# Pigouvian Tax, Abatement Policies and Uncertainty on the Environment

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**Abstract**

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## 1 Introduction

Nowadays environmental problems are a central issue for many developed and developing countries. Pollution and climate change are important challenges for human health and economic growth. Defining appropriate economic policies for containing environmental degradation and eventually improving environmental quality is now a priority for many governments.[1]

The economic effects of pollution and environment degradation have been widely analyzed in the literature. On one hand, these papers show the relevance of environmental change for different economic issues. In this field, in particular, the effects on economic growth and welfare have been both studied from a theoretical standpoint and quantified by simulations and calibrations.[2]

On the other hand, many papers examine appropriate policies in order to reduce environmental degradation. Two policy instruments are most important. The first is abatement policy, which is an intervention, financed by tax, which directly improves environmental quality. The second is the so-called 'Pigouvian tax', first introduced by PIGOU (1924), who argued that negative externality due to pollution can be internalized in a competitive market by introducing a tax equal to the social marginal damage caused by environment degradation.[3]

[1] A recent important signal of this importance is the attention paid in all developed countries to the Summit on Climate Change, Copenhagen, December 2009.

[2] For a survey of these issues see BAUMOL & OATES (1988), NORDHAUS (1994), NORDHAUS & BOYER (2000) and XEPAPADEAS (2005).

[3] For an analysis of the design and the effects of these two policies see respectively SMULDERS & GRADUS (1996), ECONOMIDES & PHILIPPOPOULOS (2008) and GUPTA & BARMAN (2009) for abatement policy and PIGOU (1924), BAUMOL & OATES (1988),

Although there is a broad consensus on the idea that the climate is changing, we also find widespread uncertainty on how this change will occur. A good example of this is the well-known issue of global warming. First, although the majority of scientists in the climate research community assert that there has been an increase in temperatures due to human activities, others disagree.[4] Second, even if we believe in global warming, there is widespread uncertainty on how temperatures will rise[5] and how this will impact on human systems.[6]

The aim of this work is to study the effects of uncertainty on climate with specific reference to its influence on the implementation of policies used to counteract environmental deterioration. For this purpose we introduce three types of uncertainty, i.e. uncertainty on the environmental quality level, uncertainty on the effect of pollution and uncertainty on the impact of the abatement policy.

To our knowledge the effects of uncertainty on the environment was studied in the literature only from a standpoint different from that considered in this work. In particular, among the papers examining this issue, Heal (1984) and Keller et al. (2004) examine the effect of uncertainty on the time of a future change in productivity caused by pollution, inserting an uncertain climate threshold in a model of optimal depletion and in a DICE growth model.[7] Ulph & Ulph (1997) and Pindyck (2000, 2002) analyse the optimal timing for the implementation of an abatement policy in a real option framework. Gollier et al. (2000) study conditions on utility functions which ensure that scientific progress reducing uncertainty on the distribution of damage related to consumption (in particular pollution damage) induce an earlier prevention effort. Finally, Soretz (2007) examines optimal pollution taxation in an endogenous growth model where productivity is affected by uncertainty on environmental quality on the basis of a relationship evolving as a stochastic Wiener process.

This paper presents three main differences with regard to the previous literature. First, the papers cited above either do not explicitly consider environmental policies, or, as in the majority of the cases, they study the effect of uncertainty on abatement tax. In this work, we analyze the effect of environmental uncertainty on both Pigouvian and abatement tax, considered both as separate instruments and joint instruments.

Second, the papers cited above make specific assumptions about the distribution of the random process describing uncertainty while our paper does not.[8] We thus obtain general conclusions which do not depend on the specific distribution introduced. In this field, we also introduce a further generalization, extending our uncertainty results to the case of the so called '$N$-th order risk change'.[9]

Finally, the existing literature studies a one-argument utility framework where utility depends only on consumption. This implies that environmental uncertainty affects agents' preferences through its effect on consumption, because it impacts on productivity. Environmental uncertainty, however, has mainly a direct effect on utility since environmental quality determines a specific dimension of agents utility, different from consumption.

This framework is clearly related to a recent strand of literature analyzing the effect of the background risk. Background risk is in fact a second source of risk, distinct from financial risk, studied in a two-argument utility model, where the second argument of the utility is related to variables such as health status

Mohtadi (1996) and Helfand & al. (2003) for Pigouvian tax. Note that other possible instruments for public intervention in environmental economics, not analysed in this work, are property rights, binding quota restrictions and markets for pollution permits.

[4] On this see, for instance, the debate between the Intergovernmental Panel on Climate Change (IPCC) and the Nongovernmental International Panel on Climate Change (NIPCC).

[5] As emphasized by the IPCC: '*Models differ considerably in their estimates of the strength of different feedbacks in the climate system, particularly cloud feedbacks, oceanic heat uptake and carbon cycle feedbacks, although progress has been made in these areas. Also, the confidence in projections is higher for some variables (e.g. temperature) than for others (e.g. precipitation), and it is higher for larger spatial scales and longer time averaging periods*' (IPCC, 2007, p. 73) and '*Projections of climate change and its impacts beyond about 2050 are strongly scenario- and model-dependent, and improved projections would require improved understanding of sources of uncertainty and enhancements in systematic observation networks*' (IPCC, 2007, p.73).

[6] On this the IPCC Fourth Assessment Report claims that: '*Effects of climate changes on human and some natural systems are difficult to detect due to adaptation and non-climatic drivers*' (IPCC, 2007, p. 72). On uncertainty on the economic effects of climatic changes see also Heal and Kriström (2002).

[7] A similar model is used to study uncertainty on future preferences about the environment (given the certain level of future environmental quality) in Beltratti et al. (1998), Ayong Le Kama (2001) and Ayong Le Kama & Schubert (2004).

[8] The only exception to this is Gollier et al. (2000), who study a framework where the distribution of a generic random variable is revised by consumers according to the Bayes rule.

[9] As defined by Ekern (1980) and Eeckhoudt & Schlesinger (2008).

or environmental quality. In this literature, Pratt (1988), Finkelshtain et al. (1999) and Courbage (2001) study the effect of background risk, and thus of environmental risk, on risk aversion. Courbage & Rey (2007), Menegatti (2009a) and Menegatti (2009b) analyse the effects of background risk on precautionary saving respectively under some specific assumptions on bivariate risk distribution and in the case of small risks. Finally Denuit et al. (2009) generalize the analysis of optimal saving and investment in the presence of two correlated risks, financial and background.

This paper is organized as follow. Section 2 introduces the model. Section 3 analyses the effect of uncertainty on optimal Pigouvian taxation. Section 4 studies the implication of uncertainty in a model with abatement policies. Section 5 examines a model where both Pigouvian tax and abatement policies are used together. Section 6 presents a generalization of our results in the case of stochastic $N$-th degree risk increase. Section 7 concludes.

# 2 The model

We assume that consumer preferences are represented by a bivariate utility function $U(c,q)$. The utility function depends on consumption $c$ and on environmental quality level $q$, where $c$ and $q$ are expressed with different unit of measurement. $U(c,q)$ is increasing and concave with regard to each argument and it is three times continuously differentiable with respect to $c$ and $q$ respectively. Letting $U_c = \partial U/\partial c$, $U_q = \partial U/\partial q$, $U_{cc} = \partial^2 U/\partial c^2$, $U_{cq} = \partial U/\partial c\partial q$ and so on, these assumptions mean $U_c > 0$, $U_q > 0$, $U_{cc} < 0$ and $U_{qq} < 0$.

Consumption $c$ is composed by exogenous income $y$ and by income derived from production activity. Production activity is financed by investment $s$ and generates an output given by $f(s)$. As usual for production functions, we assume that $f(s)$ is increasing and concave with regard to $s$, such that $f'(.) > 0$ and $f''(.) < 0$. Consumption $c$ is thus given by $c = y - s + f(s)$.[10] The value of $q$ depends on the initial environmental quality level $q_0$ and on the damage function $g(s)$, which expresses the negative influence of pollution through investment in productive activities. Since larger production implies more pollution, we assume that $g'(.) > 0$.

As usual in the literature on pollution and environment, we assume that pollution is an externality (see Helfand & al., 2003 and Xepapadeas, 2005), meaning that consumers do not take it into account when they choose the optimal level of $s$. The consumer maximisation problem, which considers environmental quality level as given and equal to $q_0$, is thus the following:

$$\max_s U(y - s + f(s), q_0)$$

Optimal investment $s^\star$ is determined as the solution of the first-order condition[11]

$$U_c(y - s^\star + f(s^\star), q_0) = f'(s^\star) U_c(y - s^\star + f(s^\star), q_0)$$

which is equivalent to

$$f'(s^\star) - 1 = 0. \tag{1}$$

Note that condition (1) simply implies the equality between marginal cost and marginal benefit of investment $s$.[12]

The externality related to pollution is neglected by the consumer, but considered by the planner. This means that, when we consider the planner's problem, we need to take the effect of $s$ on $q$ into account. So the planner's optimal choice is obtained by maximising

$$\max_s U(y - s + f(s), q_0 - g(s)).$$

[10] Note that since the model is static, we have that investment $s$ and production activity related to it are contemporaneous. This means that the agent has only to choose the optimal fraction of exogenous income $y$ to be destined to the production activity (which will depend on marginal productivity). For simplicity we call this fraction 'investment' even though the model does not exhibit any kind of capital accumulation.

