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A STATISTICAL APPROACH OF THE SPATIAL-TEMPORAL VARIABILITY OF A PHENOMENON USING A RO-EU COMPOSITE INDEX

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Abstract

This study aims at presenting a new computation index, which simultaneously measures the time and space variation of the various economic indices. The elaboration method of the space-time index is based on a matrix construct, the time and space variation of the analyzed economic index being represented via a system of vectors in plane.

The practical application of the compound index is exemplified by employing the data on the growth rate of the GDP/inhabitant, calculated for two spatial entities: Romania and the European Union, for the period 1999-2008. The obtained findings confirm the fact that the proposed space-time index is a good analysis tool for the measurement of the time and space variation related to the various economic indices, as well as for testing the economic convergence for discrete time moments.

JEL Classification: C10, C14, E27

Keywords: space-time index; convergence; economic shock.

1 Introduction

Economic disparities and country discrepancies, as well as the impact of negative shocks on economies, have been an intense debate topic for researchers and specialists during the past couple of years. The analysis frame of these phenomena is given by the economic theories that model the time and space dynamics of various economic indices. Each economic measure is characterized by its spatial and temporal dimensions, as well as by its dependent relations with other economic indices.

Given the complexity of the functional links that describe economic indices, a series of analysis tools and statistical and econometric models were elaborated in time for the purposes of studying the space-time evolution of various economic measures.

The first to have elaborated a model for analyzing the link between the two variables: time and space, were Cliff and Ord (1975). They were followed later by the contributions of Pfeiffer and Deutsch (1980), as well as Bennett (1979), who introduced a new model, namely the STARIMA model, which represents an extension of the Box – Jenkins method (identification-estimation-diagnosis), applied for the ARIMA processes. It is worth mentioning the fact that the STARIMA models constitute the basis of all the models elaborated up to now, being used especially in those applications where the space and time dimensions are built at very large scales (Kamarianakis, 2003, p. 2). Recent works have contributed to the enrichment of specialized literature by including in their analysis the non-stationary phenomenon, specific for space-time data.

Another type of modeling of the space-time variation is the panel approach, often used in the analysis of economic convergence. One of the advantages of panel modeling is that which enables the separation of the inter-spatial from the intra-spatial differences, phenomenon that cannot be studied with the help of a time series analysis. Moreover, the panel type data, due to their large number of degrees of freedom, via the possibility of eliminating the multicollinearity of variables, as well as via the bias resulted following the omission of variables, determine a greater accuracy of the estimated parameters (Cheng Hsiao, 2003, p.349).

This paper aims at presenting a new computation index, which measures the space-time dependence of the various economic indices. The motivation of creating such an index started from the idea that every economic shock (such as the current economic crisis) is reflected in the time evolution of certain economic indices, and the magnitude of the respective shock can be appreciated via the comparison of the evolution of the economic indices within certain space entities, affected by the economic shock. Thus, the simultaneous grasping of time and space changes imposes the construction of an index measure, whose synthetic value must reflect the time and space evolution of the economic index.

Such an approach suggests the use of a matrix construct. The determinant of such a (square) matrix can be interpreted as a compound index, whose size gives the space-time variation of the analyzed economic index. Given the fact that Romania and the European Union represent the two space entities which we will refer to in this analysis, we will call the proposed index the "RO-EU" index.

The paper is structured as follows: the second part presents the methodological elements used for the construction of the RO-EU index concerning the concept of determinant and its most relevant properties for our analysis. The third part offers an economic interpretation of the RO-EU index, from a theoretical and empiric perspective, occasion on which we will use the GDP/inhabitant growth rate as an economic index. The last part presents the conclusions of this study.

2. Methodological elements used for the construction of the RO-EU index

We synthetically present the concept of determinant and a series of properties of the determinant, which are of interest for our analysis.

2.1 Matrix A, its transpose and the matrix determinant

If A is a 2×2 matrix, and a_{ij} are the coefficients of the matrix, with $i = 1,2$ and $j = 1,2$:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \vec{v}_1 \\ \vec{v}_2 \end{pmatrix} \qquad (1)$$

The matrix can be also rendered by the help of vectors $\vec{v}_1 = (a_{11}, a_{12})$ and $\vec{v}_2 = (a_{21}, a_{22})$.

