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Social Groups and Economic Poverty

A Problem in Measurement

S. Subramanian*

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Abstract

This paper points to some elementary conflicts between the claims of interpersonal and intergroup justice as they manifest themselves in the process of seeking a real-valued index of poverty which is required to satisfy certain seemingly desirable properties. It indicates how 'group-sensitive' poverty measures, similar to the Anand-Sen (1995) 'Gender Adjusted Human Development Index' and the Subramanian-Majumdar (2002) 'Group-Disparity Adjusted Deprivation Index', may be constructed. Some properties of a specific 'group-sensitive' poverty index are appraised, and the advantage of having a 'flexible' measure which is capable of effecting a tradeoff between the claims of interpersonal and inter-group equality is spelt out. The implications of directly incorporating group disparities into the measurement of poverty for poverty comparisons and anti-poverty policy are also discussed.

Keywords: poverty, measurement, social groups, symmetry, transfer, subgroup sensitivity

JEL classification: D63 I32

* Madras Institute of Development Studies, 79 Second Main Road, Gandhinagar, Adyar, Chennai-600 020, Tamilnadu, India, subbu@mids.tn.nic.in

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UNU World Institute for Development Economics Research (UNU-WIDER)
Katajanokanlaituri 6 B, 00160 Helsinki, Finland

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1 Motivation

This paper combines the themes of both poverty and inequality, within a measurement setting, with a view to elucidating some of the complications that can arise, and how these might be addressed, when we allow for a certain elementary obtrusion of considerations of 'society' into routinely mainstream notions of the 'economy'. Specifically, the concern is with reckoning aspects of distributive justice from a *group* perspective, in addition to the more standardly *individualistic* perspective, with an emphasis on the sorts of conflicts which these alternative perspectives could engender, and how these conflicts might be reconciled in the process of seeking a real-valued measure of income poverty. The two perspectives of distributive justice just alluded to are handily described by Stewart (2002) in the terms, respectively, of *horizontal* inequality and *vertical* inequality. Much of received theorizing has been concerned almost exclusively with vertical inequality, which has tended to confine horizontal inequality, in a relative sense, to the unhappy status of what Stewart (op. cit.) calls 'a neglected aspect of development.' The question of why groups deserve a great deal more analytical and empirical attention than they would appear to have received in the discourse on poverty, inequality and development has been dealt with fairly exhaustively in Stewart's work, and therefore represents ground that one does not need to cover here again. Reference, in this context, must also be made to earlier work, notably from the viewpoint of measurement, by Anand and Sen (1995), Jayaraj and Subramanian (1999), Majumdar (1999), Majumdar and Subramanian (2001), and Subramanian and Majumdar (2002).

It is perhaps important to stress that the analytical content of this paper – whether in the matter of the existence results it advances or the specific poverty measures it discusses – is not motivated by any illusion as to either its revelation- or novelty-value. The arguments in this paper are, on the whole, uniformly simple and obvious. It is perhaps precisely because of this obviousness that attention needs to be drawn to the pervasive reality and centrality of groups in any assessment of social welfare; and the motivational concern of this paper is, simply, to point to the obvious so that it is not overlooked. There is nothing very paradoxical in this: it is just another instance of Edgar Allan Poe's purloined letter. If the notion of horizontal inequality has met with traditionally little engagement in exercises dealing with the assessment of overall deprivation or well-being, this fact probably has much to do with the common failure of being blind to what stares one in the face (excepting, of course, those instances of a deliberate ideological opposition to the notion of groups and their relevance in the scheme of things). The motivational objective of this paper, therefore, is to highlight an issue for reasons which arise not from its complexity, but from a combination of its importance and its relative historical neglect.

2 Measuring poverty in a stratified society

An issue of potential interest in the measurement of poverty has to do with the way in which poverty is distributed across different well-defined subgroups within the population[1]. Foster and Shorrocks (1991) have advanced and studied a property of poverty indices which they call *subgroup consistency* and which demands that, other things equal, an increase in any subgroup's poverty should increase overall poverty. In motivating their discussion of this property, the authors (1991: 687) state: 'Subgroup consistency may ... be regarded as a natural analogue of the monotonicity condition of Sen (1976), since monotonicity requires that aggregate poverty fall ... if one *person's* poverty is reduced, *ceteris paribus*, while subgroup consistency demands that aggregate poverty fall if one *subgroup's* poverty is reduced, *ceteris paribus*.' In this connection, it is immediately tempting to seek also an analogy between the conventional *transfer* axiom and a corresponding one which could be defined for subgroups.

The transfer condition requires that, *ceteris paribus*, a progressive rank-preserving transfer between two poor individuals should be accompanied by a reduction in poverty. In a similar spirit, one could require – speaking loosely for the moment – that aggregate poverty should decline with a move toward equalization, through income redistribution, of subgroup poverty levels, other things remaining the same. The requirement is formalized, in this note, through the postulation of a property called *subgroup sensitivity*.

The relevance of subgroup sensitivity is captured in the following illustration. Suppose poverty to be measured by the simple headcount ratio. Imagine that the population is partitioned into two subgroups, A and B, where A stands for a historically disadvantaged social group, say, and B stands for the rest. Suppose the headcount ratio of poverty for subgroup A to be 0.7 and that for subgroup B to be 0.3. If subgroup A's share in total population is 50 per cent, then the headcount ratio for the population as a whole would be 0.5 (= 0.5*0.7 + 0.5*0.3). If now there is a pure redistribution of income from subgroup B to subgroup A, whereby A's headcount ratio is reduced to 0.6 while B's headcount ratio is raised to 0.4, then we may be disposed to judge that such a movement toward equalization of poverty across subgroups should lead to an overall reduction in measured poverty. This, precisely, is the sort of judgment that would be endorsed by the axiom of subgroup sensitivity. Such a possibility, however, is not accommodated by the headcount ratio which, in the context of the present example, continues to remain at 0.5 (= 0.5*0.6 + 0.5*0.4). This simple example points to a possible limitation underlying conventional approaches to the measurement of poverty.

The difficulty in question resides in the fact that certain axioms for poverty measurement have been advanced on the implicit assumption that there is no more than one group – that constituted by the population as a whole – which needs to be reckoned in an overall assessment of the extent of poverty in a society. This assumption is particularly salient in the so-called 'symmetry' and 'transfer' axioms. The former property demands, in essence, that in making poverty comparisons across income profiles, the personal identities of individuals should be of no account. This property is also a standard feature of the literature in social choice theory, where it more commonly goes by the name of the 'anonymity' axiom. One can immediately see that if the identity of an individual is linked to the fact of group affiliation, then a poverty index which is

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1 For analyses of poverty measurement when different groups are perceived to have different needs, as reflected in variations in subgroup poverty lines, see Atkinson (1987) and Keen (1992). While the concern in these papers, as in the present one, is with reckoning subgroup poverty in the measurement of aggregate poverty, the underlying motivations are rather different.

sensitive to the group composition of a population could well militate against the requirement of anonymity imposed by the symmetry axiom. The axiom in question is widely regarded as being completely innocuous and self-evidently desirable from an ethical point of view: it is, indeed, so much taken for granted that its social choice version – anonymity – has come in for specifically targeted criticism in a carefully argued assessment by Loury (2000), who refers to the anonymity axiom as a stark example of 'liberal neutrality'. What could constitute a possible objection to symmetry/anonymity which, after all, echoes a requirement that is a feature of many liberal constitutions – the requirement that no person may be discriminated against on the grounds of birth, race, class, caste, religion, or sex?

