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CONSTRUCTING A COINCIDENT INDEX OF BUSINESS CYCLES WITHOUT ASSUMING A ONE-FACTOR MODEL

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SUMMARY

The Stock–Watson coincident index and its subsequent extensions assume a static linear one-factor structure for the component indicators. Such assumption is restrictive in practice, however, with as few as four indicators. In fact, such assumption is unnecessary if one defines a coincident index as an estimate of latent monthly real GDP. This paper considers VAR and factor models for latent monthly real GDP and other coincident indicators, and estimates the models using the observable mixed-frequency series. For US data, Schwartz's Bayesian information criterion selects a two-factor model. The smoothed estimate of latent monthly real GDP is the proposed index.

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1 INTRODUCTION

Since the seminal work by Stock and Watson (1989, 1991), it has been standard in the literature on business cycle indices to assume a static linear one-factor structure for coincident indicators, and use the estimated "common factor" as a coincident index; e.g., Kim and Yoo (1995), Diebold and Rudebusch (1996), Chauvet (1998), Kim and Nelson (1998), and Mariano and Murasawa (2003). This one-factor structure assumption is restrictive in practice. Indeed, Murasawa (2003) tests the autocovariance structure of the four US coincident indicators used in these works, and finds a strong evidence against the one-factor structure assumption.

This paper proposes a method for constructing a coincident index without assuming a one-factor model. The idea is simple. Many, if not all, will agree that if we observe real GDP promptly on monthly basis, then we do not need a coincident index; i.e., a coincident index is a "proxy" for latent monthly real GDP. If so, then it is natural to consider prediction of monthly real GDP directly, which does not require a one-factor model. This paper thus relates two seemingly separate issues, namely index construction and interpolation of real GDP.

Mariano and Murasawa (2003) stress that a business cycle index must have an economic interpretation, because the "amplitude" of a cycle depends on the choice of an index. Figure 1 compares the composite index (CI) of coincident indicators, released by The Conference Board, and the Stock–Watson Experimental Coincident Index (XCI), released by themselves on their home pages,[1] with seasonally-adjusted quarterly real GDP from 1979 to 1983, during which there are two peaks and two troughs. The XCI indicates that the trough in November 1982 is deeper than that in July 1980, whereas the CI indicates that the depth of the two are almost equal. In fact, real GDP is *higher* in the fourth quarter of 1982 than in the third quarter of 1980. Such inconsistency can arise because the levels of these indices have no economic interpretation. While Mariano and Murasawa (2003) include real GDP in the one-factor model to relate the common factor to monthly real GDP, this paper estimates monthly real GDP directly.

Figure 1.

Specifically, this paper considers VAR and factor models for latent monthly real GDP and other

[1] They retired their indices in June 2004. The historical values are still available.

coincident indicators, and estimates the models using the observable mixed-frequency series. The estimation procedure follows Mariano and Murasawa (2003), i.e., we derive a state-space model for the observable mixed-frequency series, and treat the mixed-frequency series as monthly series with missing observations. Using SsfPack 2.2 by Koopman, Shephard, and Doornik (1999), which runs on Ox 3.30 by Doornik (2001), ML estimation of a linear Gaussian state-space model is straightforward, even with missing observations. For US data, Schwartz's Bayesian information criterion selects a two-factor model, where the common factors jointly follow VAR(1) and the specific factors independently follow AR(1). The associated smoothed estimate of latent monthly real GDP is the proposed index.

In practice, quasi-Newton methods may fail to converge if the model has too many unknown parameters. The EM algorithm is useful in such cases. Shumway and Stoffer (1982) derive the EM algorithm for ML estimation of a linear Gaussian state-space model, allowing for missing observations. Since the EM algorithm slows down significantly near the maximum, many authors suggest switching to a quasi-Newton method at some point, i.e., using the EM algorithm to find a good starting value for a quasi-Newton method. Unfortunately, the EM algorithm does not apply directly to factor models with mixed-frequency series; hence we use it only for VAR models in this paper.

In the literature on interpolation of real GDP, the best linear unbiased interpolation by Chow and Lin (1971) is most popular, but some authors use state-space models; e.g., Bernanke, Gertler, and Watson (1997), Cuche and Hess (1999, 2000), and Liu and Hall (2001). They consider univariate linear regression models for latent monthly real GDP, where for temporal aggregation, they do not take the log transformation. This paper takes the log transformation and considers multivariate models, following the convention in the literature on index construction.

The plan of the paper is as follows. Section 2 introduces what we call a mixed-frequency VAR model, i.e., we set up a VAR model for partially latent time series, and derive a state-space model for the observable mixed-frequency series. We also derive the EM algorithm for ML estimation of a Gaussian mixed-frequency VAR model. Section 3 discusses a mixed-frequency factor model. Section 4 applies the method to US data to obtain a new coincident index. Section 5 discusses remaining issues.

2 MIXED-FREQUENCY VAR MODEL

2.1 VAR Model

Let $\{Y_{t,1}\}$ be an N_1-variate random sequence observable every third period (quarterly series) and $\{Y_{t,2}\}$ be an N_2-variate random sequence observable every period (monthly series). Let for all t, $Y_t := (Y_{t,1}', Y_{t,2}')'$ and $N := N_1 + N_2$. Assume that $\{\ln Y_t\}$ is integrated of order 1.

Let $\{Y_{t,1}^*\}$ be a latent random sequence underlying $\{Y_{t,1}\}$ such that for all t,

$$\ln Y_{t,1} = \frac{1}{3}\left(\ln Y_{t,1}^* + \ln Y_{t-1,1}^* + \ln Y_{t-2,1}^*\right), \tag{1}$$

i.e., $Y_{t,1}$ is the geometric mean of $Y_{t,1}^*$, $Y_{t-1,1}^*$, and $Y_{t-2,1}^*$. Taking the three-period differences, for all t,

$$\begin{aligned}\ln Y_{t,1} - \ln Y_{t-3,1} &= \frac{1}{3}\left(\ln Y_{t,1}^* - \ln Y_{t-3,1}^*\right) + \frac{1}{3}\left(\ln Y_{t-1,1}^* - \ln Y_{t-4,1}^*\right) \\ &\quad + \frac{1}{3}\left(\ln Y_{t-2,1}^* - \ln Y_{t-5,1}^*\right),\end{aligned}$$

or

$$\begin{aligned}y_{t,1} &= \frac{1}{3}\left(y_{t,1}^* + y_{t-1,1}^* + y_{t-2,1}^*\right) + \frac{1}{3}\left(y_{t-1,1}^* + y_{t-2,1}^* + y_{t-3,1}^*\right) \\ &\quad + \frac{1}{3}\left(y_{t-2,1}^* + y_{t-3,1}^* + y_{t-4,1}^*\right) \\ &= \frac{1}{3}y_{t,1}^* + \frac{2}{3}y_{t-1,1}^* + y_{t-2,1}^* + \frac{2}{3}y_{t-3,1}^* + \frac{1}{3}y_{t-4,1}^*, \end{aligned} \tag{2}$$

where $y_{t,1} := \Delta_3 \ln Y_{t,1}$ and $y_{t,1}^* := \Delta \ln Y_{t,1}^*$. We observe $\{y_{t,1}\}$ every third period, and never observe $\{y_{t,1}^*\}$.