The transposed matrix, A^T can be written as follows:

$$A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} \qquad (2)$$

The value of A, the determinant of the matrix, is the following:

$$\det A = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} \qquad (3)$$

2.2 Properties of the A matrix determinant

We present below those properties of the A matrix determinant which are relevant for our analysis:

a) The A matrix determinant is equal to the determinant of the transposed matrix:

$$\det A = \det\left(A^T\right) \qquad (4)$$

b) An equal to zero value of the determinant indicates the fact that the two vectors are collinear, which means that one of the vectors can be written as a product between a constant (c) and the other vector:

$$\det A \approx 0 \rightarrow \vec{v}_1, \vec{v}_2 collinear,\ \vec{v}_1 = c \cdot \vec{v}_2 \text{ where } c > 0 \text{ or } c < 0 .$$

In the graphical representation (Figure 1), the angle between the two vectors if of approximately 180^0, which means that the two vectors are parallel.

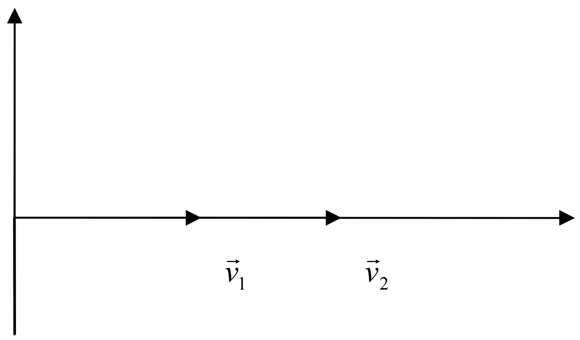

Figure 1: *The representation of the vectors when the determinant is equal to zero*

A value of the determinant which is not equal to zero indicates the fact that the two vectors are not collinear, which means, in the graphical representation, that they form an angle which is not equal to 180^0 (Figure 2).

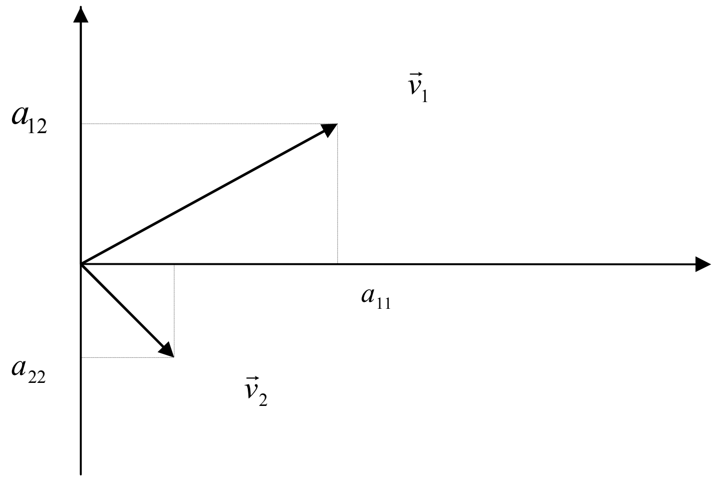

Figure 2: *The representation of the vectors when the determinant is not equal to zero*

c) Usually, what interests us in this analysis is the dependence relation of the two vectors, and not their value (length). The dependence is reflected either by the direction of the vectors or by the size of the angle between the vectors. In this respect, we must calculate the standardized value of the determinant, which is obtained by dividing the value of the matrix determinant to the product of the two vectors:

$$\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\sqrt{a_{11}^2 + a_{12}^2} \cdot \sqrt{a_{21}^2 + a_{22}^2}} \in [-1,1] \qquad (5)$$

The variation scale of the standardized value of the determinant is [-1.1]. The extreme values are obtained when the vectors are orthogonal.

d) For our analysis, it is of interest to study another property of the determinant, to which we will subsequently give an economic interpretation as well. According to this property, the determinant of a matrix linearly depends on its columns, which means that:

$$\begin{vmatrix} a_{11}+s_{11} & a_{12} \\ a_{21}+s_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} s_{11} & a_{12} \\ s_{21} & a_{22} \end{vmatrix} \qquad (6)$$

3 Economic interpretation of the A matrix determinant

3.1 Definition of the $I_{t_i,t_{i-1}}^{RO/UE}$ compound index

In this section, we plan on giving an economic interpretation to the concept of determinant. As mentioned above, the purpose of the proposed analysis is of representing the space and time variation of an economic index with the help of a single measure. This measure is reflected by the $I_{t_i,t_{i-1}}^{RO/UE}$ index, which actually represents the value of the determinant defined at point 2.1.

The index is calculated for two time moments (t_i, t_{i-1}) and for two space entities, Romania (RO) and the European Union (EU) and reflects the space-time variation of an economic index.

