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Contests with Size Effects

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Abstract

In this paper we analyze the structure of contest equilibria with a variable number of individuals. First we analyze a situation where the total prize depends on the number of agents and where every single agent faces opportunity costs of investing in the contest. Second we analyze a situation where the agents face a trade-off between productive and appropriative investments. Here, the number of agents may also influence the productivity of productive investments. It turns out that both types of contests may lead to opposing results concerning the optimal number of individuals depending on the strength of size effects. Whereas in the former case individual utility is u-shaped when the number of agents increases, the opposite holds true for the latter case. We discuss the implications of our findings for the case of anarchic societies and competition on markets.

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1 Introduction

In this paper we analyze the voluntary interaction of agents in economic environments that can be characterized as a contest. In a contest, each agent faces a trade-off between productive and appropriative activities, and individual behavior depends on the number of agents involved. A change in the size of the group has two effects on the individual perception of the economic environment. First, the individual impact on the outcome of the contest becomes smaller if the group size increases. Second, an increase in the group size may have an influence on the total amount of goods that can be distributed. These size effects can be decomposed as follows. We call any effect of group size on the total amount of goods *rent effect* and on the individual investments in appropriation *appropriation effect*. Rent effects are zero if the total amount of goods is independent of the size of the group. We are interested in the connection between group size and the structure of contest equilibria. Does an increase in the group size make the members of the group more or less aggressive? What group size maximizes individual, what group size aggregate utility? It turns out that the answers to these questions depend on the quantitative importance of rent effects as well as on the effectiveness of appropriative activities.

In addition to its general interest, the analysis of the influence of group size on the structure of equilibria in contests with size effects can be motivated by means of two examples. First, it is one of the most important results in economic theory that an increase in the number of economic agents interacting with each other is potentially welfare-improving because it allows for the specialization of individuals according to their comparative advantages. We capture this idea with our assumption that rent effects exist. These effects, however, only potentially increase aggregate utility and do not necessarily translate into actual well-being of the society. The actual gain depends on the ability of agents to impose transaction costs on other members of the society, and this ability may depend on technological as well as demographic variables. For example anthropologists argue that initially anarchic hunter-gatherer societies were confronted with an increasing potential for aggression due to an increase in population density (e.g., Diamond, 1997). The resulting increase in appropriative and defensive behavior made anarchic societies increasingly ineffective.[1] We use a model which incorporates

[1]Economists have recently started to analyze the extent of conflict and cooperation and the emergence of institutions in anarchic situations where unstable property rights create an impetus for the development of alternative ways to organize life, resulting in the emergence of chiefdoms, or more

both the positive and negative effects of an increasing population in order to gain deeper insight into the relationship between contest structure and population size. Our results show that an increase in population size does not necessarily lead to an increase in aggressiveness in the society. The development of aggressiveness rather depends on the magnitude of rent effects as well as on the relevant tradeoffs the individuals face.

Second, there exist markets which have the structure of a contest, even in modern societies with well-defined and enforced property rights. Competition for customers through advertising can create the formal analogy to the appropriation of goods in anarchy.[2] In addition, a market place often is the more attractive for customers the larger is the number of competitors supplying on this market place. When the total gross profit that can be earned is increasing in the number of competitors supplying on the market, then rent effects exist.

In order to analyze the consequences of group size on the outcome of the contest we discuss two generic forms of contests, both belonging to the class of 'common-pool' problems (Grossman, 2001). In the first contest, the total prize that can be distributed among agents is fixed and depends only on the group size. Investment in the appropriation of the prize, however, incurs a cost for the agents that can be thought of as resulting from a labor-leisure choice, or, more generally, opportunity costs of alternative uses of time. In the second contest, the total prize depends on the amount of time invested in productive activities as well as on the number of agents. The basic trade-off is between appropriative and productive activities.

Both contests differ with respect to the source of economic welfare as well as with respect to the magnitude of appropriable individual resources. In the first contest, an increase in the number of individuals has a positive impact on the total amount of goods that can be distributed, and there exists an individual resource ('leisure') that is not open to appropriation by other individuals. In the second contest, the number of agents has an influence on the marginal productivity of production, however, there are no goods to be distributed without productive investments by the individuals. The total time endowment of an agent has either to be devoted to productive activities,

generally hierarchic societies (for example Bush and Mayer, 1974; Hishleifer, 1995; Skaperdas, 1992; Grossman and Kim, 1995; Grossman, 2001, among others). We extend and complement this work to gain a better understanding of the economic mechanisms that cause the perceived inefficiency of anarchy.

[2]See Bell, Keeney, and Little (1975), Schmalensee (1976), and Monahan (1987).

which are used for the production of a good that can then be appropriated by other agents, or to appropriation.

This paper differs from the previous work on contests because it explicitly allows for size effects and extends the analysis of group size to the two different types of common-pool contests. Standard contest models either assume that the total prize is fixed and agents compete for the share they get (Nti, 1998), or that competitors can either invest in production, which increases the size of the rent, or in appropriation, which increases their share (Grossman and Kim, 1995; Skaperdas, 1992). Closest to our analysis is Hirshleifer (1995), who analyzes a variant of the first contest for the cases of absent and constant size effects.

The paper proceeds as follows: Section 2 presents the model. We derive the results in Section 3 and conclude in Section 4.

2 The model

Consider an anarchic society or group with $n \geq 2$ identical individuals indexed by i. We will analyze two different scenarios, one in which the individuals are endowed with a certain quantity of the consumption good and one where the consumption good has to be produced. Individuals compete for the final distribution in each scenario. The two types of contest share the property that the initial endowment of the consumption good (contest 1) or the individual production (contest 2) does not affect the ability to appropriate or defend. Hence, we analyze a common-pool contest (Grossman, 2001) where we do not have to distinguish between defensive and appropriative activities explicitly.

Our primary interest is to gain a better understanding of how the contest structure and the number of agents influence individual behavior in the contest. In order to do so we distinguish between two different tradeoffs the individuals face when making their decisions.

Contest 1: In the first specification we assume that the individuals can either engage in the appropriation of a rent of size R, or consume leisure, f_i.[3] The total rent R

[3]The term leisure is only a fill-in for any activity that generates utility and that is not challenged by other individuals. In a classical rent-seeking contest this activity can for example be the investment in a perfectly secure project with a return that is normalized to be equal to one.

depends on the number of individuals, n, in the following way: $R(n, g, Z) = n^g Z$, $g \in [0, 2]$,[4] where g measures the influence of the number of individuals on the total quantity of the rent, and $Z > 0$ is an arbitrary parameter. This specification generalizes the typical rent-seeking contest (Tullock, 1980) that has been applied to the analysis of anarchic situations by, among others, Grossman (2001).

Contest 2: In a second specification we assume that the individuals can either engage in the appropriation of a rent or in the production of this rent. The total quantity of the rent, R, is a function of productive investments l_i: $R(n, g, l_1, \ldots, l_n) = n^{(g-1)} \sum_{j=1}^{n} l_j$, $g \in [0, 2]$.[5] As before, g measures the effect of a change in the number of individuals on their ability to produce.[6]

Irrespective of the above specification, each individual can invest $a_i \geq 0$ units of time to appropriate part of the rent. The fraction p_i of the rent that is appropriated by individual i is given by the Tullock *contest-success function* (CSF) (Tullock, 1980),

$$p_i(a_1, \ldots, a_n) = \frac{a_i^b}{\sum_{j=1}^{n} a_j^b}, \tag{1}$$

where $b \in [0, 1]$ is the *decisiveness* parameter (Hirshleifer, 1995) of the CSF. From (1), investments do not change the fraction of the prize appropriated by competitors in the special case where $b \to 0$, and the marginal effectiveness of appropriation has a maximum if $b = 1$ (Hirshleifer, 1995). Thus, other things being equal larger values of b tend to increase the level of investment in the contest.

