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Abstract

In this note, we investigate how the depreciation of a manufactured stock and the related accumulation of a waste stock can alter the optimal path of the capital-resource economy. It is shown that the optimal consumption path is affected by how the disuse pattern of the manufactured stock in question is distributed, specifically with forward-looking terms.

I. Introduction

It is well-established in the literature that consumption prospects in economies with one manufactured capital and an exhaustible resource depend on the capital-resource substitution and technological progress. In the relevant literature of this Dasgupta-Heal-Solow-Stiglitz capital-resource model—or Ramsey model for that matter—, however, depreciation of the manufactured capital is not taken into account. Although *constant* depreciation can be easily accommodated into the stylized capital-resource economy as in Valente (2005), a more general form of depreciation, namely the delaying dynamics from the manufactured capital into the waste stock, which is the central underlying mechanism of the material flow

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analysis literature (Kleijn *et al*., 2000; Yamaguchi and Ueta, 2006), has not been treated. We address part of this incongruence between growth theory and material flow analysis, by including forward-looking terms in the optimality conditions[1].

In the present note, three additional assumptions are introduced into the capital-resource model. First, the manufactured capital depreciates each period, that at a non-constant rate. Second, along with the two stocks of the capital stock and the natural resource stock, the disused portion of the capital is considered to enter the third capital, waste stock. Third, in order to detect the effect of the waste stock on the economy, utility is dependent on the level of the waste stock, as well as on the current consumption. This is in contrast to Krautkraemer (1985), who studied the effect of a stock providing positive externality.

II. The model and the main results

We begin with the usual Ramsey-Koopmans intertemporal utility maximization of a representative agent, with the economy's waste stock level as well as current consumption affecting utility:

$$\max \int_0^\infty U[C(t), X(t)] \, e^{-\delta t} \, dt \tag{1}$$

where C is the current consumption and X is the waste stock, into which disused stock in the economy is assumed to flow. The derivatives of utility with regard to consumption and waste stock are positive and negative, respectively (i.e., $U_C > 0$, $U_X < 0$). Furthermore, the cross derivative of the utility with regard to consumption and waste stock is taken to be negative: $U_{CX} < 0$.

The increase in the manufactured capital is output net of consumption, but a varying portion of the capital depreciates. The production function F is assumed to be strictly concave and twice continuously differentiable. We therefore have

$$K(t) = \int_0^t [F(K(\tau), R(\tau)) - C(\tau)]M(t - \tau)d\tau + K(0) \tag{2}$$

where $K(t)$ is the level of the manufactured capital at t, $R(t)$ is the exhaustible resource input at t (typically a fossil fuel or mineral resource), and $K(0)$ is the initial level of the capital. $M(t)$ is the cumulative distribution function, which

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[1] A similar forward-looking term is derived in the context of habit formation in Aronsson and Löfgren (2008).

shows how much of the capital produced at t still exists and has not disappeared. By definition, it follows that $M(0) = 1$. We can consider this to be something like a time-varying discount factor; consequently, we describe the dynamics of the produced capital by way of a state equation, rather than a differential equation as usually done. What we assume happens every period in the capital stock is not only economic depreciation but at the same time physical disuse from the economy. For completeness in terms of material balance, we assume a waste stock

$$X(t) = \int_0^t [F(K(\tau), R(\tau)) - C(\tau)](1 - M(t - \tau))d\tau + X(0) \tag{3}$$

where $X(t)$ is the level of the waste stock at t that has been disused from the capital stock.

As for the resource, we simply assume

$$\dot{S}(t) = -R(t) \tag{4}$$

showing that the resource is exhaustible.

Our problem is (1) subject to (2)-(4). Following Eichner and Runkel (2005), write the present-value Lagrangian as

$$\begin{aligned}
\mathcal{L} = &\int_0^\infty U[C(t), X(t)] \, e^{-\delta t} \, dt \\
&+ \int_0^\infty \lambda_K(t)e^{-\delta t}\left[\int_0^t [F(K(\tau), R(\tau)) - C(\tau)]M(t - \tau)d\tau + K(0) - K(t)\right]dt \\
&+ \int_0^\infty \lambda_X(t)e^{-\delta t}\left[\int_0^t [F(K(\tau), R(\tau)) - C(\tau)](1 - M(t - \tau))d\tau + X(0) - X(t)\right]dt \\
&+ \int_0^\infty \lambda_S(t)e^{-\delta t}[-R(t) - \dot{S}(t)]
\end{aligned}$$

