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Abstract: This paper examines the issues of institutional quality and central bank transparency through the interaction of monetary and fiscal policies. We have found that the effects of transparency and corruption on macroeconomic performance and volatility depend on the relative importance of the marginal supply-side effects of distortionary tax and corruption, the degree of central bank conservativeness and/or the initial degree of opacity about central bank preferences. If the marginal effect of tax is relatively important, more opacity might induce higher level and volatility of inflation when the central bank is sufficiently conservative. Furthermore, opacity and tolerated corruption can mutually reinforce or weaken each other's effects on the level and volatility of inflation. Transparency is generally a better strategy when the central bank is conservative. However, there could be a case for opacity in order to compensate for the undesirable macroeconomic effects of corruption when the central bank is liberal.

Key words: Central bank transparency, central bank conservativeness, fiscal bias, distortionary tax, institutional quality (corruption).

JEL classification: D73, E52, E58, E61, E63, H50.

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1. Introduction

The lack of good governance and high levels of corruption are clearly prevalent in emerging market economies as shown by indices developed by Transparency International (TI, 2006) or the World Bank (Kaufmann *et al.*, 2007). They have significant effects on the economic development as well as international capital flows, tax evasion and stock market variability.[1]

In terms of macroeconomic policy design, these issues are very important when monetary policy decisions are not taken independently of fiscal policy and inflation tax represents an important financial resource for public expenditures (Cukierman *et al.*, 1992). In countries where central banks are not completely independent, corruption and rent seeking are correlated with higher rates of inflation and excessive public expenditures (Kaufman *et al*., 2007; Hefeker, 2010).

Several recent researches have highlighted the role of weak public institutions, including high levels of corruption, in determining monetary and fiscal policy design as well as the choice of exchange rate regime in developing countries. Huang and Wei (2006) have shown that weak institutions, modelled as exogenous erosion in the ability of government to collect revenue through formal tax channels, have important implications for the design of monetary policymaking institutions. In particular, they have found that a pegged exchange rate is inferior to a Rogoff-style conservative central banker, whose optimal degree of conservativeness is proportional to the quality of institutions. Dimakou (2006) analyses the dynamic interaction between fiscal and monetary policies under different levels of bureaucratic corruption. She has found that the government has incentive to strategically increase debt and indirectly 'force' the independent central bank to pursue an expansionary monetary policy. Hefeker (2010) considers

[1] The literature includes studies on the effects of institutions on development (Rose-Ackerman, 1975; Shleifer and Vishny, 1993; Mauro, 1995; Méon and Sekkat, 2005) as well as investigations on the consequences of corruption for international capital flows, tax evasion, and stock market volatility (Wei, 2000, 2001; Bai and Wei, 2000; Fisman and Wei, 2004; Du and Wei, 2004).

that the government endogenously chooses the optimal level of institutional quality (corruption) and has found that credibly pegging exchange rate to an anchor currency, like a currency board, can reduce corruption and improve the fiscal system. Popkova (2008), by modelling the effects of corruption on production, has shown that if corruption has a considerable positive (negative) impact on output, a tight peg will increase (decrease) the level of corruption.

Table 1. Corruption and CBT in countries with highly but incompletely independent central banks

Countries	CBI Index (maximum value=1)	CBT Index (maximum value = 15)	Corruption Index (world ranking) (maximum value = 10)
Argentina	0.74	5.5	2.9 (106)
Armenia	0.85	4	2.7 (120)
Belarus	0.73	5	2.4 (139)
Chile	0.77	7.5	6.7 (25)
Czech Republic	0.73	11.5	4.9 (52)
Estonia	0.78	6	6.6 (27)
Hungary	0.67	9.5	5.1 (46)
Israel	0.70	8.5	6.1 (32)
Lithuania	0.78	4.5	4.9 (52)
Peru	0.74	8	3.7 (75)
Poland	0.89	8	5.0 (49)
Slovak Republic	0.62	6	4.5 (56)

Sources: The Cukierman Index of Central Bank Independence (CBI), 1989-2000. Worldwide Corruption Perceptions ranking of countries published by Transparency International (2009). For the Central Bank Transparency (CBT) Index, see Dincer and Eichengreen (2007).[2]

However, the previous studies neglect one important characteristic of monetary policy decision in emerging market economies. Even though central banks in emerging market countries could be highly independent, they are characterized by different levels of transparency in communication with the public (see Table 1). Empirical evidence shows that central banks in these economies are less transparent than their counterparts in the advanced countries (Dincer and Eichengreen, 2007). One possible explanation is that they have incentive to be opaque about their monetisation operations given that the monetary financing of public deficit is important.

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[2] The CBI Index, the CBT Index and the Corruption Index are of the years 2000, 2005 and 2009 respectively. While we have not more actual data for the first two indexes, we observe in the surveys that they are generally increasing over time and the third index is quite stable.

Nevertheless, they have made much progress in this respect since the International Monetary Fund adopted a code of good conduct to increase the transparency of official operations in emerging markets, in part prompted by the 1994 peso and other emerging market crises (Wilson and Saunders, 2004; de Mendonça and Filho, 2008).

Since the pioneer work of Cukierman and Meltzer (1986), a large literature on central bank transparency has been developed, mostly for the case of developed countries and limited to the interaction between monetary authorities and private agents.[3] Most economists are instinctually of the view that more information is better than less and hence agree that openness and communication with the public are crucial for the effectiveness of monetary policy, in allowing the private sector to improve expectations and therefore to make Pareto improving decisions (Blinder, 1998; Eijffinger *et al.*, 2000; Blinder *et al.*, 2001; Hoeberichts *et al.*, 2009). Adding distortions, some authors have provided counter-examples where information disclosure instead reduces the possibility for central banks to strategically use their private information, implying that greater transparency may not lead to a welfare improvement (Sorensen, 1991; Faust and Svensson, 2001; Jensen, 2002; Sibert, 2002). In effect, according to the theory of the second best, removing one distortion may not lead to a more efficient allocation when other ones are present.[4]

The empirical literature has so far yielded mixed results on whether transparency significantly affects the average level of inflation and output gap, while it remains difficult to establish its effects on inflation and output volatility. According to Chortareas *et al.* (2002), disclosure of inflation forecasts reduces inflation but is not necessarily associated with higher output volatility. Demertzis and Hughes-Hallet (2007) have found that greater transparency benefits to inflation volatility, but has a less clear effect on output volatility and no effects on the average level of

3 See, for a survey, Geraats (2002) and Eijffinger and van der Cruijsen (2010) .
4 See Dincer and Eichengreen (2007) for a short discussion of these counter-examples.

inflation and output. The analysis of Dincer and Eichengreen (2007) suggests broadly favourable but relatively weak impacts on inflation and output volatility.

A few studies examine the interaction between monetary and fiscal policies in models which do not include the issue of imperfect institutional quality (Hughes Hallett and Viegi, 2003; Ciccarone *et al.*, 2007; Hefeker and Zimmer, 2009). By introducing supply-side fiscal policies, these studies have introduced the so-called fiscal bias. In the presence of fiscal bias, the uncertainty about central bank preferences, i.e. lack of political transparency in the sense of Geraats (2002), has important implications for macroeconomic stabilisation.

This paper provides the first theoretical study about how institutional quality (corruption) could interact with fiscal policy and central bank transparency. For this purpose, we simultaneously consider two types of endogenous distortions, i.e. distortionary tax and corruption. The issue is important because emerging market economies suffer from corruption while at the same time their central banks have low levels of transparency. In this respect, our study could give an explanation of the reasons why transparency is relatively low in emerging market economies as well as some lessons about how transparency may improve institutional quality. The implications of our study could also be relevant for developed countries where corruption is significant and central banks are not fully transparent.[5]

The remainder of the paper is structured as follows. The next section describes the model. The section after solves for the equilibrium. The fourth section studies how less transparency and more tolerance for corruption affects the level of inflation, output gap, distortionary tax rate and institutional quality. The fifth section examines the effects of opacity and tolerated corruption on macroeconomic volatility. We conclude in the last section.

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[5] For example, Greece, Italy and Portugal before they joined the Euro Zone.

2. The model

Our analytical framework is based on models used by Alesina and Tabellini (1987), De Kock and Grilli (1993), Velasco (1996) and Huang and Wei (2006) among others. We introduce endogenous institutional quality (or corruption) as in Hefeker (2010). Output gap, x, in log terms, is a positive function of surprise inflation $\pi - \pi^e$ (where π is the inflation rate and π^e the expected inflation rate), a negative function of distortionary tax rate on the total revenue of firms, τ, and a positive (or negative) function of corruption (Popkova, 2008), θ:[6]

$$x = \alpha(\pi - \pi^e) - \gamma\tau + \psi\theta, \qquad \alpha,\ \gamma > 0. \tag{1}$$

Equation (1) stipulates that taxes are systematically non-neutral in their effects on output gap and hence distortionary in the sense of depressing output gap and thus employment more than surprise inflation can improve them. The presence of τ could also represent non-wage costs associated with social security or job protection legislation, or the more general effects of supply-side deregulation (Hughes-Hallett and Viegi, 2003). To focus on the effect of uncertainty about the central bank preferences, we do not introduce supply shocks in (1).