Let for all t,

$$y_t := \begin{pmatrix} y_{t,1} \\ y_{t,2} \end{pmatrix}, \quad y_t^* := \begin{pmatrix} y_{t,1}^* \\ y_{t,2} \end{pmatrix},$$

where $y_{t,2} := \Delta \ln Y_{t,2}$. Let

$$\begin{aligned}H(L) &:= \begin{bmatrix} (1/3)I_{N_1} & 0 \\ 0 & I_{N_2} \end{bmatrix} + \begin{bmatrix} (2/3)I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L + \begin{bmatrix} I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L^2 \\ &\quad + \begin{bmatrix} (2/3)I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L^3 + \begin{bmatrix} (1/3)I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L^4,\end{aligned}$$

where L is the lag operator. Let $\mu := \mathrm{E}(y_t)$ and $\mu^* := \mathrm{E}(y_t^*)$. Then for all t,

$$y_t - \mu = H(L)(y_t^* - \mu^*). \tag{3}$$

Assume a Gaussian VAR(p) model for $\{y_t^*\}$ such that for all t,

$$\Phi(L)(y_t^* - \mu^*) = w_t, \tag{4}$$

$$\{w_t\} \sim \mathrm{NID}(0, \Sigma). \tag{5}$$

2.2 State-Space Representation

If $p \leq 5$, then we define the state vector as for all t,

$$s_t := \begin{pmatrix} y_t^* - \mu^* \\ \vdots \\ y_{t-4}^* - \mu^* \end{pmatrix}.$$

A state-space representation of the VAR model is for all t,

$$s_{t+1} = As_t + Bz_t, \tag{6}$$

$$y_t = \mu + Cs_t, \tag{7}$$

$$\{z_t\} \sim \mathrm{NID}(0, I_N), \tag{8}$$

where

$$\begin{aligned} A &:= \begin{bmatrix} \Phi_1 & \dots & \Phi_p & O_{N\times(5-p)N} \\ & I_{4N} & & O_{4N\times N} \end{bmatrix}, \\ B &:= \begin{bmatrix} \Sigma^{1/2} \\ O_{4N\times N} \end{bmatrix}, \\ C &:= \begin{bmatrix} H_0 & \dots & H_4 \end{bmatrix}. \end{aligned}$$

If $p \geq 5$, then we define the state vector as for all t,

$$s_t := \begin{pmatrix} y_t^* - \mu^* \\ \vdots \\ y_{t-p+1}^* - \mu^* \end{pmatrix}.$$

A state-space representation of the VAR model is the same except that

$$\begin{aligned} A &:= \begin{bmatrix} \Phi_1 & \dots & \Phi_{p-1} & \Phi_p \\ & I_{(p-1)N} & & O_{(p-1)N\times N} \end{bmatrix}, \\ B &:= \begin{bmatrix} \Sigma^{1/2} \\ O_{(p-1)N\times N} \end{bmatrix}, \\ C &:= \begin{bmatrix} H_0 & \dots & H_4 & O_{N\times(p-5)N} \end{bmatrix}. \end{aligned}$$

2.3 ML Estimation by a Quasi-Newton Method

Using SsfPack 2.2 by Koopman, Shephard, and Doornik (1999), which runs on Ox 3.30 by Doornik (2001), ML estimation of a linear Gaussian state-space model by a quasi-Newton method is straightforward, even with missing observations. When the number of the unknown parameters is large, however, this may fail, especially with a poor starting value.

2.4 ML Estimation by the EM Algorithm

2.4.1 Missing Observations

Following Mariano and Murasawa (2003), we fill in missing observations with random numbers independent of the model parameters, and rewrite the measurement equation accordingly, so that the Kalman filter "skips" the random numbers. Since realizations of the random numbers do not matter, we can simply put 0s for missing observations in practice.

Let for all t,

$$y_{t,1}^{+} := \begin{cases} y_{t,1} & \text{if } y_{t,1} \text{ is observable} \\ v_t & \text{otherwise} \end{cases},$$

where v_t is a random number. We assume for convenience that $\{v_t\} \sim \mathrm{NID}(0, I_{N_1})$. The measurement equation for $\{y_t\}$ is for all t,

$$\begin{pmatrix} y_{t,1} \\ y_{t,2} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} s_t.$$

We can write for all t,

$$\begin{pmatrix} y_{t,1}^{+} \\ y_{t,2} \end{pmatrix} = \begin{pmatrix} \mu_{t,1} \\ \mu_2 \end{pmatrix} + \begin{bmatrix} C_{t,1} \\ C_2 \end{bmatrix} s_t + \begin{pmatrix} D_{t,1} \\ 0 \end{pmatrix} v_t,$$

where

$$\begin{aligned} \mu_{t,1} &:= \begin{cases} \mu_1 & \text{if } y_{t,1} \text{ is observable} \\ 0 & \text{otherwise} \end{cases}, \\ C_{t,1} &:= \begin{cases} C_1 & \text{if } y_{t,1} \text{ is observable} \\ 0 & \text{otherwise} \end{cases}, \\ D_{t,1} &:= \begin{cases} 0 & \text{if } y_{t,1} \text{ is observable} \\ I_{N_1} & \text{otherwise} \end{cases}. \end{aligned}$$

Let for all t,

$$y_t^{+} := \begin{pmatrix} y_{t,1}^{+} \\ y_{t,2} \end{pmatrix}, \quad \mu_t := \begin{pmatrix} \mu_{t,1} \\ \mu_2 \end{pmatrix}, \quad C_t := \begin{bmatrix} C_{t,1} \\ C_2 \end{bmatrix}, \quad D_t := \begin{bmatrix} D_{t,1} \\ 0 \end{bmatrix}.$$

Then we have a state-space model for $\{y_t^{+}\}$ such that for all t,

$$s_{t+1} = As_t + Bz_t, \tag{9}$$

$$y_t^{+} = \mu_t + C_t s_t + D_t v_t, \tag{10}$$

$$\left\{ \begin{pmatrix} z_t \\ v_t \end{pmatrix} \right\} \sim \mathrm{NID}(0, I_{N+N_1}). \tag{11}$$

2.4.2 Likelihood Function

Assume for simplicity that we know μ^* to be 0. Let $\Phi := [\Phi_1, \ldots, \Phi_p]$, $\phi := \operatorname{vec}(\Phi')$, and $\theta := (\phi', \operatorname{vech}(\Sigma)')'$. Consider an approximate ML estimator of θ, taking s_0 as given. We derive the EM algorithm for solving this ML problem.

