Non-cooperative games with chained confirmed proposals[*]

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Abstract

We propose a bargaining process with alternating proposals as a way of solving non-cooperative games, giving rise to Pareto efficient agreements which will, in general, differ from the Nash equilibrium of the constituent games.

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1. Introduction

Since the seminal contributions by Nash (1950, 1953), bargaining models play a central role in the analysis of situations in which economic agents try to reach an agreement on the split of a certain asset. A plethora of approaches has resulted in a variety of bargaining mechanisms[1] keeping fixed the objective of splitting the pie. In a parallel and mostly independent effort, non-cooperative game theory undertook the task of determining the actions of individual agents in interactive strategic situations. While the efficiency of outcomes has been a central issue in both game theoretic paradigms[2], the role of bargaining as a determinant of individual actions in non-cooperative games has not been systematically explored.

In this paper, we illustrate the consequences of applying alternating proposal protocols as a way of solving non-cooperative games. From a technical point of view, the basic difference between our framework and that of bargaining over the split of a pie is that, in ours, two agents bargain about their strategies in a 2x2 game. Apart from the obvious departure from Rubinstein's (1982) model in that the set of possible agreements[3] is finite, in our setup, a confirmed agreement between bargaining agents concerns the pair of strategies in the *constituent* non-cooperative game. This fact increases by one the degrees of freedom and, thus, the dimension of the outcome space, allowing the use of bargaining with alternating proposals as a method of solving non-cooperative games.

Assume that two players bargain over the strategy profile to play, given that each player knows the opponent's set of possible strategies. Then, there is a constituent game whose execution leads to the two players' final payoffs and a *supergame* whose actions in each bargaining period are *proposals* of strategies for the constituent game. Games with *confirmed* proposals are interactive strategic situations in which a player, in order to give official acceptance of a contract, must confirm his/her proposed strategy combined with the strategy chosen by his/her opponent.[4] Here we focus on constituent games with complete information and finite strategy spaces. The bargaining supergame built on them is an infinite horizon game with perfect information. We show that the equilibrium outcome of the supergame can be unique even though each player's strategy space and the stages of the game itself are infinite. We call *equilibrium confirmed agreement* the corresponding equilibrium contract.

[1] While an exhaustive list of the relevant references is beyond the scope of this paper, it is worth mentioning Harsanyi (1956, 1962), Sutton (1986) and Binmore (1987).
[2] Relevant references are Harsanyi (1961), Friedman (1971), Smale (1980), and Cubitt and Sugden (1994).
[3] For a formal treatment of this issue, see the insightful analysis by Muthoo (1991).
[4] The constituent game can be a game with perfect or imperfect information and/or with complete or incomplete information. When information is incomplete, players can exploit the bargaining process to extract information on their opponent's type through their proposals.

2. The bargaining game

Throughout the paper, we assume that only two players are involved in the bargaining game and that they alternate proposals. We mainly focus on a specific family of games with confirmed proposals (GCP henceforth), those with *chained* proposals. That is, in the case of no confirmation by one player, the non confirmed strategy profile is taken as the new starting point for the subsequent negotiation. Thus, both players have the same power of confirmation and, except for the selection of the *first* mover at the beginning of the game, the rules of the game are symmetric.

Let us denote by S_h the finite strategy space for player h (with $h = i, j$) in the *constituent game*, i.e. the strategic situation whose players' strategy profiles induce all possible agreements of the GCP built on it. This is a bargaining supergame represented by a sequence of alternating proposals of the two players, which ends when a player confirms the proposal he/she made the previous stage in which he/she was active. By construction, the set of possible agreements of the GCP coincides with the set of outcomes of the constituent game, and the set of possible proposals of player h in the supergame coincides with the set of his/her strategies in the constituent game S_h. Denote by s_h^t the strategy proposed by player h in stage t, with $t = 1, 2, ..., +\infty$. Suppose that player i ("she") starts the bargaining supergame with player j ("he"). The sequence of alternating proposals is as follows:

Stage 1. Player i proposes a certain strategy $s_i^1 \in S_i$ to player j. Player i would actually play s_i^1 if (and only if) she would confirm this strategy after the reply of player j.

Stage 2. Player j proposes strategy $s_j^2 \in S_j$ to player i. This strategy would actually be played if (and only if) either i will confirm her previous strategy s_i^1 or j will confirm his proposal s_j^2 after the reply of player i.

Stage 3. Player i chooses whether or not to confirm her previous strategy s_i^1. If she confirms s_i^1, i.e. $s_i^3 = s_i^1$, then the bargaining process ends, through the sequence (s_i^1, s_j^2, s_i^1), with the confirmed agreement (s_i^1, s_j^2) and the two players receive the payoffs corresponding to the strategy profile (s_i^1, s_j^2) in the constituent game. If she does not confirm, i.e. she proposes a new strategy $s_i^3 \neq s_i^1$, the bargaining process continues with s_j^2 being player j's proposal and s_i^3 being player i's reply to j's proposal.

Stage 4. Player j chooses whether or not to confirm his previous strategy s_j^2. If he confirms s_j^2, i.e. $s_j^4 = s_j^2$, then the bargaining process ends, through the sequence (s_j^2, s_i^3, s_j^2), with the confirmed agreement (s_j^2, s_i^3) and the two players receive the payoffs corresponding to the strategy profile (s_i^3, s_j^2) in the constituent game. If he does not confirm, i.e. he proposes a new strategy $s_j^4 \neq s_j^2$, the bargaining process continues with s_i^3 being player *i*'s proposal and s_j^4 being player *j*'s reply to *i*'s proposal.