The negative effect of corruption is due to its adverse effect on the investment of firms (Mauro, 1995; Campos *et al.*, 1999), the efficiency of public expenditures (Del Monte and Papagni, 2001), the governance (Blackburn and Forgues-Puccio, 2007) and the factor requirements of firms (Dal Bó and Rossi, 2007). However, the negative effect of corruption on the output gap could be counterbalanced by the positive effect. Under the "efficient corruption" hypothesis, corruption is considered as a way to compensate the distortion caused by the burden of distortionary taxation (Leff, 1964). Leff views corruption as "grease money" to lubricate the

[6] The parameter γ could be identical to α as in Hefeker (2010), but it can also be different from α as in Huang and Wei (2006), Popkova (2008) if there is possibility of fiscal leakage.

squeaky wheels of a rigid administration. Baretto (2000) has pointed out that the efficiency-enhancing effect of corruption results from the fact that corruption can reduce bureaucratic inefficiency. Coppier and Michetti (2006) have shown that more corruption could be associated with more production, in accordance with some empirical evidence (Coppier and Piga, 2004). However, the "efficient corruption" hypothesis is not incontestable (Kaufmann and Wei, 1999; Aidt, 2003; Méon and Sekkat, 2005).[7]

Therefore, on a priori grounds, it is not always possible to make a precise guess on how corruption affects production. If the marginal effect of corruption is negative, we have $\gamma - \psi > 0$. On the contrary, i.e. when $\psi > 0$, we may have either $\gamma - \psi > 0$ or $\gamma - \psi < 0$. The latter inequality implies that the marginal effect of corruption is higher than that of distortionary tax.

To finance public expenditures (g, as a percentage of national revenue), the government has two sources of revenue: an output tax at the rate τ, which is reduced by corruption θ, and an inflation tax π.[8] The government's budget constraint is:[9]

$$g = \tau + \pi - \theta. \tag{2}$$

The government aims to simultaneously stabilize inflation and output gap as well as public expenditures around their respective targets, i.e. 0, $\bar{x}$ and $\bar{g}$. The introduction of a spending target could reflect the desire of being re-elected or other demands from interest groups that the government must satisfy. Moreover, as Hefeker (2010), we assume that the government is concerned with corruption (or leakages of fiscal revenue) and tries to control its level around the

[7] Méon and Weill (2008) have found that the grease the wheels hypothesis could be observed in countries where institutions are ineffective.

[8] There is ample evidence suggesting that seigniorage (defined as an increase in base money) is an important source of government revenue for developed countries and could account for more than ten percentage points of national revenue for developing countries (Cukierman *et al.*, 1992). While the central bank is independent, it continues to pay the seigniorage revenue to the government as it is observed empirically.

[9] We can introduce a coefficient before π in (2). For simplicity, we assume that it is equal to one.

tolerated level ($\bar{\theta}$). In effect, increasing corruption might result in protest from the population, lower foreign investment or less support from international financial organizations. Corruption fighting, through more control of public servants, reduction of the influence of interest groups and rent-seeking, and creation of better institutions like setting up independent courts and improving public administration, implies a cost for the government. On the other hand, a reduction in corruption leads to alienation of former beneficiaries of corruption, such as interest groups or bureaucrats that resist corruption fighting. Due to the personal or political costs of fighting corruption, deviations of the level of corruption in either direction from $\bar{\theta}$ are costly.

The government's objective function is:

$$L^G = \frac{1}{2}[\delta_1 \pi^2 + (x - \bar{x})^2 + \delta_2 (g - \bar{g})^2 + \delta_3 (\theta - \bar{\theta})^2]. \qquad (3)$$

Following Rogoff (1985), we assume that the government, while keeping control of its fiscal instrument, delegates the conduct of monetary policy to an independent central bank. Therefore, the central bank is unlikely to be made responsible for public expenditure deviations and is only concerned with the inflation rate and output gap. We assume that the central bank sets its policy in order to minimize the following loss function:

$$L^{CB} = \frac{1}{2}[(\mu - \varepsilon)\pi^2 + (1 + \varepsilon)(x - \bar{x})^2], \qquad \mu > 0, \qquad (4)$$

where parameter μ is the relative weight that the central bank assigns to the inflation target and it might be different from that of the government. It is therefore an index of *conservatism* (larger values of μ) versus *liberalism* or *populism* (smaller values of μ). The central bank's policy instrument is π.[10] Inserting a positive output-gap target (i.e. $\bar{x} > 0$) in the objective functions, destined to correct a shortfall in output due to the distortionary effects of taxes, introduces an

[10] In practice, the central bank would use interest rates. Since the standard theoretical models assume that nominal interest rates have no systematic long-run influence on output, we may as well use π.

inflationary bias. Since it does not significantly modify our principal analytical results concerning the effects of transparency, in the following, we assume that $\bar{x} = 0$.

The transparency issue is introduced by assuming that the weights assigned by the central bank to inflation and output gap targets are not perfectly predictable by the government and private sector.[11] Following Ciccarone *et al*. (2007), the imperfect disclosure of information about central bank preferences is represented by the fact that ε is a stochastic variable.[12] This specification of central bank's loss function is adopted in order to avoid the arbitrary effects of central bank preference uncertainty on monetary policy underlined by Beetsma and Jensen (2003). In effect, a slight change in the uncertainty specification (e.g., putting the stochastic parameter in front of one or the other argument of the central bank's objective function) can lead to radically different effects on monetary reactions.

We assume that the density function of ε is characterised by $E(\varepsilon) = 0$, $\text{var}(\varepsilon) = E(\varepsilon^2) = \sigma_\varepsilon^2$ and $\varepsilon \in [-1, \mu]$. The variance σ_ε^2 represents the degree of opacity about central bank preferences. As the random variable ε takes values in a compact set and has an expected value equal to zero, σ_ε^2 must have a well defined upper bound, i.e. $\sigma_\varepsilon^2 \in [0, \mu]$.[13] When $\sigma_\varepsilon^2 = 0$, the central bank is perfectly predictable and hence fully transparent.

3. The equilibrium

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[11] Transparency about the central bank's preferences, following the terminology defined by Geraats (2002), can be considered as political transparency which refers to openness about policy objectives and institutional arrangements. It corresponds to one of the five motives for central bank transparency, i.e. political transparency, economic transparency, procedural transparency, policy transparency and operational transparency.

[12] This formulation is similar to what is proposed by Geraats (2002) for avoiding the arbitrary effects of opacity. She assigns a weight $\alpha = \bar{\alpha} - \xi$ to the output target and $\beta = \bar{\beta} + \xi$ to the inflation target in the central bank's loss function, with $\alpha + \beta = 1$, and $\bar{\alpha}$ and $\bar{\beta}$ as their expected values. See also Hughes-Hallett and Viegi (2003).

[13] See Ciccarone *et al*. (2007) for a proof.

The timing of the game is as follows: Firstly, the private sector forms rational expectations about inflation and sets wages, then the government chooses the tax rate and corruption and the central bank makes the monetary policy decision to attain the inflation target. The fiscal and monetary authorities play a non-cooperative Nash-game.[14] The model is solved by using backward induction.

Taking account of equations (1) and (2), the minimisation of the loss function (3) leads to the following reaction functions for the government:

$$(\delta_2-\gamma\alpha)\pi+(\gamma^2+\delta_2)\tau-(\gamma\psi+\delta_2)\theta=-\gamma\alpha\pi^e+\delta_2\overline{g}\ , \tag{5}$$

$$(\alpha\psi-\delta_2)\pi-(\gamma\psi+\delta_2)\tau+(\psi^2+\delta_2+\delta_3)\theta=\alpha\psi\pi^e-\delta_2\overline{g}+\delta_3\overline{\theta}\ . \tag{6}$$

The reaction function of the central bank, which minimises the loss function (4) taking account of the economic constraint represented by equation (1), is given as:

$$[(\mu-\varepsilon)+\alpha^2(1+\varepsilon)]\pi-\alpha(1+\varepsilon)\gamma\tau+\alpha(1+\varepsilon)\psi\theta=\alpha^2(1+\varepsilon)\pi^e\ . \tag{7}$$

Taking the expected inflation rate as given, we solve equations (5)-(7) to obtain the solutions of π, τ and θ in terms of expected inflation rate and exogenous variables as follows:

$$\pi=\frac{1}{\Delta_\varepsilon}\{\alpha(1+\varepsilon)\delta_2\delta_3[(\gamma-\psi)\overline{\theta}+\gamma\overline{g}+\alpha\pi^e]\}\ , \tag{8}$$

$$\tau=\frac{1}{\Delta_\varepsilon}\begin{bmatrix}[-\psi(\gamma-\psi)(\mu-\varepsilon)+(\mu-\varepsilon)\delta_3+\alpha(1+\varepsilon)\alpha\delta_3]\delta_2\overline{g}\\-[(\mu-\varepsilon)\delta_2(\gamma-\psi)+(\mu-\varepsilon)\gamma\delta_3+\alpha(1+\varepsilon)\delta_2\delta_3]\alpha\pi^e\\+[(\mu-\varepsilon)(\gamma\psi+\delta_2)+\alpha(1+\varepsilon)(\alpha+\psi)\delta_2]\delta_3\overline{\theta}\end{bmatrix}\ , \tag{9}$$

$$\theta=\frac{1}{\Delta_\varepsilon}\begin{bmatrix}-\gamma(\mu-\varepsilon)(\gamma-\psi)\delta_2\overline{g}-(\mu-\varepsilon)(\gamma-\psi)\delta_2\alpha\pi^e\\+[(\alpha+\gamma)\alpha(1+\varepsilon)\delta_2+(\mu-\varepsilon)(\gamma^2+\delta_2)]\delta_3\overline{\theta}\end{bmatrix}\ , \tag{10}$$

