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Selling Goods of Unknown Quality: Forward versus Spot Auctions

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Abstract

We consider an environment where the sale can take place so early that both the seller and the potential buyers have the same uncertainty about the quality of the good. We present a simple model that allows the seller to put the good for sale before or after this uncertainty is resolved, , namely via forward auction or spot auction, respectively. We solve for the equilibrium of these two auctions and then compare the resulting revenues. We also consider the revenue implications of the insurance in forward auctions.

Keywords: Forward Trading, Forward Auctions, Spot Auctions

JEL classification: D44

1 Introduction

It is quite often the case that the sellers and the buyers transact when there are uncertainties about the quality of the object. Examples could be drawn from a wide range of industries. In Australia, real estates are occasionally sold 'off the plan' using auctions. In many countries, agricultural produce is often sold long before the harvest. Moreover, as a wide practice in Japanese racehorse industry, the foals are sold *before* they are born. Similarly, in many countries, including Australia, Canada, and the United States, livestock breeders sell embryos in auctions. In the UK gas industry, National Grid, the network owner that is in charge of balancing the demand and the supply of gas in the network, auctions off the transmission capacity rights - right to insert gas into the network

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- well in advance of the realization of the demand for gas, that is, before observing the availability of the capacity (See McDaniel and Neuhoff (2004)).[1]

These examples share three salient features: One, the seller uses an auction as a selling mechanism, two, the sale takes place so early that both the seller and the potential buyers have the same uncertainty (*symmetric uncertainty*) about the quality of the object, and, three, this uncertainty is resolved simultaneously for the seller and the buyers. While there is a line of literature on transactions under asymmetric uncertainty (for instance, see Lewis (2007) and Kogan and Morgan (2009)), to the best of our knowledge, there has not been any theoretical work on auctions with symmetric uncertainty.[2] This paper is a first attempt toward filling this gap.

The uncertainty present in the aforementioned examples can be captured by considering two states of the world: good state (or good quality) and the bad state (or bad quality). Both sides of the transaction anticipate that the object would create full value to its owner in the good state whereas only a fraction of it in the bad state. In extreme cases, the object would yield no value in the bad state (e.g. gas transmission and embryo auctions).

Along this line, we present a simple model in which the seller could sell the object either before the uncertainty is resolved, i.e. via *forward auction* or after, i.e. via *spot auction*. While in the former case, the terms of sale (i.e. regarding the winning bidder and winning price) are pre-committed before the ex-post values are privately observed by the bidders, in the latter case, the terms are fixed after the ex-post values are privately observed, as is usually assumed in auction theory literature.

We consider a symmetric environment with independent and private values. In this setup, symmetric and strictly increasing equilibrium bidding strategies yield the same efficient allocation in forward auction and spot auction. Therefore, if both the seller and the buyers are risk neutral, revenue equivalence principle applies, implying that the forward and spot auctions generate the same revenue. Coexistence of both types of auctions in real world, however, suggests that they could yield different revenues.

[1] Note that, in almost all of these industries, the goods are also sold after the uncertainty is resolved. For example, sale of transmission capacities also occur when there are extra capacities left after the demand is realized. Livestock and race horses are also sold after they are born.

[2] In an experimental paper Phillips, Menkhaus and Krogmeier (2001) test behavior in forward and spot *double auctions*. Their focus, however, is on the implications of inventory costs due to advance production and not on the quality uncertainty.

According to many studies buyers' preferences can exhibit risk aversion and/or loss aversion.[3],[4] In this paper, we demonstrate that these preferences could account for the revenue differences between the forward and the spot auctions. The class of utility functions we consider (i) shows risk aversion and loss aversion properties, (ii) produces closed form equilibrium bidding strategies, and (iii) boils down to risk-neurality for some parameter specifications.

In forward auctions, the price the winner commits to pay could exceed the ex-post value of the object to him, yielding a negative payoff. The seller could, therefore, naturally consider insuring the winner against such risk. For instance, National Grid guarantees to buy back certain capacity rights, if the capacity ends up being unavailable. In embryo auctions, some sellers guarantee pregnancy. To shed some light on these observations, we also look at the revenue implications of the insurance (in the form of buy-back guarantee) in forward auctions.