And so on and so forth.

If no strategy profile is ever confirmed by either player, then the outcome is the disagreement event Ω. Define with $f(s_h^{t-2}, s_{-h}^{t-1})$ the outcome of the GCP in case the agreement $(s_h^{t-2}, s_{-h}^{t-1})$ would be confirmed in stage *t*, with $t = 3, ..., +\infty$. We assume that each player *h*'s preference relation $\succsim_h$ satisfies the following conditions:[5]

(a) Disagreement is not better than any agreement: $\Omega \precsim_h f(s_h^{t-2}, s_{-h}^{t-1})$ for all $(s_h^{t-2}, s_{-h}^{t-1}) \in S_h \times S_{-h}$ and for all $t = 3, ..., +\infty$.

(b) Patience, i.e. the time of the agreement is irrelevant: if $s_h^{t-2} = s_h^{t'-2}$ and $s_{-h}^{t-1} = s_{-h}^{t'-1}$, then $f(s_h^{t-2}, s_{-h}^{t-1}) \sim_h f(s_h^{t'-2}, s_{-h}^{t'-1})$ for all $t \neq t'$, with $t, t' = 3, ..., +\infty$.

(c) Stationarity, i.e. the preference between two agreements does not depend on time: if $s_h^{t-2} = s_h^{t'-2}$, $s_{-h}^{t-1} = s_{-h}^{t'-1}$, $\tilde{s}_h^{t-2} = \tilde{s}_h^{t'-2}$ and $\tilde{s}_{-h}^{t-1} = \tilde{s}_{-h}^{t'-1}$, then $f(s_h^{t-2}, s_{-h}^{t-1}) \succsim_h f(\tilde{s}_h^{t-2}, \tilde{s}_{-h}^{t-1})$ if and only if $f(s_h^{t'-2}, s_{-h}^{t'-1}) \succsim_h f(\tilde{s}_h^{t'-2}, \tilde{s}_{-h}^{t'-1})$, for all $t \neq t'$, with $t, t' = 3, ..., +\infty$.

We refer to the extensive game with perfect information thus defined as the (bargaining) *game with chained confirmed proposals*. In the next section, we analyze the GCP version of some famous interactive strategic situations, extensively studied both in the theoretical and in the experimental literature. First, we show two examples in which the constituent game is a 2x2 static game. Then, we concentrate on three cases where the constituent game is dynamic.

3. Confirmed agreements in standard two-player games

3.1 Static Constituent Games

Consider the GCP version of the *Prisoner's Dilemma* (PD). The constituent game is a standard static PD, and the bargaining game built on it is an infinite horizon game with perfect and

[5] Conditions (a) and (c) characterize also Rubinstein (1982), while in Rubinstein's model time is valuable and a discount factor is introduced accordingly.

complete information. The sets of players' feasible proposals in the GCP coincide with their sets of actions in the constituent game: $S_i = S_j = \{Cooperate, Defect\}$, henceforth $\{C, D\}$. Figure 1, with $a > c > d > z$ shows the simultaneous-move constituent game and all possible agreements of the bargaining (super)game with confirmed proposals one can build on it.

$i \backslash j$	D	C
D	(d,d)	(a,z)
C	(z,a)	(c,c)

Figure 1. Payoff matrix of the PD game

The constituent game has the profile (D, D) as equilibrium in dominant actions. The same equilibrium outcome would be found in the standard two-stage game (without bargaining and without confirmation), where one of the two players moves first and the other observes his/her "proposal" before choosing his/her own.

Let us now calculate the subgame perfect equilibrium outcome of the GCP version of the PD game. Throughout the paper, we focus on subgame perfect equilibria in weakly dominant strategies: we assume that in equilibrium both players bargaining in the GCP choose a strategy that weakly dominates all the others.

Observe Figure 2. The set of feasible payoffs of the bargaining game is the same as in the PD in Figure 1. The first of the two payoffs always refers to player *i*, as in the constituent game. In every decision node, the active player's preferred action is marked in bold. Whenever he/she is indifferent between two or more actions, the preferred actions are marked with dotted bold lines.

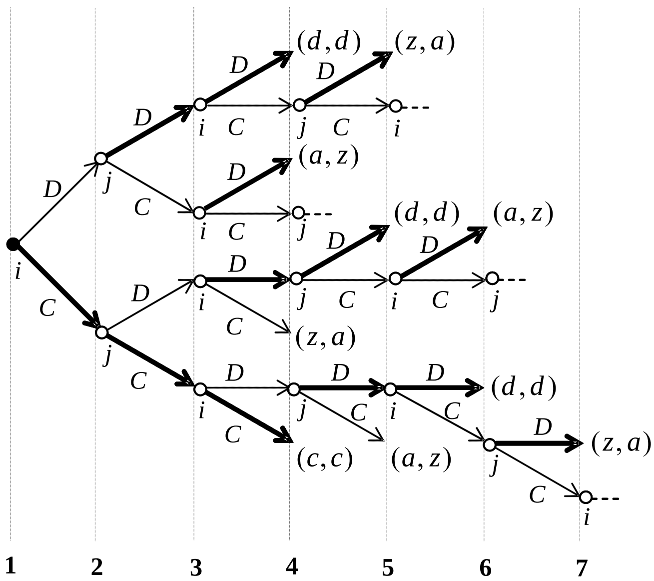

Figure 2. PD with confirmed proposals

One main result is:

Proposition 1. The PD with chained confirmed proposals has a *unique* subgame perfect equilibrium in weakly dominant strategies, inducing the cooperative confirmed agreement in the first stage in which a player is allowed to confirm his/her proposal.