[11] It is clear that, given our assumption on $U(c,q)$ and $f(s)$ the second-order condition of the consumer's problem is satisfied.

[12] Note that condition (1) is obtained in the absence of uncertainty. It is however easy to show that, unlike the other first-order conditions derived later on, this condition also holds under all the types of uncertainty studied in this work.

The first-order condition is

$$f'(\overline{s}) - 1 = \frac{U_q(y - \overline{s} + f(\overline{s}), q_0 - g(\overline{s}))}{U_c(y - \overline{s} + f(\overline{s}), q_0 - g(\overline{s}))} g'(\overline{s}) \tag{2}$$

where $\overline{s}$ is the planner optimal investment level.[13] This condition implies the equality between social marginal cost and marginal benefit of investment $s$. Note that both cost and benefit in (2) are measured in utils. The same is true for consumer's first-order condition before the simplification determining (1), due to the symmetric effect of cost and benefit on consumer's utility.

The comparison between $s^\star$ and $\overline{s}$ clearly shows that the consumer chooses a suboptimally larger investment level $s$ due to pollution externality.[14] This suboptimal result can be removed by the planner by imposing a Pigouvian tax equal to the marginal pollution damage. Furthermore, environmental quality level can be increased by means of abatement policies. The following sections compare the effects of these two instruments under uncertainty.

# 3 Pigouvian Tax and Uncertainty

The excess of investment in the consumer's optimal choice shown in the previous section can be removed by means of a Pigouvian tax, i.e. a tax proportional to investment equal to marginal pollution damage. If we introduce the Pigouvian tax $T_p$, the consumer's problem becomes

$$\max_s U(y - s + f(s) - sT_p, q_0)$$

Consumer's optimal investment $s^{\star\star}$ is thus determined in this case by the first-order condition[15,16]

$$f'(s^{\star\star}) - 1 = T_p \tag{3}$$

Given this condition and by comparing (2) and (3), it is easy to see that the consumer's choice coincides with the socially optimal investment (i.e. $s^{\star\star} = \overline{s}$) if the planner chooses the optimal level of $\overline{T_p}$ which is

$$\overline{T_p} = \frac{U_q(y - \overline{s} + f(\overline{s}), q_0 - g(\overline{s}))}{U_c(y - \overline{s} + f(\overline{s}), q_0 - g(\overline{s}))} g'(\overline{s}). \tag{4}$$

In this context we can introduce two alternative types of environmental uncertainty: 'uncertainty on environmental quality level' ($q_0$ random) or 'uncertainty on the effect of pollution' ($g(s)$ random). Note that the second case describes uncertainty on the effect of human activities on the environment, while the first considers uncertainty on environment features. For this reason, the case of $q_0$ random can represent a situation where we have uncertainty on the present level of environmental deterioration, but also a situation where we have uncertainty on the possible occurrence of natural disasters not related to human activities (like, for instance, earthquakes, tsunamis and volcanic eruptions).[17]

[13] The second-order condition for the planner problem is $U_{cc}(y-s+f(s), q_0-g(s))\,[f'(s)-1]^2 + U_c(y-s+f(s), q_0-g(s)) f''(s) + U_{qq}(y-s+f(s), q_0-g(s)) g'(s)^2 - U_q(y-s+f(s), q_0-g(s)) g''(s) - 2U_{cq}(y-s+f(s), q_0-g(s))\,[f'(s)-1]\, g'(s) < 0$. We assume that this condition is satisfied. Note however that, given the previous assumptions on $U(c,q)$ and $f(s)$, the condition is automatically satisfied if $g''(.) \geq 0$ and $U_{cq} \geq 0$. This last assumption would mean that consumption and environmental quality level are complements. A similar hypothesis was made, for example, by Mohtadi (1996), who shows that an improvement in environmental quality increases economic growth if $U_{cq} \geq 0$.

[14] The right-hand side of (2) is positive, while the right-hand side of (1) is null. Since $f'(.)$ is decreasing, this clearly implies that $s^\star \geq \overline{s}$.

[15] Obviously the condition implies equality between the new marginal cost of investment (incorporating the tax) and investment marginal benefit.

[16] Note that we introduce Pigouvian tax as a tax on polluting investment $s$. Alternatively it could be introduced as a tax on output $f(s)$. It is easy to see that, in this case, the first-order condition of consumer's problem (3) would become $(1-\widehat{T}_p) f'(s^{\star\star}) - 1 = 0$ where $\widehat{T}_p$ is the level of Pigouvian tax on output. Considering this together with (3) we get $\widehat{T}_p = 1 - \frac{1}{T_p+1}$, implying that $\widehat{T}_p$ is an increasing function of $T_p$. For this reason, the results on the larger or smaller size of $T_p$ under uncertainty, derived later on, are also correct for $\widehat{T}_p$.

[17] To formalize this idea assume, for instance, that $\widetilde{q}_0 = q_0 + \widetilde{\epsilon}$ where $\widetilde{\epsilon}$ is a random variable describing the possible occurrence of a natural disaster.

*a) Uncertainty on environmental quality*

Firstly we study the case where $q_0$ is random. We define it as $\widetilde{q_0}$ and its expected value as $\mathbb{E}(\widetilde{q_0}) = q_0$. Let us denote $\widetilde{s}$ and $\widetilde{T_p}$ the investment and tax level under uncertainty, while $\overline{s}$ and $\overline{T_p}$ are respectively investment and Pigouvian tax in the certainty case.

Under uncertainty, the planner's maximisation problem becomes

$$\max_s \mathbb{E}\left[U\left(y - s + f(s), \tilde{q_0} - g(s)\right)\right].$$

Computing the planner's first-order condition and combining it with (3) we have that

$$\tilde{T_p} = \frac{\mathbb{E}\left[U_q(y - \tilde{s} + f(\tilde{s}), \tilde{q}_0 - g(\tilde{s}))\right]}{\mathbb{E}\left[U_c(y - \tilde{s} + f(\tilde{s}), \tilde{q}_0 - g(\tilde{s}))\right]} g'(\tilde{s}). \tag{5}$$

This implies that

**Proposition 3.1.** *Under uncertainty on environmental quality level, conditions $U_{cqq} \leq (\geq) 0$ and $U_{qqq} \geq (\leq) 0$ are sufficient to have $\widetilde{T_p} \geq (\leq) \overline{T_p}$.*

*Proof.* See the Appendix for the proof. □

Proposition 3.1 shows that the comparison between the optimal level of Pigouvian tax with and without uncertainty depends on the sign of third-order derivatives of the utility function. This conclusion is not unusual in the analysis of uncertainty effects. The most important result in this field is the well-known condition ensuring precautionary saving in a univariate utility and two-period framework. As first shown by Leland (1968) and clearly explained by Kimball (1990),[18] positive precautionary saving occurs in this case if the third derivative of the utility function (with regard to its unique argument) is positive.

Moving from a univariate to a bivariate utility framework, however, the role of $U_{qqq}$ and $U_{cqq}$ (called 'cross prudence' by Eeckhoudt, Rey & Schlesinger, 2007) is shown in many works. In a bivariate utility two-period model, in particular, the signs of $U_{qqq}$ and $U_{cqq}$ are shown to determine precautionary saving in different contexts together with $U_{ccc}$ (see Courbage & Rey, 2007, Menegatti, 2009a and Menegatti, 2009b). Similarly the same derivatives are relevant for the determination of saving in the presence of correlated risks (see Denuit & al., 2009). Finally, similar conditions affect the optimal choice of labor supply in a one period bivariate utility model of choice between consumption and leisure (see Chiu & Eeckhoudt, 2010).

An interpretation of conditions $U_{qqq} \geq 0$ and $U_{cqq} \leq 0$ can be obtained starting from that of precautionary saving condition provided by Menegatti (2007).[19] In fact, following Menegatti (2007), the changes in the size of $-U_{qq}$ can be seen as a measure of the changes in the disutility due to uncertainty on environmental quality.[20] Condition $U_{qqq} \geq 0$ ensures that this disutility is reduced if we increase environmental quality and thus if we reduce pollution. Condition $U_{cqq} \leq 0$ ensures that this disutility is reduced if we decrease consumption and thus if we reduce investment ($\tilde{s} \leq \overline{s}$) below its optimal value determined by condition (2). In conclusion, both the effects described above are related to prudence. The first is a kind of 'prudence about the environment' (greater uncertainty on the environment implies that a better environmental level is desirable), while the second is a kind of negative cross-prudence (greater uncertainty on the environment implies that less consumption is desirable). These two facts together imply that, under uncertainty, we prefer less investment and less pollution.