In majority of the examples mentioned above, auctions typically use a second price rule. We hence consider auctions of the form of "second-price sealed bid auction". We solve for the equilibrium of three auction formats: spot auction, forward auction, and forward auction with insurance. We then compare the resulting revenues. First, we characterize a condition which determines when a forward auction results in higher revenues than a spot auction. It turns out that forward auction becomes less favorable to the seller if buyers are more loss averse. Next, we show that (under a mild convexity assumption) the spot auction generates higher expected revenue than the forward auction with insurance.[5] The result follows because, while the insurance induces buyers to bid more aggressively (relative to the expected bid in spot auction), it also results in inefficiencies as the seller sometimes keeps the object. It turns out that the loss of revenue due to inefficiency dominates the gain from aggressive bidding. Lastly, we compare expected revenues of forward auction with and without insurance. We find that there is no general revenue ranking between the two. Without insurance bidders might be very cautious about their bids in a forward auction. If that is the case,

[3] Seminal works on loss aversion are due to Kahneman and Tversky (1979) and Tversky and Kahneman (1991). Primary contributions on risk aversion in auctions are due to Maskin and Riley (1984) and Matthews (1987).

[4] If the buyers are risk neutral and seller is risk averse, one can easily argue that expected utility of the seller by selling via forward auction would be greater than that from spot auction.

[5] To the extent that this model captures the main aspects of the motivating examples, provision of full insurance in forward auctions is optimal only if the seller does not choose to use a spot auction for reasons not included in our model. This result conforms with practice, as full insurance is provided by the seller only in auctions for transmission capacity rights and in embryo auctions. In these two industries, forward auctions are preferred over spot auctions for the need for balancing the network in a timely fashion and the need for faster reproduction of livestock. In other three industries, insurance is typically provided not by the seller, but by third parties.

then a forward auction with insurance can be better for the seller than a forward auction.

2 Model

Consider a seller who would like to sell an object (foal), the quality of which is commonly unknown in period 1 (unborn foal), but becomes common knowledge in period 2 (after birth). The quality can take two forms: good with probability p and bad with probability $(1-p)$.

There are n bidders with the utility function,

$$u(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\gamma(-x)^{\alpha} & \text{if } x < 0, \end{cases} \tag{1}$$

where $\alpha \in (0,1]$ is the risk aversion parameter and $\gamma \geq 1$ is the loss aversion parameter. Note that, this utility specification allows for risk aversion and loss aversion. The limit cases of interest are

(i) $\alpha = 1$, $\gamma = 1$, which corresponds to risk neutrality,

(ii) $\alpha = 1$, $\gamma > 1$, which corresponds to loss aversion via kinked linear functions, and

(ii) $\gamma = 1$, $\alpha \in (0,1)$ which corresponds to constant relative risk aversion on the positive axis with no loss aversion.

Seller's value for the object is normalized to be zero. The bidders independently draw their private valuations from a atomless cumulative distribution F on $[0,1]$ with $f(\cdot) = F'(\cdot) > 0$. If the object turns out to be of good quality, then it will create the full value, x, to its buyer. On the other hand, if it is of bad quality, then it will create a fraction of the full value, kx, where $k \in [0,1)$. For the unborn foal example, x represents the value associated with the ability of the buyer to train the horse, whereas quality (1 versus $k < 1$) could represent the inherited quality of the foal.

The seller can sell the object either before the quality is realized (forward auction) or after (spot auction). In either case, he adapts the *second-price sealed bid auction.* The highest bidder gets to keep the object irrespective of the quality and pays the second highest bid.

When using a forward auction, the seller could also offer insurance in the form of "buy-back guarantee" as the winner's ex-post value for the object could exceed the price he pays (if the quality turns out to be bad), a risk that would hinder aggressive bidding. The buy-back guarantee would allow the winner to default on buying the object. Note that, in spot auction, no such insurance is needed as the price paid in equilibrium would always be smaller than the ex-post value.