[14] We can also consider a Stackelberg game as the solution concept, with the government being the Stackelberg leader and the central bank the follower.

where $\Delta_\varepsilon = (\mu - \varepsilon)[(\gamma - \psi)^2 \delta_2 + (\delta_2 + \gamma^2)\delta_3] + \alpha(\alpha + \gamma)(1+\varepsilon)\delta_2\delta_3$. Using equations (8)-(10) to eliminate π, π^e and τ in equation (1), the output gap can be expressed in terms of expected inflation rate and exogenous variables as follows:

$$x = \frac{-(\gamma - \psi)(\mu - \varepsilon)\delta_2\delta_3\overline{\theta} - \gamma(\mu - \varepsilon)\delta_2\delta_3\overline{g} - \alpha(\mu - \varepsilon)\delta_2\delta_3\pi^e}{\Delta(\varepsilon)}. \qquad (11)$$

Under the assumption of rational expectations, equation (8) implies that:

$$\pi^e = E\left\{\frac{(1+\varepsilon)\alpha\delta_2\delta_3[(\gamma - \psi)\overline{\theta} + \gamma\overline{g} + \alpha\pi^e]}{\Delta(\varepsilon)}\right\}. \qquad (12)$$

Taking account of opacity, we solve equation (12) for the expected inflation rate:

$$\pi^e = \frac{\Omega\alpha\delta_2\delta_3[(\gamma - \psi)\overline{\theta} + \gamma\overline{g}]}{1 - \Omega\alpha^2\delta_2\delta_3}, \qquad (13)$$

where Ω is defined as $\Omega = E\left[\frac{1+\varepsilon}{(\mu-\varepsilon)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] + \alpha(\alpha+\gamma)(1+\varepsilon)\delta_2\delta_3}\right] > 0$. Applying the second-order Taylor approximation to Ω leads to:

$$\Omega \approx \frac{1}{\Delta_0} + \frac{(1+\mu)[(\gamma - \psi)^2\delta_2 + (\delta_2 + \gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha + \gamma)\delta_2\delta_3]}{\Delta_0^3}\sigma_\varepsilon^2, \qquad (14)$$

with $\Delta_0 = \mu[(\gamma - \psi)^2\delta_2 + (\delta_2 + \gamma^2)\delta_3] + \alpha(\alpha + \gamma)\delta_2\delta_3$.

The equilibrium solutions of π, τ and θ can be obtained by substituting the solution of π^e into equations (8)-(11).

4. The effects of opacity and tolerated corruption on the equilibrium

Before analyzing the effects of opacity on the expected and current inflation rate, output gap, tax rate and corruption, we briefly discuss the case of full transparency (i.e. $\sigma_\varepsilon^2 = 0$ and $\varepsilon = 0$).

Introducing $\sigma_{\varepsilon}^{2}=0$ and $\varepsilon=0$ into equations (13) and (14) and taking account of the approximation of Ω, we obtain:

$$\pi^{e}=\frac{\alpha\delta_{2}\delta_{3}[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}]}{\Delta_{0}-\alpha^{2}\delta_{2}\delta_{3}}. \tag{15}$$

Using the solution of π^{e} given by equation (15) as well as equations (8)-(11), we can straightforwardly get the solutions of π, τ, θ and x.

In examining equations (8)-(11) and (15), we remark that, by assumption (i.e., $\bar{x}=0$), the well-known inflation bias problem in the standard Barro-Gordon model is absent here. However, the government introduces a *fiscal bias* through a *wage expectation effect*. In effect, as it attempts to increase output through higher public expenditures, which are financed by higher distortionary tax or inflation tax according to equation (2). As the central bank is assumed to be independent, the government cannot directly request the central bank to create an inflation tax. However, higher taxes on the revenue will generate output distortions which incite the central bank to raise the inflation rate to counterbalance the distortionary effects of taxation and hence to create some seigniorage revenue for the government. Consequently, workers claim higher nominal wages. On the other hand, the government fights corruption but admits that a certain level of corruption $\bar{\theta}$ is inevitable. This implies a loss of fiscal revenue for the government, which must be financed by a higher distortionary tax or inflation tax. If $\gamma-\psi>0$, then the induced increase of tax rate intended to compensate the loss of fiscal revenue due to corruption has a larger effect on production than that of corruption, leading the central bank to favour an increase in inflation to moderate the decrease of output gap. That explains why the expected inflation rate is positively related to $\bar{\theta}$ when $\gamma-\psi>0$.

Deriving equations (13) and (8)-(11) with respect to $\overline{\theta}$ and taking account of the approximation of Ω given by (14) as well as the definition of Δ_0 yield:

$$\frac{\partial \pi^e}{\partial \overline{\theta}} = \frac{\Omega \alpha \delta_2 \delta_3 (\gamma - \psi)\Delta_0^3}{(\Delta_0 - \alpha^2 \delta_2 \delta_3)\Delta_0^2 - \alpha^2 \delta_2 \delta_3 (1+\mu)[(\gamma-\psi)^2 \delta_2 + (\delta_2 + \gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2}. \quad (16)$$

$$\frac{\partial \pi}{\partial \overline{\theta}} = \frac{1}{\Delta_\varepsilon}\left\{\alpha(1+\varepsilon)\delta_2\delta_3[(\gamma-\psi) + \alpha\frac{\partial \pi^e}{\partial \overline{\theta}}]\right\}, \quad (17)$$

$$\frac{\partial \tau}{\partial \overline{\theta}} = \frac{1}{\Delta_\varepsilon}\begin{bmatrix}[(\mu-\varepsilon)(\gamma\psi + \delta_2) + \alpha(1+\varepsilon)(\alpha+\psi)\delta_2]\delta_3 \\ -\alpha[(\mu-\varepsilon)\delta_2(\gamma-\psi) + (\mu-\varepsilon)\gamma\delta_3 + \alpha(1+\varepsilon)\delta_2\delta_3]\dfrac{\partial \pi^e}{\partial \overline{\theta}}\end{bmatrix}, \quad (18)$$

$$\frac{\partial \theta}{\partial \overline{\theta}} = \frac{1}{\Delta_\varepsilon}\left[-(\mu-\varepsilon)(\gamma-\psi)\delta_2\alpha\frac{\partial \pi^e}{\partial \overline{\theta}} + [(\alpha+\gamma)\alpha(1+\varepsilon)\delta_2 + (\mu-\varepsilon)(\gamma^2+\delta_2)]\delta_3\right], \quad (19)$$

$$\frac{\partial x}{\partial \overline{\theta}} = -\frac{\alpha(\mu-\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial \pi^e}{\partial \overline{\theta}} < 0, \quad (20)$$

where $\Delta_0 - \alpha^2\delta_2\delta_3 = \mu[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] + \alpha\gamma\delta_2\delta_3 > 0$. Analysing the above partial derivatives under full transparency leads to the following proposition.

Proposition 1: *In the case of full transparency, an increase in the tolerated level of corruption always induces an increase in the expected and current inflation rate but reduces the output gap if $\gamma - \psi > 0$ and vice versa. It positively affects the tax rate and corruption whatever is the relative importance of γ and ψ.*

Proof: Assuming $\varepsilon = 0$ and $\sigma_\varepsilon^2 = 0$, equations (16)-(20) become:

$$\frac{\partial \pi^e}{\partial \overline{\theta}} = \frac{\alpha\delta_2\delta_3(\gamma-\psi)}{\Delta_0 - \alpha^2\delta_2\delta_3} > 0, \qquad \text{if } \gamma - \psi > 0, \quad (21)$$

$$\frac{\partial \pi}{\partial \overline{\theta}} = \frac{1}{\Delta_0}\left\{\alpha\delta_2\delta_3[(\gamma-\psi)+\alpha\frac{\partial \pi^e}{\partial \overline{\theta}}]\right\} > 0\,, \tag{22}$$

$$\frac{\partial \tau}{\partial \overline{\theta}} = \frac{1}{\Delta_0}\left[[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3 - \alpha[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3]\frac{\partial \pi^e}{\partial \overline{\theta}}\right], \tag{23}$$

$$\frac{\partial \theta}{\partial \overline{\theta}} = \frac{1}{\Delta_0}\left[-\mu(\gamma-\psi)\delta_2\alpha\frac{\partial \pi^e}{\partial \overline{\theta}} + [(\alpha+\gamma)\alpha\delta_2+\mu(\gamma^2+\delta_2)]\delta_3\right], \tag{24}$$

$$\frac{\partial x}{\partial \overline{\theta}} = -\frac{\alpha\mu\delta_2\delta_3}{\Delta_0}\frac{\partial \pi^e}{\partial \overline{\theta}} < 0\,. \tag{25}$$

