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ABSTRACT

Many labor markets share three stylized facts: employers cannot give full attention to all candidates, candidates are ready to provide information about their preferences for particular employers, and employers value and are prepared to act on this information. In this paper we study how a signaling mechanism, where each worker can send a signal of interest to one employer, facilitates matches in such markets. We find that introducing a signaling mechanism increases the welfare of workers and the number of matches, while the change in firm welfare is ambiguous. A signaling mechanism adds the most value for balanced markets.

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An online appendix is available at:
http://www.nber.org/data-appendix/w16185

1 Introduction

Job seekers in labor markets often apply for many positions, as there is a low cost for applying and a high value for being employed. Consequently, many employers face the near impossible task of reviewing and evaluating hundreds of applications. Moreover, since pursuing candidates is often costly, employers may need to assess not only the quality of an applicant, but also whether the applicant is attainable: that is, whether the candidate is likely to ultimately accept a job offer, should the employer make one. In this paper we study a mechanism that aids employers in this evaluation process by allowing applicants to credibly signal information about their preferences for positions.

In practice, in many markets that suffer from this form of application congestion, candidates communicate special interest for a select number of places. For example, in college admissions in the United States, many universities have early admission programs, where high school seniors may apply to exactly one college before the general application period. Evidence suggests that universities respond to such action in that it is easier to get into a college through early admission programs (Avery, Fairbanks and Zeckhauser, 2003).[1] Another example of applicants signaling interest can be found in the market for entry-level clinical psychologists, which in the early 1990's was organized as a telephone-based market. On "match day," program directors called applicants to make offers, and candidates were, at any moment, allowed to hold on to at most one offer. At the end of match day, all non-accepted offers were automatically declared as rejected. Due in part to its limited time frame, this market suffered from congestion, and it was common for program directors to make offers out of their preference order to applicants who credibly indicated they would accept an offer immediately (Roth and Xing, 1997).[2]

Some markets have formal, market-wide mechanisms that allow participants to signal preferences, and the formal nature of the signals ensures credibility. Since 2006, The American Economic Association (AEA) has operated a signaling service to facilitate the job search

[1]Under single early application programs, universities often require that an applicant not apply early to other schools, and this is often enforced by high school guidance counselors. In another example of colleges looking for signs of interest, many schools take great care to note whether applicants visit the campus, which presumably is costly for parents in terms of time and money. This can also be taken into account when colleges decide whom to admit.

[2]Congestion in the telephone market was costly for program directors who worried that their offer would be held the whole match day and then rejected in the last moments, leaving them to fill the position in a hectic "aftermarket" with only a few leftover candidates. As an example of offer strategy, the directors of one internship program decided to make their first offers (for their five positions) to numbers 1, 2, 3, 5, and 12 on their rank-order list of candidates, with the rationale that 3, 5, and 12 had indicated that they would accept immediately and that 1 and 2 were so attractive as to be worth taking chances on. Anecdotal evidence suggests that promises to accept an offer were binding. The market was relatively small, and as one program director mentioned: "you see these people again."

for economics graduate students. Using this service, students can send signals to up to two employers to indicate their interest in receiving an interview at the annual Allied Social Science Associations meeting. Coles et al. (2010) provide suggestive evidence that sending a signal of interest increases the chances of receiving an interview. Since interviews take place over a single weekend, departments typically interview about twenty candidates out of hundreds of applicants, which suggests that most departments must strategically choose from among their candidates that are above the bar.[3] Though not labor markets, some online dating websites allow participants to send signals to potential partners. For example in the matchmaking service of the website "Hot or Not," participants can send each other virtual flowers that purportedly increase the chances of receiving a positive response.[4] In a field experiment on a major Korean online dating website, Lee et al. (2009) study the effect of a user attaching one of a limited number of "virtual roses" to a date request. They find that users of both genders are more likely to accept a request when a virtual rose is attached.[5]

These examples all share three important features. First, in each case substantial frictions lead to market congestion: employers (or colleges or dating partners) are unable to give full attention to all possible candidates when making decisions. Second, applicants are ready to provide information about their preferences over employers. Third, employers value this preference information and are prepared to act on it.

For employers to take useful action, preference signals must be credible. But simply declaring one's interest typically bears almost no cost, and job seekers have an incentive to indicate particular interest to many employers, regardless of how strong their preferences towards these employers actually are. Hence, absent any credibility guarantee, employers may struggle to discern which preference information is sincere and which is simply cheap talk. So while candidates may wish to signal their preferences, and employers may value

[3]Similar mechanisms exist for non-academic jobs. For example, Skydeck360, a student-operated company at Harvard, offers a signaling service for MBA students in their search for internships and full-time jobs. Each registered student can send up to ten signals to employers via their secure website. (See http://skydeck360.posterous.com for detail.)

[4]In this case the number of flowers one may send is unlimited, but each flower is costly. Signals of interest may be helpful in dating markets because pursuing partners is costly. At the very least, each user may be limited in the number of serious dates she can have in a given period. "As James Hong from HotorNot tells it, his virtual flower service has three components: there's the object itself represented by a graphical flower icon, there's the gesture of someone sending the flower to their online crush, and finally, there's the trophy effect of everyone else being able to see that you got a flower. People on HotorNot are paying $10 to send the object of their affection a virtual flower – which is a staggering 3-4x what you might pay for a real flower!" (from http://www.viralblog.com/research/why-digital-consumers-buy-virtual-goods/) See http://www.hotornot.com/m/?flowerBrochure=1 for a description of HotorNot's virtual flower offerings.

[5]This dating website targets people looking for marriage partners, rather than people who want many dates. Hence, dates may be perceived as particularly costly, so users must decide carefully on whom to "spend" a date. The study found that candidates of average attractiveness, who may worry that date offers are only "safety" offers, are particularly responsive to signals of special interest.

learning candidate preferences, inability to credibly convey information may prevent any gains from preference signaling from being realized.

In this paper, we investigate how a *signaling mechanism* that limits the number of signals a job seeker may send can overcome the credibility problem and improve the welfare of market participants. We develop a model that can account for the three stylized facts mentioned above. In our model, firms make offers to workers, but the number of offers they may make is limited, so that firms must carefully select the workers to whom they make offers. We focus on the strategic question of offer choice and abstract away the question of acquiring information that determines preferences. Hence, we assume that each agent knows her own preferences over agents on the other side of the market, but is uncertain of the preferences of other agents.

In the simplest version of our model, we assume that both worker and firm preferences are idiosyncratic and uniformly distributed. Workers have the opportunity to send a signal to one firm, where each signal is binary in nature and does not transmit any further information. Firms observe their signals, but not the signals of other firms, and then each firm simultaneously makes exactly one offer to a worker. Finally, workers choose offers from those available to them. We focus on symmetric equilibria in anonymous strategies to eliminate any coordination devices beyond the signaling mechanism.

We show that, in expectation, introducing a signaling mechanism increases both the number of matches as well as the welfare of workers. Intuitively, when firms make offers to workers who send them signals, these offers are unlikely to overlap, leading to a higher expected number of matches. Furthermore, workers are not only more likely to be matched, but are also more likely to be matched to a firm they prefer the most. On the other hand, when a firm makes an offer to a worker who has signaled it, this creates strong competition for firms who would like to make an offer to that same worker because, for example, they rank that worker highest. Hence, by responding to signals, that is, being more likely to make offers to workers who have signaled them, firms may generate a negative spillover on other firms. Consequently, the effect on firm welfare from introducing a signaling mechanism is ambiguous; welfare for a firm depends on the balance between individual benefit from responding to signals and the negative spillover generated by other firms responding to signals. Furthermore, we show that the degree to which a firm responds to signals is a case of strategic complements. When one firm responds more to signals, it becomes riskier for other firms to make offers to workers who have not sent them signals. Consequently, multiple equilibria, with varying responsiveness to signals, may exist. These equilibria can be welfare ranked: workers prefer equilibria where firms respond more to signals, while firms prefer the equilibria where they respond less.

We also study an extension in which workers have correlated preferences. In this setting a worker may not necessarily signal to her overall most preferred firm. This implies that firms cannot be certain that an offer made to a worker who sent a signal will be accepted. Nonetheless, for a class of correlated preferences we show that the introduction of a signaling mechanism increases the expected number of matches and the welfare of workers.

To understand when a signaling mechanism might be most helpful, we compare performance across market settings. To do this, we focus on a simpler environment where agents care about getting a match, but not the quality of the match. Hence, the value of introducing a signaling mechanism is simply the expected increase in the number of matches. For such an environment, we find that the value of a signaling mechanism is maximal for *balanced markets*; that is, markets where the number of firms and workers are of roughly the same magnitude. We further show that the increase in the number of matches is roughly homogenous of degree one in the number of firms and workers. That is, signaling mechanisms are equally important for large and small markets in terms of the expected increase in the fraction of matched participants.

Our approach is related to several strands of literature. A standard interpretation of signaling and its effectiveness is that applicants have private information that is pertinent to how valuable an employee they would be. For example, in Spence's signaling model (Spence, 1973), applicants use wasteful costly signals, such as education, to signal their type, such as their ability.[6] More recently, Avery and Levin (2009) model early application in US college admissions as a way for students to signal college-specific quality, such as enthusiasm for a particular college. In their model, colleges explicitly derive more utility from having enthusiastic students in their freshman class than they do from other, equally able students. By contrast, in our model we abstract away from such motives and instead show how congestion, stemming from the explicit monetary or opportunity costs of making offers, can generate room for useful preference signaling.

A more closely related strand of literature is that of strategic information transmission, or "cheap talk," between a sender and receiver, introduced in Crawford and Sobel (1982). In our model, however, we consider a multi-stage game with many senders (workers) and many receivers (firms), where the structure of allowable signals plays a distinctive role. Each sender must choose the receiver to whom she will send one of her limited, identical signals, and the scarcity of signals induces credibility. Each receiver knows only whether a sender has sent a signal to it or not, and receives no additional information. While Crawford and Sobel

[6] Hoppe, Moldovanu and Sela (2009) extend this idea to an environment where agents on both sides of the market may send signals. Among other findings, they identify general conditions under which the potential increase in expected output due to the introduction of signaling is offset by the costs of signaling.

(1982) study a coordination problem between the sender and receiver, our setting includes an additional coordination problem among receivers who must decide whom to make an offer. Nevertheless, some features of Crawford and Sobel persist in our model. Signals are "cheap" in the sense that they do not have a direct influence on agent payoffs. Each agent has only a limited number of signals, so there is an opportunity cost associated with sending a signal. Finally, in our model there always exist babbling equilibria where agents ignore signals; hence, the introduction of a signaling mechanism always enlarges the set of equilibria.

While to our knowledge we are the first to introduce preference signaling in decentralized markets, papers by Abdulkadiroglu, Che and Yasuda (2008) and Lee and Schwarz (2007) deal with preference signaling in the presence of centralized clearinghouses.[7]

In summary, our paper models the introduction of a signaling mechanism in markets where interviews or offers are costly for firms, either in direct monetary terms or because of opportunity costs. Our results suggest potentially large welfare gains for workers, and an increase in the expected total number of matches. Furthermore, as the experience with the economic job market shows, introducing a signaling mechanism can be a low cost, unintrusive means of improving maket outcomes. As such we see our paper as part of the larger market design literature (c.f. Roth, 2008).

The paper proceeds as follows. Section 2 begins with a simple example, and Sections 3 and 4 discuss the offer game with and without a signaling mechanism, respectively. Section 5 considers the impact of introducing a signaling mechanism on the welfare of agents. In Section 6 we examine signaling in an environment with correlated agent preferences. Section 7 analyzes the robustness of the welfare results across various market structures. Section 8 concludes.

2 A Simple Example

In this section we lay out a simple example that shows the effects of introducing a signaling mechanism and highlights some of our main findings. Consider a market with two firms $\{f_1, f_2\}$ and two workers $\{w_1, w_2\}$. For each agent, a match with one's most preferred partner from the other side of the market yields payoff 1, while a match with one's second choice partner yields $x \in (0, 1)$. Remaining unmatched yields payoff 0.

[7] Abdulkadiroglu, Che and Yasuda (2008) show that the introduction of a signaling technology improves efficiency of the deferred acceptance algorithm in a school choice problem. Lee and Schwarz (2007) analyze preference signaling in a match formation process between firms and workers that consists of three steps: preference signaling, investments in information acquisition, and formation of matches via a centralized clearinghouse. They construct a centralized mechanism where workers communicate their complete preferences to an intermediary, and the intermediary recommends to each firm a subset of workers to interview.

Ex-ante, agent preferences are random, uniform and independent. That is, for each firm f, the probability that f prefers worker w_1 to worker w_2 is one half, as is the probability that f prefers w_2 to w_1. Worker preferences over firms are similarly symmetric. Agents learn their own preferences, but not the preferences of other agents.

We first examine behavior in a game where once agent preferences are realized, each firm may make a single offer to a worker. Workers then accept at most one of their available offers. We will examine sequential equilibria, which guarantees that workers accept their best available offer.

In the unique equilibrium of this game where firm strategies do not depend on the name of the worker,[8] each firm simply makes an offer to its most preferred worker. This follows because firms cannot discern which worker is more likely to accept an offer. In this congested market there is a fifty percent chance that both firms make an offer to the same worker, in which case there will only be one match. Hence, on average there are 1.5 matches, and the expected payoff for each firm is $\frac{3}{4}1 + \frac{1}{4}0 = 0.75$. For workers, if they receive exactly one offer, it is equally likely to be from their first or second choice firm. There is also a fifty percent chance that one worker receives two offers, hence attaining a payoff of one while the other worker receives zero. The expected payoff for each worker is then $(2 + x)/4$.

We now introduce a signaling mechanism: before firms make offers, each worker may send a *signal* to a single firm. Each signal has a binary nature: either a firm receives a signal from a particular worker or not, and signals do not not transmit any other information. We focus on non-babbling equilibria, where firms interpret a signal as a sign of being the more preferred firm of that worker, and workers send a signal to their more preferred firm.[9]

To analyze firm behavior, note that a firm that receives a signal from its top worker will make this worker an offer, since it will certainly be accepted. If on the other hand a firm receives no signals, it again optimally makes an offer to its top worker, as symmetry implies the workers are equally likely to accept an offer. The interesting strategic decision a firm must make is when it receives a signal only from its second ranked worker. In this case the other firm also receives exactly one signal. We say a firm "responds" to the signal if it makes the signaling worker an offer, and "ignores" the signal if it instead makes an offer to its top worker, which did not send it a signal.

Suppose f_1 prefers w_1 to w_2 and only w_2 sent a signal to f_1, which implies w_1 sent a

[8] See Section 3 for a formal definition of anonymous strategies.

[9] Note that there is no equilibrium where firms expect signals from workers, but interpret them as a particular lack of interest and hence reduce the probability of making an offer to a signaling worker. If this were the case, workers would simply not send any signal. There are, however, babbling equilibria where no information is transmitted, though we will not focus on those in this paper, as they are equivalent to not having a signaling device.

signal to f_2. Clearly, whenever f_1 makes an offer to w_2, f_1 receives x. Suppose f_1 instead makes an offer to w_1, who sent a signal to f_2. If f_2 responds to signals, then f_2 also makes an offer to w_1, which w_1 will accept, hence leaving f_1 a payoff of 0. If f_2 ignores signals, then there is still a fifty percent chance that w_1 is actually f_2's first choice, in which case an offer is tendered and accepted, so that f_1 again receives 0. Otherwise, f_1 receives 1. Table 1 summarizes f_1's payoffs conditional on receiving a signal from its second ranked worker, and the strategies of f_2.

Table 1: Firm f_1's payoffs conditional on receiving a signal from its second ranked worker.

$f_1 \setminus f_2$	Respond	Ignore
Respond	x	x
Ignore	0	1/2

Table 1 shows that strategies of firms are strategic complements. If a firm responds to signals, then the other firm is weakly better off from responding to signals as well. In this example, if f_2 switches from the action ignore (not making an offer to a second choice worker who has signaled) to the safe action of responding (making an offer to a second choice worker who has signaled), then f_1 optimally also takes the safe action of responding.

Turning to equilibrium analysis, note that if $x > 0.5$ there is a unique equilibrium in which both firms respond to signals. When $x < 0.5$, that is when the value of the first choice worker is much greater than that of the second ranked worker, there exist two equilibria in pure strategies. In the first, both firms respond to signals (Respond-Respond) and in the second both firm ignore signals (Ignore-Ignore).[10] Table 2 summarizes welfare properties of these equilibria. Note that the expected firm and worker payoffs, as well as the expected number of matches when signals are ignored are the same as when there is no signaling mechanism, since agent actions in these two settings are identical.[11]

Whenever there are multiple equilibria ($x < 0.5$), we can rank them in terms of firm welfare, worker welfare, and the expected number of matches. Workers and firms are opposed in

[10] There is also a mixed strategy equilibrium whenever there are two pure strategy equilibria. Properties of this equilibrium coincide with those in the equilibrium where both firms respond to signals.

[11] When both firms respond to signals, since each firm has a fifty percent chance of receiving a signal from its first choice worker, half the time this strategy yields payoff of one. Otherwise a firm has a 1/4 chance of receiving a signal from its second choice worker only, yielding a payoff of x. With a 1/4 chance a firm receives no signal, in which case it makes an offer to its first choice worker, who will accept with fifty percent probability (whenever she is not the first choice worker of the other firm). Hence, expected firm payoffs are $\frac{1}{2}1 + \frac{1}{4}x + \frac{1}{4}\frac{1}{2}1 = \frac{5+2x}{8}$. Payoffs for workers can similarly be calculated given these outcomes. Furthermore, when one firm receives all signals (which happens half the time) there is a fifty percent chance of firms making offers to the same worker, and hence, of only one match occuring, so the expected number of matches is $\frac{1}{4}1 + \frac{3}{4}2 = \frac{7}{4}$.

Table 2: Firm payoffs, worker payoffs, and number of matches when both firms use the same strategy.

	Firm Payoffs	Worker Payoffs	Number of Matches
Respond-Respond	$(5+2x)/8$	$3/4$	$7/4$
Ignore-Ignore	$3/4$	$(2+x)/4$	$3/2$

their preferences over equilibria: workers prefer the equilibrium in which both firms respond to signals while firms prefer the equilibrium in which they both ignore signals. Intuitively, while one firm may privately gain from responding to a signal, such an action may negatively affect the other firm. The expected number of matches in the equilibrium when both firms respond to signals is always greater than in the equilibrium when both firms ignore the signals.

These welfare results enable us to study the effects of introducing a signaling mechanism, as outcomes in the offer game without signals are identical to those when both firms ignore signals (even if the Ignore-Ignore equilibrium does not exist). The expected number of matches and the welfare of workers in the offer game with signals in any non-babbling equilibrium are greater than in the offer game with no signals. The welfare of firms changes ambiguously with the introduction of a signaling mechanism. We now show that these results generalize.

3 The Offer Game with No Signals

3.1 General Notation

In this paper we aim for a simple hiring model in which we can highlight the role of agents being able to credibly signal preferences in the presence of congestion. We have a market with a set of firms, a set of workers, and a distribution over firm and worker preferences. Each firm has the capacity to hire at most one worker, and each worker can fill at most one position. We examine an extreme form of congestion: each firm may make at most one offer to hire a worker, where we implicitly assume that workers have applied to all firms. In the *offer game with no signals*, firms make an offer based on the limited knowledge of the distribution of worker preferences. In the second setting, *the offer game with signals*, before offers are made, each worker has the opportunity to send one costless signal to a firm, who may use this signal to partially infer worker preferences. In the web appendix we show that the main results carry over when firms have multiple positions and workers can send several signals.

Let $\mathcal{F} = \{f_1, \ldots, f_F\}$ be the set of firms, and $\mathcal{W} = \{w_1, \ldots, w_W\}$ be the set of workers, with $|\mathcal{F}| = F$ and $|\mathcal{W}| = W$. We consider markets with at least two firms and two workers. Firms and workers have preferences over each other. For each firm f, let Θ_f be the set of all possible preference lists over workers, where $\theta_f \in \Theta_f$ is a vector of length W. We use the convention that the worker of rank one is the most preferred worker, while the worker of rank W is the least preferred worker. The set of all firm preference profiles is $\Theta_F = (\Theta_f)^F$. Similarly, we define θ_w, Θ_w and Θ_W for workers. Let $\Theta \equiv \Theta_F \times \Theta_W$, and let $t(\cdot)$ be the distribution over preference list profiles.

Firm f with preference list θ_f values a match with worker w as $u(\theta_f, w)$, where $u(\theta_f, \cdot)$ is a von-Neumann Morgenstern utility function. In our model, firms will be symmetric in the following sense: we assume that a firm's utility for a match depends only on a worker's rank. That is, for any permutation ρ of worker indices, we have $u(\rho(\theta_f), \rho(w)) = u(\theta_f, w)$.[12] Furthermore, all firms have the same utility function $u(\cdot, \cdot)$. Worker w with preference list θ_w values a match with firm f as $v(\theta_w, f)$, where match utility again depends only on rank, and all workers share the same utility function. Though not essential for our results, we will assume that workers and firms derive zero utility from being unmatched, and that any match is preferable to remaining unmatched for all participants. A *market* is given by the 5-tuple $\langle \mathcal{F}, \mathcal{W}, t, u, v \rangle$.