The economic indicator, on which our exemplification is based, is the growth rate of GDP/inhabitant, calculated according to the formula below (Duffy, 1992, p. 35):

$$\frac{PIB_t / locuitor - PIB_{t-1} / locuitor}{PIB_{t-1} / locuitor} \times 100$$

The motivation of the choice of the indicator growth rate of the GDP/inhabitant resides in the fact that any economic shock, any crisis is reflected in the growth rate of the GDP (Mankiw, 2005, p.242) and, in this respect, the analysis of the evolution of the growth rate of the GDP in time and space is of real interest.

3.2 Operationalization of the $I_{t_i,t_{i-1}}^{RO/UE}$ index

The analysis period is 1999-2008. The selected time interval is not at random. We consider that for Romania, beginning with 1999, the growth rate of the GDP is a consistent and stable indicator, without unbalanced variations.

Table 1 presents the growth rate of the GDP/inhabitant for Romania and for the European Union, as well as the values of the $I_{t_i,t_{i-1}}^{RO/UE}$ index, calculated for every two successive years (1999-2000, 2000-2001, 2001-2002 etc.)

Table 1: *Growth rate of the GDP/inhabitant and the values of the determinant*

	Growth rate of the GDP/inhabitant*			**Value of the $I_{t_i,t_{i-1}}^{RO/UE}$ index ****	**Value of the standardized determinant****
		RO	EU		
	1	**2**	**3**	**4**	**5**
1	1999	-1	2.8		
2	2000	2.1	3.6	-10.98	-0.11
3	2001	5.7	1.6	-18.03	-0.67
4	2002	5.1	0.8	-3.36	-0.11
5	2003	5.2	0.8	0.4	-0.01
6	2004	8.5	1.9	1.95	0.04
7	2005	4.2	1.5	6.5	0.16
8	2006	7.9	2.6	-2.78	-0.07
9	2007	6	2.5	3.69	0.06
10	2008	7.1	0.9	-15.01	-0.31

Source: *Eurostat
**own calculations

The vectors $\vec{v}_1$ and $\vec{v}_2$, in relation (1), represent, in this example, the growth rates registered for GDP/inhabitant every year, for the two space entities, that is space variation. For the transposed matrix (relation 2), the two vectors show the time variation of the growth rate of GDP/inhabitant for a particular space entity.

A value equal to zero of the index expresses the fact that the vectors of the corresponding matrix (we refer to the transposed matrix) are collinear and follow the same direction (possible different trajectories). In this example, this would be translated as follows: the growth rate of the GDP/inhabitant for Romania is modified (increases or decreases) in the following year with the value of the modification, in the same direction or in an opposite one, of the growth rate of the GDP/inhabitant for EU. For example, if the growth rate of the GDP/inhabitant for Romania increases in the following year by 10%, then the growth rate of the GDP/inhabitant in EU is modified, in the same direction or in the opposite one, by 10%.

The values of the $I_{t_i,t_{i-1}}^{RO/UE}$ index are presented in the fourth column of table 1. The graphical representation of the value of the index for the period 1999-2008 is given in the chart below:

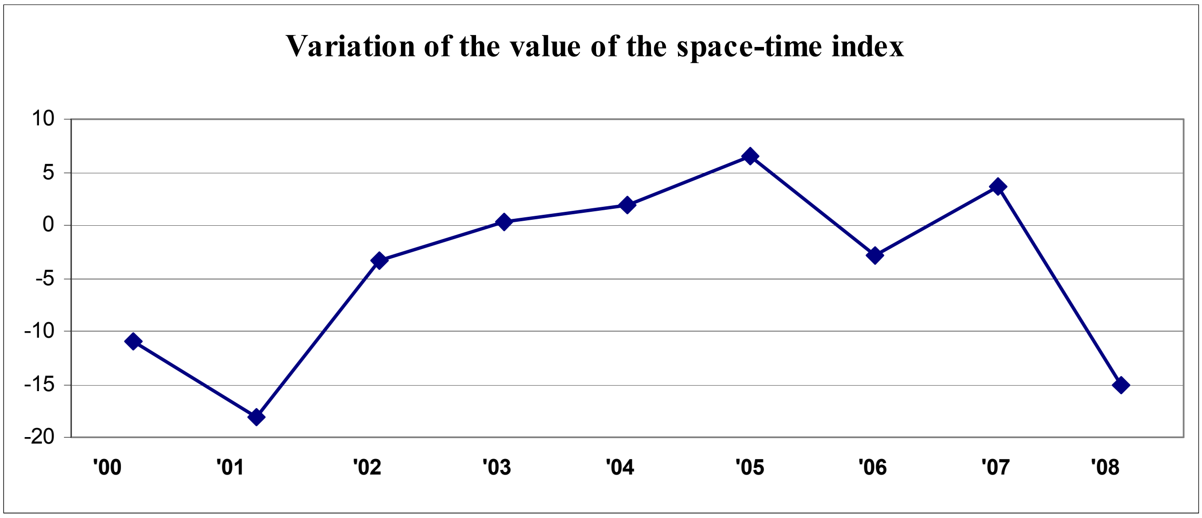

Figure 3: *Variation of the value of the* $I_{t_i,t_{i-1}}^{RO/UE}$ *index in the period 1999-2008*