Let for $t \geq 0$, $S_t := (s_0, \ldots, s_t)$. Let $Y_0^+ := \emptyset$ and for $t \geq 1$, $Y_t^+ := \left(y_1^+, \ldots, y_t^+\right)$. Let $\Omega \subset \{1, \ldots, T\}$ be the set of periods such that $y_{t,1}$ is missing. By the prediction error decomposition, we can write the joint pdf of $\left(Y_T^+, S_T\right)$ as

$$
\begin{aligned}
f\left(Y_T^+, S_T; \theta\right) &= \prod_{t=1}^{T} f\left(y_{t,1}^+, y_{t,2} | s_t, Y_{t-1}^+, S_{t-1}; \theta\right) f\left(s_t | Y_{t-1}^+, S_{t-1}; \theta\right) \\
&= \prod_{t=1}^{T} f\left(y_{t,1}^+ | s_t; \theta\right) f\left(s_t | s_{t-1}; \theta\right) \\
&= \prod_{t \in \Omega} f(v_t) \prod_{t=1}^{T} f(y_t^* | s_{t-1}; \theta).
\end{aligned}
$$

Let

$$
\begin{aligned}
F &:= \begin{cases} [\, I_N \quad O_{N \times 4N} \,] & \text{if } p \leq 5 \\ [\, I_N \quad O_{N \times (p-1)N} \,] & \text{if } p \geq 5 \end{cases}, \\
G &:= \begin{cases} [\, I_{pN} \quad O_{pN \times (5-p)N} \,] & \text{if } p \leq 5 \\ I_{pN} & \text{if } p \geq 5 \end{cases}.
\end{aligned}
$$

Then for all t,

$$
\begin{aligned}
F s_t &= y_t^* \\
&= \Phi_1 y_{t-1}^* + \cdots + \Phi_p y_{t-p}^* + w_t \\
&= \Phi G s_{t-1} + w_t \\
&= \begin{bmatrix} \phi_1' \\ \vdots \\ \phi_N' \end{bmatrix} G s_{t-1} + w_t \\
&= \begin{bmatrix} s_{t-1}' G' \phi_1 \\ \vdots \\ s_{t-1}' G' \phi_N \end{bmatrix} + w_t \\
&= \begin{bmatrix} s_{t-1}' G' & & 0 \\ & \ddots & \\ 0 & & s_{t-1}' G' \end{bmatrix} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix} + w_t \\
&= \left(I_N \otimes s_{t-1}' G'\right) \phi + w_t.
\end{aligned}
$$

The log-likelihood function of θ given $\left(Y_T^+, S_T\right)$ is

$$
\ln L\left(\theta; Y_T^+, S_T\right)
$$

$$
\begin{aligned}
&= \sum_{t\in\Omega} \ln f(v_t) - \frac{NT}{2}\ln 2\pi - \frac{T}{2}\ln\det(\Sigma) \\
&\quad -\frac{1}{2}\sum_{t=1}^{T}(Fs_t - \Phi G s_{t-1})'\Sigma^{-1}(Fs_t - \Phi G s_{t-1}) \\
&= \sum_{t\in\Omega} \ln f(v_t) - \frac{NT}{2}\ln 2\pi - \frac{T}{2}\ln\det(\Sigma) \\
&\quad -\frac{1}{2}\sum_{t=1}^{T}\left[Fs_t - \left(I_N \otimes s'_{t-1}G'\right)\phi\right]'\Sigma^{-1}\left[Fs_t - \left(I_N \otimes s'_{t-1}G'\right)\phi\right].
\end{aligned}
$$

2.4.3 Kalman Filtering and Smoothing

Initial State Let for all t, for $s \geq 0$,

$$
\begin{aligned}
s_{t|s} &:= \mathrm{E}\left(s_t | Y_s^+\right), \\
P_{t|s} &:= \mathrm{V}\left(s_t | Y_s^+\right).
\end{aligned}
$$

To start the Kalman filter, one must specify $s_{1|0}$ and $P_{1|0}$. Given stationarity, we have

$$s_{1|0} = 0, \tag{12}$$

$$\mathrm{vec}(P_{1|0}) = \left(I_{M^2} - A \otimes A\right)^{-1}\mathrm{vec}(BB'), \tag{13}$$

where M is the dimension of the state vector; see Hamilton (1994, p. 378). The second equation involves inversion of a large matrix, which can cause a computational problem. Hence we instead assume that $s_0 := 0$, which implies that

$$s_{1|0} = 0, \tag{14}$$

$$P_{1|0} = BB'. \tag{15}$$

The resulting estimator is asymptotically equivalent to the exact ML estimator.