Here is an objection: one may wish to discriminate in favour of members of an historically oppressed and consequently currently disadvantaged group; but in order to discriminate *in favour of* somebody, one will have to discriminate *against* somebody else on the ground of the latter's group affiliation – an avenue of redress for the former individual which is denied by the symmetry axiom. Briefly, symmetry cannot be reconciled with group-based principles of distributive justice such as are embodied in provisions like 'compensatory discrimination' or 'affirmative action'. The preceding discussion suggests that symmetry is an unquestionably desirable property when one is assessing inequality or poverty or welfare in the context of a *homogeneous* population; however – and possibly because of repeated, mechanical use – often the qualifying attribute of homogeneity seems implicitly to be forgotten when the axiom is invoked. Indeed, one of the few authors who are careful to rationalize the symmetry axiom on the grounds of its appeal in the context of homogeneous populations is Shorrocks (1988). One could, of course, suppose that symmetry is so widely specified as a desirable property only because of a formally unstated assumption as to the homogeneity of a population; but somehow, this is a less than convincing explanation when the axiom is also routinely invoked in the context of exercises which explicitly accommodate groups into the analysis, and therefore are concerned with heterogeneous populations.

Similarly, where the transfer axiom is concerned, one can see that considerations of inter*group* equality could, in specific cases, conflict with considerations of inter*personal* equality. Regressive transfers between members of a homogeneous group may naturally be taken to reduce welfare and enhance inequality and poverty; but should the same outcome necessarily hold for transfers between individuals belonging to different groups in a heterogeneous population? It is these sorts of difficulties in attending the measurement of poverty that are sought to be made transparent in this paper.

The rest of the paper is organized as follows. Section 3 deals with certain relevant preliminary formalities of concepts and definitions. Section 4 presents a couple of general possibility theorems on poverty indices which highlight the complications that can arise from taking the issue of group-wise poverty distribution seriously. Section 5 offers a brief interpretation and assessment of the results presented in the previous section. Examples from the literature of 'group-sensitive' poverty measures are discussed in Section 6, while Section 7 carries a brief discussion of the implications of 'group-sensitivity' for budgetary intervention in poverty redress exercises. Section 8 concludes.

3 Formalities[2]

Let N be the set of all positive integers. For every $n \in N$, let $\mathbf{X}_n$ be the set of nonnegative vectors $\{(\mathbf{x}_1,...,\mathbf{x}_i,...,\mathbf{x}_n)\}$, and define $\mathbf{X}$ to be the set $\cup_{n\in N}\mathbf{X}_n$. A typical element of the set $\mathbf{X}$ is an income vector $\mathbf{x}$, a typical element x_i of which stands for the income of person i ($i\in N$). For every $\mathbf{x}\in \mathbf{X}$, $n(\mathbf{x})$ stands for the dimensionality of $\mathbf{x}$. The *poverty line*, z, is a positive level of income such that individuals with incomes less than z are certified to be *poor*. For every $(z,\mathbf{x})\in T\times\mathbf{X}$ (where T is the set of positive reals), $\mathbf{x}_P(z,\mathbf{x})$ will stand for the vector of poor incomes; $\mathbf{x}_R(z,\mathbf{x})$ will stand for the vector of nonpoor incomes; and $\mu^P(z,\mathbf{x})$ will stand for the average income of the poor population. A vector $\mathbf{x}\in \mathbf{X}$ will be said to be derived as a *permutation* of a vector $\mathbf{y}\in \mathbf{X}$ if $\mathbf{x} = \mathbf{y}\Pi$ for some permutation matrix Π; and $\mathbf{x}^o$ is the *ordered* version of $\mathbf{x}$ if $\mathbf{x}^o$ is derived from $\mathbf{x}$ by a permutation for which $x^o_i \le x^o_{i+1}$ $(i = 1,...,n(\mathbf{x})-1)$.[3] For all $\mathbf{x},\mathbf{y}\in\mathbf{X}$, it will be said that $\mathbf{x}$ *vector-dominates* $\mathbf{y}$ — written $\mathbf{x} \vee \mathbf{y}$ — if $\mathbf{x}$ and $\mathbf{y}$ are equi-dimensional, $x^o_i \ge y^o_i$ for all i and $x^o_i > y^o_i$ for some i; $\mathbf{x}$ will be said to be derived from $\mathbf{y}$ through *an increment to a person's income* if $\mathbf{x}$ and $\mathbf{y}$ are equi-dimensional, $x_i = y_i$ for all $i \neq k$ and $x_k > y_k$ for some k; and $\mathbf{x}$ will be said to be derived from $\mathbf{y}$ through *a permissible progressive transfer* if $\mathbf{x}$ and $\mathbf{y}$ are equi-dimensional, $x_i = y_i$ for all $i \neq j,k$ for some j,k satisfying $y_j < y_k$, $x_j = y_j + \delta$ and $x_k = y_k - \delta$ where $0 \le \delta \le (y_k-y_j)/2$.

Since a major concern of this paper is with the notion of reference groups, I turn now to this latter issue. For every $n\in N$, let $\mathbf{G}_n$ be the set of all possible partitions of the set $\{1,...,n\}$, and define $\mathbf{G}$ to be the set $\cup_{n\in N}\mathbf{G}_n$. A typical element of the set $\mathbf{G}$ is a partition $\mathbf{g}$ of the population, and a typical element of $\mathbf{g}$ is a subgroup, denoted by the running index j. (It is immediate, of course, that for any partition $\mathbf{g}$ of the population, the number of subgroups must be at least one, and cannot exceed the number of individuals in the population). Notice that any $\mathbf{g}\in \mathbf{G}$ is induced by some appropriate *grouping* of the population, by which is meant some well-defined scheme of categorization (such as by height, age, gender, caste, religion, etc.), in accordance with which the population can be classified in a mutually exclusive and completely exhaustive fashion. (It is possible, of course, that two or more groupings can induce the *same* partitioning of a given population: for example, if in some society the only illiterate individuals all happen to be females, then a grouping according to gender [{male, female}] will yield up the same partition as a grouping according to literacy status [{literate, illiterate}].) For all $\mathbf{x}\in\mathbf{X}$ and $\mathbf{g}\in \mathbf{G}$, the pair $(\mathbf{x},\mathbf{g})$ will be said to be *compatible* if and only if $\mathbf{g}$ partitions a population of the same size as the dimensionality of $\mathbf{x}$. Given any compatible pair $(\mathbf{x},\mathbf{g})$ belonging to $\mathbf{X}\times\mathbf{G}$, subgroup j's vector of incomes will be represented by $\mathbf{x}^j$, for every j belonging to $\mathbf{g}$.

