Structural decomposition analysis and index number theory: an empirical application of the Montgomery decomposition

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Abstract In recent years a large number of empirical articles on structural decomposition analysis, which aims at disentangling an aggregate change into its factors, has been published in Economic Systems Research. Dietzenbacher and Los (D&L) proved that in case of n factors the number of possible decompositions is equal to n!, none of which satisfies time reversal. Averages of decompositions satisfy this requirement, such as the average of all decompositions. In index number theory this problem is known as the decomposition of an aggregate change into *symmetric* factors (usually two: price and quantity). Balk proposes to generalize the Montgomery decomposition, which obeys time reversal, to three factors. In this paper we apply this solution to a more intricate decomposition into four factors, viz. the example analyzed by D&L. We show that for most sectors the results of the Montgomery decomposition are remarkably close to those of the average of the 24 decompositions.

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1. Introduction

Ever since the seminal article of 1998 of Dietzenbacher and Los (D&L, Economic Systems Research, ESR, vol. 10, pp. 307-323) a large number of empirical articles on structural decomposition analysis, which aims at disentangling an aggregate change into its factors, has been published in ESR (we refer to the references rather than giving a long list of names and years). The major problem is that there is not a unique solution. Starting from the base period, we have the Laspeyres perspective, whereas starting from the comparison period, we have the Paasche perspective. These two are called the "polar decompositions". D&L argue that these are but two possibilities; they prove that in case of n factors the number of possible decompositions is equal to n!. In their empirical application n=4, so that there are 24 possible decompositions which, from a theoretical point of view, are equivalent. Each of these decompositions does not satisfy the requirement of time reversal which states that if base and comparison period are reversed, the decomposition should yield the reverse result. The average of the two polar decompositions satisfies time reversal, but, as argued by D&L, these two constitute but one of the (n/2)! "mirror pairs" (base and comparison period reversed); in their application 12. The average of each of the mirror pairs obeys time reversal, as well as the average of all n! (24) decompositions. D&L show that the results of the most commonly used solution, the average of the two polar decompositions, is close to those of the average of all 24 decompositions. Since the former requires two decompositions and the latter 24 it can be argued that the former is to be preferred from a computational point of view.

In index number theory this problem is known as the decomposition of an aggregate change into *symmetric* factors (usually two: price and quantity). Balk (2003) discusses the generalization to more than two factors, reviews proposals from literature, and adds a simple solution based on the work of Montgomery (1929, 1937) to the *additive* decomposition of a variable V, $V(1) - V(0)$, where the comparison period is denoted by 1 and the base period by 0. He provides the formula when the change is decomposed into three factors according to the form:

$$V = \sum_{i=1}^{n} p_i q_i r_i \tag{1}$$

In section 2 we show that in the application of D&L there are four factors of the more intricate form:

$$V_i = \sum_{j=1}^{n} \sum_{k=1}^{n} p_i q_{ij} r_{jk} s_k \qquad i = 1,\ldots,n \tag{2}$$

The organization of this article is as follows. In section 2 we discuss the model and its structural decomposition. First, we give the model that D&L analyze. Next, we discuss the problem of structural decomposition analysis and possible solutions. We start with the simplest case of two factors and derive the 2 (= 2!) polar decompositions which form a so-called "mirror pair": they follow from each other by reverting base and comparison period. None of them satisfies the requirement of time reversal, but it is easily seen that the average of the two polar decompositions satisfies this requirement. Next, we turn to the case of three factors and show that, besides the two polar decompositions, there are four other possible decompositions so that there are in total six (= 3!) decompositions (three mirror pairs). The average of each pair, as well as the average of all six decompositions, satisfies the requirement of time reversal and none of them can be preferred to the others. The argument can easily be extended: in case of four factors there are 24 (= 4!) possible solutions (12 mirror pairs). Finally, we turn to index number theory, derive the Montgomery decomposition, that satisfies the requirement of time reversal, and give the formula for the decomposition of the change in sectoral labor costs in the Netherlands between 1986 and 1992 into the effects of changes in labor costs per unit, technical changes, and changes in final demand mix and in final demand levels, i.e. the example analyzed by D&L. Section 3 is devoted to a description of the empirical application. We briefly discuss the dataset and describe how we handled zero and negative values. Next, we discuss how we handle trade and transport margins and value added tax (VAT). In our first example of the Montgomery decomposition we tried to derive tables in purchaser's prices and in the second one we used tables in basic prices and treated margins and VAT as a final demand category aggregating them with imputed bank services. In section 4 we present our results for the five sectors with the largest percentage growth and the five sectors with the largest absolute growth that D&L gave in their article. It turns out that for sectors, where the treatment of trade and transport margins is irrelevant, the results of the two versions of the Montgomery decomposition are remarkably close to the average of the 24 decompositions. Since the Montgomery decomposition has a clear theoretical underpinning, we feel that this decomposition is a good alternative to commonly used solutions that, moreover, are less attractive from a computational point of view.