For Sections 3-5, we will focus on a simple preference structure: each firm f has preferences over the workers chosen uniformly, randomly and independently from the set of all strict preference orderings over all workers. Worker preferences are analogously chosen; that is, there is no correlation in preferences. This will make the problem symmetric and easy to analyze.

In Section 6 we will relax this assumption, and consider the case in which preferences of workers over firms may exhibit correlation. In the web appendix we consider a more involved symmetric model where firms have several slots to fill, and workers can send multiple signals.

3.2 The Offer Game with No Signals

We first examine behavior in the absence of a signaling mechanism. Play proceeds as follows. After preferences of firms and workers are realized, each firm simultaneously makes an offer to at most one worker. Workers then choose at most one offer from those available to them. Sequential rationality ensures that workers will always select the best available offer. Hence,

[12] Let $\rho : \{1, \ldots, W\} \to \{1, \ldots, W\}$ be a permutation. Abusing notation, we apply ρ to preference lists, workers, and sets of workers such that the permutation applies to the worker indices. For example, suppose $W = 3$, $\rho(1) = 2$, $\rho(2) = 3$, and $\rho(3) = 1$. Then we have $\theta_f = (w_1, w_2, w_3) \Rightarrow \rho(\theta_f) = (w_2, w_3, w_1)$ and $\rho(w_1) = w_2$.

we take the workers' behavior in the last stage as given and focus on the reduced game with only firms as strategic players.

Once its preference list θ_f (f's type) is realized, firm f decides whether and to whom to make an offer. Firm f may use a mixed strategy denoted by σ_f which maps the set of preference lists to the set of distributions over the union of workers with the no-offer option, denoted by $\mathcal{N}$; that is $\sigma_f : \Theta_f \to \Delta(\mathcal{W} \cup \mathcal{N})$.[13] We denote a profile of all firms' strategies as $\sigma_F = (\sigma_{f_1}, ... \sigma_{f_F})$, and the set of firm f's strategies as Σ_f .

Let the function $\pi_f : (\Sigma_f)^F \times \Theta \to \mathbf{R}$ denote the payoff of firm f as a function of firm strategies and realized agent types. We are now ready to define the Bayesian Nash equilibrium of the offer game with no signals.

Definition 1. Strategy profile $\hat{\sigma}_F$ is a Bayesian Nash equilibrium in the offer game with no signals, if for all $f \in \mathcal{F}$ and $\bar{\theta}_f \in \Theta_f$ the strategy $\hat{\sigma}_f$ maximizes the profit of firm f of type $\bar{\theta}_f$, that is

$$\hat{\sigma}_f(\bar{\theta}_f) \in \arg\max_{\sigma_f \in \Sigma_f} \mathbb{E}_{\theta_{-f}}(\pi_f(\sigma_f, \hat{\sigma}_{-f}, \theta) \mid \bar{\theta}_f).$$

We focus on equilibria in which firm strategies are *anonymous;* that is, they depend only on workers' ranks within a firm's preference list. This rules out strategies that rely on worker indices, eliminating any coordination linked to the identity of workers. As an example, "always make an offer to my second-ranked worker" is an anonymous strategy, while "always make an offer to the worker called w_2" is not.

Definition 2. Firm f's strategy σ_f is *anonymous* if for any permutation ρ, and for any preference profile $\theta_f \in \Theta_f$, we have $\sigma_f(\rho(\theta_f)) = \rho(\sigma_f(\theta_f))$.

When deciding whom to make an offer, firms must consider both the utility from hiring a specific worker and the likelihood that this worker will accept an offer. Because preferences of both firms and workers are independently and uniformly chosen from all possible preference orderings, and since firms use anonymous strategies, an offer to any worker will be accepted with equal probability. Hence, each firm optimally makes an offer to the highest-ranked worker on its preference list. Indeed, this is the unique equilibrium when firms use anonymous strategies.

Proposition 1. *The unique equilibrium of the offer game with no signals when firms use anonymous strategies and workers accept the best available offer is* $\sigma_f(\theta_f) = \theta_f^1$ *for all* $f \in \mathcal{F}$ *and* $\theta_f \in \Theta_f$.

[13] In other words, f selects elements of a W-dimensional simplex; $\sigma_f(\theta_f) \in \Delta^W$, where $\Delta^W = \{x \in R^{W+1} : \sum_{i=1}^{W+1} x_i = 1$, and $x_i \geq 0$ for each $i\}$.

Note that the above statement requires that firm strategies be anonymous only in equilibrium. Firm deviations that do not satisfy the anonymity assumption are still allowed. As seen in the example in Section 2, in this equilibrium there might be considerable lack of coordination, leaving many firms and workers unmatched.[14]

4 The Offer Game with Signals

We now modify the game so that each worker may send a "signal" to exactly one firm. A signal is a fixed message; that is, the only decision of workers is whether and to whom to send a signal. No decision can be made about the content of the signal. Note that the signal does not directly affect the utility a firm derives from a worker, as the firm's utility from hiring a worker is determined by how high the firm ranks that worker. However, the signal of a worker may affect a firm's beliefs over whether that worker is likely to *accept* an offer. Since we have a congested market where firms can only make one offer, these beliefs may affect the firm's decision of whom to make an offer. The offer game with signals proceeds in three stages:

1. Agents' preferences are realized. Each worker decides whether to send a signal, and to which firm. Signals are sent simultaneously, and are observed only by firms who have received them.

2. Each firm makes an offer to at most one worker; offers are made simultaneously.

3. Each worker accepts at most one offer from the set of offers she receives.

Once again, sequential rationality ensures that workers will always select the best available offer. Hence, we take this behavior for workers as given and focus on the reduced game consisting of the first two stages.

In the first stage, each worker sends a signal to a firm, or else chooses not to send a signal. A mixed strategy for worker w is a map from the set of all possible preference lists to the set of distributions over the union of firms and the no-signal option, denoted by $\mathcal{N}$; that is, $\sigma_w : \Theta_w \to \Delta(\mathcal{F} \cup \mathcal{N})$. In the second stage, each firm observes the set of workers that sent it a signal, $\mathcal{W}^S \subset \mathcal{W}$, and based on these signals forms beliefs $\mu_f(\cdot|\mathcal{W}^S)$ about the preferences of workers. Each firm, based on these beliefs as well as its preferences, decides whether and to whom to make an offer. A mixed strategy of firm f is a map from the set of all possible preference lists, Θ_f, and the set of all possible combinations of received signals,

[14]Note that our model of a congested market is reminiscent of the micro-foundations for the matching function in the search literature (see e.g. Pissarides, 2000).

$2^{\mathcal{W}}$, which is the set of all subsets of workers, to the set of distributions over the union of workers and the no-offer option. That is, $\sigma_f : \Theta_f \times 2^{\mathcal{W}} \to \Delta(\mathcal{W} \cup \mathcal{N})$. We denote a profile of all worker and firm strategies as $\sigma_W = (\sigma_{w_1}, ...\sigma_{w_W})$ and $\sigma_F = (\sigma_{f_1}, ...\sigma_{f_F})$ respectively.

The payoff to firm f is a function of firm and worker strategies and realized agent types, which we again denote as $\pi_f : (\Sigma_w)^W \times (\Sigma_f)^F \times \Theta \to \mathbf{R}$. Similarly, define the payoff of workers as $\pi_w : (\Sigma_w)^W \times (\Sigma_f)^F \times \Theta \to \mathbf{R}$. As the offer game with signals is a multi-stage game of incomplete information, we consider sequential equilibrium as the solution concept.

Definition 3. The strategy profile $\hat{\sigma} = (\hat{\sigma}_W, \hat{\sigma}_F)$ and posterior beliefs $\hat{\mu}_f(\cdot|\mathcal{W}^S)$ for each firm f and each subset of workers $\mathcal{W}^S \subset \mathcal{W}$ are a sequential equilibrium if

- for any $w \in \mathcal{W}$, $\bar{\theta}_w \in \Theta_W : \hat{\sigma}_w(\bar{\theta}_w) \in \arg\max_{\sigma_w \in \Sigma_w} \mathbb{E}_{\theta_{-w}}(\pi_w(\sigma_w, \hat{\sigma}_{-w}, \theta) \mid \bar{\theta}_w)$,
- for any $f \in \mathcal{F}$, $\bar{\theta}_f \in \Theta_f$, $\mathcal{W}^S \subset \mathcal{W}$:

  $\hat{\sigma}_f(\bar{\theta}_f, \mathcal{W}^S) \in \arg\max_{\sigma_f \in \Sigma_f} \mathbb{E}_{\theta_{-f}}(\pi_f(\sigma_f, \hat{\sigma}_{-f}, \theta) \mid \bar{\theta}_f, \mathcal{W}^S, \hat{\mu}_f)$,

where $\hat{\sigma}_{-a}$ denotes the strategies of all agents except a, for $a = w, f$, and beliefs are defined using Bayes' rule.[15]

We again focus on equilibria where agents use anonymous strategies, thereby eliminating unrealistic sources of coordination.

Definition 4. Firm f's strategy σ_f is *anonymous* if for any permutation ρ, preference profile $\theta_f \in \Theta_f$, and subset of workers $\mathcal{W}^S \subset \mathcal{W}$ who send f a signal, we have $\sigma_f(\rho(\theta_f), \rho(\mathcal{W}^S)) = \rho(\sigma_f(\theta_f, \mathcal{W}^S))$. Worker w's strategy σ_w is *anonymous* if for any permutation ρ and preference profile $\theta_w \in \Theta_w$, we have $\sigma_w(\rho(\theta_w)) = \rho(\sigma_w(\theta_w))$.

4.1 Equilibrium Analysis

To analyze equilibrium behavior, we first turn to the workers' choice of whether and to whom to send a signal. In any symmetric equilibrium in which workers send signals and signals are interpreted as a sign of interest by firms and hence increase the chance of receiving an offer, *each worker sends her signal to her most preferred firm.* Since sending a signal to any firm will lead to identical probabilities of receiving an offer, it is optimal for each worker to simply send its signal to its highest ranked firm (see Proposition 4 in Section 6, which provides the analog of this statement for a more general setup).

[15] As usual in a sequential equilibrium, permissible off-equilibrium beliefs are defined by considering the limits of completely mixed strategies.

Note that babbling equilibria in which no information is transmitted via signals also exist. In one form of such equilibria, firms ignore signals and workers randomize any signals they send across firms. In another version, workers do not send signals, and firms interpret unexpected signals negatively. Note however that equilibria where firms interpret off-equilibrium signals negatively fail to survive standard equilibrium refinements (see Section 6 for details).

Finally, "perverse" equilibria, where firms interpret signals negatively, e.g. as a sign of a particular lack of interest in such a position, and workers nevertheless send such signals do not exist. This is because workers may always opt against sending a signal. We focus on non-babbling equilibria, in which each worker sends a signal only to her most preferred firm.

Hence, we have pinned down worker equilibrium behavior: workers send a signal to their highest ranked firm, and workers accept the best available offer. We now examine offers of firms in the second stage of the game, taking the strategies of workers and beliefs of firms about interpreting signals as given.[16]

Call f's most preferred worker T_f (f's top-ranked worker). Consider a firm f that has received signals from a subset of workers $\mathcal{W}^S \subset \mathcal{W}$. Call f's most preferred worker in this subset S_f (f's most preferred signaling worker).

Whenever workers signal to their most preferred firm, and other firms use anonymous strategies, f's offer choice is reduced to a binary decision between making an offer to the top ranked worker, T_f, and the most preferred (potentially) lower-ranked worker who has signaled it, S_f. When the two coincide, that is when $T_f = S_f$, there is no tradeoff, and firm f will make an offer to this worker. The expected payoff to f from making an offer to T_f or S_f (whichever yields greater payoff) is strictly greater than the payoff from making an offer to any other worker. This follows from the symmetry of worker preferences and strategies and the anonymity of firm strategies: for any two workers who sent a signal, f's expectation that these workers will accept an offer is identical. Hence, if f makes an offer to a worker who sent a signal, it should make that offer to the worker it prefers the most among them. The same logic holds for any two workers who have not sent a signal. (Propositions A2 and A3 in Appendix A.2 provide a rigorous argument for the above statements).

This suggests a special kind of strategy for firms, which we will call a *cutoff strategy*.

Definition 5 (Cutoff Strategies)**.** Strategy σ_f is a *cutoff strategy* for firm f if $\exists j_1, \ldots, j_W \in \{1, \ldots, W\}$, such that for any $\theta_f \in \Theta_f$ and any set $\mathcal{W}^S$ of workers who sent a signal,

$$\sigma_f(\theta_f, \mathcal{W}^S) = \begin{cases} S_f & \text{if } \mathrm{rank}_{\theta_f}(S_f) \leq j_{|\mathcal{W}^S|} \\ T_f & \text{otherwise.} \end{cases}$$

[16] Note that in any non-babbling symmetric equilibrium, all information sets for firms are realized with positive probability. Hence, firm beliefs are determined by Bayes' Law: if a firm receives a signal from a worker, it believes that worker ranks the firm first in her preference list.

We call $(j_1, \dots, j_W)$ f's *cutoff vector*, which has as its components *cutoffs* for each positive number $|\mathcal{W}^S|$ of received signals.

A firm f which employs a cutoff strategy need only look at the rank of the most preferred worker who sent it a signal, conditional on the number of signals f has received. If the rank of this worker is below a certain cutoff (lower ranks are better since one is the most preferred rank), then the firm makes an offer to this most preferred signaling worker S_f. Otherwise the firm makes an offer to its overall top ranked worker T_f. Cutoffs may in general depend on the number of signals the firm receives. This is because the number of signals received provides information about the signals the other firms received. This in turn affects the behavior of other firms and hence the optimal decision for firm f. Note that any cutoff strategy is, by definition, an anonymous strategy.

While we defined cutoffs as integers, we can extend the definition to include all real numbers in the range $(1, W)$ by letting a cutoff $j + \lambda$, where $\lambda \in (0, 1)$, correspond to mixing between cutoff j and cutoff $j + 1$ with probabilities $1 - \lambda$ and λ respectively.[17]

Cutoff strategies are not only intuitive but also *optimal* strategies for firms. Whenever other firms use anonymous strategies and workers signal to their most preferred firms, for any strategy of firm f there exists a cutoff strategy that provides firm f with a weakly higher expected payoff (see Proposition A3). This is due to the fact that the preferences of firms and strategies of workers are symmetric. Consequently, the probability that firm f's offer to T_f or S_f will be accepted depends only on the number of signals firm f receives, and not on the identity of the signaling workers. Hence, if f finds it optimal to make an offer to S_f, it will certainly make an offer to a more preferred S_f, provided the number of signals it receives is the same. The equilibrium results in this paper will all involve firms using cutoff strategies.

Since cutoff strategies can be represented by cutoff vectors, we can impose a natural partial order on them: firm f's cutoff strategy σ'_f is greater than cutoff strategy σ_f if all cutoffs of σ'_f are weakly greater than all cutoffs of σ_f and at least one of them is strictly greater. We say that firm f *responds more* to signals than firm f' when σ_f is greater than $\sigma_{f'}$.

We now examine how a firm should adjust its behavior in response to changes in the behavior of opponents. We find that responding to signals is a case of strategic complements.

Proposition 2 (Strategic Complements)**.** *Suppose workers send signals to their most preferred firms and accept their best available offer, and suppose all firms use cutoff strategies*

[17] This is equivalent to f making offers to S_f when S_f is ranked better than j, randomizing between T_f and S_f when S_f has rank exactly j, and making offers to T_f otherwise.

and firm f uses a cutoff strategy that is a best response. If one of the other firms responds more to signals, then the best response for firm f is to also weakly respond more to signals.

When other firms make offers to workers who have signaled to them, it is risky for firm f to make an offer to a worker who has not signaled to it. Such a worker has signaled to another firm, which is more inclined to make her an offer. The greater this inclination on the part of the firm's opponents, the riskier it is for firm f to make an offer to its most preferred overall worker T_f. Hence as a response, firm f is also more inclined to make an offer to its most preferred worker among those who sent a signal, namely S_f.

The strategic complements result allows us to apply Theorem 5 from Milgrom and Roberts (1990) to demonstrate the existence of symmetric equilibria in pure cutoff strategies with smallest and largest cutoffs (see the proof of Theorem 1 in Appendix A.1 for details).

Theorem 1 (Equilibrium Existence)**.** *In the offer game with signals, there exists a symmetric equilibrium in pure cutoff strategies where 1) workers signal to their most preferred firms and accept their best available offer and 2) firms use symmetric cutoff strategies. Furthermore, there exist pure symmetric equilibria with smallest and largest cutoffs.*

5 The Welfare Effects of Introducing a Signaling Mechanism

We have analyzed the unique equilibrium in the offer game with no signals, and we have studied symmetric equilibria in markets with a signaling mechanism. We focused on non-babbling equilibria where firms interpret signals of workers as a sign of interest, and hence each worker sends a signal to her most preferred firm. In this section we address the effect of introducing a signaling mechanism on the market outcome. We consider three outcome measures: the number of matches in the market, the welfare of firms and the welfare of workers, where for agent welfare comparisons we consider Pareto ex-ante expected utility as our criterion.

Our analysis begins with an incremental approach: we first study the effect of a single firm increasing its cutoff, that is, responding more to signals. We then rank various signaling equilibria in terms of their outcomes. Finally, we address how the introduction of a signaling mechanism impacts the three measures of welfare.

The expected welfare for a firm f and a worker w are captured by π_f and π_w respectively, where $\pi_f, \pi_w : \Sigma_w^W \times \Sigma_f^F \times \Theta \to \mathbf{R}$. Let the function $m : (\Sigma_w)^W \times (\Sigma_f)^F \times \Theta \to \mathbf{R}$ denote the expected total number of matches in the market as a function of agent strategies and types. In this section we restrict the analysis to cutoff strategies.

Consider the offer game with signals, in which workers send their signal to their first choice firms, and firms interpret these signals as signs of interest. Fix the strategies of all other firms, and assume that one firm changes its strategy to respond more to signals. How does this affect the number of matches, and the workers and firms welfare?

Proposition 3. *Consider any strategy profile in which firms use cutoff strategies, workers send signals to their most preferred firms, and workers accept their best available offer. Fix the strategies of all firms but f as σ_{-f}. Let firm f's strategy σ'_f differ from σ_f only in that σ'_f responds more to signals, that is, has higher cutoffs than σ_f. Then*

- *The expected number of matches increases. That is, $\mathbb{E}_\theta[\, m(\sigma'_f, \sigma_{-f}, \theta)\,] \geq \mathbb{E}_\theta[\, m(\sigma_f, \sigma_{-f}, \theta)\,]$.*
- *The expected payoff of each worker increases. That is, for each $w \in \mathcal{W}$, $\mathbb{E}_\theta[\, \pi_w(\sigma'_f, \sigma_{-f}, \theta)\,] \geq \mathbb{E}_\theta[\, \pi_w(\sigma_f, \sigma_{-f}, \theta)\,]$.*
- *The expected payoffs of all firms but f decrease. That is, for each $f' \in -f$, $\mathbb{E}_\theta[\, \pi_{f'}(\sigma'_f, \sigma_{-f}, \theta)\,] \leq \mathbb{E}_\theta[\, \pi_{f'}(\sigma_f, \sigma_{-f}, \theta)\,]$ (negative spillover on opponent firms).*

When at least one firm in $-f$ responds to signals, that is has a cutoff strictly greater than one for some number of received signals, then all inequalities are strict.

To understand the first result, observe that when firm f switches its offer from its first choice worker T_f to its most preferred signaling worker S_f, it is the other offers received by these two workers that determine the impact on the total number of matches. If both workers have other offers, or if neither has another offer, the number of matches is unaffected. Only when exactly one of these two workers has another offer does f's switch from T_f to S_f affect the number of matches. However, conditional on exactly one of these having another offer, it is weakly more likely to be T_f, as this worker has signaled to another firm, while S_f has not. Furthermore, T_f is *strictly* more likely than S_f to have another offer when at least one other firm responds to signals. Hence, making an offer to S_f leads to a greater expected total number of matches.

In addition to creating more matches in expectation, a firm responding more to signals unambiguously increases expected worker welfare. Note that when firm f changes its offer from T_f to S_f, then worker S_f receives an offer from her first choice firm, while worker T_f loses an offer from a firm she ranks second or worse. Hence, when the number of matches is unchanged, average worker welfare increases. Furthermore, it is more likely that the number of matches increases rather than decreases, and once more each match 'gained' is one where a worker receives her first choice firm, while each match 'lost' is one where a worker receives a firm of her second choice or worse. It follows that in expectation, each worker gains when a firm starts responding more to signals.

