Escola de ECONOMIA de São Paulo

Textos para Discussão

224

Outubro de 2009

MONEY VERSUS MEMORY

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Money versus Memory*

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August 14, 2009

Abstract

A well–established fact in monetary theory is that a key ingredient for the essentiality of money is its role as a form of memory. In this paper we study a notion of memory that includes information about an agent's past actions and trading opportunities but, in contrast to Kocherlakota (1998), does not include information about the past actions and trading opportunities of an agent's past partners. We first show that the first–best can be achieved with memory even if it only includes information about an agent's very recent past. Thus, money can fail to be essential even if memory is minimal. We then establish, more interestingly, that if information about trading opportunities is not part of an agent's record, then money can be better than memory. This shows that the societal benefit of money lies not only on being a record of past actions, but also on being a record of past trading opportunities, a fact that has been overlooked by the monetary literature.

Key Words: Money, Memory, Essentiality.

JEL Codes: E40, C73, D82

*We thank Satoru Takahashi for comments on an early draft of the paper. We also thank seminar participants at Illinois and audience members at the 2009 SAET meetings in Italy and at the 2009 Workshop on Money, Banking, and Payments at the Chicago Fed for their comments and suggestions.

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1 Introduction

An important insight of monetary theory is that money helps to keep track of past actions, i.e., money is a form of memory (Ostroy (1973), Lucas (1980), Townsend (1980), and Aiyagari and Wallace (1991)). Kocherlakota (1998) expands this point by showing that memory, appropriately defined, subsumes money in the following sense: in a broad class of environments, any allocation that can be achieved with money can also be achieved with memory.

Kocherlakota (1998) defines memory as an agent's knowledge of the history of his partners and all the agents that were directly or indirectly in contact with them. In this paper we study a weaker notion of memory that only includes information about the histories of an agent's partners. The first result we obtain is that the first–best can be achieved with memory even if it only includes information about an agent's very recent past. Intuitively, memory sustains non–autarkic allocations because it rewards the agents' willingness to produce whenever it is socially beneficial to do so. Thus, money can fail to be essential even if memory is minimal.[1]

An important fact to notice is that an agent's history includes not only his past actions, but also the nature of his past trading opportunities. For example, in random matching environments such as in Shi (1995) and Trejos and Wright (1995), the information in an agent's history includes which of his past meetings were single–coincidence meetings and which were not. Given this fact, a natural question to ask is how each of these two pieces of information help sustain desirable allocations. Clearly, memory of past actions is necessary to sustain non–autarkic allocations. If past actions cannot be observed, then an agent will never have an incentive to choose costly actions that do not entail any immediate benefit. Our second main result is that the knowledge of past trading opportunities is needed for memory to implement the first–best. The reason is that now memory cannot distinguish between an agent who does not produce because he is in a no–coincidence meeting from an agent who chooses not to produce when it his turn to do so.

The second result suggests that money can do better than memory if the latter only

[1] As is by now standard, we say that money is essential if it implements desirable allocations that cannot be achieved otherwise.

includes information about past actions. Indeed, money can achieve the first–best in a variety of settings. We give an example of this in an overlapping generations environment. More generally, Kocherlakota (2002) proves this fact in a large class of environments that includes random matching and turnpike as special cases. This shows that money outperforms memory not only because it conveys information about past actions, but also because it works as a record of past trading opportunities. This second dimension of money has been overlooked by the monetary literature.

The paper is organized as follows. In Section 2 we describe the physical environment, introduce our notion of memory, and define equilibrium. We prove our first main result in Section 3. In Section 4 we establish our second main result and discuss its implications for the relationship between money and memory. We conclude in Section 5.

2 The Environment

We first describe the physical environment. Then we describe the record–keeping technology and define equilibrium.

Physical Environment Time is discrete and indexed by $t \geq 1$. There exists a continuum of nonatomic agents and a countable set Ω of types. For each $\omega \in \Omega$ there exists a unit mass of agents who are of this type. Different types of agents can live for different lengths of time and in different periods. We denote by $N(\omega)$ the set of periods in which the agents of type ω are alive and by $\Omega_t = \{\omega \in \Omega : t \in N(\omega)\}$ the set of types who are alive in period t. Trade takes place in a decentralized market where meetings are pairwise. An agent's type is observable in any meeting he participates. Preferences are additively separable over periods and all agents maximize expected discounted utility with discount factor $\beta \in (0, 1)$.

Agents can trade one divisible and perishable good that comes in many varieties. We say that an agent in a meeting is a consumer if his partner can produce the variety that he consumes, and is a producer if his partner can consume the variety that he produces. No agent can consume the variety that he produces. For simplicity, we assume that there are no double–coincidence meetings.[2] An agent who consumes x units of the good and produces y units of it obtains instantaneous utility $u(x) - y$. We assume that $u(x) - x$ has a unique maximizer x^* and that $x^* > 0$. The first–best is achieved if in every single–coincidence meeting the producer transfers x^* units of the good to the consumer.