2. The model and its structural decomposition

2.1. The D&L model

In their application D&L used the input-output tables at basic prices for the Netherlands of 1986 and 1992. Defining the following vectors and matrices:

w: the 214x1 vector of sectoral labor costs;

u: the 214x1 vector of sectoral labor costs per unit of this sector's output (in money terms);

$\hat{u}$: the 214x214 diagonal matrix with u on the main diagonal;

q: the 214x1 vector of sectoral outputs;

A: the 214x214 matrix of technical coefficients a_{ij}, measuring the input from sector i in sector j, per unit of sector j's output;

B: the 214x5 matrix of bridge coefficients b_{jk}, measuring the fraction of the final demand in category k that is spent on products from sector i, describing the final demand mix;

f: the 5x1 vector with total final demands in each of the five categories, i.e. private consumption, government consumption, exports, investments, and imputed bank services, they consider the model:

$$w = \hat{u}q$$

$$q = Aq + Bf$$

of which the solution is:

$$w = \hat{u}LBf \qquad (3)$$

where: $L = (I - A)^{-1}$ is the Leontief inverse.

D&L decompose the change Δw in sectoral labor cost into four components:

(1) the effects of a change in the labor cost per unit ($\Delta\hat{u}$);

(2) the effects in technical changes (ΔL);

(3) the effects of changes in the final demand mix (ΔB),

(4) the effects of the changes in the final demand levels (Δf).

For further details we refer to their article (Dietzenbacher and Los, 1998).

In sum mutation (3) reads:

$$w_i = \sum_{j=1}^{n}\sum_{k=1}^{n} u_i l_{ij} b_{jk} f_k \qquad i = 1,\dots,n \qquad (4)$$

where w_i, u_i and f_i are the typical elements of the vectors w, u and f , respectively; and l_{ij} and b_{jk} the typical elements of the matrices L and B, respectively.

Consequently, the task is to decompose the change in the *multiplicative* form:

$$v_{ijk} = u_i l_{ij} b_{jk} f_k \qquad (5)$$

into the changes of the four components mentioned above.

2.2. Structural decomposition analysis (SDA): the problem and possible solutions

We start with the simplest case, the multiplicative form with two factors:

$$y = x_1 x_2$$

We wish to decompose the change in y between two points in time: the base period 0 and the comparison period 1:

$$\Delta y = y(1) - y(0)$$

into the changes of the factors x_1 and x_2, viz. into:

$\Delta x_1 = x_1(1) - x_1(0)$ and $\Delta x_2 = x_2(1) - x_2(0)$

One possibility is:

$$\begin{aligned}\Delta(x_1 x_2) &= x_1(1)x_2(1) - x_1(0)x_2(0) \\ &= x_1(1)x_2(1) - x_1(0)x_2(0) - x_1(0)x_2(1) + x_1(0)x_2(1) \\ &= (\Delta x_1)x_2(1) + x_1(0)\Delta(x_2)\end{aligned} \tag{6}$$

But we can also add and subtract the term: $x_1(1)x_2(0)$ to obtain:

$$\Delta(x_1 x_2) = (\Delta x_1)x_2(0) + x_1(1)\Delta(x_2) \tag{7}$$

Both solutions satisfy the requirements that the decomposition is

(i) *complete*, that is to say: there is no residual term;