[4] The restriction to $g \in 0, 2$ captures all qualitatively relevant scenarios as will become clear in the course of the following discussion.

[5] With one exception, using a Cobb-Douglas specification would not change the qualitative nature of the results. The exception is that with a Cobb-Douglas technology, or more generally with a production technology that has the property $R(n, g, l_1, \ldots, l_n) = 0 \Leftrightarrow \exists l_i = 0$ a full-conflict equilibrium $l_1 = \ldots = l_n = 0$ always exists in addition to an interior equilibrium. For details see Skaperdas (1992).

[6] In order to have a similar interpretation of the parameter g for both contests we have normalized the power of n to be equal to $g - 1$ because, in contest 1, $g = 0$ implies that the total rent is independent of the number of individuals. With a linear production technology, potential production would be linear in the number of individuals if the power of n is equal to zero. A constant total potential production with respect to the number of individuals therefore requires a division by the factor n. A consequence of this normalization is that for $g < 1$ output is reduced if the same aggregate input is provided by a larger number of individuals. This case corresponds to situations where more individuals actually hinder each other in the production of goods.

We assume that individuals are risk neutral.[7] Consider the case where an increase in appropriation by one unit reduces leisure by the same amount, which gives rise to the time constraint $a_i + f_i = 1$. The objective function of an individual in the first contest can then be written as $u_i(\cdot) = p_i(\cdot)\, n^g Z - a_i$, and the objective function in the second contest is $u_i(\cdot) = p_i(\cdot)\, n^{(g-1)} \sum_{j=1}^{n}(1-a_j)$. We consider Nash equilibria of the games.

In contest 1 individual i chooses a_i to solve the following problem:

$$\max_{a_i} \quad u_i(a_1, \ldots, a_n) = p_i(a_1, \ldots, a_n)\, n^g Z - a_i, \tag{2}$$

and the first-order condition is:

$$\frac{\partial p_i}{\partial a_i} n^g Z = 1, \quad i = 1, \ldots, n. \tag{3}$$

Hence, at the individual optimum the marginal increase in the fraction of the rent appropriated by individual i (LHS) has to be equal to the marginal costs caused by an increase in a_i, which is equal to 1 (RHS).

In contest 2 individual i solves:

$$\max_{a_i} \quad u_i(a_1, \ldots, a_n) = p_i(a_1, \ldots, a_n)\, n^{(g-1)} \sum_{j=1}^{n}(1-a_j), \tag{4}$$

which yields the first-order condition:

$$\frac{\partial p_i}{\partial a_i} n^{(g-1)} \sum_{j=1}^{n} l_j = p_i n^{(g-1)}, \quad i = 1, \ldots, n. \tag{5}$$

The marginal increase in the appropriated rent (LHS) has to be equal to the marginal costs of appropriation (RHS). The marginal costs consist of the loss of total production times the fraction that is appropriated by the individual.

The simultaneous solution of the optimization problems gives rise to a unique and symmetric Nash equilibrium in both contests. Contest 1 yields the following equilib-

[7] As a consequence we do not have to distinguish between the interpretation of p_i as being a share of the prize or as being a probability of winning the whole prize. See Cornes and Hartley (2003) for a detailed discussion.

rium appropriative activity, $a(n,b,g,Z)$, indirect utility, $v(n,b,g,Z)$, and aggregate utility, $W(n,b,g,Z)$:

$$a = b(n-1)n^{g-2}Z\,\forall i, \quad v = (n-(n-1)b)n^{g-2}Z\,\forall i, \quad W = (n-(n-1)b)n^{g-1}Z. \quad (6)$$

Analogously we obtain the following equilibrium appropriative activity, $a(n,b,g)$, indirect utility, $v(n,b,g)$, and aggregate utility, $W(n,b,g)$, for contest 2:

$$a = \frac{(n-1)b}{1+b(n-1)}\,\forall i, \qquad v = \frac{n^{g-1}}{1+b(n-1)}\,\forall i, \qquad W = \frac{n^g}{1+b(n-1)}. \quad (7)$$

3 Analysis of size effects

3.1 Contest 1

We first analyze the influence of g on the equilibrium investment in the contest, indirect utility, and aggregate utility. Differentiating (6) gives $\partial a/\partial g > 0, \partial v/\partial g > 0$ and $\partial W/\partial g > 0$ for all meaningful values of n. In order to understand the intuition for these properties note that an increase in g is equivalent to an increase in the prize Z. It is seen from (6) that a, v and W are proportional to Z. This makes intuitive that a higher level of g (or Z) is followed by an increase in appropriative activities, indirect utility and aggregate utility.[8] Since the marginal costs of appropriative activities are fixed to be equal to one, investments in the contest become comparatively more attractive with an increase in g. Moreover, the additional marginal prize is less than fully dissipated by the increase in investments. As a consequence, despite the fact that the individuals become more aggressive, the increase in aggressiveness does not imply a reduction of individual utility or aggregate utility.

[8]We are grateful to one of the referees for suggesting this representation.

We can then discuss the effects of an increase of the number of competitors on a, v, and W. From (6):

$$\frac{\partial a}{\partial n} = (2 + g(n-1) - n)n^{g-3} bZ, \tag{8a}$$

$$\frac{\partial v}{\partial n} = \left(p\frac{dR}{dn} + R\frac{dp}{dn}\right) - \frac{\partial a}{\partial n} = ((g-1)n - b(2 - g + (g-1)n)\, n^{g-3} Z, \tag{8b}$$

$$\frac{\partial W}{\partial n} = \frac{dR}{dn} - \frac{\partial na}{\partial n} = (gn - b(1 - g + ng))\, n^{g-2} Z. \tag{8c}$$

To interpret (8) consider the benchmark case $g = 0$. In this case a is decreasing in the number of individuals. The reason is that an increase in n reduces the marginal gain of appropriation. In the limit for $n \to \infty$ we obtain $a \to 0$, individual appropriation activities converge to zero. At the same time, aggregate investment in the contest, na, does increase with n and has a limit of bZ for $n \to \infty$. This finding shows that an increase in group size need not imply an increase in the potential for individual aggression (because of constant marginal costs and reduced marginal revenues of appropriation), but unambiguously increases the total amount of resources devoted to appropriation (because more individuals devote resources to appropriation). The positive effect of a reduction of *per-capita* investments in the contest cannot over-compensate the negative effect resulting from an increased group size, given the absence of size effects.

Let us allow for size effects now. An interesting scenario, which is based on previous work by Grossman (2001) for an arbitrary but fixed number of competitors, is the case where the marginal size effect attributed to an additional competitor is constant. With $g = 1$, (8) simplifies to

$$\frac{\partial a}{\partial n} = b\, n^{-2} Z \geq 0, \qquad \frac{\partial v}{\partial n} = -b\, n^{-2} Z \leq 0, \qquad \frac{\partial W}{\partial n} = (1 - b)\, Z > 0.$$

Competition increases investment in the contest. Competitors become more aggressive if $b > 0$. The intuition is that the marginal return from an investment in aggression increases if aggregate utility is rising, causing a reduction of individual utility: there exists a discrepancy between the interest of the single individual and the interest of the whole society. If, however, $b = 0$ and individuals cannot influence the distribution of the rent, then aggregate utility is unambiguously increasing in n, whereas individual utility is independent of group size.