Two polar cases of grouping are of interest. The first is what one may call the '*atomistic grouping*', which induces the *finest* partition $\mathbf{g}^a = \{\{1\},...,\{i\},...,\{n\}\}$ of $\{1,...,n\}$: this is the case of 'complete heterogeneity', such as might be precipitated by a classification according to 'finger-print type'. The second is what one may call the '*universal grouping*', which induces the *coarsest* partition $\mathbf{g}^u = \{\{1,...,n\}\}$ of $\{1,...,n\}$: this is the case of 'complete homogeneity', such as might be precipitated by a classification according, say, to membership to the human race.

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2 This section is heavily dependent on Jayaraj and Subramanian (1999: especially pp. 197-200).

3 See Foster and Shorrocks (1991).

We are now in a position to define a *poverty index*, by which shall be meant a mapping $P: T\times\mathbf{X}\times\mathbf{G} \rightarrow R$ (where R is the real line), such that, for all $z\in T$, and all compatible $(\mathbf{x},\mathbf{g})\in \mathbf{X}\times\mathbf{G}$, $P(z,\mathbf{x},\mathbf{g})$ is a unique real number which is intended to signify the extent of poverty that obtains in the regime $(z,\mathbf{x},\mathbf{g})$. To invest P with more structure, we need to constrain it with a set of properties that we may require the poverty index to satisfy. What follows is a restricted set of just six axioms, of which the sixth[4] is a relatively recent addition to the stock of known axioms.

Symmetry (Axiom S). For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if $\mathbf{x}$ is derived from $\mathbf{y}$ by a permutation, then $P(z,\mathbf{x},\mathbf{g}) = P(z,\mathbf{y},\mathbf{g})$.

Monotonicity (Axiom M). For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if $\mathbf{x}$ is derived from $\mathbf{y}$ by an increment to a poor person's income, then $P(z,\mathbf{x},\mathbf{g}) < P(z,\mathbf{y},\mathbf{g})$.

Respect for Income Dominance (Axiom D). (See Amiel and Cowell, 1994.) For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if $\mathbf{x}_P \vee \mathbf{y}_P$, then $P(z,\mathbf{x},\mathbf{g}) < P(z,\mathbf{y},\mathbf{g})$.

[*Note*: Amiel and Cowell (1994) point out that Axioms M and D are independent, but are rendered equivalent in the presence of the symmetry axiom].

Transfer (Axiom T). For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if $\mathbf{x}_R = \mathbf{y}_R$ and $\mathbf{x}_P$ is derived from $\mathbf{y}_P$ by a permissible progressive transfer, then $P(z,\mathbf{x},\mathbf{g}) < P(z,\mathbf{y},\mathbf{g})$.

Subgroup Consistency (Axiom SC). (See Foster and Shorrocks, 1991.) For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if $\mathbf{x}^j$ and $\mathbf{y}^j$ are of the same dimensionality for all $j\in\mathbf{g}$ and $[P(z,\mathbf{x}^j,\mathbf{g}^u) = P(z,\mathbf{y}^j,\mathbf{g}^u)$ for all $j\in\mathbf{g}\backslash\{k\}$ and $P(z,\mathbf{x}^k,\mathbf{g}^u) < P(z,\mathbf{y}^k,\mathbf{g}^u)$ for some $k\in\mathbf{g}]$, then $P(z,\mathbf{x},\mathbf{g}) < P(z,\mathbf{y},\mathbf{g})$.

Subgroup Sensitivity (Axiom SS). For all $z\in T$, all $\mathbf{x},\mathbf{y}\in \mathbf{X}$ and all $\mathbf{g}\in \mathbf{G}$ such that $(\mathbf{x},\mathbf{g})$ and $(\mathbf{y},\mathbf{g})$ are compatible pairs, if (i) $\mathbf{x}^j$ and $\mathbf{y}^j$ are of the same dimensionality for all $j\in\mathbf{g}$; (ii) $\mu^P(z,\mathbf{x}) = \mu^P(z,\mathbf{y})$; and (iii) $\mathbf{x}^j = \mathbf{y}^j$ for all $j\in\mathbf{g}\backslash\{s,t\}$ for some s,t satisfying $P(z,\mathbf{x}^t,\mathbf{g}^u) < P(z,\mathbf{y}^t,\mathbf{g}^u) \leq P(z,\mathbf{y}^s,\mathbf{g}^u) < P(z,\mathbf{x}^s,\mathbf{g}^u)$; then it is the case that $P(z,\mathbf{x},\mathbf{g}) > P(z,\mathbf{y},\mathbf{g})$.

[That is, other things remaining the same, if by a pure redistribution of poor incomes the relatively disadvantaged subgroup s becomes less poor and the relatively advantaged subgroup t becomes poorer, while maintaining the relative poverty rankings of the two subgroups, then overall poverty should decline.]

What is the class of poverty indices which satisfy the property of subgroup sensitivity in conjunction with some combination of other desirable properties discussed earlier? This question is addressed in the next section.

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4 The axiom of 'subgroup sensitivity' has been advanced in Jayaraj and Subramanian (1999).

4 Two general possibility results for poverty indices

The following two propositions are true.

Proposition 4.1. There exists no poverty index $P: T \times \mathbf{X} \times \mathbf{G} \rightarrow R$ satisfying Axioms S, M and SS.

Proof. What follows is a proof by contradiction. Consider a situation in which z = 50. Let **g** be such that the population is partitioned into exactly two subgroups which are indexed 1 and 2 respectively. Consider a pair of income vectors **x**,**y** such that $n(\mathbf{x}_P) = n(\mathbf{y}_P) = n(\mathbf{x}_R) = n(\mathbf{y}_R) = 2$. Further, assume that $\mathbf{x}^1{}_R = \mathbf{x}^2{}_R = \mathbf{y}^1{}_R = \mathbf{y}^2{}_R = (60,60)$; and that $\mathbf{x}^1{}_P = (10,20)$; $\mathbf{x}^2{}_P = (30,40)$; $\mathbf{y}^1{}_P = (10,30)$; and $\mathbf{y}^2{}_P = (20,40)$. It is immediately clear that

(4.1.1) $\mu^P(z,\mathbf{x}) = \mu^P(z,\mathbf{y})$ (= 25).