For each $t \geq 1$, there exist maps $M_t : \Omega_t \times \Omega_t \to [0, 1]$ and $\rho_t : \Omega_t \times \Omega_t \to [0, 1]$ such that: (i) $M_t(\omega, \omega')$ is the probability that an agent of type ω is matched with an agent of type ω' in period t; (ii) $\rho_t(\omega, \omega')$ is the probability that an agent of type ω who meets with an agent of type ω' in period t is a producer. Meetings are random and anonymous conditional on the types of agents who are matched.

Let $T(\omega) = \sup\{t \geq 1 : t \in N(\omega)\}$. We assume that $\rho_{T(\omega)}(\omega, \cdot) \equiv 0$ for all $\omega \in \Omega$ such that $T(\omega) < \infty$. Thus, an agent who has a last period in which he is alive cannot be a producer in this period.[3] We also assume that there exists $\varepsilon > 0$ with the property that for all $\omega \in \Omega$ and $t \in N(\omega)$ such that $M_t(\omega, \omega')\rho_t(\omega, \omega') > 0$ for some $\omega' \in \Omega_t$, there exists $\omega'' \in \Omega_{t+1}$ such that $M_{t+1}(\omega, \omega'')\rho_t(\omega, \omega'') > \varepsilon$. In other words, there is a positive ε such that if an agent can be a producer in one period, then the probability that he is a consumer in the next period is greater than ε.

Our environment contains as a special case the environments normally considered in monetary theory.

Example 1 (Random Matching): $\Omega = \{\omega_0\}$, $N(\omega_0) = \mathbb{N}$, $M_t(\omega_0, \omega_0) \equiv 1$, and $\rho_t(\omega_0, \omega_0) \equiv \rho$, with $\rho < 1/2$.

Example 2 (Overlapping Generations): $\Omega = \mathbb{Z}_+$, $N(0) = \{1\}$, and $N(t) = \{t, t + 1\}$ for all $t \geq 1$. The agents of type $t \geq 1$ are born in period t and live for 2 periods. The agents of type 0 are born in period 1 and live for one period only. As usual, an agent who is born in t is 'young' in t and 'old' in $t + 1$, except the type–0 agents, who are born old. We assume

[2] It is straightforward to extend our analysis to cover the case where double coincidences are possible.

[3] Notice that a finitely–lived agent never has an incentive to produce a positive amount of the good in his last period of life. Thus, the first–best can only be achieved if a finitely–lived agent is never a producer in his last period of activity.

that: (i) $M_1(0,1) = 1$, $\rho_1(0,1) = 1 - \rho_1(1,0) = 0$; (ii) $M_t(t,t-1) = 1$, $M_{t+1}(t,t+1) = 1$, $\rho_t(t,t-1) = 1 - \rho_t(t-1,t) = 1$, and $\rho_{t+1}(t,t+1) = 1 - \rho_{t+1}(t+1,t) = 0$ for all $t \geq 1$. Thus, in every period the young agents meet with the old agents and are the producers.

Example 3 (Turnpike): $\Omega = \{0,1\} \times \mathbb{Z}$ and $N(\omega) \equiv \mathbb{N}$. The agents of type $(0,z)$ are the so–called 'stayers' and the agents of type $(1,z)$ are the so–called 'movers'. The movers move to the right—think of the agents as located in two parallel horizontal strips. For each $t \geq 1$, $M_t((0,z+t-1),(1,z)) = 1$. When t is odd, $\rho_t((0,z+t-1),(1,z)) = 1-\rho_t((1,z),(0,z+t-1)) = 1$. When t is even, $\rho_t((0,z+t-1),(1,z)) = 1-\rho_t((1,z),(0,z+t-1)) = 0$. In other words, the stayers (movers) are producers in the odd (even) periods and consumers in the even (odd) periods.

We now describe how trade takes place. A trade in a single–coincidence meeting is a pair $\tau = (\tau_c, \tau_p)$, where τ_c (τ_p) is how much the consumer (producer) transfers to his partner. Denote the set of all possible trades by $\mathcal{T}$. A trading protocol is a map $\pi = (\pi_c, \pi_p) : \Omega^2 \to \mathcal{T}$ such that $(\pi_c(\omega_c, \omega_p), \pi_p(\omega_c, \omega_p))$ is the trade that takes place when the consumer is of type ω_c and the producer is of type ω_p. We consider an economy where the trading protocol π is the same in every single–coincidence meeting. The sequence of actions in a pairwise meeting is as follows. First, the agents learn their roles. If the meeting is a no–coincidence, then it is autarkic, i.e., there is no production.[4] If the meeting is a single–coincidence, then the agents simultaneously and independently choose from {yes, no}. If both say yes, i.e., they both agree to trade, then the trade implied by π is carried out, otherwise the meeting is autarkic.