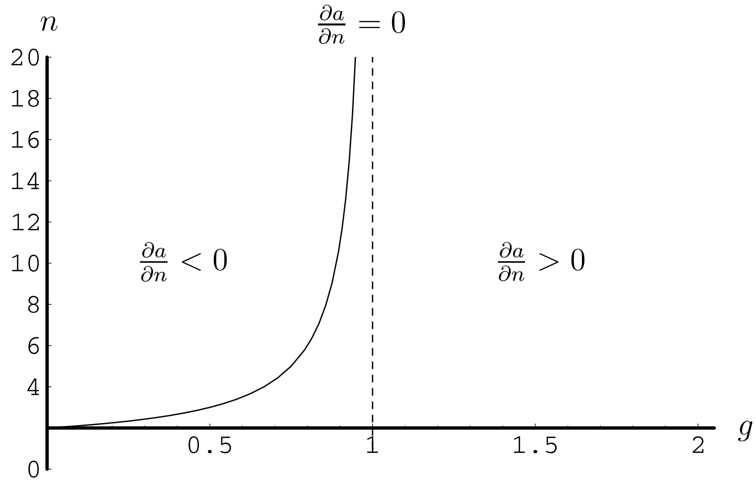

Figure 1: The effects of a change in g and n on appropriation activities in contest 1 ($b = 0.5$).

A comparison of the above cases shows that the general effect of a change in n depends in a non-trivial way on b and g. We start discussing the general case with an analysis of investment levels:

Result 1. *a. If* $b = 0$, $\partial a/\partial n = 0$.

b. If $b > 0$ *and* $g \geq 1$, $\partial a/\partial n > 0$.

c. If $b > 0$ *and* $g < 1$, $\partial a/\partial n \gtreqless 0 \Leftrightarrow n \gtreqless (2-g)/(1-g)$.

Proof: Part *a.* of the result follows from (8a) at $b = 0$. To prove parts *b.* and *c.*, let $b > 0$. For $g = 1$, $\partial a/\partial n = bZn^{g-2} > 0$. Note that $\partial a/\partial n > 0$ if and only if $n(1-g) < 2-g$. The inequality always holds for $g > 1$. For $g < 1$ it holds if and only if $n < (2-g)/(1-g)$. The RHS approaches infinity for $g \to 1$ and 2 for $g \to 0$. Hence, $\partial a/\partial n$ depends on the relationship between n and g and the borderline is defined by $n = (2-g)/(1-g)$. □

The economic intuition for the result is as follows. A larger number of individuals implies that the prize has to be shared among a larger group. Given constant marginal costs, investments in the contest are profitable for $b > 0$ if the negative effect of sharing the prize between more individuals is over-compensated by a positive rent effect. This implies that n has to be finite. If n becomes infinite, the marginal effect of a single competitor becomes negligible. However, increased competition tends to reduce *per-capita* investments in the contest in cases where the rent effect is small relative to the effect of sharing among a larger group. *Result 1 therefore shows that it is not the effect of increased competition that makes the individuals more aggressive, but the existence of sufficient rent effects.* This result highlights Diamond's (1997) claim that population pressure increases the inefficiencies of anarchy from a different perspective: if an essential aspect of anarchy is captured by contest 1, it is not the existence of population pressure *per se* that increases the potential for aggression in the society, but the implied rent effects.

Figure 1 gives a graphical representation of Result 1 for the case $b = 0.5$. In the figure g is measured along the abscissa, and n is measured along the ordinate. Investments in the contest are decreasing for all points above the dividing line $\partial a/\partial n = 0$ and increasing for all points below. Higher values of g intensify the aggressiveness of the individuals. However, if $g < 1$ then an increase in the number of individuals will finally temper the aggressiveness in the contest.

To discuss the effect of a change in the number of competitors on aggregate utility, W, it is useful to formally decompose (8c) in a rent effect and an appropriation effect. The rent effect, $\text{RE} = (dR/dn)/n^{g-2} = gnZ$, is unambiguously positive, increasing in n, and has a slope equal to gZ, starting at $2gZ$ since $n \geq 2$. The appropriation effect, $\text{AE} = (\partial(na)/\partial n)/n^{g-2} = b(1 - g + ng)Z$, is unambiguously increasing with a slope that is equal to bgZ, but depending on whether $2g/(1+g) \gtreqless b$ it is starting below or above the graph of the rent effect. The slope of the rent effect exceeds the slope of the appropriation effect. Figure 2 portrays both effects. In the figure, n is drawn along the abscissa whereas the rent and the appropriation effects are drawn along the ordinate. The two lines intersect if b and g are such that $\bar{n} = b(1 - g)/g(1 - b) \geq 2$ as in point A. This scenario describes a situation where aggregate utility is u-shaped in n, whereas aggregate utility is increasing in n in the case where $b(1 - g)/g(1 - b) < 2$.

In order to make this intuition precise let us aggregate the appropriation effect and the rent effect. This gives $b(g - 1) + ng(1 - b)$, and the sign of this term determines the sign of $\partial W/\partial n$. The following result summarizes:

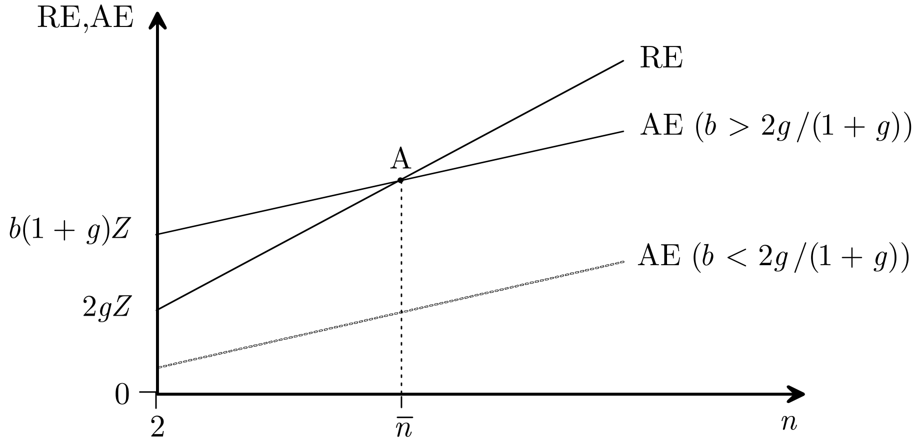

Figure 2: Impact of the rent effect (RE) and the appropriation effect (AE) on aggregate utility in contest 1.

Result 2. *a. If $b = g = 0$, or $b = g = 1$, $\frac{\partial W}{\partial n} = 0$.*

b. If $g \in (0,1)$ and $b \in [0, \frac{2g}{1+g}]$, or $g = 1$ and $b \in [0,1]$, or $g \in (1,2]$ and $b \in [0,1]$, $\frac{\partial W}{\partial n} > 0$.

c. If $g = 0$ and $b \in (0,1)$, or $g \in (0,1)$ and $b = 1$, $\frac{\partial W}{\partial n} < 0$.

d. If $g \in (0,1)$ and (i) $b \in [\frac{2g}{1+g}, 1)$, $\frac{\partial W}{\partial n} = 0 \Leftrightarrow n = \frac{b(1-g)}{g(1-b)}$,

(ii) $b \in (\frac{2g}{1+g}, 1)$, $\frac{\partial W}{\partial n} \gtrless 0 \Leftrightarrow 2 \leq n \gtrless \frac{b(1-g)}{g(1-b)}$.