Updating We have for $t \geq 1$,

$$
\begin{pmatrix} s_t \\ y_t^+ \end{pmatrix} | Y_{t-1}^+ \sim \mathrm{N}\left(\begin{pmatrix} s_{t|t-1} \\ C_t s_{t|t-1} \end{pmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1}C_t' \\ C_t P_{t|t-1} & C_t P_{t|t-1} C_t' + D_t D_t' \end{bmatrix}\right).
$$

The updating equations are for $t \geq 1$,

$$
\begin{aligned}
s_{t|t} &= s_{t|t-1} + P_{t|t-1}C_t'\left(C_t P_{t|t-1} C_t' + D_t D_t'\right)^{-1}\left(y_t - C_t s_{t|t-1}\right) \\
&= s_{t|t-1} + K_t e_t, \qquad (16)
\end{aligned}
$$

$$\begin{aligned} P_{t|t} &= P_{t|t-1} - P_{t|t-1}C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t'\right)^{-1} C_t P_{t|t-1} \\ &= (I_M - K_t C_t) P_{t|t-1}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} K_t &:= P_{t|t-1}C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t'\right)^{-1}, \\ e_t &:= y_t^+ - C_t s_{t|t-1}. \end{aligned}$$

Prediction The prediction equations are for $t \geq 1$,

$$s_{t+1|t} = A s_{t|t}, \tag{18}$$

$$P_{t+1|t} = A P_{t|t} A' + BB'. \tag{19}$$

Fixed-Interval Smoothing The following algorithm by de Jong (1989) avoids inversion of large matrices, and hence is more efficient than the standard smoothing equations; see also Durbin and Koopman (2001, sec. 4.3). Let $r_{T+1} := 0$, $R_{T+1} := 0$, and for $t = T, \ldots, 1$,

$$\begin{aligned} r_t &:= C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t'\right)^{-1} e_t + L_t' r_{t+1}, \\ R_t &:= C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t'\right)^{-1} C_t + L_t' R_{t+1} L_t, \end{aligned}$$

where

$$L_t := A(I_M - K_t C_t).$$

The smoothing equations are for $t = 1, \ldots, T$,

$$s_{t|T} = s_{t|t-1} + P_{t|t-1} r_t, \tag{20}$$

$$P_{t|T} = P_{t|t-1} - P_{t|t-1} R_t P_{t|t-1}. \tag{21}$$

The EM algorithm for ML estimation of a linear Gaussian state-space model also requires the smoothed first-order autocovariance matrix of $\{s_t\}$. Let for all t, s,

$$P_{t,s|T} := \operatorname{Cov}\left(s_t, s_s | Y_T^+\right).$$

De Jong and MacKinnon (1988) show that for $s = 1, \ldots, T-1$, for $t = 1, \ldots, T-s$,

$$P_{t+s,t|T} = \left(I_M - P_{t+s|t+s-1} R_{t+s}\right) L_{t+s-1} \cdots L_t P_{t|t-1}. \tag{22}$$

In particular, for $t = 1, \ldots, T-1$,

$$P_{t+1,t|T} = \left(I_M - P_{t+1|t} R_{t+1}\right) L_t P_{t|t-1}. \tag{23}$$

2.4.4 EM Algorithm

The score function of θ given $\left(Y_T^+, S_T\right)$ consists of

$$\begin{aligned}
&\frac{\partial \ln L\left(\theta; Y_T^+, S_T\right)}{\partial \phi} \\
&= \sum_{t=1}^{T} \left(I_N \otimes s_{t-1}' G'\right)' \Sigma^{-1} \left[F s_t - \left(I_N \otimes s_{t-1}' G'\right) \phi\right] \\
&= \sum_{t=1}^{T} \left(I_N \otimes G s_{t-1}\right) \Sigma^{-1} F s_t - \sum_{t=1}^{T} \left(I_N \otimes G s_{t-1}\right) \Sigma^{-1} \left(I_N \otimes s_{t-1}' G'\right) \phi \\
&= \sum_{t=1}^{T} \Sigma^{-1} (F s_t \otimes G s_{t-1}) - \sum_{t=1}^{T} \left(\Sigma^{-1} \otimes G s_{t-1} s_{t-1}' G'\right) \phi \\
&= \sum_{t=1}^{T} \Sigma^{-1} \mathrm{vec}(G s_{t-1} s_t' F') - \sum_{t=1}^{T} \left(\Sigma^{-1} \otimes G s_{t-1} s_{t-1}' G'\right) \phi, \\
&\frac{\partial \ln L\left(\theta; Y_T^+, S_T\right)}{\partial \Sigma^{-1}} \\
&= \frac{T}{2} \Sigma - \frac{1}{2} \sum_{t=1}^{T} (F s_t - \Phi G s_{t-1})(F s_t - \Phi G s_{t-1})' \\
&= \frac{T}{2} \Sigma - \frac{1}{2} \sum_{t=1}^{T} \left(F s_t s_t' F' - F s_t s_{t-1}' G' \Phi' - \Phi G s_{t-1} s_t' F' + \Phi G s_{t-1} s_{t-1}' G' \Phi'\right).
\end{aligned}$$

Let for $r, s = 0, 1$,

$$\begin{aligned}
M_{r,s} &:= \frac{1}{T} \sum_{t=1}^{T} \mathrm{E}\left(s_{t-r} s_{t-s}' | Y_T^+\right) \\
&= \frac{1}{T} \sum_{t=1}^{T} \left(P_{t-r,t-s|T} + s_{t-r|T} s_{t-s|T}'\right).
\end{aligned}$$

Taking the conditional expectation of the likelihood equation given Y_T^+,

$$\begin{aligned}
\Sigma^{*-1} \mathrm{vec}(G M_{1,0} F') - \left(\Sigma^{*-1} \otimes G M_{1,1} G'\right) \phi^* &= 0, \\
\Sigma^* - \left(F M_{0,0} F' - F M_{0,1} G' \Phi^{*\prime} - \Phi^* G M_{1,0} F' + \Phi^* G M_{1,1} G' \Phi^{*\prime}\right) &= 0,
\end{aligned}$$

or

$$\begin{aligned}
\phi^* &= \left(\Sigma^{*-1} \otimes G M_{1,1} G'\right)^{-1} \Sigma^{*-1} \mathrm{vec}(G M_{1,0} F') \\
&= \left[I_N \otimes (G M_{1,1} G')^{-1}\right] \mathrm{vec}(G M_{1,0} F'), \qquad (24) \\
\Sigma^* &= F M_{0,0} F' - F M_{0,1} G' \Phi^{*\prime} - \Phi^* G M_{1,0} F' + \Phi^* G M_{1,1} G' \Phi^{*\prime}. \qquad (25)
\end{aligned}$$

The EM algorithm proceeds as follows:

1. Choose a starting value $\theta^{(0)}$.

2. (E step) Compute $\left\{s_{t|T}\right\}$, $\left\{P_{t|T}\right\}$, and $\left\{P_{t,t-1|T}\right\}$.

3. (M step) Compute (ϕ^*, Σ^*), and use it as $\theta^{(1)}$.

4. Iterate until convergence.

The smoothing algorithm gives $\left\{\mathrm{E}\left(y_{t,1}^*|Y_T^+\right)\right\}$.

