0.0008 | 0.0013 |
| local-QME, $\delta = 0.5\sigma$ | 0.0227 | 0.0150 | 0.0100 | 0.0070 | 0.0052 | 0.0043 | 0.0037 | 0.0037 | 0.0044 | 0.0055 | 0.0072 | 0.0099 | 0.0137 |
| local-QME, $\delta = 1.0\sigma$ | 0.0094 | 0.0056 | 0.0035 | 0.0023 | 0.0016 | 0.0012 | 0.0010 | 0.0010 | 0.0012 | 0.0016 | 0.0022 | 0.0034 | 0.0055 |
| local-QME, $\delta = 2.0\sigma$ | 0.0033 | 0.0019 | 0.0011 | 0.0007 | 0.0005 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0004 | 0.0006 | 0.0009 | 0.0016 |
| | | | | | | | | | | | | | |
| STLS | 0.0012 | 0.0010 | 0.0007 | 0.0005 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0005 | 0.0007 | 0.0009 |
| QME, $\delta = 0.5\sigma$ | 0.0161 | 0.0125 | 0.0096 | 0.0072 | 0.0055 | 0.0043 | 0.0037 | 0.0038 | 0.0044 | 0.0057 | 0.0075 | 0.0100 | 0.0130 |
| QME, $\delta = 1.0\sigma$ | 0.0055 | 0 .0042 | 0.0032 | 0.0023 | 0.0017 | 0.0012 | 0.0010 | 0.0010 | 0.0011 | 0.0015 | 0.0021 | 0.0029 | 0.0039 |
| QME, $\delta = 2.0\sigma$ | 0.0017 | 0.0013 | 0.0010 | 0.0007 | 0.0005 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0004 | 0.0006 | 0.0008 | 0.0011 |
| | | | | | | | $n^* = 1000$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0504 | 0.0090 | 0.0018 | 0.0004 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0006 | 0.0018 |
| local-STLS | 0.0073 | 0.0017 | 0.0006 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0006 | 0.0018 |
| local-QME, $\delta = 0.5\sigma$ | 0.0479 | 0.0152 | 0.0064 | 0.0043 | 0.0037 | 0.0038 | 0.0037 | 0.0037 | 0.0039 | 0.0038 | 0.0044 | 0.0071 | 0.0140 |
| local-QME, $\delta = 1.0\sigma$ | 0.0272 | 0.0069 | 0.0024 | 0.0013 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0010 | 0.0010 | 0.0013 | 0.0027 | 0.0077 |
| local-QME, $\delta = 2.0\sigma$ | 0.0122 | 0.0027 | 0.0009 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0004 | 0.0008 | 0.0024 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0164 | 0.0051 | 0.0017 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0003 | 0.0005 | 0.0011 |
| local-STLS | 0.0027 | 0.0011 | 0.0005 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0003 | 0.0005 | 0.0011 |
| local-QME, $\delta = 0.5\sigma$ | 0.0276 | 0.0120 | 0.0065 | 0.0041 | 0.0032 | 0.0029 | 0.0027 | 0.0027 | 0.0029 | 0.0034 | 0.0043 | 0.0065 | 0.0115 |
| local-QME, $\delta = 1.0\sigma$ | 0.0117 | 0.0046 | 0.0023 | 0.0013 | 0.0009 | 0.0007 | 0.0006 | 0.0006 | 0.0007 | 0.0009 | 0.0013 | 0.0024 | 0.0053 |
| local-QME, $\delta = 2.0\sigma$ | 0.0042 | 0.0016 | 0.0007 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0004 | 0.0007 | 0.0015 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0043 | 0.0022 | 0.0012 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0004 | 0.0006 |
| local-STLS | 0.0010 | 0.0006 | 0.0004 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0004 | 0.0006 |
| local-QME, $\delta = 0.5\sigma$ | 0.0146 | 0.0092 | 0.0064 | 0.0044 | 0.0032 | 0.0025 | 0.0022 | 0.0022 | 0.0026 | 0.0033 | 0.0046 | 0.0064 | 0.0093 |
| local-QME, $\delta = 1.0\sigma$ | 0.0049 | 0.0029 | 0.0018 | 0.0012 | 0.0009 | 0.0006 | 0.0005 | 0.0005 | 0.0006 | 0.0008 | 0.0012 | 0.0019 | 0.0032 |
| local-QME, $\delta = 2.0\sigma$ | 0.0015 | 0.0009 | 0.0006 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0005 | 0.0009 |
| | | | | | | | | | | | | | |
| STLS | 0.0006 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0003 | 0.0004 |
| QME, $\delta = 0.5\sigma$ | 0.0102 | 0.0079 | 0.0060 | 0.0044 | 0.0033 | 0.0025 | 0.0022 | 0.0022 | 0.0026 | 0.0034 | 0.0046 | 0.0061 | 0.0081 |
| QME, $\delta = 1.0\sigma$ | 0.0028 | 0.0022 | 0.0016 | 0.0012 | 0.0009 | 0.0006 | 0.0005 | 0.0005 | 0.0006 | 0.0008 | 0.0011 | 0.0016 | 0.0021 |
| QME, $\delta = 2.0\sigma$ | 0.0008 | 0.0006 | 0.0005 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0004 | 0.0006 |

Table 2: Average bias at focal values in 1000 replicates of Model 1 under truncation.