Substituting $\frac{\partial \pi^e}{\partial \overline{\theta}}$ given by equation (21) and using the definition of Δ_0 into equations (23) and (24), $\frac{\partial \tau}{\partial \overline{\theta}}$ and $\frac{\partial \theta}{\partial \overline{\theta}}$ can be developed as:

$$\frac{\partial \tau}{\partial \overline{\theta}} = \frac{\begin{array}{l}\delta_2\delta_3\mu[\mu(\gamma\psi+\delta_2)+\alpha\psi\delta_2](\gamma-\psi)^2+\alpha\delta_2\delta_3^2\mu[\alpha\delta_2+\gamma\delta_2+\gamma^2\psi]\\ +\delta_3^2\mu[\mu(\gamma\psi+\delta_2)+\alpha\delta_2\psi](\delta_2+\gamma^2)+\alpha^2\delta_2\delta_3^2\psi(\gamma\delta_2+\mu\gamma+\alpha\delta_2)\end{array}}{\Delta_0(\Delta_0-\alpha^2\delta_2\delta_3)} > 0\,, \tag{26}$$

$$\frac{\partial \theta}{\partial \overline{\theta}} = \frac{\begin{array}{l}\delta_2\delta_3\mu(\gamma-\psi)^2[\alpha\delta_2\gamma+\mu(\gamma^2+\delta_2)]\\ +\delta_3^2[\mu(\delta_2+\gamma^2)+\alpha\gamma\delta_2][(\alpha+\gamma)\alpha\delta_2+\mu(\gamma^2+\delta_2)]\end{array}}{\Delta_0(\Delta_0-\alpha^2\delta_2\delta_3)} > 0\,. \tag{27}$$

Q.E.D.

If the marginal effect of tax on production is greater than that of corruption (i.e. $\gamma-\psi>0$), the effects of an increase in the level of corruption tolerated by the government on inflation expectations are positive. The more the government tolerates corruption, the more is the loss of tax revenue. Consequently, the government tends to increase the tax rate in order to balance its budget constraint. An increase in the tax rate and corruption will jointly have a negative effect on production, inciting the central bank to increase inflation in order to fulfil its objectives. Moreover, an increase in the inflation tax helps the government to balance its budget. We remark that, according to equation (16), an increase in μ (i.e. the degree of central bank conservativeness) diminishes the effects of an increase in $\bar{\theta}$ on the expected inflation rate.

On the contrary, when the marginal effect of corruption is greater than that of tax (i.e. $\gamma - \psi < 0$), the results will be reversed for the expected inflation, current inflation and output gap. In effect, when the positive marginal effect of corruption is important, it will strongly stimulate the production and hence will incite the central bank to reduce the inflation rate.

The result, according to which the effects of an increase in $\bar{\theta}$ on the tax rate and corruption are not affected by the relative importance of γ and ψ, is explained by the fact that the initial effects of an increase in $\bar{\theta}$ are not compensated by the secondary effects due to wage (inflation) expectations effect and central bank's reaction to the fiscal bias.

In the following, assuming that the central bank is opaque about its preferences, we will show that the effects of an increase in the tolerated level of corruption on endogenous variables depend on the degree of central bank conservativeness and the degree of opacity, in addition to the relative importance of marginal effects of tax and corruption on production.

Proposition 2a: *In the case of opacity, when* $\gamma - \psi > 0$, *an increase in the tolerated level of corruption always induces an increase in the expected inflation rate if either of the following conditions is verified:* 1) $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ and $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$; 2) $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$.

Proof: See Appendix A.

In Proposition 2a, $\bar{\sigma}_\varepsilon^2$ represents a critical value for opacity which is defined as $\bar{\sigma}_\varepsilon^2 = \frac{\{\mu[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]+\alpha\gamma\delta_2\delta_3\}\Delta_0^2}{(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\alpha^2\delta_2\delta_3}$. While an increase in the tolerated level of corruption has a positive effect on the expected inflation rate under the conditions specified in

Proposition 2a, we remark that the opposite effect is also possible. In particular, a higher $\overline{\theta}$ will imply a lower expected inflation rate when the degree of opacity and the degree of central bank conservativeness are high enough, i.e. $\sigma_\varepsilon^2 > \overline{\sigma}_\varepsilon^2$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$.

Proposition 2b: *In the case of opacity, under the assumption that* $\gamma-\psi>0$, *an increase in the tolerated level of corruption induces an increase in the inflation rate and a decrease in the output gap if* $\frac{\partial\pi^e}{\partial\overline{\theta}}>0$.

Proof: Under the conditions where $\frac{\partial\pi^e}{\partial\overline{\theta}}>0$, we obtain from equations (17) and (20):

$$\frac{\partial\pi}{\partial\overline{\theta}}=\frac{\alpha(1+\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}[(\gamma-\psi)+\alpha\frac{\partial\pi^e}{\partial\overline{\theta}}]>0\,;\quad \frac{\partial x}{\partial\overline{\theta}}=-\frac{\alpha(\mu-\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial\pi^e}{\partial\overline{\theta}}<0. \qquad \textbf{Q.E.D.}$$

As we have already discussed, if the degree of opacity and the degree of central bank conservativeness are high enough, we could have $\frac{\partial\pi^e}{\partial\overline{\theta}}<0$. In this case, the results concerning the output gap would be reversed. As for the effects on the inflation rate, they are ambiguous.

To determine the effects of an increase in the tolerated level of corruption under opacity, we substitute the solution of $\frac{\partial\pi^e}{\partial\overline{\theta}}$ given by equation (16) into equations (18)-(19) respectively, taking account of the approximation of Ω:

$$\frac{\partial\tau}{\partial\overline{\theta}}=\frac{1}{\Delta_\varepsilon}\left[\begin{array}{l}[(\mu-\varepsilon)(\gamma\psi+\delta_2)+\alpha(1+\varepsilon)(\alpha+\psi)\delta_2]\delta_3\\ -\dfrac{\alpha^2\delta_2\delta_3[(\mu-\varepsilon)\delta_2(\gamma-\psi)+(\mu-\varepsilon)\gamma\delta_3+\alpha(1+\varepsilon)\delta_2\delta_3]\times\{\Delta_0^2+(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\}(\gamma-\psi)}{(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2}\end{array}\right],$$

$$\frac{\partial\theta}{\partial\bar{\theta}} = \frac{1}{\Delta_\varepsilon}\left[\begin{array}{l}[(\alpha+\gamma)\alpha(1+\varepsilon)\delta_2 + (\mu-\varepsilon)(\gamma^2+\delta_2)]\delta_3 \\ -\dfrac{(\mu-\varepsilon)(\gamma-\psi)\delta_2\alpha^2\delta_2\delta_3\{\Delta_0^2 + (1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\}(\gamma-\psi)}{(\Delta_0 - \alpha^2\delta_2\delta_3)\Delta_0^2 - \alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2}\end{array}\right].$$

We notice that, in the above equations, the effects of an increase in the tolerated level of corruption also depend on the value of ε, observed by the public. But, it is not fundamental in determining the sign of $\frac{\partial\tau}{\partial\bar{\theta}}$ and $\frac{\partial\theta}{\partial\bar{\theta}}$. In the following proposition, we will neglect it (by assuming that $\varepsilon = 0$) to simplify the conditions under which $\frac{\partial\tau}{\partial\bar{\theta}}$ and $\frac{\partial\theta}{\partial\bar{\theta}}$ might be positive or negative.

Proposition 2c: *In the case of opacity, an increase in the tolerated level of corruption induces an increase in the tax rate and corruption if* $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, *independently of the relative importance of* γ *and* ψ. *The same effects can be observed if the condition concerning* μ *is reversed and* σ_ε^2 *is either sufficiently large or small.*

Proof: See Appendix B.

In the case where the central bank is sufficiently conservative, the degree of opacity about central bank preferences plays a role in determining the direction of the effects of an increase in the tolerated level of corruption on the tax rate and corruption. However, the conditions concerning σ_ε^2 are quite lengthy, and without loss of generality, they are not presented here.

Propositions 2a, 2b and 2c show that the direction of the effects of an increase in the tolerated level of corruption on the equilibrium is not altered by the introduction of a low level of opacity. If the degree of central bank conservativeness is low enough, an increase in the tolerated corruption is always associated with higher inflation expectations. Under a high degree of central bank conservativeness, it positively affects the expected inflation rate if the degree of opacity is low enough, while we are uncertain about its effects when the latter is high enough.