3 MIXED-FREQUENCY FACTOR MODEL

3.1 Factor Model

Assume a K-factor model for $\{y_t^*\}$, where $K < N$, such that for all t,

$$\begin{aligned} y_t^* &= \mu^* + \Lambda f_t + u_t, && (26) \\ \Phi_f(L) f_t &= v_t, && (27) \\ \Phi_u(L) u_t &= w_t, && (28) \\ \left\{\begin{pmatrix} v_t \\ w_t \end{pmatrix}\right\} &\sim \mathrm{NID}\left(0, \begin{bmatrix} \Sigma_{vv} & 0 \\ 0 & \Sigma_{ww} \end{bmatrix}\right), && (29) \end{aligned}$$

where $\Phi_f(.)$ is the pth-order polynomial on $\Re^{K\times K}$ and $\Phi_u(.)$ is the qth-order polynomial on $\Re^{N\times N}$.

For identification, assume that

$$\Lambda := \begin{bmatrix} I_K \\ \Lambda_2 \end{bmatrix},$$

and that $\Phi_u(.)$ and Σ_{ww} are diagonal.

3.2 State-Space Representation

Assume that $p, q \leq 5$. Let for all t,

$$s_t := \begin{pmatrix} f_t \\ \vdots \\ f_{t-4} \\ u_t \\ \vdots \\ u_{t-4} \end{pmatrix}.$$

A state-space representation of the factor model is for all t,

$$s_{t+1} = A s_t + B z_t, \qquad (30)$$

$$y_t = \mu + Cs_t, \tag{31}$$

$$\{z_t\} \sim \mathrm{NID}(0, I_{K+N}), \tag{32}$$

where

$$A := \begin{bmatrix} \begin{matrix} \Phi_{f,1} & \dots & \Phi_{f,p} & O_{K\times(5-p)K} \\ I_K & & 0 & O_{K\times K} \\ & \ddots & & \vdots \\ 0 & & I_K & O_{K\times K} \end{matrix} & O_{5K\times 5N} \\ O_{5N\times 5K} & \begin{matrix} \Phi_{u,1} & \dots & \Phi_{u,q} & O_{N\times(5-q)N} \\ I_N & & 0 & O_{N\times N} \\ & \ddots & & \vdots \\ 0 & & I_N & O_{N\times N} \end{matrix} \end{bmatrix},$$

$$B := \begin{bmatrix} \begin{matrix} \Sigma_{vv}^{1/2} \\ O_{4K\times K} \end{matrix} & O_{5K\times N} \\ O_{5N\times K} & \begin{matrix} \Sigma_{ww}^{1/2} \\ O_{4N\times N} \end{matrix} \end{bmatrix},$$

$$C := [\, H_0\Lambda \quad \dots \quad H_4\Lambda \quad H_0 \quad \dots \quad H_4 \,].$$

3.3 ML Estimation

The EM algorithm does not apply when the measurement equation has unknown parameters but does not have an error term, since these parameters do not appear in the complete-data likelihood function. According to Ruud (1991, p. 310), "the transformation τ from latent to observable data cannot depend on parameters to be estimated in such a way that the support of y^* conditional on y depends on θ."

Watson and Engle (1983, p. 397) give an alternative state-space representation of a factor model, and apply the EM algorithm. Their representation does not hold with mixed-frequency series, however. Hence we rely on a quasi-Newton method from an ad hoc starting value.

Given the parameters, we have for $t = 1, \dots, T$,

$$\mathrm{E}\left(y_t^* | Y_T^+\right) = \mu^* + \Lambda' \mathrm{E}\left(f_t | Y_T^+\right) + \mathrm{E}\left(u_t | Y_T^+\right). \tag{33}$$

The smoothing algorithm gives $\left\{\mathrm{E}\left(f_t | Y_T^+\right)\right\}$ and $\left\{\mathrm{E}\left(u_t | Y_T^+\right)\right\}$.

4 APPLICATION

4.1 Data

We apply the method described above to US coincident indicators to construct a new coincident index of business cycles. The component indicators are quarterly real GDP and the four monthly coincident indicators that currently make up the CI; see Table 1 for their descriptions. The sample period is from January 1959 to December 2002.

Table 1.

To stationarize the series, we take the first difference of the natural log of each series and multiply it 100, which is approximately equal to the quarterly or monthly percentage growth rate series. Table 2 gives summary statistics of these "growth rate" series.

Table 2.

4.2 VAR Coincident Index

We take two shortcuts in ML estimation of our state-space models, both of which are common and useful in practice. First, we "demean" the series, and delete the constant term from the model. This reduces the number of the parameters by N. Second, we use an approximate ML estimator instead of the exact one regarding the initial state for the Kalman filter, i.e., we assume that $s_0 := 0$. This avoids inversion of a large matrix, and saves the computational cost further. Recall that without missing observations one usually estimates a VAR model by applying OLS to the demeaned series. We take similar shortcuts here.

One must select p, the order of the VAR model. The two common criteria for model selection are Akaike's information criterion (AIC) and Schwartz's Bayesian information criterion (SBIC). For our model,

$$\begin{aligned} \text{AIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \left[pN^2 + \frac{N(N+1)}{2}\right]\right\}, \\ \text{SBIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \frac{\ln T}{2}\left[pN^2 + \frac{N(N+1)}{2}\right]\right\}, \end{aligned}$$

where $\hat{\theta}$ is the (approximate) ML estimator of θ.

We estimate the mixed-frequency VAR model and compute the associated AIC and SBIC for $p = 1, \ldots, 12$. Since a quasi-Newton method from an ad hoc starting value may fail to converge when p is large, we estimate the model in the following two steps:

1. Apply the EM algorithm to obtain a preliminary ML estimate.

2. Using this as the starting value, apply a quasi-Newton method to obtain the final ML estimate.

We use Ox 3.30 with SsfPack 2.2 for computation. Ox has the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm for numerical maximization.

Table 3 summarizes the result of model selection. AIC selects $p = 10$, whereas SBIC selects $p = 1$. We typically follow AIC for optimal one-step ahead prediction and SBIC for consistent model selection. Although it is unclear which is better for optimal smoothing, we follow SBIC here, preferring the simpler model.

Table 3.

The smoothing algorithm gives the smoothed estimate of the "demeaned growth rate" series of monthly real GDP. We transform it to the level series as follows:

1. Add the mean growth rate of monthly real GDP (the mean growth rate of quarterly real GDP divided by 3), and divide the series by 100.