In contrast, a firm f responding more to signals has a negative effect on the welfare of other firms. When firm f makes an offer to T_f, this offer may be rejected, as T_f may prefer other firms to firm f. But an offer from f to S_f creates "stiff" competition for competing firms, since this worker will accept f's offer with certainty, and offers from other firms will be rejected. Additionally, f's switch from T_f to S_f may only be pivotal for opponent firms making "risky" offers to their top ranked workers. Such offers are more likely to be made to S_f than to T_f since by signaling, S_f has indicated she prefers f. Hence, in addition to creating *stiffer* competition for $-f$, f's switch from T_f to S_f creates *more* competition for $-f$. The combination of these two effects gives the negative spillover result.

We now use the incremental welfare results to compare welfare across equilibria. The following corollary states that for all three of our welfare measures, there is a clear ranking of any two symmetric equilibria that can be ordered by their cutoffs.

Corollary 1. *Consider any two symmetric cutoff strategy equilibria where in one equilibrium firms have greater cutoffs (respond more to signals). Compared to the equilibrium with lower cutoffs, in the equilibrium with greater cutoffs we have the following: (i) the expected number of matches is weakly greater, (ii) workers have weakly higher expected payoffs, and (iii) firms have weakly lower expected payoffs.*

Corollary 1 states that firms and workers are *opposed* in their preferences over equilibria.[18] When multiple symmetric equilibria exist, workers prefer the equilibrium that involves firms responding the most to signals, that is the greatest cutoffs, while firms prefer the equilibrium with the lowest cutoffs.

We can now address the effect of adding a signaling mechanism to an offer game with no signals. We will assume that the equilibrium once the signaling mechanism is introduced is one of the symmetric non-babbling equilibria. Using the results above, we can show that introducing a signaling mechanism weakly increases the welfare of workers and the expected number of matches. Furthermore, the inequality is strict if firms respond to signals at all; that is, if for at least some number of signals, firms use strategies that call for an offer to a worker who signaled, S_f, even when she is not the first choice worker T_f. In contrast, firm welfare cannot be compared. As the example in Section 2 illustrates, firm welfare may be higher with or without a signaling mechanism. The following theorem encapsulates these results.

[18] Suppose that when we have a class of cutoff strategies that are strategically equivalent, we allow firms to only use the lowest one. For example, when we have W workers, a firm with $k > 1$ signals may have cutoffs of $W - k + 1$ and $W - k + 2$ that are strategically equivalent: the firm always makes an offer to S_f. When we focus on the lowest strategically equivalent cutoff, then all inequalities of Corollary 1 are strict.

Theorem 2 (Welfare)**.** *Consider any non-babbling symmetric equilibrium of the offer game with signals in which for at least some number of signals, firm strategies call for an offer to the signaling worker,* S_f*, even when she is not the first choice worker* T_f*. Then the following three statements hold.*

i. The expected number of matches is strictly greater than in the unique equilibrium of the offer game with no signals.

ii. The expected welfare of workers is strictly greater than in the unique equilibrium of the offer game with no signals.

iii. The welfare of firms may be greater or smaller than in the unique equilibrium of the offer game with no signals.

When introducing a signaling mechanism hurts firm welfare, it is because the negative externality outweighs the individual firm benefit from responding to signals. The theorem discusses the case in which firms respond to at least some degree to signals in equilibrium. Note that when there is a symmetric non-babbling equilibrium where firms ignore signals, then this equilibrium is outcome equivalent to a market without a signaling mechanism, so that agents are no worse off with the signaling mechanism. But provided firms respond even minimally to signals in equilibrium, with the introduction of a signaling mechanism, the expected number of matches and the expected welfare for workers increase unambiguously.

6 Block Correlation

So far we assumed that worker preferences are symmetric, uniform and independent. In non-babbling sequential equilibria, this implies that workers send their signal to their most preferred firm, and a firm that received a signal could be certain that an offer would be accepted. In this section we relax the assumption that worker preferences are uncorrelated. More precisely, we consider a market where firms can be partitioned in blocks, so that all workers agree which block contains the most desirable firms, which block the second most desirable set of firms and so on. However, within a block, workers may have idiosyncratic preferences over firms. Hence, for this section we consider markets where agent preferences are *block-correlated.*

Definition 6. A *block-correlated market* is a market $\langle \mathcal{F}, \mathcal{W}, t, u, v \rangle$ such that for a partition $\mathcal{F}_1, \ldots, \mathcal{F}_B$ of the firms into blocks, ordinal preferences (as encompassed in $t(\cdot)$) are such that

1. For any $b < b'$, where $b, b' \in \{1, \dots, B\}$, each worker prefers every firm in block $\mathcal{F}_b$ to any firm in block $\mathcal{F}_{b'}$;

2. Each worker's preferences within each block $\mathcal{F}_b$ are uniform and independent; and

3. Each firm's preferences over workers are uniform and independent.

We call distributions $t(\cdot)$ that satisfy the criteria in Definition 6 *block uniform.* The environment analyzed in previous sections is a special case of block-correlated markets, where there is only one block of firms. Block-correlated markets are meant to capture the notion that many two-sided markets are segmented. That is, workers may largely agree on the ranking of blocks on the other side of the market, but vary in their preferences within each block. For example, workers might agree on the set of firms that constitute the "top tier" of the market; however within that tier, preferences are influenced by factors specific to each worker.

We again focus on equilibria where agents use anonymous strategies. For firms we maintain the notion of anonymous strategies introduced in Definitions 2 and 4. For workers we only consider permutations P^B that permute firm orderings within blocks; that is, permutation $\rho \in P^B$ if for any firm f and any block b, if $f \in \mathcal{F}_b$ then $\rho(f) \in \mathcal{F}_b$

Definition 7. Worker w's strategy σ_w is *anonymous* if for any permutation $\rho \in P^B$ and preference profile $\theta_w \in \Theta_w$, we have $\sigma_w(\rho(\theta_w)) = \rho(\sigma_w(\theta_w))$.

As previously, let us first consider the offer game with no signals. Since worker preferences are still uniformly distributed there is again a unique equilibrium where firms use anonymous strategies: each firm optimally makes an offer to the highest-ranked worker on its preference list.

We now turn to the offer game with signals, where we will be interested in equilibria where firms within each block play symmetric, anonymous strategies. That is, if firm f and firm f' belong to the same block $\mathcal{F}_b$, for some $b \in \{1, ..., B\}$, they play the same anonymous strategies and have the same beliefs. We call such firm strategies and firm beliefs *block-symmetric.* We denote equilibria where firm strategies and firm beliefs are block-symmetric and worker strategies are anonymous and symmetric as *block-symmetric equilibria.* Before we can characterize the set of block-symmetric equilibria, we discuss the strategies of workers, who must choose whether to send a signal, and if so, to which firm. In block-symmetric equilibria, firms within each block $\mathcal{F}_b$ use the same anonymous strategies. Hence, we can denote the ex-ante probability of a worker w receiving an offer from a firm in block $\mathcal{F}_b$, conditional on w sending and not sending a signal to it as p_b^s and p_b^{ns} correspondingly. We also denote the equilibrium probability that a worker sends her signal to a firm in block $\mathcal{F}_b$

as α_b, where $\alpha_b \in [0, 1]$ and $\sum_{b=1}^{B} \alpha_b \leq 1$, where of course the α_b's are not independent as each worker may only send at most one signal.

The following proposition characterizes worker strategies in all block-symmetric sequential equilibria that satisfy an analog of Criterion D1 of Cho and Kreps (1987).[19]

Proposition 4 (Worker Strategies)**.** *Consider a block-symmetric sequential equilibrium that satisfies Criterion D1. Then either*

1. *Signals do not influence offers: for every $b \in \{1, ..., B\}$, $p_b^s = p_b^{ns}$ or*

2. *Signals sent in equilibrium increase the chances of receiving an offer: there exists $b_0 \in \{1, ..., B\}$ such that $p_{b_0}^s > p_{b_0}^{ns}$ and*

   (a) for any $b \in \{1, ..., B\}$ such that $\alpha_b > 0$, we have $p_b^s > p_b^{ns}$, and if a worker sends her signal to block $\mathcal{F}_b$, she sends her signal to her most preferred firm within $\mathcal{F}_b$, and

   (b) for any $b' \in \{1, ..., B\}$ such that $\alpha_{b'} = 0$, workers' strategies are optimal for any off-equilibrium beliefs of firms from block $\mathcal{F}_{b'}$.

Proposition 4 states that there are two types of block-symmetric equilibria that satisfy Criterion D1. Equilibria of the first type are babbling, where firms ignore signals. The outcomes of these equilibria coincide with the outcome in the offer games with no signals. Consequently, the signaling mechanism adds no value in this case.

In equilibria of the second type, workers send signals only to their most preferred firm in each block, possibly mixing across these top firms. We show that in equilibrium workers only send signals to blocks in which firms respond to signals, that is the chances of receiving an offer from the firm they signaled to is higher than if they had not sent that signal. Moreover, if in equilibrium worker w is not prescribed to signal to some block $\mathcal{F}_{b'}$, then w's choice of $\alpha_{b'} = 0$ is optimal for any beliefs of firms in block $\mathcal{F}_{b'}$. In particular, this strategy would be optimal even if firms in block $\mathcal{F}_{b'}$ interpreted signals in the most favorable way for worker w; i.e., upon receiving a signal from worker w each firm f in $\mathcal{F}_{b'}$ believes that it is w's most preferred firm within block $\mathcal{F}_{b'}$.

We call all strategies where a worker who sends a signal to firms in block b sends it to her most preferred firm in that block *best-in-block strategies.* We call all beliefs where a firm

[19] Criterion D1 lets us characterize beliefs when firms receive "unexpected," or off-equilibrium, signals. See the proof of Proposition 4 for the definition of our analog of Criterion D1 of Cho and Kreps (1987). Other refinements could also be used in our equilibrium characterization: for example, we could replace Criterion D1 with "universal divinity" of Banks and Sobel (1987) or by "never a weak best response" of Cho and Kreps (1987) without making a change to the statement of Proposition 4.

interprets a signal from a worker w as indicating it is the most preferred firm of w in that block *best-in-block beliefs*. We will now assume that workers use symmetric best-in-block strategies and that firms have best-in-block beliefs, and examine firm offers in the second stage of the game.[20]

An important difference between the single block and multi-block settings is that when there are multiple blocks, offers to workers who have signaled are no longer guaranteed to be accepted. This is because a firm that receives a signal knows that while it is the worker's most preferred firm in the block, the worker may receive an offer from a firm in a superior block. Nevertheless, several results about the strategies of firms carry over when we introduce block correlation. In a block-correlated market, firm f's offer choice is again reduced to a binary decision between T_f and S_f, provided workers use symmetric best-in-block strategies and firms $-f$ use anonymous strategies. Under these same conditions, cutoff strategies are again optimal for f. The strategic complements result of Proposition 2 also carries over; if firms $-f$ use cutoff strategies and workers use symmetric best-in-block strategies, then when $f' \in -f$ responds more to cutoffs, f optimally responds more to cutoffs as well (see Propositions A2, A3, and A4).

The next result establishes the existence of equilibria in block correlated settings in the offer game with signals. To prove the theorem, we first demonstrate equilibrium existence while requiring firms to use only cutoff strategies. We then invoke the optimality of cutoffs result to show that this step is not restrictive.

Theorem 3 (Equilibrium Existence under Block Correlation)**.** *There exists a block-symmetric equilibrium where 1) workers play symmetric best-in-block strategies, and 2) firms play block-symmetric cutoff strategies.*

In contrast to Theorem 1 which established equilibrium existence when there is a single block, equilibria here may involve mixed strategies for workers; that is, each worker may signal with positive probability to multiple blocks.

The final result of the section extends the welfare results of Theorem 2. Note that for the comparisons in the theorem to be strict, we require a block with at least two firms where in equilibrium, workers send signals with positive probability to that block. Without this condition, we only have weak comparisons.

[20]Note that firms have best-in-block beliefs on the equilibrium path in any block-symmetric equilibrium. In addition, a block-symmetric equilibrium satisfies Criterion D1 if and only if worker strategies remain optimal if firm off-equilibrium beliefs were best-in-block beliefs. Hence, we will focus on equilibria where firms have best-in-block beliefs even off the equilibrium path. See the proof of Proposition 4 in Appendix A.2 for details.

Theorem 4 (Welfare under Block Correlation)**.** *Consider any non-babbling block-symmetric equilibrium of the offer game with signals, in which there is a block $\mathcal{F}_b$ with at least two firms such that $\alpha_b > 0$. Then,*

i. The expected number of matches is strictly greater than in the unique equilibrium of the offer game with no signals.

ii. The expected welfare of workers is strictly greater than in the unique equilibrium of the offer game with no signals.

iii. The welfare of firms may be greater or smaller than in the unique equilibrium of the offer game with no signals.

Note that while the welfare comparisons with and without a signaling mechanism generalize to block correlated markets, the welfare comparisons across equilibria (see Corollary 1) do not generalize. In particular, when there are multiple blocks, when a single firm responds more to signals, firms in lower ranked blocks may benefit. Hence, we no longer see a purely negative spillover on other firms, which was a key step in establishing the welfare ranking.[21]

However, even when workers have correlated preferences, so that receiving a signal does not translate to a guaranteed match for a firm, we find that introducing a signaling mechanism increases the expected number of matches and the expected welfare of workers.

7 Market Structure and The Value of a Signaling Mechanism

In this section, we analyze the effects of introducing a signaling mechanism across different market structures. More precisely, we study the increase in the expected number of matches due to the introduction of a signaling mechanism.

To isolate the impact of a signaling mechanism on the number of matches in the market, we consider a special case where agents want to match, but are nearly indifferent over whom they match with. That is, firms (and workers) play an (almost) *pure coordination* game amongst themselves. Specifically, we consider the cardinal utility from being matched to a partner as being *almost* the same across partners. If agent a has a preference profile θ_a, agent a prefers to be matched with partner θ_a^k, rather than with partner $\theta_a^{k'}$, $k' > k$, though

[21]Since offers to workers who have signaled are no longer guaranteed to be accepted, firms making offers to signaling workers may be affected by f's switch from T_f to S_f. In particular, firms in the *same or higher* blocks responding to signals will not be affected, but firms in lower blocks responding to signals *prefer* that f switch from T_f to S_f. There is a positive spillover on these firms, and negative spillover on all other firms.

the difference between utility intensities is very small.[22] In addition, there is only one block of firms, so that agent preferences are uniformly distributed.[23]

Under these assumptions, there is a unique non-babbling symmetric equilibrium in the offer game with signals. Each worker sends a signal to her most preferred firm. Each firm makes an offer to its most preferred worker that has signaled provided the firm receives at least one signal; otherwise, it makes an offer to its top-ranked worker (see Proposition B1).[24] Proposition 1 also applies in this setting; that is, there is a unique equilibrium of the offer game with no signals.

We denote the expected number of matches in the unique equilibrium in the pure coordination model with signals and with F firms and W workers as $m^S(F,W)$, and without a signaling mechanism as $m^{NS}(F,W)$. The increase in expected number of matches from the introduction of the signaling mechanism, which we term the *value of the signaling mechanism*, we denote as $V(F,W) \equiv m^S(F,W) - m^{NS}(F,W)$. Figure 1 graphs $100 \cdot V(F,W)/W$ as a function of F for fixed $W = 10$ and $W = 100$, and $100 \cdot V(F,W)/F$ as a function of W for fixed $F = 10$ and $F = 100$. That is, the figure depicts the increase in the expected number of matches proportional to the size of the side of the market we keep fixed (which places an upper bound on the total number of possible matches).

The figures suggest that the value of a signaling mechanism is single peaked when varying one side of the market and holding the other constant. That is, it seems that a signaling mechanism is most beneficial for balanced markets — markets where the the number of firms and the number of workers are roughly of the same magnitude. To understand why signaling may be useful in balanced markets, it is helpful to think about the endpoints. With many workers and very few firms, firms will almost certainly match with or without the signaling mechanism, as there is no large coordination problem. With many firms and few workers,

[22] The "nearly indifferent" condition for firms is that $u(W) > \frac{W}{F}\left(1-\left(1-\frac{1}{W}\right)^F\right)u(1)$, where $u(1)$ and $u(W)$ are firm utility from matching with first and last ranked workers, respectively. A complete specification of the setup can be found in Appendix B.2.

[23] Our pure coordination model has similarities to the "urn-ball" model in the labor literature, concisely described in a survey by Petrongolo and Pissarides (2001): "Firms play the role of urns and workers play the role of balls. An urn becomes "productive" when it has ball in it. [...] In the simplest version of this process U workers know exactly the location of V job vacancies and send one application each. If a vacancy receives one or more applications it selects an applicant at random and forms a match. The other applicants are returned to the pool of unemployed workers to apply again." Our pure coordination model effectively flips the urn-ball problem around. Workers apply to all jobs, and firms propose the offers. We have a non-random selection procedure, and of course in our model we study the role of signaling. Perhaps the paper with the closest market structure to ours is Julien, Kennes and King (2000).

[24] In this case, one can view the offer game with no signals as the result of the first round of a firm-proposing deferred acceptance algorithm. When workers send signals, the result resembles one round of a worker-proposing deferred acceptance with one exception: firms who received no offer (no signal from any worker) do get to make an offer.

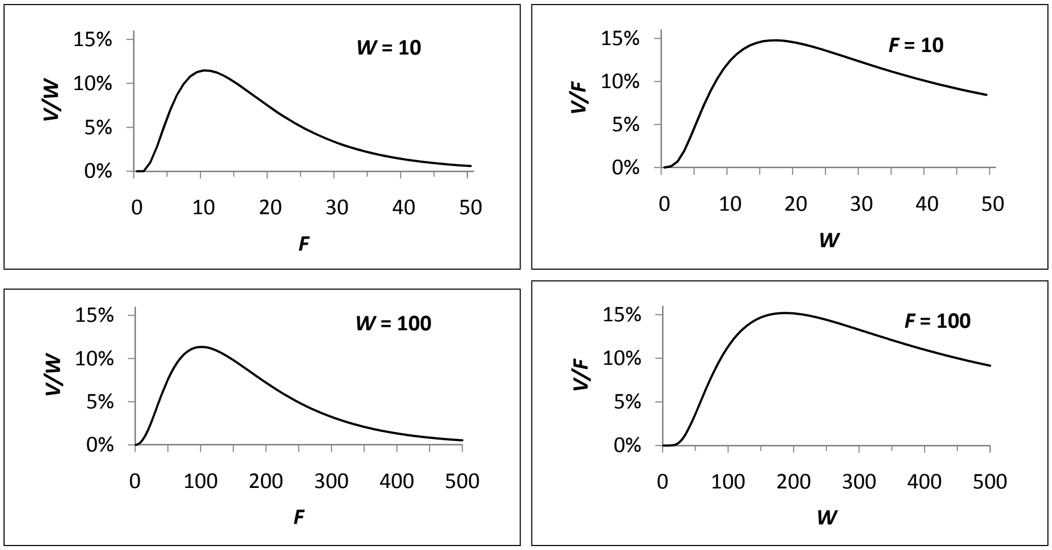

Figure 1: Balanced Markets: The proportional increase in the number of matches due to a signaling mechanism as we vary the number of firms for a fixed number of workers (left graphs) and vice versa (right graphs).

the reverse holds: most workers will get offers with or without the signaling mechanism. Hence, the signaling mechanism offers little benefit at the extremes. Furthermore, Figure 1 suggests that the proportional increase in the expected number of matches remains steady as market size increases, holding constant the ratio of workers to firms. Proposition 5 describes these observations precisely.

Proposition 5 (Balanced Markets)**.** *Consider markets with F firms and W workers. Then (i) for fixed W, $V(F,W)$ attains its maximum value at $F = x_0 W + O_W(1)$, where $x_0 \approx$* 1.01211 *and (ii) for fixed F, $V(F,W)$ attains its maximum value at $W = y_0 F + O_F(1)$, where $y_0 \approx 1.8442$.*

The proof of Proposition 5 involves the calculation of an explicit formula for $V(F,W)$. The expected increase in the number of matches can be represented as

$$V(F,W) = \alpha(\frac{W}{F})F + O_F(1)$$

or as

$$V(F,W) = \beta(\frac{F}{W})W + O_W(1),$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ are particular functions and $O_W(1)$ and $O_F(1)$ denote functions that are smaller than a constant for large W and for large F respectively. Hence, $V(F,W)$ is "almost" homogeneous of degree one for large markets. That is, the proportional increase in the number of matches, $V(F,W)/W$ and $V(F,W)/F$, is almost homogenous of degree zero.[25] As a consequence, we can evaluate the introduction of the signaling mechanism for a sample market, and its properties will be preserved for markets of other sizes, but with the same ratio of firms to workers.

For example, we can use Figure 1 to investigate maximal quantitative gains from the introduction of the signaling mechanism in large markets. For a fixed number of workers, the maximum increase in expected number of matches is approximately 15%. Furthermore, the returns to the signaling mechanism are substantial over a wide range of market conditions. For example, only when the number of firms outweighs the number of workers by more than fourfold do the gains from introducing the signaling mechanism drop to below 1%.

8 Discussion and Conclusion

Excessive applications by job market candidates lead to market congestion: employers must devote resources to evaluate and pursue potential candidates, but cannot give due attention to all. Evaluation is further complicated because employers must assess which applicants, many of whom are performing broad searches, are likely to ultimately accept a job offer.