Proof: The result follows directly from (8c), where we note that $\partial W/\partial n \gtreqless 0 \Leftrightarrow gn(1-b) \gtreqless b(1-g)$. Reformulating the latter condition yields $n \gtreqless \bar{n} = b(1-g)/(g(1-b))$. In addition, it follows from this condition that $\bar{n} \geq 2$ if and only if $b \geq 2g/(1+g)$. $\square$

Result 2 demonstrates that an increase in the number of individuals does increase aggregate utility if $g > 1$ or if the number of individuals is sufficiently large. The rent effect over-compensates every increase in appropriation if n increases. The intuition for this finding is closely related to the intuition for the change in a. First, the prize increases over-proportionally with the number of competitors at $g > 1$. This makes competitors more aggressive, but the increase in output is not fully dissipated by the increase in a. Second, an additional agent still adds to the total prize even though the rent increases underproportionally with n in the case where $g \in (0,1)$. Here, the increase in the rent to some extent mitigates the negative effect of sharing the

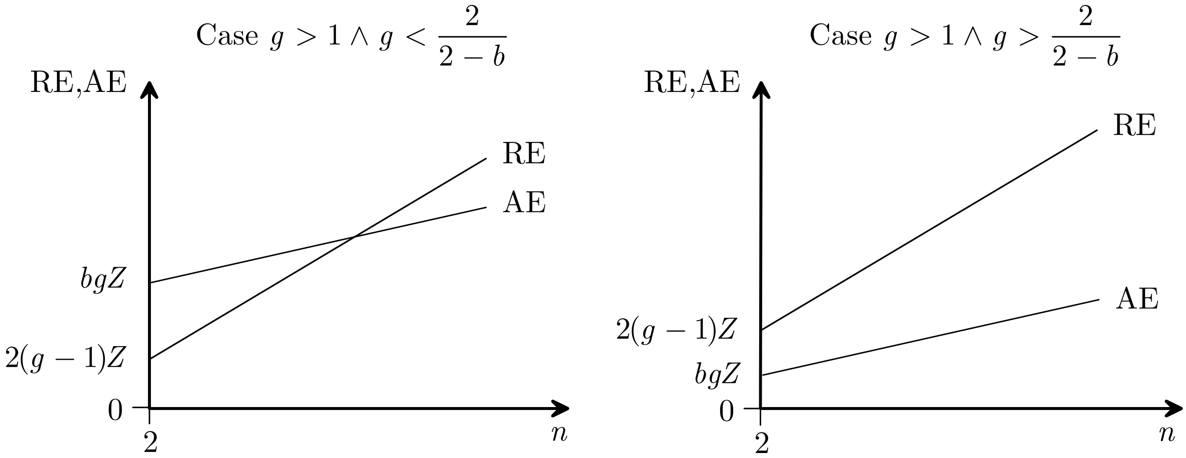

Figure 3: Impact of the rent effect (RE) and the appropriation effect (AE) on aggregate utility in contest 1.

rent among a larger group. In the extreme case $g = 0$, an increase in n reduces the marginal effectiveness of appropriation from the point of view of a single agent. As a result the agent reduces appropriation, $\partial a/\partial n = (2-n)bZ/n^3 \leq 0$ from (8a). Hence, there must exist a critical level of g where both effects exactly cancel, $\partial a/\partial n = 0 \Leftrightarrow g = (n-2)/(n-1)$.

The result shows that there is a qualitative difference between models where the rent is exogenous and models where the rent is endogenous. In a model without rent effects, aggregate utility is decreasing in the number of competitors, $\partial W/\partial n = -bZ/n^2$, and the slope depends linearly on the discriminatory power of individuals. With sufficiently large g, the slope can be positive. We will come back to this point in our discussion of the optimal number of competitors in the contest.

Next we turn to an analysis of the change in individual net utility v. We follow the line of argumentation used above and decompose (8b) in a rent effect, $\text{RE} = (dp/dn\, R + p\, dR/dn)/n^{g-3} = (g-1)nZ$, and an appropriation effect, $\text{AE} = (\partial a/\partial n)/n^{g-3} = b(2-g+(g-1)n)Z$. Aggregating the rent effect and the appropriation effect, RE$-$AE, shows that the sign of (8b) is determined by the sign of $(g-1)n - b(2-g+(g-1)n$. Figure 3 shows this relationship. If $g > 1$, the rent-effect curve is increasing in n and has a slope that is equal to $(g-1)Z$, starting at $2(g-1)Z$. The appropriation-effect curve is also increasing with a slope equal to $b(g-1)Z$, starting at bgZ. If $g > 1$, and depending on whether $2g/(1+g) \gtreqless b$, it is starting below or above the rent-effect curve. Because the slope of the rent-effect curve exceeds the slope of the appropriation-effect curve it follows immediately that v is either increasing ($2g/(1+g) < b$ and $g > 1$), u-shaped ($2g/(1+g) > b$ and $g > 1$), or decreasing (all other cases) in n. We summarize with:

Result 3. *a. If $g = 1$ and $b = 0$, or $g = 2$ and $b = 1$, $\frac{\partial v}{\partial n} = 0$.*

b. If $b = 0$ and $g \in (1, 2]$, or $b \in (0, 1)$ and $g = \frac{2}{2-b}$, or $b \in (0, 1)$ and $g \in (\frac{2}{2-b}, 2]$, $\frac{\partial v}{\partial n} > 0$.

c. If $b = 0$ and $g \in [0, 1)$, or $b = 1$ and $g \in [0, 2)$, or $b \in (0, 1)$ and $g \in [0, 1]$, $\frac{\partial v}{\partial n} < 0$.

d. If $b \in (0, 1)$ and (i) $g \in (1, \frac{2}{2-b}]$, $\frac{\partial v}{\partial n} = 0 \Leftrightarrow n = \frac{b(2-g)}{(g-1)(1-b)}$,

(ii) $g \in (1, \frac{2}{2-b})$, $\frac{\partial v}{\partial n} \gtrless 0 \Leftrightarrow 2 \leq n \gtrless \frac{b(2-g)}{(g-1)(1-b)}$.

Proof: The result follows directly from (8b), where we note that $\partial v/\partial n \gtreqless 0$ if and only if $n(1-g)(b-1) \gtreqless b(g-2) \Leftrightarrow n \gtreqless \bar{n}b(2-g)/((1-g)(b-1))$. In addition, $\bar{n} \geq 2$ if and only if $g \leq 2/(2-b)$. □

Result 3 implies that an increase in n increases the net utility of the individuals only for sufficiently large g, compared to b. The economic intuition for Result 3 is best understood if (8b) is evaluated at the boundary case where $g = 2$ and $b = 1$. At this point, rent and appropriation effects balance in a way that net utility is constant. The contest is less effective from the perspective of an individual trying to increase its share of the rent for low values of b, which implies that the individuals reduce their aggressiveness. In equilibrium this increases individual utility. By the same token, individual utility decreases if size effects are less important.[9]

An interesting implication of Result 3 is that for $g > 1$ individual utility is not monotonous in the population. It is *decreasing* up to a critical number of individuals and *increasing* thereafter. If n is relatively small, the negative appropriation effect of an increase in competition is relatively important and outweighs the rent effect. However, increasing n implies that the appropriation effect is becoming less important. There exist pairs of $\{g, b\}$ such that the appropriation effect is dominated by the rent effect. This finding shows that, for example for the case of an anarchic society, the contest

[9] To determine the slope and curvature of the condition $\partial v/\partial n = 0$ use (8b) to obtain $b = n(g-1)/((g-1)n + g - 2)$. The first and second derivatives of this function show that it is increasing and concave in g for $g > 1$, which is the relevant domain of g according to Result 3.

