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ABSTRACT

Employment Fluctuations with Downward Wage Rigidity: The Role of Moral Hazard[*]

This paper studies the cyclical dynamics of Mortensen and Pissarides' (1994) model of job creation and destruction when workers' effort is not perfectly observable, as in Shapiro and Stiglitz (1984). An occasionally-binding no-shirking constraint truncates the real wage distribution from below, making firms' share of surplus weakly procyclical, and may thus amplify fluctuations in hiring. It may also cause a burst of inefficient firing at the onset of a recession, separating matches that no longer have sufficient surplus for incentive compatibility. On the other hand, since marginal workers in booms know firms cannot commit to keep them in recessions, they place little value on their jobs and are expensive to motivate. For a realistic calibration, this last effect is by far the strongest; even a moderate degree of moral hazard can eliminate all fluctuation in the separation rate. This casts doubt on Ramey and Watson's (1997) "contractual fragility" mechanism, and means worker moral hazard only makes the "unemployment volatility puzzle" worse. However, moral hazard has potential to explain other labor market facts, because it is consistent with small but clearly countercyclical fluctuations in separation rates, and a robust Beveridge curve.

JEL Classification: C78, E24, E32, J64

Keywords: job matching, shirking, efficiency wages, endogenous separation, contractual fragility

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1 Introduction

Matching models are now widely applied in macroeconomic and microeconomic studies of unemployment, but their empirical success remains controversial, as recent discussion of the "unemployment volatility puzzle" has shown (Hall 2005a, Shimer 2005a, Costain and Reiter 2008, Hagedorn and Manovskii 2008).[1] The predominant conclusion from these debates, advocated by Hall and Shimer,[2] points to the effects of wage rigidity (in new jobs) on job creation. Wage rigidity increases the procyclicality of profits, amplifying fluctuations in hiring incentives and thus in vacancies and unemployment. Alternatively, time variation in job separation rates has also been identified as a margin that might drive unemployment fluctuations, by Mortensen and Nagypal (2007a) and Fujita and Ramey (2009), among others; but these authors do not attempt to explain *why* the separation rate varies. One theory that derives amplification entirely from the separation margin is the "contractual fragility" mechanism of Ramey and Watson (1997) and den Haan, Ramey, and Watson (1999). These papers stress incentive problems as a way of enhancing the countercyclicality of separation rates, arguing that in recessions, workers and firms may be forced to sever their relationships because match surplus is insufficient to maintain incentive compatibility.

In this paper, we study the cyclical dynamics of a version of the Mortensen and Pissarides (1994) model that incorporates aspects of both these approaches to labor market volatility. As in Shapiro and Stiglitz (1984), we consider a moral hazard problem in which firms cannot perfectly monitor workers' effort. Therefore, a firm-worker pair bargain over match surplus subject to a no-shirking condition. This constraint truncates the wage distribution from below, and since we assume the utility cost of effort varies less than labor productivity, firms may have to pay workers a larger share of match surplus in recessions, making profits more procyclical. Therefore, this form of downward wage rigidity has the potential to amplify fluctuations in hiring. By the same token, the incentive compatibility constraint may force firms to terminate jobs that still have positive surplus, if this surplus is insufficient to prevent shirking. Therefore, moral hazard also has the potential to cause a burst of inefficient separations when a negative aggregate shock occurs.

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[1] Recent surveys on the theory and applications of labor market matching models include Mortensen and Pissarides (1999), Petrongolo and Pissarides (2001), Rogerson, Wright, and Shimer (2005), and Yashiv (2007). Other empirical critiques of matching models, on grounds independent of "Shimer's puzzle", include Cole and Rogerson (1999), Fujita (2004), and Ravn (2006).

[2] See also Gertler and Trigari (2008), Walsh (2005), and Krause and Lubik (2007).

With its no-shirking condition, our model considers a form of real downward wage rigidity that applies to new as well as continuing jobs, and microfounds it on informational frictions. Our form of wage rigidity is less extreme than that in models where wage stickiness is imposed as an arbitrary constraint (like Shimer 2004 or Gertler and Trigari 2008), but this is realistic, since empirically wage rigidity for new jobs appears weak (Haefke, Sonntag, and van Rens 2008). Thus our goal here is not a new "solution" for the unemployment volatility puzzle, but rather a qualitative and quantitative analysis of search and matching under one of the canonical structural models of rigid wages. Several papers have studied the flip side of information asymmetry in labor relations, namely firms' private information about match productivity (Menzio 2005; Kennan 2008; Guerrieri, Shimer, and Wright 2008). Others have looked at moral hazard in the steady state of the Mortensen-Pissarides model (Mortensen and Pissarides 1999, Rocheteau 2001, Jansen 2001, Tawara 2008), or in a dynamic setup with exogenous separation (Bruggemann and Moscarini 2007, Park 2007).[3] However, we know of no other study of the cyclical dynamics of a matching model with endogenous separation subject to a no-shirking constraint, and we identify a number of effects absent in papers that focus on steady states or on exogenous or efficient separation.[4]

Another ongoing controversy our model naturally addresses concerns the relative importance of the hiring and separation margins for labor market fluctuations. Using new data, Shimer (2005b) and Hall (2005b) argue that changes in separation rates matter much less for unemployment dynamics than was previously thought; Fujita and Ramey (2009) and Elsby, Michaels and Solon (2009) dispute this, arguing that inflows into unemployment explain at least 35-40% of the rise in unemployment in US postwar recessions. Either way, by taking both the hiring and separation margins seriously, our model can confront a wider range of cyclical facts. As Figure 1 shows, hiring and separation rates are strongly negatively correlated, and the separation rate rises in every NBER-identified recession. Thus a complete understanding of the cyclical dynamics of unemployment requires a model with endogenous job destruction, but full wage rigidity is therefore a problematic assumption since it may imply inefficient separations that ought to be prevented by renegotiation, as Barro (1977) pointed out. Our framework takes this issue into account since it models the implications of information constraints for wages and separation simultaneously, allowing for unlimited renegotiation.

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[3]Park (2007) endogenizes temporary layoffs but treats permanent separation as exogenous.

[4]Mortensen and Nagypal (2007b) show that under certain conditions, efficient endogenous separation implies unemployment dynamics that are observationally equivalent to the case of exogenous separation. But under asymmetric information this is no longer true, since separation is inefficient and workers' surplus share varies.

Moral hazard complicates the Mortensen-Pissarides framework, because it implies that match surplus is unlikely to be a continuous function. Under full information, separation only occurs when it is mutually beneficial, so surplus goes continuously to zero at the separation threshold. But with unobservable effort, violation of the no-shirking condition drives output suddenly to zero, making the surplus function discontinuous. A related issue is that even fixing aggregate labor market conditions, the separation problem of a firm-worker pair may have multiple solutions, because the amount of surplus and therefore also the no-shirking constraint both depend on the firm's reservation thresholds. That is, if for any reason a marginal worker expects to be fired more frequently, the firm will have to pay them more to induce effort, and may therefore simply prefer to fire them. Due to this feedback between the reservation strategy and the minimum incentive-compatible wage, contraction properties of the original Mortensen-Pissarides model no longer apply. Nonetheless, using arguments like those of Rustichini (1998), we show in Section 3 that there exists a unique maximal surplus function and minimal vector of reservation thresholds that satisfy incentive compatibility. By focusing on this unique pairwise-optimal equilibrium, we effectively restrict attention to outcomes that are not subject to the Barro (1977) critique.

To preview our results, in Section 4 we show analytically in a simplified example that moral hazard may, in theory, increase or decrease the variability of the separation rate. If the probability of passing from boom to recession is sufficiently low, firms will fire their least productive workers at the start of a recession, and the fraction fired is increasing in the degree of moral hazard. But when the probability of passing from boom to recession is higher, we obtain the opposite effect: greater moral hazard decreases the spike of firing at the onset of a recession, and we calculate a parameter threshold beyond which the reservation productivities collapse to a single value, so that the separation rate is constant over the business cycle. We also show that greater moral hazard increases the procyclicality of firms' surplus share, which tends to make hiring more variable.

In Section 5, we calibrate our model to U.S. data. Surprisingly, given the ambiguity of the theoretical results, our quantitative findings are strong and robust: moral hazard decreases cyclical labor market volatility, primarily by smoothing or even eliminating fluctuations in the firing rate. The intuition behind this finding is a time-inconsistency problem. In booms some marginal jobs survive that are destroyed when the economy goes into recession. Since firms cannot commit to maintain these marginal jobs in the future, they need to pay a higher flow of surplus to workers in these jobs to prevent shirking. Hence, while the no-shirking constraint makes workers more expensive overall, raising the reservation productivities and causing inefficient job churning, it especially

4

affects the cost of marginal workers in booms. This is why it can push up the reservation threshold for booms until it coincides with the threshold for recessions, at which point all variation in separation rates is eliminated.

Thus the idea that incentive problems could amplify fluctuations of separation fails in a calibrated dynamic model; in fact, the forward-looking nature of wage bargaining actually reverses the result. Existing papers on "contractual fragility" missed this point because they either analyzed the effects of a one-time shock to productivity, or, as in Ramey and Watson (1997), assumed an *i.i.d.* idiosyncratic productivity component, so that next period's expected productivity is the same for all jobs. In contrast, in our model idiosyncratic productivity is persistent, as in the data.[5] The strong stabilizing effect of moral hazard on separation rates means that our model makes no progress on the unemployment volatility issue. Nonetheless, it is strikingly consistent with the recent claims that unemployment variability is driven mostly by job creation, not by job destruction. Moreover, because our model amplifies fluctuations in creation while diminishing those of destruction, it also exhibits a robust Beveridge curve.

2   Model

This section describes a version of the Mortensen and Pissarides (1994) model with large firms and imperfectly observable worker effort. Firms motivate workers by paying a surplus on top of the reservation wage, and threatening to fire shirkers. Jobs are severed when negative productivity shocks render them unprofitable.

2.1   Agents, preferences and technology

The economy is populated by a continuum of risk-neutral workers and firms. We normalize the mass of workers to one and we assume the number of firms is infinitesimal relative to the number of workers. Time is continuous and the infinitely-lived workers and firms discount the future at the common rate r. Firms produce a unique final good, using labor as the only input.

The lifetime utility of a worker is defined as

$$\int_0^\infty U(c_t, h_t)e^{-rt}dt = \int_0^\infty \left[c_t + (1-h_t)b\right]e^{-rt}dt, \tag{1}$$

where c_t is consumption at time t, $h_t \in \{0, 1\}$ is the fraction of time devoted to work and b is the constant value of leisure. Without loss of generality, we assume that

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[5]For a recent analysis of U.S. plant-level productivity dynamics, see Abraham and White (2007).

workers consume their entire income at all times; so when a worker is employed, c_t equals the wage, w_t. In addition, an employed worker can set effort to zero ($h_t = 0$), which we will call "shirking". Accordingly, the flow utility of a worker who exerts effort is $U(w_t, 1) = w_t$, while that of a worker who shirks is $U(w_t, 0) = w_t + b$. Unemployed workers enjoy leisure but receive no income, and so $U(0, 0) = b$.[6]

Firms have access to a constant-returns production technology. Each firm can create a continuum of jobs, which are either vacant or filled by a worker; the output of a filled job is $Y(x, y, h) = (x + y)h$. Here x is a match-specific productivity process; its realizations are observed by both the firm and the worker but not by anyone outside the match. The second component y is an aggregate productivity process, common to all jobs and observed by all agents. Finally, note that the flow output of a job drops to zero whenever the worker shirks ($h = 0$).

We assume firms cannot monitor effort perfectly. A firm always observes its total output, which reveals the average effort level of its many employees (formally a continuum). Also, any given individual worker's effort level is observed at a fixed Poisson rate φ per unit of time, but this observation is private and cannot be verified in court. Therefore, at any point in time, a firm only observes the effort of an infinitesimal fraction (formally, zero measure) of its workforce. Faced with this moral hazard problem, firms can offer incentives by paying workers a surplus above their reservation wage, if they can credibly commit to fire anyone caught shirking. That is, in a nontrivial equilibrium,[7] wages must satisfy an incentive compatibility constraint which ensures that a worker's value of exerting effort to avoid firing exceeds the value of shirking. We will see that this no-shirking condition (NSC) may or may not bind in equilibrium.[8]

In the rest of this section we explain how this version of the shirking model can be embedded into a standard matching model of unemployment with endogenous job creation and destruction.