The environment we analyze is not as general as in Kocherlakota (1998). The two main differences are that Kocherlakota allows for multilateral meetings and considers more general preferences and production technologies. With regard to the assumption of bilateral meetings, it is possible, with a substantial cost in notation, to extend our analysis to the case where agents can be matched in groups of size greater than two. Later we discuss one dimension in which we can allow for more general preferences and production technologies.

[4] The assumption that production is not possible in no–coincidence meetings is made for simplicity. We obtain the same results if production is possible in such meetings.

Record–Keeping and Equilibrium Agents have access to a technology that we label memory. This technology allows an agent to observe the 'record' of his current partner right after they meet, where an agent's record is the list of his past transfers together with a description of his role—consumer, producer, or neither—in all his past meetings.

In order to describe an agent's record in detail, let $t_1(\omega) = \min\{t \geq 1 : t \in N(\omega)\}$ be the first period in which an agent of type ω is alive. If $t \in N(\omega)$, we say that an agent of type ω is of age $t - t_1(\omega) + 1$ in period t. Consider then an agent of age s. He has no record if $s = 1$. If $s \geq 2$, his record is a list with the following information: (i) his role when he was of age $s' \in \{1, \ldots, s-1\}$; (ii) how much he produced when he was of age $s' \in \{1, \ldots, s-1\}$. Denote the event that an agent is neither a consumer nor a producer by n, the event that he is a consumer by c, and the event that he is a producer by p. Since there are no double–coincidences, the roles of two agents in any meeting are perfectly correlated. The set of possible records for an agent of type ω when he is of age s is then given by $\mathcal{H}^{\omega,s} = \{\emptyset\}$ if $s = 1$ and $\mathcal{H}^{\omega,s} = \times_{r=1}^{s-1} \mathcal{H}_r^{\omega}$ if $s \geq 2$, where $\mathcal{H}_r^{\omega} = \{(n, 0)\} \cup (\{c, p\} \times \mathbb{R}_+)$. Notice that the information included in our notion of memory is a strict subset of the information contained in the notion of memory introduced in Kocherlakota (1998).

The history of an agent includes not only his record, but also the production decisions and records of all his past partners. Since the economy is populated by a continuum of agents, there is no loss in generality if we assume that an agent only conditions his behavior on his record and role, and on the record and type of his current partner. The reason for this is that all the other information in the agent's history is private to him and is independent of his current and future partners' strategies.

Let $\mathcal{H}_t = \bigcup_{\omega' \in \Omega} \mathcal{H}^{\omega', t - t_1(\omega') + 1}$ be the set of all possible records in the population in period t. A behavior strategy for an agent of type ω is a sequence $\sigma^{\omega} = \{\sigma_1^{\omega,s}\}_{s=1}^{T(\omega) - t_1(\omega) + 1}$, where $\sigma_1^{\omega,s} : \mathcal{H}^{\omega,s} \times \mathcal{H}_{t_1(\omega)+s-1} \times \Omega_{t_1(\omega)+s-1} \times \{c, p\} \to [0, 1]$ is the probability he says yes when he is of age s if his meeting is a single–coincidence. The usual measurability constraint applies. We denote the set of all behavior strategies for an agent of type ω by Σ^{ω}.

We restrict attention to symmetric strategy profiles, i.e., strategy profiles where two agents of the same type follow the same strategy. In this case, a strategy profile can be described by a map $\Psi : \Omega \to \bigcup_{\omega\in\Omega} \Sigma^\omega$ such that $\Psi(\omega) \in \Sigma^\omega$ for all $\omega \in \Omega$. We denote the set of all (symmetric) strategy profiles by Υ. For each $\Psi \in \Upsilon$ there is associated a list $\mu = \{\mu^\omega\}_{\omega\in\Omega}$, where $\mu^\omega = \{\mu^\omega_s\}_{s=1}^{T(\omega)-t_1(\omega)+1}$ and $\mu^\omega_s : \mathcal{H}^{\omega,s} \to [0,1]$, such that $\mu^\omega_s(h)$ is the fraction of agents of type ω with record $h \in \mathcal{H}^{\omega,s}$ when they are of age s. We refer to μ as the evolution of records induced by Ψ and denote by Γ the map that takes strategy profiles into their corresponding evolution of records.