| focal values, $x_0$ | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P(y^* \le t \mid x = x_0)$ | (0.99) | (0.20) | (0.00) | (0.0) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| | | | | | | | $n^* = 500$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.2343 | 0.1026 | 0.0458 | 0.0161 | 0.0027 | 0.0002 | 0.0003 | 0.0001 | 0.0000 | 0.0003 | 0.0004 | 0.0004 | 0.0009 |
| local-STLS | -0.0005 | -0.0008 | -0.0001 | 0.0005 | 0.0004 | 0.0002 | 0.0003 | 0.0001 | 0.0000 | 0.0003 | 0.0004 | 0.0004 | 0.0009 |
| local-QME, $\delta = 0.5\sigma$ | 0.1404 | 0.0611 | 0.0257 | 0.0086 | 0.0029 | 0.0037 | 0.0023 | 0.0012 | 0.0009 | -0.0008 | 0.0001 | -0.0040 | -0.0052 |
| local-QME, $\delta = 1.0\sigma$ | 0.0260 | 0.0104 | 0.0020 | 0.0006 | 0.0006 | 0.0010 | 0.0017 | 0.0012 | 0.0008 | -0.0000 | 0.0002 | 0.0007 | 0.0007 |
| local-QME, $\delta = 2.0\sigma$ | -0.0077 | -0.0038 | -0.0187 | -0.0007 | -0.0006 | -0.0005 | -0.0001 | 0.0001 | 0.0003 | 0.0007 | 0.0006 | 0.0000 | 0.0004 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.1380 | 0.0774 | 0.0436 | 0.0236 | 0.0115 | 0.0044 | 0.0009 | 0.0002 | 0.0002 | 0.0003 | 0.0004 | 0.0004 | 0.0003 |
| local-STLS | -0.0015 | -0.0001 | 0.0006 | 0.0005 | 0.0004 | 0.0004 | 0.0003 | 0.0002 | 0.0002 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| local-QME, $\delta = 0.5\sigma$ | 0.0852 | 0.0491 | 0.0284 | 0.0157 | 0.0071 | 0.0045 | 0.0037 | 0.0016 | 0.0014 | -0.0016 | -0.0038 | -0.0066 | -0.0071 |
| local-QME, $\delta = 1.0\sigma$ | 0.0012 | 0.0029 | -0.0001 | -0.0012 | -0.0004 | 0.0006 | 0.0008 | 0.0010 | 0.0008 | 0.0004 | -0.0007 | 0.0001 | 0.0001 |
| local-QME, $\delta = 2.0\sigma$ | -0.0048 | -0.0024 | -0.0009 | -0.0008 | -0.0007 | -0.0004 | -0.0000 | 0.0001 | 0.0002 | 0.0005 | 0.0006 | 0.0004 | -0.0004 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0696 | 0.0478 | 0.0338 | 0.0243 | 0.0172 | 0.0115 | 0.0068 | 0.0032 | 0.0008 | -0.0001 | 0.0001 | 0.0003 | 0.0005 |
| local-STLS | 0.0009 | 0.0008 | 0.0006 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0005 |
| local-QME, $\delta = 0.5\sigma$ | 0.0456 | 0.0289 | 0.0205 | 0.0140 | 0.0103 | 0.0076 | 0.0037 | 0.0022 | -0.0040 | -0.0048 | -0.0068 | -0.0134 | -0.0124 |
| local-QME, $\delta = 1.0\sigma$ | -0.0039 | -0.0037 | -0.0021 | -0.0007 | -0.0007 | -0.0000 | 0.0002 | 0.0003 | 0.0004 | 0.0004 | 0.0002 | -0.0008 | -0.0015 |
| local-QME, $\delta = 2.0\sigma$ | -0.0014 | -0.0011 | -0.0009 | -0.0006 | -0.0004 | -0.0002 | -0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0005 | 0.0007 | 0.0008 |