Proof. Let us consider the infinite game in Figure 2. Each tree branch belonging to the equilibrium path is part of a weakly dominant strategy. More precisely, each player's strategy leads to the following result: the payoff obtained by the player through confirming at a stage *t* equals the highest payoff he can get by continuing the game. Moreover, in this game, for any $\hat{t} > t$ this highest payoff can be obtained only by confirming the same agreement confirmed at *t*. Observe that in Figure 2 (first 7 stages of the game) there are four decision nodes where a player can confirm the agreement yielding him/her *a*, his/her highest payoff possible. At stage $t = 4$, after the non-terminal history (D, D, C), player *j* can get *a* by choosing *D*, hence confirming his most preferred agreement. If, instead of confirming, player *j* chooses to continue the game, he can get, in any subgame in the continuation game, at most a payoff of *a*, by confirming the same agreement he could already confirm at stage $t = 4$. Therefore, for player *j* confirming (D, C) at stage $t = 4$ weakly dominates continuing the game. The same holds for player *i* at stage $t = 3$, after history (D, C), and at stage $t = 5$ after history (C, D, D, C); and for player *j* at stage $t = 6$ after history (C, C, D, D, C). Therefore, each player's equilibrium strategy prescribes confirming the favourable asymmetric agreement whenever possible. At the same time, in order to prevent the opponent from doing the same, each player's equilibrium strategy prescribes confirming also the agreement (D, D) whenever possible. This leads both players to propose *D* in each stage *t* every time at least one of the two players has proposed *D* in at least a $\tilde{t} < t$ and to propose *C* otherwise. This leads to the terminal history (C, C, C). ■

Thus, in the unique subgame perfect equilibrium of the game, player *i* starts by proposing strategy *C* to player *j*, who counter-proposes strategy *C*. Then, player *i* confirms her strategy *C*, such that the constituent game strategy profile (*C*,*C*) is the (unique) confirmed agreement. This is reached already in stage $t = 3$, after the first interaction among players takes place.

Consider now the GCP version of the *Battle of Sexes* (BS). The set of players' feasible proposals, which coincides with the set of players' actions in the constituent game, is $S_i = S_j = \{Opera, Football\}$, henceforth $\{O, F\}$. Figure 3 shows the one-shot constituent game and, also, all possible agreements of the bargaining GCP one can build on it. Parameters are such that $a > b > \max\{c_1, c_2, c_3, c_4\}$.

$i \backslash j$	O	F
O	(a,b)	(c_1,c_2)
F	(c_3,c_4)	(b,a)

Figure 3. Payoff matrix of the BS game

The constituent game has two Nash equilibria in pure strategies: (O,O) and (F,F). In the standard two-stage dynamic version of the game, the player moving first[6] has an advantage. Consider now the BS with confirmed proposals for the case in which player *i* is the first mover (Figure 4). Observe that, surprisingly, there is a *first-mover disadvantage*: the unique equilibrium confirmed agreement coincides with the Nash equilibrium of the constituent game which is preferred by player *j*.

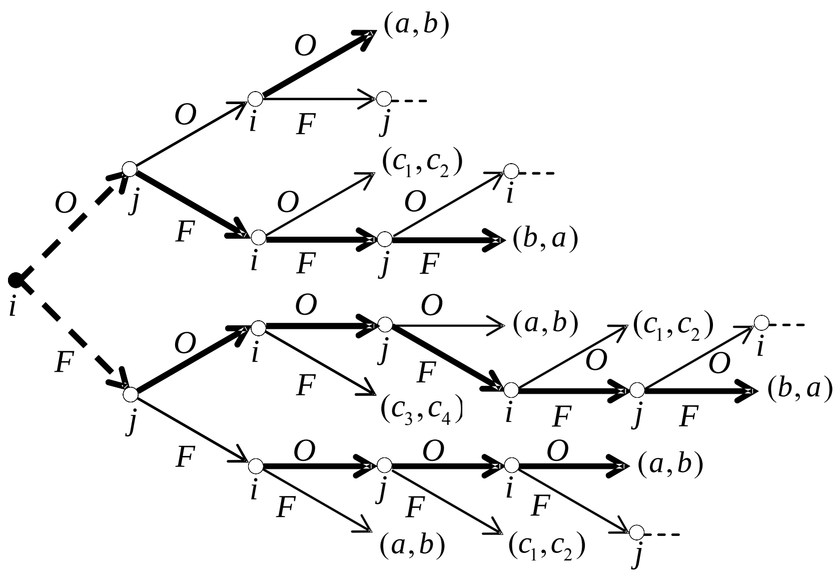

Figure 4. BS with confirmed proposals

Proposition 2. The BS with chained confirmed proposals has a *unique* equilibrium confirmed agreement in weakly dominant strategies, involving players' coordination on the constituent game equilibrium favourable to the second mover.