Consequently, applicants are often eager to convey information about their interest in particular employers, and employers stand ready to act upon such information, if it can be deemed credible. However, in many markets indicating preferences is cheap, and employers may struggle to identify which preference information is sincere. This, in turn, may prevent any potential gains from preference signaling from being realized.

In this paper we examined how a signaling mechanism can overcome this credibility problem and improve agent welfare. In our model, workers are allowed to send a costless signal to a single firm. While participation is free and voluntary, this mechanism nevertheless provides workers with a means of credibly expressing preferences. In a symmetric setting where agent preferences are uncorrelated, workers will send their signal to their most preferred firm. Firms use this information as guidance, optimally using cutoff strategies to make offers. We find that on average, introducing a signaling technology increases both the expected number of matches as well as the expected welfare of workers. The welfare of firms, on the other hand, changes ambiguously, because firms responding more to signals may impose a negative externality on other firms. The results carry over when we consider a model where firms have many positions, and workers can send multiple signals.

[25]Note that this result corroborates the stylized fact in the empirical labor literature that the matching function (the expected number of matches) has a constant return to scale. See, for example, Petrongolo and Pissarides (2001) or Rogerson, Shimer and Wright (2005).

We showed that the results hold even when workers have correlated preferences, where workers agree on the ranking of blocks of firms but vary in their preferences within each block. In this case firm offers to workers who have signaled will not result in guaranteed acceptance. This is because workers will no longer send their signal to their most preferred firm, but rather will mix among the most preferred firms from each block. We showed further that introducing a signaling mechanism adds the most value for balanced markets, that is, markets in which the number of firms and the number of workers are of roughly the same magnitude.

One path for future research would be to characterize the full set of agent preferences where signaling is beneficial. While in this paper we find that signaling mechanisms can improve agent welfare under a broad class of preferences, for some agent preferences signaling can worsen outcomes. Kushnir (2009) models a high-information setting with minimal congestion where signals disturb firms' commonly held beliefs about workers preferences, which in turn disrupts the maximal matching. Kushnir's example corroborates the intuition that signals may be more useful in low information settings than in high. Further investigation of this question could be fruitful.

Another interesting question that is beyond the scope of the current paper concerns the optimal signaling mechanism. Providing candidates with one, or else a small number of identical signals offers a tractable approach, and participants may value its simplicity. But within the realm of mechanisms that offer candidates equal numbers of identical signals, how do we identify the optimal number of signals, especially in light of the fact that multiple equilbria may exist? And might we do even better?

If we expand the class of mechanisms under study, we can potentially improve performance even more. For example, the signaling mechanism that maximizes the number of matches may be asymmetric. Consider the example in Section 2, with two firms and two workers. In the example, each worker had exactly one signal. If both workers have and send two signals that are identical, outcomes are as if each had no signal. If we offered each worker two distinct signals, e.g. a 'gold' and a 'silver' signal, analysis is as if they had one signal each.

Asymmetric signaling capacities, however, can generate a full matching. Suppose that one worker has a gold signal, while the other has two silver signals. Suppose further that firms are indifferent between the two workers. Then one equilibrium involves the first worker sending its gold signal to its preferred firm. The firm that receives the gold signal will make the signaling worker an offer, while the firm who receives no gold signal will make an offer to the worker who sent a silver signal. Both firms and workers will always be matched.

The question of the optimal signaling mechanism, as well as the question of how the benefit from signaling varies across market structures, provide interesting areas for future research.

We wish to highlight that a signaling mechanism has the potential to improve outcomes in congested markets. Importantly, since signaling mechanisms are free, voluntary, and built on top of existing labor markets, these improvements come in a reasonably non-invasive manner. As opposed to a central clearinghouse, as in the National Resident Matching Program (c. f. Roth, 1984 and Roth and Peranson, 1999), a centralized signaling mechanism requires significantly less intervention. Market designers may find it easier to get consensus from participants to introduce such a mechanism, which nevertheless can offer significant benefits. As such, we hope that in addition to furthering our understanding of how labor markets work, our paper adds to the practical literature that aims at changing and improving existing markets.

A Appendix

A.1 Markets with a single block of firms

This portion of the appendix covers proofs for Sections 3-5. In this setup workers may send at most one signal, and there is a single block of firms. We omit proofs of Propositions 1 and 2 and Theorem 2 as these are special cases of Propositions A1 and A4 and Theorem 4 respectively.

Proof of Theorem 1. As discussed in Section 4, in any symmetric non-babbling equilibrium each worker sends its signal to its most preferred firm. Consequently, all information sets for firms are realized with positive probability, so firm beliefs are determined by Bayes' Law: if a firm receives a signal from a worker, it believes that worker ranks the firm first in its preference list. We now take these worker strategies and firm beliefs as fixed, and analyze the second stage of the game when firms choose offers. We will show that this reduced game is a supermodular game, and then use the results of Milgrom and Roberts (1990) to prove our theorem.

We analyze the game where we restrict firm strategies to be cutoff strategies. Denote the set of cutoff strategy profiles as Σ_{cut}, with typical element $\sigma = (\sigma_1, ..., \sigma_F)$. Recall that a cutoff strategy for firm f is a vector $\sigma_f = (j_f^1, ..., j_f^W)$ where j_f^k corresponds to the cutoff when firm f receives k signals. We will consider only strategies where each cutoff is a natural number, i.e. $j_f^k \in \{1, ..., W\}$. As defined on p.15, vector comparison yields a natural partial order on Σ_{cut}: $\sigma \geq_{\Sigma_{cut}} \sigma' \Leftrightarrow \sigma_f \geq \sigma'_f \Leftrightarrow j_f^k \geq \left(j_f^k\right)'$ for any $f \in \mathcal{F}$ and $k \in \{1, ..., W\}$. This partial order is reflexive, antisymmetric, and transitive.

To show that the second stage is a game with strategic complementarities, we need to verify that $E_\theta(\pi_f(\sigma_f, \sigma_{-f}, \theta))$ is supermodular in σ_f, and that $E_\theta(\pi_f(\sigma_f, \sigma_{-f}, \theta))$ has increasing differences in σ_f and σ_{-f}. The former is trivially true because when f shifts of one its cutoff vector components, thisdoes not influence the change in payoff from a shift of another cutoff vector component. Namely, if we consider $\sigma_f^1 = (..., j_l, ..., j_k, ...)$, $\sigma_f^2 = (..., j_l', ..., j_k, ...)$, $\sigma_f^3 = (..., j_l, ..., j_k', ...)$, and $\sigma_f^4 = (..., j_l', ..., j_k', ...)$ for some $l, k \in \{1, ..., W\}$, then

$$E_\theta(\pi_f(\sigma_f^1, \sigma_{-f}, \theta)) - E_\theta(\pi_f(\sigma_f^2, \sigma_{-f}, \theta)) = E_\theta(\pi_f(\sigma_f^3, \sigma_{-f}, \theta)) - E_\theta(\pi_f(\sigma_f^4, \sigma_{-f}, \theta)).$$

That $E_\theta(\pi_f(\sigma_f, \sigma_{-f}, \theta))$ has increasing differences in σ_f and σ_{-f} follows from Proposition 2. Namely, for any σ_f, σ_{-f}, σ_f', and σ_{-f}' such that $\sigma_f' \geq \sigma_f$ and $\sigma_{-f}' \geq \sigma_{-f}$ we have

$$E_\theta(\pi_f(\sigma_f', \sigma_{-f}', \theta)) - E_\theta(\pi_f(\sigma_f, \sigma_{-f}', \theta)) \geq E_\theta(\pi_f(\sigma_f', \sigma_{-f}, \theta)) - E_\theta(\pi_f(\sigma_f, \sigma_{-f}, \theta)).$$

Hence the second stage of the game, when firms choose their strategies, is a game with strategic complementarities. Since in our model firms are ex-ante symmetric, Theorem 5 of Milgrom and Roberts (1990) establishes the existence of largest and smallest symmetric pure strategy equilibria. □

Proof of Proposition 3. The first two results, increase in the expected number of matches and positive spillover on workers, are demonstrated in the proof of Theorem 4 in Section 6 (which considers a more general assumption on agent preferences). To avoid repetition, we do not present the proofs here. However, the third result, that responding to signals generates a negative spillover on opponent firms, is unique to the case when agent preferences are uniformly distributed, so we present the proof below.

Let firm f strategy σ_f differ from σ_f' in that σ_f' has weakly greater cutoffs. Consider some firm $f' \in -f$. For each preference list $\theta_{f'}$ and set of signals received $\mathcal{W}^S$, firm f' either makes an offer to $S_{f'}(\theta_{f'}, \mathcal{W}^S)$ or $T_{f'}(\theta_{f'}, \mathcal{W}^S)$. Observe that a change in strategy of firm f does not affect f''s payoff from making $S_{f'}$ an offer. This follows since each worker sends her signal to her most preferred firm, so offers to signaling workers are always accepted. However, as shown in the proof of Proposition 2, the probability that $T_{f'}$ accepts firm f''s offer weakly decreases. Hence, overall the expected payoff of firm $f' \in -f$ weakly decreases when firm f responds more to signals: $E_\theta(\pi_{f'}(\sigma_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_{f'}(\sigma_f', \sigma_{-f}, \theta))$. □

Proof of Corollary 1. That the expected number of matches and the expected welfare of workers are higher in the equilibrium with higher cutoffs is a straightforward consequence of iterated application of the first and the second parts of Proposition 3. In order to show that firms have lower expected payoffs in the equilibrium with greater cutoffs, we combine the third result of Proposition 3 with a simple equilibrium property. Consider two symmetric equilibria, where firms play cutoff strategies σ and σ', with $\sigma' \geq \sigma$. From the definition of an equilibrium strategy we have $E_\theta[\pi_f(\sigma_f, \sigma_{-f}, \theta)] \geq E_\theta[\pi_f(\sigma'_f, \sigma_{-f}, \theta)]$. The third result of Proposition 3 yields $E_\theta[\pi_f(\sigma'_f, \sigma_{-f}, \theta)] \geq E_\theta[\pi_f(\sigma'_f, \sigma'_{-f}, \theta)]$. Combining these inequalities yields $E_\theta[\pi_f(\sigma_f, \sigma_{-f}, \theta)] \geq E_\theta[\pi_f(\sigma'_f, \sigma'_{-f}, \theta)]$. $\square$

A.2 General block-correlated preferences

This portion of the the appendix covers proofs for Section 6. In this setup workers may send at most one signal, and worker preferences are block-correlated. We also introduce Propositions A1-A4 which formalize statements in the text. Proofs for these propositions are in the web appendix.

Proposition A1 (Equilibrium with no signals). *The unique equilibrium of the offer game with no signals when firms use anonymous strategies and workers accept the best available offer is $\sigma_f(\theta_f) = \theta_f^1$ for all $f \in \mathcal{F}$ and $\theta_f \in \Theta_f$.*

Proposition A2 (Binary nature of optimal firm offer). *Suppose firms $-f$ use anonymous strategies and workers use symmetric best-in-block strategies. Consider a firm f that receives signals from workers $\mathcal{W}^S \subset \mathcal{W}$. Then the expected payoff to f from making an offer to S_f is strictly greater than the payoff from making an offer to any other worker in $\mathcal{W}^S$. The expected payoff to firm f from making an offer to T_f is strictly greater than the payoff from making an offer to any other worker from set $\mathcal{W}/\mathcal{W}^S$.*

Proposition A3 (Optimality of cutoff strategies). *Suppose workers use symmetric* best-in-block *strategies and firms have best-in-block beliefs. Then for any strategy σ_f of firm f, there exists a cutoff strategy that provides f with a weakly higher expected payoff than σ_f for any anonymous strategies σ_{-f} of opponent firms $-f$.*

Proposition A4 (Strategic complements under block correlation). *Suppose workers play symmetric best-in-block strategies, and firms $-f$ use cutoff strategies. If firm $f' \in -f$ increases its cutoffs (responds more to signals), firm f will also optimally weakly increase its cutoffs.*

Proof of Proposition 4. We first define an analog of criterion $D1$ of Cho and Kreps for our setting.[26] Consider some block-symmetric sequential equilibrium. Fix strategies of all agents except worker w and firm f, which we denote as $\sigma_{-f,w}$. Fix also the beliefs of firms other than firm f, which we denote as μ_{-f}. We now analyze strategies of worker w and strategies and beliefs for firm f.

There are two cases where information sets for firms might be reached with zero probability (lie "off the equilibrium path") in a block-symmetric equilibrium. First, when the symmetric worker equilibrium strategy prescribes zero probability of sending a signal to a particular block, firms in these blocks would view signals from such workers as "unexpected." Second, when a firm anticipates receiving a signal with 100% probability, then *not* receiving a signal would correspond to an off-equilibrium information set. But by the anonymous strategies assumption, this can only happen in a block-symmetric equilibrium if the firm is the only one in its block. In this case, the symmetry of worker strategies would ensure that all workers send their signals to this firm with probability 1. Since signals then would not transmit information about worker types, this equilibrium is outcome equivalent to a babbling equilibrium. We will concentrate on the first type of off-equilibrium messages – "unexpected" signals.

Consider firm f's decision at an information set that includes a (hypothetical, off-equilibrium) signal from worker w. Denote the expected equilibrium payoff of firm f as u_f^* and the expected equilibrium payoff of worker w as u_w^*. For each possible type $\bar{\theta} \in \Theta_f$ for firm f and each set of signals that firm f could receive, we denote the mixed best response of firm f that has beliefs $\bar{\mu}$ as

$$MBR_f(\bar{\theta}, \mathcal{W}^S \cup w, \bar{\mu}) = \arg\max\nolimits_{\sigma_f \in \Sigma_f} E_{\theta_{-f}}(\pi_f(\sigma_f, \sigma_{-f}, \theta) \mid \theta_f = \bar{\theta},\ \mathcal{W}_f^S = \mathcal{W}^S \cup w,\ \mu_f = \bar{\mu}).$$

We then denote the mixed best response of firm f for all possible types and all possible profiles of signals it may receive conditional on receiving worker w's signal as

$$MBR_f(w, \bar{\mu}) = \{MBR_f(\bar{\theta}, \mathcal{W}^S \cup w, \bar{\mu}) \text{ for all } \bar{\theta} \in \Theta_f,\ \mathcal{W}^S \subset \mathcal{W}\}.$$

We denote the set of best responses of firm f to probability assessments concentrated on set $\Omega \subset \Theta_w$ as

$$MBR_f(w, \Omega) = \bigcup_{\{\mu_f : \mu_f(\Omega) = 1\}} MBR_f(w, \mu_f).$$

[26]See Cho and Kreps (1987) for the original definition.

Denote for any worker's type $t \in \Theta_w$

$$\begin{aligned} D_t &= \{\phi \in MBR_f(w, \Theta_w): \ u_w^*(t) < E_{\theta_{-w}}(\pi_w(\sigma_w, \phi, \sigma_{-w,f}, \theta) \mid \theta_w = t)\} \\ D_t^0 &= \{\phi \in MBR_f(w, \Theta_w): \ u_w^*(t) = E_{\theta_{-w}}(\pi_w(\sigma_w, \phi, \sigma_{-w,f}, \theta) \mid \theta_w = t)\}. \end{aligned}$$

Intuitively, set D_t (D_t^0) is the set of firm f strategies (consistent with f best responding to strategies of firms $-f$ and to some set of beliefs that places weight 1 on w signaling f) such that by signaling f, worker w of type t would receive an expected payoff greater than (equal to) her equilibrium payoff. We say that type t may be pruned from firm f's beliefs if firm f's off-equilibrium beliefs place zero probability on worker w being type t (upon f receiving a signal from her). Using the above notation, we now state our analog of criterion $D1$ as follows:

Criterion D1. Fix strategies of workers $-w$ and strategies and beliefs of firms $-f$. If for some type $t \in \Theta_w$ of worker w there exists a second type $t' \in \Theta_w$ with $D_t \cup D_t^0 \subseteq D_{t'}$, then t may be pruned from the domain of firm f's beliefs.

The intuition behind this criterion is that whenever type t of worker w either wishes to defect and send an off-equilibrium signal to firm f or is indifferent, some other type t' of worker w strictly wishes to defect. When we prune t for worker w from firm f's beliefs, we are interpreting that firm f finds it infinitely more likely that the off-equilibrium signal has come from type t' than from type t.

We first show that there cannot be a *block-symmetric* sequential equilibrium that satisfies Criterion D1 where sending a signal to a firm in some block $\mathcal{F}_b$, $b \in \{1, ..., B\}$ *reduces* the likelihood of receiving an offer, i.e. $p_b^s < p_b^{ns}$.

Let us assume that such a block-symmetric sequential equilibrium exists. If there are at least two workers, agents use anonymous block-symmetric strategies, and agents' types are uncorrelated, each worker is unmatched with positive probability. Then in equilibrium, certainly no worker sends her signal to a firm within block $\mathcal{F}_b$; she'd prefer to simply send no signal at all. Hence, it must be that a signal would reduce the probability of an offer for firms in some block not signaled in equilibrium. Following the definition of D_t, whenever it would be beneficial for some type $\theta_w \in \Theta_w$ to deviate from the equilibrium path and send her signal to firm f (which would require firm f making an offer to worker w), then it would be beneficial for any type $\theta'_w \in \Theta_w$ of worker w such that firm f is w's most preferred firm within block $\mathcal{F}_b$, to similarly deviate. Therefore, the only types (preference profiles) of worker w that are not pruned in firms' beliefs according to Criterion D1 are those where firm f is w's most preferred firm within block $\mathcal{F}_b$. Hence, given these beliefs, if it is optimal for firm f to make an offer to worker w when it does not receive a signal from her, it is optimal for firm f to make an offer to worker w when it receives her signal. This contradicts our initial assumption, and hence $p^s_{b_0} < p^{ns}_{b_0}$ cannot be part of any block-symmetric sequential equilibrium that satisfies Criterion D1.

We have established that $p^s_b \geq p^{ns}_b$ for each $b = 1, ..., B$. It is easy to observe that there exists a block-symmetric sequential equilibrium that satisfies Criterion D1 where for any $b = 1, ..., B$, $p^s_b = p^{ns}_b$. For example, each worker may randomize her signal across all firms with equal probability, independently of her preferences, and firms simply play the equilibrium strategies of the offer game with no signals. The equilibrium beliefs are trivially block-uniform since when a firm receives a signal from worker w, its beliefs coincide with the priors. Since all blocks are reached with positive probability in equilibrium, no off-equilibrium beliefs need be specified, and the equilibrium trivially satisfies Criterion D1.

Let us now consider the case when there exists $b_0 \in \{1, ..., B\}$, such that $p^s_{b_0} > p^{ns}_{b_0}$ in some block-symmetric sequential equilibrium. Recall that the equilibrium probability that a worker sends her signal to a firm within block $\mathcal{F}_b$ is denoted as α_b, where $\alpha_b \in [0, 1]$ and $\sum_{b=1}^{B} \alpha_b \leq 1$. Let us consider some block $\mathcal{F}_b (\neq \mathcal{F}_{b_0})$ such that $\alpha_b > 0$. As mentioned, if there are at least two workers, agents use anonymous block-symmetric strategies, and agents' types are uncorrelated, each worker is unmatched with positive probability in equilibrium. Therefore, $\alpha_b > 0$ and $p^s_b = p^{ns}_b$ are incompatible in an equilibrium (worker w can benefit by signaling to block $\mathcal{F}_{b_0}$ rather than block $\mathcal{F}_b$). Hence, if $p^s_b > p^{ns}_b$ then if worker w plans to send a signal to a firm in $\mathcal{F}_b$, it should be to her most preferred firm within this block, as this delivers the greatest expected payoff to her.

Now suppose there is some block $\mathcal{F}_{b'}$, $b' \in \{1, ..., B\}$, such that $\alpha_{b'} = 0$. Consider the decision of some firm $f \in \mathcal{F}_{b'}$ at an information set that includes a (hypothetical, off-equilibrium) signal from worker w. We have two cases: either there exists type $t \in \Theta_w$ of worker w such that $D_t \neq \varnothing$, or else for any type $t \in \Theta_w$, $D_t = \varnothing$.

We will first rule out the former case. Suppose there exists type $t \in \Theta_w$ of worker w such that $D_t \neq \varnothing$. That is, if worker w sends a signal to firm f, there exists a "reasonable" firm f strategy that delivers expected payoff to worker w of type t greater than her equilibrium payoff. However, any firm f offer that delivers payoff exceeding equilibrium payoff for worker w of type t, also delivers payoff exceeding equilibrium payoff for a worker w of type t' which prefers firm f to any other firm in block $\mathcal{F}_{b'}$. Therefore, the only firm f off-equilibrium beliefs that survive Criterion D1 are such that

$$\mu_f(\{\theta_w \in \Theta_w : \, f = \max_{\theta_w}(f' \in \mathcal{F}_{b'})\} \,|\, w \subset \mathcal{W}^S_f) = 1. \tag{A.2.1}$$

But since $D_{t'}$ and $D^0_{t'}$ consist of firm f best responses, it is optimal for firm f to indeed make an offer to worker w upon receiving her signal, provided f's beliefs are restricted to (A.2.1). This means that the equilibrium strategy of worker w of type t' (not sending a signal to firm f) is not optimal if firm f has beliefs (A.2.1). Therefore, there cannot exist type $t \in \Theta_w$ of worker w such that $D_t \neq \varnothing$.

