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WHAT'S BEYOND THE
HUMAN DEVELOPMENT INDEX ?

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What's Beyond the Human Development Index ?

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Abstract: In this note, we firstly discuss Chakravarty's (2003) axiomatic
foundation of the Generalized Human Development Index (GHDI) then we pro-
pose a different set of axioms through which a class of Foster-Shorrocks-like
Human Development Measures phases out. As we will argue, this class better
deals with some recent philosofical advancements of the Capability Approach.

1   Introduction

Well being indices have drawn large attention in the last decade. Since, in
early 90ies, the *Human Development Index* (*HDI*) was proposed by a group of
scholars (among them A.Sen, 1998 Nobel laureate in economics) and the UNDP,
several non-income-oriented statistics have been normally computed by analyz-
ers, researchers and policy makers' advisors. Despite that many criticisms to
and several refinements of the HDI has been suggested[1], all these contributions
may be viewed by a policy-oriented perspective, that is constructing a sim-
ple and operatively useful index for public policies' auditing, monitoring and
reporting.

This surprising proliferation of competing indices of human develop-
ment (simply additive measures, inequality-adjusted HDI, multiplicative hu-
man development indices and so on) have raised issues related to their desirable
structure, their appropriateness with respect to human development approach's

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[1]For excellent readings on the HDI and its evolution see Fukuda-Parr and Shiva Kumar
(2003). Technical refinements of the HDI can be found in Hicks (1997), Noorbakhsh (1998),
Neumayer (2001) and Panigrahi and Sivramkrishna (2002).

philosophical foundations as well as their consistency with axiomatic properties that, at least in principle, should be satisfied. Contemporaneously, an axiomatic approach to multidimensional well being measures has been developed[2], offering a natural framework for analysing axiomatically properties of human development measures.

In a recent article, Chakravarty (2003) has started such an axiomatic foundation. His findings show that a *Generalized HDI (GHDI)* (i.e. a r-power and n-dimensions version of the HDI with $r \in [0;1]$) generally fulfills some nice properties (see below for a discussion) measuring human development better than any other multidimensional achievement index. Nevertheless, his results may be criticized using capability approach's recent advancements. This will allow us to recognize several alternative routes to refinements of a human development measure. As it will be argued, one alternative to a GHDI is to use a class of human development mesures inspired to Foster and Shorrocks' (1991) index of aggregate poverty.

The paper is organized as follows. In Section 2, Chakravarty's axioms are briefly discussed using Alkire's (2002b) foundational account of the Senian capability approach to human development. In Section 3, possible paths for extensions fo the HDI are discussed and compared, while in Section 4 an analytical set up is suggested consistently with one route of refinement. Usually, Section 5 concludes.

2 Beyond the HDI: the Generalized HDI

As well known, different approaches in defining human development co-exist[3]. Following Sen (1985) and Alkire (2002b), human development can be shortly defined as a capability set expansion which increases individuals' freedom to achieve valuable functionings. Functionings are a person's states of being, doing and becoming (like being able to be well-nourished, to move freely or to have the social bases of self-respect) through which individuals' living conditions can be described. Their achievement levels are outcome-oriented measures of human development and to each functioning a dimension of human development can be reasonably matched. These dimensions are related to some core human values able to give agents intrinsic reasons for action. Hence, as Alkire (2002a) underlines given their nature *"dimensions of human development are irreducible, non-hierarchical and incommensurable kinds of human ends"*[4].

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[2]For axiomatic foundation of multidimensional poverty measures see Tsui (2002). A class of indices for measuring well being improvements consistently with some desirable properties has been shown by Chakravarty and Mukherjee (1999).

[3]For an introduction to alternative notions of well being see Quizilbash (1998). For a survey on alternative theories of human development see Alkire (2002a).

[4]Furthermore, Crocker (1995) emphasises that, in the Capability Approach, "valuable capabilities are *incommesurable* in two senses. First, we cannote satisfy the need for one of them by giving a larger amount of another one. [...] Secondly, the plural goods are

Once this point of view is accepted, reasonable doubts about the appropriateness of Chakravarty's property of consistency in aggregation (CIA) phase out. CIA is stated as additive separability of the well being index with respect to achievement indicators. Thus, strong substitutability among dimensions of human development is entailed at a variable (depending on the degree of achievement of each functioning) social rate of substitution. The latter is defined with respect to two dimensions and generally unaffected by changes in other achievements. Our claim is: if dimensions of human development are incommensurable, how can be possible to compute continuously variable marginal rates of substitution ? More explicitly, in which way a 1000$ loss can be traded off with any increases in self-respect ? And a reduction in earned income of 1100$?