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[6]The assumption that shirking delivers the same utility from leisure as unemployment is just a normalization. See Section 2.4.1 for details.

[7]The model also has a trivial equilibrium in which firms create no jobs because they conjecture that workers never accept jobs or always shirk. We ignore this uninteresting alternative.

[8]This contrasts with the equilibrium in Shapiro and Stiglitz (1984), where a firm can costlessly adjust the size of its workforce, and therefore need not pay more than the wage at which the NSC binds. But when there are labor market frictions, a matched pair enjoys a surplus, and the worker's bargained share of surplus may always or sometimes suffice to satisfy the NSC, depending on parameters.

2.2 The productivity processes

After the creation of a match, its productivity is exposed to shocks. Shocks to the idiosyncratic productivity component arrive at the Poisson rate λ. The new values of x are *i.i.d.* draws from a distribution F with support $[\underline{x}, \bar{x}]$ and density f. For simplicity, we assume F is uniform, so that $f(x) = (\bar{x} - \underline{x})^{-1}$ on $[\underline{x}, \bar{x}]$.[9] Moreover, for the moment we assume newly-formed firm-worker pairs start to produce at the top of the distribution, with $x = \bar{x}$. The alternative of random initial match productivity is defined in Appendix B, and is one of the cases simulated in Section 5.

The shocks to the common component y represent exogenous fluctuations in aggregate labor productivity. By assumption, y follows a Markov chain across N distinct states y_i, for $i \in \{1, 2, ..., N\}$, ordered with y_1 denoting the lowest and y_N the highest possible realization. New draws arrive at Poisson rate μ, and the conditional probability of moving from y_i to y_j is denoted by $G_{y_j|y_i}$. The Markov transition matrix can thus be written as

$$G \equiv \begin{pmatrix} G_{y_1|y_1} & & G_{y_1|y_N} \\ & ... & \\ G_{y_N|y_1} & & G_{y_N|y_N} \end{pmatrix}$$

where column j lists the probabilities of the N possible successors of state y_j, implying that each column sums to one. We assume that G is irreducible (so for all i and j, y_i can eventually be reached from y_j), and that the matrix $\frac{1}{2}(I + G)$ exhibits first-order stochastic dominance.[10]

In equilibrium, sufficiently bad shocks to x or y will give rise to endogenous separations. In addition, we assume that matches separate for exogenous reasons at rate δ. All separations are permanent and there is no recall of previous offers.

2.3 Matching

Unemployed workers meet vacant jobs through a random matching technology. The gross rate of meetings at time t, m_t, is given by

$$m_t = M(u_t, v_t) \tag{2}$$

where u_t is the mass of unemployed, v_t is the mass of vacancies, and M is a constant-returns function. Therefore, a worker's probability of finding a matching opportunity,

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[9]A uniform distribution is not essential; see our earlier working paper version.

[10]This assumption guarantees that the transitions of y from one moment to the next, which are governed by $(1 - \mu dt)I + \mu G dt$, exhibit first-order stochastic dominance.

per unit of time, can be written in terms of tightness $\theta_t \equiv v_t/u_t$ as

$$p(\theta_t) = \frac{M(u_t, v_t)}{u_t} = M\left(1, \frac{v_t}{u_t}\right). \tag{3}$$

Similarly, the probability that an open vacancy meets a potential match is

$$q(\theta_t) = \frac{M(u_t, v_t)}{v_t} = M\left(\frac{u_t}{v_t}, 1\right), \tag{4}$$

so that $p(\theta_t) = \theta_t q(\theta_t)$.

2.4 The value of matching

We now state the Bellman equations that summarize match values for workers and firms. For the moment we impose two restrictions on the equilibrium that are known to be valid in related models (e.g. Mortensen and Pissarides 1994; Cole and Rogerson 1999). First, we assume that aggregate jump variables may depend on y and that match-specific jump variables may depend on x and y, but that neither depends on other state variables, like the unemployment rate or the cross-sectional distribution of productivity in existing jobs. Second, we assume firms follow a reservation strategy, summarized by an N-dimensional vector of reservation productivities R, with individual elements $R_i \equiv R(y_i)$. In other words, in aggregate state y_i only jobs with $x \geq R_i$ survive; the rest are destroyed. In Sections 4 and 5 we prove by construction that equilibria of this form exist.

We first write the Bellman equations under the assumption that workers never shirk; later we derive the NSC that guarantees this. Call the bargained wage $w(x, y)$ and let the value functions of employed and unemployed workers be $W(x, y)$ and $U(y)$, respectively. For any pair (x, y) in the set $\mathcal{C}(R) \equiv \{(x, y_i) : x \geq R_i\}$, which we will call the continuation region of a match, the function W must satisfy:

$$rW(x, y) = w(x, y) + \delta\left[U(y) - W(x, y)\right] + \lambda\left[\int_{R(y)}^{\bar{x}} W(x', y)f(x')dx' + F(R(y))U(y) - W(x, y)\right]$$

$$+ \mu\left[\sum_{y':x \geq R(y')} G_{y'|y}W(x, y') + \sum_{y':x < R(y')} G_{y'|y}U(y') - W(x, y)\right]. \tag{5}$$

This equation states that the flow of returns for a matched worker includes the wage plus several flows of expected capital gains and losses: possible losses from separation at rate δ, the gains from new idiosyncratic productivity draws x' at rate λ, and the gains from aggregate shocks that change the common productivity component from y to y' at rate $\mu G_{y'|y}$. Conditional on an idiosyncratic shock, the separation probability

8

is $F(R(y))$, while aggregate shocks cause separation whenever x lies below the new reservation productivity $R(y')$.

Unemployed workers obtain a flow payoff b from leisure and meet vacant firms at rate $p(\theta(y))$. The asset value of an unemployed worker, $U(y)$, thus satisfies:

$$rU(y) = b + p(\theta(y))N^W(y) + \mu \sum_{y'} G_{y'|y}\left[U(y') - U(y)\right] \qquad (6)$$

where $N^W(y)$ is the worker's expected value gain from a new job offer. Since we assume for now that new jobs are drawn from the top of the job distribution, the gain is

$$N^W(y) = W(\bar{x}, y) - U(y) \qquad (7)$$

and new jobs are always accepted in a nontrivial equilibrium.

Now consider the value functions associated with vacancies, $V(y)$, and filled jobs, $J(x, y)$. Inside the continuation region, the value of a filled job satisfies the following Bellman equation:

$$rJ(x,y) = x + y - w(x,y) + \delta\left[V(y) - J(x,y)\right] + \lambda\left[\int_{R(y)}^{\bar{x}} J(x',y)f(x')dx' + F(R(y))V(y) - J(x,y)\right]$$

$$+ \mu\left[\sum_{y':x\geq R(y')} G_{y'|y}J(x,y') + \sum_{y':x<R(y')} G_{y'|y}V(y') - J(x,y)\right] \qquad (8)$$

which has an interpretation analogous to that of (5).

We assume that maintaining a vacancy costs c per unit of time. Accordingly, for each possible y, the value of a vacancy must satisfy:

$$rV(y) = -c + q(\theta(y))N^F(y) + \mu \sum_{y'} G_{y'|y}\left[V(y') - V(y)\right] \qquad (9)$$

where $q(\theta(y))$ is the matching rate for vacant jobs and $N^F(y)$ is a firm's expected increase in value upon matching. Since new jobs come from the top of the productivity distribution, we have

$$N^F(y) = J(\bar{x}, y) - V(y). \qquad (10)$$

Lastly, we assume firms create jobs until the rents from vacant positions are exhausted, so at any moment in time

$$V(y) = 0. \qquad (11)$$

Next, we define match surplus as $S(x, y) \equiv J(x, y) + W(x, y) - U(y) - V(y)$. Summing the previous Bellman equations and simplifying, we can derive the Bellman equation for the match surplus:[11]

$$(r+\delta+\lambda+\mu)S(x,y) = x + y - b - p(\theta(y))N^W(y) + \lambda \int_{R(y)}^{\bar{x}} S(x',y)f(x')dx' + \mu \sum_{y':x\geq R(y')} G_{y'|y}S(x,y').$$

(12)

The division of this match surplus is determined in bilateral negotiations subject to the no-shirking condition of workers.

2.4.1 The no-shirking condition

To derive the NSC, note that a rational worker will never shirk in state (x, y) if the gain from shirking during a brief interval dt is less than the expected cost of a layoff in case of detection. The logic also works in the opposite direction. If it pays to shirk for a short time dt in state (x, y), then workers will always shirk in that state.

Formally, let $W^s(x, y)$ denote the value function of a worker who shirks during an interval dt and who plans to exert effort thereafter. Using (5), we can express the difference between $W^s(x, y)$ and $W(x, y)$ as

$$r\left[W^s(x,y) - W(x,y)\right] = bdt + \varphi dt[U(y) - W(x,y)] + o(dt)$$

(13)

where $o(dt)$ is a term that becomes negligible compared to dt as $dt \to 0$. Dividing by dt and taking the limit as $dt \to 0$, we find that workers never shirk in state (x, y) if[12]

$$W(x,y) - U(y) \geq \frac{b}{\varphi}.$$

(14)

This condition needs to be satisfied at each point in time, as long as the match continues, since we assume away temporary layoffs.[13] Finally, note that executing the threat to fire an observed shirker (off the equilibrium path) is an equilibrium strategy for the firm if failing to do so would cause all other workers to shirk. From the workers' viewpoint, this response is also an equilibrium strategy, because individual workers have

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[11]See our previous working paper version for some of the algebraic details.

[12]Note that in the NSC, the levels of shirking utility and monitoring frequency are irrelevant; only their ratio b/φ matters. Therefore there is no loss of generality in our assumption that the leisure derived from shirking is the same as the leisure derived from unemployment.

[13]Alternatively, we could rule out temporary layoffs endogenously by by introducing a sufficiently large maintenance cost for laid-off jobs. For an analysis of moral hazard in a model where temporary layoffs occur in equilibrium but endogenous separation does not, see Park (2007).

no way to prove that they are not shirking when all other workers stop exerting effort (recall that the fraction of time a firm observes any given worker's effort is negligible). Thus, effort is sustained by two credible trigger strategies: firms' threat to fire shirkers, and the threat of a firm-wide breakdown of discipline if any shirker is not fired.

2.5   Wages and turnover

We are now ready to describe the wage bargain and firms' turnover decisions. We make two strong but helpful assumptions about bargaining. First, we assume the bargaining outcome can be revised continuously at no cost on the initiative of the firm and/or the worker. This means that the wage must solve a bargaining game at all points in time, eliminating any indeterminacy about the time path of labor income.[14] Second, in the spirit of Barro (1977), we assume the firm and the worker play the equilibrium of their bilateral game that is jointly optimal, thus eliminating any indeterminacy about the result of their game. In the next section we show that this assumption makes sense— that is, given the behavior of the rest of the economy, there exists an equilibrium of the game played by a given worker-firm pair which both prefer to any other. Note that we are not assuming optimality at the aggregate level: we are simply ruling out the possibility that a given pair separate because they fail to recognize a feasible way to improve both their payoffs through renegotiation.

Subject to these assumptions, match surplus is shared through incentive-constrained Nash bargaining. That is, at all times, the wage $w(x, y)$ maximizes the Nash product

$$[W(x, y) - U(y)]^{\beta} J(x, y)^{1-\beta} \tag{15}$$

subject to (14). In equilibrium, the surplus of a worker therefore satisfies

$$W(x, y) - U(y) = \max\left[\beta S(x, y), b/\varphi\right] \tag{16}$$

where $\beta \in (0, 1)$ measures the relative bargaining strength of the worker. This wage rule can be derived as the perfect equilibrium of an alternating offer game (e.g. Rocheteau 2001). It has the desirable feature that the moral hazard problem only affects wage setting if the worker's threat of shirking is credible. Whenever this is the case, the

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[14]As in Macleod and Malcomson (1989), allowing either party to restart negotiation at any time eliminates the possibility of upfront or delayed transfers from one party to the other, because these would be made ineffectual by renegotiation. (Their paper obtained this result by assuming continuous renegotiation and constant default payoffs; in our case it is a result of continuous renegotiation and constant bargaining shares.)

firm has two options: to raise the wage until the NSC is satisfied, or to sever the relationship. Hence, in an ongoing relationship a firm's surplus satisfies

$$J(x,y) = \min\left[(1-\beta)S(x,y), S(x,y) - b/\varphi\right],\tag{17}$$

and a necessary condition for match continuation is

$$S(x,y) \geq b/\varphi.\tag{18}$$

3 Analysis

This section addresses several theoretical issues we have to understand in order to compute our model. First, we show that there is a unique bilaterally-optimal reservation policy for any firm-worker pair. Therefore it is meaningful to invoke the Barro (1977) argument that pairs will only separate if it is jointly optimal to do so. Thereafter, we describe the optimal surplus function and separation behavior and define general equilibrium. Readers who wish to skip these technical issues may jump to Section 4, where we solve the simpler case of just two aggregate states.

