E C O N O M I C S B U L L E T I N

Mixed oligopoly, productive efficiency, and spillover

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Abstract

The purpose of this paper is to examine the public sector's cost-reducing investment when there exists the effect of RDspillover. We show that the investment in the mixed oligopoly is not higher than that in the public monopoly. When the cost-reducing effect of investment for each firm is the same, the investment in the mixed oligopoly is equal to that in the public monopoly. In such a case, the emergence of private firms has a positive impact on social welfare. Our model is an extended version of Nishimori and Ogawa (2002), which study the RDinvestment by the public sector.

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1 Introduction

In recent years, the theoretical investigation of *mixed oligopoly* has been attracting considerable attention. In this field the authors study a role of the state-owned public firm in an imperfect market. De Fraja and Delbono (1989) demonstrate a possibility of the welfare-improving privatization, and Matsumura (1998) investigates the partial privatization.

More recently, Nishimori and Ogawa (2002) investigate a strategic cost-reducing investment by the public sector. They compare the public firm's investment between the public monopoly and the mixed oligopoly, and show that the emergence of private firms has a negative impact for the incentive of the investment by the public sector. Cato (2008) generalizes this result. He does not specify the demand and cost functions, and uses only mild assumptions. Further, he considers the free entry of private firms.

The purpose of this paper is to consider the public sector's cost-reducing investment when there exists the effect of R&D spillover. That is, the public firm's investment reduces not only the cost of public firm, but also that of each private firm. Many empirical studies support the existence of the effect of technological spillover, which reduces the costs of rival firms because of knowledge leaks or imperfect patenting. In this study, it is assumed that the inverse demand function is linear, and the marginal cost of each firm is constant. The public firm's investment affects the marginal cost of each firm. We assume that the marginal contribution of the public firm's investment for each private firm is not larger than that for the public firm. It is shown that the investment in the mixed oligopoly is not higher than that in the public monopoly. When the cost-reducing effect of investment for each firm is the same, the investment in the mixed oligopoly is equal to that in the public monopoly. In such a case, the emergence of private firms has a positive impact on social welfare.

Now, we mention other related literature. Brander and Spencer (1983) analyze the strategic cost-reducing investment under Nash duopoly. The two stage Cournot game with R&D spillover is studied by d'Aspremont and Jacquemin (1988) and Suzumura (1992). The R&D competition in a mixed oligopoly is studied by using a patent race model; see Delbono and Denicolò (1993), Poyango-Theotoky (1998), and Ishibashi and Matsumura (2006).

The rest of the paper is organized as follows. Section 2 introduces our model of the mixed oligopoly, and Section 3 presents the main result. Section 4 concludes this paper.

2 Basic Model

We consider $n+1$ firms that compete in Cournot fashion. The zero-th firm is a state-owned public enterprise that maximizes social welfare. Firm i $(i = 1, 2, \ldots, n)$ is private and seeks to maximize its profit. The inverse demand function is given by $P = a - Q$, where P is the price and Q is the total output. Let the cost function of the state-owned public firm be $C_0(q_0, I) = c_0(I)q_0$ and that of a private firm be $C(q_i, I) = c(I)q_i$, where I is the cost-reducing investment by the public sector. Note that the cost of each private firm depends on the public firm's investment. The cost expense for the cost-reducing investment is $f(I)$.

Social welfare comprises consumer surplus and the firms' profit:

$$W = \int_0^Q P(q)dq - PQ + \sum_{i=0}^{n} \Pi_i = \int_0^Q P(q)dq - C_0(q_0, I) - \sum_{i=1}^{n} C(q_i, I) - f(I)$$

where Π_i is firm i's profit.

We consider two regimes: (i) public monopoly and (ii) mixed oligopoly. In the public monopoly, there exists only one state-owned public firm which maximizes social welfare. In the mixed oligopoly, there exist $n+1$ firms: one public firm and n private firms. Let W^P and W^M denote welfares in public monopoly and mixed oligopoly, respectively.

The game is summarized as follows. At the first stage, the state-owned public firm chooses the investment. At the second stage, given the public firm's investment, each firm chooses output simultaneously. We use the subgame perfect Nash equilibrium as the equilibrium concept.

We present assumptions for cost functions.

Assumption 1. For all I, $c_0(I) > c(I) > 0$.

This assumption requires that for any investment level, the marginal cost of public firm is higher than that of the private firm.

Assumption 2. For all I, $c_0'(I) \leq c'(I) < 0$.

According to this assumption, a spillover reduces the cost of each private firm. Further, this implies that the marginal contribution of the investment for the public firm is larger than that for each private firm. We believe that this is realistic.

The following assumption requires that f is increasing.

Assumption 3. $f'(\cdot) > 0$.

The next assumption guarantees that the second order condition for the public firm's maximization at the first stage is satisfied.

Assumption 4. $c''(\cdot) > 0$, $f''(\cdot) > 0$, and $f''(\cdot)$ is large enough.

Moreover, additional requirements are needed to ensure the interior solutions of the first stage problem.

Assumption 5. $\lim_{I\to 0} f'(I) = 0$ and $\lim_{I\to\infty} f'(I) = \infty$.

3 Results

In this section, we compare the investment in the public monopoly to that of the mixed oligopoly. In the rest of this paper, we assume that Assumptions 1–5 are satisfied.

