A New Approach to Analyzing Convergence and Synchronicity in Growth and Business Cycles: Cross Recurrence Plots and Quantification Analysis

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Abstract

Convergence and synchronisation of business and growth cycles are important issues in the efficient formulation of euro area economic policies, and in particular European Central Bank (ECB) monetary policy. Although several studies in the economics literature address the issue of synchronicity of growth within the euro area, this is the first study to address this issue using cross recurrence analysis. The main findings are that member state growth rates had largely converged before the introduction of the euro, but there is a wide degree of different synchronisation behaviours.which appear non-linear in nature. Many of the euro area member states display what is termed here as "intermittency" in synchronization, although this is not consistent across countries or members of the euro area. These differences in synchronization behaviors could cause problems in future implementation of euro area monetary policy.

Keywords: Euro area, business cycles, growth cycles, recurrence plots, non-stationarity, complex systems, surrogate analysis.

Acknowledgements: The Bank of Finland are to be thanked for hosting Patrick Crowley during the summers of 2006 and 2007 when much of this work was originally conceived. The authors acknowledge the valuable comments from participants at the Bank of Finland Macro Workshop (August 8th, 2007) and the 2nd Recurrence Plot Workshop held in Siena, Italy (Sept 12-14th, 2007). The authors would also like to thank (the late) Joe Zbilut, Norbert Marwan and Mikael Bask for advice on recurrence plot techniques, as well as Tero Kuusi (Bank of Finland) for data assistance.

Note: A previous shortened version of the second part of this paper was previously published as Crowley (2008).

1 Introduction

The macroeconomy of the market-based economic system is probably the largest man-made complex system in existence. Although economists have created a large arsenal of time-series techniques to analyze the movement of macroeconomic variables over time, these models come with a host of assumptions (for example linearity, disributional assumptions, constant parameters) and so have limited ability to detect and characterise the non-linearities inevitably inherent in such a large system. Although a cycle in economic activity is a "stylized" fact in macroeconomics, it is less clear as to when the economic growth dynamic coincides between countries and when these dynamics synchronise. It is well known that the recessionary phase of the business cycle in most countries appears to closely follow downturns in the US economy, but often there is a variable lag to this effect, and sometimes it does not occur (- for example the euro area in the early 2000s and Australia in the current economic downturn).

According to the celebrated optimal currency area theory (Mundell (1961)), within a monetary union, given the absence of labor mobility and federal transfers as offsetting economic factors, synchronicity of growth cycles is an important pre-requisite for the optimal application of a single monetary policy. This clearly becomes an important issue for the efficacy of monetary policy as well as the establishment of policy initiatives to offset any sub-optimality of monetary policy due to lack of convergence or synchronicity.

Given the fact that business cycles are to a great extent the only cycle that economists recognize in national income data, it is perhaps natural to study the synchronicity of these cycles, and yet for monetary policy the dynamic of real GDP growth at other frequencies is also important. As monetary policy usually operates at roughly a monthly level, the dynamic of real GDP growth at even quarterly frequencies has implications for the implementation of monetary policy across different countries or jurisdictions. Because synchronisation of business and growth cycles is an important consideration for operation of a single currency, monetary policy is more easily formulated if these cycles in real GDP growth (business cycle or otherwise) are similar between member states, and in particular those that qualified for and subsequently adopted the euro.

The main purpose of this paper is to illustrate how recurrence plots can be applied to an important macroeconomic issue, in order to shed light on the non-linear dynamics present in a large economic complex system. The technique of recurrence plot analysis is used to analyse both synchronisation of economic growth at all cycle frequencies. Section 2 reviews the literature on business and growth cycles in the euro area, while section 3 introduces the methodology behind recurrence plots and recurrence quantification analysis. Section 4 describes the data and the embedding parameters used in the analysis while section 5 presents the empirical results of using recurrence plots and quantification analysis as applied to euro area growth. Section 6 discusses the results and implications for euro area monetary policy, while section 7 concludes.

2 Business and growth cycles in the euro area

There has long been recognition that business cycles propagate between countries (- the main mechnanisms being trade and capital flows). The main indicator of this propagation is the synchronicity of turning points in business cycles (noted by Backus and Kehoe (1992) and Backus, Kehoe, and Kydland (1995) in the real business cycle literature) but what is not recognized is that the economic growth dynamic between these turning points (usually the recessions or peaks of business cycles) can be radically different between countries. This observation has given rise to the notion and study of growth cycles in the context of the dynamic of economic growth between these turning points (see Kontolemis (1997) and Zarnowitz and Ozyildirim (2002)). From an empirical perspective there have been some efforts to empirically extract these cycles for measurement and comparison across countries using other time-frequency techniques (see Gallegati and Gallegati (2007), Crowley and Lee (2005) and Crivellini, Gallegati, Gallegati, and Palestrini (2004)) but only limited research has been conducted in this area.

In the euro area context, there has been a recognition for some time that with closer cooperation in monetary policy, firstly under the exchange rate mechanism (ERM) of the European Monetary System (EMS) and during the run up to Economic and Monetary Union (EMU), and then secondly during the shift to the adoption of the euro within the EMU process (- using specified economic convergence criteria), that synchronisation of euro area growth rates would likely increase. But measuring this has been more problematic for a variety of reasons - notably the short data span available and the exceptional circumstances surrounding events in the early part of this last decade (9/11, Iraq invasion, German structural problems etc). Despite these issues, there has been a variety of empirical research of different types done on this topic, with notable contributions by Artis and Zhang (1997) who first recognized the existence of a separately identifiable European business cycle, followed by Artis and Zhang (1999), and then mostly studies that have tried to measure whether the "European business cycle" has become stronger since the inception of EMU and the introduction of the euro and a single monetary policy (see Altavilla (2004), Sensier, Artis, Osborn, and Birchenhall (2004), Valle e Azevedo (2002), De Haan, Inklaar, and Sleijpen (2002), Süssmuth (2002), and more recently).

This is an important issue for the ECB for a myriad of reasons, but the most important of which are hopefully elucidated here. First, optimal currency area (OCA) theory suggests that similar growth rates in member states will ease the problems associated with the differential impact of monetary policy on these countries. Second, not only do growth rates matter, but also the dynamics of growth also matters - thus the idea that similar frequency growth cycles between countries in a monetary union will also ease the problems of implementing monetary policy across a collection of member states or countries. Third, OCA theory also suggests that even without this increased synchronicity of business and growth cycles, increased mobility of factors of production can counter this and so aid implementation of monetary policy as resources can flow from one country to another to offset the differential impact of monetary policy. With the advent of the single market in the EU after 1992, labor and capital mobility have increased, but it is still widely acknowledged that language and cultural barriers impose greater barriers to mobility of factors of production than they do in many other monetary unions (such as the US or Canada). Fifth, another offset to lack of synchronisation can be found in autonomy of fiscal policy, perhaps at a national or member state level, or at the supra-national level. This has caused considerable concerns in the euro area as the Stability and Growth pact (SGP) potentially limited member state fiscal policy so as to counterbalance ECB monetary policy, and its differential impact on certain member states is dependent largely on debt levels and any existing structural budget deficit considerations (- for example Germany). Lastly, there is also a feedback effect involved, as a single monetary policy should impact all member state growth rates across the euro area implying that an OCA might be created endogenously (- see Frankel and Rose (1997)).

3 Recurrence plots and recurrence quantification analysis (RQA)

Recurrence plot analysis is now over 20 years old (see Eckmann, Oliffson Kamphorst, and Ruelle (1987) for the first contemporary application) and the quantification of these plots is much more recent (see Zbilut and Webber Jr. (1992) and Webber and Zbilut (1994)) but the notion of recurrence has a much longer pedigree in mathematics (see Feller (1950)). Recurrence plots first originated from work done in mathematics and physics but now has a considerable following in a variety of fields[1]. There are several excellent introductions available to RQA and recurrence plots, not least those by Marwan, Romano, Thiel, and Kurths (2007) and Webber Jr. and Zbilut (2005). There are very few papers that apply recurrence plot techniques to macroeconomic issues, the notable exceptions being Zbilut (2005), Kyrtsou and Vorlow (2005), and Crowley (2008) and with the exception of the latter, none of these papers look at cross recurrence or the particular issue of convergence and synchronicity.

[1] Norbert Marwan's website catalogues all the articles published using recurrence plots and RQA, and is a veritable mine of information on this topic. See http://www.recurrence-plot.tk

3.1 A simple example of auto-recurrence

To illustrate that basic ideas behind recurrence analysis, using the example from Webber Jr. and Zbilut (2005)[2] we consider the system of waves on the sea as measured from buoys. With a dataset of 226 hourly measurements of the wave heights over roughly 9.4 days, it is clear from figure 1A that tidal flows have some form of periodicity, although the movement clearly does not follow any simple deterministic function. To illustrate the basic premise behind recurrence plots and RQA, if dots are drawn to indicate whenever the wave height is at 0.9 feet, 25 points are obtained indicating each occasion that the wave height returns to 0.9 feet. By plotting these points in figure 1B for this series against itself, we obtain the "auto-recurrence" plot.

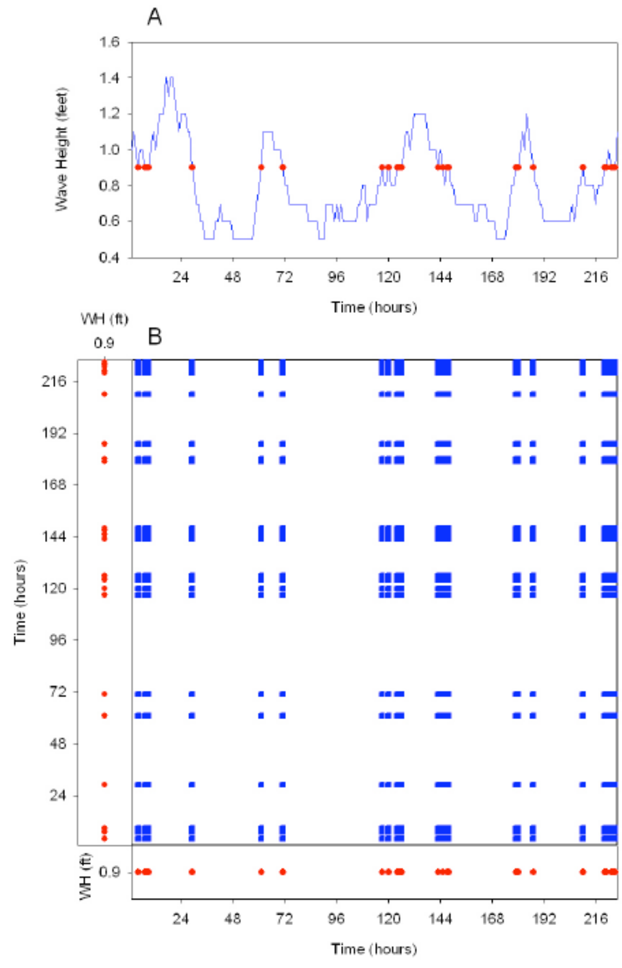


Figure 1: Wave height measurements recorded hourly from byoys located 33 nautical miles south of Islip, LI, NY, USA.

[2]This extremely useful introduction to RQA is also available from the NSF website at www.nsf.gov/sbe/bcs/pac/mnbs/mnbs.jsp

Of course, just looking at one particular point probably doesn't indicate much about the dynamics of the series, so assume that we are interested in how close to a particular wave height the wave dynamic is at any specific point in time - that is, a "local" recurrence plot. But recurrence plots can be generalized so as to visually display proximity to any particular wave height, not just one specific wave height, and these "global" recurrence plots are shown in figure 2 below, in plot C. One parameter which measures the proximity to any particular wave height is also varied so as to give a "spectrum" of how close another section of the time series is to the wave height in another section is. The parameter for detecting proximity ("radius") is increased in plot B to 0.9 feet, so the blue areas show all parts of the auto-recurrence that are within 0.9 feet of one another. Clearly the white areas of the plot indicate significant departures of over 0.9 feet from any other part of the series. In plot C, the radius parameter is varied to five different ranges: blue (0-0.2 feet), cyan (0.2-0.4 feet), green (0.4-0.6 feet), yellow (0.6-0.8 feet) and red (0.8 feet and over) - this is known as a "multi-thresholded" plot.

3.2 A simple example of cross recurrence

Cross recurrence plots were introduced into the physics literature by Marwan, Thiel, and Nowaczyk (2002) and Romano, Thiel, Kurths, Kiss, and Hudson (2005) and are a simple way of looking at recurrence between time series. From this, one can study the synchronization of non-linear time series. For example take two trigonometric series, one representing the movement of a pendulum, and the other the velocity of the pendulum. This can be represented by a sine and cosine function respectively as per equation 1 below:

$$x_t = \sin(2\pi t) \tag{1}$$

$$y_t = \cos(2\pi t) \tag{2}$$

which graphically looks as per figure 3. Clearly both x and y are not synchronous, but are phased by a quarter of a cycle.

When preparing the recurrence plot, as both series possess the same scale, a similar exercise can be used as per the auto-recurrence plot, except that we will be comparing the level of x_t against the level of y_t and then blackening the area of the recurrence plot when these levels are similar. Figure 4 shows the resulting lattice-type plot that shows when these two series are at similar levels. If the series were synchronous in time, then this would result in a dark diagonal line up the middle of the plot (- implying that at any point in time, the first series was close in value to the second series). In this instance it is clear that if lagged by a quarter cycle, the series would be synchronous.