3.1 Partial equilibrium

Some previous papers have argued that match surplus sharing under an incentive compatibility constraint may not pin down a unique bargain (Den Haan et al. 1999; Mortensen and Pissarides 1999). The intuition is simple. For given aggregate labor market conditions, workers' match surplus depends negatively on the anticipated layoff rate. Hence, if workers expect a rise in job destruction, a higher wage will be required to ensure effort, but this reduces profits and so firms may choose to fire more frequently, validating workers' expectations. Thus, the non-cooperative choice of effort and reservation thresholds creates scope for multiple equilibria.

However, contrary to appearances, this potential multiplicity relates to partial equilibrium, not to general equilibrium. That is, it arises in the game played by a specific firm-worker pair, taking as given aggregate market conditions. Moreover, we will now show that for any aggregate conditions $\theta, N^W \in \mathcal{R}_+^N$, there exists a unique bilateral equilibrium reservation policy which the pair jointly prefers.

To analyze the pair's separation behavior, we must look beyond Bellman equations (5)-(12), which only describe values *inside* the continuation region $\mathcal{C}(R)$. Instead, we now calculate the value T of continuing in an arbitrary state (x,y)— possibly outside of

$\mathcal{C}(R)$— until the next aggregate or idiosyncratic shock occurs.[15] To be precise, suppose that after any change in x or y, the worker and firm expect to follow a given reservation strategy R, and they expect the value of their match to be $S(x, y)$, where S is a given non-negative function, weakly increasing in x and defined for $x \geq \underline{x}$. Then the value of staying together temporarily can be calculated from (12) as follows:[16]

$$T(x, y; S, R, \theta, N^W) \equiv (r + \delta + \lambda + \mu)^{-1} \left[x + y - b - p(\theta(y)) N^W(y) \right.$$

$$\left. + \lambda \int_{R(y)}^{\overline{x}} S(x', y) f(x') dx' + \mu \sum_{y': x \geq R(y')} G_{y'|y} S(x, y') \right]. \quad (19)$$

Incentive compatibility remains satisfied as long as $T(x, y; S, R, \theta, N^W)$ is at least equal to b/φ. But note that T itself depends on the reservation strategy R. Therefore, given any candidate R, we can calculate a new reservation strategy $\widetilde{R}$ as follows:

$$\widetilde{R}(y) = \min\{x \geq \underline{x} : T(x, y; S, R, \theta, N^W) \geq b/\varphi\}. \quad (20)$$

The true reservation strategy associated with a given surplus function S must be a fixed point of the mapping (20). We show in Appendix C that under weak regularity conditions, a fixed point exists; moreover, there is an unambiguously lowest fixed point.

Like R, we can also think of the surplus S as the solution to a fixed point problem. Inside $\mathcal{C}(R)$, surplus equals T; outside, by definition, it is zero. But T depends on S. Therefore given any candidate S, we can define a new surplus function $\widetilde{S}$ as follows:[17]

$$\widetilde{S}(x, y) = \begin{cases} 0 & \text{for} \quad x < R(y) \\ T(x, y; S, R, \theta, N^W) & \text{for} \quad x \in [R(y), \infty) \end{cases} \quad (21)$$

The surplus function must be a fixed point of (21). The following proposition shows that there is a unique fixed point of (21) which maximizes surplus; associated with it is a unique, lowest possible vector of reservation productivities. In other words, for any aggregate conditions, there is a unique reservation strategy that is jointly optimal for the worker-firm pair.

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[15]Echoing arguments from Section 2.4.1, the pair prefers to continue for an arbitrary length of time in state (x, y) if and only if they prefer to continue in state (x, y) for a brief interval dt. So there is no need to consider any off-equilibrium deviations other than the one analyzed here.

[16]Since for now we are only considering the partial equilibrium behavior of an individual pair, we do not yet impose mutual consistency between S and R and θ and N^W.

[17]It is convenient to define S for $x \geq \overline{x}$, even though these values of x never occur, because this ensures that the "min" in (20) is always well-defined.

Proposition 1. *For any $\theta, N^W \in \mathcal{R}_+^N$, there exists a unique pair $\overline{S}$ and $\underline{R}$ such that:*

1. *$\underline{R}$ is a fixed point of (20) given surplus function $\overline{S}$.*

2. *$\overline{S}$ is a fixed point of (21) given reservation vector $\underline{R}$.*

3. *If there exists another fixed point (S', R') of (20)-(21), then $\underline{R}(y) \leq R'(y)$ and $\overline{S}(x,y) \geq S'(x,y)$ for all x and y.*

Proof. See Appendix C. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The proof of Proposition 1 adapts Rustichini's (1998) method for problems with incentive constraints to address the bounding of R and S simultaneously.[18] It constructs a monotone sequence S^j, for $j \in \{0, 1, 2, ...\}$, by iterating on (21). The initial function S^0 is weakly increasing in x and y. Mapping (21) obviously preserves monotonicity in x, strengthening it to strict monotonicity on $\mathcal{C}(R)$. Given our first-order stochastic dominance assumption for y, (21) also preserves monotonicity in y as long as $x + y - b - p(\theta(y))N^W(y)$ is strictly increasing in y, meaning that for all i,

$$y_{i+1} - p(\theta(y_{i+1}))N^W(y_{i+1}) > y_i - p(\theta(y_i))N^W(y_i). \tag{22}$$

Finally, since the monotonicity properties of S are preserved at each step j, they also hold in the limit, and this implies that the limiting reservation productivities are increasing in y. Therefore we have the following corollary:

Corollary 1. *Suppose that $\theta, N^W \in \mathcal{R}_+^N$ satisfy (22). Then the jointly optimal fixed point pair $(\overline{S}, \underline{R})$ of (20)-(21) has the following properties:*

1. *Function $\overline{S}$ is strictly increasing in x for $x \in [R(y), \overline{x}]$.*

2. *Function $\overline{S}$ is weakly increasing in y.*

3. *The vector of reservation productivities $\underline{R}$ is weakly decreasing in y.*

From here on, we will assume that (22) holds. Hence, arranging the reservation productivities in ascending order we can write

$$R_{N+1} \leq R_N \leq \quad ... \quad \leq R_1 \leq R_0 \tag{23}$$

where we have defined $R_{N+1} \equiv \underline{x}$ and $R_0 \equiv \overline{x}$. Using this notation we can divide the support of x into $N+1$ (possibly empty) intervals of the form $I_i \equiv [R_i, R_{i-1})$, plus an upper "interval" $I_0 \equiv \{\overline{x}\}$ that contains newly created jobs. The monotonicity of the reservation productivities implies that jobs with $x \in I_i$ survive as long as $y \geq y_i$.

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[18]In the absence of moral hazard, existence of a unique solution to the Bellman equation for the surplus function can be proved using the contraction mapping theorem. But here, the discontinuity in mapping (21) means that it fails to be a contraction.

3.2 Characterizing the surplus function

Next, we characterize the solution to (12), noting that the the surplus function is piecewise linear, with jumps at the endpoints of the intervals defined in (23).

Inside the intervals I_i, equation (12) permits us to differentiate the surplus function with respect to x:

$$(r + \delta + \lambda + \mu)\frac{\partial S(x,y)}{\partial x} = 1 + \mu \sum_{y':x \geq R(y')} G_{y'|y}\frac{\partial S(x,y')}{\partial x}. \tag{24}$$

Notice that this equation contains just one value of x. Therefore, the equations on any segment I_i can be solved independently from those on all the other segments, and the possible existence of empty segments is irrelevant for the solution. The equations for the slopes on segment I_i can be simplified as follows:

$$\begin{pmatrix} \frac{\partial S(x,y_i)}{\partial x} \\ \cdots \\ \frac{\partial S(x,y_N)}{\partial x} \end{pmatrix} = ((r + \delta + \lambda + \mu)\,I - \mu G_i')^{-1} \begin{pmatrix} 1 \\ \cdots \\ 1 \end{pmatrix}. \tag{25}$$

where I is an identity matrix of order $N + 1 - i$ and G_i is the matrix formed from rows and columns $j \geq i$ of G (that is, the last $N - i + 1$ rows and columns of G). An inspection of (25) demonstrates that for each $i \in \{1, 2, \ldots, N\}$, the surplus function $S(x, y_i)$ gets steeper as x approaches the top of the productivity distribution. The reason is that jobs with high realizations of x are relatively stable over the cycle. By contrast, jobs in the interval I_N are destroyed after any negative shock, so the revenues from these fragile jobs are discounted at a higher rate.

The fact that the slope of S increases as we move right across intervals I_i is a standard property of models with endogenous separations. The novel feature introduced by moral hazard is the presence of discontinuities in the surplus function. As equation (21) shows, surplus $S(x, y_i)$ must be discontinuous at $x = R_i$ because an infinitesimal reduction in x would make the match unsustainable, implying a loss of at least b/φ. Moreover, the prospect of future inefficient separations also causes a jump at R_i in all higher states $y_j \geq y_i$. The size of these "secondary" jumps depends on the probability that the economy will enter aggregate state i at some point in the future.

To be more precise, for any i and j define the jump in the surplus function $S(x, y_j)$ at $x = R_i$ as

$$\Delta(R_i, y_j) \equiv \lim_{dx \to 0} S(R_i + dx, y_j) - S(R_i - dx, y_j).$$

If there is continuation on both sides of R_i in state j, we can use (12) to calculate the following formula for the jump:

$$(r + \delta + \lambda + \mu)\Delta(R_i, y_j) = \mu \sum_{y':R(y') \leq R(y_i)} G_{y'|y_j}\Delta(R_i, y'). \tag{26}$$

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Since $\Delta(R_i, y_i) \geq b/\varphi$ and G is irreducible, (26) implies that the jumps $\Delta(R_i, y_j)$ at points in the interior of $\mathcal{C}(R)$ are nonzero as long as there is moral hazard (that is, as long as $b/\varphi > 0$).

Next, consider the jump in surplus associated with a marginal job, $\Delta(R_i, y_i)$. Bearing in mind our assumption that matched pairs play their jointly optimal equilibrium, two things may occur at the reservation productivity R_i for any state i. First, suppose the segment $I_i = [R_i, R_{i-1})$ is non-empty. Then the jump in $S(x, y_i)$ at $x = R_i$ cannot exceed b/φ, because if it did, the firm and worker would benefit from continuing at some strictly lower productivity $x - dx$, and could do so without violating the NSC (recall that $T(x, y_i; S, R, \theta, N^W)$ is continuous everywhere except at the reservation productivities). On the contrary, if the segment I_i is empty, then the surplus of the marginal job can exceed b/φ. The possibility of an empty interval arises because $S(x, y_i)$ is discontinuous at R_{i-1}, so there may be a situation in which $T(R_{i-1} + dx, y_i; S, R, \theta, N^W) > b/\varphi$ while $T(R_{i-1} - dx, y_i; S, R, \theta, N^W) < b/\varphi$ for any arbitrarily small value of dx. In this case the reservation productivities of states i and $i - 1$ collapse, $R_i = R_{i-1}$. The reason is that in state i, jobs with productivity marginally less than R_{i-1} would be too short-lived to generate a surplus of at least b/φ.

In sum, it cannot be the case that $\Delta(R_i, y_i)$ exceeds b/φ when R_i is distinct from R_{i-1}, a fact which can be summarized as a set of N complementary slackness conditions:

$$dR_i dS_i \equiv (R_i - R_{i-1}) \left(S(R_i, y_i) - \frac{b}{\varphi} \right) = 0 \tag{27}$$

Equivalently, we can combine (27) with the match surplus equation (12), to obtain the following the job destruction conditions:

$$(r+\delta+\lambda+\mu)\frac{b}{\varphi} \leq R_i + y_i - b - p(\theta(y_i))N^W(y_i) + \lambda \int_{R_i}^{\bar{x}} S(x', y_i) f(x') dx' + \mu \sum_{y_j : R_i \geq R_j} G_{y_j | y_i} S(R_i, y_j) \tag{28}$$

with equality if $i = 1$ or if $R_i < R_{i-1}$ strictly.