Changing the order of integration, this is equivalent to

$$
\mathcal{L} = \int_0^\infty U[C(t), X(t)] \, e^{-\delta t} \, dt
$$

$$
+ \int_0^\infty \left[\int_t^\infty \lambda_K(s) e^{-\delta(s-t)} [F(K(t), R(t)) - C(t)] M(s-t) ds + \lambda_K(t) e^{-\delta t} [K(0) - K(t)] \right]
$$

$$
+ \int_0^\infty \left[\int_t^\infty \lambda_X(s) e^{-\delta(s-t)} [F(K(t), R(t)) - C(t)] (1 - M(s-t)) ds + \lambda_X(t) e^{-\delta t} [X(0) - X(t)] \right]
$$

$$
+ \int_0^\infty \left[-\lambda_S(t) R(t) + (\dot\lambda_S(t) - \delta \lambda_S(t)) S(t) \right] e^{-\delta t} \, dt + \lambda_S(0) S(0)
$$

$$
= \int_0^\infty \mathcal{H} e^{-\delta t} dt - \int_0^\infty \lambda_K(t) e^{-\delta t} [K(t) - K(0)] dt
$$

$$
- \int_0^\infty \lambda_X(t) e^{-\delta t} [X(t) - X(0)] dt + \int_0^\infty [\dot\lambda_S(t) - \delta \lambda_S(t)] S(t) e^{-\delta} dt + \lambda_S(0) S(0)
$$

where the current-value Hamiltonian $\mathcal{H}$ is defined by

$$
\mathcal{H} \equiv U[C(t), X(t)] + \int_t^\infty \lambda_K(s) e^{-\delta(s-t)} [F(K(t), R(t)) - C(t)] M(s-t) ds
$$

$$
+ \int_t^\infty \lambda_K(s) e^{-\delta(s-t)} [F(K(t), R(t)) - C(t)] (1 - M(s-t)) ds - \lambda_S(t) R(t)
$$

and λ_K, λ_X and λ_S represent the shadow prices of the stocks K, X and S, respectively. The integral terms in the Hamiltonian show that we are now dealing with a forward-looking problem.

Kamien and Muller (1976), along the line of Arrow (1964), formulated the optimal control problem when the constraints are integral. Following them, the optimality conditions are $0 = \partial \mathcal{L}/\partial C = \partial \mathcal{H}/\partial C$; $0 = \partial \mathcal{L}/\partial R = \partial \mathcal{H}/\partial R$; $0 = \partial \mathcal{L}/\partial K = \partial \mathcal{H}/\partial K - \lambda_K$; $0 = \partial \mathcal{L}/\partial X = \partial \mathcal{H}/\partial X - \lambda_X$; and $0 = \partial \mathcal{L}/\partial S = \partial \mathcal{H}/\partial S + \dot\lambda_S - \delta \lambda_S$. With the control variables C and R and the state variables K, X and S, the first-order necessary conditions are

$$
U_C = \int_t^\infty \lambda_K(s) e^{-\delta(s-t)} M(s-t) ds + \int_t^\infty \lambda_X(s) e^{-\delta(s-t)} (1 - M(s-t)) ds
$$

$$
U_C F_R = \lambda_S
$$

$$
U_C F_K = \lambda_K
$$

$$
U_X = \lambda_X
$$

$$
0 = \dot\lambda_X - \delta \lambda_S
$$

F_R is the derivative of $F(K, R)$ with regard to R, and so on. Note that U_C is defined in terms of a forward-looking perspective as in the model of Eichner and Runkel (2005), and the interpretation is straightforward: the marginal utility of the current consumption is the sum of the shadow price of its forgone investment in the manufactured capital weighted by the depreciation, plus the shadow price of the stock of its delayed waste outflow. Taking the time derivative of the marginal utility of consumption, they collectively lead to

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta + \int_t^\infty \frac{U_C(s)}{U_C(t)} F_K(s) e^{-\delta(s-t)} \dot{M}(s-t) ds - \int_t^\infty \frac{U_X(s)}{U_C(t)} e^{-\delta(s-t)} \dot{M}(s-t) ds \tag{5}$$

$$\frac{\dot{F}_R}{F_R} = F_K + \int_t^\infty \frac{U_C(s)F_K(s) - U_X(s)}{U_C(t)} e^{-\delta(s-t)} \dot{M}(s-t) ds \tag{6}$$