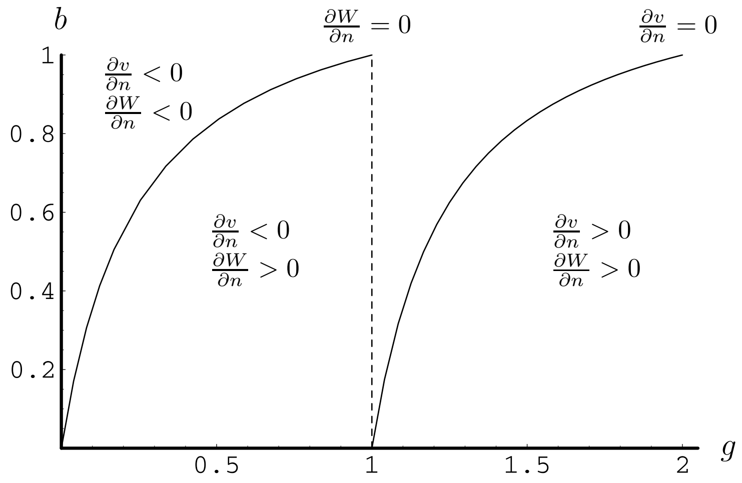

Figure 4: Comparison of changes in aggregate utility and individual utility in contest 1 ($n = 5$).

creates a u-curve effect that may hamper a further development of the society if it starts at the decreasing region of the v-curve.

A comparison of Result 2 and Result 3 demonstrates that there may exist a qualitative difference between the effects on individual utility and the effects on aggregate utility of a growing number of individuals. Figure 4 unifies both conditions. The change in individual and aggregate utility has the same sign for extreme values of b and g. However, there exists an interval of 'intermediate' values for b and g where the aggregate utility increases but individual utility decreases. The intuition for this discrepancy is as follows. Recall that $W = nv$. Thus, $\partial W/\partial n = v + n\,\partial v/\partial n$. The first term measures the effect of an additional agent on aggregate utility, which is always positive. The second term measures the effect of an additional competitor on other agents. This effect can be either positive or negative. Hence, if the individual utility is increasing, aggregate utility has to increase by definition. However, there is a region for which individual utility is decreasing but this decrease does not over-compensate the effect that a 'new' competitor adds to the aggregate. The former effect is dominant in cases where g is small and b is large.

A straightforward question is about the optimal number of individuals. There are two perspectives from which we can determine this number. (i) From the individuals' perspective it is given by the number of individuals that maximizes net utility. (ii) From a social point of view the optimal number of competitors maximizes aggregate utility. Both measures do not necessarily lead to the same results. We start our discussion for the case of potential interior solutions. It follows directly from Results 2.d. and 3.d. that any interior solution has to be a minimum both, from the aggregate and the individual point of view: indirect utility and aggregate utility are decreasing if n is below a critical level and increasing thereafter. This implies that the optimal number of agents is either 2 or infinity from both perspectives, depending on g and b. In order to determine the optimal number of agents we calculate the limit of W and v for n approaching infinity. We obtain the following corollary to Result 2:

Corollary 1. *a. If $b = g = 0$ and if $b = g = 1$, aggregate utility is independent of n. Net aggregate utility is $W = Z$.*

b. If $b = 0$ and $g > 0$ or if $b \in (0, 1)$ and $g \geq 1$, or if $b = 1$ and $g > 1$, the aggregate utility maximizing number of agents converges to infinity. The resulting level of aggregate utility converges to infinity.

c. If $b < 1$ and $g < 1$, the aggregate utility maximizing number of agents converges to infinity. The resulting level of aggregate utility converges to infinity as well.

d. If $b = 1$ and $g < 1$ the aggregate utility maximizing number of agents is equal to 2, the resulting level of aggregate utility is $2^{g-1}Z$.

Proof: Parts *a.* and *b.* follow from Result 2 *a.* to *c.*. In order to prove parts *c.* and *d.* we have to take the limit of $W = (n + (1 - n)b)n^{g-1}Z$ for $n \to \infty$. It follows that for $b < 1$, $\lim_{n\to\infty} W$ is equal to ∞ for $g < 1$. If $b = 1$ we obtain $\lim_{n\to\infty} W$ is equal to 0 for $g < 1$. On the other hand, $W(n = 2) = 2^{g-1}Z$. □

The above finding shows that the net effect of an increase in n on aggregate utility is positive if g is large relative to b, even though large values of g make aggressive behavior more profitable. The marginal rent effect of an additional agent is increasing, and this outweighs the costs caused by the increase in appropriative activities.

The fact that we obtain a corner solution even for relatively small size effects at $g < 1$ follows from the assumption that size effect are globally decreasing in the number of agents. As a consequence, the marginal size effect is large in the case where the number of agents is small and it is small if this number is large, which implies that agents react differently, depending on the initial number of agents. For small numbers they increase appropriation, whereas they reduce appropriation for large n. This explains the u-shaped structure of aggregate utility. A maximal number of agents is then optimal if $b < 1$ and $g > 0$ or $b = 1$ and $g > 1$.

The effect on individual utility can be summarized as follows.

Corollary 2. *a. If $g < 1$, or if $g = 1$ and $b > 0$, or if $g \in (1, 2)$ and $b = 1$, the utility-maximizing number of agents is equal to 2 and the utility level is equal to $2^{g-2}(2 - b)Z$.*

b. If $g = 1$ and $b = 0$ and if $g = 2$ and $b = 1$, the utility level is independent of the number of agents and equal to Z.

c. If $g > 1$ and $b \in [0, 1)$, the utility-maximizing number of agents converges to ∞. The utility level converges to ∞ as well.

Proof: Parts *a.*, *b.*, and the case $b = 0$ of part *c.* follow directly from Result 3. To show cases $b \in (0, 1)$ of part *c.* first note that $v = 2^{g-2}(2 - b)Z$ for $n = 2$, which is a finite number. Taking the limit of v for $n \to \infty$ shows that it always converges to ∞.□

It follows from our previous results that a maximal number of agents is optimal if $b < 1$ and $g > 0$ or $b = 1$ and $g > 1$. Utility is equal to zero if $b \in [0, 1)$ and $g \in [0, 1)$ or if $b = 1$ and $g \in [0, 2)$. An increase in the number of agents tends to reduce utility if g is relatively small compared to b.

A comparison of Corollaries 1 and 2 reveals that the public and private evaluation of the optimal number of agents coincides for $b \in [0, 1)$ and $g \in (1, 2]$, and $b = 1$ and $g \in [0, 1)$ respectively. They differ for $b \in [0, 1)$ and $g \in [0, 1]$, and $b = 1$ and $g \in [1, 2]$.