*b) Uncertainty on the pollution effect*

[18]In this field, see also Sandmo (1970), Drèze & Modigliani (1972) and Menegatti (2001).

[19]On this see also Eeckhoudt & Schlesinger (2006).

[20]Following Menegatti (2007) $\mathbb{E}\left[U\left(y - s + f(s), \tilde{q_0} - g(s)\right)\right]$ can be written as $U\left(y - s + f(s), q_0 - g(s)\right) + \Gamma\left(y - s + f(s), \tilde{q_0} - g(s)\right)$, where $\Gamma\left(y - s + f(s), \tilde{q_0} - g(s)\right) = \mathbb{E}\left[U\left(y - s + f(s), \tilde{q_0} - g(s)\right)\right] - U\left(y - s + f(s), q_0 - g(s)\right)$. The index $\Gamma(.,.)$ is called utility premium (or generalized risk measure) and it is a measure of the disutility caused by uncertainty. Following Stone (1970) and Menegatti (2007) we have that, for small risks, $\Gamma(.,.) \cong -\frac{1}{2}U_{qq}(.,.)\sigma^2$.

We now turn to uncertainty on the pollution effect. For simplicity, we assume that the damage function $g(s)$ is linear defining $g(s) = ks$. Note that under this assumption, Equation (4) becomes

$$\overline{T_p} = \frac{kU_q(y - \overline{s} + f(\overline{s}), q_0 - k\overline{s})}{U_c(y - \overline{s} + f(\overline{s}), q_0 - k\overline{s})}. \tag{6}$$

Under this assumption we introduce uncertainty on parameter $k$. We denote $\widetilde{k}$ the random value of $k$ and its expected value as $\mathbb{E}(\widetilde{k}) = \overline{k}$. As before, we denote as $\widetilde{s}$ and $\widetilde{T_p}$ investment and Pigouvian tax under uncertainty. The planner's maximisation problem now becomes

$$\max_s \mathbb{E}\left[U(y - s + f(s), q_0 - \tilde{k}s)\right].$$

implying that

$$\tilde{T_p} = \frac{\mathbb{E}\left[\widetilde{k}U_q(y - \tilde{s} + f(\tilde{s}), q_0 - \widetilde{k}\tilde{s})\right]}{\mathbb{E}\left[U_c(y - \tilde{s} + f(\tilde{s}), q_0 - \widetilde{k}\tilde{s})\right]} \tag{7}$$

This result implies in turn

**Proposition 3.2.** *Under uncertainty on the effect of pollution, conditions $U_{cqq} \leq (\geq) 0$ and $-ks\frac{U_{qqq}}{U_{qq}} \geq (\leq) -2$ are sufficient to have $\widetilde{T_p} \geq (\leq) \overline{T_p}$.*

*Proof.* See the Appendix for the proof. □

In the case of uncertainty on the pollution effect, we have a higher Pigouvian tax if $U_{cqq} \leq 0$ and $-ks\frac{U_{qqq}}{U_{qq}} \geq -2$. The first condition is the same as obtained in the case of uncertainty on environmental quality, and its interpretation is the same too. The second condition is different from the previous case. In order to explain it, it must first be emphasised that the index $-ks\frac{U_{qqq}}{U_{qq}}$ is a relative prudence index.[21] The level of relative prudence has been proved to be relevant in cases where a return is uncertain. In particular, Rothschild & Stiglitz (1971) showed that uncertainty on the interest rate (the return of saving) increases saving if the index of relative prudence is larger than 2. Similarly Chiu & Eeckhoudt (2010) showed that the same conditions in Proposition 3.2 ensure that uncertainty on wages raises the labor supply.[22] These two results together indicate that a condition of relative prudence is related to uncertainty on returns (either the returns from saving or the return from labor).

The interpretation of condition $-ks\frac{U_{qqq}}{U_{qq}}$ in Proposition 3.2 is consistent with these conclusions. Indeed, in our framework, parameter $k$ measures the effect of a unit of investment on pollution, and it is thus a kind of 'negative return' of investment in terms of pollution. For this reason, as in the problems cited above, the effect of uncertainty on it depends on the index of absolute prudence. Furthermore, the difference in our condition compared to that in Rothschild & Stiglitz (a threshold level of -2 instead of 2) occurs since the effect is here a negative return instead of a positive return (we like interest and we dislike pollution).[23]

# 4 Abatement Policies and Uncertainty

In the previous section we introduced a Pigouvian tax in order to reduce pollution. We now turn to a different policy to improve environmental quality: abatement policy. Abatement (or cleanup) activities can

[21] Following Pratt (1964) and Kimball (1990) for an univariate utility function $U(x)$ we can define the following indexes: absolute risk aversion $-U_{xx}(x)/U_x(x)$, relative risk aversion $-xU_{xx}(x)/U_x(x)$, absolute prudence $-U_{xxx}(x)/U_{xx}(x)$ and relative prudence $-xU_{xxx}(x)/U_{xx}(x)$.

[22] Chiu & Eeckhoudt (2010) showed that these conditions are sufficient for an increase in labor supply, in case of a mean-preserving spread in wages. The comparison between a known level of wage and a random one can be seen as a specific case of mean-preserving spread.

[23] On this last conclusion see also the results in next section.

in general be seen as a policy, financed by a lump sum tax, aimed at reducing environmental degradation due to pollution and increasing environmental quality. [24]

It is important to note that this instrument is completely different from the previous one since it does not affect the level of investment $s$ and the level of pollution. The policy introduced here is simply a policy which improves environmental quality, and its optimal implementation does not influence agent's decision problem.[25] For this reason the terms $f(s^\star) - s^\star$ and $g(s^\star)$ in the maximization problem could be removed in this context. Furthermore implementation of the abatement policy is not specifically linked to the presence of an externality related to pollution. In fact, an optimal abatement policy could also be introduced in a framework where consumers take pollution into account (i.e. in the absence of externalities), if the planner can intervene in order to improve environmental quality.

Nevertheless, in order for the paper to have a unified framework, we retain the terms $f(s^\star) - s^\star$ and $g(s^\star)$ in our specification. We emphasize, however, that the problems in Section 3 and in Section 4 are different and distinct problems. The two instruments (and the two problems) will be considered together in Section 5.

We assume that the effect of the abatement policy is described by a linear function so that for each tax unit paid we have an increase of $m$ in environmental quality.[26] Given consumer's choice, the planner defines the optimal abatement tax level solving the following problem

$$\max_{T_a} U\left(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + mT_a\right).$$

The first-order condition of this problem is[27]

$$-U_c\left(y - s^\star + f(s^\star) - \overline{T_a}, q_0 - g(s^\star) + m\overline{T_a}\right) + mU_q\left(y - s^\star + f(s^\star) - \overline{T_a}, q_0 - g(s^\star) + m\overline{T_a}\right) = 0. \quad (8)$$

where $\overline{T_a}$ is the optimal abatement tax.

In this context, we can introduce two alternative types of environmental uncertainty: 'uncertainty on environmental quality level' ($q_0$ random) or 'uncertainty on the effect of the abatement policy' ($m$ random).

*a) Uncertainty on environmental quality*

First we study the case when $q_0$ is random. We define it as $\widetilde{q_0}$ and its expected value as $\mathbb{E}(\widetilde{q_0}) = \overline{q_0}$. We also denote as $\widetilde{T_a}$ the abatement tax under uncertainty. Under uncertainty the previous maximisation problem becomes

$$\max_{T_a} \mathbb{E}U(y - s^\star + f(s^\star) - T_a, \widetilde{q_0} - g(s^\star) + mT_a)].$$

The first-order condition is now

$$-\mathbb{E}[U_c(y - s^\star + f(s^\star) - \widetilde{T_a}, \widetilde{q_0} - g(s^\star) + m\widetilde{T_a})] +$$
$$+ m\mathbb{E}[U_q(y - s^\star + f(s^\star) - \widetilde{T_a}, \widetilde{q_0} - g(s^\star) + m\widetilde{T_a})] = 0. \quad (9)$$

Comparing (8) and (9) we have

**Proposition 4.1.** *Under uncertainty on environmental quality level, conditions $U_{cqq} \leq (\geq) 0$ and $U_{qqq} \geq (\leq) 0$ are sufficient to have $\widetilde{T_a} \geq (\leq) \overline{T_a}$.*

*Proof.* See the Appendix for the proof. □

[24] The United Nations defines pollution abatement as technology applied or measure taken to reduce pollution and/or its impacts on the environment (see United Nations, 1997).