(ii) *zero value robust*, that is to say that it can deal with zero values;

but they do not satisfy the third requirement of structural decomposition:

(iii) *time reversal*, that is to say that if the time period is reversed, the decomposition yields the reverse result, i.e. $y(0) - y(1) = -[y(1) - y(0)]$,

see for instance Hoekstra and Van der Bergh (2003).
A solution is to take the average of these two so-called *polar decompositions*.
Next, we turn to the case of the decomposition of a multiplicative form with three factors:

$$y = x_1 x_2 x_3$$

Along the same lines as for the case of two factors, we have:

$$\Delta(x_1 x_2 x_3) = (\Delta x_1)x_2(0)x_3(0) + x_1(1)(\Delta x_2)x_3(0) + x_1(1)x_2(1)(\Delta x_3)$$

and by interchanging 0 and 1:

$$\Delta(x_1 x_2 x_3) = (\Delta x_1)x_2(1)x_3(1) + x_1(0)(\Delta x_2)x_3(1) + x_1(0)x_2(0)(\Delta x_3)$$

These two polar decompositions are but 2 out of 3! = 6 possibilities:

	$(\Delta x_1)x_2x_3$	+	$x_1(\Delta x_2)x_3$	+	$x_1x_2(\Delta x_3)$
1	0 0		1 0		1 1
2	0 0		1 1		1 0
3	1 0		0 0		1 1
4	1 1		0 0		0 1
5	0 1		1 1		0 0
6	1 1		0 1		0 0

Each of them satisfies the requirements of completeness and of zero value robustness, but not of time reversal.

A solution is to take the average of the two polar decompositions (i.c. of 1 and 6), but also the average of the combinations 2 and 4, and of 3 and 5, satisfies the requirement of time reversal, as does the average of all six possibilities.

Dietzenbacher & Los (1998) prove that in the general case of n factors, the number of equivalent decompositions is equal to n! In their example there are four factors, so that the number of equivalent decompositions is equal to 24. None of them satisfies the requirement of time reversal, but there are 12 "mirror" pairs (0 and 1 interchanged) of which the average satisfies time reversal. The average of the 24 decompositions obviously satisfies the requirement of time reversal as well.

The problem is that each of these 13 possibilities is a solution and none of them can be preferred to the others. D&L show that in their empirical example the average of the 24 decompositions is close to the average of the two polar decompositions so that from a computational point of view the latter may be preferred to the former.

As an alternative, we look at the theory of index numbers, where a similar problem exists.

2.3. Index number theory: the Montgomery decomposition

Let $p_i(1)$ and $p_i(0)$ denote the prices of commodity i (= 1,…,n) in comparison and base period, and let $q_i(1)$ and $q_i(0)$ be the corresponding quantities. Then,

$$V(1) = \sum_{i=1}^{n} p_i(1)q_i(1) \text{ and } V(0) = \sum_{i=1}^{n} p_i(0)q_i(0)$$

are total consumption expenditure in comparison and in base period.

If we decompose the change in total expenditure into the price factor and the quantity factor, we have to obtain the difference of the expenditure on commodity i:

$$v_i = p_i q_i$$

(a multiplicative form), in terms of the price change and the change in quantities.

The solution of Montgomery (1929,1937) makes use of the logarithmic mean that for two *positive* numbers a and b is defined as:

$$L(a,b) = \frac{a-b}{\ln(a/b)} \text{ and } L(a,a) = a \qquad (8)$$

The properties (Balk, 2003)[1] are:

(i) $\min(a,b) \le L(a,b) \le \max(a,b)$

(ii) $L(a,b)$ is continuous

(iii) $L(\lambda a, \lambda b) = \lambda L(a,b)$

(iv) $L(a,b) = L(b,a)$

(v) $\sqrt{ab} \le L(a,b) \le \frac{a+b}{2}$

Property (iv), that can be proved straightforwardly from the definition (8), implies that the logarithmic mean is *symmetric* in a and in b.
Consider the logarithmic mean of $v_i(1)$ and $v_i(0)$:

$$L[v_i(1), v_i(0)] = \frac{v_i(1) - v_i(0)}{\ln[v_i(1)/v_i(0)]} \tag{9}$$

We rewrite (9) to:

$$v_i(1) - v_i(0) = L[v_i(1), v_i(0)]\ln[v_i(1)/v_i(0)] =$$

$$= L[v_i(1), v_i(0)]\ln[p_i(1)/p_i(0)] + L[v_i(1), v_i(0)]\ln[(q_i(1)/q_i(0)]$$

where the second equality directly follows from $\ln[v_i(t)] = \ln[p_i(t)] + \ln[q_i(t)] \quad (t = 0,1)$.
The decomposition of the change in consumption expenditure in its price and quantity factors reads:

$$\begin{aligned} V(1) - V(0) &= \sum_{i=1}^{n} [v_i(1) - v_i(0)] = \\ &= \sum_{i=1}^{n} L[v_i(1), v_i(0)]\ln[p_i(1)/p_i(0)] + \sum_{i=1}^{n} L[v_i(1), v_i(0)]\ln[q_i(1)/q_i(0)] \end{aligned} \tag{10}$$

Since the logarithmic mean is symmetric, i.c. property (iv), it follows from (10) that, interchanging 0 and 1:

$$V(0) - V(1) = -[V(1) - V(0)],$$

i.e. the Montgomery decomposition satisfies the requirement of *time reversal*. Moreover, in (10) there is no residual term, consequently, the Montgomery decomposition also satisfies the requirement of *completeness*.

The advantage of the Montgomery decomposition, compared to taking the average of the two polar decompositions or the average of 24 decompositions, is that we only need one decomposition. Moreover, it has a sound foundation in index number theory.
Let us revisit the original problem, the decomposition of (4):

$$w_i = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{ijk} = \sum_{j=1}^{n}\sum_{k=1}^{n} u_i l_{ij} b_{jk} f_k \qquad i = 1,\dots,n$$

We can decompose it by using the logarithmic mean $L[v_{ijk}(1), v_{ijk}(0)]$ to yield[2]:

$$\Delta w_i = \sum_{j=1}^{n}\sum_{k=1}^{n} L[v_{ijk}(1), v_{ijk}(0)].\ln[u_i(1)/u_i(0)] + \sum_{j=1}^{n}\sum_{k=1}^{n} L[v_{ijk}(1), v_{ijk}(0)].\ln[l_{ij}(1)/l_{ij}(0)] + \\ + \sum_{j=1}^{n}\sum_{k=1}^{n} L[v_{ijk}(1), v_{ijk}(0)].\ln[b_{jk}(1)/b_{jk}(0)] + \sum_{j=1}^{n}\sum_{k=1}^{n} L[v_{ijk}(1), v_{ijk}(0)].\ln[f_k(1)/f_k(0)] \tag{11}$$

3. The empirical application

3.1. The dataset used by D&L

Dietzenbacher and Los (1998) used the input-output tables for the Netherlands at basic prices of 1986 and 1992. For purpose of comparison we use the very same tables. Each table consists of 214 sectors and 16 categories of final demand which have been aggregated by D&L to 5. It is not clear from their description[3] how they handled changes in stocks, trade margins, transport margins and VAT. In the data sets there are zeros and negative values that might cause problems when using the Montgomery decomposition.

3.2. Handling zero's and negative values

The logarithmic mean is defined for positive real numbers a and b so that zero's and negative values might cause problems.
Ang, Zhang and Choi (1998) dealt with the zeros and proved that in an empirical application zeros can be replaced by epsilon small positive numbers. Consequently, the Montgomery decomposition satisfies the requirement of *zero-robustness* as well.
If in both base and comparison period the values are negative, there is no problem. An example can be found in our empirical application where in both 1986 and 1992 the value of agricultural investment was negative due to a reduction in live stock. Their ratio is positive and the logarithm can readily be taken. If one value is negative in one period and non-negative in the other, then there is a problem[4]. This is frequently the case for the change in stocks. But since this change does not constitute a fundamental element of the change in the composition of final demand between 1986 and 1992, we propose to eliminate stock changes from our empirical application. A popular way is to add it to investment. However, in the very detailed classification that we use in over 30 cases the sum of investment and stock change turned out to remain negative in one period and positive in the other. Therefore we have decided[5] to split them over all other items of a row according to the pertinent shares in total output. Since the column sums are not any

longer equal to total output, we have added a row in which we record the adjustment for stocks. (This row does not play a role in the decomposition).