Further, since P satisfies Axioms S and M, it must satisfy Axiom D as well; and by Axiom D, given that $n(\mathbf{x}^1) = n(\mathbf{x}^2) = n(\mathbf{y}^1) = n(\mathbf{y}^2)$ and $\mathbf{x}^2{}_P \vee \mathbf{y}^2{}_P \vee \mathbf{y}^1{}_P \vee \mathbf{x}^1{}_P$, we have:

(4.1.2) $P(z,\mathbf{x}^2,\mathbf{g}^u) < P(z,\mathbf{y}^2,\mathbf{g}^u) < P(z,\mathbf{y}^1,\mathbf{g}^u) < P(z,\mathbf{x}^1,\mathbf{g}^u)$.

In view of (4.1.1) and (4.1.2), Axiom SS will dictate that

(4.1.3) $P(z,\mathbf{x},\mathbf{g}) > P(z,\mathbf{y},\mathbf{g})$.

Next, notice that $\mathbf{x} = (\mathbf{x}^1{}_P,\mathbf{x}^2{}_P,\mathbf{x}^1{}_R,\mathbf{x}^2{}_R)$ is just a permutation of $y = (\mathbf{y}^1{}_P,\mathbf{y}^2{}_P,\mathbf{y}^1{}_R,\mathbf{y}^2{}_R)$: if the person with income 20 in $\mathbf{x}^1{}_P$ swaps places with the person with income 30 in $\mathbf{x}^2{}_P$, then $\mathbf{x}^1{}_P$ becomes $\mathbf{y}^1{}_P$ and $\mathbf{x}^2{}_P$ becomes $\mathbf{y}^2{}_P$. By Axiom S, one must have:

(4.1.4) $P(z,\mathbf{x},\mathbf{g}) = P(z,\mathbf{y},\mathbf{g})$.

(4.1.3) and (4.1.4) are mutually incompatible, and this completes the proof of the proposition. (*Q.E.D.*).

Proposition 4.2. There exists no poverty index $P: T \times \mathbf{X} \times \mathbf{G} \rightarrow R$ satisfying Axioms D, T and SS.

Proof. Again we have a proof by contradiction. As in the proof of Proposition 4.1, imagine a situation in which z = 50, and **g** is such as to partition the population into two subgroups, 1 and 2. Let **x** and **y** be two income vectors satisfying $n(\mathbf{x}_P) = n(\mathbf{y}_P) = n(\mathbf{x}_R) = n(\mathbf{y}_R) = 2$, and let it be the case that $\mathbf{x}^1{}_R = \mathbf{x}^2{}_R = \mathbf{y}^1{}_R = \mathbf{y}^2{}_R = (60,60)$; $\mathbf{x}^1{}_P = (10,20)$; $\mathbf{x}^2{}_P = (20,30)$; $\mathbf{y}^1{}_P = (10,25)$; and $\mathbf{y}^2{}_P = (15,30)$. Notice first that

(4.2.1) $\mu^P(z,\mathbf{x},\mathbf{g}) = \mu^P(z,\mathbf{y},\mathbf{g})$ (= 20).

Further, since $n(\mathbf{x}^1) = n(\mathbf{x}^2) = n(\mathbf{y}^1) = n(\mathbf{y}^2)$, and $\mathbf{x}^2{}_P \vee \mathbf{y}^2{}_P \vee \mathbf{y}^1{}_P \vee \mathbf{x}^1{}_P$, Axiom D will require that

(4.2.2) $P(z,\mathbf{x}^2,\mathbf{g}) < P(z,\mathbf{y}^2,\mathbf{g}) < P(z,\mathbf{y}^1,\mathbf{g}) < P(z,\mathbf{x}^1,\mathbf{g})$.

In view of (4.2.1) and (4.2.2), Axiom SS will dictate that

(4.2.3) $P(z,\mathbf{x},\mathbf{g}) > P(z,\mathbf{y},\mathbf{g})$.

Next, it is easy to see that $\mathbf{x}_P = (\mathbf{x}^1{}_P,\mathbf{x}^2{}_P)$ has been derived from $\mathbf{y}_P = (\mathbf{y}^1{}_P,\mathbf{y}^2{}_P)$ by a permissible progressive transfer (of 5 from the person with income 25 in the vector $\mathbf{y}^1{}_P$ to the person with income 15 in the vector $\mathbf{y}^2{}_P$). Given, additionally, that $n(\mathbf{x}) = n(\mathbf{y})$, Axiom T will demand that

(4.2.4) $P(z,\mathbf{x},\mathbf{g}) < P(z,\mathbf{y},\mathbf{g})$.

From (4.2.3) and (4.2.4), we obtain a contradiction. This completes the proof of the proposition. (*Q.E.D.*).

5 Assessment

The two non-existence results presented in the preceding section confirm the wisdom of a moral (suitably translated to the poverty-measurement context) that has been upheld by Sen in his discussion of impossibility results in social choice theory. This moral (Sen, 1970: 178) points to 'the sever[ity] of the problem of postulating absolute principles … that are supposed to hold in every situation'. Confronted with an impossibility theorem, it is not always a simple matter to be able to convincingly identify the 'villain of the piece', namely the guilty axiom that is driving the result under review. Specifically, given the present context, and by way of illustration, neither the transfer nor the subgroup sensitivity axiom is as persuasive as either may appear in a context-free environment. For example, in particular cases the antecedents of Axiom SS can be satisfied by a very regressive transfer (from an acutely impoverished person belonging to a relatively advantaged subgroup to a much better-off individual belonging to a relatively disadvantaged subgroup), and in such cases we may find it hard to endorse Axiom SS. By the same token, *any* permissible progressive transfer between two poor individuals would be encouraged by the transfer axiom, and in particular cases, wherein such transfers exacerbate inter-group poverty differentials, we may find it hard to accept Axiom T.

Given this general difficulty of discerning an unqualified virtue in any given axiom under all plausible circumstances, a possible way out may be to restrict the applicability of the axiom to a domain that is relatively non-controversial. In the specific instance of the *symmetry* axiom, for example, there may be a case for requiring only that *within* any subgroup, swapping incomes across members of the subgroup should leave the value of the poverty index for the subgroup unchanged. In a *between-group* context, one might further wish to impose the restriction that measured poverty should be invariant with respect to the precise labels that are attached to subgroups. Similarly, in the case of the *transfer* axiom, one may wish to restrict its operation, in its conventional form, only to interpersonal redistributions of income *within* each subgroup. Is there also a way of ensuring some requirement of equity, from a *between-group* perspective, in the distribution of poverty across subgroups? How this can be done is perhaps best exemplified by means of an illustration, discussed in the following section.