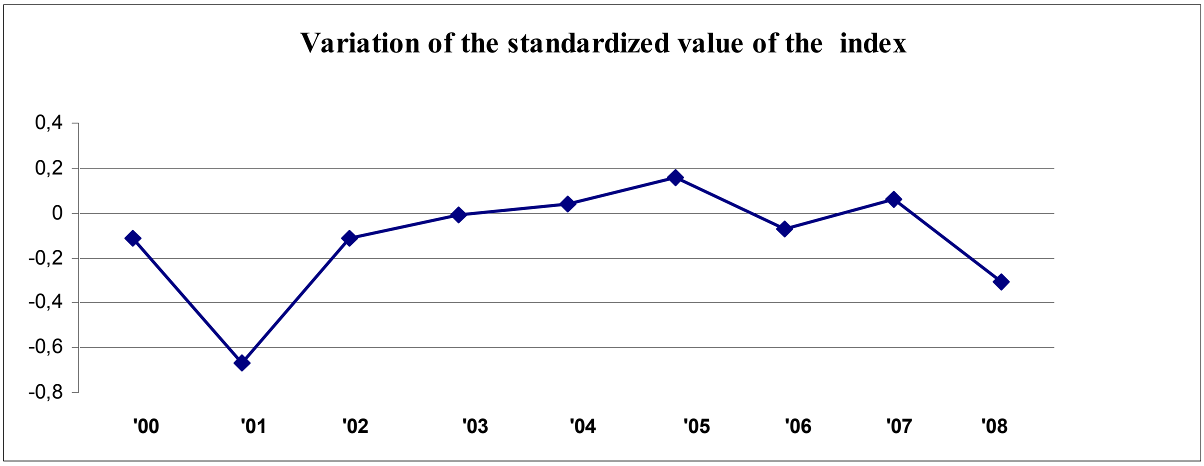

Figure 4: *Variation of the standardized value of the* $I_{t_i,t_{i-1}}^{RO/UE}$ *index in the period 1999-2008*

Just as we would have expected, the two charts above are similar, only the variation scale is different. The $I_{t_i,t_{i-1}}^{RO/UE}$ index has a value close to zero in a single case, when calculated for 2002-2003. Consequently, the growth rates of the GDP/inhabitant for the two space entities either remained constant or modified proportionally (increasing or decreasing). According to the data in table 1, the growth rated of the GDP registered almost the same value for 2002-2003.

For the other years we may observe that the value of the determinant is different from zero, which means that the vectors have different directions (see Figure 4). Consequently, the growth rates of the GDP/inhabitant for the two space entities, taken separately, evolved in a different manner from one year to another. The biggest variation between space entities, calculated for a specific time period, is given by a large value of the determinant (in absolute value). Thus, in the chart of the standardized index, which best surprises the variations, we notice that the biggest values are registered in 2001 as compared to 2000, in 2005 compared to 2004, as well as in 2008 compared to 2007. In these cases, the increase or decrease of the growth rate of the GDP/inhabitant for a space entity is much bigger than the variation registered by the other space entity. For example, if the growth rate of the GDP/inhabitant for Romania modifies in the following year by 10%, then the growth rate of the GDP/inhabitant in the European Union is also modified, in the same direction or in the opposite one, by more than 10%.

There is another explanation for the significant variation registered in the $I_{t_i,t_{i-1}}^{RO/UE}$ space-time index. In this case, we can refer to two situations. The first supposes that the factors which determined the changes in the evolution of the growth rate of the GDP/inhabitant in a space entity did not influence the growth rate of the GDP/inhabitant in the other region as well. In other words, if we are dealing with an economic shock, this occurred at a regional level and did not simultaneously affect the two entities. The second situation-scenario is that the economic shock is not a regional, but a global one, and one of the space entities is affected to a greater extent than the other one.

If the absolute value of the size of the space-time index was analyzed above, it would be interesting to see which is the significance of the sign (plus or minus) which accompanies the index.

From a mathematical perspective, a negative value of the determinant (Δ) means:

$$\left|\frac{a_{21}}{a_{11}}\right| > \left|\frac{a_{22}}{a_{12}}\right| \tag{7}$$

From an economic standpoint, in what concerns this example, we can state that the variation, in absolute value, of the growth rate of the GDP/inhabitant in Romania is **bigger** than the increase or decrease, in absolute value, of the growth rate of the GDP/inhabitant in the EU.