In Sections 3, 4 and 5, we analyze spot auction, forward auction, and forward auction with insurance, respectively. We then compare the respective revenues in Section 6 and conclude in Section 7.

3 Spot auction

In a spot auction, in both contingencies (good quality or bad quality), bidders bid their values in (weakly dominant) equilibrium, therefore seller's expected revenue is given by

$$(p + (1-p)\,k)\,E\left[Y_2^{(n)}\right] \tag{2}$$

where $Y_2^{(n)}$ is the random variable representing the second highest of n independent and identical draws from distribution F.

4 Forward auction

Here we consider a seller who uses a forward auction but does not offer any insurance. Then, the expected utility of a bidder with value x who bids as if his value is z is given by

$$\begin{aligned} & G(z)\,E\left[pu\left(x-\beta^F(y)\right)+(1-p)\,u\left(kx-\beta^F(y)\right) \mid y<z\right] \\ = & \int_0^z \left(pu\left(x-\beta^F(y)\right)+(1-p)\,u\left(kx-\beta^F(y)\right)\right) g(y)\,dy \end{aligned}$$

where $G(z) = F(z)^{n-1}$ represents the distribution of the highest value of the remaining $n-1$ bidders, i.e. $Y_1^{(n-1)}$, $g(z) = G'(z)$ represents the corresponding density, and $\beta^F(\cdot)$ is the symmetric strategy they follow. Differentiating with respect to z and evaluating at $z = x$ gives the following necessary first order condition:

$$\left(pu\left(x-\beta^F(x)\right)+(1-p)\,u\left(kx-\beta^F(x)\right)\right) g(x) = 0$$

Since $g(x) > 0$, it follows that

$$pu\left(x - \beta^F(x)\right) + (1-p)\,u\left(kx - \beta^F(x)\right) = 0.$$

For this to be true, we should have

$$x \geq \beta^F(x) \geq kx.$$

Since u is assumed to be of the form in (1), we obtain

$$p\left(x - \beta^F(x)\right)^{\alpha} = (1-p)\,\gamma\left(\beta^F(x) - kx\right)^{\alpha},$$

or

$$\frac{x - \beta^F(x)}{\beta^F(x) - kx} = \left(\frac{\gamma(1-p)}{p}\right)^{\frac{1}{\alpha}} \equiv t.$$

Rearranging this expression gives us the following equilibrium bidding strategy,

$$\beta^F(x) = \frac{1+tk}{1+t}x$$

or

$$\beta^F(x) = \left(\frac{1}{1+t} + \frac{t}{1+t}k\right)x.$$

Note that, this satisfies $x \geq \beta^F(x) \geq kx$.[6]

We see that, in equilibrium, bidders calculate a weighted average of the qualities and bid proportional to this weighted average. For example, when bidders are risk neutral, namely when $\alpha = \gamma = 1$, the weights attached to respective qualities correspond to bidders' priors p and $(1-p)$, implying that the risk neutral bidders would bid their expected valuations (i.e. $(p + (1-p)\,k)\,x$) in a forward auction. Yet, in general, depending on the parameter values, bidders might bid more or less than their expected valuations. We elaborate on this below when comparing the forward and spot auction revenues.

[6] Note that we obtain the equilibrium bidding strategy by using only the necessary condition. It can be confirmed that for this strategy, global deviations are not profitable and $\beta^F(\cdot)$ constitutes a symmetric Bayesian Nash equilibrium.

For above equilibrium, the seller's expected revenue is given by

$$\frac{1+tk}{1+t}E\left[Y_2^{(n)}\right]. \tag{3}$$

5 Forward auction with insurance

We showed in the previous section that, in equilibrium, the winner in a forward auction could end up with a negative payoff, which hinders aggressive bidding. To induce more aggressive bidding, the seller could offer an insurance. Here, we consider the insurance in the form of a buy-back guarantee: the seller guarantees to buy-back the object at the selling price if the winner demands so.