3.3 General equilibrium

So far, we have analyzed the behavior of a matched pair as a function of the productivities (x, y). To define general equilibrium, it suffices to ensure that tightness and the value of searching for new jobs are consistent with the representative pair's surplus function and continuation behavior:

Definition 1. *A* **no-shirking equilibrium** *can be summarized by a surplus function* $S(x, y)$, *a vector* R *of reservation productivities, a tightness vector* θ, *and a vector of new job values* N^W *that satisfy the following conditions:*

1. *For each* y, *the surplus function satisfies (12) for* $x \in [R(y), \overline{x}]$, *and* $S(x, y) = 0$ *for* $x \in [\underline{x}, R(y))$.

2. *For each* y, *the surplus function satisfies the job destruction condition (28) at the reservation productivity* $R(y)$.

3. *Labor market tightness* $\theta(y)$ *and the new job value* $N^W(\theta)$ *are given by*

$$c = q(\theta(y)) \min \left\{ (1 - \beta)S(\overline{x}, y), S(\overline{x}, y) - b/\varphi \right\}, \tag{29}$$

$$N^W(y) = \max \left\{ \beta S(\overline{x}, y), b/\varphi \right\}. \tag{30}$$

To calculate equilibrium we solve a root-finding problem to find numbers dR_i and dS_i, for $i \in \{1, 2, \ldots, N\}$, consistent with this definition. See Appendix A for details.

Finally, after calculating S, N^W, θ and R, it is straightforward to simulate employment and productivity dynamics because $\theta(y)$ and $R(y)$ jump immediately to their new equilibrium values each time a shock hits the economy. Formally, let $e_t(I_i)$ denote the measure of employed workers with productivity in the set I_i at time t and let $e_t = 1 - u_t$ denote total employment. These employment aggregates evolve as follows:[19]

$$de_t(I_0) = p(\theta(y_t))u_t dt - \lambda e_t(I_0) dt \tag{31}$$

$$de_t(I_i) = \begin{cases} [(F(R_i) - F(R_{i-1}))e_t - e_t(I_i)] \lambda dt & \text{when } y_{t+dt} \geq y_i \\ - e_t(I_i) & \text{when } y_{t+dt} < y_i \end{cases} \tag{32}$$

$$du_t = -de_t = -\sum_{i=0}^{N} de_t(I_t) \tag{33}$$

where we have dropped all terms negligible relative to dt. Equation (31) defines the evolution of the mass of jobs in the top productivity interval I_0. Over a brief interval dt, the economy creates $p(\theta(y_t))u_t dt$ new jobs, and the outflow from I_0 is $\lambda e_t(I_0) dt$. These jobs continue if they draw a new x satisfying $x \geq R(y)$, and are destroyed otherwise. The mass of continuing jobs evolves according to (32). In the absence of aggregate shocks the mass $e_t(I_i)$ evolves smoothly towards a conditional steady state, but $e_t(I_i)$ drops abruptly to zero if the aggregate productivity component falls below y_i.

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[19]We briefly abuse notation by subscripting y in an inconsistent but hopefully transparent way.

4 Example: two aggregate states

Worker moral hazard can be expected to distort both the hiring and separation margins; shortly we will quantify its effects in an example calibrated for the U.S. But before moving to the numerical analysis, we want to anticipate some of these effects in a simple example with two aggregate states, booms and recessions, and a symmetric aggregate transition matrix:

$$G \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We set $\delta = 0$, so there are only endogenous separations, and assume that new jobs start at the top of the distribution, with productivity $\bar{x}$.

Mortensen and Pissarides (1994) showed that an economy of this type generates counter-cyclical fluctuations in job destruction when effort is perfectly observable. By continuity, this will still be true in the presence of a small amount of moral hazard. Hence, firms will have a core of *stable* jobs (with $x \in [R_1, \bar{x})$) that survive through booms and recessions, and in a boom they will also build up a fringe of *fragile* jobs (with $x \in [R_2, R_1)$) which will be destroyed when the economy enters a recession.

4.1 Two states: calculating the surplus function

From Section 3.2 we know that the surplus function is piecewise linear, with slopes given by (25). If there is no moral hazard problem, so that $b/\varphi = 0$, then the surplus function is continuous, as illustrated in Figure 2a; this is the model analyzed by Mortensen and Pissarides (1994). With moral hazard, there are also discontinuities at the reservation productivities, given by formula (26). Simplifying (25) and (26), we can state the surplus function as follows in terms of the reservation thresholds:

$$S(x, y_1) = \frac{x - R_1}{r + \lambda} + \frac{b}{\varphi}. \tag{34}$$

$$S(x, y_2) = \begin{cases} \frac{x - R_2}{r + \lambda + \mu} + \frac{b}{\varphi} & \text{for } x \in [R_2, R_1) \\[2mm] \frac{x - R_1}{r + \lambda} + \frac{R_1 - R_2}{r + \lambda + \mu} + \frac{r + \lambda + 2\mu}{r + \lambda + \mu} \frac{b}{\varphi} & \text{for } x \in [R_1, \bar{x}] \end{cases} \tag{35}$$

Fig. 2b illustrates this discontinuous surplus function (it reduces to Fig. 2a if $b/\varphi = 0$). In recessions the surplus has slope $\frac{1}{r+\lambda}$. In contrast, in booms it has slope $\frac{1}{r+\lambda+\mu}$ to the left of R_1 and $\frac{1}{r+\lambda}$ to the right of R_1. Intuitively, a marginal increase in idiosyncratic productivity x is less valuable in the interval of "fragile" jobs, because the match is not expected to last so long as it would if it lay in the "stable" interval.

Table 1: *Firing rates and surplus flows required by NSC (assuming $R_2 < R_1$)*

	Fragile jobs	Stable jobs
Recessions		
Current firing rate	∞	$\lambda F(R_1)$
Expected firing rate	n.a.	$\lambda F(R_1)$
Required surplus flow	n.a.	$(r + \lambda F(R_1))b/\varphi$
Booms		
Current firing rate	$\lambda F(R_2)$	$\lambda F(R_2)$
Expected firing rate	$\mu + \lambda F(R_2)$	$\lambda F(R_1)$
Required surplus flow	$(r + \mu + \lambda F(R_2))b/\varphi$	$(r + \lambda F(R_2))b/\varphi$

According to (35), the jump in the surplus function for booms, $S(x, y_2)$, that occurs at the stability threshold R_1, must equal

$$\Delta(R_1, y_2) = \left(\frac{\mu}{r + \lambda + \mu} \right) \frac{b}{\varphi} \tag{36}$$

which is the expected loss of surplus associated with separation in case of a future recession. However, this jump formula depends only on the degree of moral hazard, not on the productivity difference between booms and recessions. Therefore, (35) cannot be satisfied under all conceivable parameter values, and a different equilibrium configuration will arise as moral hazard increases, in which the reservation productivities collapse to a single value. At R_1, in order to have $R_2 < R_1$ strictly, it must be the case that $S(x, y_2) - S(x, y_1) \geq \frac{\mu b / \varphi}{r + \lambda + \mu}$. Otherwise, if booms cause only a small upward shift in the surplus function (small relative to the degree of moral hazard, $\frac{b}{\varphi}$), firms will find it unprofitable to maintain workers with $x < R_1$ in booms. This possibility is illustrated in Figure 2c.

4.2 Two states: characterizing fluctuations in separation

The possibility that the reservation productivities may collapse in the presence of moral hazard arises because workers in fragile jobs are particularly hard to motivate. To see this, consider Table 1, which compares separation rates across various situations, and analyzes their implications for the wage required to prevent shirking.

As the table shows, current firing rates are higher in recessions for all jobs than they are in booms. Fragile jobs separate immediately in recessions (indicated as an infinite firing rate in the table), and while the recession continues stable jobs separate at rate $\lambda F(R_1)$, which is at least as high as the separation rate in booms, $\lambda F(R_2)$, since $R_2 \leq R_1$. However, what matters for incentives is the *expected* rate of firing, taking into account the fact that the state may change. For fragile jobs in booms the

expected firing rate is $\mu + \lambda F(R_2)$, which factors in the rate μ at which booms end. In contrast, for marginal jobs in recessions the expected rate of firing equals the current rate $\lambda F(R_1)$, regardless of μ.

This difference in expected firing rates affects the no-shirking condition, which says that workers must expect to earn b/φ above the value of unemployment over the life of their jobs. In flow terms this means the wage must be high enough to generate the per-period flows of surplus shown in the bottom row of each section of the table. Note that if μ is close to zero, the required surplus flow in marginal jobs is higher in recessions than in booms (because $F(R_1) \geq F(R_2)$). This makes marginal workers more expensive in recessions, which raises R_1 even further relative to R_2. Therefore, when $\mu \approx 0$, including moral hazard in the model makes separation more variable, by causing a larger wave of firing at the start of any recession.

However, when μ is sufficiently large, this argument is reversed. In particular, for $\mu \geq \lambda(F(R_1) - F(R_2))$, the expected duration of marginal jobs is shorter in booms than in recessions. This forces firms to pay marginal workers a higher flow surplus in booms, encouraging them to fire more in booms, and thus tends to make the difference in the reservation productivities smaller. In fact, when μ is sufficiently large relative to other parameters, firms will raise R_2 until it coincides with R_1. Then the separation rate will be constant over the cycle; there will no longer be any fragile jobs, and there will be no burst of firing when a recession occurs. In Appendix D, we perform a comparative statics analysis of $R_2 - R_1$ in terms of $y_2 - y_1$ to derive the following criterion for time variation in the separation rate.

Proposition 2. *Consider the two-state case, assuming $y_2 - y_1$ is small, and that b/φ is small enough so that $\beta S(\overline{x}, y_i) \geq b/\varphi$ for each i. Then R_2 cannot be strictly less than R_1 unless the following inequality is satisfied:*

$$\frac{\mu b/\varphi}{r + \lambda + \mu} < \frac{y_2 - y_1}{r + \lambda F(R) + \beta q \theta/(1 - \alpha)}$$

Thus, separation varies countercyclically if the transition rate from booms to recessions is low relative to the productivity difference between booms and recessions, or if there is no moral hazard; otherwise it collapses to a constant rate.[20] This way in which moral hazard decreases the volatility of separation is obviously missed by any analysis that considers steady states only. It is also absent in the discrete-time model of Ramey and Watson (1997), because they assume idiosyncratic shocks have no persistence, which means no jobs are more 'fragile' than any others.

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[20]The fact that we are assuming only two aggregate states is not essential for this result, because the calculation just involves comparing the probability of transition between two neighboring aggregate states with the productivity gap between those states. A similar argument can be made about the possible collapse of two or more neighboring reservation thresholds when there are many aggregate states, and even as we go to the limit of a continuous aggregate productivity distribution.

Table 2: *Fluctuations in workers' initial surplus share*

	Constant shares $b/\varphi < \beta S(\overline{x}, y_1)$	Countercyclical worker share $\beta S(\overline{x}, y_1) < b/\varphi < \beta S(\overline{x}, y_2)$	Rent rigidity $\beta S(\overline{x}, y_2) < b/\varphi$
Worker share (recessions)	$\beta S(\overline{x}, y_1)$	b/φ	b/φ
Worker share (booms)	$\beta S(\overline{x}, y_2)$	$\beta S(\overline{x}, y_2)$	b/φ
Amplification of hiring?	No	Yes	Yes

4.3 Two states: characterizing fluctuations in hiring

By raising the reservation thresholds, the NSC decreases the expected duration of all jobs. Therefore it decreases match surplus, and hiring incentives, both in recessions and booms. But the degree to which hiring is reduced varies over time, especially since the NSC causes surplus shares to vary with the cycle. Table 2 shows how our current example, with two aggregate states and new jobs at the top of the distribution, generates three possible sharing regimes for the surplus of new jobs.

In the zero moral hazard case of Mortensen and Pissarides (1994), workers obtain a constant share β of initial match surplus. By continuity, this remains true for a small degree of moral hazard b/φ. As moral hazard increases, the NSC may eventually bind in new jobs. Since surplus is monotonic in y, the NSC must bind on new jobs in recessions before booms, that is, $\beta S(\overline{x}, y_1) < b/\varphi < \beta S(\overline{x}, y_2)$, which implies that newly hired workers will receive an efficiency wage in recessions and a bargained wage in booms. This makes worker's surplus share countercyclical, and firms' share procyclical. Finally, moral hazard may be so severe that new jobs receive an efficiency wage even in booms, $\beta S(\overline{x}, y_2) < b/\varphi$, which requires firms to pay workers the same rent b/φ in recessions and booms,[21] and makes firms' surplus share even more procyclical than the previous case. Thus in the last two cases, moral hazard tends to amplify fluctuations in job creation: firms profit more from hiring in booms both because aggregate productivity is higher and because they receive a higher share of match surplus.