3.3. Handling trade, transport margins and VAT

We would like to use the matrices of trade and transport margins and VAT to arrive at the tables valued at purchaser's prices. These tables are not readily available for 1986 and 1992 on paper, let alone in an electronic version. Since we have expository purposes only, we first have decided to split them over the rows in the very same way as we split the changes in stocks over the rows: we used the shares (including the stock changes) and we added a row to record the adjustment for margins and VAT (like before, this row does not play a role in the decomposition either).
Since Dietzenbacher & Los, in their application, might have aggregated them with the imputed bank services into one final demand category, we have also performed the Montgomery decomposition combining margins and VAT with imputed bank services[6].

4. Results

In table 1 we present results for the five sectors with the largest percentage growth and the five sectors with the largest absolute growth that D&L used in their table 1. The figures reported in the columns "$\min_i$", "$\max_i$" and "μ_i" are taken from D&L and give the minimum, the maximum, and the average of the 24 possible decompositions. In the column "Montgomery (margins split over rows)" we give the results of the first Montgomery decomposition, where trade and transport margins and VAT have been split over the rows in the same way as stock changes, whereas in the column "Montgomery (margins as final demand)" we give the results for the decomposition where we aggregated trade and transport margins and VAT with imputed bank services to one final demand category.
It follows from table 1 that for the five sectors with the largest percentage growth (the sectors 156, 153, 205, 127 and 157) and for two out of the five sectors with the largest absolute growth (the sectors 171 and 162) the results of the average of the 24 decompositions of D&L are remarkably close to those of both versions of the Montgomery decomposition. For the wholesale trade (sector 121), retail trade (sector 123) and, to a lesser extent, railways, communication services, taxi and coach enterprises (sector 146) there are some substantial differences. But these are just the sectors for which the treatment of trade and transport margins is crucial!
For the wholesale trade (sector 121) the effect of the change in labor cost per unit ($\Delta\hat{u}$) only differs marginally between the three decompositions, but there are huge differences for the effect of technical change (ΔL); the change in the final demand mix (ΔB) and in the change in the final demand levels (Δf). If we split the margins over the rows, the effect of technical change is substantially higher than if we consider the average of the 24 decompositions, while for the effect of the change in the final demand mix the reverse is true; the effect of the change in final demand level being close to each other. If we treat margins as a final demand category, the effects of technical change and of the change in the final demand mix are considerably lower for the Montgomery decomposition than for the average of the 24 decompositions, while the effect of the change in the final demand level is considerably higher.
For the retail trade (sector 123) the difference between the three decompositions of the effect of the change in labor cost per unit is marginal again; the difference between the Montgomery decomposition, where margins are treated as a final demand category, and

the average of the 24 decompositions is very small as well, but the results of the Montgomery decomposition, where margins are split over the rows, are quite different. This might be an indication that Dietzenbacher and Los treated trade and transport margins and VAT as a final demand category. For the sector 146, railways etc., finally, the differences are less pronounced than for the wholesale and retail trade.