There exists an interesting formal similarity between the literature on the optimal size of a population and our approach.[10] It is a well-established result in the theory of optimal population size that in a world with finite resources, sum-utilitarianism implies an infinite population with arbitrarily low individual utilities. This property of utilitarian aggregate utility functions has been called the 'repugnant conclusion' by Parfit (1984), see also Razin and Sadka (1995). On the other hand, average utilitarianism implies a minimal population with maximal individual utility.

The logic of this conclusion abstracts from any institutional details and holds for the case that the potential rent is independent of the size of the population and not

[10] We stress that the similarity is a formal one because from a normative point of view the problem to determine an optimal group size of already living individuals is different from determining an optimal population of individuals who still have to be born. See Dasgupta (1993) for further details.

dissipated in a contest. Our results can be interpreted in the spirit of the repugnant conclusion to test its relevance in anarchic environments with positive size effects. For this purpose let us interpret individual utility, v, as average utilitarianism and aggregate utility, W, as sum utilitarianism. It is clear that the repugnant conclusion does not hold in our model as long as $g \in [0, 1)$. In contrast, the repugnant conclusion holds when $g \in (1, 2)$, since then individual utility and aggregate utility converge in opposite directions. However, in the case where $g = 2$, individual utility and aggregate utility one again converge in the same direction. It follows that the logic of the repugnant conclusion is obtained as a special case in our model. It does apply in cases where the growth of the potential rent exceeds population growth only moderately, whereas the repugnant conclusion cannot be obtained when the contestable rent grows at a slower rate as the number of individuals. The reason is that the dissipation of part of the potential rent in the contest requires a minimum size effect in order to guarantee increasing aggregate or individual utilities.[11]

Let us summarize the most important results we have obtained in this section:

- First, if $g \in (0, 1)$ and starting at $n = 2$ an increase in the number of individuals increases aggressiveness if the population is small and decreases aggressiveness if it is large. The larger g, the larger becomes the critical number of individuals from which on appropriative behavior is finally reduced. If $g \in [1, 2]$ the individuals unambiguously increase their appropriative behavior if the population increases.

- This implies that for an increase in the population and $g \in (0, 1)$, aggregate utility is first decreasing and then increasing. It is unambiguously increasing if $g \geq 1$.

- Individual utility is decreasing in the number of individuals if $g \leq 1$. However, if $g > 1$ there exists a critical size of the population below which individual utility is decreasing and from which on individual utility is increasing.

[11] It is straightforward to show that, for $b = 0$ (there exists no conflict in the economy), $\lim_{n\to\infty} v = 0, g < 1; Z, g = 1, \infty, g > 1$ and $\lim_{n\to\infty} W = Z, g = 0, \infty, g > 0$, which replicates the repugnant conclusion in its standard formulation for $g \leq 1$.

3.2 Contest 2

In this section we analyze how the results of the last section change if individuals face a trade-off between appropriation and production of the rent. From (7), an increase in g has the following influence on a, v, and W.

$$\frac{\partial a}{\partial g} = 0, \tag{9a}$$

$$\frac{\partial v}{\partial g} = \frac{\ln(n) n^{(g-1)}}{1 + b(n-1)} > 0, \tag{9b}$$

$$\frac{\partial W}{\partial g} = \frac{\ln(n) n^{g}}{1 + b(n-1)} > 0. \tag{9c}$$

In the second contest appropriative investment is independent of g when competitors can directly influence the size of the rent from (9a). The result is explained by the fact that an increase in g results in higher marginal revenue of investments. However, the increase in marginal costs exactly offsets the effects of an increase in g, leaving the marginal rate of transformation between appropriative and productive activities unaffected. Since these activities are independent of g, the increase in the gross prize caused by the increase in g is equal to the increase in the net prize, which is equally divided among competitors in equilibrium.

Next we analyze the consequences of an increase in the number of agents. From (7),

$$\frac{\partial a}{\partial n} = \frac{b}{(1 + b(n-1))^2} \geq 0, \tag{10a}$$

$$\frac{\partial v}{\partial n} = g n^{g-2}(1-a) - n^{g-1}\frac{\partial a}{\partial n} = \frac{n^{g-2}}{(1 + b(n-1))^2}\left((g-1)(1 + b(n-1)) - bn\right), \tag{10b}$$

$$\frac{\partial W}{\partial n} = (g-1) n^{g-1}(1-a) - n^{g}\frac{\partial a}{\partial n} = \frac{n^{g-1}}{(1 + b(n-1))^2}\left(g(1 + b(n-1)) - bn\right). \tag{10c}$$

It follows from (10a) that an increase in n intensifies aggressiveness. The intuition is that productive investments create a positive externality for other competitors in an anarchic economy: the individuals internalize the total (marginal) costs of production (in terms of foregone appropriation of the already existing rent), but get only a fraction of the (marginal) surplus, (which is equal to $1/n$ in equilibrium). Clearly, the associated free-rider problem is the more severe the larger is the number of individuals. In other words, the marginal rate of transformation between appropriative and productive activities (in absolute terms) is an increasing function of the number of competitors, which implies that aggressiveness is intensified if the population increases.

As in the previous section, we next analyze two benchmark cases, $g = 0$ and $g = 1$, before we turn to the analysis of the general case. For $g = 1$, equations (10b) and (10c) become

$$\begin{aligned} \frac{\partial v}{\partial n} &= -\frac{b}{(1+b(n-1))^2} \leq 0, \\ \frac{\partial W}{\partial n} &= \frac{1-b}{(1+b(n-1))^2} \geq 0. \end{aligned}$$

The indirect utility of the individuals is weakly decreasing in n. It remains constant in the case where $b = 0$, which implies that individual net utility is independent of the number of agents. However, $b > 0$ implies that v is decreasing in n. Aggregate utility is constant at $b = 1$, hence, individual utility has to decrease. By the same token, individual utility decreases as long as aggregate utility increases by less than one unit per individual. The increase in aggregate utility is smaller than one for all $b > 0$ since each agent devotes more time to appropriative activities. Aggregate utility is weakly increasing in n because the potential increase in additional wealth is not completely offset by a reallocation of investments. Only $b = 1$ gives that aggregate utility is not increasing with the number of individuals.