Here, $U_C(s)$ is $\partial U/\partial C(s)$, $F_K(s)$ is $\partial F/\partial K(s)$, etc. Obviously they correspond to Euler equation and Hotelling Rule, respectively. Furthermore, since the waste stock affects the utility function, the former suggests that the optimal path of consumption grows according to

$$\frac{\dot{C}}{C} = \frac{1}{\eta(C)}\left[F_K - \delta + \frac{1}{U_C(t)} \int_t^\infty [U_C(s)F_K(s) - U_X(s)]e^{-\delta(s-t)} \dot{M}(s-t) ds + \frac{U_{CX}(t)}{U_C(t)} \dot{X}(t)\right] \tag{7}$$

where $\eta(C)$ represents the elasticity of the marginal utility, and $\dot{X}(t)$ is actually $\int_0^t [F(K(\tau), R(\tau)) - C(\tau)]\dot{M}(t-\tau)d\tau$ from (3). The first two terms in the bracket in the RHS of (7) are familiar: the manufactured capital's own rate of interest subtracted by the rate of impatience. Since $\dot{M} \leq 0$, the third term of the RHS with an integral is likely to be non-positive, and represents the present value of the foregone benefit entailed with the depreciation of the capital stock ($U_C(s)F_K(s)$) and the stock disutility once it comes to an end of life and enters the stock in the environment ($-U_X(s)$). Both parts are weighted by the associated depreciation, $\dot{M}(s-t)$.

The final term of the RHS of (7), which is also negative by the assumption $U_{CX} < 0$, shows the cross elasticity of the marginal consumption due to the waste stock.

Both of the defining conditions of the optimal path (6) and (7) are now affected by the delayed depreciation of the capital stock and its associated waste stock. Note that when $\dot{M} = 0$ and $U_X = 0$, they are reduced to the usual Euler equation and the Hotelling Rule.

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For a long-lasting durable good like vehicle or nuclear fuel, the point of depreciation or discarding is sometimes in a far distant future, and time distribution possibly more dispersed, and the impact of the delayed distribution on the economy is determined by the relative effect of net benefit and discount rate.

III. Conclusion

In this note, we have incorporated into the capital-resource economy the depreciation of the manufactured capital and the related accumulation of the waste stock. In doing so, we have set up integral constraints to capture the distribution effect. The resulting optimality conditions are modified to include forward-looking terms, underlining the significance of the intertemporal disuse pattern of capital in question, which is usually dismissed in the literature to be simplified as a constant rate of depreciation. Considering that the delaying mechanism of the active stock being disused into the waste stock can be generalized not just to durable goods but also to such super-durable stocks as nuclear waste, this is a feature of capital-resource economy worth paying attention to.

References

[1] Aronsson, T. and Löfgren, K.-G. (2008) Welfare equivalent NNP and habit formation, *Economics Letters*, **98**, 84-88.

[2] Arrow, K. (1964) Optimal capital policy, the cost of capital, and myopic decision rules, *Annals of the Institute of Statistical Mathematics*, **16**, 21-30.

[3] Eichner, T. and Runkel, M. (2005) Efficient policies for green design in a vintage durable good model, *Environmental and Resource Economics*, **30**, 259-278.

[4] Kamien, M. and Muller, E. (1976) Optimal control with integral state equations, *Review of Economic Studies*, **43**, 469-473.

[5] Kleijn, R., Huele, R. and van der Voet, E. (2000) Dynamic substance flow analysis: the delaying mechanism of stocks, with the case of PVC in Sweden, *Ecological Economics*, **32**, 241-254.

[6] Krautkraemer, J. (1985) Optimal growth, resource amenities and the preservation of natural environments, *Review of Economic Studies*, **52**, 153-170.

[7] Valente, S. (2005) Sustainable development, renewable resources and technological progress, *Environmental and Resource Economics*, **30**, 115-125.

[8] Yamaguchi, R. and Ueta, K. (2006) Substance flow analysis and efficiency conditions: a case of lead, Discussion Paper No.119, 21COE Interfaces for Advanced Economic Analysis, Kyoto University, Kyoto.