Table 1. Results of the decomposition by Dietzenbacher and Los (1998) and of the two versions of the Montgomery decomposition*

Sector	$\min_i$ minimum of all decompositions	$\max_i$ maximum of all decompositions	μ_i average of all decom-positions	Montgomery (margins split over rows)	Montgomery (margins final as demand)
Sectors with largest percentage growth					
156 economic advising agents	$w_i(86) = 539$	$w_i(92) = 1436$	$\Delta w_i = 897$	$\Delta\% = 166.4$	
$\Delta\hat{u}$	8.2	21.4	14.4	13.7	13.7
ΔL	392.5	517.3	454.2	458.9	455.3
ΔB	176.7	238.8	207.2	208.1	213.6
Δf	142.9	301.1	221.2	216.3	214.5
153 computer services	$w_i(86) = 1245$	$w_i(92) = 2842$	$\Delta w_i = 1597$	$\Delta\% = 128.3$	
$\Delta\hat{u}$	119.1	248.1	180.4	176.7	175.6
ΔL	584.4	822.9	700.0	730.5	675.5
ΔB	209.2	301.7	253.4	234.8	273.7
Δf	333.2	599.3	463.3	456.1	472.2
205 gambling and betting services	$w_i(86) = 77$	$w_i(92) = 172$	$\Delta w_i = 95$	$\Delta\% = 123.4$	
$\Delta\hat{u}$	1.3	2.8	2.0	1.9	1.9
ΔL	0	0	0	0	0
ΔB	50.8	68.4	59.6	61.6	61.1
Δf	24.9	42.1	33.5	31.5	32.0
127 beverage serving services (no lodging)	$w_i(86) = 189$	$w_i(92) = 416$	$\Delta w_i = 227$	$\Delta\% = 120.1$	
$\Delta\hat{u}$	16.1	32.6	23.9	23.4	23.4
ΔL	19.0	27.4	23.1	24.5	22.9
ΔB	83.1	120.3	101.3	104.1	104.1
Δf	59.3	99.3	78.8	74.9	76.6
157 other business services	$w_i(86) = 1032$	$w_i(92) = 2249$	$\Delta w_i = 1217$	$\Delta\% = 117.9$	
$\Delta\hat{u}$	5.0	10.8	7.8	7.4	7.4
ΔL	451.6	578.2	514.6	511.6	508.0
ΔB	285.0	355.7	320.1	335.6	338.1
Δf	279.2	470.0	374.5	362.4	363.5

Sectors with the largest absolute growth					
121 wholesale trade	$w_i(86) = 13212$	$w_i(92) = 21712$	$\Delta w_i = 7500$	$\Delta\% = 56.8$	
$\Delta\hat{u}$	1902.6	2607.2	2249.5	2237.1	2242.7
ΔL	387.9	614.1	492.8	**624.6**	**244.1**
ΔB	597.3	911.5	745.0	659.3	**524.2**
Δf	3606.2	4428.4	4012.7	3979.0	**4489.0**
123 retail trade	$w_i(86) = 7726$	$w_i(92) = 12225$	$\Delta w_i = 4499$	$\Delta\% = 58.2$	
$\Delta\hat{u}$	1235.6	1685.6	1458.5	1447.3	1454.1
ΔL	6.6	9.2	7.8	**342.1**	**-8.7**
ΔB	244.9	375.7	308.1	**169.1**	335.7
Δf	2482.3	2971.0	2724.6	2540.5	2717.9
146 railways, communication services, taxi and coach enterprises	$w_i(86) = 5385$	$w_i(92) = 8232$	$\Delta w_i = 2847$	$\Delta\% = 52.9$	
$\Delta\hat{u}$	363.0	519.9	439.8	**317.9**	**317.9**
ΔL	291.9	405.5	346.3	395.3	382.0
ΔB	300.5	425.0	360.3	**479.9**	424.3
Δf	1564.6	1838.9	1700.6	1653.9	1722.8
171 special (primary) education (for handicapped children)	$w_i(86) = 8221$	$w_i(92) = 10863$	$\Delta w_i = 2642$	$\Delta\% = 32.1$	
$\Delta\hat{u}$	1169.0	1525.4	1341.4	1338.4	1338.4
ΔL	-7.9	-5.6	-6.7	-6.0	-6.3
ΔB	-714.4	-505.8	-607.2	-600.9	-601.2
Δf	1723.4	2117.1	1914.4	1910.5	1911.2
162 local government	$w_i(86) = 6933$	$w_i(92) = 9417$	$\Delta w_i = 2484$	$\Delta\% = 35.8$	
$\Delta\hat{u}$	654.2	856.1	751.9	748.7	748.7
ΔL	-279.1	-177.7	-226.1	-218.0	-222.0
ΔB	205.3	306.7	252.3	244.3	239.7
Δf	1578.1	1843.2	1706.0	1709.0	1709.0

* $\Delta w_i = w_i(92) - w_i(86)$ is the change in the labor cost which is decomposed into the change in labor cost per unit ($\Delta\hat{u}$), technical change (ΔL), the change in the final demand mix (ΔB) and the change in the final demand levels (Δf). In bold we report the figures that are either below the minimum or above the maximum of the 24 decompositions that Dietzenbacher and Los calculated.