It is to notice that the results reported in Proprositions 2a, 2b are derived under the condition that $\gamma - \psi > 0$. When this condition is reversed, i.e. $\gamma - \psi < 0$, the results will also be reversed if other conditions remain unchanged. Furthermore, the effects of an increase in the tolerated level of corruption on the inflation rate and output gap depend on the sign of $\frac{\partial \pi^e}{\partial \bar{\theta}}$ and hence that of $(\gamma - \psi)$ as it is straightforward to see from equation (16).

The equilibrium solutions allow us to examine the effects of opacity on the level of π^e, π, τ, θ and x. Opacity influences macroeconomic variables through its effects on the expected inflation rate. Hence, we firstly derive the effects of opacity on π^e using equation (13) and taking account of (14). Then, we use this result and equations (8)-(11) to examine the effects of opacity on the level of π, τ, θ and x respectively.

Proposition 3a*: An increase in opacity has a positive effect on the expected and current inflation rate and a negative effect on the output gap if* $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi > 0$ *and* $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$*. It has the opposite effects if the last condition is reversed.*

Proof: Deriving π^e given by equation (13) with respect to σ_ε^2 and taking account of the approximation of Ω given by (14), we obtain:

$$\frac{\partial \pi^e}{\partial \sigma_\varepsilon^2} = \frac{\Delta_0^3 \alpha \delta_2 \delta_3 (1+\mu)[(\gamma-\psi)\bar{\theta} + \gamma\bar{g}][(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]}{\left\{\Delta_0^3 - \{\Delta_0^2 + (1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\}\alpha^2\delta_2\delta_3\right\}^2}.$$

Using then equations (8) and (11), we derive π and x with respect to σ_ε^2 as follows:

$$\frac{\partial \pi}{\partial \sigma_\varepsilon^2} = \frac{\alpha^2(1+\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial \pi^e}{\partial \sigma_\varepsilon^2}; \quad \frac{\partial x}{\partial \sigma_\varepsilon^2} = \frac{-\alpha(\mu-\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial \pi^e}{\partial \sigma_\varepsilon^2}.$$

For $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi > 0$, using the definition of Δ_0, we can show that $\frac{\partial \pi^e}{\partial \sigma_\varepsilon^2} > 0$, $\frac{\partial \pi}{\partial \sigma_\varepsilon^2} > 0$ and $\frac{\partial x}{\partial \sigma_\varepsilon^2} < 0$ if $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3}$ (i.e. $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$) and *vice versa*. **Q.E.D.**

Proposition 3a is formulated under the condition that $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi > 0$. As we have already discussed, the target of public expenditures is closely related to the tax rate chosen by the government, which induces a fiscal bias. An increase in the tax rate positively affects the wage (and hence inflation) expectations. On the other hand, the fight against corruption allows reducing the tax rate, and hence the fiscal bias and inflation expectations, for a given level of public expenditures. Depending on the relative importance of these opposite effects, inflation expectations may increase or decrease. If other conditions remain unchanged, the results will be reversed if $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi < 0$.

Furthermore, we also remark that the degree of central bank conservativeness plays an important role in determining the equilibrium effects of opacity on the expected and current inflation rate, and output gap. Consider the case where $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi > 0$, which implies that the fiscal policies will initially have a negative effect on the output gap and a positive effect on the inflation rate. If the central bank is sufficiently conservative on average, i.e. $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3}$, the public will believe that it will not strongly fight against an increase in the inflation rate under opacity. This leads to a higher expected (and current) inflation rate and a lower output gap comparing to the case of full transparency. Therefore, a more conservative central bank has incentive to communicate more clearly with the public about its preferences.

Proposition 3b*: An increase in opacity has a negative effect on the tax rate if* $\gamma > \max\{\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}, \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}\}$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. *It has a positive effect on the tax rate if the last condition is reversed.*

Proof: Using equations (9), we derive τ with respect to σ_ε^2 as follows:

$$\frac{\partial\tau}{\partial\sigma_\varepsilon^2} = \frac{-\alpha[(\mu-\varepsilon)\delta_2(\gamma-\psi)+(\mu-\varepsilon)\gamma\delta_3+\alpha(1+\varepsilon)\delta_2\delta_3]}{\Delta_\varepsilon}\frac{\partial\pi^e}{\partial\sigma_\varepsilon^2}.$$

Conditions $\gamma > \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ ensure that $\frac{\partial\pi^e}{\partial\sigma_\varepsilon^2} > 0$. We then have $\frac{\partial\tau}{\partial\sigma_\varepsilon^2} < 0$ if $\gamma > \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}$. Combining next the conditions concerning γ leads to $\gamma > \max\{\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}, \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}\}$. Given the last condition and Proposition 3a, we obtain $\frac{\partial\tau}{\partial\sigma_\varepsilon^2} > 0$ if $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. **Q.E.D.**

In Proposition 3b, we have put in evidence the effects of opacity on the tax rate by altering the degree of central bank conservativeness while the condition concerning γ remains unchanged. Under the latter condition, if the central bank is sufficiently conservative, an increase in opacity implies a higher inflation rate, allowing the government to reduce the tax rate thanks to an increase in the inflation tax. The results will not be modified if we substitute the condition concerning γ by $\gamma < \min\{\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}, \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}\}$. However, in the case where we have $\min\{\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}, \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}\} < \gamma < \max\{\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}, \frac{(\mu-\varepsilon)\delta_2\psi-\alpha(1+\varepsilon)\delta_2\delta_3}{(\mu-\varepsilon)(\delta_2+\delta_3)}\}$ while the condition concerning μ is unchanged, the results will be reversed.

Proposition 3c*: An increase in opacity has a negative effect on the level of corruption if* $\gamma - \psi > 0$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3}$. *It has a positive effect on the level of corruption if the last condition is reversed.*

Proof: Using equation (10), we derive θ with respect to σ_ε^2 as follows:

$$\frac{\partial\theta}{\partial\sigma_\varepsilon^2} = \frac{-\alpha(\mu-\varepsilon)(\gamma-\psi)\delta_2}{\Delta_\varepsilon}\frac{\partial\pi^e}{\partial\sigma_\varepsilon^2}.$$

Taking account of Proposition 3a, it leads to Proposition 3c. **Q.E.D.**

According to Proposition 3c, the effects of opacity on corruption depend on the degree of central bank conservativeness. For high enough values of γ, if the central bank is sufficiently conservative, an increase in opacity can help reducing corruption. On the contrary, in the case of a liberal central bank, opacity will induce the government to effectively tolerate more corruption.

In Proposition 3c, the effects of opacity on the level of corruption vary with the degree of central bank conservativeness given that $\gamma - \psi > 0$. The results remain the same if we substitute the condition concerning γ by $\gamma < \frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}$. However, if $\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}} < \gamma < \psi$ and without altering the condition concerning μ, the results will be reversed.

Proposition 4: *Under the assumption that* $\gamma - \psi > 0$, *an increase in the tolerated level of corruption reinforces the sensibility of the expected and current inflation rate to opacity as well as that of output gap if* $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ *and vice versa.*

Proof: Deriving twice π^e given by equation (13) with respect to σ_ε^2 and $\bar{\theta}$, and taking account of the approximation of Ω and rearranging the terms lead to:

$$\frac{\partial^2 \pi^e}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}} = \frac{\alpha\delta_2\delta_3(\gamma-\psi)\{(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\}\Delta_0^3}{\{(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\}^2}.$$

We have $\frac{\partial^2 \pi^e}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}} > 0$ if $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$, i.e. $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, and *vice versa*.

Using equations (8) and (11), we derive twice π and x with respect to σ_ε^2 and $\overline{\theta}$ as follows:

$$\frac{\partial^2 \pi}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}} = \frac{\alpha^2(1+\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial^2 \pi^e}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}} \ ; \quad \frac{\partial^2 x}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}} = \frac{-\alpha(\mu-\varepsilon)\delta_2\delta_3}{\Delta_\varepsilon}\frac{\partial^2 \pi^e}{\partial \sigma_\varepsilon^2 \partial \overline{\theta}}.$$ **Q.E.D.**

The results presented in Proposition 4 can be interpreted in another way. More precisely, under the assumption that $\gamma-\psi>0$, an increase in opacity reinforces the sensibility of the expected and current inflation rate, and output gap to the tolerated level of corruption if the central bank is sufficiently conservative and *vice versa*.

Even if the central bank is conservative enough, the public, uncertain about the degree of central bank conservativeness, will believe that it will not strongly fight against an increase in the inflation rate. This leads to higher expected inflation comparing with the case of full transparency. If $\gamma-\psi>0$, this effect is reinforced by an increase in the tolerated level of corruption since a higher level of corruption will imply a higher tax rate to compensate the loss of tax revenue. Higher tax rate and corruption jointly have negative effects on the output gap and positive effects on the inflation rate. This leads the public, facing an opaque and quite conservative central bank, to anticipate a further increase in the expected inflation rate. Therefore, if the government tolerates more corruption, for a conservative central bank, the incentive to communicate more clearly with the public about its preferences becomes stronger.

5. The effects of opacity and tolerated corruption on macroeconomic volatility

The volatility of inflation and output gap is generated by the shock ε which affects central bank preferences. Central bank opacity affects the volatility of these variables through inflation expectations. In effect, it is through the latter channel that opacity interacts with the decisions of the government and private sector.