| | | | | | | | | | | | | | |
| STLS | 0.0006 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0003 | 0.0003 | 0.0003 | 0.0002 | 0.0002 | 0.0002 |
| QME, $\delta = 0.5\sigma$ | 0.0228 | 0.0194 | 0.0161 | 0.0128 | 0.0095 | 0.0062 | 0.0028 | -0.0005 | -0.0038 | -0.0071 | -0.0104 | -0.0138 | -0.0171 |
| QME, $\delta = 1.0\sigma$ | -0.0015 | -0.0012 | -0.0010 | -0.0008 | -0.0005 | -0.0003 | -0.0000 | 0.0002 | 0.0005 | 0.0007 | 0.0010 | 0.0012 | 0.0015 |
| QME, $\delta = 2.0\sigma$ | -0.0011 | -0.0009 | -0.0008 | 0.0006 | -0.0004 | -0.0003 | -0.0001 | 0.0001 | 0.0003 | 0.0004 | 0.0006 | 0.0008 | 0.0009 |
| | | | | | | | $n^* = 1000$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.2179 | 0.0902 | 0.0388 | 0.0133 | 0.0022 | 0.0032 | 0.0006 | 0.0009 | 0.0009 | 0.0005 | -0.0002 | -0.0011 | -0.0023 |
| local-STLS | -0.0034 | -0.0011 | -0.0006 | -0.0005 | -0.0001 | 0.0003 | 0.0006 | 0.0009 | 0.0009 | 0.0005 | -0.0002 | -0.0011 | -0.0023 |
| local-QME, $\delta = 0.5\sigma$ | 0.1039 | 0.0423 | 0.0173 | 0.0041 | 0.0007 | 0.0013 | 0.0010 | 0.0001 | -0.0009 | -0.0041 | -0.0026 | -0.0027 | -0.0018 |
| local-QME, $\delta = 1.0\sigma$ | 0.0091 | 0.0008 | -0.0005 | 0.0003 | 0.0002 | 0.0008 | 0.0005 | 0.0009 | 0.0008 | 0.0006 | 0.0003 | -0.0009 | -0.0017 |
| local-QME, $\delta = 2.0\sigma$ | -0.0086 | -0.0035 | -0.0014 | -0.0008 | -0.0002 | 0.0005 | 0.0009 | 0.0012 | 0.0012 | 0.0006 | -0.0001 | -0.0008 | -0.0018 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.1217 | 0.0666 | 0.0374 | 0.0205 | 0.104 | 0.0042 | 0.0010 | 0.0006 | 0.0005 | 0.0003 | -0.0002 | -0.0010 | -0.0019 |
| local-STLS | -0.0011 | -0.0009 | -0.0007 | -0.0004 | -0.0000 | 0.0003 | 0.0005 | 0.0006 | 0.0005 | 0.0003 | -0.0003 | -0.0010 | -0.0019 |
| local-QME, $\delta = 0.5\sigma$ | 0.0060 | 0.0030 | 0.0013 | 0.0041 | 0.0011 | 0.0019 | 0.0000 | -0.0013 | -0.0014 | -0.0037 | -0.0055 | -0.0071 | -0.0071 |
| local-QME, $\delta = 1.0\sigma$ | -0.0005 | -0.0013 | -0.0008 | -0.0001 | 0.0001 | 0.0004 | 0.0007 | 0.0010 | 0.0009 | 0.0005 | 0.0003 | -0.0005 | -0.0018 |
| local-QME, $\delta = 2.0\sigma$ | -0.0037 | -0.0020 | -0.0013 | -0.0082 | - 0.0002 | 0.0004 | 0.0006 | 0.0007 | 0.0007 | 0.0004 | -0.0002 | -0.0010 | -0.0017 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0599 | 0.0416 | 0.0299 | 0.0216 | 0.0154 | 0.0103 | 0.0061 | 0.0029 | 0.0007 | -0.0002 | -0.0001 | -0.0003 | -0.0009 |
| local-STLS | -0.0014 | -0.0010 | -0.0004 | -0.0003 | -0.0000 | 0.0001 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0001 | -0.0003 | -0.0009 |