Proof. The game ends with the confirmation of the strategy profile (*F*, *F*), whatever is player *i*'s initial proposal. In each of the two subgame perfect equilibria in pure strategies, *j* replies to *i*'s first proposal by indicating the opposite proposal (*F* if *O* and *O* if *F*). By doing that, *j* obliges player

[6] In fact, this is equivalent to a commitment.

i to propose the same action already proposed by him (otherwise, *i* would confirm her initial action and would get $c_1 < b$ or $c_3 < b$). If this action is *F*, then *j* confirms *F* and gets his highest payoff possible. If instead this action is *O*, then *j* proposes *F* and *i* finds convenient to propose *F,* since, otherwise, she would get $c_1 < b$; then *j* confirms *F* and gets his highest payoff possible. Therefore, in the first stage player *i* is indifferent between her two possible proposals. ■

Thus, in the BS with chained confirmed proposals the second mover is able to confirm the coordinating equilibrium outcome of the constituent game more favourable to him.

3.2 Dynamic Constituent Game

When the constituent game is dynamic, the confirmed proposals structure can be built relying on the strategic form of the constituent game. The set of strategies of each player in the constituent game corresponds to the set of possible proposals in the corresponding GCP.

Consider, for example, the *Trust Game* (TG). In the constituent game, player *i* (the *truster*) chooses whether to *Trust* (*T*) or to *Not trust* (*N*) player *j* (the *trustee*). In case *i* trusts *j*, total profits are higher. In that case, *j* would decide whether to *Grab* (*G*) or to *Share* (*S*) the higher profits. The strategic form of the game in Figure 5, where $\underline{x}$:="*x* if *T*", with $x = G, S$, and $c_i > d_i > z \geq 0$, $a > c_j > d_j > 0$, $a + z = c_i + c_j$, represents all possible agreements of the bargaining GCP built on this dynamic game.[7]

$i \backslash j$	$\underline{G}$	$\underline{S}$
N	(d_i, d_j)	(d_i, d_j)
T	(z, a)	(c_i, c_j)

Figure 5. Payoff matrix of the TG

In the unique subgame perfect equilibrium of the constituent game, *i* does not trust *j*, while the latter would choose to grab if *i* had trusted him in the first place.

Given players' role asymmetries in the constituent game, the resulting GCP involves two possible versions: one in which the *truster* in the constituent game (*i*) is the first mover of the bargaining sequence, and one in which the *trustee* in the constituent game (*j*) is the first mover of

[7] Notice that, in order for *j* to confirm an agreement, he has to re-propose the same strategy in two subsequent stages in which he is active. According to this rule, for example $(\underline{S}, T, \underline{G})$ is not a terminal history of the GCP, even though both strategy profiles $(T, \underline{S})$ and $(T, \underline{G})$ induce the same terminal history in the constituent game.

the bargaining sequence. In this last case, *j* begins the GCP by announcing his intention to grab or to share the higher total profits in case *i* would trust him. The two versions of the TG with chained confirmed proposals are represented in Figure 6.a and 6.b respectively. Recall that the first of the two payoffs always refers to player *i*, as in the constituent game.

Figure 6.a

Figure 6.b

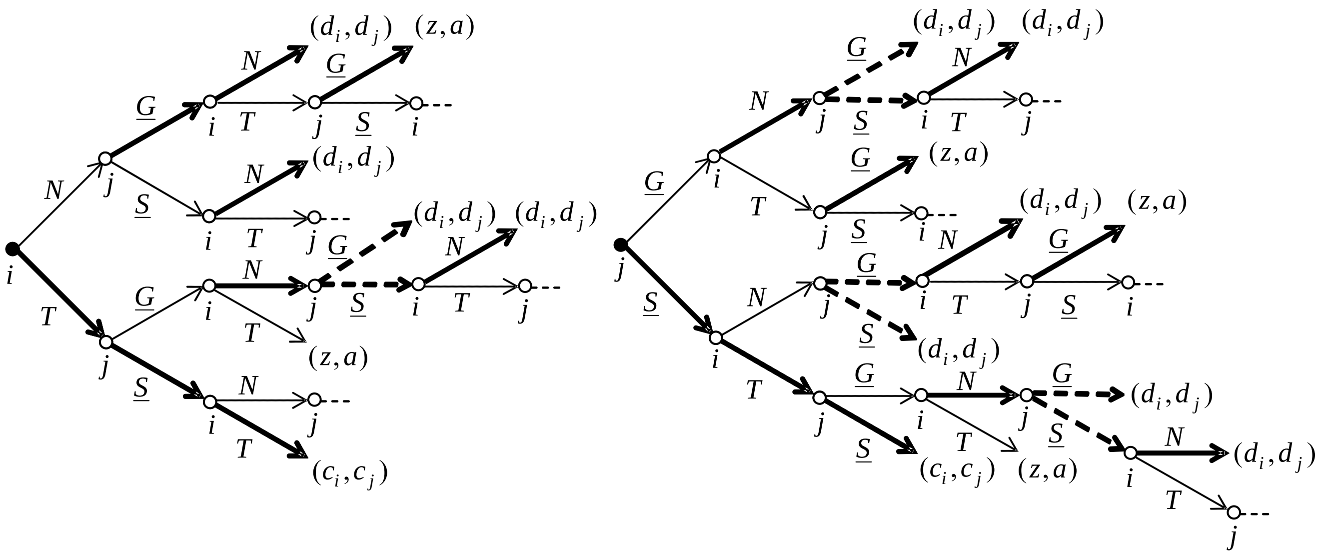

Figure 6. TG with confirmed proposals, with *i* (Figure 6.a) or *j* (Figure 6.b) as first mover

For the two GCP in Figure 6, the following result holds.