Consider a bidder with value x who pretends as if his value is z, then his expected payoff is given by

$$\begin{aligned} & G(z)\, E\left[pu\left(x-\beta^{FI}(y)\right)+(1-p)\max\{u\left(kx-\beta^{FI}(y)\right),0\} \mid y<z\right] \\ = & \int_0^z \left(pu\left(x-\beta^{FI}(y)\right)+(1-p)\max\{u\left(kx-\beta^{FI}(y)\right),0\}\right) g(y)\, dy, \end{aligned}$$

where $G(z)=F(z)^{n-1}$, $g(z)=G'(z)$, and $\beta^{FI}(\cdot)$ is the symmetric strategy that the remaining bidders follow. Differentiating with respect to z and evaluating at $z=x$, yields the following first order condition,

$$\left(pu\left(x-\beta^{FI}(x)\right)+(1-p)\max\{u\left(kx-\beta^{FI}(x)\right),0\}\right) g(x)=0.$$

Since $g(\cdot)>0$, it follows that

$$pu\left(x-\beta^{FI}(x)\right)+(1-p)\max\{u\left(kx-\beta^{FI}(x)\right),0\}=0.$$

Hence, we should have

$$\beta^{FI}(x)=x.$$

That is , the buyers bid truthfully. Note that, this equilibrium holds for any utility specification (with $u(0)=0$). Note also that, in equilibrium, the seller buys the object back with positive probability. Hence, with a buy-back guarantee, the outcome of the forward auction is not fully efficient, whereas the standard forward auction and the spot auction are both efficient.

Seller's expected revenue from the forward auction with insurance is then given by

$$p\int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1 + (1-p)\int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1 \tag{4}$$

where x_1 represents highest order statistic and x_2 represent second highest order statistic of n random draws and $f_{1,2}(x_1,x_2)$ is the corresponding joint density. Note that, in equilibrium, the winner pays the second highest value whenever he keeps the object, and he does so *unless* the quality is bad and the price exceeds his expost value for the bad quality object, kx_1, hence the upper bound of the second integral in the second term.

6 Comparison of revenues in different auction formats

6.1 Spot auction versus forward auction

Proposition 1 *Expected revenue of forward auction is greater than that of spot auction if and only if*

$$p > \frac{\gamma^{\frac{1}{1-\alpha}}}{1+\gamma^{\frac{1}{1-\alpha}}}.$$

Proof. From (3) and (2), expected revenue in a forward auction is greater than that of spot auctions when

$$\frac{1}{1+t} + \frac{t}{1+t}k > p + (1-p)\,k$$

where

$$t = \left(\frac{\gamma(1-p)}{p}\right)^{\frac{1}{\alpha}}.$$

Rearranging, we obtain

$$\frac{1-p}{p} > \left(\frac{\gamma(1-p)}{p}\right)^{\frac{1}{\alpha}}$$

which is true when $p > \frac{\gamma^{\frac{1}{1-\alpha}}}{1+\gamma^{\frac{1}{1-\alpha}}}$. ■

That is, forward auction is better if and only if the probability of the object being good is greater than the amount $\frac{\gamma^{\frac{1}{1-\alpha}}}{1+\gamma^{\frac{1}{1-\alpha}}}$. Note that, this condition is independent of k, the quality measure of the bad object. This follows from the fact that, in forward auction bidders bid according to some weighted average of the two qualities (1 and k) and, in spot auction, ex-ante average bid is also a weighted average of these qualities.[7] While the weight attached to the good quality is $\frac{1}{1+t}$ in the former, it is p in the latter. Thus, which average would be greater depends only on the attached weights, but not the qualities.

Note also that, $\frac{\gamma^{\frac{1}{1-\alpha}}}{1+\gamma^{\frac{1}{1-\alpha}}}$ is increasing in γ. This implies that the equilibrium bid in forward auction decreases with the degree of loss aversion (γ), therefore forward auction becomes less favorable to the seller the more loss averse the bidders.

Remark 1 *If $\gamma = 1$, the forward auction is better than the spot auction iff $p > \frac{1}{2}$. If $\alpha = 1$, and $\gamma > 1$, that is, if the utilities are loss averse via kinked linear functions, then it turns out that the spot auction is always better than the forward auction. If $\alpha = \gamma = 1$, that is, if the bidders are risk neutral, then the spot and the forward auctions are revenue equivalent. This equivalence could also be obtained by the revenue equivalence principle as both auctions result in efficient allocations.*

6.2 Spot auction versus forward auction with insurance

Remember that, in the spot auction and in the forward auction with insurance, bidders bid their true valuations and that the corresponding expected revenues are given by (2) and (4). We obtain the following revenue ranking.