6 'Group-sensitive' poverty indices: an example

Let **P*** be the set of all symmetric, monotonic, transfer-satisfying, and decomposable poverty indices. (A poverty index is said to be *decomposable* – see Foster, Greer and

Thorbecke, 1984 – if overall poverty can be expressed as a population-share weighted sum of subgroup poverty levels). Let ($\mathbf{x}$,$\mathbf{g}$) be a compatible pair belonging to $\mathbf{X}\times\mathbf{G}$, and let $\mathbf{g}$ partition the population into K distinct subgroups. The poverty line, as usual, is given by z. Consider $P^*\in\mathbf{P}^*$, and let P^*_j serve as a shorthand for $P^*(z,\mathbf{x}^j,\mathbf{g}^u)$, which is the poverty level, as measured by the index P*, of the jth subgroup (j = 1,…,K). Assume, further, that the subgroups have been indexed in non-increasing order of poverty, so that $P^*_j \geq P^*_{j+1}$, j = 1,…,K–1. Let θ_j be the population share of the jth subgroup, and Θ_j the proportion of the population that belongs to groups whose poverty levels are less than or equal to the poverty level of the jth subgroup. Then, two examples of aggregate poverty measures which directly incorporate considerations of 'inter-group equity' are the indices ${}_1P$ and ${}_2P$ below:

(6.1) $\quad {}_1P(z,\mathbf{x},\mathbf{g}) = [1/(K-1)][\Sigma_{j=1}^{K}[(K-1-j)\theta_j + \Theta_j]P^*_j$ and

(6.2) $\quad {}_2P(z,\mathbf{x},\mathbf{g}) = [\Sigma_{j\in\mathbf{g}}\theta_j(P^*_j)^2]^{1/2}$

The index ${}_1P$ is of a type discussed in Jayaraj and Subramanian (1999), Majumdar and Subramanian (2001), and Subramanian and Majumdar (2002), while the index ${}_2P$ is of a type discussed in Anand and Sen (1995). In what follows, we shall confine attention to the class of indices embodied in (6.2); and, in the interests of specificity, it would be useful to particularize the index P* to a familiar poverty measure. To this end, consider the so-called P_α class of indices proposed by Foster, Greer and Thorbecke (1984) (FGT P_α), and given, for all $z\in T$ and all compatible $(\mathbf{x},\mathbf{g}^u)\in\mathbf{X}\times\mathbf{G}$, by:

(6.3) $\quad P_\alpha(z,\mathbf{x},\mathbf{g}^u) = (1/n(\mathbf{x}))\Sigma_{i\in Q(\mathbf{x})}[(z-x_i)/z]^\alpha, \alpha \geq 0$

where Q($\mathbf{x}$) is the set of poor individuals in $\mathbf{x}$. Each member of the class of indices P_α (see FGT, 1984) is known to be symmetric and decomposable; and the index P_2, in addition, satisfies the monotonicity and transfer axioms. Let us designate by F the index P_2. A specialization of the class of 'between-group equity-conscious' poverty measures encompassed in (6.2) is yielded by the following index,[5] F^a, which is given, for all $z\in T$, and all compatible $(\mathbf{x},\mathbf{g})\in\mathbf{X}\times\mathbf{G}$, by

(6.4) $\quad F^a(z,\mathbf{x},\mathbf{g}) = [\Sigma_{j\in\mathbf{g}}\theta_j F^2_j]^{1/2}$

In what sense does F^a attend to the concern for inter-group equity? One way of seeing this is, first, to note that $C^2 \equiv [(1/F^2)\Sigma_{j\in\mathbf{g}}\theta_j F^2_j - 1]$ is the squared coefficient of variation in the distribution of the group-specific poverty levels F_j. Then, it is clear that $\Sigma_{j\in\mathbf{g}}\theta_j F^2_j = F^2(1 + C^2)$ whence, in view of (6.4), and after making the appropriate substitution, we have:

(6.5) $\quad F^a(z,\mathbf{x},\mathbf{g}) = F(1 + C^2)^{1/2}$

Notice from (6.5) that the index F^a is just the average (across subgroups) level of poverty, as measured by the index F, enhanced by a factor incorporating the squared coefficient of variation in the inter-group distribution of poverty as measured by the F_j. F^a is mean poverty 'adjusted' for inter-group inequality. (It may be noted that for the

5 The underlying logic of the poverty index F is motivationally similar to that of the 'gender-adjusted human development index' of Anand and Sen (1995), which has been discussed in UNDP (1995).

class of poverty indices subsumed in (6.1), the 'adjustment' for inequality in the inter-group distribution of poverty levels is *via* a 'Gini-type', rather than 'coefficient of variation-type', inequality measure.)

It is not difficult to check that the index F^a satisfies both the 'within-group' and the 'between-group' versions of the symmetry property discussed earlier. 'Within-group', because it is a known property of the FGT P_α class of indices that each member satisfies symmetry, and each of the F_j is just the FGT index realized for $\alpha = 2$. 'Between-group', because one can see from inspection of (6.4) that switching around the labels of the subgroups will make no difference to the value of F^a. Further, F^a clearly also satisfies the 'within-group' version of the transfer property, since each of the F_j is known to be transfer-respecting (indeed, the P_α indices all satisfy transfer for every $\alpha > 1$); while in a 'between-group' context, as the expression for F^a in (6.3) makes clear, it *is* sensitive to inter-group inequality in the distribution of poverty: when the latter (as measured by C^2) rises, other things equal, F^a also registers an increase in value.

An advantage with a poverty index such as F^a resides in a specific sort of 'flexibility' it possesses, in that it effects a tradeoff between the conventional transfer axiom and Axiom SS, by allowing the former to 'trump' the latter when interpersonal transfers across groups are rather more than less 'progressive', and allowing the latter to 'trump' the former when interpersonal transfers across groups are rather less than more 'progressive'. A simple example, similar to one in Jayaraj and Subramanian (1999), might help to elucidate this point.