[25] A very simple example of the kind of policy introduced in this section can be, for instance, a public intervention (financed by a lump sum tax) which plants new trees.

[26] A linear abatement is used for instance by Economides & Philippopoulos (2008) and by Gupta & Barman (2009).

[27] The second-order condition for this problem is $U_{cc}\left(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + mT_a\right) - 2mU_{cq}\left(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + mT_a\right) + m^2 U_{qq}\left(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + mT_a\right) < 0$. Given our assumption on $U(c, q)$ it is surely satisfied if $U_{cq} \geq 0$.

The sufficient conditions in Proposition 4.1 are the same as Proposition 3.1, and their interpretation is similar too. Conditions $U_{cqq} \leq 0$ and $U_{qqq} \geq 0$ ensure that the disutility due to uncertainty is reduced when consumption decreases and environmental quality is improved. This implies that we choose larger abatement under uncertainty.

*b) Uncertainty on the impact of abatement policy*

We now study the second type of uncertainty: uncertainty on the effect of abatement policy. We thus consider $m$ random and we denote it as $\tilde{m}$ and its expected value $E(\widetilde{m})$ as $\overline{m}$. In this case the maximisation problem and the first-order condition become respectively

$$\max_{T_a} \mathbb{E}[U\left(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + \tilde{m}T_a\right)].$$

and

$$\begin{aligned} &-\mathbb{E}[U_c(y - s^\star + f(s^\star) - \widetilde{T_a}, q_0 - g(s^\star) + \widetilde{m}\widetilde{T_a})] + \\ &+\mathbb{E}[\widetilde{m}U_q(y - s^\star + f(s^\star) - \widetilde{T_a}, q_0 - g(s^\star) + \widetilde{m}\widetilde{T_a})] = 0. \end{aligned} \tag{10}$$

Comparing (8) and (10) we have

**Proposition 4.2.** *Under uncertainty on abatement effect, conditions $U_{cqq} \leq (\geq)0$ and $-mT_a\frac{U_{qqq}}{U_{qq}} \geq (\leq)2$ are sufficient to have $\widetilde{T_a} \geq (\leq)\overline{T_a}$.*

*Proof.* See the Appendix for the proof. □

Sufficient conditions obtained in this case are similar to those in the case of the effect of uncertainty on pollution. In particular, condition $U_{cqq} \leq 0$ is exactly the same and it has an analogous interpretation. Condition $-mT_a\frac{U_{qqq}}{U_{qq}} \geq (\leq)2$ is similar to that obtained in Proposition 3.2 and is equivalent to that derived by Rothschild & Stiglitz (1971). As explained in Section 2, this condition is related to uncertainty on returns, and, consistently, abatement performance (now uncertain) is the return of the abatement policy. Furthermore, unlike the case of $k$ in Section 3, return here is positive (policy improves environmental quality level), so the threshold level is exactly the same as determined by Rothschild & Stiglitz.

# 5 A Pigouvian Tax and Abatement Policy Model

In this section we introduce jointly both instruments, using a framework where the policy maker uses both an abatement policy and a Pigouvian tax. In this case, the planner chooses socially optimal levels of investment ($s$) and abatement policies ($T_a$), solving the following problem

$$\max_{s, T_a} U(y - s + f(s) - T_a, q_0 + mT_a - ks).$$

Note that this problem seems similar to that in Section 4. The difference between them is that, in the model in this section, $s$ is chosen by the planner (since it is determined by the Pigouvian tax $T_p$) while, in the model in Section 4, the value of $s$ is exogenously given. The maximisation implies

$$-U_c\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, q_0 + m\overline{T_a} - k\overline{s}\right) + mU_q\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, q_0 + m\overline{T_a} - k\overline{s}\right) = 0 \tag{11}$$

$$\begin{aligned} &(f'(\overline{s}) - 1)\, U_c\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, q_0 + m\overline{T_a} - k\overline{s}\right) - \\ &+kU_q\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, q_0 + m\overline{T_a} - k\overline{s}\right) = 0 \end{aligned} \tag{12}$$

From (11) and (12) we get[28]

$$\frac{U_c(.,.)}{U_q(.,.)} = m \tag{13}$$

[28] In order to simplify, we omit the two arguments of the utility function.

and

$$\frac{U_c(.,.)}{U_q(.,.)} = \frac{k}{f'(\overline{s}) - 1}. \tag{14}$$

Given (13) and (14) we have that

$$(f'(\overline{s}) - 1) = \frac{k}{m}. \tag{15}$$

Once the optimal investment $\overline{s}$ is obtained, we compute $\overline{T_a}$ by substituting it in (11). Finally, given (3), the optimal level of $T_p$ is obtained by $\overline{T_p} = f'(\overline{s}) - 1$.

Previous conditions clearly indicate a separation of the objectives of Pigouvian tax and abatement policy. Pigouvian tax is used in order to guarantee a level of investment $s$ which satisfies condition (15). The left-hand side of this equation can be seen as the ratio of monetary net marginal positive return of investment $s$ on consumption (which is equal to $f'(s)-1$) and monetary marginal cost of abatement policy on consumption (which is equal to 1). The right-hand side is the ratio between the marginal negative effect of pollution due to environment (equal to $k$) and the marginal positive effect of abatement policy on environment (equal to the investment $m$). This condition can thus be seen as a condition requiring the equality between the 'relative marginal positive return' of the two policies in term of consumption and the 'relative marginal negative effect/cost' of the two policies in terms of environmental quality. The Pigouvian tax is thus used for this purpose. Once this efficiency condition is satisfied, we need to balance the effects in terms of utils of the two instruments. This is guaranteed by the optimal choice of $T_a$ in (11). Abatement policy is thus focused on this second goal.

Given these premises, we can now introduce different kinds of uncertainty. In this model we can have three types of environmental uncertainty: uncertainty on environmental quality level ($q_0$ random), uncertainty on the effect of pollution ($k$ random) and uncertainty on the effect of the abatement policy ($m$ random). In all three cases, we use notation from previous sections in order to indicate random variables (i.e., $\widetilde{q}_0$, $\widetilde{m}$ and $\widetilde{k}$) and we denote as $\widetilde{s}$, $\widetilde{T_p}$ and $\widetilde{T_a}$ investment, Pigouvian tax and abatement tax under uncertainty respectively.

*a) Uncertainty on environmental quality*

We first examine the case when $q_0$ is random. The planner's maximisation problem will be:

$$\max_{s,T_a} \mathbb{E}\left[U(y - s + f(s) - T_a, \widetilde{q}_0 + mT_a - ks)\right].$$

The first-order conditions are

$$-\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, \tilde{q}_0 + m\widetilde{T_a} - k\tilde{s})] +$$
$$+m\mathbb{E}[U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, \tilde{q}_0 + m\widetilde{T_a} - k\tilde{s})] = 0 \tag{16}$$
$$(f'(\tilde{s}) - 1)\,\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, \tilde{q}_0 + m\widetilde{T_a} - k\tilde{s})] -$$
$$+k\mathbb{E}[U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, \tilde{q}_0 + m\widetilde{T_a} - k\tilde{s})] = 0 \tag{17}$$

Simple computations show that in this case, condition (15) still holds, so that

$$f'(\tilde{s}) - 1 = \frac{k}{m} \tag{18}$$

By comparing conditions (11), (12), (15) and (16), (17), (18) we have

**Proposition 5.1.** *Given uncertainty on environmental quality: a)* $\widetilde{T_p} = \overline{T_p}$ *and b) conditions* $U_{cqq} \leq (\geq) 0$ *and* $U_{qqq} \geq (\leq) 0$ *are sufficient to have* $\widetilde{T_a} \geq (\leq) \overline{T_a}$.

*Proof.* See the Appendix for the proof. □

We analyze the interpretation of these results after studying the other types of uncertainty.

*b) Uncertainty on the impact of abatement*

When $m$ is random, the maximisation problem becomes

$$\max_{s,T_a} \mathbb{E}\left[U(y - s + f(s) - T_a, q_0 + \tilde{m}T_a - ks)\right].$$

The first-order conditions are

$$-\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + \tilde{m}\widetilde{T_a} - k\tilde{s})] +$$

$$+\mathbb{E}[\tilde{m}U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + \tilde{m}\widetilde{T_a} - k\tilde{s})] = 0 \tag{19}$$

$$(f'(\tilde{s}) - 1)\,\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + \tilde{m}\widetilde{T_a} - k\tilde{s})] -$$

$$+k\mathbb{E}[U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + \tilde{m}\widetilde{T_a} - k\tilde{s})] = 0 \tag{20}$$

Simple computations show that in this case, condition (15) is replaced by

$$f'(\tilde{s}) - 1 = \frac{k\mathbb{E}\ [U_q(.,.)]}{\mathbb{E}\ [\tilde{m}U_q(.,.)]}. \tag{21}$$

By comparing conditions (11), (12), (15) and (19), (20) and (21) we get

**Proposition 5.2.** *Given uncertainty on abatement, a)* $\tilde{T}_p > \overline{T_p}$ *and b) conditions* $U_{cqq} \geq 0$ *and* $U_{qqq} \leq 0$ *are sufficient to have* $\tilde{T}_a < \overline{T_a}$.