Using equations (8), (11) and (13), the variances of π and x are obtained as follows:

$$\operatorname{var}(\pi)=E(\Phi^2)\left\{\frac{\alpha\delta_2\delta_3[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}]}{1-\Omega\alpha^2\delta_2\delta_3}\right\}^2;\ \operatorname{var}(x)=E(\Xi^2)\left\{\frac{\delta_2\delta_3[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}]}{1-\Omega\alpha^2\delta_2\delta_3}\right\}^2;$$

where $\Phi=\frac{1+\varepsilon}{\Delta_\varepsilon}$, $\Xi=\frac{\mu-\varepsilon}{\Delta_\varepsilon}$ with $\Delta_\varepsilon=(\mu-\varepsilon)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]+\alpha(\alpha+\gamma)(1+\varepsilon)\delta_2\delta_3$.

Opacity affects $\operatorname{var}(\pi)$ through Ω and $E(\Phi^2)$ and $\operatorname{var}(x)$ through Ω and $E(\Xi^2)$. Deriving $\operatorname{var}(\pi)$ and $\operatorname{var}(x)$ with respect to σ_ε^2 yields:

$$\frac{\partial\operatorname{var}(\pi)}{\partial\sigma_\varepsilon^2}=\left\{\frac{\alpha\delta_2\delta_3[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}]}{1-\Omega\alpha^2\delta_2\delta_3}\right\}^2\left\{\frac{\partial E(\Phi^2)}{\partial\sigma_\varepsilon^2}+2E(\Phi^2)\left(\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}\right)\right\}. \tag{28}$$

$$\frac{\partial\operatorname{var}(x)}{\partial\sigma_\varepsilon^2}=\left\{\frac{\delta_2\delta_3[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}]}{1-\Omega\alpha^2\delta_2\delta_3}\right\}^2\left\{\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2}+2E(\Xi^2)\left(\frac{\alpha^2\delta_2\delta_3}{(1-\Omega\alpha^2\delta_2\delta_3)}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}\right)\right\}. \tag{29}$$

We remark that the direction of the effects of opacity on the volatility of inflation and output gap does not depend on the relative importance of marginal effects of tax and corruption on production. In the following, we will examine the conditions under which $\frac{\partial E(\Phi^2)}{\partial\sigma_\varepsilon^2}$ (or $\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2}$) and $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}$ have the same sign. When this is the case, we obtain some closed-form conditions under which the effects of opacity on the volatility of inflation are clearly determined. When they are of opposite sign, there will not be clear-cut conditions given the complexity of their respective expressions.

Proposition 5a: *An increase in opacity positively affects the volatility of inflation if* $\mu > \max\left\{\frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}-3; \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}\right\}$ and $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$. *It negatively affects the volatility of inflation if one of the following conditions is verified:*

i) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \mu < \frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} - 3$ and $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$.

ii) $\mu < \min\left\{\frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}-3; \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}\right\}$.

Proof: See Appendix C.

The condition concerning μ given in i) is verified only when $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > 3$, which implies that the central bank is highly conservative. According to Proposition 5a, an increase in opacity implies higher inflation volatility if the central bank is sufficiently conservative and the initial degree of opacity small enough. Conversely, an increase in opacity allows reducing the volatility of inflation when the central bank is highly conservative given a sufficiently high initial degree of opacity. The latter effects are observed when the central bank is liberal enough, independently of the initial degree of opacity.

Proposition 5b: *For* $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > \frac{2}{3}$, *an increase in opacity has a positive effect on the volatility of output gap if* $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ *and* $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$. *For* $\frac{1}{3} < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$, *an increase in opacity has a positive effect on the volatility of output gap if* $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}$ *and* $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$. *It has a negative effect on the volatility of output gap if one of the following conditions is verified:*

i) $\mu > \max\left\{\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3};\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}\right\}$ *and* $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$, *for* $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$.

ii) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3} < \mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, *for* $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{1}{3}$.

Proof: See Appendix D.

If the central bank is conservative, an increase in opacity implies higher volatility of output gap when the initial degree of opacity is sufficiently low. However, an increase in opacity reduces the volatility of output gap when the central bank is sufficiently conservative but the initial degree of opacity is sufficiently high as well as when the central bank is liberal enough. In the latter case, the effects are independent of the initial degree of opacity.

Proposition 6*: An increase in the tolerated level of corruption has a positive effect on the volatility of inflation and output gap if* $\gamma - \psi > 0$ or $\gamma < \frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}}$. The effects are reversed if $\frac{\bar{\theta}\psi}{\bar{\theta}+\bar{g}} < \gamma < \psi$.

Proof: We derive $\text{var}(\pi)$ and $\text{var}(x)$ with respect to $\bar{\theta}$. It follows straightforwardly that the sign of $\frac{\partial\,\text{var}(\pi)}{\partial\bar{\theta}}$ and $\frac{\partial\,\text{var}(x)}{\partial\bar{\theta}}$ depends on that of the expression $[(\gamma-\psi)\bar{\theta}+\gamma\bar{g}](\gamma-\psi)$. This leads to the conditions given in Proposition 6. **Q.E.D.**

It is to notice that the direction of the effects of an increase in the tolerated level of corruption on the volatility of inflation and output gap depend on the relative importance of γ and ψ. For extreme (intermediate) values of γ, the corruption is positively (negatively) linked to the volatility of inflation and output gap, independently of the degree of central bank conservativeness and the degree of opacity.

Having shown how opacity affects the level and volatility of inflation and output gap, we will examine whether there is a case for more opacity. When the central bank decided not to reveal private information about its preferences, it accepted lower equilibrium inflation (and higher output gap) in exchange of greater macroeconomic instability. If the equilibrium level and volatility of inflation were both increasing (or decreasing) in opacity, there would be no such trade-off with respect to the degree of opacity. In the case where both inflation level and volatility were increasing in opacity, the most desirable situation is that the central bank should be fully transparent ($\sigma_\varepsilon^2 = 0$). Inversely, if both of them were decreasing in opacity, there would be a case for monetary policy opacity.

According to Proposition 3a, we have $\frac{\partial \pi}{\partial \sigma_\varepsilon^2} > 0$ if $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi > 0$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ or if $\gamma - \frac{\bar{\theta}}{\bar{\theta}+\bar{g}}\psi < 0$ and $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. Meanwhile, as shown in Proposition 5a, we could have $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} < 0$ if $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, independently of the relative importance of γ and ψ. Therefore, under some conditions, we can simultaneously have $\frac{\partial \pi}{\partial \sigma_\varepsilon^2} > 0$ and $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} < 0$ for any initial degree of opacity. However, under certain conditions for which the initial degree of opacity plays a role, we can simultaneously obtain $\frac{\partial \pi}{\partial \sigma_\varepsilon^2} < 0$ and $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} > 0$. In these two cases, the trade-off is possible since the central bank that desires to reduce the volatility of inflation could accept an increase in the level of inflation and *vice versa*.

On the other hand, Propositions 4a and 5a implies that we can simultaneously have $\frac{\partial \pi}{\partial \sigma_\varepsilon^2} > 0$ and $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} > 0$ under certain conditions imposed on the degree of central bank conservativeness and the initial degree of opacity. In this case, more transparency is preferable to less if initially the central bank is not fully transparent. Nevertheless, we can also have $\frac{\partial \pi}{\partial \sigma_{\varepsilon}^{2}} < 0$ and $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_{\varepsilon}^{2}} < 0$ at the same time. Hence, there is a case for opacity under certain conditions.

Without giving the detailed calculations and using the results given in Propositions 5a and 6, we can deduce from equation (28) that opacity and tolerated corruption can mutually reinforce or weaken each other's effects on the volatility of inflation. For example, when the degree of central bank conservativeness is sufficiently high and the initial degree of opacity is sufficiently low, their effects are mutually reinforced if the marginal effect of tax on production is strong enough or weak enough, or mutually weakened if the marginal effect of tax is at intermediate levels. Taking account of Proposition 4, we can state that in some circumstances, central bank transparency becomes more compelling, notably when the central bank is sufficiently conservative. In these circumstances, more transparency allows reducing the effects of corruption on the level and volatility of inflation. Under other conditions, there may be a case for opacity in order to compensate for the undesirable effects of corruption on macroeconomic performance and volatility, mainly when the central bank is liberal.

6. Conclusion

In this paper, we have examined the relationship between institutional quality and central bank transparency and their implications for macroeconomic performance and volatility through the interaction of monetary and fiscal policies. We have found that the results depend on the relative importance of the marginal effects of distortionary tax and corruption on production, the degree of central bank conservativeness as well as the degree of opacity about central bank preferences.

In the case of full transparency, when the marginal effect of tax on production is greater than that of corruption, an increase in the tolerated level of corruption has positive effects on the current and expected inflation rate, tax rate and corruption and a negative effect on the output gap. On the contrary, if the marginal effect of tax is smaller than that of corruption, an increase in the tolerated level of corruption will result in a higher output gap and hence will incite the central bank to reduce the inflation rate, reversing thus the previous effects.