In the case where $g = 0$ equations (10b)-(10c) become

$$\begin{aligned} \frac{\partial v}{\partial n} &= \frac{(b-1) - 2bn}{(1+b(n-1))^2 n^2} < 0, \\ \frac{\partial W}{\partial n} &= -\frac{b}{(1+b(n-1))^2} \leq 0. \end{aligned}$$

Indirect utility is unambiguously decreasing if the number of individuals has no effect on total production. At $b = 0$ agents choose to invest in productive activities only, but total production is independent of the number of agents and has to be shared among a larger number of individuals. This also implies that aggregate utility is independent of n in this case. If $b > 0$ there are appropriative as well as productive activities, but it follows from (10a) that appropriation is increasing in the number of individuals. Therefore, it is not only that the potential production has to be shared among a larger number of agents, the *realized* production is actually reduced because more resources

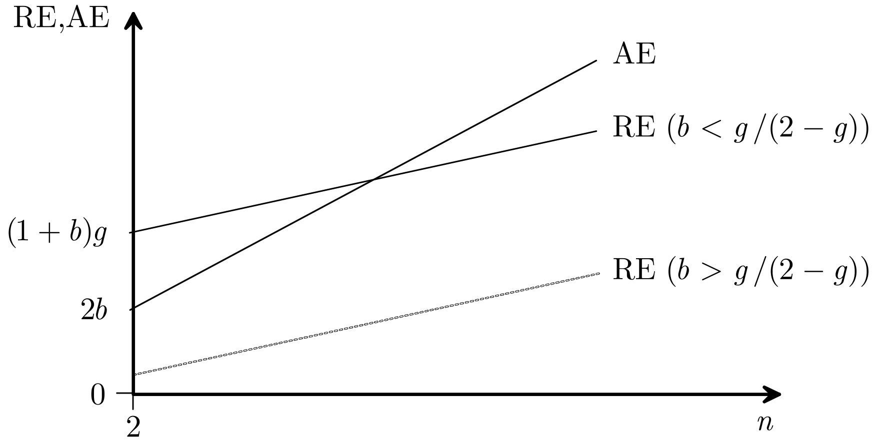

Figure 5: Impact of the rent effect (RE) and the appropriation effect (AE) on aggregate utility in contest 2.

are devoted to appropriation. This implies that individual utility and aggregate utility is decreasing in n.

We now turn to the general effects. (10c) can again be divided into a rent and an appropriation effect, $(\mathrm{RE} - \mathrm{AE})\, n^{g-1} / (1 + b(n-1))^2$. Following the line of argumentation of the last section, the rent effect is equal to $\mathrm{RE} = g(1 - b + bn)$ whereas the appropriation effect is equal to $\mathrm{AE} = bn$. The graph of the rent effect starts at $(1 + b)g > 0$ for $n = 2$ and has a slope of gb. The graph of the appropriation effect starts at $2b > 0$ and has a slope of b. Hence, the slope of the rent effect exceeds the slope of appropriation effect if and only if $g > 1$. In this case, $(b+1)g - 2b > 0$, which implies that W is monotonically increasing in n. By the same token, if $g < 1$ and $(b + 1)g - 2b > 0$, contrary to contest 1 we obtain an interior maximum for W. This case is indicated in the figure. More precisely, the general effect of a change of n on W is as follows:

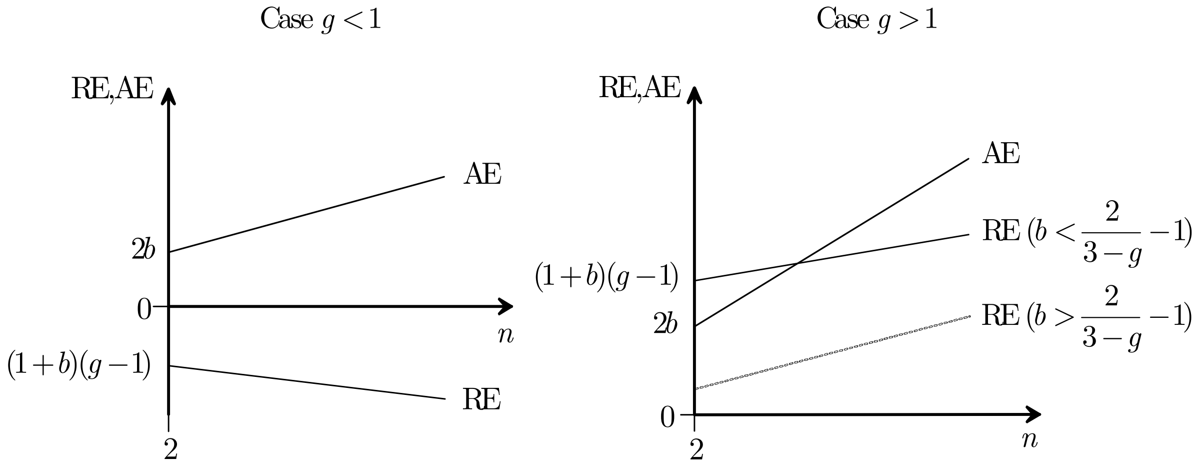

Figure 6: Impact of the rent effect (RE) and the appropriation effect (AE) on individual utility in contest 2.

Result 4. *a. If $g = b = 0$, or $g = b = 1$, $\frac{\partial W}{\partial n} = 0$.*

b. If $b = 0$ and $g \in (0, 2]$, or $b \in (0, 1)$ and $g \in [1, 2]$, or $b = 1$ and $g \in (1, 2]$, $\frac{\partial W}{\partial n} > 0$.

c. If $b \in (0, 1)$ and $g \in [0, \frac{2b}{1+b})$, or $b \in (0, 1)$ and $g = \frac{2b}{1+b}$, or $b = 1$ and $g \in [0, 1)$, $\frac{\partial W}{\partial n} < 0$.

d. If $b \in (0, 1)$ and (i) $g \in [\frac{2b}{1+b}, 1)$, $\frac{\partial W}{\partial n} = 0 \Leftrightarrow n = \frac{(1-b)g}{(1-g)b}$,

(ii) $g \in (\frac{2b}{1+b}, 1)$, $\frac{\partial W}{\partial n} \gtrless 0 \Leftrightarrow 2 \le n \lesseqgtr \frac{(1-b)g}{(1-g)b}$.

Proof: The result follows immediately from (10c). One can use (10c) to show that $\partial W/\partial n \gtreqless 0$ for $n \lesseqgtr \bar{n} \equiv (1-b)g/((1-g)b)$. The resulting level of $\bar{n}$ is only larger or equal to 2 if and only if $g \ge 2b/(1+b)$. The term on the right-hand side of the inequality is smaller or equal to 1 from $b < 1$. In the other case, $n \ge 2$ is a binding restriction. $\square$

In order to analyze the effect of n on individual utility we again decompose (10b) into a rent and a appropriation effect, RE $= (g-1)(1-b+bn)$, AE $= bn$. The relationship is displayed in Figure 6. The graph of the rent effect starts at $(b+1)(g-1) \gtreqless 0 \Leftrightarrow g \gtreqless 1$ for $n = 2$ and has a slope of $(g-1)b \gtreqless 0 \Leftrightarrow g \gtreqless 1$. The appropriation effect starts at $2b > 0$ and has a slope of b. Hence, the slope of the appropriation effect exceeds the slope of rent effect for all $g \in [0, 2)$. If $g = 2$, both slopes are the same.

The example in the figure has the following structure. For the case $g < 1$ it follows immediately that the marginal utility is smaller than zero. If $g > 1$ we have assumed that $(b-1)(g-1) - 2b > 0$, which implies that the marginal utility is positive for small and negative for large n. Again this shows that there is an interior value for the utility-maximizing population size. The general effect of a change of n on v is:

Result 5. *a. If $g = 1$ and $b = 0$, or $g = 2$ and $b = 1$, $\frac{\partial v}{\partial n} = 0$.*

b. If $g \in (1, 2)$ and $b = 0$, or $g = 2$ and $b \in [0, 1)$, $\frac{\partial v}{\partial n} > 0$.

c. If $g \in (1, 2)$ and $b \in [0, 1]$, or $g = 1$ and $b \in (0, 1]$, or $g \in (1, 2)$ and $1 + b - \frac{2}{3-g} = 0$, or $b \leq 1$ and $1 + b - \frac{2}{3-g} > 0$, $\frac{\partial v}{\partial n} < 0$.

d. If $g \in (1, 2)$ and (i) $b \in (0, \frac{2}{3-g} - 1]$, $\frac{\partial v}{\partial n} = 0 \Leftrightarrow n = \frac{(1-b)g}{(1-g)b}$,

(ii) $b \in (0, \frac{2}{3-g} - 1)$, $\frac{\partial v}{\partial n} \gtrless 0 \Leftrightarrow 2 \leq n \lessgtr \frac{(1-b)g}{(1-g)b}$.