*Proof.* See the Appendix for the proof. ☐

In this case too, the interpretation of the proposition is proposed later.

*c) Uncertainty on the effect of pollution*

Finally we examine the case where $k$ is random. In this case, the maximisation problem becomes

$$\max_{s,T_a} \mathbb{E}[U(y - s + f(s) - T_a, q_0 + mT_a - \tilde{k}s)].$$

The first-order conditions are

$$-\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + m\widetilde{T_a} - \tilde{k}\tilde{s})] +$$

$$+m\mathbb{E}[U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + m\widetilde{T_a} - \tilde{k}\tilde{s})] = 0 \tag{22}$$

$$(f'(\tilde{s}) - 1)\,\mathbb{E}[U_c(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + m\widetilde{T_a} - \tilde{k}\tilde{s})] -$$

$$+\mathbb{E}[\tilde{k}U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + m\widetilde{T_a} - \tilde{k}\tilde{s})] = 0 \tag{23}$$

Simple computations show that in this case condition (15) is replaced by

$$f'(\tilde{s}) - 1 = \frac{\mathbb{E}\ [\tilde{k}U_q(.,.)]}{m\mathbb{E}\ [U_q(.,.)]}. \tag{24}$$

By comparing conditions (11), (12), (15) and (22), (23), (24) we have

**Proposition 5.3.** *Given uncertainty on the pollution effect, a)* $\tilde{T}_p > \overline{T_p}$ *and b) conditions* $U_{cqq} \geq 0$ *and* $U_{qqq} \leq 0$ *are sufficient to have* $\tilde{T}_a < \overline{T_a}$.

*Proof.* See the Appendix for the proof. ☐

Propositions 5.1, 5.2 and 5.3 show the effects on $q_0$, $k$ and $m$ respectively under uncertainty. Considering them together, we can now interpret the conclusions obtained. We start by considering $T_p$. As we mentioned, the Pigouvian tax is used by the planner in order to guarantee the equality between the relative marginal positive return in terms of consumption and the relative marginal negative effect/cost in terms of pollution of the two policies. This fact justifies the different results obtained in point a) of Propositions 5.1, 5.2 and 5.3.

In fact, when $q_0$ is random, uncertainty affects neither marginal returns of the policies nor their marginal costs. This implies that, the choice of $T_p$ in this case is the same as the one under certainty.

On the contrary, when $k$ is random, we have that the expected marginal cost of pollution rises (See the role of the covariance in the proof of Proposition 5.3). The increase in the expected marginal cost requires an increase in marginal return of investment. This implies in turn a smaller $s$ and a larger $T_p$.

Similarly, when $m$ is random, the expected marginal abatement performance decreases (See the role of the covariance in the proof of Proposition 5.2) becoming smaller than the expected marginal cost. This also requires an increase in marginal return of investment in order to recover the equality between relative marginal return and relative marginal cost. This implies in turn a smaller $s$ and a larger $T_p$.

Consider now the effect on $T_a$. As shown at the beginning of this section, $T_a$ is used in order to guarantee the balance of the effect of the two instruments used by the planner in terms of utils. Whatever the type of uncertainty, $q_0$, $k$ or $m$, this balance cannot be guaranteed by the pair of values, $\overline{T_p}$, $\overline{T_a}$, since the disutility due to uncertainty must somehow be counteracted.

As shown before, when $q_0$ is random, the value of $T_p$ does not change. This implies that the disutility due to uncertainty must be totally counteracted by the change in $T_a$. Given this conclusion change in the value of $T_a$ depends on the features of the third-order derivatives of the utility function, for the reasons explained in the comment to Proposition 3.1.

When either $k$ or $m$ is random, the picture is more complicated. In fact, as shown before, in these two cases, the level of $T_p$ increases. This obviously has an effect on marginal utility. When $U_{cqq} \geq 0$ and $U_{qqq} \leq 0$, the change of $T_p$ strengthens the effect of disutility due to uncertainty. So the level of $T_a$ must surely decrease in order to counteract it. When $U_{cqq} \leq 0$ and $U_{qqq} \geq 0$, on the other hand, the increase in $T_p$ counteracts the effect of the disutility due to uncertainty. This implies that, in general, we cannot say if $T_a$ has to increase, decrease or remain unchanged. This last reasoning explains the asymmetry in the inequalities in Propositions 5.2 b) and 5.3 b) with regard to the results in the previous propositions.

# 6 Generalization to the $N$-th Order Risk Changes

In the previous sections we considered three kinds of uncertainty related to variables $q_0$, $k$ and $m$. In particular, we compared the optimal choice when one of these variables is certain with the optimal choice when the same variable is random, under the assumption that the expected value of the variable in the uncertainty case is equal to the given value of the variable in the certainty case. This means that the comparison is in fact a comparison between two random variables with same expected value and different variance (zero in one case, positive in the other). It can thus be interpreted as a change in the second-order moment of the distribution of the random variable given the first-order moment.

This interpretation is relevant if it is related to a taxonomy of risk changes first introduced by Ekern (1980) and recently reconsidered by Eeckhoudt & Schlesinger (2008). In particular, Ekern (1980) introduced the general concepts of $N$-th degree stochastic dominance and $N$-th degree increase in risk, related to an increase in the $N$-th moment of a risky distribution, given the invariance of the $N-1$ previous moments. Formal definitions of the concepts of $N$-th degree stochastic dominance and $N$-th degree increase in risk are as follows.

Let $F$ and $G$ be the cumulative distribution of the two random variables $\widetilde{y}$ and $\widetilde{x}$, defined on a probability support contained within an open interval $(a, b)$. Define $F_1 = F$, $G_1 = G$, $F_{n+1}(x) = \int_a^x F_n(t)dt$ for $n \geq 1$ and similarly define $G_{n+1}$. Given these definitions, $\widetilde{y}$ dominates $\widetilde{x}$ via $N$-th degree stochastic dominance if $F_N(x) \leq G_N(x)$ $\forall x$ and $F_n(b) \leq G_n(b)$ for $n = 1, ..., N-1$. Furthermore $\widetilde{x}$ is defined as an increase in $N$-th degree risk over $\widetilde{y}$ if $\widetilde{y}$ dominates $\widetilde{x}$ via $N$-th degree stochastic dominance and the first $N-1$ moments of the distributions of $\widetilde{y}$ and $\widetilde{x}$ coincide.

It is clear that in this framework the comparisons presented in previous sections are second-degree risk changes. This suggests that the previous results can be generalized to the case of a $N$-th degree risk change. In particular, Eeckhoudt & Schlesinger (2008) provide a characterisation of $N$-th degree risk changes related to the sign of the derivates until order $N$ of an expected univariate utility function. In a recent paper, Chiu & Eeckhoudt (2010) generalise these results to the case of a bivariate utility. Their characterisation can be summarised as follows[29]

**Lemma 6.1.** *Defining $U_{(i),(j)}$ as the i-th order partial derivate with regard to the first argument and the j-th order partial derivate with regard to the second argument of $U(.,.)$, we have that*

*a) for $\widetilde{x}$ being an N-th degree risk increase over $\widetilde{y}$, $\mathbb{E}U(a,\widetilde{x}) \leq (\geq)\mathbb{E}U(a,\widetilde{y})$ if and only if $(-1)^N U_{(0),(N)} \leq (\geq)0$;*

*b) for $\widetilde{y}$ dominating $\widetilde{x}$ via N-th degree stochastic dominance, $\mathbb{E}U(a,\widetilde{x}) \leq (\geq)\mathbb{E}U(a,\widetilde{y})$ if and only if $(-1)^n U_{(0),(n)} \leq (\geq)0$ for all $n = 1, 2, \ldots, N$.*

This Lemma can be used to generalise some from the results in the previous sections, as in the following propositions:

**Proposition 6.2.** *Assume $q_0$ random and that $\widetilde{q_2}$ is an N-th degree risk increase of $\widetilde{q_1}$. Conditions $(-1)^N U_{(0),(N+1)} \geq (\leq)0$ and $(-1)^N U_{(1),(N)} \leq (\geq)0$ are sufficient for*

*a) $\widetilde{T_p}(\widetilde{q_2}) \geq (\leq)\widetilde{T_p}(\widetilde{q_1})$ in Pigouvian tax model (Section 3) and in Pigouvian tax and abatement policy model (Section 5).*