Let the poverty line z be given by 10. Let $\mathbf{x}$ and $\mathbf{y}$ be two 4-dimensional income vectors, and let $\mathbf{g}^o$ be such as to partition the population into two subgroups 1 and 2 respectively, with each subgroup having two persons in it. Similarly, let $\mathbf{u}$ and $\mathbf{v}$ be any other two 4-dimensional vectors, and let $\mathbf{g}^o$ again partition the population into the subgroups 1 and 2, with each subgroup having two members. The vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{u}$ and $\mathbf{v}$ can be written, respectively, as $\mathbf{x} = (\mathbf{x}^1,\mathbf{x}^2)$, $\mathbf{y} = (\mathbf{y}^1,\mathbf{y}^2)$, $\mathbf{u} = (\mathbf{u}^1,\mathbf{u}^2)$, and $\mathbf{v} = (\mathbf{v}^1,\mathbf{v}^2)$. Let it be the case that $\mathbf{x}^1 = (1,9)$, $\mathbf{x}^2 = (1.5,3.5)$, $\mathbf{y}^1 = (0,9)$ and $\mathbf{y}^2 = (1.5,4.5)$; and $\mathbf{u}^1 = (1,9)$, $\mathbf{u}^2 = (1.5,2)$, $\mathbf{v}^1 = (0,9)$ and $\mathbf{v}^2 = (1.5,3)$. Suppose we measure poverty by the 'adjusted' index F^a of (5.2). Some routine computation will confirm that $F(z,\mathbf{x}^1,\mathbf{g}^u) = 0.205$; $F(z,\mathbf{x}^2,\mathbf{g}^u) = 0.28625$; $F(z,\mathbf{y}^1,\mathbf{g}^u) = 0.2525$; $F(z,\mathbf{y}^2,\mathbf{g}^u) = 0.25625$; $F^a(z,\mathbf{x},\mathbf{g}^o) = 0.2490$; $F^a(z,\mathbf{y},\mathbf{g}^o) = 0.2541$; $F(z,\mathbf{u}^1,\mathbf{g}^u) = 0.205$; $F(z,\mathbf{u}^2,\mathbf{g}^u) = 0.340625$; $F(z,\mathbf{v}^1,\mathbf{g}^u) = 0.2525$; $F(z,\mathbf{v}^2,\mathbf{g}^u) = 0.303125$; $F^a(z,\mathbf{u},\mathbf{g}^o) = 0.2811$; and $F^a(z,\mathbf{v},\mathbf{g}^\mathbf{o}) = 0.2790$. Notice now that $\mathbf{y}$ is derived from $\mathbf{x}$ through a regressive transfer, exactly as $\mathbf{v}$ is derived from $\mathbf{u}$ through a regressive transfer; in both cases, the transfer is from a poor person belonging to a relatively advantaged group to a richer poor person belonging to a relatively disadvantaged group. The transfer axiom will dictate that $F^a(z,\mathbf{y},\mathbf{g}^o) > F^a(z,\mathbf{x},\mathbf{g}^o)$, while the subgroup sensitivity axiom will dictate that $F^a(z,\mathbf{y},\mathbf{g}^o) < F^a(z,\mathbf{x},\mathbf{g}^o)$; in exactly similar fashion, the transfer axiom will dictate that $F^a(z,\mathbf{v},\mathbf{g}^o) > F^a(z,\mathbf{u},\mathbf{g}^o)$, while the subgroup sensitivity axiom will dictate that $F^a(z,\mathbf{v},\mathbf{g}^o) < F^a(z,\mathbf{u},\mathbf{g}^o)$. What actually obtains is a situation in which $F^a(z,\mathbf{y},\mathbf{g}^o)$ (= 0.2541) > $F^a(z,\mathbf{x},\mathbf{g}^o)$ (= 0.2490), and $F^a(z,\mathbf{v},\mathbf{g}^o)$ (= 0.2790) < $F^a(z,\mathbf{u},\mathbf{g}^o)$ (= 0.2811). That is to say, in the transition from $\mathbf{x}$ to $\mathbf{y}$, Axiom T is upheld and Axiom SS violated, while in the transition from $\mathbf{u}$ to $\mathbf{v}$, Axiom T is violated and Axiom SS upheld. In both cases an interpersonally regressive income transfer has been accompanied by a diminution in the inter-group poverty-differential; only, in the first case the transfer has been more regressive than in the second (the income difference between those involved in the transfer is greater in the first case than in the second), and the poverty index F^a has effected a tradeoff in favour of the transfer axiom in the first case, and a tradeoff in favour of the subgroup sensitivity axiom in the second case. This does not accord ill with intuition for, as was pointed out in Section 5,

> in particular cases the antecedents of Axiom SS can be satisfied by a very regressive transfer (from an acutely impoverished person belonging to a relatively advantaged subgroup to a much better-off individual belonging to a relatively disadvantaged subgroup), and in such cases we may find it hard to endorse Axiom SS, [while], by the same token, *any* permissible progressive transfer between two poor individuals would be encouraged by the transfer axiom, and in particular cases, wherein such transfers exacerbate inter-group poverty differentials, we may find it hard to accept Axiom T.

Additionally, F^a has the convenient property of precipitating the index F as a special case, which happens when the grouping employed is the *universal* grouping which induces the coarsest partition $\mathbf{g}^u$ of the population. At the other extreme, when the grouping employed is the *atomistic* one, which induces the finest partition $\mathbf{g}^a$ of the population, it can be verified that F^a just becomes $(P_4)^{1/2}$, where P_4 is the P_α index for $\alpha = 4$. (The details are available in Jayaraj and Subramanian, 1999.)

Finally, a concrete empirical illustration of the information-value of an index such as F^a may be useful. Making use of data on the cross-country distribution of per capita gross national product (GNP) available in the United Nations Development Programme's *Human Development Report* (*HDR*), one can construct a picture of global poverty. Such an exercise has been carried out in Subramanian (2003), and we draw on the results of that exercise. The *HDR 1999* provides information on per capita GNP, in 'purchasing power parity dollars', for each of 174 countries for the year 1997. The global average per capita GNP works out to a little in excess of PPP$6,000, and we shall take the 'international poverty line' to be PPP$3,000 per capita per annum – which is less than one-half of the global average per capita GNP. We shall designate this poverty line by z^*; and let $\mathbf{x}^*$ denote the country-wise distribution of income as presented in the *HDR 1999*. Since information on *intra*-country distribution is unavailable, we shall simply assume that each person in each country receives the country's average per capita GNP. A grouping of countries resorted to in the *HDR 1999* is a classification comprising the following seven groups: sub-Saharan Africa, Asia and the Pacific, the Arab states, Latin America and the Caribbean, Eastern Europe and the Commonwealth of Independent States, Southern Europe, and Industrialized countries. Let us denote by $\mathbf{g}^*$ the partition of the world's population induced by this particular grouping. Poverty for each country-group j will now be measured by an index which one may call the *triage headcount ratio* h_j: for a poor country-group j (that is, a country-group whose per capita GNP is less than the poverty line), h_j is simply the proportion of the country-group's population that must be allocated an income of zero so that the average income of the rest of the population in the country-group is enabled to rise to the poverty line level of income z^*; and for a nonpoor country-group j, we shall take h_j to be zero. By this reckoning, the proportion of the world's population — call it h — that must just cease to exist in order that every remaining person may receive an income of z^* works out to 18.6 per cent – the details are provided in Table 1. More specifically, this is the value of the triage headcount ratio corresponding to the *universal* grouping $\mathbf{g}^u$: $h(z^*,\mathbf{x}^*,\mathbf{g}^u) = 0.186$. What if we employed the grouping $\mathbf{g}^*$ resorted to in the *HDR 1999*? The *adjusted* triage

Table 1 Grouped data on global poverty (1997)

Country Group	Population (in millions)	Population share	Triage headcount ratio
Sub-Saharan Africa	555.20	0.0967	0.6048
Asia and the Pacific	3140.30	0.5467	0.2126
Arab states	252.30	0.0439	0.1012
Latin America and the Caribbean (including Mexico)	490.50	0.0854	0.0167
Eastern Europe and the Commonwealth of Independent States	398.90	0.0695	0.0823
Southern Europe	64.20	0.0112	0.0000
Industrialized countries	842.30	0.1466	0.0000
World	5743.70	1.0000	0.1863

Note: The poverty line is taken to be PPP$3,000 per capita per annum (roughly one-half the average per capita GNP). The 'triage headcount ratio' is the proportion of the population that must be allowed to perish so that each member of the surviving population is enabled to just achieve a poverty-line level of income.