Proposition 3. The TG with chained confirmed proposals has a *unique* equilibrium confirmed agreement in weakly dominant strategies, the cooperative one. This agreement is immediately confirmed by the first mover in the GCP.

Proof. Given that both players follow strategies that are weakly dominant, in both GCP in figure 6, at each stage *t* each player would: (1) confirm his/her most preferred agreement if he/she is given the possibility in that stage; (2) confirm agreements other than his/her most preferred if: (2.1) in some stage $\hat{t} > t$ (with $\hat{t} < \infty$) of the equilibrium continuation path, his/her opponent would confirm an agreement not better for him/her than the one he/she could confirm in *t*; (2.2) by not confirming in *t*, neither (1) nor (2.1) applies to any stage *t* + *k*, with $k = 1,...,+\infty$, and the best agreement he/she could confirm when he/she is active in the continuation subgame is the same he/she could confirm in stage *t*. When the first mover is player *i*, in stage 3 she would confirm $(T,\underline{S})$ because of (1). She would confirm also $(N,\underline{G})$ because of (2.1), and $(N,\underline{S})$ because of (2.2). Instead, she would not confirm $(T,\underline{G})$, given that none of the above mentioned cases applies. Hence, she would propose *N* after history $(T,\underline{G})$. In stage 4, after $(T,\underline{G},N)$, player *j* is indifferent between confirming the agreement $(N,\underline{G})$ and proposing $\underline{S}$. In both cases the payoffs are (d_i,d_j), since, if he proposes $\underline{S}$, in the subsequent stage, player *i* would confirm $(N,\underline{S})$ because of (2.2) (as we have previously seen after history $(N,\underline{S})$). Thus, the subgame perfect equilibrium path is $(T,\underline{S},T)$, with *i* confirming the agreement $(T,\underline{S})$ in stage 3. When the first mover is player *j*, in equilibrium he confirms the same agreement in stage 3. This follows from the fact that in stage 3 he would confirm the agreement $(T,\underline{G})$ because of (1), $(T,\underline{S})$ because of (2.1) and he is indifferent between confirming or not the agreements $(N,\underline{G})$ and $(N,\underline{S})$ because of (2.2) (as we have already seen when the first mover is the player *i*, after history $(T,\underline{G},N)$). ■

Notice that, as in the example of the BS, in both versions of the TG with chained confirmed proposals, the second mover reciprocates in stage 2 the first-mover's proposal: he/she cooperates if the first-mover's proposal is cooperative ($\underline{S}$ if *T* and *T* if $\underline{S}$, respectively) and does not cooperate otherwise ($\underline{G}$ if *N* and *N* if $\underline{G}$, respectively).

Consider now the dynamic *Entry Game* (EG). In the constituent game *i* (the potential *entrant*) chooses whether to *Enter* (*E*) or to *Stay Out* (*S*) of the market, with *j* (the *incumbent*) deciding whether to *Accommodate* (*A*) or *Fight* (*F*) if the entrant decides to enter. The strategic form of the game in Figure 7, where $\underline{x}$:="*x* if *E*", with $x = A, F$, and $a > d > 0 > z$, represents all possible agreements of the bargaining GCP built on this dynamic game. In the unique subgame perfect equilibrium of the constituent game, *i*'s entry takes place, with *j* accommodating it.

$i \backslash j$	$\underline{A}$	$\underline{F}$
E	(d,d)	(z,z)
S	$(0,a)$	$(0,a)$

Figure 7. Payoff matrix of the EG

Instead, in all subgame perfect equilibria of the correspondent GCP, the entrant stays out. The two possible versions of the EG with chained confirmed proposals are represented in Figure 8. The first version, in Figure 8.a, represents the case in which player *i*, the potential *entrant* in the constituent game, moves first in the corresponding GCP. In the second version - Figure 8.b - player *j*, the *incumbent* in the constituent game, is the first mover.

For both GCP in Figure 8, the following result holds.

Proposition 4. The EG with chained confirmed proposals has *two payoff-equivalent* confirmed agreements in weakly dominant strategies, which involve the entrant to stay out.

Proof. For the version of the game in Figure 8.a, where the first mover is player *i*, the proposition can be proved using the same reasoning as in the proof of Proposition 2. Notice that in the two GCP (Figure 4 and Figure 8.a) a player *h* (with $h = i, j$) has the same equilibrium strategy: *j* replies to *i*'s "kind" ("unkind") proposal by indicating the "unkind" ("kind") proposal. By doing that, *j* obliges player *i* to propose in stage 3 the action she did not propose in stage 1. And so on and so forth. The only difference with respect to the BS is that in the EG, after history $(S,\underline{A})$, player *i* is indifferent between confirming this agreement and proposing *E*, because condition (2.1) (see proof of Proposition 3) applies. Thus, besides the agreement $(S,\underline{F})$, also $(S,\underline{A})$ is an equilibrium agreement, equivalent in payoff to $(S,\underline{F})$. More precisely, if player *i* starts the bargaining process, there are three equilibrium terminal histories: $(E,\underline{F},S,\underline{F})$, $(S,\underline{A},E,\underline{F},S,\underline{F})$ and $(S,\underline{A},S)$. When the first mover is player *j* (Figure 8.b), there are two equilibrium terminal histories: $(\underline{A},E,\underline{F},S,\underline{F})$ and $(\underline{A},S,\underline{A})$. This follows from the fact that: each player always confirms agreements leading to his/her highest payoff possible, i.e. (d,d) for player *i* and $(0,a)$ for player *j*; players' optimal behavior after history $(S,\underline{A},E)$ in Figure 8.b is the same as after history $(\underline{A},E)$ in Figure 8.a; after history $(\underline{A})$, player *i* is indifferent between *E* and *S* because condition (2.1) (see proof of Proposition 3) applies. ■