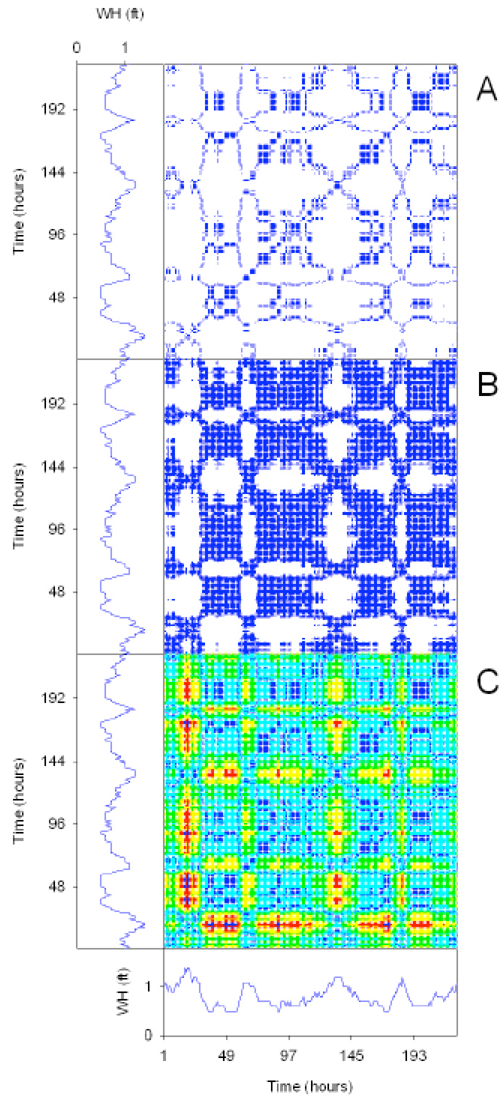


Figure 2: Recurrence plots of wave height data

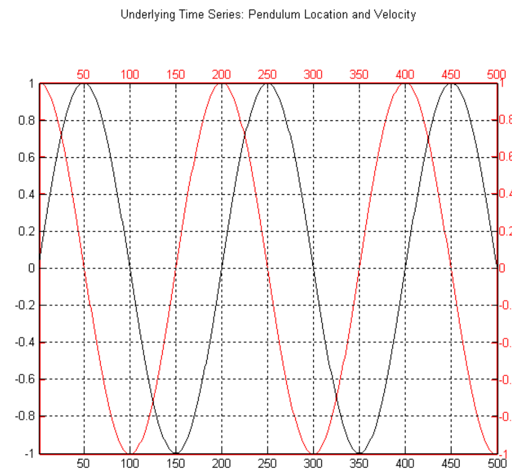


Figure 3: Pendulum location and velocity against time

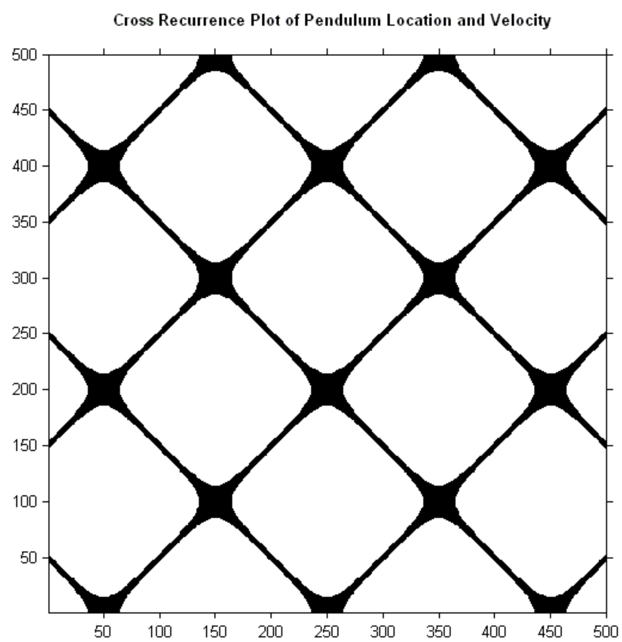


Figure 4: Cross recurrence plot of pendulum location and velocity

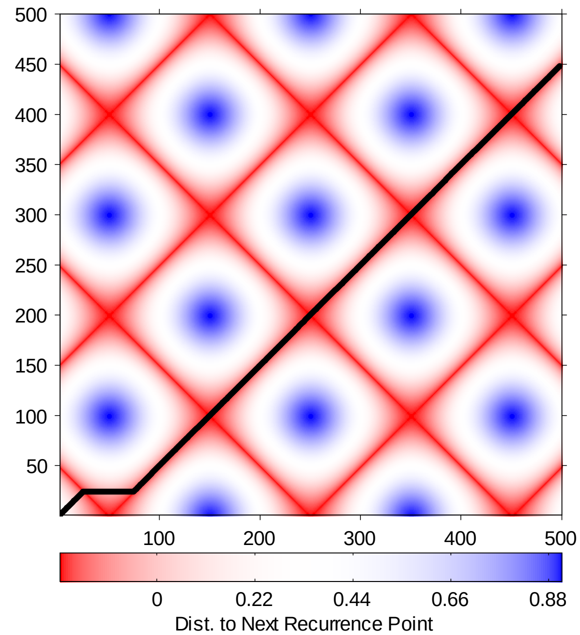


Figure 5: Unthresholded recurrence plot for pendulum with line of synchronicity

To enhance these results and to find how the series are synchronized together, the recurrence plot can first be unthresholded - in other words rather than using a specific criteria to assess whether a point is similar between the two series, to measure the distance between the points in one series and another series at any given moment in time. To best illustrate this, figure 5 shows red for points which are very similar between the series, blue for points that are very distant between the two series, and white for all points in between. Clearly these periodic series are completely deterministic and yield a beautiful lattice pattern reminiscent of the patterns created by looking into a child's toy kaleidoscope from the mid twentieth century. A line of synchronicity can be estimated for the data and this is plotted using a thick black line. As can easily be seen, the line of synchronicity shows that a half sample delay in the first (sine) series will be consistent with synchronicity between the two series.

The start of the line of synchronicity is at the origin, but this is just a default starting point - the program used for this analysis could start at (50,0) instead, which would yield a straight line for the line of synchronicity. Obviously the results above are for deterministic systems, and when systems are non-linear and stochastic, these neat results will no longer appear. Nevertheless, in applications in the physical sciences and elsewhere, recurrence plots are able to reveal interesting features of dynamical systems, so clearly they possess potential to reveal interesting features of economic time series.

3.3 Mathematical background

Pretty pictures are all well and good, but analysing these plots for what they contain is now a major part of recurrence analysis, in the form of RQA. RQA was initiated by Zbilut and Webber Jr. (1992) and has now been introduced into mainstream physics through the study of nonlinear dynamics. A good summary is available in Webber Jr. and Zbilut (2005). As our focus is on cross recurrence plots, following Marwan and Kurths (2002), the emphasis is on measures for cross RQA.

First, using Takens' embedding theorem (see Takens (1981)), the recurrence plot is a way of analysing the dynamics of phase space trajectories in deterministic systems. Takens' embedding theorem states that the dynamics can be approximated from a time series x_k sampled every t by using an embedding dimension m, and a time delay, τ, by a reconstruction of the phase-space trajectory $\overrightarrow{y}_t$, where:

$$\overrightarrow{y}_t = (x_t, x_{t+\tau}, ..., x_{t+(m-1)\tau}) \tag{3}$$

The choice of m and τ are based on methods for approximating these parameters, such as the method of false nearest neighbors and mutual information for m and τ respectively. When using cross recurrence plots, the choice of m and τ are assumed to be the same.

Second, following Marwan, Thiel, and Nowaczyk (2002) the cross recurrence plot is definied by:

$$\mathbf{CR}_{i,j} = \Theta(\varepsilon - \|y_i - z_j\|) \tag{4}$$

where $i, j = 1, ..., N$, y_i and z_i are two embedded series, ε is the predefined "threshold", $\|\|$.is the norm (for example a Euclidean norm) and Θ is the Heaviside function. This gives a cross recurrence matrix $\mathbf{CR}_{i,j}$ which contains either 0s (the white areas in the plots) or 1s.(the black areas in the plots). To get the contoured plots shown above, ε is varied to predetermined values.

Third, in an auto-recurrence plot, the main diagonal is always present, as every point in the series is identical to the same point in the series, so there will always be a diagonal line (1's down the main diagonal of the $\mathbf{R}_{i,j}$ matrix), once all points in the series are considered (- see figure 2 for example). In the cross recurrence plot if this line is present, the two series are identical, but this is obviously a special case. A line, if it appears in the cross-recurrence plot, implies similar dynamics, but these maybe offset from the main diagonal, implying phasing of the two cycles (-

see figure 5 for example)..This line, if it can be identified, is termed the "line of synchronization" or LOS.

Fourth, complexity measures can be derived to characterize the cross-dynamics of a given series. For two series these will be characterized as diagonal lines (not necessarily on the main diagonal), which demonstrate similar dynamics maybe at different points in time. Following Marwan and Kurths (2002) the distributions of the diagonal line lengths can be written as $P_t(l)$ for each diagonal parallel to the main diagonal, where $t = 0$ denotes the main diagonal, $t > 0$ denotes diagonals above the main diagonal (a lead) and $t < 0$ denotes diagonals below the main diagonal (a lagged dynamic).

3.4 Cross recurrence quantification analysis (CRQA)

Given that we are interested in cross recurrence analysis, the following measures can be extracted from the cross recurrence plot:

i) the recurrence rate

$$PRET(t) = \frac{1}{N-t} \sum_{l=1}^{N-t} l P_t(l) \tag{5}$$

which represents the probability of similar dynamics occurring between the series with delay t.

ii) a determinism measure

$$PDET(t) = \frac{\sum_{l=l_{\min}}^{N-t} l P_t(l)}{\sum_{l=1}^{N-t} l P_t(l)} \tag{6}$$

which represents the proportion of long sequences of dynamics between the series in all similar dynamics. A deterministic system will have a high $PDET$ while a stochastic system will have a low $PDET$.

iii) maximum diagonal line length, $max(L(t))$, which gives the maximum length of time that similar dynamics occur.

iv) average diagonal line length, $MeanLL(t)$:

$$MeanLL(t) = \frac{\sum_{l=l_{\min}}^{N-t} l P_t(l)}{\sum_{l=l_{\min}}^{N-t} P_t(l)} \tag{7}$$

which shows the average duration of the similar dynamics in the two series.

v) an entropy measure which refers to the Shannon entropy of the probability $p(l)$, and is defined as:

$$ENTR = \left| - \sum_{l=l_{\min}}^{N} p(l) \ln p(l) \right| \quad (8)$$

This is a measure of the complexity of the recurrence plot.

vi) laminarity, which is defined as:

$$LAM = \frac{\sum_{v=v_{\min}}^{N-t} vP_t(v)}{\sum_{v=1}^{N-t} vP_t(v)} \quad (9)$$

This is a measure of tangential motion, and refers to the distributions of the vertical line lengths in the recurrence plot, which can be written as $P_t(v)$, analogous to the diagonal lines in the plot. where high values of LAM denote motions that are not opposite in direction of trajectory but are not similar in direction either.

vii) trapping time, which is defined as:

$$TT = \frac{\sum_{v=v_{\min}}^{N-t} vP_t(v)}{\sum_{v=v_{\min}}^{N-t} P_t(v)} \quad (10)$$

This refers to the average length of these vertical line segments and is analogous to $L(t)$ above.

viii) maximum vertical line, $vMax$.

High values of $PRET(t)$ indicate a high probability of occurence of the same state in both systems, and high values of $PDET(t)$ and $L(t)$ indicate a long time span for these common dynamics.

4 Data and embedding

4.1 Data

Given that real GDP is non-stationary, quarterly real GDP data transformed into log quarter over quarter changes was used. The data was sourced from the respective country's national statistical office, while the euro area aggregate was obtained from the ECB's euro area quarterly model database[3] and updated using Eurostat. The transformed data runs from 1971 quarter 2 (1971Q2) through 2009 quarter 1 (2009Q1) giving 156 datapoints.