| local-QME, $\delta = 0.5\sigma$ | 0.0162 | 0.0067 | 0.0036 | 0.0029 | 0.0038 | 0.0024 | -0.0007 | -0.0020 | -0.0027 | -0.0039 | -0.0056 | -0.0079 | -0.0104 |
| local-QME, $\delta = 1.0\sigma$ | -0.0012 | -0.0005 | -0.0005 | 0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0006 | 0.0007 | 0.0008 | 0.0006 | 0.0002 | -0.0002 |
| local-QME, $\delta = 2.0\sigma$ | -0.0026 | -0.0019 | -0.0011 | -0.0005 | -0.0001 | 0.0001 | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0001 | -0.0003 | -0.0010 |
| | | | | | | | | | | | | | |
| STLS | -0.0000 | -0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0003 | 0.0003 |
| QME, $\delta = 0.5\sigma$ | 0.0080 | 0.0066 | 0.0051 | 0.0037 | 0.0023 | 0.0009 | -0.0005 | -0.0020 | -0.0034 | -0.0048 | -0.0062 | -0.0076 | -0.0090 |
| QME, $\delta = 1.0\sigma$ | -0.0003 | -0.0002 | -0.0001 | 0.0000 | 0.0001 | 0.0002 | 0.0003 | 0.0004 | 0.0005 | 0.0006 | 0.0007 | 0.0008 | 0.0009 |
| QME, $\delta = 2.0\sigma$ | -0.0001 | -0.0001 | -0.0001 | -0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0004 | 0.0004 |

Table 3: Average MSE at focal values in 1000 replicates of Model 2 under truncation.

| focal values, $x_0$<br>$P(y^* \le t \mid x = x_0)$ | 2.0<br>(0.00) | 2.5<br>(0.00) | 3.0<br>(0.00) | 3.5<br>(0.00) | 4.0<br>(0.01) | 4.5<br>(0.04) | 5.0<br>(0.04) | 5.5<br>(0.00) | 6.0<br>(0.00) | 6.5<br>(0.00) | 7.0<br>(0.00) | 7.5<br>(0.00) | 8.0<br>(0.00) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $n^* = 500$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0112 | 0.0018 | 0.0006 | 0.0082 | 0.0414 | 0.0797 | 0.0804 | 0.0394 | 0.0059 | 0.0021 | 0.0094 | 0.0020 | 0.0332 |
| local-STLS | 0.0112 | 0.0018 | 0.0008 | 0.0023 | 0.0030 | 0.0193 | 0.0180 | 0.0026 | 0.0018 | 0.0023 | 0.0094 | 0.0020 | 0.0332 |
| local-STLS2 | 0.0077 | 0.0014 | 0.0016 | 0.0013 | 0.0014 | 0.0015 | 0.0016 | 0.0013 | 0.0018 | 0.0010 | 0.0017 | 0.0015 | 0.0078 |
| local-QME, $\delta = 0.5\sigma$ | 0.0344 | 0.1076 | 0.0064 | 0.0132 | 0.0388 | 0.0699 | 0.0653 | 0.0334 | 0.0106 | 0.0067 | 0.0120 | 0.0109 | 0.0654 |
| local-QME, $\delta = 1.0\sigma$ | 0.0202 | 0.0055 | 0.0027 | 0.0058 | 0.0144 | 0.0470 | 0.0419 | 0.0114 | 0.0045 | 0.0037 | 0.0107 | 0.0069 | 0.0456 |
| local-QME, $\delta = 2.0\sigma$ | 0.0122 | 0.0021 | 0.0011 | 0.0016 | 0.0049 | 0.0720 | 0.0492 | 0.0030 | 0.0015 | 0.0024 | 0.0090 | 0.0025 | 0.0345 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0094 | 0.0039 | 0.0005 | 0.0315 | 0.1519 | 0.2779 | 0.2730 | 0.1494 | 0.0273 | 0.0014 | 0.0119 | 0.0010 | 0.0854 |