2. Take the cumulative sum; then take the exponential transformation.

Figure 2 plots the resulting estimate of monthly real GDP, which we call the VAR coincident index. It appears rather volatile, but captures the NBER business cycle reference dates fairly well.

Figure 2.

4.3 K-Factor Coincident Index

A factor model may predict monthly real GDP better than VAR models with less parameters. AIC and SBIC for a K-factor model are

$$\begin{aligned} \text{AIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \left[(N-K)K + pK^2 + \frac{K(K+1)}{2} + qN + N\right]\right\}, \\ \text{SBIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \frac{\ln T}{2}\left[(N-K)K + pK^2 + \frac{K(K+1)}{2} + qN + N\right]\right\}. \end{aligned}$$

Since the EM algorithm does not apply directly to mixed-frequency factor models, we use an ad hoc starting value for the BFGS algorithm. We find that numerical singularity tends to occur as

K increases, which may be an identification problem as well as a numerical problem; hence we consider only up to two-factor models.

Table 4 and 5 summarize the result of model selection. AIC selects $(K, p, q) = (2, 1, 3)$, whereas SBIC selects $(K, p, q) = (2, 1, 1)$. Both criteria select two-factor models rather than one-factor models. Among both VAR and factor models, AIC selects the VAR(1) model, whereas SBIC selects the above two-factor model. We follow SBIC again, preferring the simpler model.

Table 4 and 5.

A factor model gives two different coincident indices: the smoothed estimate of monthly real GDP and its common factor component (or the first common factor according to our identification restriction). Mariano and Murasawa (2003) propose the latter as a natural extension of the Stock–Watson coincident index, assuming a one-factor model. Figure 3 shows that the two indices differ substantially, although both capture the NBER business cycle reference dates.

Figure 3.

4.4 Comparison

We have three candidate coincident indices:

1. smoothed estimate of monthly real GDP based on the VAR(1) model (VAR coincident index),
2. smoothed estimate of monthly real GDP based on the two-factor model (2-factor coincident index),
3. smoothed estimate of the common factor component of monthly real GDP in the two-factor model.

Figure 4 plots the three indices with the CI from 1979 to 1983. The two estimates of monthly real GDP are close to each other and to quarterly real GDP in Figure 1, although they are rather volatile. The first common factor is smoother, but differs significantly from monthly real GDP; hence it is not a good proxy for monthly real GDP.

Figure 4.

Note a "dip" in January 1982. The dip is deeper than the trough in November 1982 for the two estimates of monthly real GDP, but it is the opposite for the other two indices. Although quarterly real GDP increased, the four monthly coincident indicators all decreased between the two months, which explains the slight decline in the common factor component of monthly real GDP.

In general, the behavior of the common factor component strongly depends on the choice of the component indicators; hence monthly real GDP and its common factor component can give different turning points. This paper thus proposes using an estimate of monthly real GDP, the 2-factor coincident index selected by SBIC, as a new coincident index of business cycles.

5 DISCUSSION

Index construction is interpolation of real GDP; i.e., a coincident index of business cycles must be an estimate of monthly real GDP. This paper uses VAR and factor models for predicting monthly real GDP, but one can try other models as well, e.g., a univariate regression model.

Dating turning points in the estimated monthly real GDP is another issue. The dating algorithm by Bry and Boschan (1971) is popular. Fitting a Markov-switching model to the estimated monthly real GDP is an alternative; see Hamilton (1989). Harding and Pagan (2003) compare the two approaches, and criticize the latter.

ML estimation of a mixed-frequency factor model could be difficult when there are many common factors. One solution is to extract major principal components from quarterly and monthly indicators respectively, and estimate a mixed-frequency VAR model for quarterly real GDP and the principal components. Another solution is to apply the Bayesian method, perhaps using simulation.

One can extend this paper in several ways. First, construction of leading and lagging indices is straightforward by defining a leading index as, say, a six-month ahead forecast of monthly real GDP and a lagging index as the final estimate of the past monthly real GDP. Second, other models are worth considering, since they may predict better with less parameters; e.g., VARMA models, dynamic factor models, Markov-switching models, cointegration, etc. Third, to exploit information in various indicators, one can consider a VAR model for latent monthly real GDP and major principal components, or "diffusion indices," extracted from many indicators; see Stock and Watson

(2002a, 2002b). Fourth, it seems useful to predict monthly real GDP from the production, expenditure, and distribution sides respectively, and combine the three forecasts. Fifth, our framework has other interesting applications; e.g., one can estimate monthly impulse response functions using mixed-frequency series.

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Table 1: US Coincident Indicators

Indicator	Description
	Quarterly
GDP	Real GDP (billions of chained 2000 dollars, SA, AR)
	Monthly
EMP	Employees on nonagricultural payrolls (thousands, SA)
INC	Personal income less transfer payments (billions of chained 1996 dollars, SA, AR)
IIP	Index of industrial production (1997 = 100, SA)
SLS	Manufacturing and trade sales (millions of chained 1996 dollars, SA)

Note: SA means "seasonally-adjusted" and AR means "annual rate."

Table 2: Summary Statistics

Indicator	Mean	S.D.	Min.	Max.
		Quarterly		
GDP	0.82	0.88	−2.04	3.86
		Monthly		
EMP	0.17	0.23	−0.88	1.23
INC	0.27	0.56	−4.95	3.70
IIP	0.26	0.83	−3.66	6.00
SLS	0.27	1.05	−3.21	3.54

Note: Statistics are for the first difference of the natural log times 100.