Analogously for a positive value of the determinant:

$$\Delta > 0 \Rightarrow \left|\frac{a_{21}}{a_{11}}\right| < \left|\frac{a_{22}}{a_{12}}\right| \tag{8}$$

Thus, the variation, in absolute value, of the growth rate of the GDP/inhabitant in Romania is **smaller** than the increase or decrease, in absolute value, of the growth rate of the GDP/inhabitant in the EU.

If we take into consideration the above mentioned results, the analogy between the space-time index and the concept of convergence is natural, especially if we refer to the analogy with the beta convergence type. According to the type of convergence, poor countries grow, on average, faster than rich countries (Islam, 2003). In this case, when we analyze the growth rate of the GDP/inhabitant for Romania in relation to the same rate for the EU, the phenomenon of convergence is all the more probable.

Consequently, in this example, the beta convergence hypothesis is confirmed when the growth rate of the GDP/inhabitant in Romania is **bigger** in the absolute value than the growth rate of the

GDP/inhabitant in the European Union. In order to test this hypothesis, it is necessary to verify the relations (7) and (8), which indicate the sign of the determinant (and implicitly of the index). For our data, we obtain the following table (Table 2):

Table 2: *Analysis of the sign of the determinant*

No.	**Year**	**RO vs.EU**
1	1999	
2	**2000**	I+I > I+I
3	**2001**	I+I > I-I
4	2002	I-I > I-I
5	2003	=
6	2004	I+I < I+I
7	2005	I-I < I-I
8	**2006**	I+I > I+I
9	2007	I-I < I-I
10	**2008**	I+I > I-I

Source: own calculations

According to the data in table 2, we can talk about convergence (of a beta type) only in the following years: 2000, 2001, 2006 and 2008 (the respective values were written in bold), when the variation of the growth rate of the GDP/inhabitant in Romania is **bigger** in absolute value than the growth rate of the GDP/inhabitant in the EU.

Following the analysis of charts no. 3 and 4 we can make one more remark. We notice that the variation of the value of the RO-EU index oscillates around the same measure, which indicates that the trajectory of the index value is a *stationary* one. Moreover, the respective series tends towards the zero value, which means that the series tends towards that value of the determinant (index) for which the growth rate of the GDP/inhabitant for Romania is modified (increases or decreases) in the following year with the value of the modification, in the same direction or in the opposite one, of the growth rate of the GDP/inhabitant for the EU.

To conclude, we would like to present the economic implications of property d) of matrix A (relation 6), presented in paragraph 2.2. Let us suppose that, within a country/region, we deal with a negative shock which affects a particular economy. Given the space-time dependence of economic indices, this shock may also have an impact on other countries/regions. We can analyze the extent to which economic shocks act at a local or global level by the help of the proposed method, taking into consideration the property expressed by relation 6. The impact of the shock on the employed economic indicator is given via the variable s_{ij}, which can have positive values (for a positive shock) or negative ones (for a negative shock). The approach under the form of a determinant of the space-time dependence of economic indicators thus enables us to calculate not only the impact of the shock on local indicators but also the connection between the economic shock produced in a country/region and the economy of the other analyzed space entity (see the right side element in relation 6).

The above mentioned analysis can also be performed for matrices with bigger ranks, which implies the application of more complex methods for the calculation of the value of the determinants.

4 Conclusions

The propagation of the positive or negative shocks of an economy is closely related to the space and time dependence of the various economic indicators. The measurement of the interdependence of the two dimensions – the spatial and the temporal one – which characterizes an economic indicator –is of real interest for the economic analysis.

This paper presents a new calculation index, whose measure reflects the simultaneous action of the space and time coordinates. By using the growth rate of the GDP/inhabitant as an economic indicator, we showed that the $I_{t_i,t_{i-1}}^{RO/UE}$ index is a good analysis tool in order to reflect, via a single measure, the time evolution of an economic indicator in a space entity in relation to the evolution of the same indicator in the other analyzed entity. Moreover, with the help of the $I_{t_i,t_{i-1}}^{RO/UE}$ index, we can test the (absolute) convergence hypothesis between two entities for discrete time moments.

The calculation method proposed in this paper also has some disadvantages. On the one hand, it can only be applied for discrete time moments, and on the other hand the employed matrix construct imposes that the number of time units be equal to the number of space entities subjected to the analysis.

Our future research aims at expanding the analysis to include other space entities and in comparing the results obtained following the application of the calculation index with classical methods for the testing of economic convergence.

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