Since the agents are non–atomic, their behavior does not affect the evolution of records. This means that when an agent computes his expected payoff from following a given strategy, he takes the evolution of records to be independent of his own strategy. This also implies that there exists no distinction between a strategy profile and a strategy profile for all but one agent. We assume that agents believe that the evolution of records is also independent of their own record. In particular, an agent with a record that has zero probability under the postulated evolution of records does not change his belief about the evolution of records. This corresponds to the assumption that agents believe that any off–the–equilibrium–path behavior that they observe is caused by a deviation initiated by a finite number of agents, which then has no impact on aggregate behavior.

Consider an agent of type ω who follows a strategy σ and suppose the strategy profile for the other agents is Ψ. Let $v^\omega_t(\sigma,\Psi|h,h',\omega',r)$ and $x^\omega_t(\sigma,\Psi|h,h',\omega',r)$ be, respectively, the agent's flow payoff and transfer in a period–t meeting with an agent of record h' and type ω' when his record is h and his role is r. Notice that both v^ω_t and x^ω_t depend on the trading protocol π and that $v^\omega_t(\sigma,\Psi|h,h',\omega',n) = x^\omega_t(\sigma,\Psi|h,h',\omega',n) = 0$. Now let $\eta^\omega_t(\omega'',h'')$ be the probability that an agent of type ω meets with an agent of type $\omega'' \in \Omega_t$ and record $h'' \in \mathcal{H}^{\omega'',t-t_1(\omega'')+1}$ in period t. By construction, $\eta^\omega_t(\omega'',h'') = M_t(\omega,\omega'')\mu^{\omega''}_{t-t_1(\omega')+1}(h'')$, where $\mu = \Gamma(\Psi)$. Moreover, let $\xi_t(r|\omega,\omega')$ be the probability that an agent of type ω in a period–t meeting with an agent of type ω' has role $r \in \{n,c,p\}$. Finally, let $U^\omega_{t,r}(\sigma,\Psi|h,h',\omega',\mu)$ be period–t normalized lifetime payoff to the agent if: (i) his record is h; (ii) his period–t partner is of type ω' and has record h'; (iii) his role in t is r; and (iv) the evolution of records is μ. Notice that $U^{\omega}_{T(\omega)+1,r} \equiv 0$ if $T(\omega) < \infty$. A standard argument shows that

$$U^{\omega}_{t,r}(\sigma, \Psi | h, h', \omega', \mu) = (1-\beta) v^{\omega}_t(\sigma, \Psi | h, h', \omega', r)$$
$$+\beta \sum_{\omega'', h''} \eta^{\omega}_{t+1}(\omega'', h'') U^{\omega}_{t+1}(\sigma, \Psi | (h, r, x^{\omega}_t(\sigma, \Psi | h, h', \omega', r)), h'', \omega'', \mu),$$

where $U^{\omega}_{t+1}(\sigma, \Psi | h, h', \omega', \mu) = \sum_r \xi_t(r | \omega, \omega') U^{\omega}_{t,r}(\sigma, \Psi | h, h', \omega', \mu)$.

Definition 1: *A strategy profile Ψ is a population equilibrium given a trading protocol π if for each $\omega \in \Omega$, $\Psi(\omega)$ is sequentially rational given Ψ, the evolution of records $\mu = \Gamma(\Psi)$, and π.*

The above equilibrium notion generalizes to our setting the equilibrium notion introduced in Takahashi (2008). Notice that an agent's behavior is sequentially rational if, taking into account the continuation payoffs $\{U^{\omega}_t\}$, in all of his single–coincidence meetings his decision of whether to agree to trade or not is optimal given his partner's behavior.

3 First–Best with Memory

Here we show how memory can be used to construct population equilibria that achieve the first–best. For this, we consider the trading protocol π^* such that $\pi^*(\omega_c, \omega_p) = (0, x^*)$.

First–best with one–period memory We start by constructing a population equilibrium that is informationally minimal in the sense that the only part of an agent's record that is required to determine behavior is the information from the last period. Our equilibrium construction borrows ideas from Takahashi (2008).[5] An important difference between our environment and Takahashi's environment is that in the latter the stage game in a pairwise meeting is symmetric.

We say that an agent is in state b if in the previous period he was a producer, but did not transfer x^* to his partner. Otherwise, we say that the agent is in state g. Our candidate equilibrium is the strategy profile Ψ^* where: (i) all agents start in state g; (ii) a consumer

[5] See also Rosenthal (1979), Kalai et. al (1988), Bhaskar (1998), and Olszewski (2007) for similar equilibrium constructions.

always agrees to trade; (*iii*) a producer agrees to trade if his partner is in state g; (*iv*) in period t, a producer agrees to trade with probability $q^t_{b,\omega'}$ if his partner is of type ω' and is in state b. Observe that Ψ^* implements the first–best and that the probabilities $q^t_{b,\omega'}$ only need to be defined for $t \geq 2$.