The introduction of a low level of opacity will not modify the direction of the effects of an increase in the tolerated level of corruption on endogenous variables. If the marginal effect of tax on production is greater than that of corruption, under a low degree of conservativeness, more tolerance for corruption by the government is always associated with higher expected inflation. Under a high degree of conservativeness, the tolerated level of corruption is positively linked to the expected inflation rate if the degree of opacity is sufficiently low, while the effects are indeterminate when the degree of opacity is high enough.

In terms of macroeconomic performance, we have found that when the marginal effect of tax on production is sufficiently large, an increase in opacity has a positive effect on the expected and current inflation rate and a negative effect on the output gap, tax rate and corruption if the central bank is conservative enough. These effects are reversed if the central bank is sufficiently liberal and/or the marginal effect of tax on production sufficiently low. Central bank opacity and the tolerated level of corruption mutually reinforces (weakens) each other's effects on the equilibrium if the degree of central bank conservativeness is sufficiently high (low).

Finally, when the central bank is sufficiently conservative, an increase in opacity might induce higher (lower) volatility of inflation and output gap if the initial degree of opacity is low (high) enough. An increase in opacity allows reducing the volatility of inflation and output gap when the central bank is conservative enough, given a high initial degree of opacity. The negative effects of opacity are also observed when the central bank is liberal enough, independently of the initial degree of opacity. Furthermore, opacity and tolerated corruption can mutually reinforce or weaken each other's effects on the volatility of inflation.

Central bank transparency could become more compelling when the central bank is sufficiently conservative. However, there could be a case for opacity in order to compensate for the undesirable effects of corruption on macroeconomic performance and volatility when the central bank is liberal.

Appendix A: Proof of Proposition 2a

In the case of opacity, when $\gamma - \psi > 0$, according to equation (16), we have $\frac{\partial \pi^e}{\partial \bar{\theta}} > 0$ if:

$$\begin{aligned} &\{\mu[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] + \alpha\gamma\delta_2\delta_3\}\Delta_0^2 \\ &-(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]\alpha^2\delta_2\delta_3\sigma_\varepsilon^2 > 0. \end{aligned} \qquad \text{(A.1)}$$

Taking account of the definition of Δ_0, we have:

$$\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 = \mu[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] - \alpha(\alpha+\gamma)\delta_2\delta_3.$$

If $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$, i.e. $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, for condition (A.1) to be checked, we must impose that $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$. If $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 < 0$, i.e. $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, condition (A.1) is always verified. **Q.E.D**

Appendix B: Proof of Proposition 2c

To determine the sign of $\frac{\partial \tau}{\partial \bar{\theta}}$, we substitute $\frac{\partial \pi^e}{\partial \bar{\theta}}$ given by equation (16) and the approximation of Ω given by (14) into equation (18) as follows:

$$\frac{\partial\tau}{\partial\overline{\theta}}=\frac{1}{\Delta_\varepsilon}\left[\begin{array}{l}[(\mu-\varepsilon)(\gamma\psi+\delta_2)+\alpha(1+\varepsilon)(\alpha+\psi)\delta_2]\delta_3 \\ -\dfrac{\alpha^2\delta_2\delta_3[(\mu-\varepsilon)\delta_2(\gamma-\psi)+(\mu-\varepsilon)\gamma\delta_3+\alpha(1+\varepsilon)\delta_2\delta_3]\times \{\Delta_0^2+(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\}(\gamma-\psi)}{(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2}\end{array}\right].$$

Departing from the case of full transparency, evaluating $\frac{\partial\tau}{\partial\overline{\theta}}$ for $\varepsilon=0$ and $\sigma_\varepsilon^2>0$ and rearranging the terms of the resulting equation yield:

$$\frac{\partial\tau}{\partial\overline{\theta}}=\frac{1}{\Delta_0}\frac{\begin{array}{l}[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\{\alpha^2\delta_2\delta_3[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3](\gamma-\psi)\Delta_0^2 \\ -\left[\begin{array}{l}[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3 \\ +[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3](\gamma-\psi)\end{array}\right]\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\end{array}}{(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2} \tag{B.1}$$

Since equation (B.1) can be reduced to equation (26) when $\sigma_\varepsilon^2=0$, it follows that:

$$[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3(\Delta_0-\alpha^2\delta_2\delta_3)-\alpha^2\delta_2\delta_3[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3](\gamma-\psi)>0.$$

If $\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3<0$, i.e. $\mu<\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, the denominator and numerator of equation (B.1) are both positive and hence $\frac{\partial\tau}{\partial\overline{\theta}}>0$.

If $\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3>0$, i.e. $\mu>\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, to obtain $\frac{\partial\tau}{\partial\overline{\theta}}>0$, the denominator and numerator of equation (B.1) must be both positive or both negative. We distinguish two cases.

First case: The denominator and numerator of equation (B.1) are both positive. For that to be possible, we must simultaneously have $\sigma_\varepsilon^2<\overline{\sigma}_\varepsilon^2$ and

$$\sigma_\varepsilon^2<\frac{[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3](\gamma-\psi)\Delta_0^2}{\{\mu(\delta_2+\gamma^2)\delta_3+(\alpha+\gamma)\alpha\delta_2\delta_3+\mu\delta_2(\gamma-\psi)^2\}\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]}. \tag{B.2}$$

To have $\frac{\partial\tau}{\partial\bar{\theta}} > 0$, it is sufficient to impose the condition (B.2), which is more restrictive than the condition $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$ given that $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$.

Second case: The denominator and numerator of equation (B.1) are both negative. Consequently, we must simultaneously have $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$ and

$$\sigma_\varepsilon^2 < \frac{[\mu(\gamma\psi+\delta_2)+\alpha(\alpha+\psi)\delta_2]\delta_3(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3[\mu\delta_2(\gamma-\psi)+\mu\gamma\delta_3+\alpha\delta_2\delta_3](\gamma-\psi)\Delta_0^2}{\{\mu(\delta_2+\gamma^2)\delta_3+(\alpha+\gamma)\alpha\delta_2\delta_3+\mu\delta_2(\gamma-\psi)^2\}\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]}. \tag{B.3}$$

In order to obtain $\frac{\partial\tau}{\partial\bar{\theta}} > 0$, it is sufficient to impose the condition $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$ as the latter is more restrictive than condition (B.3).

To determine now the sign of $\frac{\partial\theta}{\partial\bar{\theta}}$, we substitute $\frac{\partial\pi^e}{\partial\bar{\theta}}$ given by equation (16) and the approximation of Ω given by (14) into equation (19). Evaluating the resulting equation at $\varepsilon = 0$ and $\sigma_\varepsilon^2 > 0$ and rearranging the terms, we obtain:

$$\frac{\partial\theta}{\partial\bar{\theta}} = \frac{1}{\Delta_0}\frac{\left\{\begin{array}{l}[(\alpha+\gamma)\alpha\delta_2+\mu(\gamma^2+\delta_2)](\Delta_0-\alpha^2\delta_2\delta_3)\delta_3\Delta_0^2-\mu(\gamma-\psi)^2\delta_2\alpha^2\delta_2\delta_3\Delta_0^2\\ -\{[(\alpha+\gamma)\alpha\delta_2+\mu(\gamma^2+\delta_2)]\delta_3+\mu(\gamma-\psi)^2\delta_2\}\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2\end{array}\right\}}{(\Delta_0-\alpha^2\delta_2\delta_3)\Delta_0^2-\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0-2\alpha(\alpha+\gamma)\delta_2\delta_3]\sigma_\varepsilon^2} \tag{B.4}$$

If $\sigma_\varepsilon^2 = 0$, equation (B.4) is equivalent to equation (27). Consequently, we have:

$$[(\alpha+\gamma)\alpha\delta_2+\mu(\gamma^2+\delta_2)](\Delta_0-\alpha^2\delta_2\delta_3)\delta_3-\mu(\gamma-\psi)^2\delta_2\alpha^2\delta_2\delta_3 > 0.$$

If $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 < 0$, i.e. $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, then we have $\frac{\partial\theta}{\partial\bar{\theta}} > 0$.

If $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$, i.e. $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, to obtain $\frac{\partial\theta}{\partial\bar{\theta}} > 0$, the denominator and numerator of equation (B.4) must be both positive or negative. Two cases are distinguished.

First case: The denominator and numerator of equation (B.4) are both positive. Hence, we must simultaneously have $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$ and

$$\sigma_\varepsilon^2 < \frac{[(\alpha+\gamma)\alpha\delta_2 + \mu(\gamma^2+\delta_2)](\Delta_0 - \alpha^2\delta_2\delta_3)\delta_3\Delta_0^2 - \mu(\gamma-\psi)^2\delta_2\alpha^2\delta_2\delta_3\Delta_0^2}{\{[(\alpha+\gamma)\alpha\delta_2 + \mu(\gamma^2+\delta_2)]\delta_3 + \mu(\gamma-\psi)^2\delta_2\}\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]}. \quad \text{(B.5)}$$

It is sufficient to impose condition (B.5) in order to have $\frac{\partial\theta}{\partial\bar{\theta}} > 0$ since it is more restrictive than the condition $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$.