[3] This data can be obtained from the Euro Area Business Cycle Network at www.eabcn.org

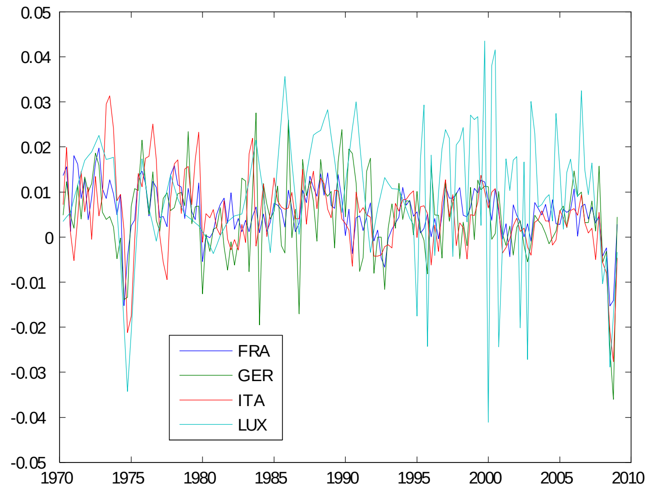
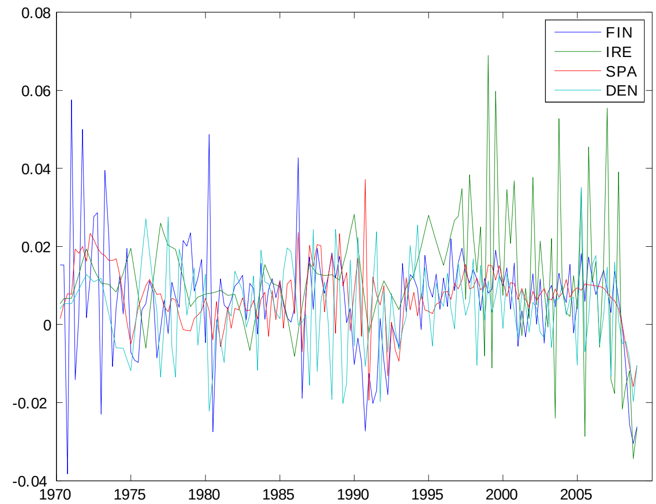


Figure 6: Selected core euro area member state growth rates (quarterly log change in real GDP)

Figure 6 shows the data for selected core euro area member states - then peripheral euro area and non-euro area European Union member state growth rates are shown in figure **??**. Figure 7 also shows growth rates for other non-euro area EU member states and other European countries while figure 8 shows the euro area aggregate growth rate together with the US growth rate. There are 12 countries included in the dataset, plus the euro area aggregate.

With the exception of Luxembourg and Ireland, the growth rates of the euro area core countries are very similar and since the inception of EMU in the early 1990s appear to be even less dispersed. With the peripheral euro area members the dispersion is much greater although since the early 1990s (coincident with the establishment of convergence criteria for membership of EMU) the growth rates have become more tightly bunched. In economic terms the dynamics of economic growth in European Union (EU) countries is going to be affected according to the time period under consideration - the pre-EMU exchange rate mechanisms ("snake", early EMS, "new" EMS) and the situation post-ERM crisis in 1993 through to full-blown euro area in 1999 can all be thought of as different institutional regimes. Lastly there are significant similarities and differences between euro area growth cycles and those of the US - sometimes recessions are synchronised (for example in 1974 because of the common oil price shock) but in other instances recessions seem not to be phase synchronised (for example in the early 1990s). It is notable is that all countries have been affected in the most recent downturn - none of the countries under consideration here have avoided contraction.



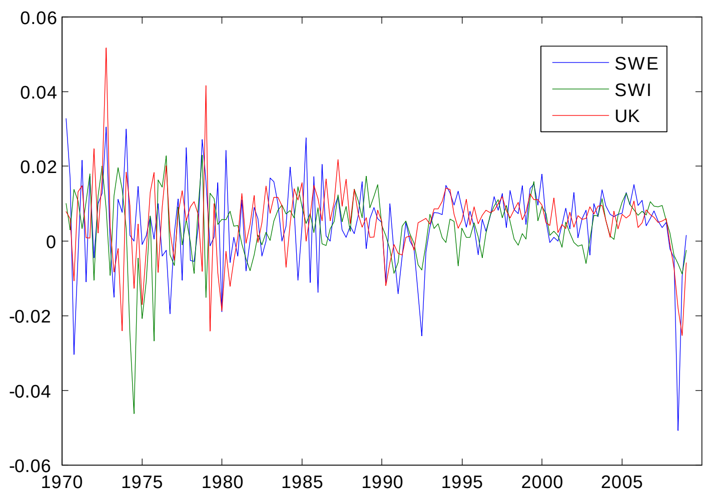


Figure 7: Swedish, Swiss and UK growth rates (quarterly log change in real GDP)

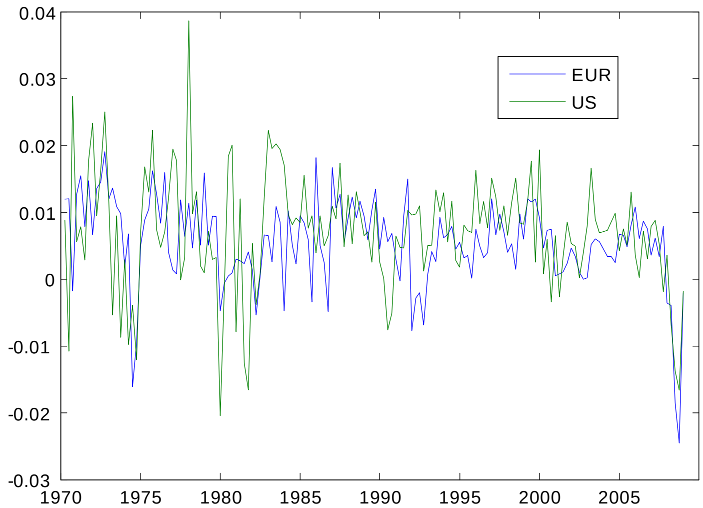


Figure 8: Euro-area aggregate and US growth rates (quarterly log change in real GDP)

4.2 Data embedding

Two parameters need to be determined for recurrence plot analysis, namely m and τ, the embedding and time delay parameters[4]. Kyrtsou and Vorlow (2005) set these two parameters to be 1 in all their recurrence plots, following Iwanski and Bradley (1998) who suggests that experimental data does not need to be embedded. In terms of setting the radius parameter, ε, they follow Webber and Zbilut (1994) and set a threshold level of the lower 10% of the maximum distance between the embedded vectors. As Kyrtsou and Vorlow (2005) analysed only consumer prices, industrial production, interest rates and unemployment, more traditional methods for setting embedding and time delay parameters are used here. Embedding parameters are determined by the method of false nearest neighbors and the delay by the mutual information, and the results displayed in table 1 for the euro area member states and table 2 for non-euro area countries and the euro area aggregate.

	DEN	FIN	FRA	GER	IRE	ITA	LUX	SPA
m	4	5	4	4	4	4	6	4
τ	1	1	1	1	1	1	1	1

Table 1: Embedding and time delay parameters for Euro area countries

Given the results above, an embedding parameter of 4 with a time delay of 1 is used in all the

[4] All the empirical research done in this paper was done using Norbert Marwan's CRP MATLAB toolbox.

	SWE	SWI	UK	EU	US
m	3	4	5	4	4
τ	1	1	1	1	1

Table 2: Embedding and time delay parameters for non euro area countries

analysis that follows[5].

5 Cross recurrence analysis for EU member states

In the euro area, as there is a single monetary policy, there are two desirable features in economic growth dynamics:

a) phase synchronicity - this indicates that monetary policy is acting on business cycles that are similarly phased and in the recurrence plot context is measured by the length of diagonal lines at or near the leading diagonal. Given two variables, x_t and y_t, then in mathematical terms:

$$\varphi(x_t) - \varphi(y_t) \leq \pm\omega \tag{11}$$

where φ is a phase function and ω represents a critical phase shift. In recurrence plots, this will be indicated by lines running up the leading diagonal of the plot. Two completely phase synchronous series would appear as a single line up the leading diagonal of the cross recurrence plot. Note that the unthresholded plots are going to be best at indicating this, as they can detect phase synchronous dynamics even when growth rates are at different levels (- that is, have not converged);

b) convergence - this indicates that the nominal interest rates implicit in monetary policy are having a similar impact on economic growth (through the transmission mechanism of monetary policy) across the euro area. In a recurrence quantification context this is indicated by the recurrence measure. In mathematical terms:

$$d(x - y) \leq \varepsilon; \forall t \tag{12}$$

where d is the distance function and ε is some critical distance. Convergence shows up in recurrence plots as dynamics that occur to the same series within some critical distance, and is indicated in the unthresholded plots by red lines (given the colour scheme chosen for the plots) and in the thresholded plots by black points.

[5] Clearly m differs between some of the countries but as the euro area measure has an embedding parameter of 4 and this paramter has to be the same for both variables under consideration when using cross-recurrence analysis, for ease of exposition and cross-country comparisons $m = 4$ is used for all variables.

The degree of phase synchronicity and convergence within the core of the euro area is first analysed, and then those in the periphery of the euro area (which have smaller weights in the euro area aggregate - for example Ireland and Finland) are analysed. Lastly for those outside of the euro area (such as the UK), the objective is to see if synchronicity and convergence have increased or decreased. As both convergence and synchronicity are relevant, both thresholded and non-thresholded plot are used in what follows.

5.1 Core euro area member states

5.1.1 France

With France, we first use an unthresholded plot with a jet scale in figure 9 to show overall distance between the two series through time. The jet scale below the plot shows that if the distance is small, red points should appear in the plot, whereas for increasing distances between points in the two series, the colors change through until large distances are in violet and then border on red again. It is clear that from around 1993 growth rates have been quite similar between the two series (shown by the yellow/red box in the upper right corner) with a clear red diagonal line appearing through most of the box and beyond. In fact there is a small box in the upper right corner of the yellow/red box that is almost all red (from around 2003 onwards), indicating that all growth rates have been similar over this period. It is also noteworthy that in the early period under consideration there were periods when there were constant gaps between growth rates as evidenced by the short blue diagonal lines (mainly in the early 1970s) situated on the off-diagonals.

In figure 10 we calibrate the plot for a particular level of convergence by setting a certain threshold (here one standard deviation), and then repeating the plot. The black points and areas represent points that satisfy the threshold and white areas do not (as per figure 1). It is clear that there are periods of synchronicity down the leading diagonal, but these are not consistent, although they are now more closely bunched together. To analyse this phenomenon of "intermittent synchronicity" we now turn to CRQA.

The cross recurrence quantification is shown in figure 11 where a window of 8 periods long (2 years) is passed through the thresholded cross recurrence plot to accomplish an "epoch" analysis, with surrogate analysis[6] being used with 100 repetitions to provide both 90 and 95 percent significance levels (- the area between these levels is blocked in yellow so that they can be distinguished from the CRQ measures). The recurrence rate appears to have increased since 1992 and has been continuously significant since, but fluctuates with highs in 1989, 1996 and 2005. The determinism

[6] Surrogate analysis is a form of bootstrapping where the frequency domain properties of the time series are left unaltered.

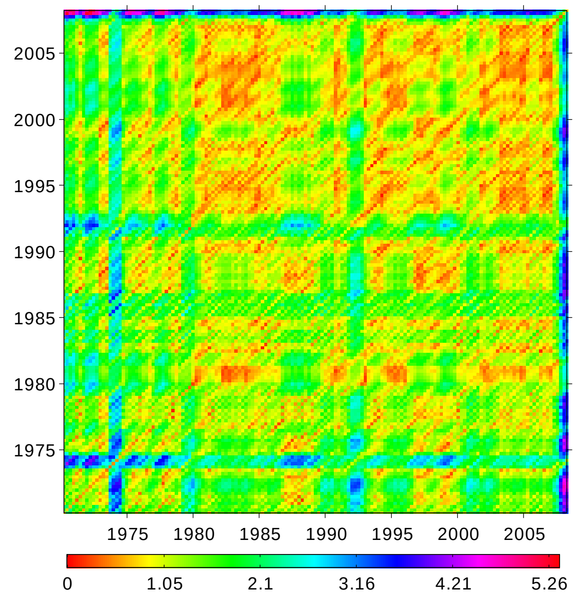


Figure 9: Unthresholded cross recurrence plot for France vs euro area growth rates.

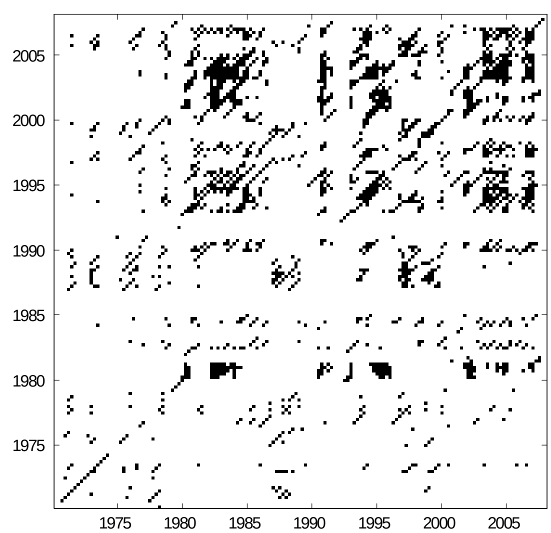


Figure 10: Thresholded cross recurrence plot for French vs euro area growth rates (ε=σ).