4.4 Two states: wage compression

Another way moral hazard affects newly-created jobs is by compressing wages across those jobs, as we now show. We have emphasized how a binding NSC truncates the wage distribution from below, to maintain surplus at or above b/φ. But in fact, the NSC implies a degree of wage rigidity even when it is not binding. The reason is simple: in their initial negotiations, workers and firms foresee the possibility that the NSC will

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[21]Bruggemann and Moscarini (2007) have referred to this extreme case as 'rent rigidity'.

bind in the future. When it binds, the worker's share of the flow proceeds will exceed β, so these redistributive effects must be undone during the initial negotiations.

To see this, let $\widehat{x}_i$ denote idiosyncratic productivity level at which $\beta S(\widehat{x}_i, y_i) = b/\varphi$. Assuming for a moment that $\mu = 0$, and that $\widehat{x}_i < \overline{x}$ for $i = 1, 2$, we can derive the following expression for initial wages:

$$w(\overline{x}, y_i) = (1 - \beta)rU(y_i) + \beta(\overline{x} + y_i) - \lambda \int_{R(y_i)}^{\widehat{x}_i} \frac{b/\varphi - \beta S(x', y_i)}{\overline{x} - \underline{x}} dx'$$

The first two terms represent the bargained wage in the absence of moral hazard, and the last term is an implicit transfer from the worker to the firm which compensates for the fact that the the worker will earn more than $\beta S(x, y_i)$ whenever x falls below $\widehat{x}_i$.

This shows that the NSC compresses the cross-sectional wage distribution, for a given aggregate state. But the wage distribution is also compressed over time by a similar mechanism. For example, suppose that $\beta S(\overline{x}, y_1) < b/\varphi < \beta S(\overline{x}, y_2)$, the case of countercyclical surplus shares. Then the wage in booms equals

$$w(\overline{x}, y_2) = (1 - \beta)rU(y_2) + \beta(\overline{x} + y_2) - \lambda \int_{R(y_2)}^{\widehat{x}_2} \frac{b/\varphi - \beta S(x', y_2)}{\overline{x} - \underline{x}} dx' - \mu \left[\frac{b}{\varphi} - \beta S(\overline{x}, y_1) \right]$$

This equation shows that the worker makes two implicit transfers to the firm. As before, the integral term is a transfer which serves to compensate the firm for the possibility that may x fall below $\widehat{x}_2$ in the future. The last term is a transfer that compensates the firm for the fact that it will need to pay the worker an efficiency wage if the economy enters into a recession.

5   Calibrated results

Section 4 showed that moral hazard amplifies hiring volatility by causing time variation in firms' share of surplus, and that it affects the volatility of separation in an ambiguous way, depending on the size and frequency of aggregate shocks. We now wish to look at these effects quantitatively, and explore how all margins of labor market volatility are affected by moral hazard when the model is calibrated to U.S. data.

5.1   Data and calibration

Relevant moments from U.S. data are reported in Table 3. The series for unemployment (u), vacancies (v), and average productivity ($\overline{y}$) are taken from the FRED database;

Table 3: *Summary statistics, quarterly U.S. data, 1951:1-2006:2*

		u	v	θ	p	s	$\overline{y}$
Mean		0.0565	0.0610	0.926	1.350	0.0808	—
Standard deviation		0.193	0.197	0.374	0.164	0.0667	0.0164
Autocorrelation		0.941	0.945	0.946	0.910	0.623	0.864
Correlations							
	u	1	-0.886	-0.958	-0.946	0.577	-0.259
	v	—	1	0.976	0.920	-0.536	0.170
	θ	—	—	1	0.962	-0.581	0.202
	p	—	—	—	1	-0.461	0.194
	s	—	—	—	—	1	-0.421
	$\overline{y}$	—	—	—	—	—	1

tightness is $\theta = v/u$.[22] The data on the probability of job finding (p) and separation (s) are the series used in Shimer (2007).[23] To compute the second moments the data are logged and HP filtered with a smoothing parameter of 10^5, as in Shimer (2005).

The recent debate on labor market volatility mainly stresses the cyclical behavior of the job finding rate p and the unemployment rate u. In the data p is roughly 10 times as volatile as productivity, and fluctuations in p explain a large part of the fluctuations in u. Like p, vacancies v are also very volatile and strongly negatively correlated with unemployment. Nonetheless, Table 3 clearly indicates that the job finding rate is not the only relevant margin of adjustment. The rate at which workers enter unemployment, s, is more than 4 times as volatile as $\overline{y}$ and is negatively correlated with p and $\overline{y}$. In line with this evidence, Fujita and Ramey (2009) estimate that job destruction accounts for at least a third of the fluctuations in u.

With exogenous separation, it is straightforward to identify the matching parameters from the means of various labor market flows. Calibration is more difficult if separation is assumed to be driven by match-specific productivity shocks, so that labor flows depend on the parameters of the shock processes in complex ways. Where we cannot calculate parameters directly from observables, our calibration strategy largely follows that of Pissarides (2007).

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[22]The unemployment series is UNRATE, the unemployment rate of persons aged 16 and over, which has mean $u^* = 0.0565$ in our sample; the vacancy series is HELPWANT, the Conference Board's series of help-wanted advertising; and productivity is OPHNFB, output per hour in nonfarm business. The series were downloaded from http://research.stlouisfed.org/fred2/.

[23]Robert Shimer (2007) pointed out that it is important to correct for time aggregation bias when calculating transition probabilities from data on unemployment stocks. We use the corrected series that he constructed. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley. For additional details, please see Shimer (2007) and his webpage http://robert.shimer.googlepages.com/flows.

Our targets for θ, p, s and u come directly from Shimer's data. In his series, mean matching rates for workers and jobs are $p^* = 1.35$ and $q^* = 1.25$ per quarter, so mean tightness is $\theta^* \equiv p^*/q^* = 0.9259$. The mean rate at which employed workers separate is $s^* = 0.0808$ per quarter. Shimer also pointed out that if matching has constant returns, and all new jobs are accepted, then regressing the job finding rate on tightness gives the elasticity of matches with respect to vacancies. In our data this coefficient is 0.42, so for the matching function $m_t = m_0 v_t^{1-\alpha} u_t^\alpha$ we need $\alpha = 0.58$, and we can deduce m_0 from $p^* = m_0 (\theta^*)^{1-\alpha}$. We then parameterize bargaining so that Hosios' efficiency condition is satisfied in the absence of moral hazard, $\beta = \alpha$.

Our model postulates both an exogenous separation rate δ and an endogenous rate $\lambda F(R)$. Findings in Davis, Haltiwanger, and Schuh (1996, Chap. 2.5) offer a possible way to separate these two components. In their quarterly data, job reallocations represent only 32%-53% of worker reallocations (and they cite other studies of matched worker-firm data in which this fraction is roughly 40%). Since vacancies are filled quickly, they argue that quarterly changes in a firm's workforce must mainly reflect changes in its demand for labor, rather than workers' responses to individual factors. We feel that the idiosyncratic productivity variations in our model are best interpreted as changes in firms' need for specific types of labor, and we therefore map the fraction of endogenous separations into the ratio of job reallocation to worker reallocation. Taking this to be 40%, we have $\delta = 0.6s^*$ and $\lambda F(R) = 0.4s^*$.

No more parameters can be inferred directly from observed steady state labor market flows. In particular, endogenous separation is the product of λ and $F(R)$, neither of which is easily observable. We finish the calibration following Pissarides (2007), who assumes idiosyncratic shocks are uniformly distributed and arrive at a quarterly rate of $\lambda = 0.1$. Writing the support of these shocks as $[1 - \epsilon, 1 + \epsilon]$, steady state endogenous separations are

$$\lambda \left(\frac{R - (1 - \epsilon)}{2\epsilon} \right) = 0.4s^* \tag{37}$$

Also, Pissarides uses Hall's (2006) calibration that the worker's cost of labor b is 71% of average productivity, which requires[24]

$$b = 0.71 \frac{1 + \epsilon + R}{2} \tag{38}$$

assuming that the aggregate shock process y has mean zero.

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[24]Here we compute b as a fraction of the average x after a job is hit by a shock. This measures average productivity exactly if the initial productivity of a job is drawn from F. When jobs start at the top of the distribution, as we assume here, average productivity is a weighted average of the productivity of new and continuing jobs, but the difference is small for realistic parameter values.

Table 4: *Calibrated parameters for the U.S.: efficient benchmark*

Parameter	Value	Source
moral hazard b/φ	0	Efficiency
matching parameter m_o	1.394	CPS; Shimer (2005)
matching elasticity α	0.580	CPS; Shimer (2005)
tightness θ	0.926	CPS; Shimer (2005)
bargaining power β	0.580	Hosios condition
leisure value b	0.720	Hall (2005); calibrated
exogenous destruction rate δ	0.048	CPS; Davis et al. (1996)
risk free rate r	0.010	
vacancy cost c	0.217	calibrated
uniform distribution F	$[x - \overline{x}]/[\overline{x} - \underline{x}]$	
domain $[\overline{x} - \underline{x}]$	[0.948,1.052]	calibrated
arrival rate λ	0.100	Pissarides (2007)
arrival rate μ	1	normalization

Our parameterization must also be consistent with the job creation and job destruction conditions governing steady state flows. These equations depend on the match surplus, which is

$$S(x) = \frac{x - R}{r + \delta + \lambda} + \frac{b}{\varphi}$$

in steady state. Using this surplus function, the productivity $\hat{x}$ at which the NSC just starts to bind is given by

$$\hat{x} = \min\left\{1 + \epsilon, R + \frac{1 - \beta}{\beta}(r + \lambda + \delta)\frac{b}{\varphi}\right\}.$$

Then, using the fact that $w'(x) = \beta$ for all $x \geq \hat{x}$, and assuming new jobs start with the maximum productivity $\overline{x} = 1 + \epsilon$, the job creation condition can be written as

$$\frac{c(\theta^*)^\alpha}{m_0} = \frac{(1 - \beta)(1 + \epsilon) + \beta\hat{x} - R}{r + \delta + \lambda} \tag{39}$$

Likewise, the reservation threshold R must satisfy the job destruction condition. This condition depends on the worker's flow value of search, pN^W, which is

$$pN^W = m_0\theta^{1-\alpha}\max\left\{\frac{b}{\varphi}, \beta\left(\frac{b}{\varphi} + \frac{1 + \epsilon - R}{r + \lambda + \delta}\right)\right\}$$

if jobs start with maximum productivity. The job destruction condition is then

$$R = b + pN^W - y + (r + \delta + \lambda F(R))\frac{b}{\varphi} - \frac{\lambda\epsilon}{r + \lambda + \delta}(1 - F(R))^2 \tag{40}$$

Plugging in $\hat{x}$ and pN^W, equations (37)-(40) jointly determine ϵ, b, c, and R conditional on λ, b/φ, and the interest rate r. We initially calibrate the model abstracting

Table 5: *Simulated moments: efficient benchmark calibration*

	Variable					
	u	v	θ	p	s	$\overline{y}$
Mean	0.0595	0.0548	0.9260	1.2751	0.0806	—
Standard deviation	0.0751	0.0351	0.0561	0.0223	0.0798	0.0172
Autocorrelation	0.722	0.416	0.850	0.851	0.268	0.845
Correlations						
u	1	0.697	-0.889	-0.884	0.758	-0.891
v	—	1	-0.307	-0.299	0.662	-0.312
θ	—	—	1	0.994	-0.588	0.995
p	—	—	—	1	-0.559	0.992
s	—	—	—	—	1	-0.597
$\overline{y}$	—	—	—	—	—	1

from moral hazard ($b/\varphi = 0$), and then explore how it behaves as we raise b/φ. Table 4 shows the implied parameterization if we set quarterly rates $r = 0.01$ and $\lambda = 0.1$ (as in Pissarides, 2007). Finally, to match the standard deviation and quarterly autocorrelation of our U.S. labor productivity data we set μ to one and we assume that y takes nine evenly-spaced discrete values spanning plus or minus two standard deviations, setting the probabilities by Tauchen's method.