Proposition 2 *Under a mild convexity assumption–the assumption that $xf(x)$ is increasing–the expected revenue from the spot auction is greater than that from the forward auction with insurance.*

Proof. The difference in the two revenues is given by

$$\begin{aligned}
& (p + k(1-p)) E\left[Y_2^{(n)}\right] - \left(pE\left[Y_2^{(n)}\right] + (1-p)\int_0^1 \int_0^{kx_1} x_2 f_{1,2}(x_1, x_2)\, dx_2 dx_1\right) \\
= \; & (1-p)\left(kE\left[Y_2^{(n)}\right] - \int_0^1 \int_0^{kx_1} x_2 f_{1,2}(x_1, x_2)\, dx_2 dx_1\right) \\
= \; & (1-p)\left(k\int_0^1 \int_0^{x_1} x_2 f_{1,2}(x_1, x_2)\, dx_2 dx_1 - \int_0^1 \int_0^{kx_1} x_2 f_{1,2}(x_1, x_2)\, dx_2 dx_1\right)
\end{aligned}$$

[7] Ex-ante average bid in a spot auction is the same as the equilibrium bid in a forward auction with risk neutral bidders. Thus, in forward auction, the behavior of a risk neutral bidder can be compared to the non risk neutral loss averse bidders along the same lines.

Define $A(k) = k\int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1 - \int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1$ and note that $A(0) = A(1) = 0$. Differentiate $A(k)$ once to obtain

$$A'(k) = \int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1 - \int_0^1 x_1 kx_1 f_{1,2}(x_1,kx_1)\,dx_1,$$

and by differentiating twice we get

$$A''(k) = -\int_0^1 (x_1)^2 f_{1,2}(x_1,kx_1) + k(x_1)^3 \frac{\partial}{\partial x_2} f_{1,2}(x_1,kx_1)\,dx_1.$$

Since the valuations are independent, we have

$$f_{1,2}(x_1,x_2) = n(n-1) f(x_1) f(x_2) F(x_2)^{n-2}$$

and

$$\begin{aligned}\frac{\partial}{\partial x_2} f_{1,2}(x_1,x_2) &= n(n-1) f(x_1)\left[f'(x_2) F(x_2)^{n-2} + (f(x_2))^2 (n-2) F(x_2)^{n-3}\right]\\ &= n(n-1) f(x_1) f(x_2) F(x_2)^{n-2}\left[\frac{f'(x_2)}{f(x_2)} + \frac{f(x_2)}{F(x_2)}(n-2)\right].\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}A''(k) &= -\int_0^1 (x_1)^2 \left(f_{1,2}(x_1,kx_1) + kx_1\frac{\partial}{\partial x_2} f_{1,2}(x_1,kx_1)\right) dx_1\\ &= -\int_0^1 (x_1)^2 n(n-1) f(x_1) f(kx_1) F(kx_1)^{n-2}\left(1 + kx_1\left(\frac{f'(kx_1)}{f(kx_1)} + \frac{f(kx_1)}{F(kx_1)}(n-2)\right)\right) dx_1.\end{aligned}$$

Note that, $(x_1)^2 n(n-1) f(x_1) f(kx_1) F(kx_1)^{n-2}$ is positive for all k and x_1. Moreover,

$$1 + kx_1\left(\frac{f'(kx_1)}{f(kx_1)} + \frac{f(kx_1)}{F(kx_1)}(n-2)\right) \geq 1 + kx_1\frac{f'(kx_1)}{f(kx_1)}$$

and $1 + kx_1\frac{f'(kx_1)}{f(kx_1)}$ is greater than zero for the distributions that satisfy $(f(x)x)' = f'(x)x + f(x) \geq 0$.