| local-STLS | 0.0118 | 0.0021 | 0.0008 | 0.0027 | 0.0024 | 0.0383 | 0.0246 | 0.0035 | 0.0021 | 0.0033 | 0.0113 | 0.0010 | 0.0854 |
| local-STLS2 | 0.0113 | 0.0022 | 0.0036 | 0.0033 | 0.0015 | 0.0082 | 0.0088 | 0.0041 | 0.0045 | 0.0010 | 0.0033 | 0.0019 | 0.0116 |
| local-QME, $\delta = 0.5\sigma$ | 0.0354 | 0.0112 | 0.0068 | 0.0287 | 0.1143 | 0.1773 | 0.1749 | 0.0737 | 0.0151 | 0.0069 | 0.0142 | 0.0140 | 0.1286 |
| local-QME, $\delta = 1.0\sigma$ | 0.0197 | 0.0061 | 0.0027 | 0.0069 | 0.0218 | 0.0822 | 0.0605 | 0.0143 | 0.0056 | 0.0045 | 0.0127 | 0.0069 | 0.1033 |
| local-QME, $\delta = 2.0\sigma$ | 0.0137 | 0.0022 | 0.0012 | 0.0017 | 0.0034 | 0.0959 | 0.0433 | 0.0022 | 0.0016 | 0.0036 | 0.0107 | 0.0017 | 0.0885 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0707 | 0.1108 | 0.0165 | 0.0525 | 0.3403 | 0.6461 | 0.6661 | 0.3860 | 0.0857 | 0.0020 | 0.0532 | 0.0292 | 0.0407 |
| local-STLS | 0.0094 | 0.0024 | 0.0007 | 0.0029 | 0.0024 | 0.2350 | 0.0090 | 0.0060 | 0.0024 | 0.0037 | 0.0121 | 0.0010 | 0.1175 |
| local-STLS2 | 0.0957 | 0.0031 | 0.0247 | 0.0059 | 0.0068 | 0.0365 | 0.0350 | 0.0049 | 0.0113 | 0.0519 | 0.0397 | 0.0020 | 0.0270 |
| local-QME, $\delta = 0.5\sigma$ | 0.0216 | 0.0200 | 0.0118 | 0.0677 | 0.2920 | 0.5136 | 0.3978 | 0.1593 | 0.0250 | 0.0097 | 0.0319 | 0.0143 | 0.1138 |
| local-QME, $\delta = 1.0\sigma$ | 0.0152 | 0.0070 | 0.0027 | 0.0218 | 0.2746 | 0.2867 | 0.0928 | 0.0254 | 0.0065 | 0.0049 | 0.0148 | 0.0070 | 0.1304 |
| local-QME, $\delta = 2.0\sigma$ | 0.0136 | 0.0022 | 0.0012 | 0.0017 | 0.4558 | 0.2866 | 0.0161 | 0.0039 | 0.0017 | 0.0041 | 0.0112 | 0.0018 | 0.1299 |
| | | | | | | | $n^* = 1000$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0083 | 0.0016 | 0.0004 | 0.0092 | 0.0451 | 0.0803 | 0.0742 | 0.0363 | 0.0053 | 0.0023 | 0.0110 | 0.0021 | 0.0264 |
| local-STLS | 0.0083 | 0.0016 | 0.0005 | 0.0022 | 0.0019 | 0.0131 | 0.0101 | 0.0019 | 0.0015 | 0.0026 | 0.0110 | 0.0021 | 0.0264 |
| local-STLS2 | 0.0034 | 0.0011 | 0.0009 | 0.0009 | 0.0009 | 0.0010 | 0.0011 | 0.0009 | 0.0013 | 0.0005 | 0.0013 | 0.0010 | 0.0043 |
| local-QME, $\delta = 0.5\sigma$ | 0.0215 | 0.0077 | 0.0042 | 0.0113 | 0.0349 | 0.0592 | 0.0515 | 0.0253 | 0.0071 | 0.0053 | 0.0110 | 0.0077 | 0.0532 |
| local-QME, $\delta = 1.0\sigma$ | 0.0129 | 0.0034 | 0.0014 | 0.0048 | 0.0100 | 0.0398 | 0.0322 | 0.0079 | 0.0033 | 0.0031 | 0.0108 | 0.0045 | 0.0364 |
| local-QME, $\delta = 2.0\sigma$ | 0.0087 | 0.0017 | 0.0007 | 0.0012 | 0.0028 | 0.0603 | 0.0398 | 0.0015 | 0.0010 | 0.0026 | 0.0105 | 0.0022 | 0.0275 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0077 | 0.0036 | 0.0002 | 0.0358 | 0.1624 | 0.2804 | 0.2599 | 0.1343 | 0.0232 | 0.0018 | 0.0139 | 0.0007 | 0.0761 |