Source: Based on UNDP (1999).

headcount ratio $h^a(z^*,\mathbf{x}^*,\mathbf{g}^*) \equiv [\Sigma_{j\in \mathbf{g}^*}\theta_j h^2{}_j]^{1/2}$ turns to be 0.247, as can be confirmed from the figures presented in Table 1. The rise in the triage headcount ratio from 18.6 per cent to 24.7 per cent is a substantial one, and an indicator of the considerable inequality in the cross-country distribution of deprivation. If this outcome is an unattractive one, the picture, presumably, would be even worse if the grouping we employed classified the world not into 7 groups, but into 174 groups – each group being represented by an individual country. Incorporating inter-group differentials into an overall assessment of deprivation certainly shows up in the deeply stratified world in which we live.

7 Two implications of 'group-sensitivity' in a poverty measure

The particular grouping of a population we resort to must be informed by an appreciation of the sociological salience of the classificatory scheme we adopt. More than one classificatory scheme may be relevant, depending on the precise context in, and purpose for which, deprivation is being sought to be measured. It is therefore a matter of some importance, in making poverty comparisons, to be explicit not only about the distributions under comparison and the poverty line(s) employed, but also about the grouping invoked. A certain ranking, valid for poverty measured with a particular grouping in mind, could in principle be inverted by a ranking which is valid for poverty measured with some other grouping in mind. An example of such rank reversal has in fact already been considered in Section 6. Harking back to the income vectors **u** and **v** reviewed in Section 6, it can be verified that, when z = 10, $F^a(z,\mathbf{u},\mathbf{g}^u)$ (= 0.2728) < $F^a(z,\mathbf{v},\mathbf{g}^u)$ (= 0.2778), but $F^a(z,\mathbf{u},\mathbf{g}^o)$ (= 0.2811) > $F^a(z,\mathbf{v},\mathbf{g}^o)$ (= 0.2790). A first implication of working with 'group-sensitive' poverty measures, therefore, is that the particular grouping we employ can make a substantial difference to the evaluative outcome of poverty comparisons.

Second, the grouping that is employed also has implications for ‘targeting’ in poverty redress schemes. Again, a simple numerical illustration might be helpful in explicating the idea. Let the poverty line be $z' = 10$, and let **a** be a 4-dimensional income vector (namely $n(\mathbf{a}) = 4$). Let $\mathbf{g}'$ be a partition of the population, based, let us say, on a grouping by caste, which divides it into two groups, 1 and 2 respectively. We shall write $\mathbf{a} = (\mathbf{a}^1,\mathbf{a}^2)$, with $\mathbf{a}^1 = (a_{11},a_{12})$ and $\mathbf{a}^2 = (a_{21},a_{22})$, where a_{ji} stands for the income of the ith person in the jth group (j = 1,2 and i = 1,2). Suppose, for specificity, that $\mathbf{a}^1 = (4,6)$ and $\mathbf{a}^2 = (3,9)$. If θ_j is the population share of subgroup j (j = 1,2), then it is clear that in the present instance $\theta_1 = \theta_2 = 1/2$. Let $\mathbf{g}^u$ be an alternative partitioning of the population, induced by the universal grouping, which recognizes only one group, that constituted by the grand coalition of individuals. Suppose a budgetary allocation of T = 5 is available for poverty alleviation. Which is the best way of allocating the budget among the individuals in the population?

To complicate matters, we shall imagine that there are two policymakers, A and B, of whom A – who has no time for sociological affectations – believes that $\mathbf{g}^u$ is the only valid partitioning of the population, while B – who herself is a member of an underprivileged caste – believes that $\mathbf{g}'$ is a meaningful partition of the population. Policymaker A is comfortable with using the poverty index $F(z,\mathbf{x})$ (which, to recall, is the same as the index $F^a(z,\mathbf{x},\mathbf{g}^u)$), while policymaker B is comfortable with using the index $F^a(z,\mathbf{x},\mathbf{g})$. Let t_{ji}, in B’s notation, be the amount of the budgetary allocation T which goes to the ith individual in the jth group (j = 1,2 and i = 1,2), and denote by **t** the vector $(t_{11},t_{12},t_{21},t_{22})$; further, let $\mathbf{t}^1$ and $\mathbf{t}^2$ stand for the vectors (t_{11},t_{12}) and (t_{21},t_{22}) respectively. (Of course, the individuals that B calls 11, 12, 21 and 22 will probably be called just 1, 2, 3 and 4 by A, but since the latter believes in a thoroughgoing version of the symmetry axiom, his philosophy of ‘what’s in a name?’ should be compatible with an acceptance of B’s eccentric mode of labelling the individuals.)

A’s objective is to solve the following programming problem:

Problem A

Minimize $F^a(z',\mathbf{a} + \mathbf{t},\mathbf{g}^u) = [1/n(\mathbf{a})(z')^2][\{z'-(a_{11} + t_{11})\}^2 + \{z'-(a_{12} + t_{12})\}^2 + \{z'-(a_{21} + t_{21})\}^2 + \{z'-(a_{22} + t_{22})\}^2]$

$\{t_{11},t_{12},t_{21},t_{22}\}$

subject to

$t_{11} + t_{12} + t_{21} + t_{22} \leq T$ and

$0 \leq t_{11} \leq z'-a_{11}$, $0 \leq t_{12} \leq z'-a_{12}$, $0 \leq t_{21} \leq z'-a_{21}$, and $0 \leq t_{22} \leq z'-a_{22}$

The optimal solution to this problem is the so-called ‘lexicographic maximin’ solution (see Bourguignon and Fields, 1990, and Gangopadhyay and Subramanian, 1992). The solution consists in raising the poorest person’s income to the income of the second poorest person if the budget will permit, or to the highest feasible level not exceeding the second poorest person’s income; if the budgetary outlay is thereby exhausted, we stop the exercise here, and if not, the incomes of the two poorest individuals are raised to the income of the third poorest person if the budget will permit, or to the highest feasible level not exceeding the third poorest person's income; and so on, until we reach that marginal individual with whom the budget is exhausted. Given the specific numerical values we have assigned to the poverty line z′, the income vector **a**, and the budgetary outlay T, it can be verified that the optimal solution to Problem A is provided by

(7.1) $t^*_{11} = 2$, $t^*_{12} = 0$, $t^*_{21} = 3$, and $t^*_{22} = 0$

The resulting, post-transfer income vector is given by

(7.2) $\mathbf{a}^* = (a_{11} + t^*_{11}, a_{12} + t^*_{12}, a_{21} + t^*_{21}, a_{22} + t^*_{22}) = (6,6,6,9)$

Next, policymaker B's problem can be written as follows.