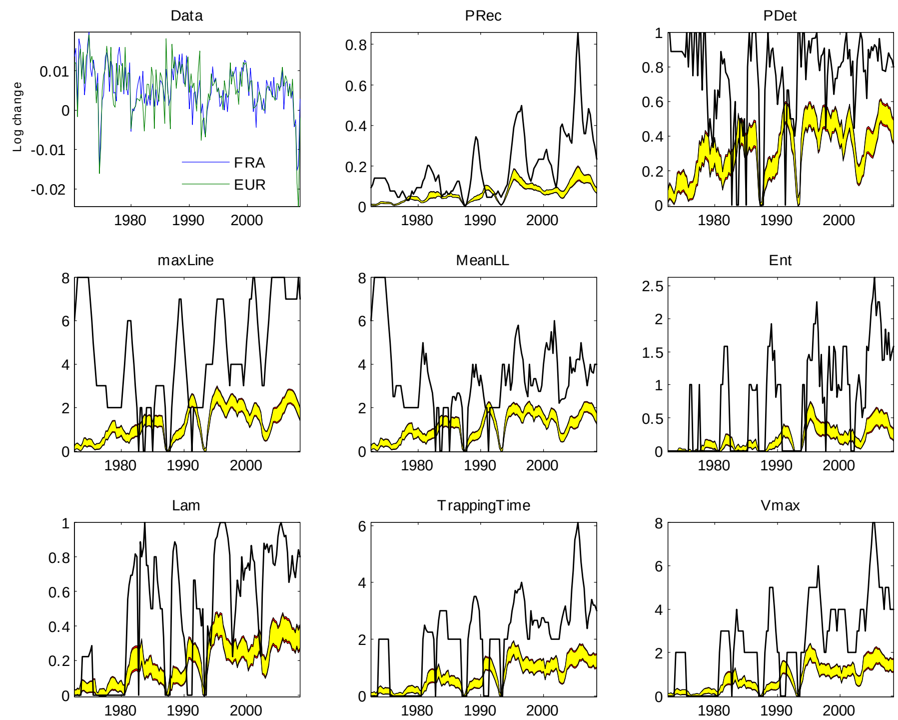


Figure 11: CRQA for French vs euro area growth.

measure (PDet), maximum diagonal line length (maxLine)[7] and the mean diagonal line length (MeanLL) have also been significant since between 1992, but the Entropy measure has only been significant periodically. Maximum diagonal line length (maxLine) peaked in around 2005, while mean line length peaked in around 2001, both after the introduction of the euro in 1999. Since 1992, apart from a short period in around 2002 laminarity (Lam) and trapping time were significant since around 1993, perhaps indicating that business cycle fluctuations have been dampened over the last cycle.

5.1.2 Germany

Figure 12 shows the unthresholded plot for German growth. It shows quite clearly that phase convergence weakened between 1987 and 1991, but interestingly German economic growth did not depart by a large amount from euro area economic growth levels for long during German reunification. The period of German reunification is clearly delineated in the plot, with largely

[7]Note that maximum line lengths can never exceed 8 as the width of the window used with the epoch analysis is 8x8 in dimension.

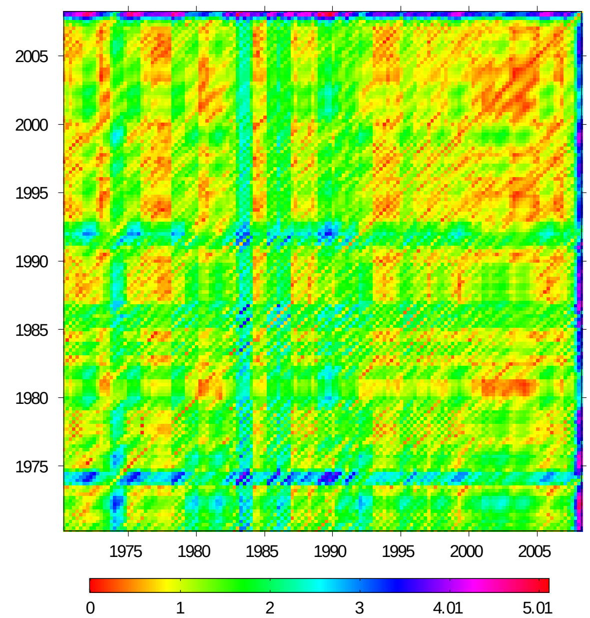


Figure 12: Unthresholded cross recurrence plot for German vs euro area growth rates.

green rather than yellow or red areas dominating from 1990 to 1993. In more recent years, it is clear that there is a "red square" showing high convergence with laminar regions from 1999 to around 2005, but after this time another different pattern appears. Figure 13, the thresholded version of the recurrence plot, clearly shows that the effects of reunification fades in 1993 and then economic convergence fairly consistently occurs. Similarly to French GDP growth, German GDP growth also becomes more laminar after 1999.

The thresholded plot also tells a slightly different story about when synchronization consistently occurs with the euro area, as the diagonal only becomes consistently black about half way through 1998, and then it appears to be lagging that of the euro area, but from that point onwards it has been continuous.

The plot in figure 14 shows the German CRQA against the euro area aggregate. The recurrence rate is significant during most of the 1970s and up until 1985, and then recurrence only recovers again in 1993, but then collapses in roughly 1996 through 1998, but after this point recurrence becomes highly significant. Determinism also follows a similar pattern (although it doesn't seem to fall quite as rapidly as the recurrence rate does), as does the maximum diagonal line (maxLine) and mean diagonal line length (MeanLL). The entropy measure (Ent) is somewhat different, and is not significant between roughly 1975 and 1978, and then is not significant from around 1983

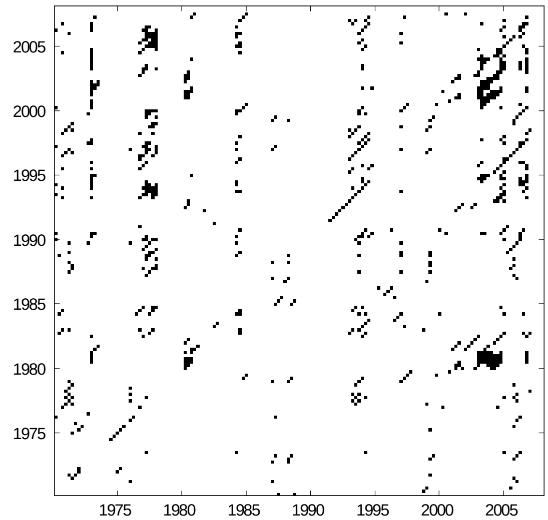


Figure 13: Thresholded cross recurrence plot for German vs euro area growth rates (ε=σ).

through 1993, after which it follows a similar pattern to the other measures already mentioned. The laminar, trapping time and maximum vertical line measures on the bottom row all show a similar pattern, perhaps most importantly staying significant after 1999.

5.1.3 Italy

In figure 15 we see the unthresholded cross recurrence plot for Italian vs euro area growth. Here there are clearly parallel diagonal lines on and on both sides of the leading diagonal running up the plot, with some of these showing convergence and others with synchronized activity but not such close convergence (- yellow diagonal lines can clearly be seen in the mid-1980s and early 1990s, for example).The laminar "red box" appears in around 2000 for Italy, and there is a clear red leading diagonal line from around 1996 onwards, indicating convergence in growth rates with the euro area aggregate. There are clearly quite large gaps in the years governed by the ERM of the EMS and notable gaps in 1990-1992 and 1995.

In the unthresholded plot in figure 16, a clearer picture of convergence in growth rates between Italy and the euro area emerges. In most of the 1970s and 1980s there was little convergence, but then in the 1990s we see some convergence in 1990 and then again through 1993 to 1995. Interestingly in the late 1980s convergent dynamics appear to occur with Italy leading the rest of the euro area, but this dissolves in 1990 with German reunification and then the collapse of the ERM of the EMS. From around 1994 there is very little indication of any convergence with

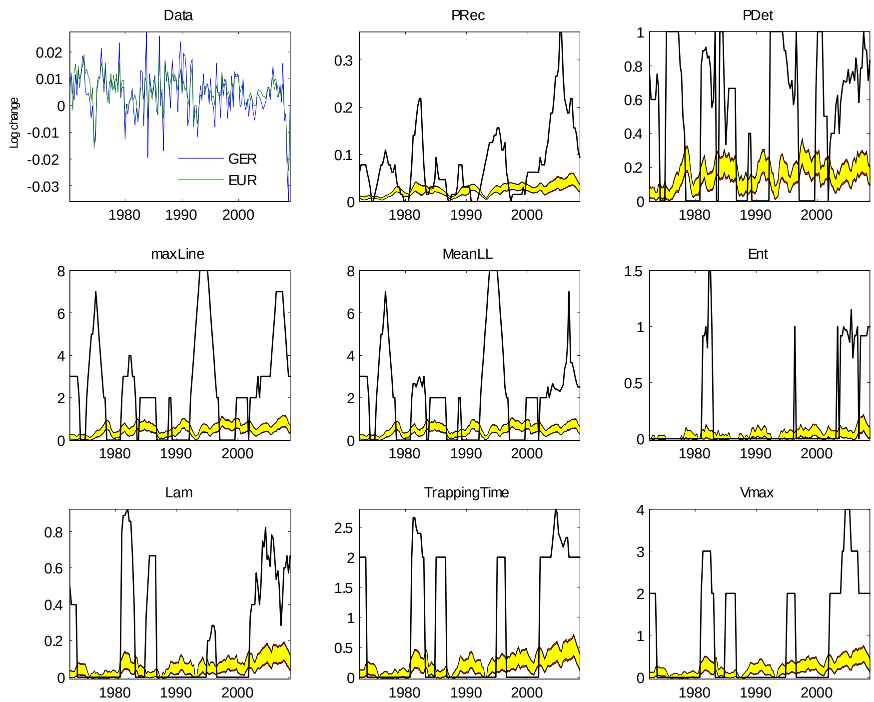


Figure 14: CRQA for German vs euro area growth.

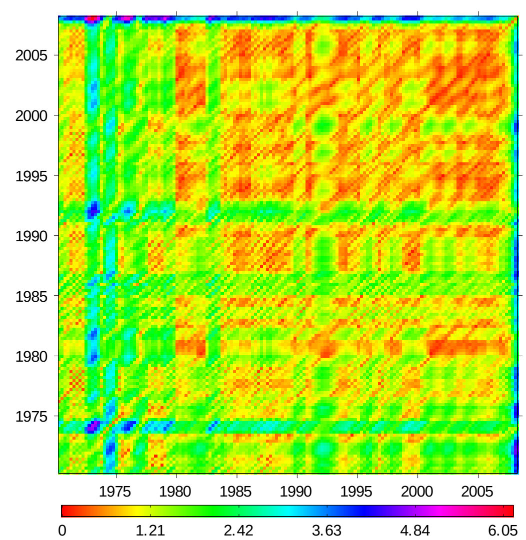


Figure 15: Unthresholded cross recurrence plot for Italian vs euro area growth rates.

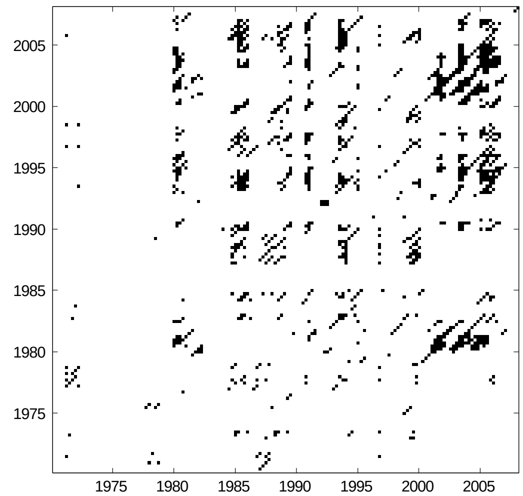


Figure 16: Thresholded cross recurrence plot for Italian vs euro area growth rates (ε=σ).

the euro area aggregate. Only in 1999 does convergence really occur and then it occurs with a lag, which appears to correct itself in about 2003. Since 2006 there appears to be little consistent convergence in growth rates, but interestingly the last point in the plot (upper right hand corner) is a convergent, which is likely due to the common growth dynamics of euro area member states during the current economic downturn and emergent recovery.

Figure 17 shows the cross recurrence quantification analysis for Italian real GDP growth. Here there are no significant recurrence rates recorded in the 1970s, significant rates recorded between 1982 and 85, then a short period in the late 1980s followed by significant recurrence rates from around 1989 through to around 1997, and then highly significant recurrence from then on until recently, when the recurrence rate appears to be dropping precipitously. Similar patters are once again observed for the determinism measure, (PDet), the maximum line length (maxLine), mean line length (MeanLL) , entropy and the vertical line measures.