| local-STLS | 0.0096 | 0.0019 | 0.0005 | 0.0026 | 0.0015 | 0.0337 | 0.0205 | 0.0031 | 0.0018 | 0.0038 | 0.0133 | 0.0007 | 0.0761 |
| local-STLS2 | 0.0113 | 0.0022 | 0.0036 | 0.0033 | 0.0015 | 0.0082 | 0.0088 | 0.0041 | 0.0045 | 0.0010 | 0.0033 | 0.0019 | 0.0116 |
| local-QME, $\delta = 0.5\sigma$ | 0.0236 | 0.0086 | 0.0045 | 0.0241 | 0.0988 | 0.1504 | 0.0955 | 0.0445 | 0.0108 | 0.0056 | 0.0137 | 0.0087 | 0.1160 |
| local-QME, $\delta = 1.0\sigma$ | 0.0124 | 0.0039 | 0.0013 | 0.0059 | 0.0092 | 0.0686 | 0.0273 | 0.0086 | 0.0038 | 0.0040 | 0.0128 | 0.0036 | 0.0904 |
| local-QME, $\delta = 2.0\sigma$ | 0.0107 | 0.0018 | 0.0008 | 0.0013 | 0.0018 | 0.0821 | 0.0390 | 0.0014 | 0.0011 | 0.0041 | 0.0125 | 0.0009 | 0.0798 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0657 | 0.0962 | 0.0106 | 0.0614 | 0.3524 | 0.6505 | 0.6603 | 0.3746 | 0.0778 | 0.0033 | 0.0581 | 0.0300 | 0.0378 |
| local-STLS | 0.0075 | 0.0021 | 0.0004 | 0.0027 | 0.0016 | 0.0998 | 0.0066 | 0.0060 | 0.0020 | 0.0043 | 0.0142 | 0.0006 | 0.1041 |
| local-STLS2 | 0.0957 | 0.0031 | 0.0247 | 0.0059 | 0.0069 | 0.0365 | 0.0350 | 0.0049 | 0.0113 | 0.0519 | 0.0397 | 0.0020 | 0.0270 |
| local-QME, $\delta = 0.5\sigma$ | 0.0154 | 0.0124 | 0.0055 | 0.0601 | 0.2851 | 0.4228 | 0.1937 | 0.0651 | 0.0128 | 0.0071 | 0.0268 | 0.0114 | 0.1092 |
| local-QME, $\delta = 1.0\sigma$ | 0.0090 | 0.0047 | 0.0013 | 0.0071 | 0.0995 | 0.2072 | 0.0149 | 0.0110 | 0.0042 | 0.0044 | 0.0143 | 0.0037 | 0.1173 |
| local-QME, $\delta = 2.0\sigma$ | 0.0108 | 0.0018 | 0.0008 | 0.0013 | 0.0139 | 0.2733 | 0.0135 | 0.0033 | 0.0012 | 0.0048 | 0.0131 | 0.0008 | 0.1161 |

Table 4: Average MSE at focal values in 1000 replicates of Model 3 under truncation.

| focal values, $x_0$ <br> $P(y^* \leq t \mid x = x_0)$ | 2.0 <br> (0.00) | 2.5 <br> (0.00) | 3.0 <br> (0.00) | 3.5 <br> (0.02) | 4.0 <br> (0.21) | 4.5 <br> (0.47) | 5.0 <br> (0.45) | 5.5 <br> (0.16) | 6.0 <br> (0.01) | 6.5 <br> (0.00) | 7.0 <br> (0.00) | 7.5 <br> (0.00) | 8.0 <br> (0.00) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $n^* = 500$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0107 | 0.0019 | 0.0005 | 0.0177 | 0.0899 | 0.1797 | 0.1789 | 0.0812 | 0.0123 | 0.0010 | 0.0090 | 0.0020 | 0.0332 |
| local-STLS | 0.0111 | 0.0018 | 0.0011 | 0.0018 | 0.0096 | 0.1360 | 0.0895 | 0.0061 | 0.0017 | 0.0019 | 0.0091 | 0.0020 | 0.0332 |