Table 3: Model Selection (VAR Model)

p	Log-likelihood	AIC	SBIC
1	−1825.2	−3.5109	−3.6121
2	−1766.5	−3.4469	−3.6493
3	−1723.8	−3.4133	−3.7170
4	−1697.6	−3.4110	−3.8159
5	−1674.5	−3.4146	−3.9207
6	−1636.5	−3.3900	−3.9973
7	−1607.3	−3.3819	−4.0904
8	−1570.2	−3.3590	−4.1687
9	−1554.1	−3.3759	−4.2868
10	−1516.7	−3.3523	−4.3644
11	−1495.4	−3.3594	−4.4728
12	−1475.2	−3.3684	−4.5830

Table 4: Model Selection (One-Factor Model)

K	p	q	Log-likelihood	AIC	SBIC
1	0	0	−2025.9	−3.8631	−3.9036
1	0	1	−1940.9	−3.7113	−3.7720
1	0	2	−1884.0	−3.6129	−3.6938
1	0	3	−1865.6	−3.5874	−3.6886
1	0	4	−1860.4	−3.5871	−3.7086
1	0	5	−1853.7	−3.5839	−3.7256
1	1	0	−1953.0	−3.7268	−3.7714
1	1	1	−1883.6	−3.6045	−3.6693
1	1	2	−1827.3	−3.5073	−3.5923
1	1	3	−1808.1	−3.4802	−3.5855
1	1	4	−1803.4	−3.4807	−3.6063
1	1	5	−1796.6	−3.4775	−3.6233
1	2	0	−1949.7	−3.7225	−3.7710
1	2	1	−1881.6	−3.6028	−3.6716
1	2	2	−1826.1	−3.5069	−3.5960
1	2	3	−1807.1	−3.4802	−3.5895
1	2	4	−1802.3	−3.4806	−3.6101
1	2	5	−1795.3	−3.4769	−3.6267
1	3	0	−1949.3	−3.7235	−3.7762
1	3	1	−1881.5	−3.6044	−3.6773
1	3	2	−1825.7	−3.5079	−3.6010
1	3	3	−1806.8	−3.4816	−3.5950
1	3	4	−1803.6	−3.4851	−3.6187
1	3	5	−1795.1	−3.4783	−3.6321
1	4	0	−1949.3	−3.7254	−3.7821
1	4	1	−1881.4	−3.6061	−3.6830
1	4	2	−1825.2	−3.5088	−3.6060
1	4	3	−1806.8	−3.4834	−3.6008
1	4	4	−1802.1	−3.4840	−3.6217
1	4	5	−1794.9	−3.4799	−3.6378
1	5	0	−1949.3	−3.7273	−3.7880
1	5	1	−1881.3	−3.6078	−3.6887
1	5	2	−1825.0	−3.5104	−3.6117
1	5	3	−1806.3	−3.4844	−3.6059
1	5	4	−1801.4	−3.4847	−3.6264
1	5	5	−1794.4	−3.4807	−3.6427

Table 5: Model Selection (Two-Factor Model)

K	p	q	Log-likelihood	AIC	SBIC
2	0	0	−2018.7	−3.8571	−3.9138
2	0	1	−1896.1	−3.6339	−3.7108
2	0	2	−1874.2	−3.6019	−3.6990
2	0	3	−1855.6	−3.5760	−3.6934
2	0	4	−1853.5	−3.5816	−3.7193
2	0	5	−1853.7	−3.5915	−3.7494
2	1	0	−1861.3	−3.5661	−3.6390
2	1	1	−1769.6	−3.4015	−3.4947
2	1	2	−1757.4	−3.3879	−3.5012
2	1	3	−1750.6	−3.3844	−3.5180
2	1	4	−1746.0	−3.3851	−3.5390
2	1	5		singular	
2	2	0	−1852.8	−3.5575	−3.6465
2	2	1	−1766.9	−3.4040	−3.5133
2	2	2	−1754.8	−3.3904	−3.5200
2	2	3	−1747.0	−3.3852	−3.5350
2	2	4	−1742.3	−3.3858	−3.5558
2	2	5		singular	
2	3	0	−1835.8	−3.5328	−3.6381
2	3	1	−1766.1	−3.4101	−3.5356
2	3	2	−1753.5	−3.3957	−3.5414
2	3	3	−1745.3	−3.3896	−3.5556
2	3	4	−1741.8	−3.3924	−3.5787
2	3	5	−1735.5	−3.3899	−3.5964
2	4	0	−1834.2	−3.5373	−3.6588
2	4	1	−1754.0	−3.3947	−3.5364
2	4	2	−1744.3	−3.3858	−3.5478
2	4	3	−1739.3	−3.3858	−3.5680
2	4	4	−1735.9	−3.3888	−3.5912
2	4	5		singular	
2	5	0	−1828.0	−3.5331	−3.6708
2	5	1	−1760.4	−3.4145	−3.5723
2	5	2	−1742.5	−3.3899	−3.5681
2	5	3	−1737.6	−3.3901	−3.5885
2	5	4	−1733.6	−3.3921	−3.6107
2	5	5	−1728.8	−3.3924	−3.6313

Note: "singular" means that BFGS algorithm fails because of numerical singularity.

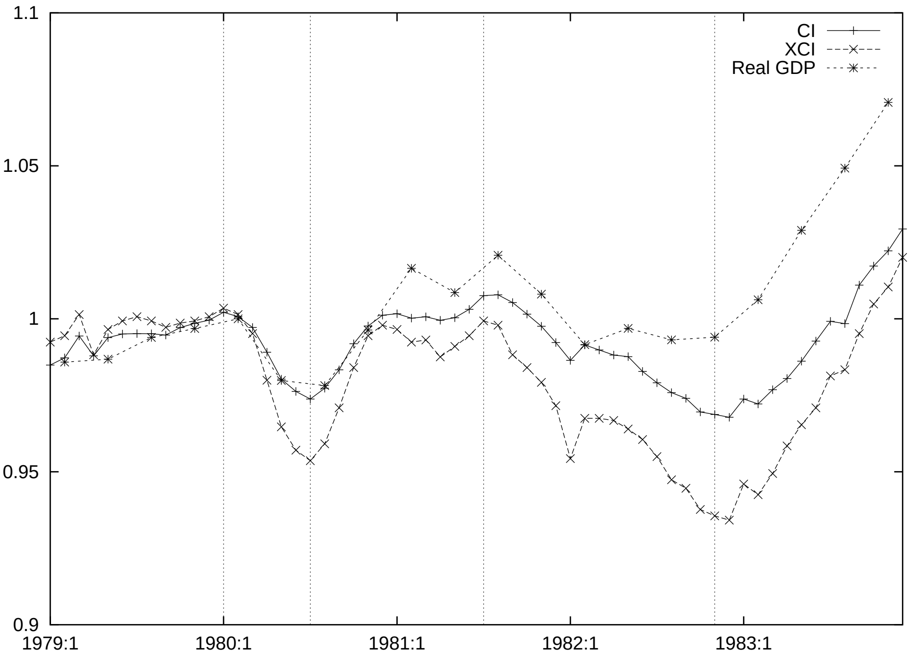

Figure 1: The CI, the Stock–Watson XCI, and Seasonally-Adjusted Quarterly Real GDP from 1979 to 1983 (1980:Q1=1). The vertical lines are the NBER business cycle reference dates. *Sources*: The NBER, the home page of James Stock, and the Bureau of Economic Analysis.