5.1.4 Luxembourg

The unthresholded recurrence plot for Luxembourg is shown in figure 18 The Luxembourg growth rate has clearly been quite different from the euro area growth rate for much of the period under scrutiny, although here the issue appears to be different levels of convergence at different points in time. The vertical bands throughout the plot appear to be a direct manifestation of this, with clearly very different growth rates in 1974-75, 1994-96 and 1999-2001. Looking closely at the plot though, there are lines running across these bands, for example between 1994 and 1996 when

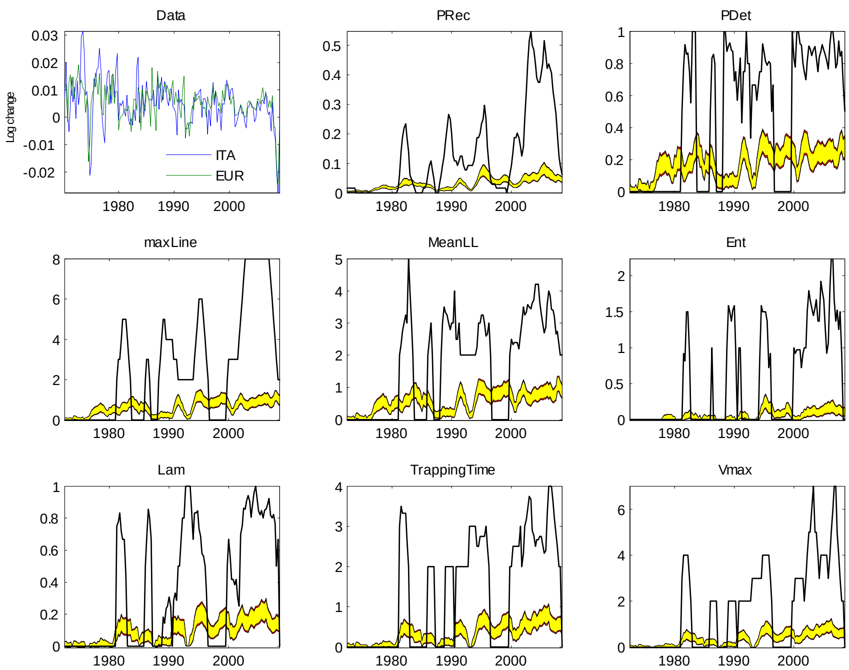


Figure 17: CRQA for Italian vs euro area growth.

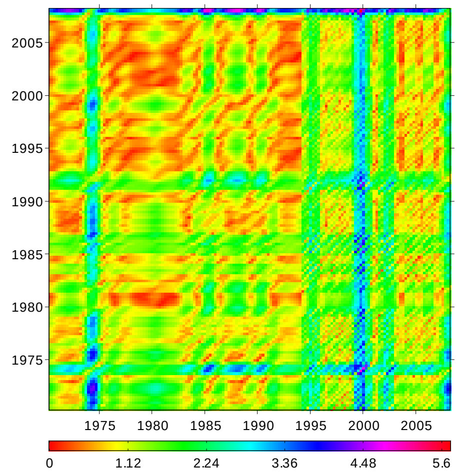


Figure 18: Unthresholded cross recurrence plot for Luxembourg vs euro area growth rates.

there is clearly a red leading diagonal line, indicating synchronicity and convergence did occur even when growth rates were very different from other periods. If anything in the first part of the plot there appears to be some curvy synchronized diagonal bands (from around 1974 to 1984) which then reoccurs from about 1986 to 1992 - unfortunately these curvy lines are not picked up using this methodology - only straight diagonal lines are. Only from around the middle of 2002 does any consistent convergence occur. Here we omit the thresholded plot, as it gives no useful additional information compared to the unthresholded plot.

Figure 19 shows the quantification analysis for Luxembourg. It shows that in the late 1970s followed almost immediately in the early 1980s there was a highly significant rate of recurrence and then nothing until short period in the early 1990s. Then in the mid 1990s the recurrence rate was significant again, and then in the euro period only from around 2004 to 2006 was recurrence significant. Interestingly determinism wasn't significant in the 1970s, but was highly significant in the other periods. This pattern follows for the maxLine and MeanLL measures, and the entropy measures are only significant in the early 1980s and the mid-1990s. The vertical line measures match the recurrence significance patterns.

5.2 Peripheral euro area member states

Now we turn to the case of peripheral euro area member states in our sample, namely Finland, Ireland and Spain..

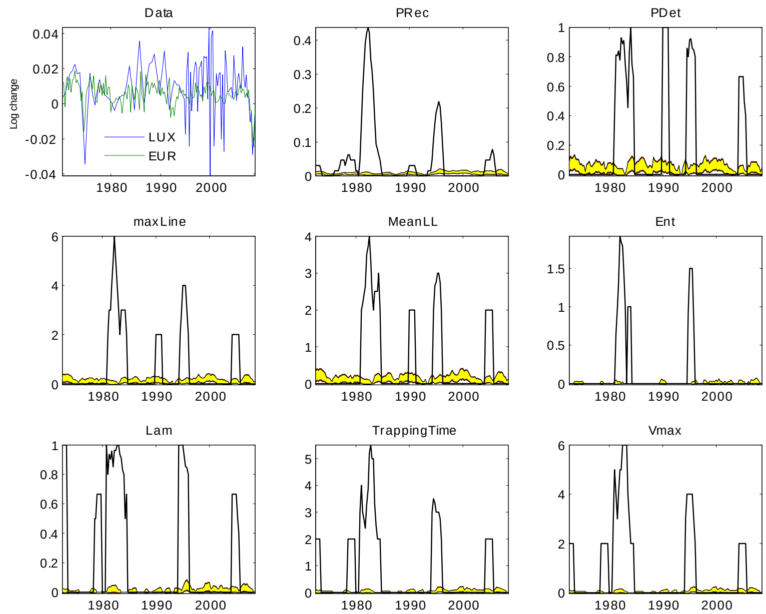


Figure 19: CRQA for Luxembourg vs euro area growth.

5.2.1 Finland

In this instance for the thresholded plots as well as the quantification analysis we widen out the threshold to 1.5 times the standard deviation of the euro growth rate. For Finland, in figure 20 the major recession in the early 1990s is clearly notable in terms of the blue/green vertical band across the plot. The most recent data used in this study appears to suggest that these was a divergence in growth rates (in around 2001-02) after a period of increased synchronisation in the early years of EMU. Also it is notable that in the late 1990s if anything Finland appears to lead the euro area in terms of its growth rate as the red diagonal line sags below the leading diagonal from around 1996 until around 2000.

In figure 21 the thresholded plot is shown and illustrates when convergence took place - clearly convergence was sporadic with only brief episodes, notable periods being 1997-1999 (- as stated above with a slight lead over the euro area) and 2006-2008.

Lastly, the thresholded plot gives rise to the quanitifcation analysis shown in figure 22. Although significant at 90% levels in the 1970s, only in the early 1980s did recurrence become significant until about 1988, and then briefly again in 1989. Around the time of the launch of the euro in 1999, recurrence clearly becomes highly significant again until about 2003, and then although still significant (- it lies within the yellow band), it once again becomes highly signifi-

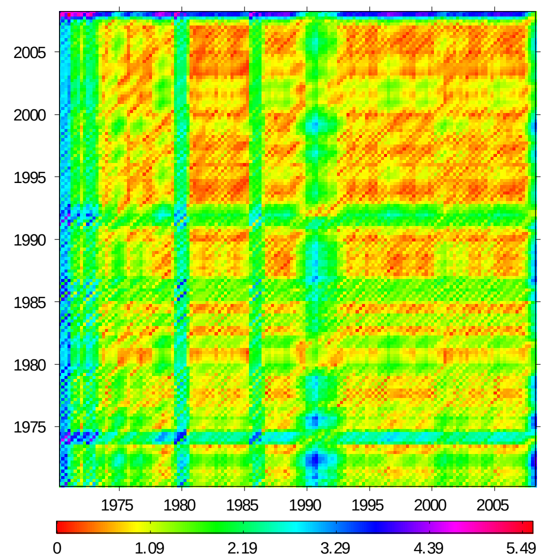


Figure 20: Unthresholded cross recurrence plot for Finnish vs euro area growth rates.

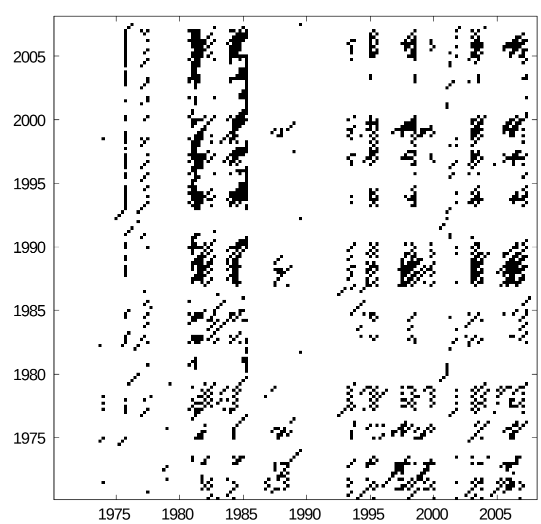


Figure 21: Thresholded cross recurrence plot for Finnish vs euro area growth rates (ε=1.5σ).

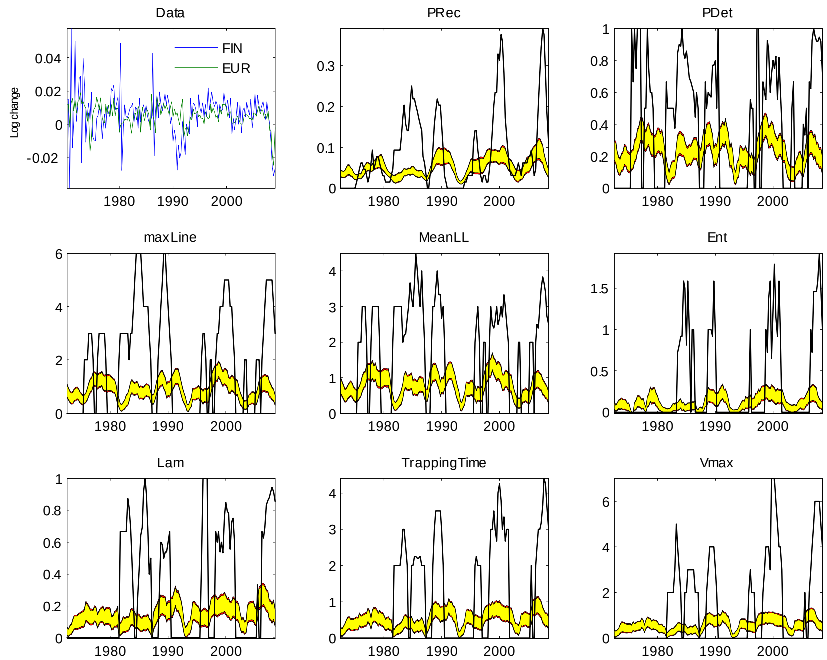


Figure 22: CRQA for Finnish vs euro area growth.

cant from about 2007 onwards. Determinism follows similar patterns although the 1970s period is more significant than with the recurrence measure, and this also goes for maxLine and MeanLL. The vertical line measures in the bottom row appear to follow somewhat different patterns in the 1980s with three different periods of significant values, reflecting the small off-diagonal laminar sequences evident in the thresholded plot.

5.2.2 Ireland

Ireland is an interesting case, as this has likely been one of the least convergent of the euro area member states. This shows up clearly in the unthresholded recurrence plot in figure 23, where there are lines of synchronicity along the leading diagonal particularly in the early 1980s, and early 1990s, but there is little sign of synchronous dynamics after the inception of the euro apart from two brief episodes in 2004 and 2006. In this instance no threshold plot is shown as this adds little to the analysis.

This is also reflected not only in the series for Ireland, which becomes very volatile in the late 1990s (- maybe because of a change in the method of data compilation), but as figure 24 shows, convergence fell in the mid-1990s and only increased again to significant levels for a short period

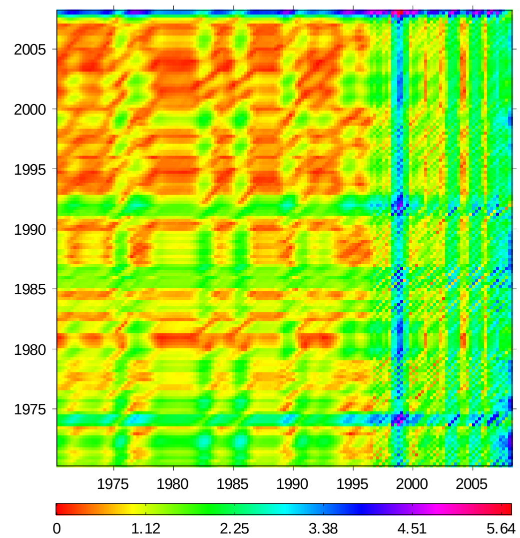


Figure 23: Unthresholded cross recurrence plot for Irish vs euro area growth rates.

of time after 2005. In the earlier period though convergence was high in the early 1970s and then again in the late 1980s. This pattern is also followed for most of the CRQA variables, with the exception of entropy, where there were no significant values in the late 2000s.

5.2.3 Spain

On the other hand, Spanish real GDP growth has been relatively synchronous with euro area real GDP growth from 1993 through until 2009. Figure 25 shows the unthresholded cross recurrence plot and shows that a definite red/yellow square in the upper right hand part of the plot indicates much closer convergence with a leading digonal through the first part of the square and the last part of the square. In between the leading diagonal diverges indicating a short lead in Spanish growth rates up until around 2001 followed by a laminar area from around 2004 onwards. The early part of the 1980s shows some convergence.By looking at the thresholded plot in figure 26 the pattern of synchronization and convergence is revealed in more detail. Clearly there was a period of lagged synchronicity in the 1980s, and then very little convergence from 1985 through 1992, after which there is a definite synchronous pattern from 1992 to 1997. In the post-euro area inception period there is a short period of synchronous dynamics in 2001 and then from around 2004 onwards.