Proof: The result follows immediately from (10b). In addition, one can use (10b) to show that $\partial v/\partial n \gtreqless 0$ for $n \lesseqgtr \bar{n} = (g-1)(1-b)/((2-g)b)$. The resulting level of $\bar{n}$ is only larger or equal to 2 if and only if $g \geq (3b+1)/(1+b)$. $\square$

As for contest 1 we finally turn to the analysis of the optimal number of individuals in the contest. We will clarify the implications of Result 4 in the following discussion. The next two corollaries follow straightforwardly from Results 4 and 5:

Corollary 3. *a. If $g > 1$, the aggregate utility maximizing number of agents is $n \to \infty$.*

b. If $g = 1$ and (i.) $b = 1$ the aggregate utility maximizing number of agents is $n \in [2, \infty)$ whereas for (ii.) $b < 1$ the aggregate utility maximizing number of agents is $n = 2$.

c. If $g < 1$ and (i.) $g > 2b/(1+b)$ the aggregate utility maximizing number of agents is $n = (1-b)g/\left(b(1-g)\right)$, whereas for (ii.) $g \leq 2b/(1+b)$ the aggregate utility maximizing number of agents is $n = 2$.

Corollary 4. *a. If $g \leq 1$, the utility-maximizing number of agents is $n = 2$.*

b. If $g > 1$ and (i.) $b \geq \frac{g-1}{3-g}$ the utility-maximizing number of agents is $n = 2$, whereas if (ii) $b < \frac{g-1}{3-g}$ the utility-maximizing number of agents is $n = \frac{(b-1)(1-g)}{b(2-g)}$.

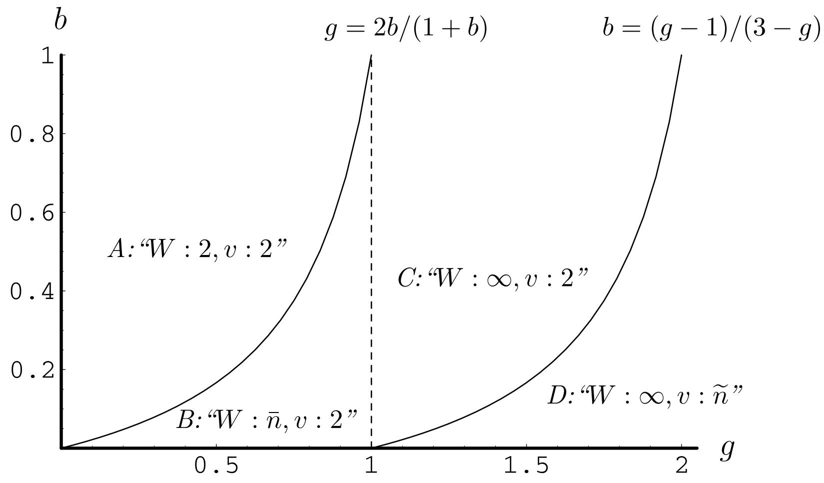

Figure 7: Comparison of changes in aggregate utility and individual utility in contest 2.

Corollaries 3 and 4 show that there exists a conflict of interest between the individual and the aggregate perspective in general. Figure 7 shows this difference. The parameter space is divided in four different areas. In area A, both, individual utility and aggregate utility are maximized at $n = 2$. In area B individual utility is still maximized at $n = 2$, but aggregate utility maximization requires $n = (b-1)g/(g-1)b$. In area C the individual optimum is still unchanged, but maximization of aggregate utility requires $n \to \infty$. Finally, in area D the individual utility maximum yields $n = (b-1)(1-g)/(b(2-g)$, whereas aggregate utility is maximized at $n \to \infty$. This implies that there exists a conflict of interest in areas B, C, and D. Only in area A g is sufficiently small and b is sufficiently large that aggregate and individual utility maximization coincide with respect to the resulting optimal number of individuals.

It is the externality created by productive activities that explains the result: starting at $n = 2$ with $a = b/(1+b)$, an increase in the number of individuals increases appropriation. Since a converges to 1 for $n \to \infty$, the total time endowment is invested in appropriation. If g is relatively large compared to b, an increase in n has a positive effect on individual utility. The reason is that the reduction of productive activities is over-compensated by the increase in marginal productivity of these investments. If b, however, is sufficiently large, the reduction in productive investments immediately over-compensates the increase in marginal productivity causing a net utility loss. If, on the other hand, b is moderate but the number of competitors is large, the increase in marginal productivity cannot over-compensate the decrease in productive investments since $g < 2$. The implication is that there exists an interior optimal number of individuals from the point of view of the single individual.

The finding extends the first contest model in two respects. First, without contested productive activities by the individuals, individual utility is decreasing if $g < 1$ and may be either increasing or decreasing if $g > 1$. Extending the model by incorporating productive investments reduces the extent to which size effects are necessary in order to create positive effects of competition for the individual competitor. Second, we have seen that there exists a u-curve effect of increasing competitiveness if size effects are sufficiently strong, whereas this effect is inverted in the presence of productive investments.

4 Conclusions

Population size matters for the extent of conflict in an anarchic society. However, it is not population size alone that matters but also the associated effects on productivity. Our results suggest that the production technology as well as the conflict technology are important explanatory factors for the implications of population size in anarchy. In order to obtain a better understanding of the implications of our findings we will interpret population growth as a continuous variable in the following.

If size effects merely increase the stock of goods that can be distributed, our model implies that for large size effects individuals tend to either organize in very small or very large groups (families and nations, respectively). If the initial group is small and the group size has to be extended continuously, a growth trap may exist because of the u-curved shape of individual utility in population size. As a consequence there exists a collective-decision problem in anarchy that is similar to the one analyzed in Grossman (2002). He argues that such a collective decision problem may explain the emergence of a ruling elite that need not be benevolent but that is able to internalize the externalities of decentralized defensive activities. The emergence of hierarchies can be explained as a solution to the same type of problem in our model. Hence, according to this view the transformation of hunter-gatherer societies into hierarchical societies has two roots, sufficient size effects and a collective decision problem. If size effects are small, however, it is optimal from an individual point of view to organize in small units if possible, even if population growth would maximize aggregate utility.

In the extended model with productive investments, size effects may merely increase the return from productive investments. If size effects are sufficiently small but still large enough compared to the discriminatory power of the contest, our model implies that there exists a finite group size larger than 2 that maximizes individual utility. This finding can be interpreted as the formation of a non-hierarchical tribe or a village. The mechanisms that are responsible for aggregate utility improvements if the group size increases are comparable to those discussed in the literature on agglomeration (see for example Krugman, 1995), namely returns to scale or sufficiently strong size effects. However, the counterbalancing effect that explains a finite group size differs in our approach. The literature on agglomeration focuses on transportation costs and crowding. In contrast, our focus is on the incentives to engage in appropriative activities. As we have demonstrated, appropriative activities are an alternative explanation for a finite group size, even in the presence of global size effects.

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