By construction, an agent's action in Ψ^* is independent of his record and type. This implies that in every period an agent's expected payoff does not depend on the record and type of his partner. Since producers need to vary their actions (yes or no) depending on their partners' record, it must be that a producer is always indifferent between agreeing to trade or not if Ψ^* is to be an equilibrium.

Let $V^\omega_t(\theta) = U^\omega_t(\sigma, \Psi | h, h', \omega', \mu)$, where θ is the state implied by the record h and $\mu = \Gamma(\Psi^*)$. First notice that given μ and the behavior of the producers in Ψ^*, it is strictly optimal for consumers to say yes in any meeting. Let us now consider the producers. From the previous paragraph, we need that

$$-(1-\beta)x^* + \beta V^\omega_{t+1}(g) = \beta V^\omega_{t+1}(b) \tag{1}$$

for each $t \geq 1$ and $\omega \in \Omega_t$ such that there exists $\omega' \in \Omega_t$ with $M_t(\omega, \omega')\rho_t(\omega, \omega') > 0$. If condition (1) is satisfied, we then have that

$$\begin{aligned} V^\omega_t(\theta) &= \sum_{\omega' \in \Omega_t} M_t(\omega, \omega')\xi_t(n|\omega, \omega')\beta V^\omega_{t+1}(g) \\ &\quad + \sum_{\omega' \in \Omega_t} M_t(\omega, \omega')\xi_t(p|\omega, \omega')\beta V^\omega_{t+1}(b) \\ &\quad + \sum_{\omega' \in \Omega_t} M_t(\omega, \omega')\xi_t(c|\omega, \omega')\{(1-\beta)q^t_{\theta,\omega}u(x^*) + \beta V^\omega_{t+1}(g)\}, \end{aligned}$$

where $q^t_{g,\omega} \equiv 1$. Indeed, the period–t lifetime payoff to an agent of type ω who is in state θ is $\beta V^\omega_{t+1}(g)$ if his period–t meeting is a no–coincidence and $(1-\beta)q^t_{\theta,\omega}u(x^*) + \beta V^\omega_{t+1}(g)$ if he is a consumer in his period–t meeting—in both cases the agent's state in $t+1$ is g. Similarly, since a producer must always be indifferent between agreeing to trade or not, the period–t lifetime payoff to an agent of type ω who is a producer in period t is $\beta V^\omega_{t+1}(b)$. Thus, the indifference condition (1) is satisfied if for all $t \geq 1$ and $\omega, \omega' \in \Omega_t$ such that

$M_t(\omega, \omega')\rho_t(\omega, \omega') > 0$,

$$1 - q_{b,\omega}^{t+1} = \frac{x^*}{\beta \left[\sum_{\omega' \in \Omega_{t+1}} M_{t+1}(\omega, \omega')\rho_{t+1}(\omega', \omega)\right] u(x^*)}. \tag{2}$$

A necessary condition for (2) to be satisfied is that its right–hand side is smaller than one when $\beta = 1$. Let $\widetilde{\Omega}_t = \{\omega \in \Omega_t : \exists \omega' \in \Omega_t \text{ s.t. } M_t(\omega, \omega')\rho_t(\omega, \omega') > 0\}$ be the set of types who can be producers in period t. Notice that if $\omega \in \widetilde{\Omega}_t$, then an agent of type ω lives at least until period $t+1$. Now let $\kappa = \inf\{\sum_{\omega' \in \Omega_{t+1}} M_{t+1}(\omega, \omega')\rho_{t+1}(\omega', \omega) : \omega \in \widetilde{\Omega}_t,\ t \geq 1\}$. Notice that $\kappa \geq \varepsilon > 0$ by assumption. We then have the following result.

Proposition 1. *Suppose that $x^*/u(x^*) < \kappa$. The strategy profile Ψ^* with the probabilities $q_{b,\omega}^t$ given by* (2) *is a population equilibrium as long as $\beta \geq x^*/\kappa u(x^*)$.*

Notice that $\kappa = 1$ in the overlapping generations environment of Example 2 and in the turnpike environment of Example 3. Thus, the condition $x^*/u(x^*) < \kappa$ is automatically satisfied in both cases (since $u(x^*) > x^*$ by the definition of x^*). In the random matching environment of Example 1, $\kappa = \rho$, the probability that an agent is a consumer in a meeting.

One dimension in which our environment is less general than the environment in Kocherlakota (1998) is that we don't allow heterogenous preferences and production technologies. It is possible to extend Proposition 1 to the case where this type of heterogeneity is present if we modify our notion of memory to also include the utility functions of an agent's past partners.[6] Notice that this type of information is already present in the notion of memory introduced in Kocherlakota (1998).