| local-STLS2 | 0.0080 | 0.0013 | 0.0016 | 0.0023 | 0.0215 | 0.0085 | 0.0093 | 0.0204 | 0.0024 | 0.0013 | 0.0013 | 0.0015 | 0.0078 |
| local-QME, $\delta = 0.5\sigma$ | 0.0319 | 0.0091 | 0.0075 | 0.0104 | 0.0238 | 0.0904 | 0.0808 | 0.0316 | 0.0108 | 0.0092 | 0.0172 | 0.0129 | 0.0464 |
| local-QME, $\delta = 1.0\sigma$ | 0.0209 | 0.0051 | 0.0035 | 0.0050 | 0.0216 | 0.1691 | 0.1153 | 0.0173 | 0.0049 | 0.0041 | 0.0114 | 0.0069 | 0.0420 |
| local-QME, $\delta = 2.0\sigma$ | 0.0122 | 0.0021 | 0.0014 | 0.0030 | 0.0187 | 0.2472 | 0.1522 | 0.0117 | 0.0032 | 0.0015 | 0.0087 | 0.0025 | 0.0342 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0068 | 0.0040 | 0.0008 | 0.0597 | 0.2515 | 0.4425 | 0.4308 | 0.2407 | 0.0514 | 0.0004 | 0.0111 | 0.0012 | 0.0800 |
| local-STLS | 0.0131 | 0.0018 | 0.0011 | 0.0018 | 0.0049 | 0.0924 | 0.0500 | 0.0032 | 0.0030 | 0.0025 | 0.0108 | 0.0011 | 0.0805 |
| local-STLS2 | 0.0081 | 0.0013 | 0.0016 | 0.0025 | 0.0144 | 0.0322 | 0.0365 | 0.0275 | 0.0033 | 0.0015 | 0.0014 | 0.0016 | 0.0088 |
| local-QME, $\delta = 0.5\sigma$ | 0.0281 | 0.0107 | 0.0084 | 0.0109 | 0.0267 | 0.0891 | 0.2972 | 0.0546 | 0.0173 | 0.0098 | 0.0221 | 0.0120 | 0.0933 |
| local-QME, $\delta = 1.0\sigma$ | 0.0218 | 0.0056 | 0.0035 | 0.0050 | 0.0143 | 0.1164 | 0.0879 | 0.0092 | 0.0045 | 0.0051 | 0.0135 | 0.0067 | 0.0947 |
| local-QME, $\delta = 2.0\sigma$ | 0.0140 | 0.0022 | 0.0014 | 0.0029 | 0.0079 | 0.1036 | 0.0525 | 0.0072 | 0.0060 | 0.0017 | 0.0102 | 0.0017 | 0.0815 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0782 | 0.0899 | 0.0026 | 0.1166 | 0.5052 | 0.8729 | 0.8886 | 0.5437 | 0.1516 | 0.0010 | 0.0419 | 0.0324 | 0.0294 |
| local-STLS | 0.0137 | 0.0018 | 0.0011 | 0.0019 | 0.0041 | 0.0738 | 0.2954 | 0.0086 | 0.0038 | 0.0028 | 0.0114 | 0.0011 | 0.1086 |
| local-STLS2 | 0.0262 | 0.0042 | 0.0153 | 0.0004 | 0.0317 | 0.0910 | 0.0908 | 0.0311 | 0.0006 | 0.0292 | 0.0015 | 0.0017 | 0.0114 |
| local-QME, $\delta = 0.5\sigma$ | 0.0341 | 0.0580 | 0.0503 | 0.0538 | 0.2550 | 0.7571 | 1.0556 | 0.6362 | 0.2310 | 0.0424 | 0.0335 | 0.0254 | 0.0766 |
| local-QME, $\delta = 1.0\sigma$ | 0.0222 | 0.0057 | 0.0035 | 0.0062 | 0.0565 | 0.4057 | 0.6077 | 0.2127 | 0.0406 | 0.0065 | 0.0138 | 0.0082 | 0.1345 |
| local-QME, $\delta = 2.0\sigma$ | 0.0144 | 0.0022 | 0.0014 | 0.0030 | 0.0064 | 0.0822 | 0.2944 | 0.0170 | 0.0078 | 0.0018 | 0.0106 | 0.0017 | 0.1007 |
| | | | | | | | $n^* = 1000$ | | | | | | |
| $h = 2$ | | | | | | | | | | | | | |
| LPR | 0.0079 | 0.0017 | 0.0003 | 0.0199 | 0.0972 | 0.1840 | 0.1697 | 0.0762 | 0.0124 | 0.0011 | 0.0105 | 0.0021 | 0.0264 |