*b) $\widetilde{T_a}(\widetilde{q_2}) \geq (\leq)\widetilde{T_a}(\widetilde{q_1})$ in abatement policy model (Section 4).*

*Proof.* See the Appendix for the proof. □

**Corollary 6.3.** *Assume $q_0$ random and that $\tilde{q_1}$ dominates $\tilde{q_2}$ via N-th degree stochastic dominance. Conditions $(-1)^n U_{(0),(n+1)} \geq (\leq)0$ and $(-1)^n U_{(1),(n)} \leq (\geq)0$ for $n = 1, \ldots, N$ are sufficient for a) and b) in Proposition 6.2.*

*Proof.* See the Appendix for the proof. □

**Proposition 6.4.** *Assume m random and that $\tilde{m_2}$ is an N-th degree risk increase of $\tilde{m}_1$. Conditions $-mT_a\frac{U_{(0),(N+1)}}{U_{(0),(N)}} \geq (\leq)N$ and $(-1)^N U_{(1),(N)} \leq (\geq)0$ are sufficient for $\widetilde{T_a}(\tilde{m_2}) \geq (\leq)\tilde{T}_a(\tilde{m_1})$ in the abatement policy model (Section 4).*

*Proof.* See the Appendix for the proof. □

**Corollary 6.5.** *Assume m random and that $\tilde{m_1}$ dominates $\tilde{m_2}$ via N-th degree stochastic dominance. Conditions $-mT_a\frac{U_{(0),(n+1)}}{U_{(0),(n)}} \geq (\leq)n$ and $(-1)^n U_{(1),(n)} \leq (\geq)0$ for all $n = 1, \ldots, N$ are sufficient for $\widetilde{T_a}(\tilde{m_2}) \geq (\leq)\overline{T_a}(\tilde{m_1})$ in the abatement policy model (Section 4).*

*Proof.* See the Appendix for the proof. □

**Proposition 6.6.** *Assume k random and that $\tilde{k_2}$ is an N-th degree risk increase of $\tilde{k_1}$. Conditions $(-1)^N (ks)\cdot \frac{U_{(0),(N+1)}}{U_{(0),(N)}} \leq (\geq)(-1)^N N$ and $(-1)^N U_{(1),(N)} \leq (\geq)0$ are sufficient for $\widetilde{T_p}(\tilde{k_2}) \geq (\leq)\widetilde{T_p}(\tilde{k_1})$ in the Pigouvian tax model (Section 3).*

*Proof.* See the Appendix for the proof. □

**Corollary 6.7.** *Assume k random and that $\tilde{k_1}$ dominates $\tilde{k_2}$ via N-th degree stochastic dominance. Conditions $(-1)^n (ks)\frac{U_{(0),(n+1)}}{U_{(0),(n)}} \leq (\geq)(-1)^n n$ and $(-1)^N U_{(1),(n)} \leq (\geq)0$ are sufficient for $\widetilde{T_p}(\tilde{k_2}) \geq (\leq)\overline{T_p}(\tilde{k_1})$ for all $n = 1, \ldots, N$ in Pigouvian tax model (Section 3).*

*Proof.* See the Appendix for the proof. □

[29] Our Lemma reproduces Lemma 1 by Chiu & Eeckhoudt (2010). See, for instance, Ingersoll (1987) for the proof.

Propositions 6.2, 6.4 and 6.6 generalise some results obtained in the previous sections. Note that this generalisation does not involve the results obtained in the model with both Pigouvian tax and abatement policy in the cases of random $k$ and $m$. The reason is that we cannot generalise the conclusions on the changes of $T_p$ obtained in these two cases, specifically related to the sign of covariances to an $N$-th order risk change.

Once the generalisation is performed, it has various possible applications. The literature on risks provides the interpretation of risk changes of many different orders.[30] Some of these interpretations can be simply applied to the case of uncertainty on the environment.

First, consider the case of a first-degree increase in risk ($N = 1$ in the previous propositions) and assume $q_0$ is random. This case is a situation where we consider a reduction in first-order moment of the distribution of $\widetilde{q}_0$. This occurs for example if climate change implies a lower expected environmental quality.[31]

Second, consider the case of a third-degree increase in risk ($N = 3$ in the previous propositions) and assume again $q_0$ is random. MENEZES, GEISS & TRESSLER (1980) study a third-degree risk increase as a downside risk increase, i.e. as a increase in the left skewness of a distribution.[32] In our framework, a case of a third-degree increase in risk in $\widetilde{q}_0$, could thus be seen as an increase in the probability of very bad outcomes in environmental quality, for given expected value and variance of $\widetilde{q}_0$.[33]

Finally, consider a fourth degree risk increase case ($N = 4$ in the previous proposition). In this case, for given expected value, variance and skewness we have an increase in the kurtosis of the distribution (see MENEZES & WANG, 2005). For example, this could be climate change which affects temperature level determining an increase in the probability of either very high or very low temperatures, but keeping constant the expected value, the variance and the skewness of the temperature distribution.

In all this cases, conditions obtained in Propositions 6.2, 6.4 and 6.6 could thus give an indication on optimal policy reactions to shocks.

# 7 Conclusions

This paper studies the implication of three kinds of environmental uncertainty (uncertainty on environmental quality, uncertainty on the effect of pollution and uncertainty on the impact of abatement) for two different environmental policies (Pigouvian tax and abatement policy) used either alternatively or contemporaneously.

In the two cases where we use either Pigouvian tax or abatement policy, we determine the conditions ensuring that the introduction of uncertainty brings about an increase in the size of public intervention (a higher Pigouvian tax or a larger abatement) and vice versa. In the case of uncertainty on environmental quality, these conditions involve the signs of two third-order derivatives of the utility function ($U_{qqq}$ and $U_{cqq}$). The results for $U_{qqq}$ are linked to a wide strand of literature, starting from LELAND (1968), SANDMO (1970) and KIMBALL (1990), showing the role of prudence for precautionary saving in a one-argument utility framework.[34] Similarly, the role of $U_{cqq}$ in decisions under uncertainty has recently been emphasized by many papers (e.g. COURBAGE & REY, 2007, MENEGATTI, 2009a and 2009b, DENUIT ET AL., 2009, CHIU & EECKHOUDT, 2010) which consider a two-argument utility. In this field, our conclusions confirm the role of third order cross derivatives of the utility function in the determination of optimal choices under uncertainty. Furthermore, the interpretation of the conditions obtained shows that, as in the case of precautionary saving analysis (see MENEGATTI, 2007 and EECKHOUDT & SCHLESINGER, 2006), the reason that the signs of these

[30] On this see MENEZES, GEISS & TRESSLER (1980), MENEZES & WANG (2005), EECKHOUDT & SCHLESINGER (2006, 2008).

[31] EECKHOUDT & SCHLESINGER (2008), who study a two-period consumption problem, suggest that a first-degree risk change can occur in that framework when the probability of becoming unemployed increases, reducing future expected income. Similarly, in our case, a first-degree risk change can occur when the probability of an environmental accident increases, and future expected environmental quality is lowered.

[32] MENEZES, GEISS & TRESSLER (1980) show that this can be obtained by applying a mean preserving spread and a mean preserving contraction with specific feature.

[33] Note that, in a very recent paper, PINDYCK (2009) studies the economic impact of global warming, considering not the expected climate change but the implications of the different features of the distribution of the random variable describing future changes in temperature (represented by a three-parameter gamma distribution). His calibrations show that a change in the skewness of the distribution for given expected value and variance (i.e. a third-degree change in risk) can significantly affect agents optimal behaviour.

[34] Recently prudence was proved to be crucial also in the determination of optimal prevention (see EECKHOUDT & GOLLIER, 2005 and MENEGATTI, 2009c).

derivates are relevant is their ability to indicate the direction of the effect on the disutility due to uncertainty of change in consumption level or environmental quality.

The effects of uncertainty on pollution effect and abatement effect (either Pigouvian tax or abatement policy) are, on the other hand, shown to depend on the size of relative prudence. This result is consistent with those obtained by Rothschild & Stiglitz (1971) in the field of optimal saving under uncertainty on the interest rate (the return of saving) and by Chiu & Eeckhoudt (2010) in the field of optimal labour supply under uncertainty on wage (the return of labour). We showed that this parallelism occurs, because in our analysis uncertainty involves two kinds of returns: the positive return of abatement policy or the negative return of investment in terms of pollution.

When Pigouvian tax and abatement policy are used together we have a kind of separation of the two policy instruments. In particular, the Pigouvian tax is used to guarantee equality between the relative marginal positive return in terms of consumption and the relative marginal negative effect/cost in terms of pollution of the two policies. This conclusion implies that the Pigouvian tax level is increased by uncertainty on both pollution effect and abatement effect, since the former raises the expected marginal cost of pollution and the latter reduces the expected marginal return of investment. Finally, uncertainty on environmental quality affects neither expected marginal returns nor expected marginal costs of the two policies, so that it does not influence Pigouvian tax.