Since producers must always be indifferent between agreeing to trade or not in the equilibrium of Proposition 1, the punishment to an agent with a bad record cannot be too severe. This implies that the discount factor needed to sustain this equilibrium cannot be too small. If, as in Kocherlakota (1998), an agent had not only access to his partners' records, but also to the record of all the agents that had direct or indirect contact with his partners, then it

[6] The information about production disutilies is not necessary since it has no impact on an agent's decision of whether to agree to trade or not when he is a producer.

is possible to construct an equilibrium where producers who do not agree to trade suffer the worst punishment possible, permanent autarky.[7] Naturally, the discount factor β_* necessary to sustain the first–best under the threat of permanent autarky is lower than the discount factor of Proposition 1. We compute β_* below.

Consider a strategy profile that achieves the first–best. The period–t normalized expected lifetime payoff to an agent of type ω is

$$\widetilde{V}_t^{\omega} = (1-\beta)\sum_{s=1}^{\infty}\beta^{s-1}\sum_{\omega'\in\Omega_{t+s-1}} M_{t+s-1}(\omega,\omega')\gamma_t(\omega,\omega'), \tag{3}$$

where $\gamma_t(\omega,\omega') = \rho_t(\omega',\omega)u(x^*) - \rho_t(\omega,\omega')x^*$ is the agent's expected flow payoff in period t if he meets with an agent of type ω'. A producer of type $\omega \in \Omega_t$ agrees to trade in period t only if $-(1-\beta)x^* + \beta\widetilde{V}_t^{\omega} \geq 0$. Let $\beta_{\omega,t}$ be the lowest value of β for which the last inequality holds. Then, $\beta_* = \sup_{\omega,t}\beta_{\omega,t}$. It is simple to show that $\beta_* \leq x^*/\kappa u(x^*)$. Moreover, a straightforward consequence of the reasoning leading to the derivation of β_* is that if $\beta < \beta_*$, then there exists no equilibrium that implements the first–best.

In the remainder of this section we consider environments where agents do not know their roles before they are matched. We show that if agents have the choice of opting out of a meeting before they learn their roles and this choice is part of their records, then there exists a population equilibrium that achieves the first–best as long as $\beta \geq \beta_*$. This equilibrium, unlike that of Proposition 1, makes use of all the information in the record of an agent.

First–Best with Full Memory Suppose the sequence of events is as follows. First, agents observe their partners' records and simultaneously and independently announce 'in' or 'out'. If at least one agent opts out, then the meeting is autarkic. If both agents announce in, then the sequence of events in the meeting is as before: agents observe their partners' types and learn their roles and then decide whether they want to trade or not.

[7] The equilibrium is as follows. There are two possible states for an agent, g or b. All agents start in g. A consumer agrees to trade regardless of his and his partner's state. A producer in state b never agrees to trade. A producer in state g agrees to trade if, and only if, his partner is in state g. State transitions are as follows. The state b is absorbing. Consider now an agent in state g. His state stays the same if he is not a producer. If he is a producer, his state stays the same if, and only if, he behaves as prescribed above.

An agent's record is now a list with the following information: his past announcements (in or out), roles, and transfers. Notice that the role of an agent in a meeting is determined whether he opts out of the meeting or not. As before, we denote the set of possible records for an agent of type ω and age s by $\mathcal{H}^{\omega,s}$ and a behavior strategy for an agent of type ω by σ^{ω}. Notice that now $\sigma^{\omega} = \{(\sigma_0^{\omega,s}, \sigma_1^{\omega,s})\}_{s=1}^{T(\omega)-t_1(\omega)+1}$, where $\sigma_1^{\omega,s}$ is the same as when the opt–out option is not present and $\sigma_0^{\omega,s} : \mathcal{H}^{\omega,s} \times \mathcal{H}_{t_1(\omega)+s-1} \to [0,1]$ is the probability that an agent of type ω opts out of his age–s meeting as a function of his and his partner's record. The notion of a population equilibrium is identical to that of Definition 1. In particular, it does not depend on what information is included in an agent's record

First observe that there still exists a one–period–memory population equilibrium that achieves the first–best as long as agents are patient enough. For this, say that an agent is in state b if in the previous period he either announced out or he announced in and was a producer, but did not transfer x^* to his partner. Otherwise, say that the agent is in state g. Consider now the strategy profile where agents announce in in every meeting and then behave as in Ψ^* with the probabilities $q_{b,\omega}^t$ given by (2) and suppose that $\beta > x^*/\kappa u(x^*)$. From the proof of Proposition 1, the strategy profile just described is an equilibrium as long as it is optimal for agents to always announce in. This, however, is immediate given that the lowest continuation payoff to an agent who announces in is when he is producer, which is the same payoff he obtains if he announces out.