Problem B

Minimize $F^a(z',\mathbf{a} + \mathbf{t},\mathbf{g}') = [\theta_1\{F^a(z',\mathbf{a}^1 + \mathbf{t}^1,\mathbf{g}^u)\}^2 + \theta_2\{F^a(z',\mathbf{a}^2 + \mathbf{t}^2,\mathbf{g}^u)\}^2]^{1/2}$

$\{t_{11},t_{12},t_{21},t_{22}\}$

subject to

$t_{11} + t_{12} + t_{21} + t_{22} \leq T$ and

$0 \leq t_{11} \leq z'-a_{11}$, $0 \leq t_{12} \leq z'-a_{12}$, $0 \leq t_{21} \leq z'-a_{21}$ and $0 \leq t_{22} \leq z'-a_{22}$

Will the transfer schedule **t*** presented in (7.1) also be an optimal solution to Problem B? It can be verified, given the numerical assumptions we have made regarding θ_1, θ_2, z′ and T, that $F^a(z',\mathbf{a} + \mathbf{t}^*,\mathbf{g}') = 0.1281$. If there is no other allocation **t**** such that $F^a(z',\mathbf{a} + \mathbf{t}^{**},\mathbf{g}') < 0.1281$, then **t*** is an optimal solution to Problem B. However, consider the transfer schedule **t**** given by

(7.3) $t^{**}_{11} = 2.4558$, $t^{**}_{12} = 0.4558$, $t^{**}_{21} = 2.0884$ and $t^{**}_{22} = 0$.

It is easy to check that $F^a(z',\mathbf{a} + \mathbf{t}^{**},\mathbf{g}') = 0.1256 < F^a(z',\mathbf{a} + \mathbf{t}^*,\mathbf{g}') = 0.1281$. Since poverty is lower with the transfer schedule **t**** than with the schedule **t***, it is clear that **t*** is *not* an optimal solution to Problem B. (In fact, **t**** solves Problem B. This claim will not be proved here, but it may be noted that an intuitive sufficient condition for an optimum is a feasible transfer schedule which (a) respects the lexicographic maximin principle of allocation *within* each subgroup, and (b) simultaneously ensures equalization of poverty levels *between* subgroups. In the present instance, note that $\mathbf{a}^1 + \mathbf{t}^{1**} = (6.4558,6.4558)$ and $\mathbf{a}^1 + \mathbf{t}^{2**} = (5.0884,9)$: the lexicographic maximin outcome obtains within each subgroup, and further, subgroup poverty levels are equalized since, as can be confirmed, $F^a(z',\mathbf{a}^1 + \mathbf{t}^{1**},\mathbf{g}^u) = F^a(z',\mathbf{a}^2 + \mathbf{t}^{2**},\mathbf{g}^u) = 0.1256$.)

Briefly, policymakers A and B have a quarrel – and a substantial one at that – on their hands. While quarrels over the choice of the poverty line z have been numerous, quarrels over the choice of **g** have been relatively muted, with the implicit consensus favouring policymaker A's approach to the problem. Yet, both choices have implications not only for poverty comparisons but also for the proper targeting of scarce resources in poverty alleviation programmes. There would thus appear to be a case for an *explicit* statement of the precise choice of **g** that is made, and for a justification of that choice. The issue assumes a particular salience in the context of societies characterized by stratification arising from the historically cumulated maldistribution of burdens and benefits across identifiable subgroups of the population. For analysts concerned with the measurement of poverty and anti-poverty policy based on such measurement, the issue resolves itself into not just a problem in logic but into a larger problem in social ethics.

8 Concluding observations

In much of mainstream economic theorizing, the only 'marker' of identity is income. This is quite clearly evident in, for example, standard approaches to the measurement of poverty. The point is made explicit in Sen's (1976) seminal paper dealing with the derivation of an ordinal measure of poverty, in which he draws specific attention to an assumption which is at the welfare basis of many poverty measures, and which he calls the '*Monotonic Welfare*' axiom. According to this axiom, given any income vector **x** and any pair of individuals j and k with incomes x_j and x_k respectively, if $x_j > x_k$, then $W_j(\mathbf{x}) > W_k(\mathbf{x})$, where W_j (respectively, W_k) is the welfare level of individual j (respectively, individual k). It should be emphasized that Sen is himself sceptical of the universal validity of this axiom, and employs it largely in the spirit of assembling material for a characterization theorem. In a richer framework of welfare, the latter would presumably be a function of arguments other than just income. Specifically, room would have to be made for the notion, as Akerlof and Kranton (2000: 718) put it, that 'identity is based on social categories…', and the fact that income classes do not exhaust social categories. If a person's identity, and the welfare she experiences, depends not only on her income but also, for example, on the colour of her skin, then it is entirely conceivable that, given an income vector **x** and an n-tuple *s* describing each individual's skin colour, one can have a pair of individuals j and k such that $x_j > x_k$, j is black and k is white, and $V_j(\mathbf{x},s) < V_k(\mathbf{x},s)$, where V_i (i = 1,…,n) stands for person i's welfare level, and each person's welfare level is assumed to be increasing in her/his income, other things equal. In terms of the standard symmetry axiom, aggregate welfare and poverty levels should remain unchanged if j and k were to swap their incomes; however, in terms of the welfare index V, one can easily see that black j would be rendered worse off and white k would be rendered better off if j and k were to swap incomes. Similarly, the standard transfer axiom would endorse a permissible progressive income transfer from j to k; however, such a transfer would only serve to widen the welfare-gap (when welfare is measured by V) between the two individuals. It is clear then that allowing for a plurality of groups in society does have non-trivial implications for the measurement of society-wide deprivation, a point that is emphasized by Thurow (1981: 179, 180, 182):

> Is the correct economic strategy to resist group welfare measures and group redistribution programmes wherever possible? Or do groups have a role to play in economic justice? … [I]t is not possible for society to determine whether it is or is not an equal opportunity society without collecting and analyzing economic data on groups … Individuals have to be judged based on group data … A concern for groups is unavoidable.

This note has been concerned to explore an aspect of the analytics of poverty measurement as a specific application of the exercise of complicating mainstream accounts of the economy by allowing for the pervasive reality of the stratification of society into groups. In the process, it points to two issues that could be salient in a consideration of how to accommodate subgroup poverty in the aggregation exercise of measuring income-deprivation. First, it suggests the desirability of the poverty index being a variable, rather than trivial or constant, function of the precise grouping that is employed in partitioning a population into subgroups. By entering the grouping explicitly as an argument in the poverty function, the domain of the function is informationally expanded in a way that enriches a group-sensitive assessment of poverty. Second, it suggests that if it is considered desirable to directly incorporate into the measurement of poverty considerations relating to the inter-group distribution of poverty, then certain conventional axioms of poverty measurement may have to be modified, *via* restrictions on their domains of applicability, in order to avoid problems of internal consistency in the aggregation exercise.

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