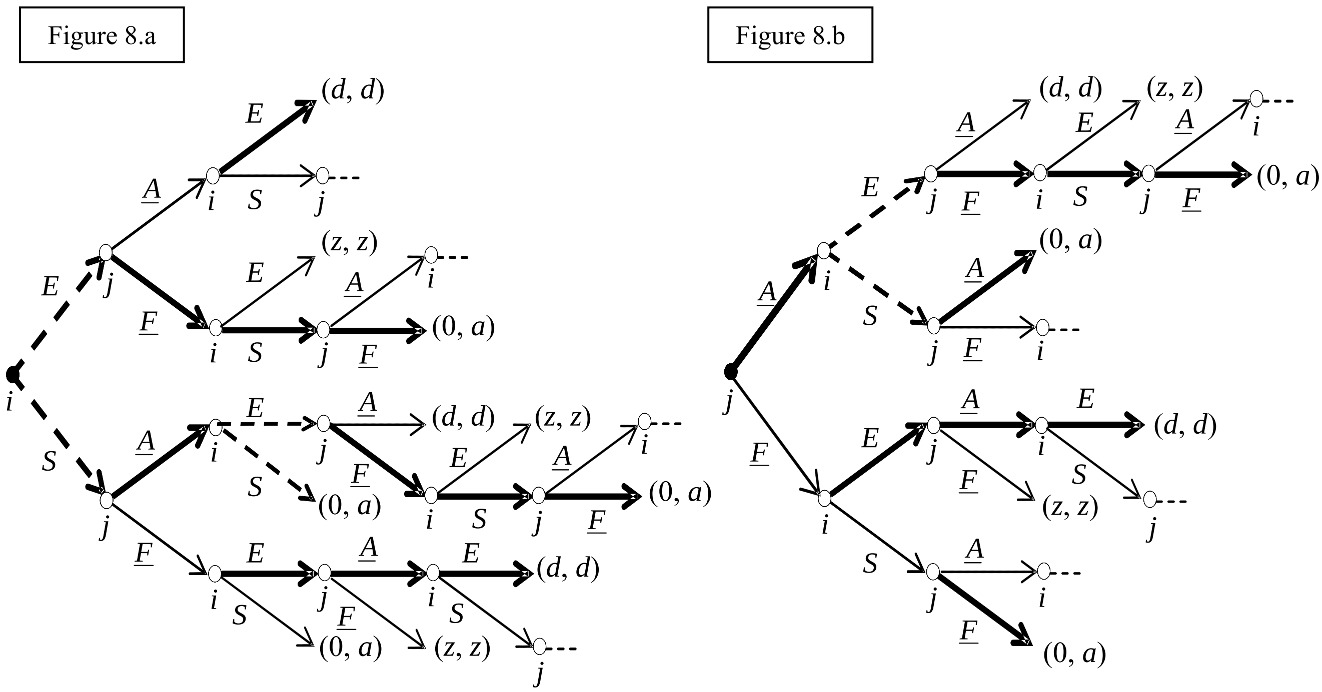

Figure 8. EG with confirmed proposals, with *i* (Figure 8.a) or *j* (Figure 8.b) as first mover

Therefore, in both GCP in Figure 8, there is an equilibrium confirmed agreement in which the incumbent threatens to fight, $(S, \underline{F})$, and an additional one in which he would accommodate in case his opponent would enter, $(S, \underline{A})$. In both agreements the potential entrant accepts to stay out. If player *i* is the first mover (Figure 8.a), in equilibrium she will either immediately confirms the agreement $(S, \underline{A})$ or she will no longer be able to confirm any agreement at all: only player *j* would confirm from stage 4 onwards. If player *i* is not the first mover (Figure 8.b), in equilibrium, she will never be able to confirm any agreement: only player *j* can confirm in stage 3, 4 or 5. Notice that, in the GCP version of the EG, the following properties hold:

(i) only two agreements can be confirmed in equilibrium;

(ii) the two equilibrium confirmed agreements are payoff-equivalent;

(iii) both of them are Pareto efficient;

(iv) none of them is a subgame perfect equilibrium of the constituent game;

(v) in one of the two equilibrium agreements player *i*'s strategy is not even a best reply in the constituent game;

(vi) the second mover in the constituent game (*j*) is able to benefit from the confirmed proposals structure, getting his highest payoff possible;

(vii) in the equilibrium path, player *i* is indifferent between her two possible proposals in the first stage in which she is active;

(viii) properties (i) – (vii) hold independently of whether the player is assigned the role of first mover in the GCP.

We show below that the same features emerge when analyzing the confirmed proposals version of a totally different strategic interaction setting: the *Ultimatum Game* (UG). In the constituent game, *i* (*proposer*) can offer a fair (*F*) or unfair (*U*) division to *j* (*respondent*); the latter, after having received *i*'s offer, may either accept (*A*) or reject (*R*). In the confirmed proposals version of this game, the set of *i*'s possible proposals coincides with her actions in the constituent game, while the set of *j*'s possible proposals coincides with his strategies in the constituent game, i.e. $S_j = \{\underline{AA}, \underline{AR}, \underline{RA}, \underline{RR}\}$, with $\underline{x}\,\underline{y}$:="*x* if *F* and *y* if *U*", with $x, y = A, R$.