Now we construct a full–memory population equilibrium that achieves the first–best as long as $\beta \geq \beta_*$. For this, say that an agent is in state b if in any of his previous meetings he announced in and was a producer, but did not transfer x^* to his partner. Otherwise, say that the agent is in state g. Consider then the strategy profile Ψ^{**} where: (i) all agents start in state g; (ii) an agent announces in only if he and his partner are in state g, otherwise he announces out; (iii) if neither he nor his partner opts out, an agent always agrees to trade in a single–coincidence meeting. Observe that Ψ^{**} makes use of all the information present in an agent's record.

Proposition 2. Ψ^{**} *is a population equilibrium if* $\beta \geq \beta_*$.

Proof: Let $V_t^\omega(\theta)$ be the period–t normalized expected lifetime payoff to an agent of type ω who is in state $\theta \in \{b, g\}$, computed before meetings take place. Then, $V_t^\omega(b) \equiv 0$ and $V_t^\omega(g)$ is given by (3). Let us begin with a producer's decision of whether to trade in a period–t meeting in which he and his partner did not opt out. If his type is ω, he agrees to trade if, and only if,

$$-(1-\beta)x^* + \beta V_{t+1}^\omega(g) \geq \beta V_{t+1}^\omega(b) = 0.$$

A necessary and sufficient condition for this is that $\beta \geq \beta_{\omega,t}$. Since this inequality must be satisfied for all $t \geq 1$ and all $\omega \in \Omega$, Ψ^{**} is an equilibrium only if $\beta \geq \beta_*$. Consider now an agent's decision of whether to opt out of a period–t meeting. There are three cases to consider. First suppose that he and his partner are in state g. Since, his continuation payoff is the same whether he announces in or out (his state in next period will be g), it is optimal for him to announce in. Now suppose that he has a bad record. Since state b is absorbing and he expects his partner to announce out, he weakly prefers to announce out. Finally, suppose that he is in state g, but his partner is in state b. Since he expects his partner to announce out, his flow payoff is zero regardless of his announcement. Moreover, if he announces in and turns out to be a producer, his record will be bad in the next period. Thus, the agent strictly prefers to announce out. Thus, Ψ^{**} is an equilibrium if $\beta \geq \beta_*$. $\square$

4 Deconstructing Memory and the Role of Money

Monetary theory emphasizes two frictions that are necessary for the essentiality of money, limited commitment and limited record–keeping. Proposition 1 shows that as long as agents are patient enough, money fails to be essential even if the record–keeping technology registers much less information than in Kocherlakota (1998).[8] In this sense, Proposition 1 parallels the main result of Araujo and Camargo (2009), who study a competing notion of memory introduced in Kocherlakota and Wallace (1998).

[8]The equilibrium of Proposition 2 uses more information than the equilibrium of Proposition 1. Nevertheless, it is still the case that no information about the histories of the direct and indirect partners of an agent's past partners is needed.

The notion of memory we consider contains information about an agent's past actions and roles. Information about past actions is clearly necessary for memory to implement non–autarkic allocations: if an agent cannot observe the previous actions of his current partner, then autarky is the only possible population equilibrium. In this section we show that if memory does not contain information about past roles, then the first–best is not an equilibrium outcome in a large class of environments. We then discuss how in such environments money can be used to implement the first–best. Thus, money can be better than memory if the latter only contains information about past actions.

We start with an example. Consider the overlapping generations economy of Example 2 in Section 2. In every $t \geq 1$ a mass one of agents who live for two periods enters the economy. An agent is young in his first period of life and old in his second. There is also a mass one of agents in the economy in $t = 1$ who live for one period only, the initial old. In each period the old agents are randomly and anonymously matched with the young agents and every such match is a single–coincidence with the young agent as the producer. All old agents derive utility $u(x)$ from consuming x units of the good and all young agents incur disutility x from producing x units of it.

Proposition 1 shows that as long as the agents are patient enough, there exists a population equilibrium that achieves the first–best. Since in this particular example every meeting is a single–coincidence, the information about roles in an agent's record is redundant. Thus, the first–best can be achieved even if the record of an old agent in $t \geq 2$ only includes how much he produced when young.

Suppose now that with probability $1 - \rho$, with $0 < \rho < 1$, a match between a young agent and an old agent is a no–coincidence. Proposition 1 is still valid in this case. However, regardless of β, the first–best cannot be achieved if an old agent's record does not include his role when young. The reason is simple: a young agent in a match cannot distinguish an old agent who did not produce in a single–coincidence meeting when young from an old agent who was in a no–coincidence meeting when young, i.e., defectors cannot be identified. This implies that the only way to sustain production in single–coincidences is to punish young agents who participate in no–coincidences, i.e., to have inefficient punishments.[9]

The next proposition shows that the intuition from the above example can be extended to any environment in which not all meetings are single–coincidences.