| local-STLS | 0.0081 | 0.0016 | 0.0007 | 0.0011 | 0.0063 | 0.1106 | 0.0762 | 0.0038 | 0.0010 | 0.0021 | 0.0105 | 0.0021 | 0.0264 |
| local-STLS2 | 0.0034 | 0.0008 | 0.0008 | 0.0013 | 0.0174 | 0.0075 | 0.0084 | 0.0101 | 0.0013 | 0.0009 | 0.0008 | 0.0009 | 0.0043 |
| local-QME, $\delta = 0.5\sigma$ | 0.0233 | 0.0069 | 0.0052 | 0.0078 | 0.0179 | 0.0823 | 0.0590 | 0.0181 | 0.0073 | 0.0064 | 0.0147 | 0.0094 | 0.0399 |
| local-QME, $\delta = 1.0\sigma$ | 0.0132 | 0.0033 | 0.0019 | 0.0030 | 0.0131 | 0.1348 | 0.0912 | 0.0087 | 0.0025 | 0.0033 | 0.0110 | 0.0043 | 0.0340 |
| local-QME, $\delta = 2.0\sigma$ | 0.0086 | 0.0018 | 0.0008 | 0.0018 | 0.0124 | 0.1894 | 0.1300 | 0.0075 | 0.0017 | 0.0016 | 0.0100 | 0.0022 | 0.0273 |
| $h = 3$ | | | | | | | | | | | | | |
| LPR | 0.0051 | 0.0037 | 0.0009 | 0.0672 | 0.2676 | 0.4488 | 0.4185 | 0.2219 | 0.0450 | 0.0004 | 0.0128 | 0.0011 | 0.0714 |
| local-STLS | 0.0104 | 0.0016 | 0.0008 | 0.0013 | 0.0025 | 0.0829 | 0.0487 | 0.0023 | 0.0023 | 0.0030 | 0.0127 | 0.0009 | 0.0718 |
| local-STLS2 | 0.0038 | 0.0009 | 0.0008 | 0.0014 | 0.0157 | 0.0325 | 0.0357 | 0.0244 | 0.0015 | 0.0011 | 0.0009 | 0.0012 | 0.0054 |
| local-QME, $\delta = 0.5\sigma$ | 0.0227 | 0.0080 | 0.0059 | 0.0078 | 0.0163 | 0.0867 | 0.1956 | 0.0195 | 0.0088 | 0.0083 | 0.0201 | 0.0094 | 0.0881 |
| local-QME, $\delta = 1.0\sigma$ | 0.0153 | 0.0034 | 0.0020 | 0.0030 | 0.0078 | 0.0982 | 0.0593 | 0.0046 | 0.0026 | 0.0046 | 0.0130 | 0.0036 | 0.0866 |
| local-QME, $\delta = 2.0\sigma$ | 0.0104 | 0.0018 | 0.0009 | 0.0021 | 0.0042 | 0.0916 | 0.0504 | 0.0042 | 0.0042 | 0.0021 | 0.0118 | 0.0013 | 0.0742 |
| $h = 5$ | | | | | | | | | | | | | |
| LPR | 0.0731 | 0.0764 | 0.0007 | 0.1299 | 0.5218 | 0.8829 | 0.8882 | 0.5357 | 0.1436 | 0.0003 | 0.0461 | 0.0331 | 0.0274 |
| local-STLS | 0.0110 | 0.0016 | 0.0008 | 0.0014 | 0.0020 | 0.0635 | 0.5164 | 0.0079 | 0.0031 | 0.0034 | 0.0134 | 0.0007 | 0.0971 |
| local-STLS2 | 0.0113 | 0.0042 | 0.0145 | 0.0002 | 0.0341 | 0.0949 | 0.0930 | 0.0306 | 0.0007 | 0.0339 | 0.0010 | 0.0013 | 0.0075 |
| local-QME, $\delta = 0.5\sigma$ | 0.0210 | 0.0177 | 0.0123 | 0.0163 | 0.1130 | 0.5773 | 1.0219 | 0.6507 | 0.1950 | 0.0228 | 0.0226 | 0.0163 | 0.0898 |
| local-QME, $\delta = 1.0\sigma$ | 0.0159 | 0.0034 | 0.0020 | 0.0030 | 0.0102 | 0.2483 | 0.6560 | 0.2493 | 0.0134 | 0.0050 | 0.0134 | 0.0039 | 0.1244 |
| local-QME, $\delta = 2.0\sigma$ | 0.0109 | 0.0018 | 0.0009 | 0.0022 | 0.0034 | 0.0633 | 0.5050 | 0.0134 | 0.0057 | 0.0022 | 0.0124 | 0.0012 | 0.0930 |

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