The strategic form of the UG in Figure 9 (with $a > f > b > 0$) represents all possible agreements of the bargaining GCP built on this dynamic game.[8]

[8] Recall that confirmation is achieved through re-proposal of the same strategy, thus histories like $(\underline{AR}, F, \underline{AA})$ are not terminal for the GCP, even though both strategy profiles $(F, \underline{AR})$ and $(F, \underline{AA})$ induce the same terminal history in the constituent game.

$i \backslash j$	$\underline{AA}$	$\underline{AR}$	$\underline{RA}$	$\underline{RR}$
F	(f,f)	(f,f)	$(0,0)$	$(0,0)$
U	(a,b)	$(0,0)$	(a,b)	$(0,0)$

Figure 9. Payoff matrix of the UG

In the unique subgame perfect equilibrium of the constituent game, unfair division takes place, with j accepting both i's offers.

Figure 10 shows the two possible versions of the UG with chained confirmed proposals. Figures 10.a refers to the case in which the proposer in the constituent game moves first. Figure 10.b shows the version with the responder in the constituent game being the first mover in the UG with confirmed proposals.

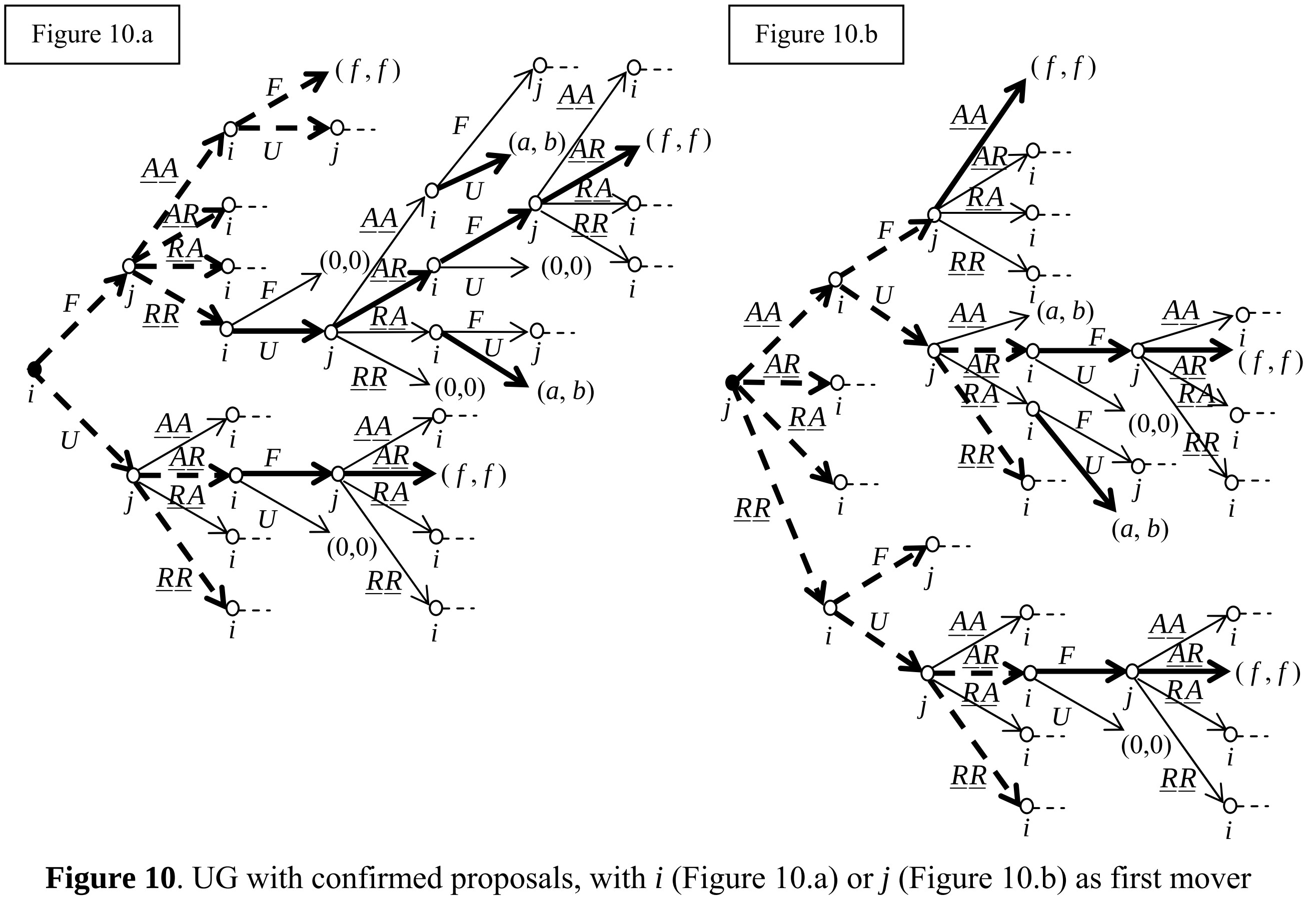

Figure 10. UG with confirmed proposals, with i (Figure 10.a) or j (Figure 10.b) as first mover

For both GCP in Figure 10, the following result holds.

Proposition 5. The UG with chained confirmed proposals has an infinite number of subgame perfect equilibria, all leading to *two payoff-equivalent* confirmed agreements in weakly dominant strategies, which involve the egalitarian outcome.