Proposition 3. *Suppose that an agent's record only includes his past production decisions. If there exists $t \geq 1$ and $\omega \in \Omega_t$ such that $0 < \sum_{\omega' \in \Omega_t} M_t(\omega, \omega')\rho_t(\omega, \omega') < 1$, then there exists no population equilibrium that achieves the first–best.*

Proof: Suppose not, i.e., in every single–coincidence meeting on the path of play the producer transfers the efficient amount of the good to the consumer. Let $t \geq 1$ and $\omega \in \Omega_t$ be such that $0 < \sum_{\omega' \in \Omega_t} M_t(\omega, \omega')\rho_t(\omega, \omega') < 1$. This implies that in period t the event that an agent of type ω is a producer and the event that an agent of the same type is a consumer have both positive probability. Since records only include information about production decisions, the continuation payoff to an agent of type ω who participates in a single–coincidence meeting in t cannot depend on whether he produces the efficient quantity or produces zero. Thus, this agent has no incentive to produce a positive amount of the good, a contradiction. $\square$

Proposition 3 shows that if memory only includes information about past actions, then only inefficient equilibria are possible in an environment in which not all meetings are single–coincidences. This inefficiency can be ameliorated if there exists a technology that allows an agent to somehow communicate his past trading opportunities to his current partner. This communication is trivially possible if we directly incorporate the information about past trading opportunities in the definition of memory. We now argue that money is another technology that enables this communication, a dimension of money that has been overlooked by the literature.

[9]For example, the argument leading to Proposition 1 shows that the following strategy profile is an equilibrium as long as $\beta > x^*/\rho u(x^*)$, where $x^* > 0$ is the unique maximizer of $u(x) - x$: (i) a young agent in a single–coincidence meeting in $t = 1$ produces x^* to his partner; (ii) a young agent in a single–coincidence meeting in $t \geq 2$ produces x^* with probability one if his partner produced x^* in the previous period and produces x^* with probability $q = 1 - x^*/\rho u(x^*)$ if his partner produced $x \neq x^*$ in the previous period. If production were possible in no–coincidence meetings, an alternative way to sustain non–autarkic allocations would be to have production taking place in every meeting, i.e., to have inefficient production.

We illustrate our point in the context of the overlapping generations example given above where no–coincidences occur with positive probability in every meeting. Assume that all the initial old are now endowed with one unit of money. For this particular example, whether money is divisible or not is of no consequence. Consider then the following trading arrangement. A young agent in a single–coincidence transfers the efficient amount to his partner in exchange for one unit of money. An old agent always transfers one unit of money to his partner, unless he is in a single–coincidence and his partner does not produce to him. Notice that old agents with money are indifferent between the amount of money they transfer to their partners. Thus, it is optimal for them to behave as described. Given the behavior of the old agents, it is optimal for a young agent to produce the efficient amount to his partner whenever he is in a single–coincidence, otherwise he never consumes when old. Clearly, this arrangement implements the first–best as long as agents are patient enough. In particular, if memory is just a record of past actions, then money does better than memory.

In the above example money helps achieving the first–best since, unlike memory of past actions, it allows one to distinguish between an old agent who could not produce when young from an old agent who could produce when young but chose not to. Money can play this role because an old agent does not care about the amount of money he keeps to himself. This, however, is particular to an overlapping generations economy. The key question is whether money can act as a memory of past trading opportunities in other environments. Kocherlakota (2002) shows that this is possible in a class of environments that contains the ones we consider as long as money is perfectly divisible. The mechanism that supports the first–best constructs a one–to–one mapping between individual histories and the decimal expansions of money holdings, allowing money to become a complete record of an agent's history. In particular, an agent's money holdings at any given point in time reveal whether he always produced the efficient amount in every single–coincidence meeting in which he was a producer.

An aspect of Kocherlakota's construction is that monetary transfers in any match must converge to zero over time, so that individuals never run out of money. More recently, Hu, Kennan, and Wallace (2009) show that in the Lagos–Wright environment, see Lagos and Wright (2005), there exists a monetary equilibrium in which monetary transfers are constant over time that achieves the first–best. The difference from Kocherlakota (2002) is that the agents can use the centralized market to replenish their money holdings, thus eliminating the need for monetary transfers to diminish over time. Even though we don't consider the Lagos–Wright environment in this paper, it is straightforward to extend our analysis to cover such environment. The key observation is that individual actions cannot be observed in the centralized market, and so have no impact on prices. Thus, in the absence of money, the presence of a centralized market does not change the set of allocations one can achieve in the decentralized market.

5 Conclusion

This paper investigates the relationship between money and memory. For this we study a notion of memory that only includes information about an agent's past actions and trading opportunities. Our first result is that money can fail to be essential even if record–keeping is minimal. We then show that if information about trading opportunities is not part of an agent's record, then money can outperform memory. This shows that the societal benefit of money lies not only on being a record of past actions, but also on being a record of past trading opportunities.

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