Again, when Pigouvian tax and abatement policy are used together, the abatement policy is applied in order to guarantee the balance between the effects of the two policy instruments in terms of utils and thus also counteracts the disutility caused by uncertainty. When environmental quality is uncertain, given that the Pigouvian tax does not change, abatement tax is the only instrument used to counteract the disutility due to uncertainty. It thus exhibits a change in a direction clearly determined by the features of the third-order derivatives of the utility function. When uncertainty is on either pollution effect or abatement effect, the level of Pigouvian tax increases, so that a clear indication on the change of the abatement tax can be obtained only where the change in Pigouvian tax strengthens the disutility due to uncertainty, making necessary an increment in the level of the other instrument.

The last part of the paper shows how some of our results can be generalized to the case of $N$-th order risk change. This kind of shock, first studied by Ekern (1980), has recently been rediscovered by literature (eg. Eeckhoudt & Schlesinger, 2006 and 2008) because of its applicability to cases of change in random variables distribution other than increases in variance (i.e. increases in uncertainty). Our analysis shows which conditions, involving $N$-th order derivatives of the utility function, allow us to determine the directions of the effects of these shocks on some optimal policy choices.

# Appendix

*Proof of Proposition 3.1.* By Jensen's inequality, $U_{cqq} \leq 0$ implies $\mathbb{E}\left[U_c(y-\overline{s}+f(\overline{s}),\widetilde{q_0}-\ g(\overline{s})\right) \leq U_c(y-\overline{s}+f(\overline{s}),\overline{q_0}-g(\overline{s}))$ and $U_{qqq} \geq 0$ implies $\mathbb{E}\left[U_q(y-\overline{s}+f(\overline{s}),\widetilde{q_0}-g(\overline{s}))\right] \geq U_q(y-\overline{s}+f(\overline{s}),\overline{q_0}-g(\overline{s}))$. Given (4) and (5), this implies $\widetilde{T_p} \geq \overline{T_p}$. The proof with reversed inequalities is analogous. □

*Proof of Proposition 3.2.* By Jensen's inequality $U_{cqq} \leq 0$ implies $\mathbb{E}[U_c(y-\overline{s}+f(\overline{s}),q_0-\widetilde{k}\overline{s})] \leq U_c(y-\overline{s}+f(\overline{s}),q_0-\overline{k}\overline{s})$. Given this result, $\widetilde{T_p} \geq \overline{T_p}$ if $\mathbb{E}[\widetilde{k}U_q(y-\overline{s}+f(\overline{s}),q_0-\widetilde{k}\overline{s})] \geq \overline{k}U_q(y-\overline{s}+f(\overline{s}),q_0-\overline{k}\overline{s})$. By Jensen's inequality, this last inequality is satisfied if the function $\Psi(k) = kU_q\left(y-s+\ f(s),q_0-ks\right)$ is convex with respect to $k$. After simple calculus we find that $\Psi''(k) = -2sU_{qq}+s^2kU_{qqq} \geq 0$, which (since $U_{qq} < 0$) is equivalent to $-ks\frac{U_{qqq}}{U_{qq}} \geq -2$. Proof with reversed inequalities is analogous. □

*Proof of Proposition 4.1.* By Jensen's inequality, $U_{cqq} \leq 0$ implies $\mathbb{E}[U_c(y-s^\star+f(s^\star)-\ \overline{T_a},\widetilde{q_0}-g(s^\star)+m\overline{T_a})] \leq U_c(y-s^\star+f(s^\star)-\overline{T_a},\overline{q_0}-g(s^\star)+m\overline{T_a})$ and $U_{qqq} \geq 0$ implies $\mathbb{E}[U_q(y-s^\star+f(s^\star)-\overline{T_a},\ \widetilde{q_0}-g(s^\star)+m\overline{T_a})] \geq U_q(y-s^\star+f(s^\star)-\overline{T_a},\overline{q_0}-g(s^\star)+m\overline{T_a})$. Since, by second-order condition left-hand sides of (8) and (9) are decreasing in $T_a$, the two previous inequalities imply $\widetilde{T_a} \geq \overline{T_a}$. The proof with reversed inequalities is analogous. □

*Proof of Proposition 4.2.* By Jensen's inequality $U_{cqq} \leq 0$ implies $\mathbb{E}\left[U_c(y-s^\star+f(s^\star)-\ \overline{T_a},q_0-g(s^\star)+\tilde{m}\overline{T_a}\right] \leq U_c(y-s^\star+f(s^\star)-\overline{T_a},q_0-g(s^\star)+\overline{m}\overline{T_a})$. Given this result and since, by second-order condition, left-hand sides of (8) and (9) are decreasing in $T_a$ so $\widetilde{T_a} \geq \overline{T_a}$ is satisfied if $\mathbb{E}[\widetilde{m}U_q(y - s^\star + f(s^\star) - \overline{T_a}, q_0 - g(s^\star) + \tilde{m}\overline{T_a}] \geq \overline{m}U_q(y - s^\star + f(s^\star) - \overline{T_a}, q_0 - g(s^\star) + \overline{m}\overline{T_a})$. By Jensen's inequality, this last condition is verified if the function $\Phi(m) = mU_q\left(y - s^\star + f(s^\star) \ - T_a, q_0 - g(s^\star) + mT_a\right)$ is convex with respect to $m$. After simple calculus we have $\Phi''(m) = 2T_aU_{qq} + mT_a^2U_{qqq} \geq 0$, which is equivalent to $-mT_a\frac{U_{qqq}}{U_{qq}} \geq 2$. Proof with reversed inequalities is analogous. □

*Proof of Proposition 5.1.* Since (15) and (18) are equal a) is automatically proved. With reference to b), given that $\tilde{s} = \overline{s}$ and by Jensen's inequality, $U_{cqq} \leq 0$ implies $\mathbb{E}\left[U_c\left(y - \overline{s} + f(\overline{s}) \ -\overline{T_a}, \widetilde{q_0} + m\overline{T_a} - k\overline{s}\right) \leq U_c\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, \overline{q_0} + m\overline{T_a} - k\overline{s}\right)\right.$ while $U_{qqq} \geq 0$ implies $\mathbb{E}\left[U_q\left(y - \ \overline{s} + f(\overline{s}) - \overline{T_a}, \widetilde{q_0} + m\overline{T_a} - k\overline{s}\right)\right] \geq U_q\left(y - \overline{s} + f(\overline{s}) - \overline{T_a}, \overline{q_0} + m\overline{T_a} - k\overline{s}\right)$. These two conditions together imply that the left-hand side of (11) is smaller than the left-hand side of (16). Since, by second-order conditions, (11) and (16) are decreasing in $T_a$, $U_{cqq} \leq 0$ and $U_{qqq} \geq 0$ prove $\widetilde{T_a} \geq \overline{T_a}$. Proof with reversed inequalities is analogous. □

*Proof of Proposition 5.2.* In order to prove a) note that (15) can be written as

$$f'(\overline{s}) - 1 = \frac{k\mathbb{E}\ U_q(.,.)}{m\mathbb{E}\ U_q(.,.)}. \tag{25}$$

Also note that $\mathbb{E}\ [\tilde{m}U_q(.,.)] = \overline{m}\mathbb{E}\ \ [U_q(.,.)] + cov\,[\tilde{m}, U_q(.,.)]$. Given the concavity of the utility function ($U_{qq} < 0$), $cov[\tilde{m}, U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + \widetilde{m}\widetilde{T_a} - k\tilde{s})] < 0$. This implies $\mathbb{E}\ [\tilde{m}U_q(.,.)] < \overline{m}\mathbb{E}\ [U_q(.,.)]$, implying, in turn, that the right-hand side of (25) is smaller than the right-hand side of (21). Since $f'(s)$ is decreasing this means that $\widetilde{s} < \overline{s}$, implying in turn $\tilde{T_p} > \overline{T_p}$.