Let us now consider the case where for any type $t \in \Theta_w$, we have $D_t = \varnothing$. That is, it is never beneficial for any type of worker to send an off-equilibrium signal, as no reasonable offers can be expected for any firm beliefs. Therefore, $\alpha_{b'} = 0$ is an equilibrium strategy for worker w independently of off-equilibrium beliefs of firm f. In particular, worker w's strategy is optimal for any off-equilibrium beliefs of firms in block $\mathcal{F}_{b'}$, even if each firm f has the most favorable possible beliefs about worker w, such as in (A.2.1).

Note that if there are at least two workers, the interaction between worker w and some firm f (fixing the strategies and beliefs of other agents) is a monotonic signaling game of Cho and Sobel (1990). The assumption of monotonicity is satisfied in our environment because each type of worker w prefers the same action of firm f, i.e. firm f making an offer to worker w. As a consequence, Criterion D1 is equivalent to "never a weak best response" of Cho and Kreps (1987) and "universal divinity" of Banks and Sobel (1987) in our setting. More detailed discussion of monotonic signaling games can be found in Cho and Sobel (1990). $\square$

Proof of Theorem 3. We first prove the theorem while requiring firms to use cutoff strategies and workers to use best-in-block strategies, and then show that this assumption is not restrictive. Denote a typical such strategy profile as $\sigma = (\sigma_F, \sigma_W)$ that consists of firm cutoff strategies $\sigma_F = (\sigma_{f_1}, ..., \sigma_{f_F})$ and worker best-in-block strategies strategies $\sigma_W = (\sigma_{w_1}, ..., \sigma_{w_W})$.

A strategy of firm f is a vector of real numbers of size W that specifies cutoff points for each positive number of signals firm f could receive, $\sigma_f = (j_f^1, ..., j_f^W)$, where j_f^l is a real number from the interval $[1, W]$ for each $l = 1, ..., W$. Denote the set of possible firm cutoff strategies as $\Sigma_f^{cut} = [1, W]^W$.

A best-in-block strategy of worker w is a vector of size B that specifies the probability that she sends her signal to her top firm of specific block $\sigma_w = (\alpha_w^1, ..., \alpha_w^B)$, where $\alpha_w^b \geq 0$ for each $b = 1, ..., B$ and $\sum_{b=1}^{B} \alpha_w^b \leq 1$. We denote the set of possible worker best-in-block strategies as $\Sigma_w^{block} = \{(\alpha^1, ..., \alpha^B) : \alpha^b \geq 0 \text{ and } \sum_{b=1}^{B} \alpha^b \leq 1\}$.

Let us also denote the expected payoff of worker w when she uses best-in-block strategy σ_w and the other agents use strategy σ_{-w} as[27]

$$U_w(\sigma_w, \sigma_{-w}) = E_\theta(\pi_w(\sigma_w, \sigma_{-w}, \theta))$$

[27] Note that the strategy of agents are anonymous. Therefore, they do not depend on particular realization of preferences.

and the expected payoff of firm f when it uses strategy σ_f and the other agents use strategies σ_{-f} as

$$U_f(\sigma_f, \sigma_{-f}) = E_\theta(\pi_f(\sigma_f, \sigma_{-f}, \theta)).$$

We introduce best reply correspondence $g : (\Sigma_f^{cut})^F \times (\Sigma_w^{block})^W \to 2^{(\Sigma_f^{cut})^F \times (\Sigma_w^{block})^W}$ such that

$$g_f(\sigma) = \arg\max_{\beta \in \Sigma_f^{cut}} U_w(\beta, \sigma_{-w})$$

for each $f \in \mathcal{F}$ and

$$g_w(\sigma) = \arg\max_{\beta \in \Sigma_w^{block}} U_a(\beta, \sigma_{-w})$$

for each $w \in \mathcal{W}$.

An immediate consequence of the above definitions is that Σ_f^{cut} and Σ_w^{block} are *non-empty, convex,* and *compact*. Also, $U_w(\sigma_w, \sigma_{-w})$ is a linear function of its first argument. Namely, let us denote the expected payoff of worker w from sending a signal to some block $\mathcal{F}_b$ given the strategies of agents σ_{-w} as $\Pi_b(-\sigma_w)$. If worker w employs strategy $\sigma_w = (\alpha_w^1, ..., \alpha_w^B)$, her payoff equals

$$U_w(\sigma_w, \sigma_{-w}) = \sum_{b=1}^{B} \alpha^b \Pi_b(-\sigma_w).$$

Therefore, $g_w(\sigma)$ is a continuous correspondence with closed graph.

Let us now consider function $U_f(\sigma_f, \sigma_{-f})$. Similarly, let us consider some realized preference profile θ when firm f receives $|\mathcal{W}^S|$ signals. Given the strategies σ_{-f} of other agents, we denote the expected payoff of firm f from making an offer to T_f as Π_T, and the expected payoff of firm f from making an offer to S_f as Π_S. We then evaluate the payoff for firm f from using cutoff strategy $j_{|\mathcal{W}^S|}$, $\sigma_f = (..., j_{|\mathcal{W}^S|}, ...)$ as

$$\pi_f(\sigma_f, \sigma_{-f}, \theta) = \begin{cases} \Pi_T & \text{if } j_{|\mathcal{W}^S|} \leq rank(S_f) - 1 \\ (\lceil j_{|\mathcal{W}^S|} \rceil - j_{|\mathcal{W}^S|})\Pi_S + (j_{|\mathcal{W}^S|} - \lfloor j_{|\mathcal{W}^S|} \rfloor)\Pi_T & \text{if } j_{|\mathcal{W}^S|} \in (rank(S_f) - 1, rank(S_f)) \\ \Pi_S & \text{if } j_{|\mathcal{W}^S|} \geq rank(S_f) \end{cases}$$

where $\lceil j_{|\mathcal{W}^S|} \rceil$ and $\lfloor j_{|\mathcal{W}^S|} \rfloor$ denote the closest integer larger and smaller than $j_{|\mathcal{W}^S|}$ correspondingly.

Function $\pi_f(\sigma_f, \sigma_{-f}, \theta)$ is a quasi-concave function of cutoff $j_{|\mathcal{W}^S|}$. Therefore, the expected payoff from using cutoff $j_{|\mathcal{W}^S|}$, $E_\theta[\pi_f(\sigma_f, \sigma_{-f}, \theta) || \mathcal{W}_f^S| = |\mathcal{W}^S|]$, is also a quasi-concave function of cutoff $j_{|\mathcal{W}^S|}$ as it is a linear combination of quasi-concave functions. Therefore, $U_f(\sigma_f, \sigma_{-f})$ is a quasi-concave function of its first argument. It follows that $g_f(\sigma)$ is a continuous correspondence with closed graph.

Since $g(\sigma)$ is a continuous correspondence with closed graph, $g(\sigma)$ has a fixed point by

Kakutani's theorem (see Kakutani, 1941).

Until now we have required cutoff strategies for firms. However, Proposition 4 and Proposition A3 allow us to conclude that the above equilibrium is also an equilibrium when we allow any deviations, not simply deviations in cutoff strategies. Hence, we have established the existence of an equilibrium when workers use symmetric best-in-block strategies and firms use symmetric cutoff strategies and have best-in-block beliefs. □

Proof of Theorem 4. We will use following lemma, proved in the web appendix.

Lemma A1 (Incremental welfare)**.** *Assume firms use cutoff strategies and workers use best-in-block strategies. Fix the strategies of firms $-f$ as σ_{-f}. Let firm f's strategy σ_f differ from σ'_f only in that σ'_f has greater cutoffs (more response more to signals). Then $E_\theta(m(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(m(\sigma_f, \sigma_{-f}, \theta))$ and $E_\theta(\pi_w(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_w(\sigma_f, \sigma_{-f}, \theta))$.*

Let us denote firm strategies in the unique equilibrium of the offer game with no signals as σ_F^0. Now consider a block-symmetric equilibrium of the offer game with signals when agent use strategies (σ_F, σ_W). If agents employ strategies (σ_F^0, σ_W), the expected number of matches and the welfare of workers equal the corresponding parameters in the offer game with no signals. Therefore, the result that the expected number of matches and the expected welfare of workers in a block-symmetric equilibrium in the offer game with signals are *weakly* greater than the corresponding parameters in the unique equilibrium of the offer game with no signals is a consequence of sequential application of Lemma A1.

Let us now consider a non-babbling block-symmetric equilibrium (σ_F, σ_W) of the offer game with signals such that there exists block $\mathcal{F}_b$ with at least two firms where $\alpha_b > 0$. Proposition 4 shows that firms from block $\mathcal{F}_b$ respond to signals in the equilibrium, i.e. make offers to signaling workers with positive probability, so that $p_b^s > p_b^{ns}$.

Select some firm f from block $\mathcal{F}_b$. Using a construction similar to that in the proof of Lemma A1 we consider two sets of preference profiles:

$$\bar{\Theta}_+ \equiv \{\theta \in \Theta \mid \ m(\sigma_f^0, \sigma_{-f}, \theta) < m(\sigma_f, \sigma_{-f}, \theta)\}$$
$$\bar{\Theta}_- \equiv \{\theta \in \Theta \mid \ m(\sigma_f^0, \sigma_{-f}, \theta) > m(\sigma_f, \sigma_{-f}, \theta)\}.$$

Consider some realized profile of preferences, $\theta \in \Theta$, and denote $T_f = w'$ and $S_f = w$. Define mapping $\psi : \Theta \to \Theta$ so that $\psi(\theta)$ is the profile in which workers have preferences as in θ, but firms $-f$ all swap the positions of workers w' and w in their preference lists. Note that $\psi(\psi(\theta)) = \theta$ and ψ is a bijection on Θ. A direct consequence of Lemma A1 is that $|\bar{\Theta}_+| \geq |\bar{\Theta}_-|$. Let us now show that there exist $\theta \in \bar{\Theta}_+$ such that $\psi(\theta) \notin \bar{\Theta}_-$.

There are at least two firms, f and f', in block $\mathcal{F}_b$ that respond to signals. Consider some profile θ from $\bar{\Theta}_+$. We again denote $T_f = w'$ and $S_f = w$. Therefore, worker w does not have an offer from any other firm for profile θ from $\bar{\Theta}_+$, but worker w' has at least two offers. Since worker w' sends her signal to firm f' with positive probability and firm f' responds to signals, i.e. makes offers to its top signaling workers, there exist $\theta^* \in \bar{\Theta}_+$ such that worker w' is the top signaling worker of firm f', and firm f' makes an offer to worker w'.

However, worker w for profile $\psi(\theta^*)$ does not have any other offer, because she is neither T_f nor S_f for profile $\psi(\theta^*)$. Therefore, $\psi(\theta^*)$ cannot belong to $\bar{\Theta}_-$. Therefore, we have found a profile from $\bar{\Theta}_+$ that does not belong to $\bar{\Theta}_-$. As a result, $|\bar{\Theta}_+| > |\bar{\Theta}_-|$ and we have that

$$E_\theta[m(\sigma_f^0, \sigma_{-f}, \theta)] < E_\theta[m(\sigma_f, \sigma_{-f}, \theta)].$$

In addition, we know that

$$E_\theta[m(\sigma_f^0, \sigma_{-f}^0, \theta)] \leq E_\theta[m(\sigma_f^0, \sigma_{-f}, \theta)],$$

which gives us

$$E_\theta[m(\sigma_f^0, \sigma_{-f}^0, \theta)] < E_\theta[m(\sigma_f, \sigma_{-f}, \theta)].$$

Overall, the expected number of matches in the offer game with signals when agents use strategies (σ_F, σ_W) is strictly greater than the expected number of matches in the offer game with no signals.

Using the above construction and the logic of the proof of Lemma A1 we obtain the result for worker welfare. The example presented in Section 2 illustrates that signals can ambiguously influence the welfare of firms. Specifically, Table 2 shows that firm welfare increases upon introduction of a signaling mechanism only if the value of a second ranked worker is sufficiently high, in this case when $x > 0.5$. □

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B Web Appendix: Proofs and extensions.

B.1 Proofs of results about block correlated markets.

This subsection provides proofs for Propositions A1, A2, A3 and A4, and the proof of Lemma A1.

Proof of Proposition A1 (Equilibrium with no signals)**.** Consider some agent preference profile $\theta \in \Theta$. We will compare two strategies for firm f given its profile of preferences θ_f: strategy σ_f of making an offer to its top worker, and strategy σ'_f of making an offer to its nth ranked worker, $n > 1$. We have $\sigma_f(\theta) = \theta^1_f \equiv w$ and $\sigma'_f(\theta) = \theta^n_f \equiv w^n$. We will show that for any anonymous strategies σ_{-f} of opponent firms $-f$, these two strategies yield identical probabilities of f being matched, so that f optimally makes its offer to its most preferred worker. The proposition straightforwardly follows.

Denote a permutation that changes the ranks of w and w^n in a firm preference list (or profile of firm preference lists) as

$$\rho : (..., w, ..., w^n, ...) \longrightarrow (..., w^n, ..., w, ...).$$

We now construct preference profile $\theta' \in \Theta$ from θ as follows:

- firm f preferences are the same as in $\theta : \theta'_f = \theta_f$,
- workers w and w^n are exchanged in the preference lists of firms $-f$: $\forall f' \in -f$, we have $\theta'_{f'} = \rho(\theta_{f'})$
- worker w and worker w^n preference profiles are exchanged: $\theta'_w = \theta_{w^n}$, $\theta'_{w^n} = \theta_w$, and
- $\theta_{w'} = \theta'_{w'}$ for any other $w' \in \mathcal{W} \setminus \{w, w^n\}$.

Define function $m_f : (\Sigma_w)^W \times (\Sigma_f)^F \times \Theta \to \mathbf{R}$ as the probability of firm f being matched as a function of agent strategies and types. Since firm $-f$ strategies are anonymous we have

$$\sigma_{-f}(\theta'_{-f}) = \sigma_{-f}(\rho(\theta_{-f})) = \rho\left(\sigma_{-f}(\theta_{-f})\right)$$

Therefore, the probability of firm f', $f' \in -f$, making an offer to worker w for profile θ equals the probability of making an offer to worker w^n for profile θ'. Moreover, since we exchange worker w and w^n preference lists for profile θ', whenever it is optimal for worker

w to accept firm f offer for profile θ, it is optimal for worker w^n to accept firm f's offer for profile θ'. Therefore,

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta')$$

In other words, given θ_f, for each θ_{-f} there exists θ'_{-f} such that the probability of f's offer to θ_f^1 being accepted when opponent preferences are θ_{-f} equals the probability of f's offer to θ_f^n being accepted when opponent preferences are θ'_{-f}.[28] Moreover θ'_{-f} is different for different θ_{-f} by construction. Since θ_{-f} and θ'_{-f} are equally likely, we have

$$E_{\theta_{-f}} m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta_f) = E_{\theta_{-f}} m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta_f)$$

and

$$E_\theta m_f(\sigma_f, \sigma_{-f}, \theta) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta).$$

That is, the expected probability of getting a match from firm f's top choice equals the expected probability of getting a match from firm f's nth ranked choice. Since the utility from obtaining a top match is greater, the strategy of firm f of making an offer to its top worker is optimal. □

Proof of Proposition A2 (Binary nature of firm optimal offer)**.** Consider firm f from some block $\mathcal{F}_b$, $b \in \{1, ..., B\}$ that has realized preference profile $\theta^* \in \Theta_f$ and that receives signals from the set of workers $\mathcal{W}^{\mathcal{S}} \subset \mathcal{W}$. Denote worker S_f as w and select arbitrary other worker $w' \in \mathcal{W}^{\mathcal{S}}$. We first prove that the expected payoff to f from making an offer to worker w is strictly greater than the expected payoff from making an offer to worker w'. We denote the strategies of firm f that correspond to these actions as $\sigma_f(\theta^*, \mathcal{W}^{\mathcal{S}}) = w$ and $\sigma'_f(\theta^*, \mathcal{W}^{\mathcal{S}}) = w'$.

Workers use symmetric best-in-block strategies and firms have best-in-block beliefs. Specifically, firm f believes that it is the top firm within block $\mathcal{F}_b$ in the preference lists of workers w and w'. Denote the set of all possible agents' profiles consistent with firm f beliefs as[29]

$$\bar{\Theta} \equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^{\mathcal{S}}\}$$

[28] In this context, θ_{-f} is a preference profile for all agents – both workers and firms – other than f.

[29] For the case of one block of firms, firm f beliefs also exclude preference profiles where firm f is a top firm for those workers that did not send signal to firm f.

$$\bar{\Theta} \equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*,\ f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^{\mathcal{S}},\ \text{and } f \neq \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W} \backslash \mathcal{W}^{\mathcal{S}}\}.$$

For simplicity, we assume that there are at least two blocks. All the derivations are also valid without change for the case of one block.

As in the proof of Proposition A1, we denote a permutation that changes the ranks of w and w' in a firm preference list (or profile of firm preference lists) as

$$\rho : (..., w, ...w', ...) \rightarrow (..., w', ...w, ...).$$

We now construct preference profile $\theta' \in \Theta$ from θ^* as follows:

- firm f preferences are the same as in θ^*: $\theta'_f = \theta^*$,
- workers w and w' are exchanged in the preference lists of firms $-f$: $\forall f' \in -f$, we have $\theta'_{f'} = \rho(\theta_{f'})$,
- worker w and worker w' preference profiles are exchanged $\theta'_w = \theta_{w'}, \ \theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \backslash \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Since firm f 's preference list is unchanged and since w,$w' \in \mathcal{W}^{\mathcal{S}}$, profile θ' belongs to $\bar{\Theta}$. Since strategies of firms $-f$ are anonymous, then for any $f' \in -f$ and for any $\mathcal{W}^{\mathcal{S}}_{f'} \subset \mathcal{W}$ we have

$$\sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^{\mathcal{S}}_{f'})) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^{\mathcal{S}}_{f'})\right).$$

Worker w and w' send their signals to firm f under both profile θ and θ'. Therefore, they do not send their signals to firms $-f$, i.e. $\rho(\mathcal{W}^{\mathcal{S}}_{f'}) = \mathcal{W}^{\mathcal{S}}_{f'}$. Since $\theta'_f = \rho(\theta_f)$ we have

$$\sigma_{f'}(\theta'_{f'}, \mathcal{W}^{\mathcal{S}}_{f'}) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^{\mathcal{S}}_{f'})\right).$$

This means that the probability of firm f' making an offer to worker w for profile θ equals the probability of making an offer to worker w' for profile θ'. Moreover, since we exchange worker w and w' preference lists for profile θ', whenever it is optimal for worker w to accept firm $f's$ offer under profile θ, it is optimal for worker w' to accept an offer from firm f' under profile θ'. Since firm types are independent, the probability of firm f being matched when it uses strategy σ_f for profile θ equals the probability of firm f being matched when it uses strategy σ'_f for profile θ' :

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta').$$

Therefore, for each $\theta \in \bar{\Theta}$ there exists $\theta' \in \bar{\Theta}$ such that the probability that firm f gets an offer from worker w equals the probability that firm f gets an offer from worker w'. Moreover, profile θ' is different for different θ by our construction. Since θ and θ' are equally likely,

$$E_\theta m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta \in \bar{\Theta}) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta \in \bar{\Theta}).$$

Therefore, the expected probability that firm f gets a match if it makes an offer to some worker in $\mathcal{W}^S$ is the same across all workers in $\mathcal{W}^S$. But within this set, a match with S_f offers the greatest utility, so the expected payoff to f from making an offer to S_f is strictly greater than the payoff from making an offer to any other worker in $\mathcal{W}^S$.

A similar construction is valid for the workers in set $\mathcal{W} \backslash \mathcal{W}^S$. That is, the probability that firm f's offer is accepted is the same across all workers in $\mathcal{W} \backslash \mathcal{W}^S$. Hence, firm f prefers making an offer to its most valuable worker, T_f, than to any other worker in $\mathcal{W} \backslash \mathcal{W}^S$.[30] □

Proof of Proposition A3 (Optimality of Cutoff Strategies)**.** If workers use best-in-block strategies and firms have best-in-block beliefs, the optimal choice of firm f for each set of received signals is either S_f or T_f (or some lottery between them) (see Proposition A2). In light of this, we break the proof into two parts. First we show that the identities of workers that have sent a signal to firm f influence neither the expected payoff of making an offer to S_f nor the expected payoff of making an offer to T_f, conditional on the total number of signals received by f remaining constant. Second we prove that if it is optimal for firm f to choose S_f when it receives signals from some set of workers, then it still optimal for firm f to choose S_f if the number of received signals does not change and S_f has a smaller rank (S_f is more valuable to f).