5.2   Results

In Table 5 we report the implications of our efficient benchmark model for the same moments shown in Table 3.[25] As can be seen, the shocks to y generate roughly 40% of the observed volatility in unemployment. However, the separation rate is more volatile than in the data, whereas fluctuations in p account only for 13.6% of the observed volatility in the job finding rate. Hence, a disproportionately large share of the fluctuations in u are driven by changes in the job destruction rate.[26]

An even more striking failure of the benchmark model is its prediction for the correlation between unemployment and vacancies. In the data these variables have a strong negative contemporaneous correlation, (with an even stronger correlation between vacancies and the one-quarter lag of unemployment, not shown in the table).

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[25]Since the model is defined in continuous time, the simulations are performed with short periods (two weeks) after an appropriate rescaling of the parameters. Simulation results in Tables 5-11 are generated by simulating 1000 histories of 240 quarters, discarding the first 40 quarters, so that the remaining 200 quarters correspond roughly to the length of the U.S. post-war period.

[26]Mortensen and Nagypál (2007b) report similar results. They also obtain a countercyclical vacancy rate but they do not discuss the implications for the Beveridge curve correlation.

Our model instead generates a strong positive contemporaneous Beveridge correlation. As explained in previous studies, this counterfactual result is due to a so-called "echo effect" (e.g. Fujita, 2004). The spike in u_t at the start of a recession immediately causes a spike in v_t since the large inflow into unemployment makes it easy for firms to locate workers. Furthermore, the model generates much less persistence in u_t and v_t than is observed in the data.

While our one-shock model generates an excessive correlation between the job finding probability and productivity, its search and matching setup does a good job with the negative correlation between job finding and separation. It is particularly successful in predicting that separation leads job finding by one quarter. In the data the negative correlation between separation and job finding increases from -0.461 contemporaneously to -0.602 with a one-period lead in separation; in the model it increases from -0.559 to -0.664 (only the contemporaneous correlations are shown in the table).

Effects of worker moral hazard in the benchmark economy

We now study the effects of a gradual increase in b/φ when all other parameters are held constant at their benchmark values. The results are reported in Table 6. The first two columns contain the data for the U.S. and our efficient benchmark and in the rest of the table we report a subset of moments for five increasing degrees of moral hazard. The values of b/φ are chosen so that they span all the surplus sharing cases described in Table 2, from constant shares to 'rent rigidity'.

Theoretically, Section 4 suggested that introducing moral hard might amplify or smooth the cyclical fluctuations in separation. But this calibration exercise suggests that the overwhelming quantitative effect of moral hazard is to make separation smoother. The log standard deviation σ_s decreases monotonically as we raise b/φ, and beyond a value of 0.05 firms choose the same reservation productivity in all nine aggregate states, so that the separation rate is constant. On the other hand, the standard deviations of p and u follow an inverted-U pattern in response to the changes in b/φ. The initial reduction in σ_p reflects the losses from inefficient churning which is more intense in good states than in bad states. The reason that the volatility of p picks up suddenly in the last column is that at this level of moral hazard, job creation is driven by the NSC in all nine states. Hence, the data reported in the last column correspond to an efficiency wage model in which the workers earn a constant rent b/φ in all jobs.[27] The implied procyclicality of firms' surplus share substantially increases the volatility

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[27]At any given point in time the cross-sectional wage distribution is degenerate because all workers earn the same wage, but the variance of the wage is non-zero because the efficiency wage moves along the cycle.

Table 6: *Effects of worker moral hazard in the benchmark economy*

	Data	Model					
Mean		0	0.01	0.02	0.05	0.1	0.2
u	0.0565	0.0595	0.0620	0.0645	0.0699	0.0764	0.0902
θ	0.9260	0.9260	0.9258	0.9255	0.9240	0.9208	0.0892
p	1.3500	1.2760	1.2760	1.2758	1.2749	1.2732	1.2565
s	0.0808	0.0806	0.0842	0.0879	0.0957	0.1053	0.1245
Standard deviation							
y	0.0164	0.0172	0.0174	0.0176	0.0177	0.0177	0.0176
u	0.1934	0.0751	0.0554	0.0381	0.0196	0.0194	0.0292
v	0.1974	0.0351	0.0225	0.0237	0.0376	0.0377	0.0586
θ	0.3737	0.0561	0.0561	0.0560	0.0557	0.0555	0.0852
p	0.1637	0.0223	0.0223	0.0222	0.0221	0.0220	0.0339
s	0.0667	0.0798	0.0539	0.0303	0	0	0
Correlations							
(v_t, u_t)	-0.8841	0.6974	0.1667	-0.6143	-0.8740	-0.8756	-0.8662
(u_t, y_t)	-0.2532	-0.8909	-0.9125	-0.9355	-0.9412	-0.9416	-0.9071
(p_t, s_t)	-0.4608	-0.5590	-0.5503	-0.5300	$---$	$---$	$---$

of p. Nonetheless, this effect is too weak to compensate the initial fall in σ_u; in the last column the standard deviation of p is 52% higher than in the efficient benchmark, but the standard deviation of u is down to 0.0292 which is just to 15% of the volatility observed in the data.

One noteworthy improvement in the performance of the model is the change in the sign of the Beveridge curve. In the last three columns of Table 6 the model replicates the strongly negative correlation between u and v. But even for a minimum employment surplus of 0.02 (10% of the efficient value of $N^W(y_9)$) the model is able to generate a Beveridge curve relationship with a $corr(v_t, u_t) = -0.6143$. Moreover, as soon as the correlation of u and v becomes strongly negative, the correlation structure changes. For example, with $b/\varphi = 0.02$, the strongest correlation is $corr(v_{t-1}, u_t) = -0.6372$, so vacancies lead unemployment by one quarter, as they do in U.S. data.

Inspection of Table 6 suggests that the sign reversal of $corr(v, u)$ is largely due to the fall in the volatility of s. This reduces the spike in unemployment when the economy is hit by a negative shock, making echo effects weaker as we raise the value of b/φ. The countercyclical fluctuations in workers' surplus share also tend to smooth the echo effects, since they discourage job creation in bad states, but this effect seems small. In fact, in our results the negative correlation between u and v becomes slightly weaker as we raise b/φ beyond the value at which s becomes a constant.

These experiments have kept all parameters the same except for the level of b/φ, in order to explore the effects of moral hazard on second moments. Of course, changing b/φ also affects means; the unemployment rate rises by roughly one half as we increase b/φ to 0.2. Therefore we have also run an experiment in which we repeat the calibration procedure at each value of b/φ, to keep first moments fixed as far as possible. Quantitatively, when recalibrating, the standard deviations of the logs of unemployment and vacancies increase to 0.0539 and 0.1043 at $b/\varphi = 0.2$, about 40% higher than the figures in Table 6. Qualitatively, though, the effects are very similar to those observed in Table 6, so we do not report them here.

5.3 Robustness

The effects of moral hazard seen in Table 6 are quite robust across a variety of parameterizations. Here we consider changes in our specification for initial match productivity, and also alternative values of b and λ.

Random initial match value
Our baseline model assumed that new jobs are created at the top of the distribution. A natural alternative is to suppose that a job's initial productivity is drawn from the same distribution as subsequent shocks to x; this setup is spelled out in Appendix B. In this case the model offers two additional margins for volatility in job finding. The first margin is the acceptance probability. At any given point in time only a share $(1 - F(R(y_t)))$ of the matches are accepted and this probability varies cyclically in the opposite direction of the firing probability. The second margin is the expected surplus share of the newly hired workers. In aggregate states with a low realized y the NSC binds in a larger proportion of jobs ($[F(\widehat{x}_i) - F(R_i)]/[1 - F(R_i)]$) than in good aggregate states with a high y. For any positive value of b/φ, the expected surplus share of the firms is therefore procyclical. This second effect becomes stronger as we tighten the NSC. In contrast, fluctuations in the acceptance probability should become weaker as we raise the value of b/φ, since moral hazard dampens fluctuations in R.

To assess the quantitative effects of the fluctuations in the hiring margin and the surplus shares, we recalibrate the model following the same procedure as before, adjusted to take into account the changed zero-profit condition of firms. In Table 7 we report the results of increasing moral hazard, holding fixed the rest of the parameters.[28]

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[28]For this setup too, we have run an experiment in which we recalibrate to keep first moments roughly constant as we increase b/φ, but the results are qualitatively similar to those in Table 7, so we omit them.

Table 7: *Effects of moral hazard: random initial match value*

Mean	Data	Model					
		0	0.01	0.02	0.05	0.1	0.2
u	0.0565	0.0611	0.0642	0.0673	0.0750	0.0848	0.1077
Standard deviation							
u	0.1934	0.0828	0.0631	0.0448	0.0186	0.0194	0.0224
v	0.1974	0.0402	0.0261	0.0236	0.0411	0.0441	0.0541
p	0.1637	0.0464	0.0390	0.0316	0.0213	0.0227	0.0273
s	0.0667	0.0775	0.0566	0.0364	0	0	0
Correlations							
(v, u)	-0.8841	0.7540	0.3756	-0.3871	-0.8620	-0.8521	-0.8300
(p, s)	-0.4608	-0.3633	-0.3528	-0.3466	$---$	$---$	$---$

Holding fixed all parameters, making new jobs random has a large effect on mean unemployment and greatly increases labor market volatility. But here, recalibrating the parameters according to the same criteria as before, random initial match productivity raises unemployment volatility somewhat at low levels of moral hazard, but the quantitative effects are small.[29] Moreover, the overall pattern of effects from moral hazard is unchanged.

The opportunity cost of employment
One parameter that has been central to the debate about the volatility puzzle is the opportunity cost of employment, b. Our simulations so far have used an intermediate value for b that amounts to 71% of labor productivity. In Tables 9 and 10 (see Appendix E) we present results assuming ratios of 40 and 80 per cent, respectively,[30] holding the remaining parameters fixed at their efficient benchmark values. As expected from previous studies, labor market volatility depends positively on the value of b; a higher opportunity cost of labor leads to stronger fluctuations in job creation because it reduces the match surplus. Nonetheless, once again we find the same pattern of results. Sufficient moral hazard eliminates the cyclical fluctuations in the separation rate, with little amplification of the fluctuations in job creation.

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[29]In our benchmark model from Table 5 the expected value of x is falling with tenure. Consequently, in new jobs the bargained wage places a lower weight on the value of forgone leisure than in existing jobs. This effect, which tends to dampen the fluctuations in job creation, is eliminated when all the realizations of x are drawn from the same distribution. For details on the relationship between tenure effects and volatility, see Mortensen and Nagypal (2007b).

[30]We avoid going as far as the Hagedorn and Manovskii (2008) calibration in order to consider a wide range of values of b/φ.

Persistence of the idiosyncratic shock process

Finally, we consider a change in the frequency of idiosyncratic shocks. As we saw in Sec. 5.1, this parameter cannot be calibrated from aggregate flow data, and we are unaware of any microeconomic estimates for the persistence and variance of match-specific productivity consistent with our setup. Therefore we have simply used the value of λ from Pissarides (2007). His model replicates the quasi-elasticity of the mean layoff rate with respect to y if λ is set to 0.1. Yet, in our model this parameter choice leads to an efficient benchmark with relatively too much volatility in separation. Therefore we explore the implications of a more persistent shock process, with $\lambda = 0.07$, which delivers smaller cyclical fluctuations in separation. The results are reported in Table 11. By decreasing fluctuations in s, this parameterization also decreases the fluctuations in u, and it slightly improves the Beveridge curve correlation since the spikes in s are smaller. Overall, though, the qualitative effects of moral hazard are the same we have seen under other parameter configurations.

6   Conclusions

This paper has characterized the dynamics of a matching model with imperfectly observable worker effort. At a theoretical level, we showed that moral hazard may increase unemployment volatility through two channels. It can amplify the fluctuations in job creation by making firms' surplus share procyclical, and it may amplify the fluctuations in the job destruction rate when aggregate shocks are relatively infrequent.

Nonetheless, when we calibrate the model to U.S. data we find that moral hazard strongly decreases the cyclical volatility of the main labor market variables. In all our experiments the introduction of moral hazard causes a gradual reduction, and eventually elimination, of the fluctuations in the job destruction rate, and this effect dominates the rise in fluctuations in the job creation rate.