In figure 27 the probability of recurrence measure (PRec) essentially describes the convergence of the Spanish growth rate with the euro area growth rate, but synchronicity within the established level of convergence is shown in the max and mean line length measure (MaxLine and MeanLL).

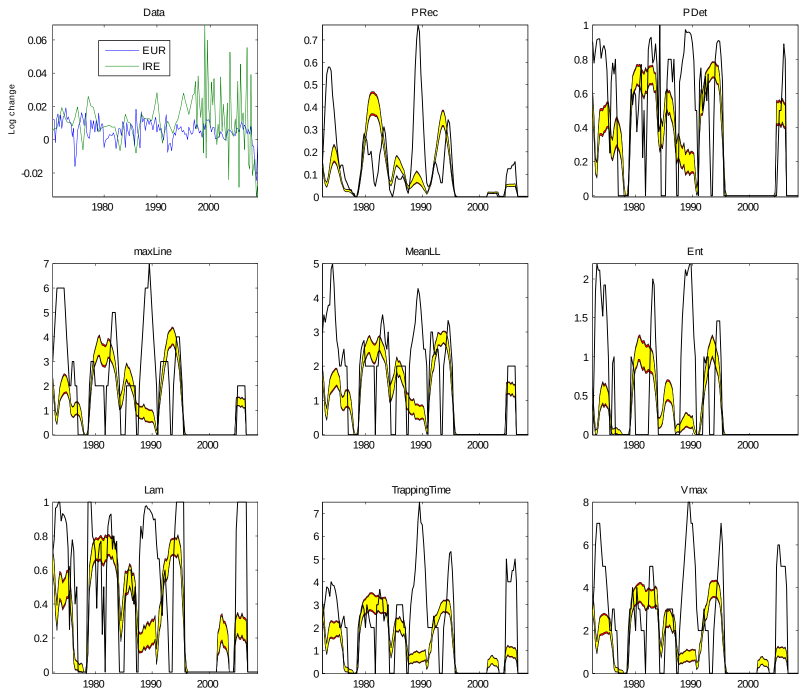


Figure 24: CRQA for Irish vs euro area growth.

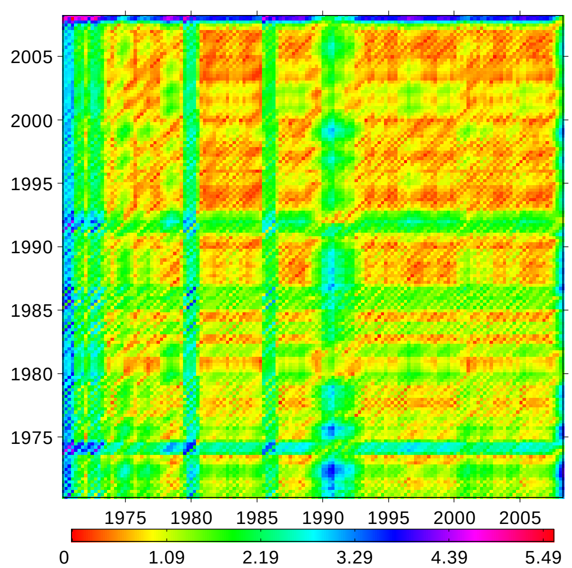


Figure 25: Unthresholded cross recurrence plot for Spanish vs euro area growth rates.

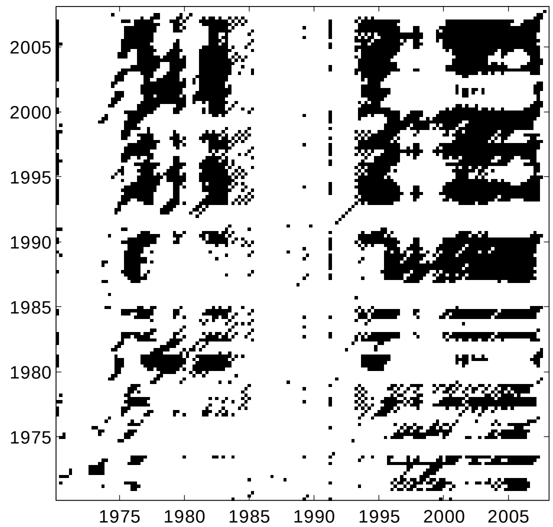


Figure 26: Thresholded cross recurrence plot for Spanish vs euro area growth rates (ε=1.5σ).

Clearly synchronicity has remained significant since the early 1990s although with a brief period in the early 2000s when it became insignificant. Most of the other measures follow a similar pattern, although laminar regions are clearly detected during this period as shown by the Lam measure and also the Vmax measure.

5.3 Non-euro-area EU member states

5.3.1 Denmark

The Danish cross recurrence plots show distinctly different patterns compared with the majority of the euro area member states. Here in the unthresholded recurrence plot in figure 28 there are clear synchronous lines running diagonally up the recurrence plot, although at times these lines clearly show little convergence. On the other hand there is clearly convergence from around 1993 through until 2004, but after this time the two series clearly diverge.

When we look at the thresholded plot in figure 29, which detects both synchronous and converged dynamics, it is clear that only during the early 1970s, and from 1999 to 2003 (where there is a slight lag in dynamics) was there both convergence and synchronicity. This suggests that although Denmark's growth rate was quite synchronous with the euro area rate, it was rarely both synchronous and converged.

In the CRQA plots shown in figure 30 the observations noted above are clearly reflected in the recurrence measures, with only signifcant recurrence from the early 2000s onwards until around

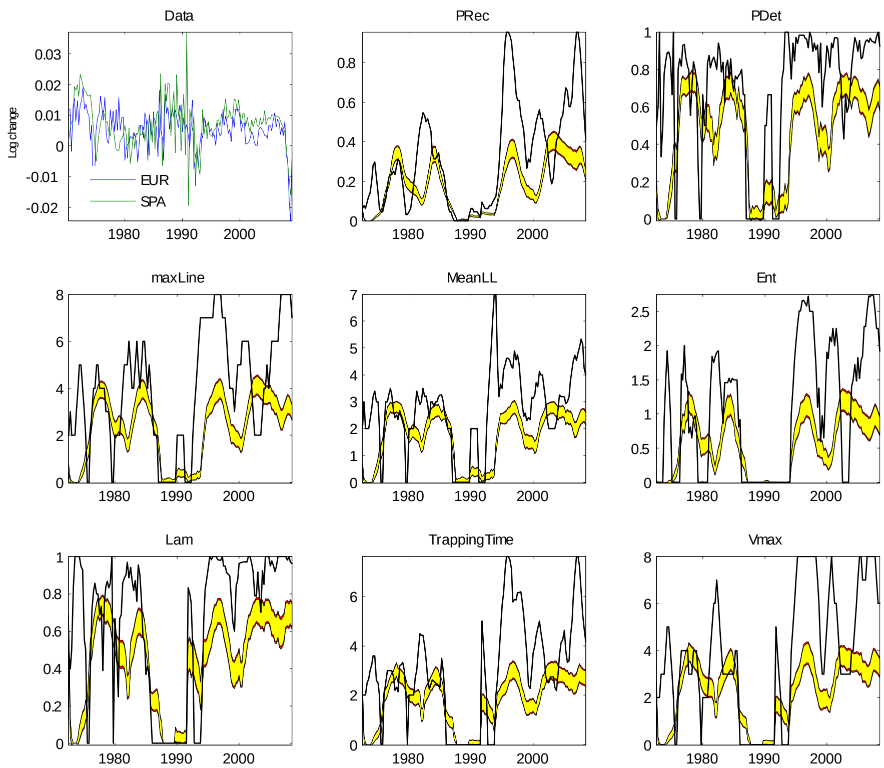


Figure 27: CRQA for Spanish vs euro area growth.

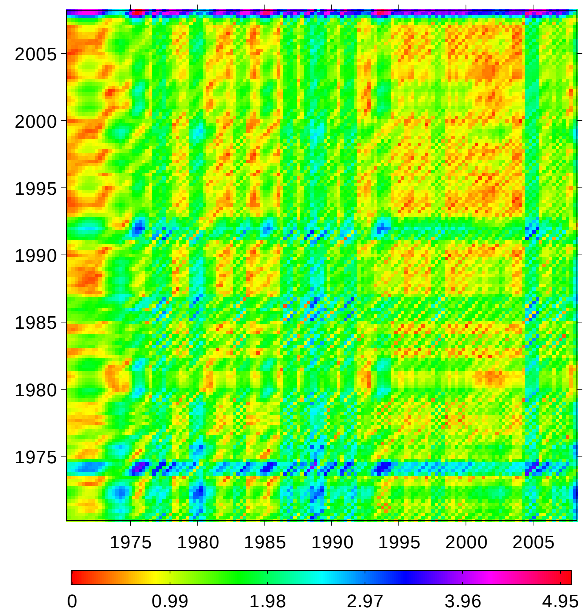


Figure 28: Unthresholded cross recurrence plot for Danish vs euro area growth rates.

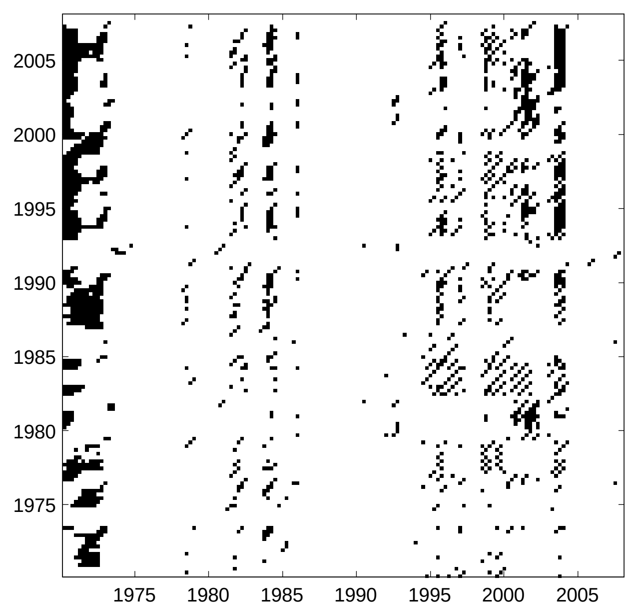


Figure 29: Thresholded cross recurrence plot for Danish vs euro area growth rates (ε=1.5σ).

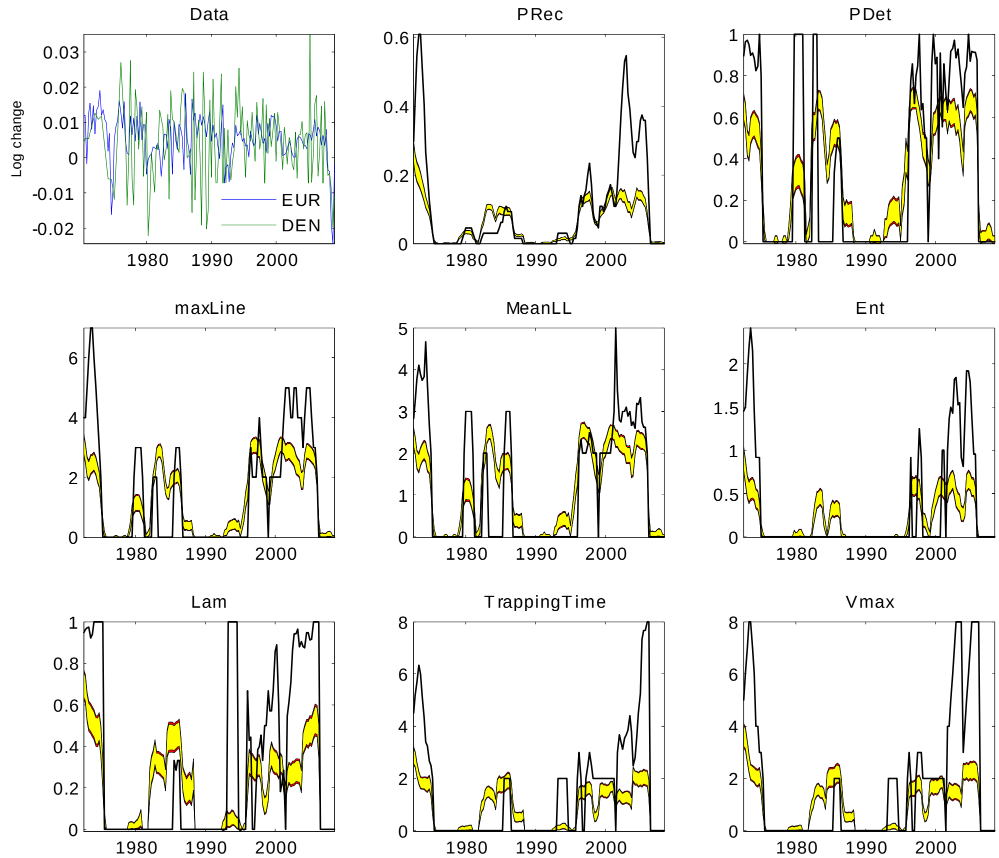


Figure 30: CRQA for Danish vs euro area growth

2006 and in the 1970s.

5.3.2 Sweden

Sweden's unthresholded recurrence plot in figure 31 displays similar patterns to those of other euro area member states, suggesting that Sweden might be a good candidate for the euro area. Convergence clearly occurs in around 1993 and then apart from the period from 2001 until 2003, the leading diagonal is almost consistently red, indicating convergence. Outside of this period there are clear diagonal lines indicating synchronous dynamics, although not at the same degree of convergence.