Surely, weak substitutability between dimensions of human development may exists within societies since the conversion of individual resources and accesses in achieved functionings is a *socially-embedded transformation* process culturally, socially and institutionally (in a neo-institutional economics sense) shaped. For instance, in western cultures renouncing at a certain degree of participative social activities for an higher labor status is seen as perfectly consistent with human flourishing. The same cannot be said for world wide rural cultures. Nevertheless, it is unlikely that functionings' substitutability might vary continuously. Social inertia may be at work.

Moreover, weak substituability among functionings it seems to be an acceptable assumption for developing countries in which survival is under discussion and thus humans may decide to neglect one dimension of human development for higher attainments in others, more strategical functioning achievements. The same cannot be said for wealthy countries where increasing one functioning achievement level without augmenting the others may frustrate human development (an idea made explicit in the UNDP's assumption of decreasing returns of income). In these cases, non-sostituability among functionings might be a more appropriate assumption.

Chakravarty (2003) also imposes a symmetry condition (SYM), i.e. given some achievement indicators, any permutation of these does not affect the humand development measure. Once more, how may it be argued that socially embedded transformation precesses works symmetrically for a well-endowed but severely ill person and for poor but healthy agents? If it was the case, institutions and conversion processes should be functioning neutral something far reached to be verified[5]. This axiom is used in Theorem 3's proof (see p.111) where a multidimensional well being index is assumed to be well-specified with respect to each component by a symmetric scalar function[6]. Thus, taken to-

incommensurable in the sense that they are *irreducible* to some common and deeper measure such as utility". Hence, irreducibility suggests us to employ *fuzzy* measures of functionings achievement instead of more structured measures of deprivation.

[5] For instance, Nussbaum (2003) argues that institutionally located patterns of human development largely depend on agents' level of education.

[6] Additionally, in Theorem 3's proof identical attainments are assumed in order to apply a

gether, additive separability and symmetry of a human development index involve that each dimension can be autonomously improved and high levels of development are attainable even if some functionings are completely non-achieved. Nevertheless, this position not only neglects human development dimensions dynamics (i.e. an increase in one dimension may increases other achievements that allow to better exploit the first functioning), but also ignores that a *balanced pattern of human development* is strongly recommended by social philosophers[7]. As it will be clearer below, the GHDI is simply unable to give value to balanced paths of development because of its technical structure.

3   Alternative Extensions of the HDI

Deeper research is needed, as Sen (2003) underlines, for indices of human development. After more than ten years from their introduction, HDIs has to be refined in order to face some unpleasant idiosyncrasies with recent advancements of the Capability Approach. Our discussion shows some of these.. Firstly, it might be enlarged the informative basis of a measure of human development, including a larger set of dimensions in the index. Secondly, it might be stressed that satisfying measures has to ensure non-sustituability among dimensions since their underlying values are largely non-commensurable. Finally, it might be enclosed in human development measures a positive sensitivity to balanced patterns of human development.

Existing literature on well being measures suggests several routes to refine the HDI. To map these alternative paths we can use three usefull dichotomies: (a) *internal* vs *external extensions*; (b) *multi-level* vs *mono-level refinements* and (c) *axiomatic* vs *pragmatic foundations.*
Internal extensions (like Chakravarty (2003)'s one) usually enlarges informational bases of the HDI to a n-tuple of relevant development dimensions. Each dimension (i.e. being healthy) is characterized through several variables (health indicators) and for each of them a fuzzy measure[8] of achievement is defined and computed. Then, an aggregation procedure is undertaken using set operators ($\cap, \cup$,etc.). In contrast, external extensions maintain an elementary HDI's structure, with only few dimensions included, accompanying it with additional simple indices focused on other crucial featues of human development

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normalization axiom (N) such that any well being index determined over identical achievement indicators is equal in value to them. Unfortunately, this assumption is surprisingly hidden some rows later for getting the result.

[7]See, among the others, Nussbaum (1998).

[8]Fuzzy set theory was originally proposed by Zadek (1965). For an application to human development see Chiappero-Martinetti (2000). For a fuzzy-approach to multidimensional poverty see Cheli and Lemmi (1995).

processes (like, gender equity, political voice etc.)[9]. Computational simplicity is traded-off with methodological appropriateness.

By describing human development through attainments in n relevant dimensions of human flourishing, from basic (like being well-nourished) to more complex ones (like being able to get social recognition), internal extensions normally propose an horizontally-oriented and mono-level enlargement of the HDI. However, also a vertically-oriented and multi-level route can be followed. For instance, Anand and Sen (1994) propose a multi-level refinement of the HDI in order to deal with *non-basic* human capabilities. Their proposal is to add to each HDI's dimension one or more indicators of human development (i.e. not only adult literacy but also secondary or terziary school enrollment, not only life expectacy but also under-5 and maternal mortality). The idea here is not to enlarge observed dimensions, but to refine and deepen our comprehension and description of existing ones. Similarly, additional indices used in external extensions of the HDI can be focused on different fields of human empowerment (like political voice) or on specific adjustments of what reported by the HDI (i.e. adjusted human development with respect to gender inequalities).