Second case: The denominator and numerator of equation (B.4) are both negative. Therefore, we must simultaneously have $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$ and

$$\sigma_\varepsilon^2 > \frac{[(\alpha+\gamma)\alpha\delta_2 + \mu(\gamma^2+\delta_2)](\Delta_0 - \alpha^2\delta_2\delta_3)\delta_3\Delta_0^2 - \mu(\gamma-\psi)^2\delta_2\alpha^2\delta_2\delta_3\Delta_0^2}{\{[(\alpha+\gamma)\alpha\delta_2 + \mu(\gamma^2+\delta_2)]\delta_3 + \mu(\gamma-\psi)^2\delta_2\}\alpha^2\delta_2\delta_3(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]}. \quad \text{(B.6)}$$

For ensuring $\frac{\partial\theta}{\partial\bar{\theta}} > 0$, it is sufficient to impose the condition $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$, which is more restrictive than condition (B.6). **Q.E.D.**

Appendix C: Proof of Proposition 5a

Using the second-order Taylor approximation, we obtain:

$$E(\Phi^2) \approx \frac{1}{\Delta_0^2} + \frac{(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3]\{(3+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] - 2\alpha(\alpha+\gamma)\delta_2\delta_3\}}{\Delta_0^4}\sigma_\varepsilon^2.$$

Deriving $E(\Phi^2)$ with respect to σ_ε^2 yields:

$$\frac{\partial E(\Phi^2)}{\partial\sigma_\varepsilon^2} = \frac{(1+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3]\{(3+\mu)[(\gamma-\psi)^2\delta_2 + (\delta_2+\gamma^2)\delta_3] - 2\alpha(\alpha+\gamma)\delta_2\delta_3\}}{\Delta_0^4}. \quad \text{(C.1)}$$

According to (28), to obtain $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} > 0$, it is sufficient to have $\frac{\partial E(\Phi)^2}{\partial \sigma_\varepsilon^2} > 0$ and $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} > 0$. Using (C.1), we can easily show that a sufficient condition for $\frac{\partial E(\Phi)^2}{\partial \sigma_\varepsilon^2} > 0$ is that $\mu > \frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} - 3$.

To have $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} > 0$, we distinguish two possibilities, i.e. $1-\Omega\alpha^2\delta_2\delta_3$ and $\frac{\partial \Omega}{\partial \sigma_\varepsilon^2}$ are simultaneously positive or negative. Using (14), we obtain:

$$\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} = \frac{(1+\mu)[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3][\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3]}{\Delta_0^3}. \quad \text{(C.2)}$$

It follows that $\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} > 0$ if $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 > 0$, i.e. $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. Given the latter condition, in order to obtain $1-\Omega\alpha^2\delta_2\delta_3 > 0$, we must also have $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$. Under these conditions, both $1-\Omega\alpha^2\delta_2\delta_3$ and $\frac{\partial \Omega}{\partial \sigma_\varepsilon^2}$ are positive.

The case where these two terms are simultaneously negative is not possible. In effect, if $\Delta_0 - 2\alpha(\alpha+\gamma)\delta_2\delta_3 < 0$, i.e. $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, we have $\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} < 0$ and $1-\Omega\alpha^2\delta_2\delta_3 > 0$.

Summarizing the above sufficient conditions for obtaining $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} > 0$ yields: $\mu > \max\left\{\frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} - 3; \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}\right\}$ and $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$.

In order to obtain $\frac{\partial \operatorname{var}(\pi)}{\partial \sigma_\varepsilon^2} < 0$, it is sufficient to have $\frac{\partial E(\Phi)^2}{\partial \sigma_\varepsilon^2} < 0$ and $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} < 0$. Using (C.1), we can easily show that a sufficient condition for $\frac{\partial E(\Phi)^2}{\partial \sigma_\varepsilon^2} < 0$ is that $\mu < \frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} - 3$. To have $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} < 0$, we distinguish two cases.

First case: $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}>0$ and $1-\Omega\alpha^2\delta_2\delta_3<0$. To ensure that $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}>0$, we must have $\mu>\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. To ensure that $1-\Omega\alpha^2\delta_2\delta_3<0$, we must also impose that $\sigma_\varepsilon^2>\bar{\sigma}_\varepsilon^2$.

Second case: $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}<0$ and $1-\Omega\alpha^2\delta_2\delta_3>0$. To ensure that $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}<0$, we must have $\mu<\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$. The last condition also guarantees that $1-\Omega\alpha^2\delta_2\delta_3>0$, $\forall\sigma_\varepsilon^2$.

Summarizing the sufficient conditions for having $\frac{\partial\,\mathrm{var}(\pi)}{\partial\sigma_\varepsilon^2}<0$ leads to two conditions:

i) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}<\mu<\frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}-3$ and $\sigma_\varepsilon^2>\bar{\sigma}_\varepsilon^2$.

ii) $\mu<\min\left\{\frac{2\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}-3;\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}\right\}$.

For condition i) to be verified, we must have $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}>3$. **Q.E.D.**

Appendix D: Proof of Proposition 5b

Using the second-order Taylor approximation yields:

$$E(\Xi^2)=\frac{\mu^2}{\Delta_0^2}+\frac{\{(1+3\mu)\alpha(\alpha+\gamma)\delta_2\delta_3-2\mu[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]\}\alpha(\alpha+\gamma)\delta_2\delta_3(1+\mu)\sigma_\varepsilon^2}{\Delta_0^4}.$$

Deriving $E(\Xi^2)$ with respect to σ_ε^2 leads to:

$$\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2}=\frac{\{(1+3\mu)\alpha(\alpha+\gamma)\delta_2\delta_3-2\mu[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]\}\alpha(\alpha+\gamma)\delta_2\delta_3(1+\mu)}{\Delta_0^4}.$$

According to (29), in order to obtain $\frac{\partial\,\mathrm{var}(x)}{\partial\sigma_\varepsilon^2}>0$, it is sufficient to have $\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2}>0$ and $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2}>0$. If $(1+3\mu)\alpha(\alpha+\gamma)\delta_2\delta_3-2\mu[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]>0$, it is easy to show that $\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2}>0$. The previous inequality is true if one of the following conditions is verified:

a) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$ and $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}$;

b) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > \frac{2}{3}$, $\forall\mu$.

To simultaneously have $1-\Omega\alpha^2\delta_2\delta_3 > 0$ and $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2} > 0$ and hence $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2} > 0$, we must impose that $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ and $\sigma_\varepsilon^2 < \overline{\sigma}_\varepsilon^2$. Meanwhile, the case where $1-\Omega\alpha^2\delta_2\delta_3 < 0$ and $\frac{\partial\Omega}{\partial\sigma_\varepsilon^2} < 0$ is impossible (see the proof of Proposition 5a).

Combining the conditions obtained here and the conditions a) and b), we get $\frac{\partial \operatorname{var}(x)}{\partial\sigma_\varepsilon^2} > 0$ when $\sigma_\varepsilon^2 < \overline{\sigma}_\varepsilon^2$ and if one of the following conditions is verified:

i) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > \frac{2}{3}$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$;

ii) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$ and $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}$.

We note that the last condition concerning μ is verified only if $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > \frac{1}{3}$.

On the contrary, we have $\frac{\partial \operatorname{var}(x)}{\partial\sigma_\varepsilon^2} < 0$ if $\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2} < 0$ and $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2} < 0$. We can show that $\frac{\partial E(\Xi^2)}{\partial\sigma_\varepsilon^2} < 0$ if $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$ and $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}$. The case where $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} > \frac{2}{3}$ implies that μ must be negative and that is impossible.

We obtain $\frac{\alpha^2\delta_2\delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial\Omega}{\partial\sigma_\varepsilon^2} < 0$ under two conditions (see the proof of Proposition 5a)

a) $\mu > \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$ and $\sigma_\varepsilon^2 > \overline{\sigma}_\varepsilon^2$;

b) $\mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, $\forall\sigma_\varepsilon^2$.

Combining the conditions ensuring $\frac{\partial E(\Xi^2)}{\partial \sigma_\varepsilon^2} < 0$ and $\frac{\alpha^2 \delta_2 \delta_3}{1-\Omega\alpha^2\delta_2\delta_3}\frac{\partial \Omega}{\partial \sigma_\varepsilon^2} < 0$, we find that $\frac{\partial \operatorname{var}(x)}{\partial \sigma_\varepsilon^2} < 0$

when $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{2}{3}$ and if one of the following conditions is verified:

i) $\mu > \max\left\{\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3};\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3}\right\}$ and $\sigma_\varepsilon^2 > \overline{\sigma}_\varepsilon^2$;

ii) $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{2[(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3]-3\alpha(\alpha+\gamma)\delta_2\delta_3} < \mu < \frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3}$, $\forall \sigma_\varepsilon^2$.

We remark that the last condition can be verified when $\frac{\alpha(\alpha+\gamma)\delta_2\delta_3}{(\gamma-\psi)^2\delta_2+(\delta_2+\gamma^2)\delta_3} < \frac{1}{3}$. **Q.E.D.**

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