In figure 32 the thresholded plot for Sweden is shown, and it is clear that at this level of convergence the synchronicity was intermittent after 1993, with perhaps the greatest amount of

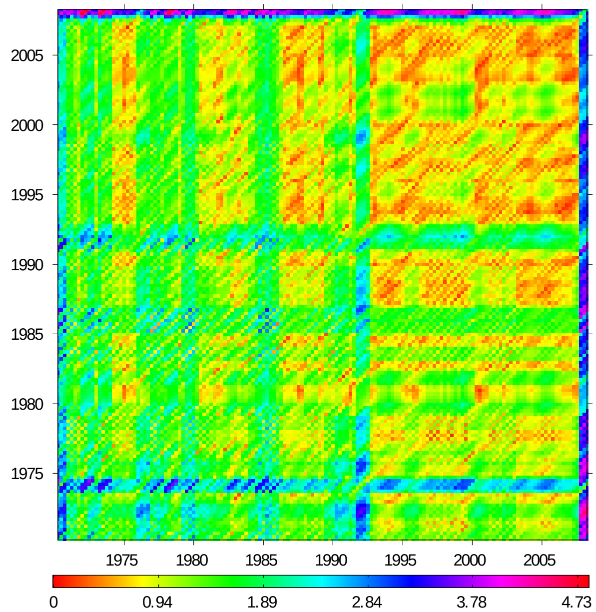


Figure 31: Unthresholded cross recurrence plot for Swedish vs euro area growth rates

in-phase synchronicity shown from 1995 to 1997. In the earlier decades there is little synchronicity evident.

When looking at the CRQA measures in figure 33 the recurrence clearly increases to significant levels post-1995 to about 2001 and then there is another short period of significant recurrence in 2002 followed by much higher levels of recurrence in 2005 onwards. As the thresholded plot showed, there is a substantial amount of laminarity evident after 1993 and this is evident in the laminarity, trapping time and the maximum vertical distance measures shown in the bottom row.

5.3.3 UK

In the case of the UK, the results are somewhat surprising. There has clearly been considerable convergence in growth rates and this occurred from roughly 1990 onwards. Also synchronous dynamics appear in the recurrence plot but they are off the leading diagonal, and appear to lag those of the euro area, particularly in the late 1980s and early 1990s (the time when the UK was a member of the ERM of the EMS and shadowing Bundesbank monetary policy).

In figure 35, there is a clear laminar square formed from 1994 onwards, although this has clearly not been evident in the last year of the data series. Further the lagged synchronous dynamic is now clearly evident in the early 1990s and from 1984 to around 1986.

The CRQA measures in figure 36 basically mirror the observations made from the thresholded plot, showing significant recurrence from the mid-1990s to present, and also the laminar region in

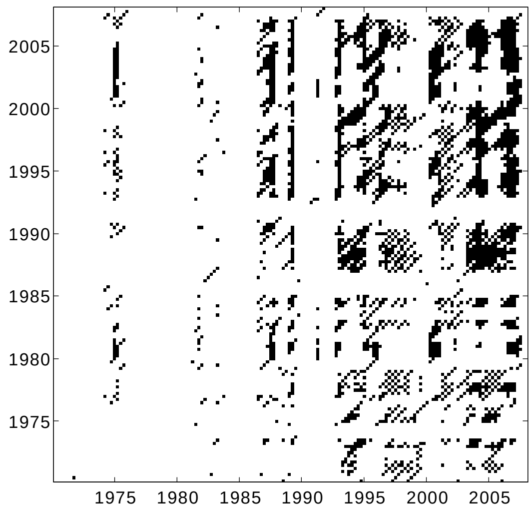


Figure 32: Thresholded cross recurrence plot for Swedish vs euro area growth rates (ε=1.5σ).

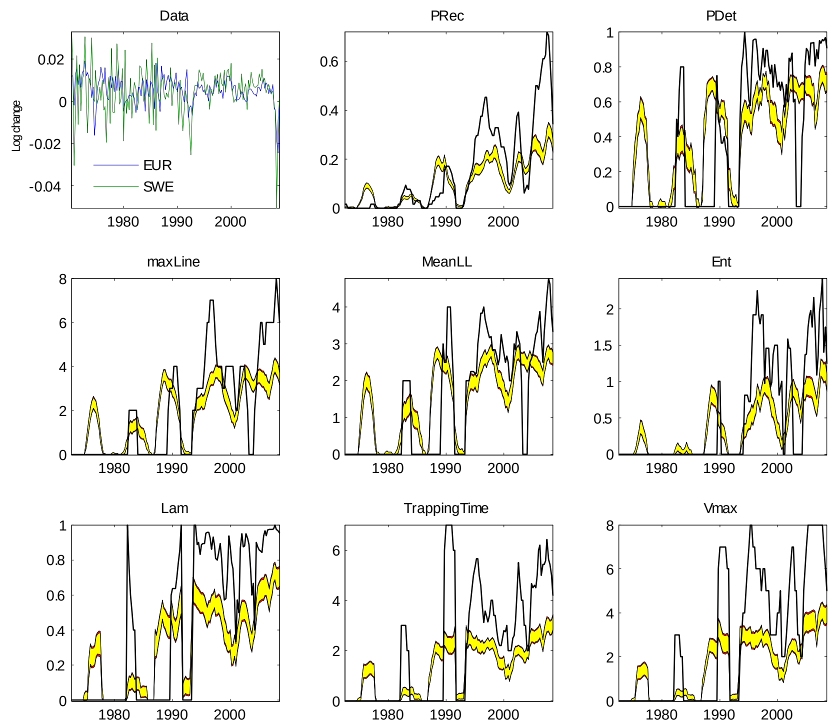


Figure 33: CRQA for Swedish vs euro area growth

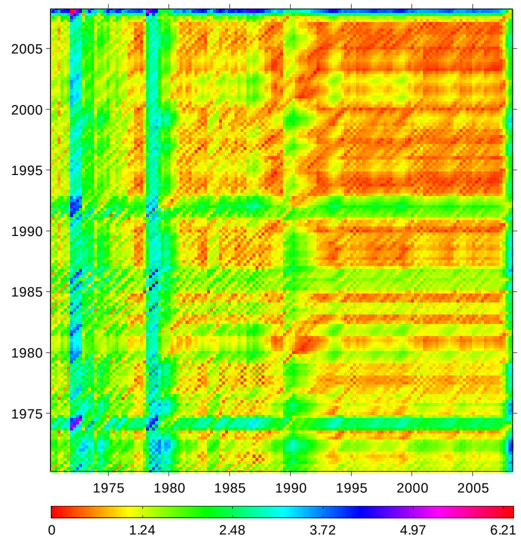


Figure 34: Unthresholded cross recurrence plot for UK vs euro area growth rates

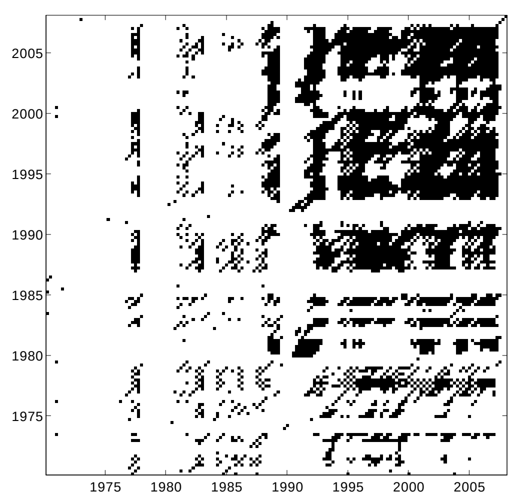


Figure 35: Thresholded cross recurrence plot for UK vs euro area growth rates (ε=1.5σ).

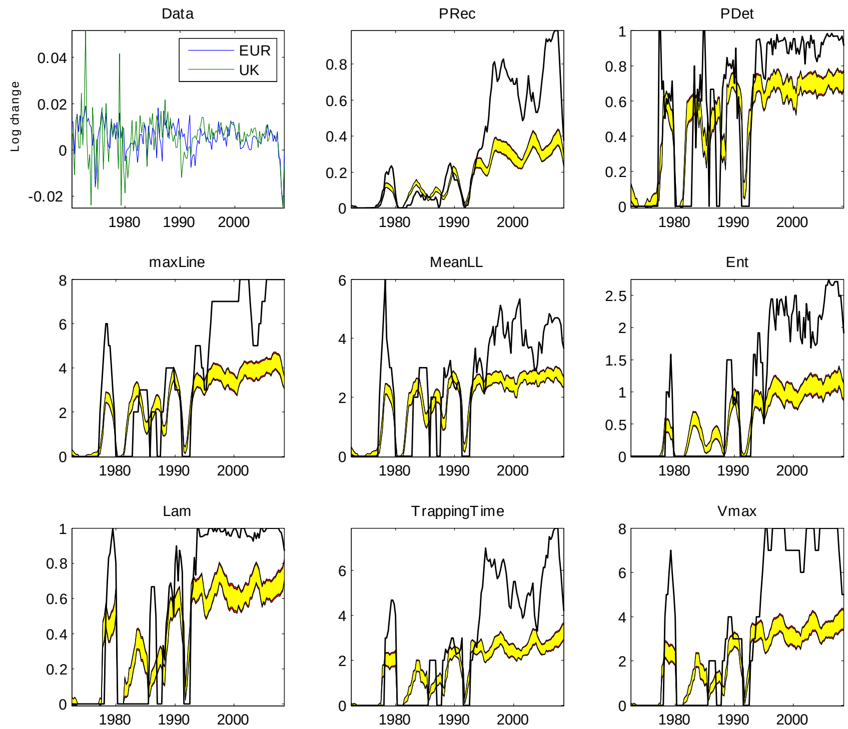


Figure 36: CRQA for UK vs euro area growth

recent years gives rise to high laminarity and vertical distance measures in the bottom row of the figure.

5.3.4 Switzerland

In figure 37 for Switzerland, we have much more obvious diagonal line segments going up the unthresholded recurrence plot, even in periods before 1992, and even when there was little convergence (as in the early 1970s). In fact for Switzerland greater convergence appears to have occurred in the late 1970s with some intermittency in both synchronicity and convergence occurring during the 1980s and into the early 1990s, but beyond 1992 more consistent convergence appears to have occurred as with other euro area member states.

This is mirrored in the thresholded plot in figure 38 where convergent synchronous dynamics can be detected along the leading diagonal from around 1998 onwards, but before this time there are both leading and lagged effects (lagged in 1993-95 and leading in 1997), and then only patchy convergence through the 1980s.

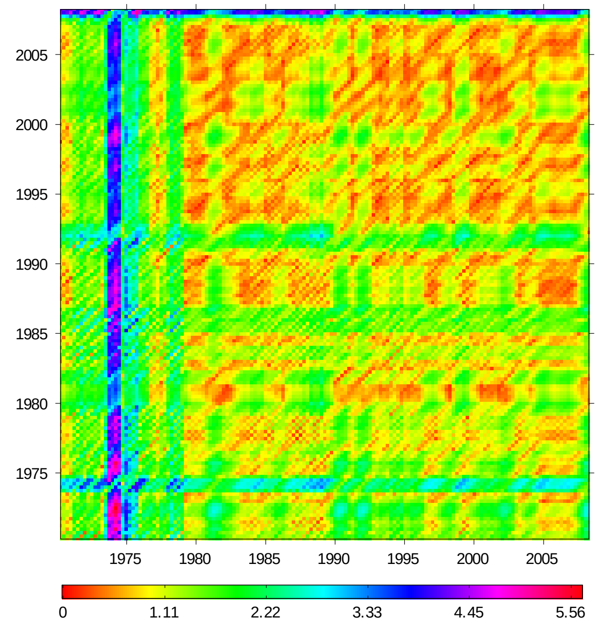


Figure 37: Unthresholded cross recurrence plot for Swiss vs euro area growth rates

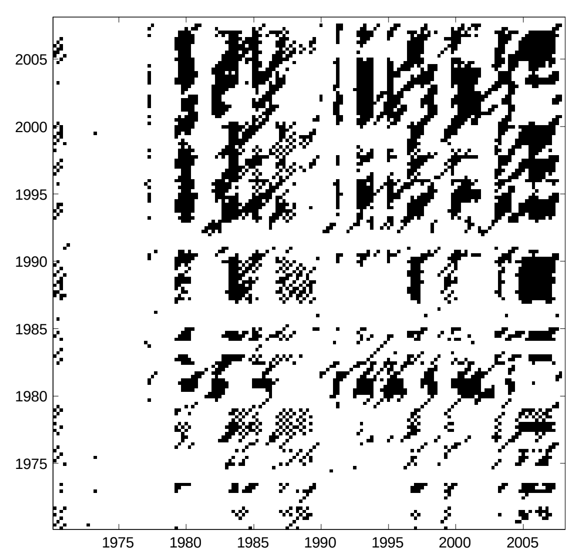


Figure 38: Thresholded cross recurrence plot for Swiss vs euro area growth rates (ε=1.5σ).