In order to prove b) we define (12) as $\Psi\left(s, T_a, m, k\right) = 0$. By Jensen's inequality, we have that $U_{cqq} \geq 0$ and $U_{qqq} \leq 0$ ensure

$$\mathbb{E}\ \left[\Psi\left(\overline{s}, \overline{T_a}, \widetilde{m}, \overline{k}\right)\right] \geq \Psi\left(\overline{s}, \overline{T_a}, \overline{m}, \overline{k}\right). \tag{26}$$

By a) we know that $\widetilde{s} < \overline{s}$. Since $\frac{\partial\Psi}{\partial s} < 0$, this together with (26) implies $\mathbb{E}\ \left[\Psi\left(\widetilde{s}, \overline{T_a}, \widetilde{m}, \overline{k}\right)\right] > \Psi\left(\overline{s}, \overline{T_a}, \overline{m}, \overline{k}\right)$. Since $\frac{\partial\Psi}{\partial T_a} > 0$ and given (12) and (20), this last result implies $\tilde{T_a} < \overline{T_a}$. □

*Proof of Proposition 5.3.* In order to prove a), given condition (25), note that $\mathbb{E}\ [\tilde{k}U_q(.,.)] = \overline{k}\mathbb{E}\ [U_q(.,.)] + cov[\tilde{k}, U_q(.,.)]$, where $cov[\tilde{k}, U_q(y - \tilde{s} + f(\tilde{s}) - \widetilde{T_a}, q_0 + m\widetilde{T_a} - \tilde{k}\tilde{s})] \geq 0$ for the concavity of the utility function ($U_{qq} \leq 0$). This implies that $\mathbb{E}\ [\tilde{k}U_q(.,.)] > \overline{k}\mathbb{E}\ U_q(.,.)$ implying, in turn, that the right-hand side of (25) is smaller than the right-hand side of (24). Since $f'(s)$ is decreasing this means that $\widetilde{s} < \overline{s}$, implying in turn $\tilde{T_p} > \overline{T_p}$.

In order to prove b) we define (11) as $\Theta\left(s, T_a, m, k\right) = 0$. By Jensen's inequality, we have that $U_{cqq} \geq 0$ and $U_{qqq} \leq 0$ ensure

$$\mathbb{E}\ [\Theta(\overline{s}, \overline{T_a}, \overline{m}, \widetilde{k})] \leq \Theta(\overline{s}, \overline{T_a}, \overline{m}, \overline{k}). \tag{27}$$

By a) we know that $\widetilde{s} < \overline{s}$. Since $\frac{\partial\Theta}{\partial s} > 0$, this together with (27) implies $\mathbb{E}\ [\Theta(\widetilde{s}, \overline{T_a}, \overline{m}, \widetilde{k})] < \Theta(\overline{s}, \overline{T_a}, \overline{m}, \overline{k})$. Since $\frac{\partial\Theta}{\partial T_a} < 0$ and given (11) and (22), this last result implies $\tilde{T_a} < \overline{T_a}$. □

*Proof of Proposition 6.2.* We prove a). By Lemma 6.1 a), $(-1)^N U_{(1),(N)} \leq (\geq) 0$ implies $\mathbb{E}\left[U_{(1),(0)}(y - \tilde{s_1} + f(\tilde{s_1}), \tilde{q_2} - g(\tilde{s_1})) \leq (\geq)\mathbb{E}\left[U_{(1),(0)}(y - \tilde{s_1} + f(\tilde{s_1}), \tilde{q_1} - g(\tilde{s_1}))\right]\right.$ and $(-1)^N U_{(0),(N+1)} \geq (\leq) 0$ implies $\mathbb{E}\left[U_{(0),(1)}(y - \tilde{s_1} + f(\tilde{s_1}), \tilde{q_2} - g(\tilde{s_1})) \geq (\leq)\mathbb{E}\left[U_{(0),(1)}(y - \tilde{s_1} + f(\tilde{s_1}), \ \tilde{q_1} - g(\tilde{s_1}))\right.\right.$. These results, together with (5), imply that $\widetilde{T_p}(\tilde{q_2}) \geq (\leq)\widetilde{T_p}(\tilde{q_1})$ for the model with Pigouvian tax (Section 3). The same result, given (11) and (16) and since (by second-order condition) the left-hand side of (16) is decreasing in $T_p$, implies that $\widetilde{T_p}(\tilde{q_2}) \geq (\leq)\widetilde{T_p}(\tilde{q_1})$ in a model with Pigouvian tax and abatement policy (Section 5).

We now prove b). By Lemma 6.1 a), $(-1)^N U_{(1),(N)} \leq (\geq) 0$ implies $\mathbb{E}\left[U_{(1),(0)}(y - s^\star + \ f(s^\star) - \widetilde{T_{1a}}, \tilde{q_2} - g(s^\star) + m\widetilde{T_{1a}})\right] \leq (\geq)\mathbb{E}[U_{(1),(0)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, \tilde{q_1} - g(s^\star) + m\widetilde{T_{1a}})]$ and $(-1)^N U_{(0),(N+1)} \geq (\leq) 0$ implies $\mathbb{E}[U_{(0),(1)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, \tilde{q_2} - g(s^\star) + m\widetilde{T_{1a}})] \geq (\leq)\mathbb{E}[U_{(0),(1)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, \tilde{q_1} - g(s^\star) + m\widetilde{T_{1a}})]$. Given (9) and since (by second-order condition) the left-hand side of (9) is decreasing in $T_a$, these results imply that $\widetilde{T_a}(\tilde{q_2}) \geq (\leq)\tilde{T_a}(\tilde{q_1})$ in the abatement policy model (Section 4). □

*Proof of Corollary 6.3.* The corollary is proved by applying Lemma 6.1 b) and following the same steps in the proof of Proposition 6.2. □

*Proof of Proposition 6.4.* By Lemma 6.1 a), $(-1)^N U_{(1),(N)} \leq (\geq) 0$ implies $\mathbb{E}\left[U_{(1),(0)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, q_0 - g(s^\star) + \tilde{m_2}\widetilde{T_{1a}}) \leq (\geq)\mathbb{E}[U_{(1),(0)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, q_0 - g(s^\star) + \tilde{m_1}\widetilde{T_{1a}})\right]$ Given this result and (8) and (10), we have that $\widetilde{T_a}(\tilde{m_2}) \geq \widetilde{T_a}(\tilde{m_1})$ in the model with abatement policy (Section 4) if

$$\mathbb{E}[\tilde{m_2}U_{(0),(1)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, q_0 - g(s^\star) + \tilde{m_2}\widetilde{T_{1a}})] \geq (\leq)\mathbb{E}[\tilde{m_1}U_{(0),(1)}(y - s^\star + f(s^\star) - \widetilde{T_{1a}}, q_0 - \\ + g(s^\star) + \tilde{m_1}\widetilde{T_{1a}})]. \quad (28)$$

Defining the function $L(m) = mU_{(0),(1)}(y - s^\star + f(s^\star) - T_a, q_0 - g(s^\star) + mT_a)$ we get that, by Lemma 6.1 a), (28) is satisfied if $(-1)^N L(m)_{(0),(N)} \geq (\leq)0$. By differentiating $L(m)$, it is easy to see that this inequality is equivalent to $-mT_a \frac{U_{(0),(N+1)}}{U_{(0),(N)}} \geq (\leq)N$. □

*Proof of Corollary 6.5.* The corollary is proved by applying Lemma 6.1 b) and following the same steps as in the proof of Proposition 6.4. □

*Proof of Proposition 6.6.* As before by Lemma 6.1 a), $(-1)^N U_{(1),(N)} \leq (\geq)0$ implies $\mathbb{E}\left[U_{(1),(0)}(y - \tilde{s_1} + f(\tilde{s_1}), q_0 - \tilde{k_2}\tilde{s_1})\right] \leq (\geq)\mathbb{E}[U_{(1),(0)}(y - \tilde{s_1} + f(\tilde{s_1}), q_0 - \tilde{k_1}\tilde{s_1})]$. Given this result, together with (4), (5), we have that $\widetilde{T_p}(\tilde{k_2}) \geq \widetilde{T_p}(\tilde{k_1})$ in the Pigouvian tax model (Section 3) if

$$\mathbb{E}[\tilde{k_2}U_{(0),(1)}(y - \tilde{s_1} + f(\tilde{s_1}), q_0 - \tilde{k_2}\tilde{s_1})] \geq (\leq)\mathbb{E}[\tilde{k_1}U_{(0),(1)}(y - \tilde{s_1} + f(\tilde{s_1}), q_0 - \tilde{k_1}\tilde{s_1})]. \quad (29)$$

Defining the function $Z(k) = kU_{(0),(1)}(y - s + f(s), q_0 - ks)$ we get that, by Lemma 6.1 a), (29) is satisfied if $-(1)^N Z(k)_{(0),(N)} \geq (\leq)0$. By differentiating $Z(k)$, it is easy to see that this inequality is equivalent to $(-1)^N (ks)\frac{U_{(0),(N+1)}}{U_{(0),(N)}} \leq (\geq)(-1)^N N$. The same result, given (11) and (16) and since (by second-order condition) the left-hand sides of (11) and (16) are decreasing in $T_p$, implies that $\widetilde{T_p}(\tilde{k_2}) \geq (\leq)\widetilde{T_p}(\tilde{k_1})$ for Pigouvian tax model and abatement policy model (Section 5) respectively. □

*Proof of Corollary 6.7.* The corollary is proved by applying Lemma 6.1 b) and following the same steps as in the proof of Proposition 6.6. □

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