Finally, in all above cases researchers may follow an *axiomatic approach*, dealing with desirable properties formally stated, or a *pragmatic approach*, by trying to improve the algebraic structure of the HDI in order to accomodate some revealed statistical problems like spread dependence or limits dependence (i.e. HDI ranking that changes .whether, respectively, indicator value spread or upper and lowe bounds change). The *Generalized HDI* discussed in the previous section is grounded through axiomatization, while the *Adjusted HDI*, recently suggested by Panigrahi and Sivramkrishna (2002), is thought as a measure to practically overcome existing weaknesses of the HDI.

In the next section, we suggest an *internal/mono-level/axiomatic* path through which it will be possible to get foundations for a class of human development measures different by the GHDI. As it will be clearer below, a straightforward reformulation of well-known results of the poverty measurment literature will lead us to a Foster and Shorrocks' (1991)-inspired class of human development measures.

4 A Foster/Shorrock Class of Human Development Measures

Behind its drawbacks, Charavarty's contribution opens a new interesting route of research. In what follows, we suggest a slightly different set up for an axiomatic foundation of human development measures. As it will be argue,

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[9]For example, UNDP computes different simple indices (i.e. the Gender Empowerment Measure or Political freedom Index) for dimensions of human development not represented in the HDI.

this set up is more consistent than Charavarty's one with respect to recent advancements in (Sen's) human development theory.

Firstly, instead of assuming a lower and an upper bound for our measure (indeed a critical assumption as the HDI's history shows !), we use *fuzzy* indicators of functioning achievement derived from a choosen membership function. This allows us to rule out unpleasant sensivity of measurements to changes in indicators' limits as those originally discussed in Kelly (1991). Secondly, we propose a *population-symmetry axiom* instead of a *achievement-symmetry* one. Consistently with what discussed above, it is our opinion that permutations only over who achieves and not over what is achieved can be seen as irrelevant for an overall measure of HD. Thirdly, we consider a *sub-group-consistency axiom* which ensures consistency in aggregation between reference groups not among human development dimensions[10]. This property of a HD measure may result useful in targeted human poverty reduction interventions, a natural scope of human development analyses and measures. Finally, an axiom which gives value to *balanced achievements of human development* dimensions (through a slight modification of the well-known Pigou-Dalton principle) is proposed. As we will see, this set of axioms leads, with very few analytic, to a new version of a fairly famous multidimensional poverty measure.

Let us suppose that N individuals recognized as valuable K functionings. For any of these, and with respect to any agent $(i = 1, ..., N)$, a fuzzy indicator $e_{ij} \in [0; 1]$ shows achievement levels with 0 for a totally non-achieved functioning and 1 for its full achievement. Hence, the class of $n \times k$ matrices $\Lambda(n; k)$ such that $e_{ij} \in R_+$ is defined as:

$$\Lambda := \left\{ F \in \bigcup_n \bigcup_k \Lambda(n; k) \right\} \tag{1}$$

with $F := [e_{ij}]_{\substack{i=1,...,n \\ j=1,...,k}}$ named *functioning achievement matrix*. Hence, a *human development measure (HDM)* is defined as a map $H : \Lambda \to R_+$ supposed twice continuously differentiable. As discussed above, our axioms for a HDM are the following:

(a) POPULATION SYMMETRY (PSYM): Given $F \in \Lambda$, $H(F) = H(\Pi F)$ where Π is a $n \times n$ permutation matrix

(b) REPLICATION INVARIANCE (RI): Given $F \in \Lambda$, $H(F) = H(F^\alpha)$ where $F^\alpha = \alpha F$ with $\alpha \geq 1$

(c) MONOTONICITY (M): Given F and $\widetilde{F} \in \Lambda$ where F is obtained from $\widetilde{F}$ increasing any one attribute, then $H(F) \geq H(\widetilde{F})$

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[10] Sub-groups inconsistency of the HDI has been recognized as one of its main limits by Anand and Sen (1994).