Since $A(k)$ is zero at $k = 0$ and $k = 1$ and concave in between, we conclude that $A(k) \geq 0$ for all k. ■

Note that, all convex distributions and all power distributions (whether convex or concave) satisfy the above mentioned mild-convexity assumption. The intuition behind the result is the following. When the quality of the object is good, both spot auction and forward auction with insurance obtain the same revenue. When the quality is bad, however, spot auction always obtains an ex-post revenue of $kY_2^{(n)}$, whereas forward auction with insurance obtains ex-post revenue of either $Y_2^{(n)}$ or 0. When the value distribution becomes sufficiently convex, the revenue becomes more likely to be 0. Therefore spot auction obtains a higher expected revenue.

6.3 Forward auction versus forward auction with insurance

Lastly, we compare expected revenues of forward auctions with and without insurance.

Proposition 3 *There is no general revenue ranking between forward auction and forward auctions with insurance.*

Proof. The expected revenue from forward auctions with insurance is given by (4). For uniform value distributions, $F(x) = x$, $f_{1,2}(x_1, x_2) = n(n-1)(x_2)^{n-2}$ and (4) simplifies to $(p + (1-p)k^n) E\left[Y_2^{(n)}\right]$. Therefore, for uniform distributions, expected revenue from forward auction is greater than that with full insurance if

$$\begin{aligned} \frac{1+tk}{1+t} &> p + (1-p)k^n \\ \frac{1 + \left(\frac{\gamma(1-p)}{p}\right)^{\frac{1}{\alpha}} k}{1 + \left(\frac{\gamma(1-p)}{p}\right)^{\frac{1}{\alpha}}} &> p + (1-p)k^n \end{aligned}$$

we conclude that there is no unambiguous ranking: Even for the two extreme cases, namely for $\alpha = 1$, and for $\gamma = 1$, we find two sets of parameter values that show either auction format can be better.

For $\alpha = 1$, $\{n = 2, p = \frac{1}{2}, k = \frac{1}{2}, \gamma = 2\}$ gives 0.67 versus 0.625 (left hand side versus right hand side) and $\{n = 2, p = \frac{1}{2}, k = \frac{1}{2}, \gamma = 4\}$ gives us 0.6 versus 0.625. Other one is for $\gamma = 1$, $\{n = 2, p = \frac{1}{3}, \alpha = \frac{1}{2}, k = 0.1\}$ gives 0.28 versus 0.34 and $\{n = 2, p = \frac{1}{3}, \alpha = \frac{1}{2}, k = 0.4\}$ gives 0.52 versus 0.44. ■

Remark 2 *When buyers are risk neutral, i.e.* $\alpha = \gamma = 1$, *(with the convexity assumption), both the forward and the spot auctions are revenue superior to the forward auctions with insurance.*

With insurance (of the form buy-back guarantee), it might seem that the seller would be worse off as he is offering insurance for free. But it is not quite the case, as without this insurance bidders might be very cautious for their bids. That is why, for some parameter values, providing insurance in a forward auction is preferable for the seller.

7 Conclusion

We observe wide use of forward transactions (auctions-markets-contracts) in real world, yet theoretical analysis of forward auctions is virtually missing. Emergence of forward auctions might stem from different reasons such as credit constraints, risk hedging, or scheduling purposes. We provide an alternative explanation in the absence of these possible reasons, namely the revenue implications of buyers' risk attitudes. We present a simple model with risk (and loss) averse buyers and compare the equilibria of forward versus spot auctions on the basis of expected revenue. We also consider the revenue effects of the insurance provided by the seller. We observe that either of the spot or the forward auction can be revenue superior to the other. Interestingly, this superiority is independent of the quality of the "bad object." We also observed that spot auction is always revenue superior to forward auction with insurance, whereas forward auctions with insurance can be revenue superior to forward auctions.

Extension of our work to the multi-unit setting is the next natural step. Forward auctions for transmission capacity rights, for instance, would more naturally be modelled as a multi-unit auction where buyers have multi-unit demand. In a multi-unit setting, one would also be interested in seeing how the seller endogenously determines the number of units she sells in a forward auction. Insights from multi-unit forward auctions would also help us better understand forward markets in general.

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