Let us consider some firm f from block $\mathcal{F}_b$, $b \in \{1, ..., B\}$ and some realization θ^* of its preference list. Assume that it is optimal for firm f to make an offer to S_f if it receives a set of signals $\mathcal{W}^S \subset \mathcal{W}$. We want to show that if firm f receives the set of signals $\mathcal{W}^{S'}$ such that $S_f(\theta^*, \mathcal{W}^S) = S_f(\theta^*, \mathcal{W}^{S'})$ and $\left|\mathcal{W}^{S'}\right| = \left|\mathcal{W}^S\right|$, it is still optimal for firm f to make an offer to S_f. For simplicity, we only consider the case when $\mathcal{W}^S$ and $\mathcal{W}^{S'}$ differ only in one signal. (The general case then follows straightforwardly.) That is, there exist worker w and worker w' such that w belongs to set $\mathcal{W}^S$, but not to set $\mathcal{W}^{S'}$; while w' belongs to $\mathcal{W}^{S'}$, but not to $\mathcal{W}^S$. We consider two firm f strategies for realization of signals $\mathcal{W}^S$ and $\mathcal{W}^{S'}$.

$$\sigma_f(\theta^*, \cdot) = S_f(\theta^*, \cdot)$$
$$\sigma'_f(\theta^*, \cdot) = T_f(\theta^*, \cdot).$$

We denote the set of possible agents' profiles that are consistent with firm f having received

[30] It is certainly possible that $T_f = S_f$. In this case the statement of the proposition is still valid. Firm f believes that it is T_f's top firm within block $\mathcal{F}_b$ and firm f prefers making an offer to $T_f = S_f$ rather than to any other worker in $\mathcal{W}$.

signals from $\mathcal{W}^S$ and $\mathcal{W}^{S'}$ as[31]

$$\begin{aligned}\bar{\Theta}^S &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^S\} \\ \bar{\Theta}^{S'} &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^* \text{ and } f = \max_{\theta_w}(f' \in \mathcal{F}_{b'}) \text{ for each } w \in \mathcal{W}^{S'}\}\end{aligned}$$

correspondingly. We now construct a bijection between $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$. Denote a permutation that changes the ranks of w and w' in a firm preference profile as

$$\rho : (..., w, ...w', ...) \longrightarrow (..., w', ...w, ...).$$

For any profile $\theta \in \bar{\Theta}^S$ we construct profile $\theta' \in \Theta$ as follows:

- firm f preferences are the same as in θ: $\theta'_f = \theta^*$,
- the ranks of workers w and w' are exchanged in the preference lists of firms $-f$: $\forall f' \in -f,\ \theta'_f = \rho(\theta_f)$,
- the preference lists of worker w and worker w' are exchanged: $\theta'_w = \theta_{w'},\ \theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \setminus \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Since this construction leaves the preference list of firm f unchanged, and since workers w and w' swap preference lists, we have that if $\theta \in \bar{\Theta}^S$, then $\theta' \in \bar{\Theta}^{S'}$. By construction, profile θ' is different for different θ. Finally, since the cardinality of sets $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$ are the same, the above correspondence is a bijection.

Since firm $-f$ strategies are anonymous, for any $f' \in -f$ and $\mathcal{W}^S_{f'} \subset \mathcal{W}$

$$\sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^S_{f'})) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right).$$

This means that the probability of firm f' making an offer to worker w for any profile θ equals the probability of firm f' making an offer to worker w' for corresponding profile θ'. Moreover, since we exchange worker w and w' preference lists for profile θ', whenever it is optimal for worker w to accept firm f offer for profile θ, it is optimal for worker w' to accept firm f's offer for profile θ'. Since firms types are independent, the probability of firm f being matched when it uses strategy $\sigma_f(\theta^*, \cdot)$ for profile θ equals the probability of firm f being matched when it uses strategy $\sigma_f(\theta^*, \cdot)$ for profile θ':

$$m_f(\sigma_f, \sigma_{-f}, \theta) = m_f(\sigma_f, \sigma_{-f}, \theta').$$

[31]See footnote 29 for the definition of firm beliefs for the case of one block.

Similarly, for strategy $\sigma'_f(\theta^*, \cdot)$ we have

$$m_f(\sigma'_f, \sigma_{-f}, \theta) = m_f(\sigma'_f, \sigma_{-f}, \theta').$$

Since our construction is a bijection between $\bar{\Theta}^S$ and $\bar{\Theta}^{S'}$, and since θ and θ' are equally likely, we have

$$\begin{aligned} E_\theta m_f(\sigma_f, \sigma_{-f}, \theta \,|\, \theta &\in \bar{\Theta}^S) = E_\theta m_f(\sigma_f, \sigma_{-f}, \theta' \,|\, \theta' \in \bar{\Theta}^{S'}) \\ E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta \,|\, \theta &\in \bar{\Theta}^S) = E_\theta m_f(\sigma'_f, \sigma_{-f}, \theta' \,|\, \theta' \in \bar{\Theta}^{S'}). \end{aligned}$$

Therefore, if firm f optimally makes an offer to S_f (T_f) when it has received set of signals $\mathcal{W}^S$, it also should optimally make an offer to S_f (T_f), which is the same worker, for the set of signals $\mathcal{W}^{S'}$.

We now prove that if firm f optimally chooses $S_f(\theta^*, \mathcal{W}^S)$ when it receives signals from $\mathcal{W}^S$, then it should still optimally choose $S_f(\theta^*, \mathcal{W}^{S'})$ for set of signals $\mathcal{W}^{S'}$, if the number of received signals is the same $\left|\mathcal{W}^{S'}\right| = \left|\mathcal{W}^S\right|$ and $S_f(\theta^*, \mathcal{W}^{S'})$ has a smaller rank, that is, when the signaling worker is more valuable to f. We consider set $\mathcal{W}^{S'}$ that differs from $\mathcal{W}^S$ only in the best (for firm f) worker and the difference between the ranks of top signaled workers equals one. (The general case follows straightforwardly.) That is,

$$\begin{aligned} w \in \mathcal{W}^S / S_f(\theta^*, \mathcal{W}^S) \Leftrightarrow w \in \mathcal{W}^{S'} / S_f(\theta^*, \mathcal{W}^{S'}) \quad \text{and} \\ rank_f(S_f(\theta^*, \mathcal{W}^{S'})) = rank_f(S_f(\theta^*, \mathcal{W}^S)) - 1. \end{aligned}$$

The construction in the first part of the proof works again in this case. Using sets of profiles and a correspondence similar to the one above, we can show that the probabilities of firm f being matched with S_f (T_f) are the same for $\mathcal{W}^S$ and $\mathcal{W}^{S'}$. Observe that if firm f's offer to T_f is accepted, naturally firm f gets the same payoff for sets $\mathcal{W}^S$ and $\mathcal{W}^{S'}$. If firm f's offer to S_f is accepted, firm f gets strictly greater payoff for set $\mathcal{W}^{S'}$ compared to set $\mathcal{W}^S$, because by definition $S_f(\theta^*, \mathcal{W}^{S'})$ has smaller rank than $S_f(\theta^*, \mathcal{W}^S)$. Hence, if it is optimal for firm f to make an offer to $S_f(\theta^*, \mathcal{W}^S)$ when it receives set of signals $\mathcal{W}^S$, it is optimal for firm f to make an offer to $S_f(\theta^*, \mathcal{W}^{S'})$ when firm f receives set of signals $\mathcal{W}^{S'}$.

Combined, the two statements we have just proved allow us to conclude that if firms $-f$ use anonymous strategies, firm f's optimal strategy can be represented as some cutoff strategy.[32] □

[32] Note that there can be other optimal strategies. If firm f is indifferent between making an offer to S_f and making an offer to T_f for some set of signals, firm f could optimally make its offer to S_f or to T_f for any set of signals conditional on maintaining the same rank of the most preferred signaling worker and cardinality of signals received.

Proof of Proposition A4 (Strategic complements under block correlation). Consider some firm f from some block $\mathcal{F}_b$, $b \in \{1, ..., B\}$. We consider two strategy profiles, σ_{-f} and σ'_{-f}, for firms $-f$ that vary only in the strategy for firm f' . For simplicity, we assume that $\sigma'_{f'}$ differs from $\sigma_{f'}$ only for some profile $\bar{\theta}_{f'}$ and some set of received signals $\overline{\mathcal{W}^S_{f'}}$

$$\begin{aligned} \sigma_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha S_{f'} + (1-\alpha) T_{f'} \\ \sigma'_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha' S_{f'} + (1-\alpha') T_{f'} \end{aligned}$$

such that $\alpha' > \alpha$. Formally, this means σ'_f is not a cutoff strategy, because a cutoff strategy requires the same behavior for any profile of preferences (anonymity) when firms receive the same number of signals. We will prove the statement using our simplifying assumption about strategies for firms $-f$, and the extension to the full proposition follows from iterated application of this result.

Consider some realized firm f preference profile $\theta^*_f \in \Theta$ and some set of signals $\mathcal{W}^S \subset \mathcal{W}$. We want to show that firm f's payoff from making an offer to T_f (weakly) decreases whereas firm f's payoff from making an offer to S_f (weakly) increases when firm f' responds more to signals, i.e. plays strategy $\sigma'_{f'}$ instead of $\sigma_{f'}$. That is,

$$\begin{aligned} I)\ E_\theta(\pi_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \geq E_\theta(\pi_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \\ II)\ E_\theta(\pi_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \leq E_\theta(\pi_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S). \end{aligned}$$

Since firm f's offer can only be either accepted or declined, the above statements are equivalent to

$$\begin{aligned} I)\ E_\theta(m_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \geq E_\theta(m_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \\ II)\ E_\theta(m_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f &= \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \leq E_\theta(m_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S). \end{aligned}$$

That is, we wish to show that the probability of being matched to T_f weakly decreases, and the probability of being matched to S_f weakly increases.

We first prove *I)* first. Define the sets of agent profiles that lead to the increase and decrease in the probability of getting a match given the change in firm f' strategy as

$$\begin{aligned} \bar{\Theta}_+ &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S \text{ and } m_f(T_f, \sigma_{-f}, \theta) < m_f(T_f, \sigma'_{-f}, \theta)\} \\ \bar{\Theta}_- &\equiv \{\theta \in \Theta \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S \text{ and } m_f(T_f, \sigma_{-f}, \theta) > m_f(T_f, \sigma'_{-f}, \theta)\} \end{aligned}$$

correspondingly. If set $\bar{\Theta}_+$ is empty, the statement has been proved. Otherwise, select arbitrary $\theta \in \bar{\Theta}_+$ and denote $T_f \equiv w$. Since in this case, f''s strategy change pivotally reduces competition to f's offer to w, we must have $T_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) = w$ and $S_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) = w' \neq w$, and

$$\begin{aligned} \sigma_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha w' + (1-\alpha)w \\ \sigma'_{f'}(\bar{\theta}_{f'}, \overline{\mathcal{W}^S_{f'}}) &= \alpha' w' + (1-\alpha')w. \end{aligned}$$

Note that it cannot be that firm f is from a higher ranked block than firm f' , i.e. $f' \in \mathcal{F}_{b'}$ where $b' > b$. If f were from a higher ranked block, an offer from firm f' is always worse than the offer of firm f and could not influence the probability that firm f obtains a match. Therefore, firm f is from a block that is weakly worse than $\mathcal{F}_{b'}$, i.e. $b' \leq b$.

Note that under θ, worker w has sent a signal neither to firm f nor to firm f'. This will allow us to construct element $\theta' \in \bar{\Theta}_-$. Consider a permutation that changes the ranks of w and w' in a firm preference profile

$$\rho : (..., w, ...w', ...) \longrightarrow (..., w', ...w, ...).$$

For any profile $\theta \in \bar{\Theta}_+$ we construct profile $\theta' \in \Theta$ as follows:

- $\theta'_f = \theta^*_f$
- the ranks of workers w and w' are exchanged in the preference lists of firms $-f$: for each firm $f' \in -f$, $\theta'_{f'} = \rho(\theta_{f'})$
- worker w and worker w' preference profiles are exchanged: $\theta'_w = \theta_{w'}$, $\theta'_{w'} = \theta_w$, and
- for any other $w^0 \in \mathcal{W} \backslash \{w, w'\}$, $\theta_{w^0} = \theta'_{w^0}$.

Note that under θ and θ', firm f has the same preferences θ^*_f and receives the same set of signals.

Since firm strategies are anonymous we have that

$$\begin{aligned} \sigma_{f'}(\theta'_{f'}, \mathcal{W}^{S'}_{f'}) &= \sigma_{f'}(\rho(\theta_{f'}), \rho(\mathcal{W}^S_{f'})) \text{ (by our construction)} \\ &= \alpha\rho(w') + (1-\alpha)\rho(w) \text{ (by anonymity)} \\ &= \alpha w + (1-\alpha)w' \end{aligned}$$

and similarly

$$\sigma'_{f'}(\theta'_{f'}, \mathcal{W}^{S'}_{f'}) = \alpha' w + (1-\alpha')w'.$$

We will now argue that $\theta' \in \bar{\Theta}_-$. Since $\theta \in \bar{\Theta}_+$, the strategy change for firm f' reduces the likelihood of firm f being matched with worker w (when f makes T_f an offer under profile θ). Under profile θ', firm f' makes an offer to worker w more frequently when using strategy $\sigma'_{f'}$ rather than $\sigma_{f'}$. Furthermore, worker w prefers firm f' to firm f under profile θ'. (We have already shown that f' cannot be in a lower ranked block than f. If firm f' is in a higher ranked block $\mathcal{F}_{b'}$, $b > b'$, worker w always prefers firm f' to firm f. If firm f and firm f' are from the same block, $b = b'$, worker w prefers f to f', since worker w sends a signal to firm f' under profile θ').

To finish our proof, we must also investigate the behavior of a firm that receives worker w's signal for profile θ, say firm f_y. If firm f_y makes an offer to worker w for profile θ, since the change of firm f' strategy changes firm f's payoff, firm f_y must be lower ranked than both firms f and f' in worker w's preferences. Hence, firm f_y's offer cannot change the action of worker w. If worker w' sends her signal to firm f_y then firm f_y either makes an offer to worker w' or to worker T_{f_y}, which are both different from worker w.

Hence, firm f_y does not influence the behavior of the agents in question, and the overall probability that firm f's offer to worker w is accepted is smaller when firm f' uses strategy $\sigma'_{f'}$ rather than $\sigma_{f'}$. That is, $\theta' \in \bar{\Theta}_-$.

Note that the above construction gives different profiles in $\bar{\Theta}_+$ for different profiles of $\bar{\Theta}_-$. Hence, our construction is an injective function from $\bar{\Theta}_+$ to $\bar{\Theta}_-$, so $\left|\bar{\Theta}_-\right| \geqslant \left|\bar{\Theta}_+\right|$.[33] Since profiles θ and θ' are equally likely, we have

$$E_\theta(m_f(T_f, \sigma_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \geq E_\theta(m_f(T_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S).$$

We now prove inequality *II)*. That is, we will show that if firm f' responds more to signals, the probability of firm f being matched to S_f (upon making S_f an offer) weakly increases. If firm f, $f \in \mathcal{F}_b$, receives a signal from worker w it believes it is the best firm in block $\mathcal{F}_b$ according to worker w's preferences. That is, worker w prefers the offer of firm f to an offer from any other firm f' from any block $\mathcal{F}_{b'}$ with $b' \geq b$. Therefore, the change of the behavior of any firm f' from block $\mathcal{F}_{b'}$, $b' \geq b$, does not influence firm f's payoff.

If we consider some firm f' from group $\mathcal{F}_{b'}$, $b' < b$, it can draw away worker w's offer from firm f only if it makes an offer to worker w. However, firm f' makes an offer to worker w, conditionally on firm f receiving a signal from worker w, only when worker w is $T_{f'}$. However, if firm f' responds more to signals, it makes an offer to its T_f more rarely. This means that firm f' draws worker w away from firm f less often. Therefore, the probability that firm f's offer is accepted by S_f increases:

$$E_\theta(m_f(S_f, \sigma_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S) \leq E_\theta(m_f(S_f, \sigma'_{-f}, \theta) \,|\, \theta_f = \theta^*_f, \mathcal{W}^S_f = \mathcal{W}^S).$$

[33] One may show by example that this is not, in general, a bijection.

As a corollary of *I)* and *II)*, if firm f' increases its cutoff point for some set of signals, firm f will also optimally (weakly) increase its cutoff points. The above logic is valid for the change of cutoff points for any set of signals of the same size and any profile of preferences, so the statement of the proposition immediately follows. □

Proof of Lemma A1. We prove the first statement first. Let us consider firm f cutoff strategies σ_f and σ'_f such that σ'_f has weakly greater cutoffs. We consider two sets of preference profiles

$$\bar{\Theta}_+ \equiv \{\theta \in \Theta \mid m(\sigma_f, \sigma_{-f}, \theta) < m(\sigma'_f, \sigma_{-f}, \theta)\}$$
$$\bar{\Theta}_- \equiv \{\theta \in \Theta \mid m(\sigma_f, \sigma_{-f}, \theta) > m(\sigma'_f, \sigma_{-f}, \theta)\}.$$

For each profile θ from set Θ^+, it must be the case that without firm f's offer, T_f has an offer from another firm and worker S_f does not:

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = 1. \tag{B.1.1}$$

Similarly, if profile θ is from set Θ^-, it must be the case that without firm f offer, S_f has an offer from another firm, and T_f does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = -1. \tag{B.1.2}$$

We will now show that $|\bar{\Theta}_+| \geq |\bar{\Theta}_-|$. Equations (B.1.1) and (B.1.2), along with the fact that each $\theta \in \Theta^+ \cup \Theta^-$ occurs equally likely, will then be enough to prove the result.

Let us denote $T_f = w'$ and $S_f = w$. We construct function $\psi : \Theta \to \Theta$ as follows. Let $\psi(\theta)$ be the profile in which workers have preferences as in θ, but firms $-f$ all swap the positions of workers w' and w in their preference lists. If profile θ belongs to $\bar{\Theta}_-$, without firm f's offer, worker w has an offer from another firm, and worker w' does not. Therefore, when preferences are $\psi(\theta)$, without firm f's offer the following two statements must be true: *i)* worker w' ***must*** have another offer and *ii)* worker w ***cannot*** have another offer.

To see *i)*, note that under θ, worker w sends a signal to firm f, so his outside offer must come from some firm f' who has ranked him first. Under profile $\psi(\theta)$, firm f' ranks worker w' first. If worker w' has not sent a signal to firm f', then by anonymity, w' gets the offer of firm f'. If worker w' has signaled to firm f', worker w' again gets firm f''s offer.

To see *ii)*, suppose to the contrary that under $\psi(\theta)$, worker w does in fact receive an offer from some firm $f' \neq f$. Since worker w sends a signal to firm f, worker w must be $T_{f'}$ under $\psi(\theta)$, so that worker w' is $T_{f'}$ under θ. But then by anonymity w' receives the offer of firm f' under θ, a contradiction.

From $i)$ and $ii)$, we have

$$\theta \in \bar{\Theta}_- \Rightarrow \psi(\theta) \in \bar{\Theta}_+.$$

Since function ψ is injective, we have $|\bar{\Theta}_+| \geq |\bar{\Theta}_-|$.

In order to prove the second statement note that the expected number of matches of each worker increases when firm f responds more to signals. Using the construction presented above, one can show that whenever worker w "loses" a match with firm f for profile θ (worker w is T_f) it is possible to construct profile θ' when worker w obtains a match (worker w is S_f). The function that matches these profiles is again injective. Moreover, worker w values more greatly the match with firm f when she has signaled it (S_f) rather when she is simply highest ranked (T_f). Therefore, the ex-ante utility of worker w increases when firm f responds more to signals. $\square$

B.2 Market Structure and the Value of a Signaling Mechanism — Proofs and Extensions

This set of results pertains to Section 7: Market Structure and the Value of a Signaling Mechanism. In this section, we denote as $u(j)$ the utility of a firm from matching with its jth ranked worker.

Proposition B1. *Under the assumption that*

$$u(W) > \tfrac{W}{F}\left(1 - \left(1 - \tfrac{1}{W}\right)^F\right) u(1) \tag{B.2.1}$$

there is a unique non-babbling equilibrium in the offer game with signals. Each worker sends her signal to her top firm. Each firm f makes an offer to S_f if it receives at least one signal; otherwise, firm f makes an offer to T_f.

Proof. We will show that under condition (B.2.1) even if S_f is the worst ranked worker in firm f preferences, firm f still optimally makes her an offer.

Proposition 2 shows that if firms $-f$ respond more to signals, i.e. increase their cutoffs, it is also optimal for firm f to respond more to signals. Therefore, if firm f optimally responds to signals when no other firm does, it will certainly optimally respond to signals when other firm respond. Hence, it will be enough to consider the incentives of firm f when firms $-f$ do not respond to signals and always make an offer their top ranked workers.