(d) SUBGROUP CONSISTENCY (SGC): For any n and k such that F_1 and $\widetilde{F}_1$ are $n \times k$ matrices and F_2 and $\widetilde{F}_2$ too with $F^T := \left[F_1^T ; F_2^T \right]$ and $\widetilde{F}^T := \left[\widetilde{F}_1^T ; \widetilde{F}_2^T \right]$ then $H(F) > H\left(\widetilde{F}\right)$ whenever $H(F_1) > H\left(\widetilde{F}_1\right)$ and $H(F_2) = H\left(\widetilde{F}_2\right)$

Hence, adapting Foster and Shorrocks' (1991) Proposition 1, we can easily remind that:

Claim 1 *A HDM satisfies PSYM, RI, M and SGC if and only if it has the following form*

$$H(F) = G \left[\frac{1}{n} \sum_{i=1}^{n} \theta\left(\underline{e}_i\right) \right] \qquad (2)$$

where $\underline{e}_i$ is the individual achievement vector, G a strictly increasing and continuous function and $\theta := \Lambda(1) \rightarrow R_+$ is a continuous and non-decreasing function with respect to each attribute. Furthermore, $\theta\left(\underline{e}_i\right) = 0$ when $e_{ij} = 0$ for all $j = 1, ... k$.[11]

Whereas we add an axiom related to balanced human development, our measure in (2) takes a particular form. The next property employs an adapted version of the Pigou-Dalton principle in order to give value to balance patterns of development.

(e) BALANCED HUMAN DEVELOPMENT (BHD): For all $i = 1, ..., n$, given F and $\widetilde{F} \in \Lambda$ with F obtained from $\widetilde{F}$ through a Pigou-Dalton change in functioning achievements (that is, given $\underline{e}_i$, $\underline{e}_i'$ and two achievements m and p with $e_{ih} = e_{ih}'$ for any $h \neq m, p$ with $e_{im} + e_{ip} = e_{im}' + e_{ip}'$ and $\left| e_{im}' - e_{ip}' \right| < \left| e_{im} - e_{ip} \right|$) then $H(F) \geq H\left(\widetilde{F}\right)$.

By adding (e) to previous axioms, it can be stressed the following:

Claim 2 *The class of HDM in (2) satisfies BHD if and only if θ is a concave function*[12].

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[11] The proof is a simple modification of Foster and Shorrocks' (1991) one. The main difference is in axiom M which determines that θ is a non-decreasing function of each e_{ij}. In order to show this, we have to consider that the domain of θ is a closed and compact set. In extreme cases (*i.e.* $\underline{e}_i = \underline{0}$ and $\underline{e}_i = \underline{1}$), M holds only if $\theta(\underline{0}) = 0$, $\theta(\underline{1}) = 1$ and θ is a continuous function. The last conditions are sufficient for θ be non-decreasing in its arguments. Necessity comes straightforward.

[12] Once more, the proof is a slight modification of Kolm's (1977) result for the case of multidimensional inequality indices. The concavity of θ (instead of its convexity) can be shown since, in our set up, a mean-preserving Pigou-Dalton transfer among functioning achievements has to increase the HDM instead of decresing measured inequality.

As it may be easily checked, the GHDI (and *a fortiori* the HDI) does not satisfies axiom BHD given the additive form of θ [13]. On the contrary, a class of *Foster-Shorrocks-like HDM* deals better with requirements of weak (or null) substitutability among functionings and balanced patterns of human development. An operatively useful example, originally suggested by Sagar and Najam (1998), of an index of human development which belongs to our class is:

$$HDM = \frac{1}{N} \sum_{i=1}^{N} \left[\prod_{j=1}^{K} e_{ij} \right] \tag{3}$$

As these authors point out, its multiplicative form radically changes HDI-ranking among countries.

5   A Final Remark

In this note possible extensions of the HDI have been discussed and compared. By starting from the recently proposed Generalized HDI, some routes to overcome existing indices of human development's drawbacks has been explored as well. This has led us to a taxonomy of possible reaserch directions. Among them, axiomatic foundations for a human development measure can be searched taking as reference recent advancements in Sen's capability approach. In particular, we argue that by imposing sub-group consistency and a balanced patterns of HD condition a Foster-Shorrock's class of measures easily emerge.

Finally, our set up suggests that different classes of human development indices can be axiomatically characterized by using different sets of axioms and relative normative judgements on what human flourishing actually means. Such a *non-invariance property* not only constitues a peculiar fuzziness and complexity of measuring an hostic concept like human development, but it seems perfectly consistent with the high variety of methodological viewpoints and analytical persepctives which distinguish the Capability Approach. Economists, sociologists or phsycologists very likely have different visions about what really mean human empowerment and well.being. Hence, what they need beyond the HDI are classes of measures able to emphasize several aspects of human development processes. An axiomatic approach to measurement, by precisely defining and intersecting axioms, would guarantee adequate parsimony of assumptions and no excessive proliferation of partially overlapping indeces.

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[13] The same can be said for measure inspired to the general fuzzy index of poverty proposed by Cerioli and Zani (1990) where θ is not concave.

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