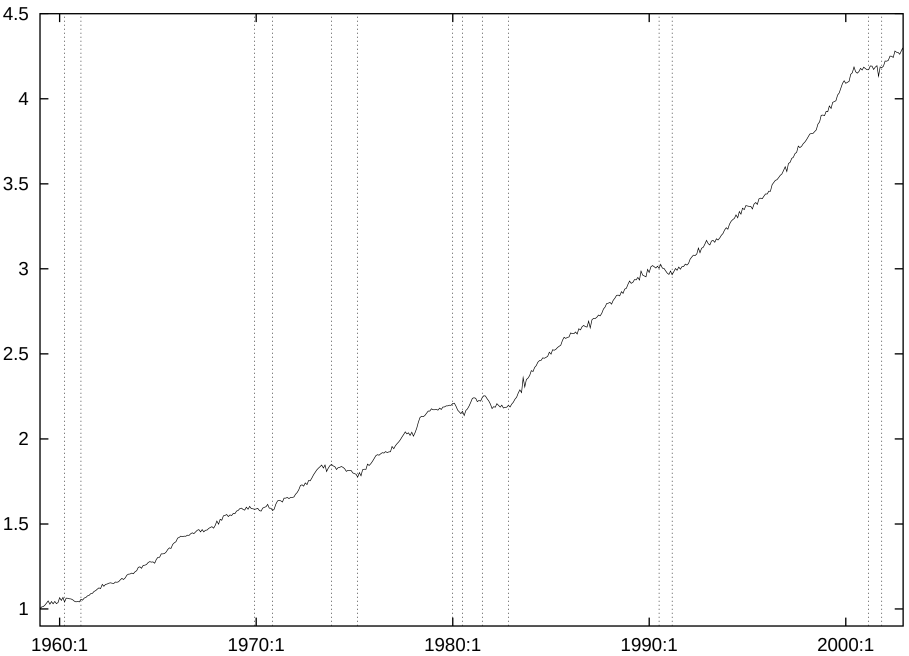

Figure 2: Historical Plot of the VAR Coincident Index (1959:1=1). The vertical lines are the NBER business cycle reference dates.

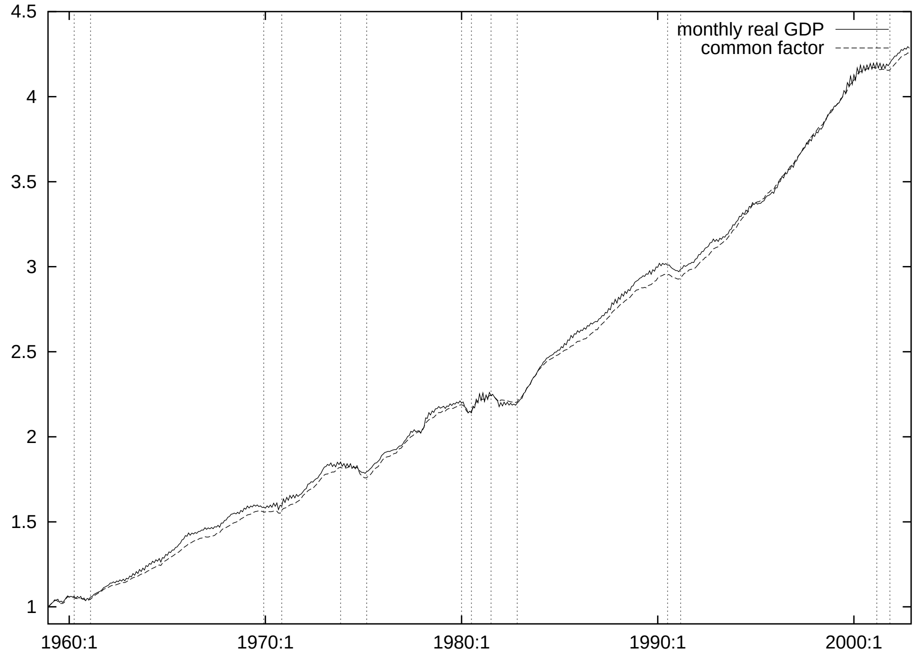

Figure 3: Historical Plot of the Two-Factor Coincident Index (1959:1=1). The vertical lines are the NBER business cycle reference dates.

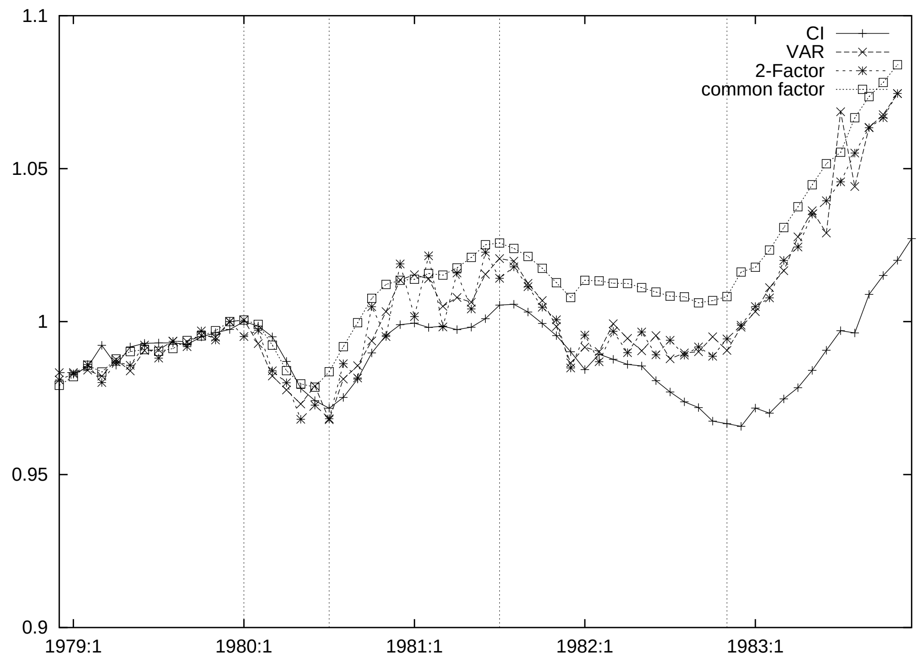

Figure 4: Comparison of Alternative Indices from 1979 to 1983 (1980:1=1). The vertical lines are the NBER business cycle reference dates.