Proof. The proof is similar for the two versions of the GCP. Consider the first version of the game, where *i* is the first mover (Figure 10.a). Player *i* is never able to confirm her most preferred agreement $(U,\underline{AA})$ or $(U,\underline{RA})$. In fact, if she proposed an unfair offer, *j* would reply with a proposal not incorporating the acceptance of *U* ($\underline{AR}$ or $\underline{RR}$), otherwise *i* would confirm *U* in the subsequent stage. Therefore, in every subgame after *i* has proposed *U*, *j* would never propose $\underline{AA}$ or $\underline{RA}$. Then, both after each history of the type $(...,\hat{s}_j^t,U,\underline{AR})$ with $\hat{s}_j^t \neq \underline{AR}$ and after each $(...,\tilde{s}_j^t,U,\underline{RR})$ with $\tilde{s}_j^t \neq \underline{RR}$, *i* would propose *F*, otherwise she would confirm *j*'s rejection of her unfair offer, hence getting 0. Then, after each $(...,\hat{s}_j^t,U,\underline{AR},F)$ with $\hat{s}_j^t \neq \underline{AR}$, *j* would propose $\underline{AR}$, thus confirming his most preferred agreement $(F,\underline{AR})$; after each $(...,\tilde{s}_j^t,U,\underline{RR},F)$ with $\tilde{s}_j^t \neq \underline{RR}$, *j* would never propose $\underline{RR}$, because that would lead to confirm an agreement involving the rejection of *i*'s fair offer, hence getting 0. Moreover, player *j* is always able to avoid confirming the agreements $(U,\underline{RR})$ and $(U,\underline{AR})$. In fact, after each history $(...,\underline{RR},U)$, he would propose the strategy $\underline{AR}$, thus avoiding to confirm the zero-payoff agreement $(U,\underline{RR})$ or to give player *i* the possibility to confirm $(U,\underline{AA})$ or $(U,\underline{RA})$ in the subsequent stage; after each history $(...,\underline{AR},U)$, he would propose $\underline{RR}$, thus avoiding to confirm the zero-payoff agreement $(U,\underline{AR})$, or to give player *i* the possibility to confirm $(U,\underline{AA})$ or $(U,\underline{RA})$ in the subsequent stage. This explains why whenever an agreement is confirmed by *j* in a stage $t>3$, this agreement is $(F,\underline{AR})$ and each terminal history is of the type $(...,U,\underline{AR},F,\underline{AR})$. All previous considerations about player *j*'s optimal behavior conditional on *i*'s feasible proposals explain why player *i* could never obtain the confirmation of an agreement which would impose player *j* to accept an unfair offer. Therefore, whenever in a stage $t>3$ she has to choose between continuing the game through the terminal sub-history $(U,\underline{AR},F,\underline{AR})$ and confirming an agreement $(F,\underline{Ax})$, with $x=A,R$, she is indifferent according to condition (2.2) (see proof of Proposition 3): both agreements allow player *i* to get her second highest payoff possible and player *j* to get his highest payoff possible. Moreover, given that *i* would always propose *F* in each decision node where she could confirm $(U,\underline{RR})$, then, after every sequence of proposals $(F,\bar{s}_j^t,U,\underline{RR})$ with $\bar{s}_j^t = \underline{AA},\underline{AR},\underline{RA}$, some equilibrium loops could emerge, thus leading to an infinite number of equilibrium terminal histories, all ending with player *j*'s confirmation of the agreement $(F,\underline{AR})$, or with player *i*'s confirmation of the agreement $(F,\underline{Ax})$, with $x=A,R$. The only case in which player *j* can confirm, in equilibrium, an agreement different from $(F, \underline{AR})$ is in stage 3 of the GCP (Figure 10.b), where he confirms the payoff-equivalent agreement $(F, \underline{AA})$. ■

Therefore, in every subgame perfect equilibrium of the UG with chained confirmed proposals, *i* offers a fair division to *j*, and *j* accepts. In each GCP in Figure 10 we indicated the unique equilibrium terminal history leading to confirm the agreement $(F, \underline{AA})$ in stage 3, and two among the infinite possible equilibrium terminal histories leading to confirm the agreement $(F, \underline{AR})$ in a stage $t > 3$. Both kind of agreements lead to a fair division.

Quite surprisingly, the equilibrium agreements of the confirmed proposals version of the UG satisfy the same features (i) – (viii) characterizing the equilibria in the EG with confirmed proposals. In this regard, note also the strong similarity between the subgame perfect equilibrium paths of TG with confirmed proposals in Figure 6.a and those of the PD with confirmed proposals in Figure 2. The same holds for the EG with confirmed proposals in Figure 8.a and the BS with confirmed proposals in Figure 4. Finally, for the three GCP in Figure 4 (BS), 8.a (EG) and 10.a (UG) it is common that a first-mover disadvantage exists, while instead the relative constituent games are all characterized by a first-mover advantage.

All these results suggest that the confirmed proposal mechanism works in the same way for dynamic constituent games with different strategic structures.

4 Conclusions

Throughout the paper, we have defined Games with Confirmed Proposals (GCP) and shown their effect on agents' ability to coordinate on Pareto efficient outcomes even in cases in which they are not equilibrium outcomes of the constituent non-cooperative game. Our focus was on a confirmed proposal mechanism with a *chain,* requiring that each non-confirmed strategy profile becomes the starting point for the next negotiation round. One could discuss the implications of breaking this chain on the main features of the confirmed proposal mechanism. We leave this for future research.

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