This surprising effect is the result of a time-inconsistency problem. Workers in marginal jobs in booms turn out to be especially expensive to motivate, since they place little value on maintaining jobs they expect to lose quickly anyway. Therefore, having no way to commit to a long-term contract, firms may choose not to hire into such "fragile" jobs in the first place. This finding calls into question the robustness of the contractual fragility mechanism advocated by Ramey and Watson (1997). Persistence in idiosyncratic match characteristics is the crucial element absent from their work which reverses the effect of asymmetric information problems on fluctuations in match separation.

For the same reason, the introduction of worker moral hazard fails to help in solving Shimer's unemployment volatility puzzle. However, moral hazard does appear consistent with other features of labor market dynamics. It may help explain the relative smoothness of separation rates over time, as compared with rates of hiring. Nonetheless separation remains a highly countercyclical variable in our model (as long as some fluctuation in separation remains). Our model is also successful with the several other labor market correlations, like the fact that separation leads hiring, and the fact that vacancies lead unemployment. Furthermore, by partially suppressing the volatility of job destruction, moral hazard decreases the echo effects in vacancy formation. This helps strengthen the negative correlation of unemployment with vacancies, in contrast with the failure of the Beveridge curve in a number of previous papers with time-varying separation.

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A   Details of algorithm

We define an N-dimensional vector Q to summarize the complementary slackness conditions (27). Given $R_0 \equiv \overline{x}$, for $i \in \{1, 2, \ldots, N\}$ we define

$$Q_i \equiv dR_i \equiv R_i - R_{i-1} \text{ if } R_i < R_{i-1} \tag{41}$$

$$Q_i \equiv dS_i \equiv S(R_i, y_i) - b/\varphi \text{ if } R_i = R_{i-1} \tag{42}$$

Thus $Q_i < 0$ indicates that R_i is distinct from R_{i-1}, whereas $Q_i > 0$ indicates that thresholds R_i and R_{i-1} collapse to a single value.

To calculate general equilibrium it suffices to solve an N-dimensional root-finding problem for Q, as follows.

0. Guess an initial vector Q.

1. Loop over aggregate states y_i, for $i = 1$ to N, using the information in Q to calculate R_i and $S(R_i, y_i)$.

2. For each y_i, loop over intervals $I_j = [R_j, R_{j-1})$ for $j \in \{i, i-1, \ldots, 2\}$ and finally $I_1 = [R_1, R_0]$:

   (a) If $R_{j-1} < R_j$ strictly, solve (25) to calculate the increase in S on interval I_j.

   (b) If $R_{j-1} < R_j$ strictly, and $j \geq 2$, use equations (26) to calculate the jump in $S(x, y_i)$ at $x = R_{j-1}$.

At this point we have constructed an increasing, upper semi-continuous surplus function S consistent with Q. The next steps are:

3. Use equations (29), (3), and (4) to calculate tightness θ and the probabilities p and q.

4. Use (30) to calculate the worker's value N^W of a new job.

We now know all the objects that appear in the surplus equation (12). On the left-hand side of (12), Q tells us directly the value of $S(R_i, y_i)$:

$$S(R_i, y_i) = \begin{cases} b/\varphi & \text{if } Q_i < 0 \\ b/\varphi + Q_i & \text{if } Q_i \geq 0 \end{cases} \tag{43}$$

To see whether separation is optimal, we can now check whether (12) holds with the desired accuracy at the reservation productivity $x = R_i$ for each i:[31]

$$(r + \delta + \lambda + \mu) S(R_i, y_i) = R_i + y_i - b + \lambda \int_{R_i}^{\overline{x}} S(x', y_i) \, dF(x') + \mu \sum_{y_j : R_i \geq R_j} G_{y_j | y_i} S(R_i, y_j) - p(\theta(y_i)) N^W(y_i) \tag{44}$$

If we find a vector Q that satisfies the job destruction condition (44), then we have found the equilibrium surplus S, as well as R, θ, and all other equilibrium quantities.

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[31] Checking this equation involves integrating $S(x, y_i)$. The integral can be evaluated piecewise using the derivative information from step 2a.

B   Random productivity of new jobs

In the main text, we assumed all new jobs start with the maximum idiosyncratic productivity $\overline{x}$. Here we consider an alternative model with random initial idiosyncratic productivity, drawn from the same uniform distribution F as continuing jobs. This implies the following changes relative to the main text. Equation (7), which describes the worker's value $N^W(y)$ from a new job offer, is replaced by

$$N^W(y) = \int_{R(y)}^{\overline{x}} \left(W(x', y) - U(y) \right) f(x')dx'$$

This formula reflects the fact that some new jobs are rejected. Likewise, for the firm's value of a new matching opportunity, equation (10) is replaced by

$$N^F(y) = \int_{R(y)}^{\overline{x}} \left(J(x', y) - V(y) \right) f(x')dx'$$

Also, in Section 3.1, if new jobs are random there is no need to define the interval $I_0 \equiv \{\overline{x}\}$ consisting of the best jobs only. Instead, productivity $\overline{x}$ should be included in the first interval, defining $I_1 \equiv [R_1, \overline{x}]$.

In the definition of a no-shirking equilibrium, point 3. is replaced by:

$3'$. *Labor market tightness $\theta(y)$ and the new job value $N^W(y)$ are given by*

$$c = q(\theta(y)) \int_{R(y)}^{\overline{x}} \min[S(x', y) - b/\varphi, (1-\beta)S(x', y)]f(x')dx' \quad (45)$$

$$N^W(y) = \int_{R(y)}^{\overline{x}} \max[b/\varphi, \beta S(x', y)]f(x')dx' \quad (46)$$

Thus in steps 3 and 4 of the computational algorithm, q and N^W are calculated using (45) and (46). Evaluating the integrals in (45)-(46) requires us to calculate the cutoffs $\widehat{x}(y)$ at which the NSC starts to bind, because the integrals are evaluated differently to the left and right of $\widehat{x}(y)$.

Finally, the employment dynamics equations (31)-(33) are replaced by these two:

$$de_t(I_i) = \begin{cases} \left[(F(R_i) - F(R_{i-1})) \left(\lambda e_t + p(\theta(y_t))u_t \right) - \lambda e_t(I_i) \right] dt & \text{when } y_{t+dt} \geq y_i \\ - e_t(I_i) & \text{when } y_{t+dt} < y_i \end{cases} \quad (47)$$

$$du_t = -\sum_{i=1}^{N} de_t(I_i) \quad (48)$$

36

C   Proof of Proposition 1

We begin with a lemma that constructs a unique minimal reservation vector R^* conditional on any appropriate (equilibrium or nonequilibrium) surplus function S.

Lemma 1. *Given $\theta, N^W \in \mathcal{R}_+^N$, and given any nonnegative surplus function $S(x,y)$ that is weakly increasing in x for $x \geq \underline{x}$, there exists a unique vector $\underline{R}$ such that:*

1. *$\underline{R}$ is a fixed point of (20) given S, θ and N^W.*

2. *If there exists another fixed point R' of (20) then $\underline{R}(y) \leq R'(y)$ for all y.*

Proof of Lemma 1. We prove Lemma 1 by constructing a monotone, bounded sequence R^i of reservation productivity vectors.

Define the N-dimensional vector $R^0 \equiv (\underline{x}, \underline{x}, ...\underline{x})$. Given S, θ, and N^W, define a new vector R^1 by iterating once on (20) evaluated at $R = R^0$. By construction, since the minimum in (20) is selected from $x \geq \underline{x}$, we have $R^1(y) \geq R^0(y)$ for each y.

Define R^2 by iterating once on (20) evaluated at $R = R^1$. Since S is weakly increasing in x, (19) shows that T is strictly increasing and unbounded in x. Also, since S is nonnegative, T is weakly decreasing in R. Since $R^1 \geq R^0$, these monotonicity properties of T imply that $R^2(y)$ exists, and satisfies $R^2(y) \geq R^1(y)$, for all y. By induction, if we define R^{i+1} by iterating once on (20) evaluated at $R = R^i$, we obtain $R^{i+1}(y) \geq R^i(y)$ for all y and all $i \geq 0$.

We can find an upper bound for R by constructing a lower bound for T. Since S is nonnegative, each element $R(y_i)$ is less than or equal to $\hat{R}(y_i)$, defined as follows:

$$\hat{R}(y_i) \; = \; \min \left\{ x \in [\underline{x}, \infty) : \frac{x + y_i - b - p(\theta(y_i))N^W(y_i)}{r + \delta + \lambda + \mu} \geq b/\varphi \right\} \qquad (49)$$

So the increasing sequence of vectors R^i is bounded above by the vector $\hat{R}$, and therefore the sequence R^i converges to a limit $\underline{R}$.

Finally, suppose there is another fixed point R' of (20). By construction, $R' \geq R^0$. Applying (20) once to both sides of this inequality, we obtain $R' \geq R^1$. Applying (20) repeatedly to both sides, we obtain $R' \geq R^i$ for all i, and therefore $R' \geq \underline{R}$. **Q.E.D.**

We will use the notation $R^*(S, \theta, N^W)$ to indicate the minimal fixed point $\underline{R}$ identified in Lemma 1, showing explicitly its dependence on S, θ and N^W. Note that an increase in S increases T, causing $R^*(S, \theta, N^W)$ to (weakly) decrease.

Proof of Prop. 1. Rustichini (1998) advocates solving dynamic incentive-constrained models by constructing a bounded, monotone sequence of value functions. This proof adapts Rustichini's method to deal with our surplus function and reservation vector simultaneously. It is formally similar to the proof of Lemma 1.

Note that $S^0(x,y) \equiv \max\{\frac{\bar{x}+y_N}{r}, \frac{x+y_N}{r}\}$ is nonnegative, and is an upper bound to all fixed points of (21) and hence to the true surplus function. Let $R^*(S^0, \theta, N^W)$ be the minimal fixed point of (20) identified in Lemma 0. Set $R^0 \equiv R^*(S^0, \theta, N^W)$.

Define $S^1(x,y)$ by iterating once on (21), evaluated at $S = S^0$ and $R = R^0$. By construction, $S^1(x,y) \leq S^0(x,y)$ for all x and y. Also, by construction of R^0,

$T(x, y, S^0, R^0, \theta, N^W) \geq b/\varphi$ for $x \in [R^0(y), \bar{x}]$; moreover T is increasing in x. Therefore S^1 is a nonnegative function, weakly increasing in $x \in [\underline{x}, \infty)$. By Lemma 0, there exists a fixed point $R^1 \equiv R^*(S^1, \theta, N^W)$ of the mapping (20) evaluated at $S = S^1$. Since $S^1 \leq S^0$, $R^1 \geq R^0$.

Now define $S^2(x, y)$ by iterating once on (21), evaluated at $S = S^1$ and $R = R^1$. Since T is increasing in S, and $S^1 \leq S^0$, and since T is decreasing in R, and $R^1 \geq R^0$, we conclude that $S^2(x, y) \leq S^1(x, y)$ for all x and y. Also, for $R^1(y) \leq x \leq \bar{x}$, $T(x, y, S^1, R^1, \theta, N^W) \geq b/\varphi$; and T is increasing in x. Therefore S^2 is a nonnegative function, weakly increasing in $x \in [\underline{x}, \infty)$. By Lemma 0, there exists a fixed point $R^2 \equiv R^*(S^2, \theta, N^W)$ of the mapping (20) evaluated at $S = S^2$. Since $S^2 \leq S^1$, $R^2 \geq R^1$. By induction, we can define a decreasing sequence of surplus functions $S^{i+1} \leq S^i$ which all satisfy the assumptions of Lemma 0, and are therefore associated with an increasing sequence of reservation vectors $R^{i+1} \geq R^i$.

The functions S^i are all bounded below by zero. Therefore the sequence S^i converges to a limit $\bar{S}$, which is also a nonnegative function, weakly increasing in $x \in [\underline{x}, \infty)$, which has associated with it a reservation vector $\underline{R} \equiv R^*(\bar{S}, \theta, N^W)$. Since $\bar{S} \leq S^i$ for all i, and R^* is decreasing in S, $\underline{R} \geq R^i$ for all i.

Now suppose there exists another fixed point pair (S', R'). Since S^0 is an upper bound for all other fixed points of (21), and since R^* is decreasing in S, we have $S^0 \geq S'$ and $R^0 \leq R'$. Note that the mapping defined by (21) is increasing in S and decreasing in R. Iterating once on (21), we obtain $S^1 \geq S'$ and $R^1 \leq R'$. Now by induction, S^i and R^i bound S' and R' for all i, and thus in the limit we have $\bar{S}(x, y) \geq S'(x, y)$ for all x and y and $\underline{R}(y) \leq R'(y)$ for all y. **Q.E.D.**

D   Comparative statics underlying Proposition 2

To analyze the effects of moral hazard on separation rates we can perform comparative statics on the reservation thresholds R_i with respect to the aggregate shock y_i. The derivative dR/dy shows the difference in reservation cutoffs implied by a small difference in y_i across states, and this determines the mass of firing that occurs when aggregate productivity decreases.