Let us consider some realized profile of preferences of firm f and denote T_f as w. If firms $-f$ do not respond to signals, then some firm among $-f$ makes an offer to worker w with probability $q = \frac{1}{W}$. Therefore, the probability that the offer of firm f to worker w is accepted equals

$$(1-q)^{F-1} + ... + C^j_{F-1} q^j (1-q)^{F-1-j} \tfrac{1}{j+1} + ... + q^{F-1} \tfrac{1}{F} \tag{B.2.2}$$

where $C^y_x = \frac{x!}{y!(x-y)!}$. Intuitively, j firms among the other $F-1$ firms simultaneously make an offer to worker w with probability $C^j_{F-1} q^j (1-q)^{F-1-j}$. Therefore, firm f is matched with worker w only with probability $\frac{1}{j+1}$ because worker w's preferences are uniformly distributed. The sum over all possible j from 0 to $F-1$ gives us the overall probability of firm f's offer being accepted. We can simplify this expression as follows:

$$\begin{aligned}
& \sum\nolimits_{j=0}^{F-1} C^j_{F-1} q^j (1-q)^{F-1-j} \tfrac{1}{j+1} && \text{(B.2.3)}\\
= & \sum\nolimits_{j=0}^{F-1} \tfrac{(F-1)!}{j!(F-1-j)!} q^j (1-q)^{F-1-j} \tfrac{1}{j+1} && \text{(B.2.4)}\\
= & \sum\nolimits_{j=0}^{F-1} \tfrac{1}{Fq} \tfrac{F!}{(j+1)!(F-(1+j))!} q^{j+1} (1-q)^{F-(1+j)} && \text{(B.2.5)}\\
= & \tfrac{1}{Fq} \sum\nolimits_{j=1}^{F} \tfrac{F!}{j!(F-j)!} q^j (1-q)^{F-j} && \text{(B.2.6)}\\
= & \tfrac{1}{Fq} \left(\sum\nolimits_{j=0}^{F} \tfrac{F!}{j!(F-j)!} q^j (1-q)^{F-j} - (1-q)^F \right) && \text{(B.2.7)}\\
= & \tfrac{1}{Fq} \left(1-(1-q)^F\right) = \tfrac{W}{F} \left(1-\left(1-\tfrac{1}{W}\right)^F\right). && \text{(B.2.8)}
\end{aligned}$$

Alternatively, if firm f makes an offer to its top signaling worker, its offer is accepted with probability one. Therefore, it is optimal for the firm to make an offer to the signaling worker only if $u(W) > \frac{W}{F}\left(1-\left(1-\frac{1}{W}\right)^F\right) u(1)$. We conclude that under assumption B.2.1 there is no other non-babbling symmetric equilibrium in the offer game with signals. □

Proof of Proposition 5. We first calculate an explicit formula for the increase in the expected number of matches from the introduction of the signaling mechanism.

Lemma B1. *Consider a market with W workers and $F > 2$ firms. The expected number of matches in the offer game with no signals equals*

$$m^{NS}(F,W) = W\left(1-\left(1-\tfrac{1}{W}\right)^F\right). \tag{B.2.9}$$

The expected number of matches in the offer game with signals equals

$$m^S(F,W) = F \begin{pmatrix} 1-\left(\frac{F-1}{F}\right)^W + \frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}}\left(1-\frac{F-1}{W}\left(1-\left(\frac{F-2}{F-1}\right)^W\right)\right) * \\ * \left(1-\left(1-\frac{1}{W}\left(\frac{F-2}{F-1}\right)^{W-1}\right)^{F-1}\right) \end{pmatrix}. \tag{B.2.10}$$

Proof of Lemma B1. Let us first calculate the expected number of matches in the pure coordination game with no signals. Proposition A1 establishes that the unique symmetric non-babbling equilibrium when agents use anonymous strategies is as follows. Each firm makes an offer to its top worker and each worker accepts the best offer among those available. We have already calculated the probability of firm f being matched to its top worker in Proposition B1. The probability of this event is

$$\frac{W}{F}\left(1-\left(1-\frac{1}{W}\right)^{F}\right)$$

.Therefore, the expected total number of matches in the game with no signals equals

$$m^{NS}(F,W)=W\left(1-\left(1-\frac{1}{W}\right)^{F}\right) \tag{B.2.11}$$

Let us now calculate the expected number of matches in the offer game with signals. Proposition B1 derives agent strategies in the unique symmetric non-babbling equilibrium in the pure coordination game with signals. Each worker sends her signal to her top firm and each firm makes its offer to its top signaling worker if it receives at least one signal, otherwise it makes an offer to its top ranked worker.

We first calculate ex-ante probability of being matched by some firm f. We denote the set of workers that send her signal to firm f as $\mathcal{W}_f^S \subset \mathcal{W}$. If firm f receives at least one signal, $|\mathcal{W}_f^S|>0$, it is guaranteed a match because each worker sends her signal to her top firm. If firm f receives no signals, it makes an offer to its top ranked worker T_f. This worker accepts firm f's offer only if this offer is the best one among those she receives. Let us denote the probability that T_f accepts firm f's offer (under the condition that firm f receives no signals) as

$$P_{T_f,|\mathcal{W}_f^S|=0} \equiv P(T_f \text{ accepts firm } f'\text{s offer}|\left|\mathcal{W}_f^S\right|=0).$$

The ex-ante probability that firm f is matched then equals

$$Prob\_match_f(F,W)=P(|\mathcal{W}_f^S|>0)*1+P(|\mathcal{W}_f^S|=0)*P_{T_f,|\mathcal{W}_f^S|=0}. \tag{B.2.12}$$

If firm f receives no signals, $|\mathcal{W}_f^S|=0$, it makes an offer to T_f, which we will call worker w. Worker w receives an offer from its top ranked firm, say firm f_0, conditional on firm f receiving no signals, $|\mathcal{W}_f^S|=0$, with probability equal to

$$G = P(|\mathcal{W}_{f_0}^S|=1||\mathcal{W}_f^S|=0)*1+...+P(|\mathcal{W}_{f_0}^S|=W||\mathcal{W}_f^S|=0)*\frac{1}{W} \tag{B.2.13}$$

$$= \sum_{j=0}^{W-1} C_{W-1}^{j}\left(\frac{1}{F-1}\right)^{j}\left(1-\frac{1}{F-1}\right)^{W-j-1}\frac{1}{j+1}. \tag{B.2.14}$$

Intuitively, firm f_0 receives a signal from a particular worker with probability $\frac{1}{F-1}$ (note that firm f receives no signals). Then, if firm f_0 receives signals from j other workers, worker w receives an offer from firm f_0 with probability $\frac{1}{j+1}$. Similarly to equation (B.2.3) the expression for G simplifies to

$$G = \frac{F-1}{W}\left(1 - (1 - \frac{1}{F-1})^W\right). \tag{B.2.15}$$

Firm f can be matched with worker w only if worker w does not receive an offer from its top firm, which happens with probability $1 - G$. If worker w does not receive an offer from her top firm $-$ firm f_0 $-$ firm f competes with other firms that have received no signals from workers. The probability that some firm f' among firms $\mathcal{F}\backslash\{f, f_0\}$ receives no signals conditional on the fact that worker w sends her signal to firm f_0 and firm f receives no signals ($|\mathcal{W}_f^S| = 0$) equals $r = (1 - \frac{1}{F-1})^{W-1}$. Note that the probability that firm f' does not receive a signal from a worker equals $1 - \frac{1}{F-1}$, because firm f receives no signals. There are also only $W - 1$ workers that can send a signal to firm f', because worker w sends her signal to firm f_0.

Therefore, the probability that some firm f' among firms $\mathcal{F}\backslash\{f, f_0\}$ receives no signals and makes an offer to worker w, conditional on the fact that worker w sends her signal to firm f_0, equals $\frac{r}{W}$. Therefore, the probability that worker w prefers the offer of firm f to other offers (conditional on the fact that firm f receives no signals and worker w sends her signal to firm f_0) equals[34]

$$\sum_{j=0}^{F-2} C_{F-2}^j \left(\frac{r}{W}\right)^j (1 - \frac{r}{W})^{F-2-j}\frac{1}{j+1} = \frac{W}{(F-1)r}\left(1 - \left(1 - \frac{r}{W}\right)^{F-1}\right). \tag{B.2.16}$$

The probability that worker w accepts firm f's offer then equals

$$P_{T_f, |\mathcal{W}_f^S|=0} = (1 - G)\left(\frac{W}{(F-1)r}\left(1 - \left(1 - \frac{r}{W}\right)^{F-1}\right)\right).$$

Taking into account that firm f receives no signals with probability $P(|\mathcal{W}_f^S| = 0) = (1-\frac{1}{F})^W$, the probability of firm f being matched in the offer game with signals is then

$$\begin{aligned} Prob\_match_f(F, W) &= 1 - (1 - \tfrac{1}{F})^W + (1 - \tfrac{1}{F})^W * P_{T_f, |\mathcal{W}_f^S|=0} \\ &= 1 - (1 - \tfrac{1}{F})^W + (1 - \tfrac{1}{F})^W \tfrac{W}{(F-1)r} * \\ &\quad \left(1 - \tfrac{F-1}{W}\left(1 - (1 - \tfrac{1}{F-1})^W\right)\right) * \left(1 - \left(1 - \tfrac{r}{W}\right)^{F-1}\right) \end{aligned} \tag{B.2.17}$$

[34]Note that the maximum number of offers worker w could get equals to $M - 1$ as it does not receive an offer from its top firm f_0.

where $r = (1 - \frac{1}{F-1})^{W-1}$. The expected total number of matches in the offer game with signals equals $m^S(F,W) = F * Prob\_match_f(F,W)$. □

Lemma B1 establishes the expected total number of matches in the offer game with and without signals. Let us first fix W and calculate where the increase in the expected number of matches from the introduction of the signaling mechanism, $V(F,W) = m^S(F,W) - m^{NS}(F,W)$, attains its maximum. In order to obtain the proposition, we consider large markets (markets where the number of firms and the number of workers are large) and we use Taylor's expansion formula:

$$(1-a)^b = \exp(-ab + O(a^2 b)). \tag{B.2.18}$$

where $O(a^2b)$ is a function that is smaller than a constant for large values of a^2b. Setting $x \equiv \frac{F}{W}$, the expected number of matches in the offer game with no signals can be approximated as

$$m^{NS}(F,W) = W\left(1 - \left(1 - \tfrac{1}{W}\right)^F\right) = W(1 - e^{-x+O(x/W)}).$$

Let us now consider the expected number of matches in the offer game with signals. Using the result of Lemma B1 we get

$$m^S(F,W) = Wx\left(1 - e^{-1/x + O\left(1/(x^2W)\right)} + A * B\right)$$

where

$$\begin{aligned} A &= \left(1 - \tfrac{F-1}{W}\left(1 - (\tfrac{F-2}{F-1})^W\right)\right) \text{ and} \\ B &= \tfrac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}}\left(1 - \left(1 - \tfrac{1}{w}(\tfrac{F-2}{F-1})^{W-1}\right)^{F-1}\right). \end{aligned}$$

We first calculate an approximation of A for large markets. Using (B.2.18) we have that

$$1 - (1 - \tfrac{1}{F-1})^W = 1 - e^{-x+O(1/(x^2W))}$$

and

$$A = 1 - x\left(1 - e^{-1/x+O(1/(x^2W))}\right) + O(1/(xW)).$$

We now calculate an approximation of B for large markets:

$$\begin{aligned}
\frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}} &= \frac{W}{F}\left(\frac{F-1}{F}\right)^{W-1}\left(\frac{F-1}{F-2}\right)^{W-1} \\
&= \frac{1}{x}e^{-(W-1)/F+O(1/(x^2W))}e^{(W-1)/(F-1)+O(1/(x^2W))} \\
&= \frac{1}{x}e^{O(1/(x^2W))}.
\end{aligned}$$

Also, we have that

$$\begin{aligned}
\left(1-\left(1-\frac{Z}{W}\right)^{F-1}\right) &= 1-e^{-Z(F-1)/W+O(x/W)} \\
&= 1-e^{-Zx+O(x/W)}
\end{aligned}$$

where $Z=(\frac{F-2}{F-1})^{W-1}=e^{-1/x+O(1/(x^2W))}$. Finally, we have

$$\begin{aligned}
B &= \frac{W(F-1)^{2W-2}}{F^W(F-2)^{W-1}} * \left(1-\left(1-\frac{1}{W}(\frac{F-2}{F-1})^{W-1}\right)^{F-1}\right) \\
&= \frac{1}{x}e^{O(1/(x^2W))}(1-e^{-xe^{-1/x}+O(x/W)}).
\end{aligned}$$

Putting it all together, we have

$$\begin{aligned}
V(F,W) &= Wx\left(\begin{array}{c} 1-e^{-1/x+O(1/W)}+\left(1-x\left(1-e^{-1/x+O(1/W)}\right)+O(1/W)\right)* \\ *\frac{1}{x}e^{O(1/W)}(1-e^{-xe^{-1/x}+O(1/W)}) \end{array}\right)- \\
&\quad -W(1-e^{-x+O(1/W)}) \\
&= W\left(x-xe^{-1/x}+\left(1-x\left(1-e^{-1/x}\right)\right)(1-e^{-xe^{-1/x}})-1+e^{-x}\right)+O(1) \\
&= W\alpha(x)+O(1)
\end{aligned}$$

where $\alpha(x)$ is a positive quasi-concave function that attains maximum at $x_0 \simeq 1.012113$. Therefore, for fixed W, $V(F,W)$ attains its maximum value at $F=x_0W+O(1)$.

Similar to the previous derivation, we can fix F and calculate the value of W where $V(F,W)$ attains its maximum:

$$\begin{aligned}
V(F,W) &= F\left(\begin{array}{c} 1-e^{-1/x+O(1/W)}+\left(1-x\left(1-e^{-1/x+O(1/W)}\right)+O(1/W)\right) \\ *\frac{1}{x}e^{O(1/W)}(1-e^{-xe^{-1/x}+O(1/W)}) \end{array}\right) \\
&\quad -\frac{F}{x}(1-e^{-x+O(1/F)}) \\
&= F\left(1-e^{-1/x}+\left(1-x\left(1-e^{-1/x}\right)\right)\frac{1}{x}(1-e^{-xe^{-1/x}})-\frac{1}{x}\left(1-e^{-x}\right)\right)+O(1) \\
&= F\beta(x)+O(1)
\end{aligned}$$

where $\beta(x)$ is a positive quasi-concave function that attains maximum at $x_{00} \simeq 0.53074$.

Therefore, for fixed F, $V(F,W)$ attains its maximum value at $W = y_0 F + O(1)$, where $y_0 = 1/x_{00} = 1.8842$. □

B.3 Extension: Signaling with Many Positions and Many Signals

In this section we consider a model of matching markets in a symmetric environment that is similar to the one in Sections 3 and 4. The difference is that each firm now has the capacity to hire up to L workers, and each worker may send up to K identical costless private signals. We assume that the number of signals each worker may send is less than the number of firms, $K < F$, and each worker can send at most one signal to a particular firm.

We assume that firm utilities are additive, i.e. firm f with preferences θ_f over individual workers values a match with a subset of workers $\mathcal{W}_0 \subset \mathcal{W}$ as $u(\theta_f, \mathcal{W}_0) = \sum_{w \in \mathcal{W}_0} u(\theta_f, w)$, where $u(\theta_f, \cdot)$ is a von-Neumann Morgenstern utility function. Worker w with preference list θ_w values a match with firm f as $v(\theta_w, f)$. We keep all assumptions of Sections 3 and 4 regarding agent utilities $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$.

The timing and strategies of agents of the offer game without signals can be adopted from Section 3:

1. Agents' preferences are realized. In the case of a signaling mechanism, each worker sends up to K private, identical, costless signals to firms. Signals are sent simultaneously, and are observed only by firms who have received them.

2. Each firm makes an offer to at most L workers; offers are made simultaneously.

3. Each worker accepts at most one offer from the set of offers she receives.

Once again, sequential rationality ensures that workers will always select the best available offer. Hence, we take this behavior for workers as given and focus on the reduced game consisting of the first two stages.

A mixed strategy for worker w when deciding wether and to whom to send signals is a map from the set of all possible preference lists to the set of distributions over subsets of firms of size K or less that we denote as $\Delta(2^{\mathcal{F}_\mathrm{K}})$, i.e. $\sigma_w : \Theta_w \to \Delta(2^{\mathcal{F}_\mathrm{K}})$. Similarly, a mixed strategy of firm f is a map from the set of all possible preference lists, Θ_f, and the set of all possible combinations of received signals, $2^{\mathcal{W}}$, to the set of distributions over subsets of workers of size L or less, which we denote as $\Delta(2^{\mathcal{W}_\mathrm{L}})$. That is, $\sigma_f : \Theta_f \times 2^{\mathcal{W}} \to \Delta(2^{\mathcal{W}_\mathrm{L}})$.

Preferences of both firms and workers are independently and uniformly chosen from all possible preference orderings. Similarly to Sections 3 and 4 we define σ_W, σ_F, Σ_w,Σ_f, π_w, and π_f. The definition of sequential equilibrium and anonymous strategies can also be adopted in an analagous manner.

We first consider an offer game without signals. If firms use anonymous strategies, the chances of hiring any worker, conditional on making her an offer, are the same. Therefore, each firm optimally makes its offers to the L highest-ranked workers on its preference list. This is the unique symmetric equilibrium of the offer game without signals when firms use anonymous strategies (see Proposition B3).

We now turn to the analysis of the offer game with signals. In any symmetric equilibrium in which workers send signals and signals are interpreted as a sign of interest by firms and hence increase the chance of receiving an offer, each worker sends her K signals to her K most preferred firms (see Proposition B4). As in the case of one signal and each firm only having one position, we pin down the behavior of workers in equilibrium: workers send their signals to their highest ranked firms, and workers accept the best available offer. We now examine offers of firms in the second stage of the game, taking the strategies of workers and beliefs of firms about interpreting signals as given.[35]

In Section 4 each worker could send up to one signal, and each firm had $L = 1$ positions to fill. Then, when all other firms used anonymous strategies, firm f decided between making an offer to f's most preferred worker T_f (or T_f^1) and f's most preferred worker in the subset of signaled workers S_f (or S_f^1). Now, when all other firms use anonymous strategies, firm f can make up to L offers. When deciding whom to make the first offer, firm f, once more, decides between the most preferred worker T_f (or T_f^1) and the most prefered worker among those who sent a signal S_f (or S_f^1) where that decision may depend on the total number of signals received. So, if firm f received $|\mathcal{W}^S|$ signals and uses a cutoff strategy with corresponding cutoff $j_{|\mathcal{W}^S|}$, then f makes an offer to S_f^1 if and only if the rank of S_f^1 is lower or equal than $j_{|\mathcal{W}^S|}$. If firm f made an offer to S_f^1, then, for the second position, the firm decides between T_f^1 and S_f^2 the most preferred worker among those that sent a signal to whom firm f has not made an offer yet. Furthermore, firm f will use the *same* cutoff strategy as before: Firm f still received $|\mathcal{W}^S|$ signals and hence will make an offer to S_f^2 compared to T_f^1 if and only if the rank of S_f^1 is lower than $j_{|\mathcal{W}^S|}$.

If the firm made its first offer to T_f^1, then for the second offer, firm f decides between T_f^2 and S_f^1, where f can use a new cutoff strategy, since the alternative to a signaling worker is now T_f^2, the overall second most preferred worker, and not T_f^2. We can show that in equilibrium, the cutoff for T_f^2 will be greater than for T_f^1 for any number if received signals (see Proposition B5). We can now define the notion of cutoff strategies for this setting.

[35]Note that in any non-babbling symmetric equilibrium, all information sets for firms are realized with positive probability. Hence, the beliefs of firms are determined by Bayes' Law: if a firm receives a signal from a worker, it believes that it is on of the kth top firms, $k \in \{1, ..., K\}$, in the workers' preference list and the probability of having rank k is identical across ranks $\{1, ..., K\}$.

Definition B1 (Cutoff Strategies in Case of Many Positions and Multiple Signals)**.** Strategy σ_f is a *cutoff strategy* for firm f if there are L vectors $J^l = (j_1^l, ..., j_W^l)$, $l = 1, ..., L$ such that for any $\theta_f \in \Theta_f$ and any set $\mathcal{W}^S$ of workers who sent a signal to firm f we have the following: For any number m of offers already made, let the most preferred worker to whom firm f has not yet made an offer be T_f^r of rank $1 \leq r < L$ and let the most preferred worker who sent a signal and to whom f has not yet made an offer be S_f^q of rank $1 \leq q < L$, where $m = q + r - 2$. Then firm f makes its next offer to

$$\begin{cases} S_f^q & \text{if } \text{rank}_{\theta_f}(S_f^q) \leq j^r_{|\mathcal{W}^S|} \\ T_f^r & \text{otherwise.} \end{cases}$$

We call $(J^1, \ldots, J^L)$ *a cutoff matrix* that has cutoff vectors for each of the top L ranked workers as its components. Note that the probability of a firm's offer being accepted by any worker who has signaled to it is the same as in a symmetric equilibrium. Similarly, the probability of a firm's offer being accepted by any worker who has not signaled to the firm is also the same across such workers (see Lemma B2).