5. Concluding remarks

In this paper we have used the Montgomery decomposition which has been introduced in index number theory for the symmetric decomposition of the change of the expenditure on commodities between the base and comparison period over two factors, price and quantity. We have applied it to the decomposition of the change in labor cost over four factors: labor cost per unit, technical change, final demand mix and final demand levels. We have demonstrated that only a single decomposition, computationally easily implementable, is needed to arrive at a decomposition that satisfies the requirement of time reversal, whereas the methods used in practice, such as the average of the two polar decompositions (or the average of the 24 possible decompositions) require more computational effort and do not have a sound theoretical underpinning.

We have shown in our empirical example that for those sectors where the treatment of trade and transport margins and of VAT is unessential, the results of the average of the 24 possible decompositions and the Montgomery decompositions that we have considered are remarkably close to each other. Since Dietzenbacher and Los report that the results of the average of the 24 decompositions are quite close to those of the average of the two polar decompositions, we conclude that our results are close to the latter average as well. In view of the computational advantage and of the sound theoretical background we prefer to use the Montgomery decomposition, but since the average of the 24 decompositions and the average of the two polar decompositions satisfy time reversal and yield largely the same results, these methods can be used as well, of course.

It follows from the empirical example that for the decomposition of the sectors wholesale and retail trade (for which the treatment of trade margins is important), and, to a lesser extent for the sector railways, etc. (for which the treatment of transport margins is important), the results depend on how we treat margins. There are (at least) three different ways from which one can choose:

1. using input-output tables in purchaser's prices, where trade and transport margins and VAT are included in the figures (preferably by using the pertinent matrices in order to avoid the crude assumption, as in this paper, that they are split over the rows using the pertinent shares);

2. using input-output tables in basic prices, treating trade and transport margins and VAT as a final demand category; and

3. input-output tables where trade and transport margins and VAT are included as an additional row and column.

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[1]We give the footnote 1 of Balk (2003): "The logarithmic mean was introduced in the economics literature by Törnqvist in 1935 in an unpublished memo of the Bank of Finland; see Törnqvist, Vartia and Vartia (1985) …..A proof of the fact that $(ab)^{1/2} \le L(a,b) \le (a+b)/2$ was provided by Lorenzen (1990)". Substitution of $b = a$ in (i) or (iv), leads directly to $L(a,a) = a$

[2] Ang, Zhang and Choi (1997) obtained this decomposition along a different route (Divisia index number theory) and named it: "logarithmic mean Divisia index method". Hoekstra and Van der Bergh (2003) named it:" Refined Divisia".

[3] The objective of Dietzenbacher and Los was to apply SDA to a large dataset; not to deal thoroughly with the decomposition of the change in labor costs into its four components.

[4] If the agricultural investment had been positive in 1986 and negative in 1992, we would have solved the problem by splitting investment (-216 million guilders in 1992) in both periods over the meat-packing industry (10502 million guilders in 1992) and exports (14231 million guilders in 1992).

[5] National Account statisticians with a thorough knowledge of supply and use tables might prefer attributing the stock change for each sector to the most important item(s). We, however, do not dispose of this knowledge. Since our objective is a comparison with the results of Dietzenbacher & Los, we refrained from seeking pertinent advice.

[6] Alternatively, we might introduce an additional row and column representing trade and transport margins, like in the Norwegian input-output tables, see Peters & Hertwich (2006). Since our objective is a comparison with the results of Dietzenbacher & Los, who did not introduce an additional row and column, we refrained from this refinement.