Consider the simplified case analyzed in Section 4. Assume for concreteness that b/φ is small enough so that the firm's and worker's surplus shares are $1 - \beta$ and β in equations (29)-(30). Assume the productivity difference between booms and recessions, $dy \equiv y_2 - y_1$, is small enough for linearization to be accurate; also abbreviate $dR \equiv R_2 - R_1$ and $dS \equiv S(R_1, y_2) - S(R_1, y_1)$.

We can then linearize the job destruction equation by subtracting (28) evaluated at $y = y_1$ from the same equation at $y = y_2$. If we do so *assuming* that $R_2 < R_1$ strictly, so that $S(R_i, y_i)) = b/\varphi$, we obtain

$$0 = dR + dy - dp N^W + \lambda \int_{R_2}^{R_1} S(x', y_2) f(x') dx' + \lambda \int_{R_1}^{\bar{x}} dS f(x') dx' - \mu(b/\varphi + dS)$$

$$\approx dR + dy - dp N^W + \lambda F'(R)(b/\varphi) dR + (1 - F(R)) dS - \mu(b/\varphi + dS)$$

Table 8: *Two aggregate states: relation between y_i and thresholds R_i*

Case	Surplus at threshold	Difference in thresholds
$\mu = 0,\ b/\varphi = 0$	$S(R_1, y_1) = S(R_2, y_2) = 0$	$dR = -\frac{r+\lambda}{r+\lambda F(R)+\beta q\theta/(1-\alpha)}dy$
$\mu = 0,\ b/\varphi > 0$	$S(R_1, y_1) = S(R_2, y_2) = b/\varphi$	$dR = -\frac{r+\lambda}{r+\lambda F(R)+\beta q\theta/(1-\alpha)-(r+\lambda)\lambda F'(R)b/\varphi}dy$
$\mu > 0,\ b/\varphi > 0$	$S(R_1, y_1) = S(R_2, y_2) = b/\varphi$	$dR = \frac{[r+2\mu+\lambda F(R)+\beta q\theta/(1-\alpha)]\mu b/\varphi-(r+\lambda+\mu)dy}{r+\mu+\lambda F(R)+\beta q\theta/(1-\alpha)-(r+\lambda+\mu)\lambda F'(R)b/\varphi}$
$\mu > 0,\ b/\varphi > 0$	$dS = \frac{1}{r+2\mu+\lambda F(R)+\beta q\theta/(1-\alpha)}dy$	$R_1 = R_2 \equiv R$

On the other hand, if we linearize *assuming* that $R_2 = R_1 \equiv R$, we obtain

$$(r + \lambda + \mu)dS \ = \ dy - dpN^W + \lambda \int_R^{\bar{x}} dS f(x')dx' - \mu dS$$

Using the other equilibrium relationships to solve out for dR or dS, we find the multipliers summarized in Table 8.

The table analyzes several cases in order of increasing complexity. If $\mu = b/\varphi = 0$ (no moral hazard, and no transitions across aggregate states), an increase in aggregate productivity lowers the reservation threshold unambiguously: $dR/dy < 0$. In the second line of the table, by allowing for a small amount of moral hazard, $b/\varphi > 0$, the denominator of the multiplier becomes smaller, so dR/dy becomes *more negative*. That is, when $\mu = 0$, adding moral hazard increases the difference between the reservation thresholds, so there would be a larger boost in firing if the economy were to move from boom to recession (though at $\mu = 0$, this transition occurs with zero probability).

In the third line of the table, we continue to assume that $R_2 < R_1$ strictly, but we allow for $\mu > 0$ and $b/\varphi > 0$. Note, though, that dR is no longer proportional to dy, because the non-negligible quantity $\left[r + 2\mu + \lambda F(R) + \frac{\beta q\theta}{1-\alpha}\right]\frac{\mu b}{\varphi}$ enters the numerator of the multiplier formula. But note therefore that it is impossible to have $R_2 < R_1$ strictly unless the numerator in the third line is positive. Simplifying, we find that $R_2 < R_1$ requires

$$\frac{\mu b/\varphi}{r + \lambda + \mu} < \frac{y_2 - y_1}{r + \lambda F(R) + \beta q\theta/(1 - \alpha)}$$

as stated in Prop. 2.

Intuitively, the reservation productivities may differ if there is no moral hazard ($b/\varphi = 0$), or if the probability of moving from recession to boom is low (μ small), or if aggregate productivity is sufficiently large in booms compared with recessions (dy large). But if these conditions are not satisfied, then firms prefer not to maintain

any workers with $x < R_1$ in booms, because the wage that must be paid to these workers in order to maintain the no-shirking incentive is too high to justify continuation. In that case, the two thresholds collapse, $R_1 = R_2 \equiv R$. At $x = R$, the surplus equals b/φ in recessions, but it is strictly higher in booms, since the worker is more productive. The comparative statics equations then no longer serve to determine dR; instead, they determine the difference between the surpluses in booms and recessions, $dS \equiv S(R, y_2) - S(R, y_1) = S(R, y_2) - b/\varphi$, as shown in the last line of the table.

E   Additional tables

Table 9: *Low opportunity cost of employment*

	Data	Model					
Mean		0	0.01	0.02	0.05	0.1	0.2
u	0.0565	0.5935	0.0605	0.0617	0.0643	0.0674	0.0737
Standard deviation							
u	0.1934	0.0360	0.0270	0.0187	0.0094	0.0093	0.0092
v	0.1974	0.0172	0.0112	0.0111	0.0179	0.0179	0.0179
p	0.1637	0.0106	0.0106	0.0106	0.0105	0.0105	0.0105
s	0.0667	0.0393	0.0270	0.0154	0	0	0
Correlations							
(v, u)	-0.8841	0.7030	0.2286	-0.5634	-0.8730	-0.8738	-0.8754
(p, s)	-0.4608	-0.5420	-0.5383	-0.5229	$---$	$---$	$---$

Table 10: *High opportunity cost of employment*

	Data	Model					
Mean		0	0.01	0.02	0.05	0.1	0.2
u	0.0565	0.0597	0.0634	0.0667	0.0748	0.0844	0.1352
Standard deviation							
u	0.1934	0.1091	0.0794	0.0544	0.0284	0.0281	0.0690
v	0.1974	0.0501	0.0319	0.0351	0.0551	0.0552	0.1518
p	0.1637	0.0326	0.0326	0.0325	0.0323	0.0322	0.0862
s	0.0667	0.1135	0.0754	0.0419	0	0	0
Correlations							
(v, u)	-0.8841	0.6917	0.1105	-0.6521	-0.8745	-0.8767	-0.8327
(p, s)	-0.4608	-0.5720	-0.5593	-0.5354	$---$	$---$	$---$

Table 11: *High persistence of idiosyncratic shocks*

	Data	Model					
Mean		0	0.01	0.02	0.05	0.1	0.2
u	0.0565	0.0595	0.0610	0.0624	0.0652	0.0689	0.0767
Standard deviation							
u	0.1934	0.0543	0.0406	0.0287	0.0196	0.0195	0.0270
v	0.1974	0.0242	0.0230	0.0294	0.0375	0.0375	0.0531
p	0.1637	0.0221	0.0221	0.0221	0.0220	0.0220	0.0310
s	0.0667	0.0538	0.0347	0.0170	0	0	0
Correlations							
(v, u)	-0.8841	0.1527	-0.4917	-0.8318	-0.8728	-0.8841	-0.8635
(p, s)	-0.4608	-0.5087	-0.5024	-0.4744	$---$	$---$	$---$

Fig. 1a. Fluctuations in hiring and separation rates
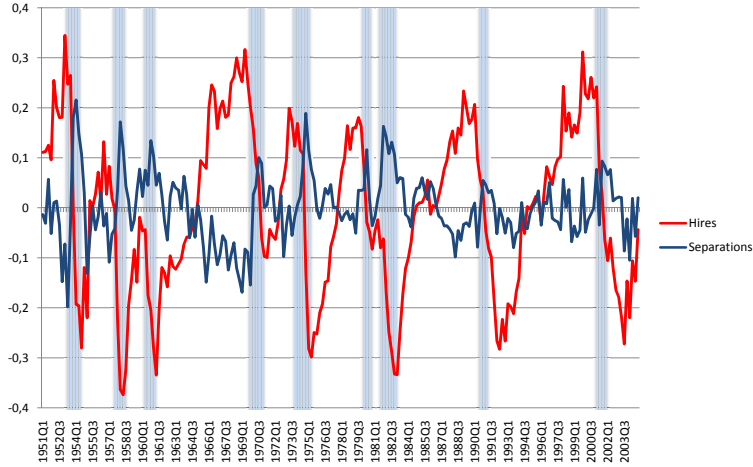
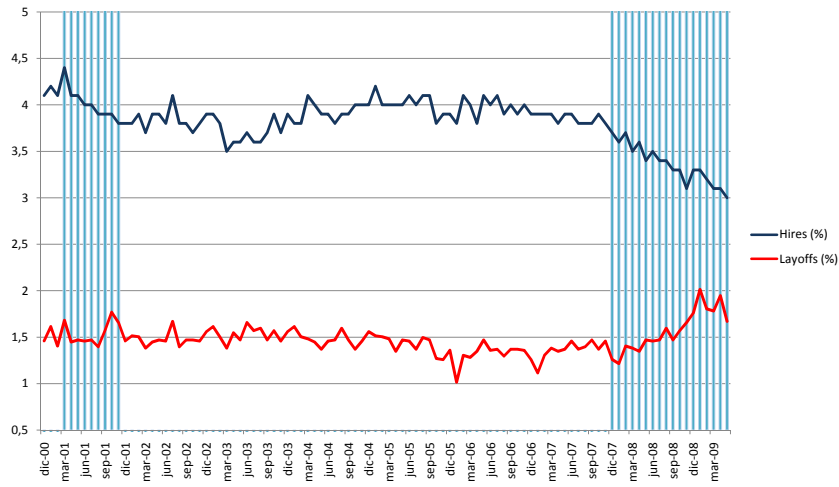**Fig. 1b. JOLTS data for hiring and layoff rates**

Fig. 1a. reports data for the cyclical fluctuations in the quarterly transition rates between employment and unemployment for the U.S. economy for the period 1951 - 2004. The original data are constructed by Shimer and are corrected for possible time-aggregation biases. For more details, see footnote 25. Fig. 1b. reports JOLTS data on hiring and layoff rates for the total non-farming sector available at www.bls.gov/jlt/. We have adjusted the layoff data for seasonal effects.

Fig. 2a. Surplus functions without moral hazard
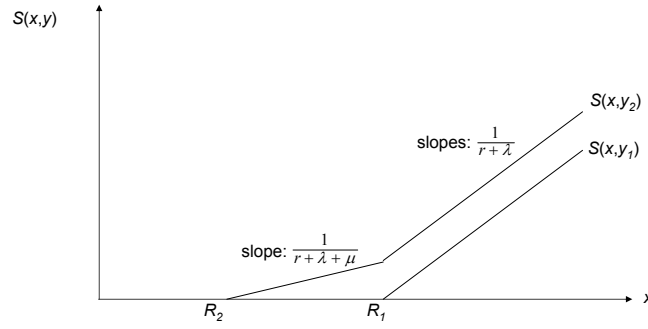
Fig. 2b. Surplus functions under moral hazard: Countercyclical job destruction
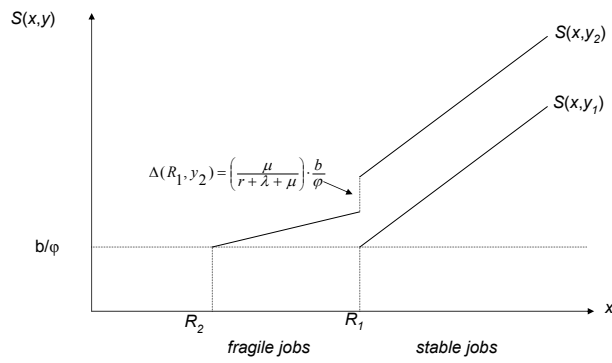
Fig. 2c. Surplus functions under moral hazard:
Acyclical job destruction