Using an argument similar to the case of one position and one signal, we show that cutoff strategies are optimal strategies for firms (see Proposition B6). We can also impose a partial order on the cutoff strategies as in Section 4. However, strategies of firms are no longer necessarily strategic complements. When other firms respond more to signals, this decreases the payoff from making an offer to both workers who have and workers who have not signaled to the firm. This is because receiving a signal does not guarantee acceptance in case an offer is tendered to that worker. We can, however, assure the existence of symmetric mixed strategy equilibrium.

Theorem B1 (Equilibrium Existence)**.** *There exists a symmetric equilibrium of the offer game with signals where 1) workers send their signals to top K firms, and 2) firms play symmetric cutoff strategies.*

We now address the welfare implications from the introduction of a signaling mechanism. Proposition B2 and Theorem B2 formally restate our welfare results from previous chapters for the case when firms have many positions and workers can send multiple signals. The logic of their proofs again begins with an incremental approach: we first study the effect of a single firm increasing its cutoff, that is, responding more to signals. We then rank various signaling equilibria in terms of their outcomes. Finally, we show how the introduction of a signaling mechanism impacts our three measures of welfare.

Proposition B2 (Welfare Across Equilibria)**.** *Consider any two symmetric cutoff strategy equilibria where in one equilibrium firms have greater cutoffs. Compared to the equilibrium with lower cutoffs, in the equilibrium with greater cutoffs we have the following:*

- *the expected number of matches is weakly greater,*
- *workers have weakly higher expected payoffs, and*
- *firms have weakly lower expected payoffs.*

Theorem B2 (Welfare Impact of a Signaling Mechanism)**.** *Consider any non-babbling symmetric equilibrium of the offer game with signals. Then the following three statements hold.*

i. The expected number of matches is strictly greater than in the unique equilibrium of the offer game with no signals.

ii. The expected welfare of workers is strictly greater than in the unique equilibrium of the offer game with no signals.

iii. The welfare of firms may be greater or smaller than in the unique equilibrium of the offer game with no signals.

Proofs: Signaling with Many Positions and Many Signals

In addition to providing proofs for the above results, this section introduces Propositions B3-B6 and Lemma B2 which help establish the main findings.

Proposition B3. *The unique equilibrium of the offer game with no signals when firms use anonymous strategies and workers accept the best available offer is* $\sigma_f(\theta_f) = (\theta_f^1, ..., \theta_f^L)$ *for all* $f \in \mathcal{F}$ *and* $\theta_f \in \Theta_f$.

Proof. The proof repeats the argument of Proposition 1. □

Proposition B4. *In any symmetric non-babbling equilibrium of the offer game with signals each worker sends signals to her* K *top firms.*

Proof. Select an arbitrary worker. Firms use symmetric anonymous strategies, signals are identical, and the worker can send at most one signal to a given firm. Hence, from the worker's perspective the probability of getting an offer from a firm depends only on whether the worker has sent a signal to this firm or not. Similar to the argument of the proof of

Proposition 4 the probability of getting an offer from a firm that receives the worker's signal is greater than the probability of getting an offer from a firm that does not receive the worker's signal. Since this probability does not depend on the identity of the firm in a symmetric equilibrium we conclude that the worker optimally sends her signals to her K top firms. □

Proposition B5. *Suppose firms $-f$ use anonymous strategies and workers send their signals to their top K firms. Then firm f makes offers to its $L^{NS} \in \{0, ..., L\}$ top workers who have signaled to it and to its $L^S = L - L^{NS}$ top workers who have not signaled to it in any non-babbling symmetric sequential equilibrium.*

Proof. Note that firms use anonymous strategies, workers send their signal to their top K firms, and workers accept the best available offer. We first prove a lemma that states that from point of view of firm f, the probability that workers who have and have not signaled to it accept its offer depends only on the number of signals firm f receives.

Lemma B2. *Suppose firms $-f$ use anonymous strategies and workers send their signals to their top K firms. Consider two events, A and B. Event A is that firm f receives the set of signals $\mathcal{W}^S$. Event B is that firm f receives the set of signals $\check{\mathcal{W}}^S$, where $|\mathcal{W}^S| = |\check{\mathcal{W}}^S|$. Then*

- *the probability that worker $w \in \mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that worker $w' \in \check{\mathcal{W}}^S$ accepts firm f offer conditional on event B;*
- *the probability that worker $w \in \mathcal{W} \backslash \mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that worker $w' \in \mathcal{W} \backslash \check{\mathcal{W}}^S$ accepts firm f offer conditional on event B.*

Proof. Let us consider firm f with realized preference profile $\theta_f^* \in \Theta_f$ that receives signals from the set of workers $\mathcal{W}^S$. We first prove that the probability that a worker from $\mathcal{W}^S$ accepts firm f's offer conditional on event A equals the probability that a worker from $\check{\mathcal{W}}^S$ accepts firm f's offer conditional on event B.

Note that firm f believes that it is one of the top K firms in worker preference list if it receives an offer from her. Let us denote the set of possible agent profiles consistent with firm f beliefs in both events as

$$\Theta^A \equiv \{\theta \in \Theta | \theta_f = \theta_f^* \text{ and } rank_{\theta_{w^s}}(f) \in \{1, ..., K\} \text{ for each } w^s \in \mathcal{W}^S\}$$

$$\Theta^B \equiv \{\theta \in \Theta | \theta_f = \theta_f^* \text{ and } rank_{\theta_{w^s}}(f) \in \{1, ..., K\} \text{ for each } w^s \in \check{\mathcal{W}}^S\}$$

Since firm f receives the same number of signals for both events, i.e. $|\mathcal{W}^S| = |\check{\mathcal{W}}^S|$, for each worker $w_a \in \mathcal{W}^S$ we pair some worker $w'_a \in \check{\mathcal{W}}^S$, $a = 1, ..., |\mathcal{W}^S|$. Let us denote $rank_{\theta_{w_a}}(f) = k_a$ and $rank_{\theta_{w'_a}}(f) = k'_a$. Therefore, $k_a, k'_a \in \{1, ..., K\}$ for each a. We denote a permutation that changes k_a and k'_a's positions in a worker's preference list as

$$\rho^{w_a} : (..., k_a, ..., k'_a, ...) \to (..., k'_a, ..., k_a, ...).$$

We also denote a permutation that changes the ranks of w_a and w'_a for every a in a firm preference lists as

$$\rho^f : (..., w_a, ..., w'_a, ...) \to (..., w'_a, ..., w_a, ...).$$

Beginning with arbitrary profile of preferences $\theta \in \Theta^A$, we construct a profile of preferences θ' as follows:

- we do not change firm f preference list, i.e. $\theta'_f = \theta^*_f$,
- the ranks of workers w_a and w'_a are exchanged in the preference lists of firms $-f$ for each a: for each firm $f' \in -f$, $\theta'_f = \rho^f(\theta_f)$,
- firms in positions k_a and k'_a in worker w_a and worker w'_a preference profiles are exchanged for each a:
$$\theta'_{w_a} = \rho^{w_a}(\theta_{w_a}),\ \theta'_{w'_a} = \rho^{w_a}(\theta_{w'_a}), \quad \text{and}$$
- for any other $w^0 \in \mathcal{W} \backslash (\mathcal{W}^S \bigcup \check{\mathcal{W}}^S)$, $\theta_{w^0} = \theta'_{w^0}$.

Since firm f 's preference list is unchanged, $\theta'_f = \theta^*$, and firm f receives signals from the set $\check{\mathcal{W}}^S$ for profile θ', this profile belongs to Θ^B. Since firm $-f$ strategies are anonymous for any $f' \in -f$ and for any $\mathcal{W}^S_{f'} \subset \mathcal{W}$, we have that

$$\sigma_{f'}(\rho^f(\theta_{f'}), \rho^f(\mathcal{W}^S_{f'})) = \rho^f\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right).$$

Workers in $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ send their signals to the same firms among $-f$ for both profiles θ and θ'. Therefore, i.e. $\rho^f(\mathcal{W}^S_{f'}) = \mathcal{W}^S_{f'}$. Since $\theta'_f = \rho^f(\theta_f)$ we have that

$$\sigma_{f'}(\theta'_{f'}, \mathcal{W}^S_{f'}) = \rho\left(\sigma_{f'}(\theta_{f'}, \mathcal{W}^S_{f'})\right)$$

This means that the probability of firm f' making an offer to worker $w_a \in \mathcal{W}^S$ for profile θ equals the probability of making an offer to a worker in $w'_a \in \check{\mathcal{W}}^S$ for profile θ'. Moreover, since we exchange worker w_a and w'_a preference lists for profile θ', whenever it is optimal for worker w_a to accept firm f offer for profile θ, it is optimal for worker w'_a to accept firm f's offer for profile θ'.

Since firm types are independent the probability of firm f being matched when it makes an offer to w_a for profile θ equals the probability of firm f being matched when it makes an offer to worker w'_a for profile θ'. Therefore, for each $\theta \in \Theta^A$ there exists $\theta' \in \Theta^B$ such that the probability that firm f gets an offer from worker w_a equals the probability that firm f gets an offer from worker w'_a. Moreover, profile θ' is different for different θ by the construction. Therefore, we have constructed a bijection between sets Θ^A and Θ^B. Since θ and θ' are equally probable, the likelihood that firm f 's offer is accepted by worker w_a in the event A equals the probability that firm f's offer is accepted by worker w'_a in the event B.

An analagous construction works for the proof of the second statement that involves workers in sets $\mathcal{W}\backslash\mathcal{W}^S$ and $\mathcal{W}\backslash\check{\mathcal{W}}^S$. Therefore, the probability that worker $w \in \mathcal{W}\backslash\mathcal{W}^S$ accepts firm f offer conditional on event A equals the probability that worker $w' \in \mathcal{W}\backslash\check{\mathcal{W}}^S$ accepts firm f offer conditional on event B. □

The statement of the proposition follows directly from the lemma. Since the probability that the worker who has sent a signal to firm f accepts its offer is independent of the identity of the worker, firm f prefers to make offers to its top workers among those who signaled to it. Similarly, firm f prefers to make offers to its top workers among those who has not signaled to it. Finally, firm f prefers to make all L offers. □

Proposition B6. *Suppose workers send their signals to their top K firms. Then for any strategy σ_f of firm f, there exists a cutoff strategy that provides f with a weakly higher expected payoff than σ_f for any anonymous strategies σ_{-f} of opponent firms $-f$.*

Proof. Let us consider two sets of workers that firm f might receive $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ such that $\mathcal{W}^S$=$\check{\mathcal{W}}^S$. Firm f makes an offer to workers $\mathcal{W}_{offer} = \mathcal{W}^S_{offer} \bigcup \mathcal{W}^{NS}_{offer}$ such that$\mathcal{W}^{NS}_{offer} \subset \mathcal{W}^S$ and $\mathcal{W}^{NS}_{offer} \subset \mathcal{W}\backslash\mathcal{W}^S$ in equilibrium. Lemma B2 proves that identities of workers who have sent a signal to firm f do not influence the probability that workers accept the firm's offer provided that the total number of signals firm f receives is constant. Therefore, if workers $\mathcal{W}^S_{offer}$ are among$\check{\mathcal{W}}^S$, i.e. $\mathcal{W}^S_{offer} \subset \check{\mathcal{W}}^S$, it is still optimal for firm f to make its offers to workers $\mathcal{W}_{offer}$.

Let us again consider two sets of signals with the same power, i.e. $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ such that $\mathcal{W}^S$=$\check{\mathcal{W}}^S$. However, these sets differ now in one worker: there exist $w \in \mathcal{W}^S$ and $w' \in \check{\mathcal{W}}^S$ such that $\mathcal{W}^S\backslash w$=$\check{\mathcal{W}}^S\backslash w'$. Moreover, firm f prefers worker w' to worker w, i.e. $rank_{\theta_f}(w') > rank_{\theta_f}(w)$. As a consequence of Lemma B2, if firm f makes an offer to worker

w when it receives the set of signals $\mathcal{W}^S$ in equilibrium, it should make an offer to w' when it receives the set of signals $\check{\mathcal{W}}^S$. Let us consider the case when sets $\mathcal{W}^S$ and $\check{\mathcal{W}}^S$ differ in more than one worker. There are some workers in $\check{\mathcal{W}}_0 \subset \check{\mathcal{W}}^S$ who are better than workers in $\mathcal{W}_0 \subset \mathcal{W}^S$ who receive an offer from firm f when it recieves signals from $\mathcal{W}^S$. Similar argument shows that firm f should then optimally make an offer to $\check{\mathcal{W}}_0$ when it receives signals from $\check{\mathcal{W}}^S$.

The two arguments presented above allows us to conclude that if firm $-f$ use anonymous strategies, firm f's optimal strategy could be represented as some cutoff strategy. □

Proof of Theorem B1.

The proof repeats the steps of the proof of Theorem 3. □

Lemma B3. *Assume firms use cutoff strategies and workers send their signals to their top K firms. Fix the strategies of firms $-f$ as σ_{-f}. Let firm f's strategy σ_f differ from σ'_f only in that σ'_f has greater cutoffs (responds more to signals). Then we have*

$$E_\theta(m(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(m(\sigma_f, \sigma_{-f}, \theta))$$
$$E_\theta(\pi_w(\sigma'_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_w(\sigma_f, \sigma_{-f}, \theta))$$

where $m(\cdot)$ denotes the total number of matches.

Proof. Let us consider firm f cutoff strategies σ_f and σ'_f such that σ'_f has weakly greater cutoffs for profile θ_f:

$$\sigma_f(\theta_f, \mathcal{W}_f^S) = \mathcal{W}_{offer}^S \bigcup \mathcal{W}_{offer}^{NS}$$
$$\sigma'_{f'}(\theta_f, \mathcal{W}_f^S) = \check{\mathcal{W}}_{offer}^S \bigcup \check{\mathcal{W}}_{offer}^{NS}$$

In order to preserve anonymity firm f also should have the corresponding increase in cutoff strategies for any profile of preferences and any set of received signals of the same power. Firm f responds more to signals for profile θ_f means that $\mathcal{W}_{offer}^S \subset \check{\mathcal{W}}_{offer}^S \subset \mathcal{W}_f^S$ and $\check{\mathcal{W}}_{offer}^{NS} \subset \mathcal{W}_{offer}^{NS} \subset \mathcal{W} \backslash \mathcal{W}_f^S$. Proposition B5 shows that $|\mathcal{W}_{offer}^S \bigcup \mathcal{W}_{offer}^{NS}| = |\check{\mathcal{W}}_{offer}^S \bigcup \check{\mathcal{W}}_{offer}^{NS}| = L$. We consider only the case when $\mathcal{W}_{offer}^S \backslash \check{\mathcal{W}}_{offer}^S = w^S$ and $\check{\mathcal{W}}_{offer}^{NS} \backslash \mathcal{W}_{offer}^{NS} = w^{NS}$. More general case directly follows.

We denoter two sets of preference profiles

$$\Theta_+ \equiv \{\theta \in \Theta | \ m(\sigma_f, \sigma_{-f}, \theta) < m(\sigma'_f, \sigma_{-f}, \theta)\}$$
$$\Theta_- \equiv \{\theta \in \Theta | \ m(\sigma_f, \sigma_{-f}, \theta) > m(\sigma'_f, \sigma_{-f}, \theta)\}$$

For each profile θ from set Θ^+ it must be the case that without firm f offer w^{NS} has an offer from another firm, and worker w^S does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = 1. \tag{B.3.1}$$

Similarly, if profile θ is from set Θ^-, it must be the case that without firm f offer w^S has an offer from another firm and w^{NS} does not

$$m(\sigma'_f, \sigma_{-f}, \theta) - m(\sigma_f, \sigma_{-f}, \theta) = -1. \tag{B.3.2}$$

We will now show that $|\Theta_+| \geq |\Theta_-|$. Equations (B.3.1) and (B.3.2) along with the fact that each $\theta \in \Theta_+ \cup \Theta_-$ happens equally likely will then be enough to prove the result.

If profile θ belongs to Θ_-, without firm f's offer, worker w^S has an offer from another firm, name this firm f', and worker w^{NS} does not. We construct function $\psi : \Theta \to \Theta$ as follows. Let us considerLet $\psi(\theta)$ be the profile such that

- firms swap the positions of workers w^{NS} and w^S in their preference lists.
- if both w^S and w^{NS} send signals to firm f' for profile θ their preferences remain unchanged
- if woker w^S (w^{NS}) sends her signal to firm f' but worker w^{NS} (w^S) does not for profile θ, find a firm f_y such that worker $w^S(w^{NS})$ does not send her signal to firm f_y, and worker w^{NS} (w^S)does. Exchange the positions of firm f' and firm f_y in worker w^{NS} and worker w^S preference lists.

Note that firm f_y exists because each worker sends exactly K signals in any non-babling symmetric equilibrium. We need the latter modification because each worker can send several signals, and the fact that worker w^Ssends her signal to firm f does not guarantee that she does not send another signal to firm f'.

If profile θ belongs to Θ_-, without firm f's offer, worker w^S has an offer from firm f', and worker w^{NS} does not. Therefore, when preferences are $\psi(\theta)$, without firm f's offer the following two statements should be true i) worker w^{NS} ***must*** have another offer and ii) worker w^S ***cannot*** have another offer.

To see i), note that under θ, worker w^S his outside offer comes from firm f'. Under $\psi(\theta)$ worker w^{NS} take position of worker w^S in firm f' preference list, and worker w^{NS} sends a signal to firm f' for profile $\psi(\theta)$ whenever worker w^S sends a signal to firm f' for profile θ. Anonymity of firm strategies guarantee that firm f' makes an offer to worker w^{NS}.

To see ii), suppose to the contrary that under $\psi(\theta)$, worker w does in fact receive an outside offer from some firm f''. This cannot be firm f'. Otherwise worker w^{NS} should get an offer from firm f' for profile θ by anonymity. This cannot be firm f_y because worker w^{NS} would get an offer from firm f_y for profile θ.

The main idea of the construction preserves the logic of Theorem 4. Specifically, if a worker receives firm's offer when she does not send a signal to the firm, she will definitely receives an offer if she sends a signal to the firm.

From i) and ii), we have

$$\theta \in \Theta_- \Rightarrow \psi(\theta) \in \Theta_+.$$

Since function ψ is injective, we have $|\Theta_+| \geq |\Theta_-|$.

In order to prove the second statement note that the expected number of matches of each worker increases when firm f responds more to signals. Using the construction presented above, one could show whenever worker w looses a match with firm f for profile θ (worker w ranks firm f low) it is possible to construct profile θ' when worker w obtains the match (worker w ranks firm f high). The function that matches these profiles is again injective. Moreover, worker w values more the match with high ranked firms. Therefore, ex-ante utility of worker w increases when firm f responds more to signals. □

Proof of Proposition B2.

The result that the expected number of matches and the expected welfare of workers is higher in the equilibrium with higher cutoffs is an immediate consequence of Lemma B3.

In order to show that firms have lower expected payoffs in the equilibrium with greater cutoffs we first consider the following situation. We take some firm f such that its strategy σ_f differs from σ'_f only in that σ'_f has weakly greater cutoffs. Let us consider some firm $f' \in -f$. For each profile of preferences $\theta_{f'}$ and a set of signals $\mathcal{W}^S$, firm f' either makes an offer to $S_{f'}(\theta_{f'}, \mathcal{W}^S)$ or $T_{f'}(\theta_{f'}, \mathcal{W}^S)$. If firm f responds more to signals this decreases the probability that both $T_{f'}$ and $S_{f'}$ accept firm f' offer. Therefore, the expected payoff of firm $f' \in -f$ weakly decreases when firm f responds more to signals.

$$E_\theta(\pi_{f'}(\sigma_f, \sigma_{-f}, \theta)) \geq E_\theta(\pi_{f'}(\sigma'_f, \sigma_{-f}, \theta)).$$

Let us now consider two symmetric equilibria where firms play cutoff strategies $\tilde{\sigma}$ and $\bar{\sigma}$ correspondingly such that $\tilde{\sigma} \geq \bar{\sigma}$. From the definition of an equilibrium strategy we have:

$$E_\theta[\pi_f(\bar{\sigma}_f, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma_f}, \bar{\sigma}_{-f}, \theta)]$$

Using the result proved above we proceed with

$$E_\theta[\pi_f(\tilde{\sigma_f}, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma_f}, \tilde{\sigma}_{-f}, \theta)]$$

Therefore

$$E_\theta[\pi_f(\bar{\sigma}_f, \bar{\sigma}_{-f}, \theta)] \geq E_\theta[\pi_f(\tilde{\sigma}_f, \tilde{\sigma}_{-f}, \theta)]$$

□

Proof of Theorem B2.

Denote firm strategies in the unique equilibrium of the offer game with no signals as σ_F^0. Now consider a symmetric equilibrium of the offer game with signals where agents use strategies (σ_F, σ_W). If agents employ strategies (σ_F^0, σ_W), the expected number of matches and the welfare of workers equal the corresponding parameters in the offer game with no signals. Therefore, the result that the expected number of matches and the expected welfare of workers in a symmetric equilibrium in the offer game with signals are *weakly* greater than the corresponding parameters in the unique equilibrium of the offer game with no signals is a consequence of sequential application of Lemma B3. The result for worker and firm welfare, and the argument that the comparison is strict are analagous to those in Theorem 4. □