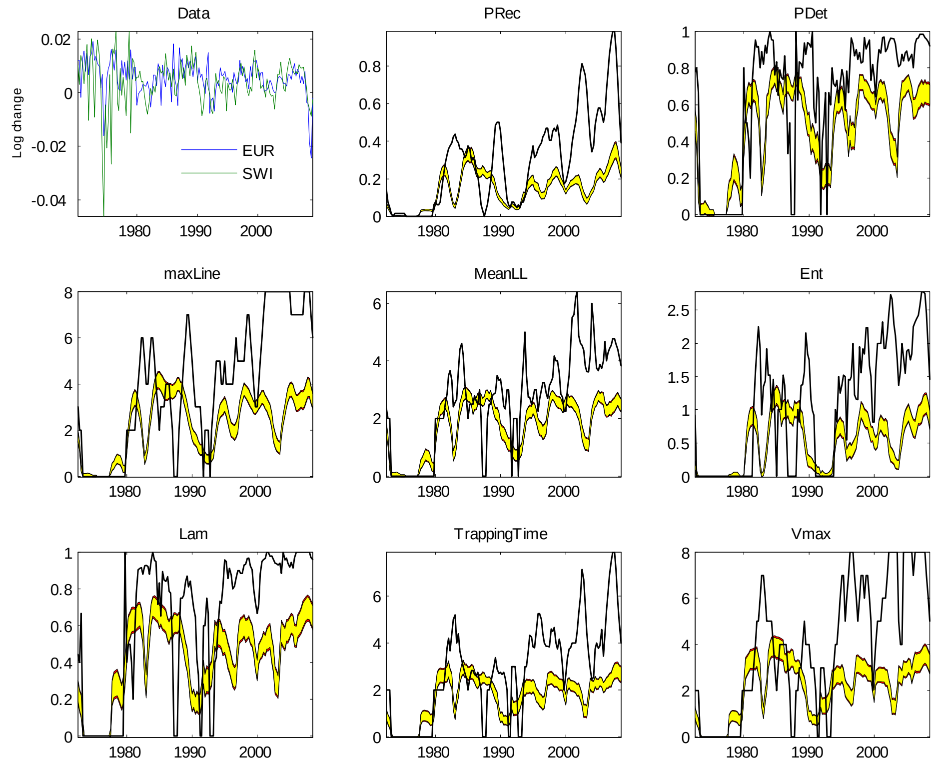


Figure 39: CRQA for Swiss vs euro area growth

Once again the results from the thresholded plot are mirrored in the results for the CRQA measures in figure 39. Here there is clearly significant recurrence in the mid-1980s, and then again from the mid-1990s until 1999 and then from 2000 onwards. A similar pattern appears for detrerminism and also the digaonal line length measures. Laminar areas are significant from 1992 onwards and this is also reflected in the vertical distance measures in the last row of the plot.

5.3.5 US

Lastly, we consider the US and the euro area growth rates. In figure 40 it is clear that up until 1992 the US growth rate was much more volatile compared with the euro area growth rate, as the vertical green bars up the first part of the plot suggest that none of the US observations in this period were similar to any euro area growth rate observations. Beyond 1992 once again there is increased convergence and synchronicity with some complex dynamics in the red areas and definite intermittency in synchronization occurring.. This is an important result, as it suggests that the increased degree of both convergence in growth rates and synchronization of movement

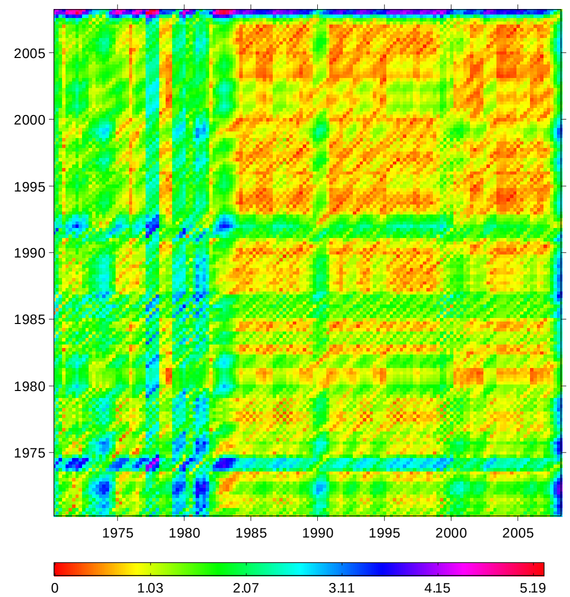


Figure 40: Unthresholded cross recurrence plot for US vs euro area growth rates

in these rates was a global phenonmenon, and had little to do with European integration during this period. It is also apparent from this unthresholded recurrence plot that growth rates have become more similar between the euro area and the US except during recessions. The yellow and red areas of the plot are concentrated in a rectangle bounded by the early 1990s in the US and the 1992/93 period in the euro area (- presumably coincident with German reunification and the ERM crisis). Furthermore, the way vertical bands of red and yellow are structured suggests that it was the US that had dissimilar growth rates to the euro area up until 1983, but after this time US growth becomes similar to euro area growth rates for the whole period and in particular those from 1993 onwards.

The thresholded plot in figure 41 shows the rather ragged dynamics between the US and euro area growth rates after 1992, with little apparent phase-synchronicity at these levels of convergence (- both "intermittent" leading and lagging dynamics can be detected though) and with laminar regions appearing in around 2003. Given that these laminar regions hide the mask the dynamics, it is hard to tell whether these are phase-synchronous.

The CRQA plots for the US are shown in figure 42. If the PRec is indicative of synchronicity in specific dynamics, then it is clear that there have been 5 episodes of significant synchronous dynamics (note - not necessarily phase synchronous) since the late 1980s. What is particularly interesting about these dynamics is that they don't appear to be centred on turning points in the

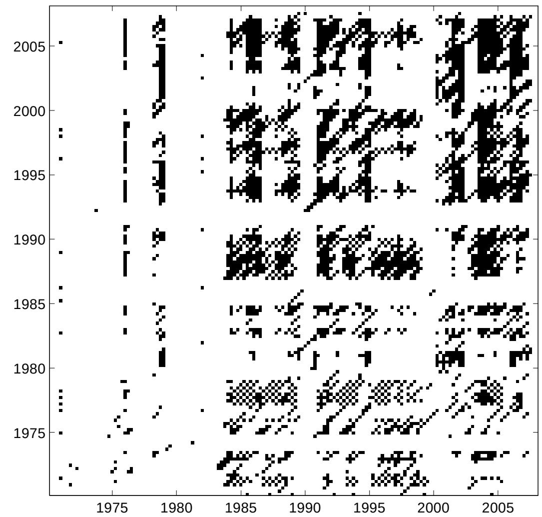


Figure 41: Thresholded cross recurrence plot for US vs euro area growth rates (ε=1.5σ).

business cycle (as one might expect), but appear to occur intermittently at reasonaly regularly spaced intervals of around 3 to 4 years.

6 Is lack of convergence and/or phase-synchronicity a problem for the ECB?

With a single monetary policy there are four possible outcomes for convergence and synchronisation of real GDP growth rates:

i) both convergence and (either full or "intermittent") synchronisation (e.g. France, Germany, Italy, Spain)

ii) converged but not synchronized (e.g. Finland, UK);

iii) not converged but synchronized (e.g. Ireland, Sweden); and

iv) neither converged nor synchronized (e.g. US).

Although this is admittedly a rough classification, it shows that there are some member states which are better suited to the ECB's monetary policy than others, and this will obviously imply a differential impact across the euro area. One of the interesting features of this result is that the "core vs periphery" distinction (made by Artis and Zhang (1997)) is not as clear cut as it was

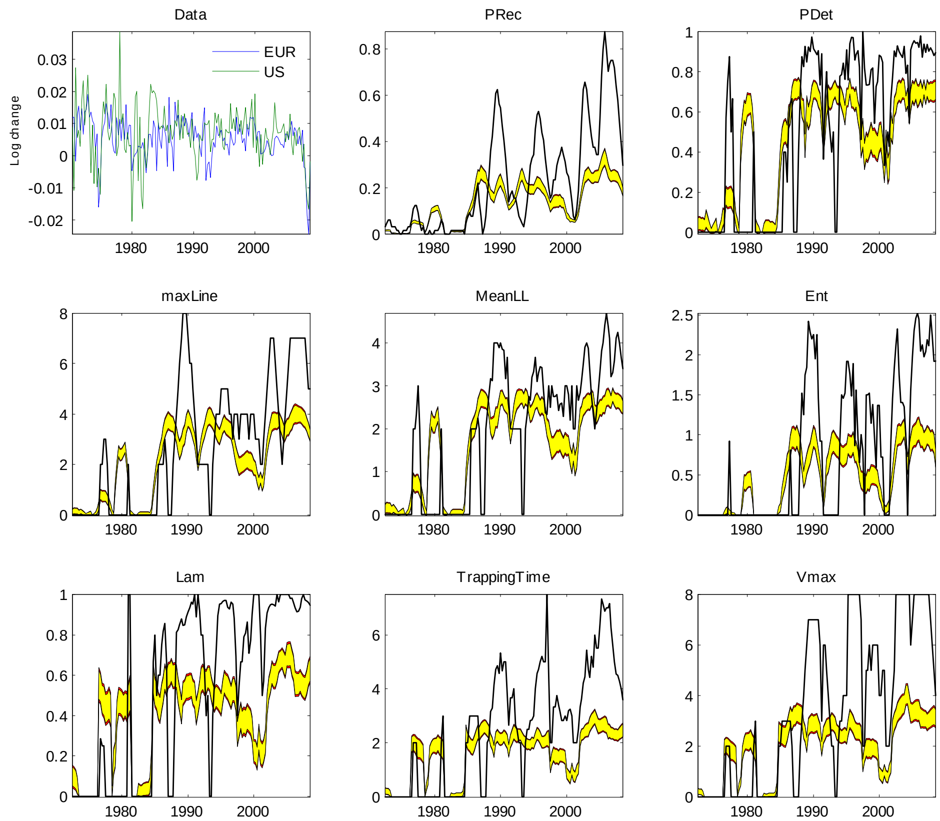


Figure 42: CRQA for US vs euro area growth

in the 1990s - one core member state (Netherlands) is not grouped with the other core member states, and one peripheral member state (Spain) is.

Given this classification of growth and business cycles in relation to the euro area aggregate there are three important questions that are relevant for ECB policy:

a) is convergence sufficient for ECB monetary policy to be optimal or is a reasonable degree of synchronicity necessary?;

b) is "intermittent" synchronicity a potential problem for ECB monetary policy? If so, what degree of "intermittency" is a problem?

c) are recent trends in the euro area core indicative of a change in regime occurring in late 2005?

The first question asks whether ECB monetary policy could prove wholly inappropriate for some member states if synchronisation is not achieved, which in turn could lead to problems for the long term sustainability of the euro area. According to OCA theory, convergence is not a sufficient condition so non-phase synchronised member states might eventually decide to either leave the euro area or insist on an implementation of supra-national policies to counter a non-optimal monetary policy.

The second question applies to certain member states (such as Germany) where there are intermittent periods where synchronous growth dynamics are absent. If these are due to asymmetric "shocks" hitting the system, which cause divergence for a short period of time then this is only a short-term problem, but if this is a systematic and repetitive dynamic then this could be problematic for ECB monetary policy at some point in the future.

The third question points to some worrying developments in the most recent data used in the study (to 2009), which shows that the core of the euro area appears to have diverged from the growth rates experienced by the average euro area member (- represented by the euro area aggregate). This could be a temporary divergence (- a differential response to the US slowdown, for example), but also it could be more permanent and represent a change in regime. Given the events of early 2010 this divergence in growth rates between the core (and in particular Germany) and the southern periphery (in particular Spain, Portugal and Greece) is only likely to have exacerbated this problem perhaps leading to a longer period of "growth dislocation".

7 Conclusions

Cross recurrence plots and quantification analysis offers a unique non-linear approach to studying economic time series and their interaction over time. There are few existing studies of macroeconomic dynamics which utilize this methodology, and so the application in this contribution serves to illustrate the potential for this approach in the study of macroeconomic dynamics and policy analysis.

In this paper cross recurrence plots were used to analyse real GDP growth between a euro area growth aggregate and a sample of European member states. Synchronicity and convergence were found to be high between most of the core euro area member states, suggesting that monetary policy is not having a major differential impact on these countries and that they are increasingly suited to being part of a monetary union. Both inside and outside of this core, however, there is no consistency for many member states in terms of their synchronicity or convergence onto the euro area growth rate. This should clearly be a concern for the future membership of the euro area, and as other studies have suggested, this might decrease the public's perceptions of the desirability of being part of the euro area in these countries in the future. When repeating the exercise for the US against the euro area growth rate the nature of the international propagation of business and growth cycles was found to have increased in the 1980s, but this approach suggests that there has been no significant change since then. In terms of the phasing of synchronous episodes, the euro area also clearly lags behind the US.

In terms of the study of non-linear dynamics, there is ample scope for further research: first, the relationship between convergence and synchronisation is clearly of interest in the context of propagation of business and growth cycles and European integration, and secondly recurrence plots reveal the "intermittency" of synchronisation observed in several instances. Both these issues clearly merit further investigation.

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