Public Monopoly The case of the public monopoly can be calculated along the same line as Nishimori and Ogawa (2002). In the second stage, given I, the public firm decides its output to maximize social welfare, and, thus, the optimality condition is: $P(q_s^*) - \partial C_0(q_s^*, I)/\partial q = 0$, where q_s^* is the function of I. This implies that $a - q_s^* = c_0(I)$. In the first stage, the public firm decides its investment. The optimality condition in the first stage is as follows:

$$\frac{dW^P}{dI} = \Big[P(q_s^*) - \frac{\partial C_0(q_s^*, I)}{\partial q}\Big]\frac{dq_s^*}{dI} - \frac{\partial C_0(q_s^*, I)}{\partial I} - f'(I) \tag{1}$$

$$= -c_0'(I)q_s^* - f'(I) = 0. \tag{2}$$

The second order condition is satisfied by Assumption 4, and Assumption 5 ensures the interia solution. Let I^P denote the optimal investment in the public monopoly. That is, $-c_0'(I^P)q_s^* - f'(I^P) = 0$. Since $a - q_s^* = c_0(I^P)$, we have: $-c_0'(I^P)\{a - c_0(I^P)\} - f'(I^P) = 0$.

Mixed Oligopoly We consider the public firm's investment under the mixed monopoly when the effect of R&D spillover exists. We can solve this game by backward induction. In the second stage, given the investment of the public firm, $n+1$ firms choose their outputs. We focus the equilibrium in which the outcomes of private firms are symmetric, i.e., $q_1 = q_2 =, \ldots, = q_n$. Let q_s^{**} denote the equilibrium output of the public firm and q_p^{**} denote that of each private firm. Furthermore, Q^{**} denote the equilibrium total output. From $(n+1)$ first order conditions, we obtain the following equilibrium conditions:

$$P(Q^{**}) + q_p^{**} P'(Q^{**}) - \frac{\partial C(q_p^{**}, I)}{\partial q} = 0,$$
$$P(Q^{**}) - \frac{\partial C_0(q_s^{**}, I)}{\partial q} = 0.$$

By solving these equations, we obtain the equilibrium outputs:

$$q_p^{**} = c_0(I) - c(I),$$
$$q_s^{**} = a - (1+n)c_0(I) + nc(I),$$
$$Q^{**} = a - c_0(I).$$

Note that Assumption 1 guarantees the positive production of each private firm.

In the first stage, the public firm decides its investment. The first order condition in the first stage is as follows:

$$\frac{dW^M}{dI} = n\Big[P(Q^{**}) - \frac{\partial C(q_p^{**}, I)}{\partial q}\Big]\frac{dq_p^{**}}{dI} - \frac{\partial C_0(q_s^{**}, I)}{\partial I} - \frac{\partial C(q_p^{**}, I)}{\partial I} - f'(I), \tag{3}$$
$$= 2n\big\{c_0(I) - c(I)\big\}\big\{c_0'(I) - c'(I)\big\} - c_0'(I)\big\{a - c_0(I)\big\} - f'(I) = 0. \tag{4}$$

Since $f''(\cdot)$ is large enough, the second order condition is satisfied. Further, Assumption 5 ensures the interia solution. Let I^M denote the optimal investment in the mixed oligopoly. That is, the following equation holds: $2n\{c_0(I^M) - c(I^M)\}\{c_0'(I^M) - c'(I^M)\} - c_0'(I^M)\{a - c_0(I^M)\} - f'(I^M) = 0$.

Comparison In order to compare investments in two regimes, we derive the derivative of social welfare in the mixed oligopoly at I^P (the optimal investment in the public monopoly). From equation (4), we have the following:

$$\frac{dW^M}{dI}\Big|_{I=I^P} = 2n\big\{c_0(I^P) - c(I^P)\big\}\big\{c_0'(I^P) - c'(I^P)\big\} - c_0'(I^P)\big\{a - c_0(I^P)\big\} - f'(I^P),$$
$$= 2n\big\{c_0(I^P) - c(I^P)\big\}\big\{c_0'(I^P) - c'(I^P)\big\}.$$

By Assumption 1 and 2, we obtain:

$$\frac{dW^M}{dI}\Big|_{I=I^P} \leq 0.$$

From this equation, we obtain the following result.

Proposition 1. *Suppose that Assumption 1–5 are satisfied. Then, $I^P \geq I^M$. The equality holds if and only if $c_0'(I^P) = c'(I^P)$.*

According to this proposition, the investment in the public monopoly is higher than that in the mixed oligopoly except for a special case. This result means the robustness of Nishimori and Ogawa's (2002) result.

Now, we consider the case where the cost-reducing effect of the investment for each firm is the same, i.e., $c_0'(\cdot) = c'(\cdot)$. In such a case, we obtain the following irreverence results.

Proposition 2. *Suppose that Assumption 1–5 are satisfied. When* $c_0'(\cdot) = c'(\cdot)$,

(i) the investment in the mixed oligopoly is equal to that in the public monopoly,

(ii) the total output in the mixed oligopoly is equal to that in the public monopoly, and

(iii) the consumer surplus in the mixed oligopoly is equal to that in the public monopoly.

The proof of this proposition is very straightforward. First, since $c_0'(\cdot) = c'(\cdot)$, Proposition 1 implies $I^M = I^P$. Second, since the price is equal to the public firm's marginal cost, the total outputs are the same between two regimes. Third, when the total outputs are the same, the consumer surplus is unchanged.

Furthermore, we obtain the following result.

Proposition 3. *Suppose that Assumption 1–5 are satisfied. When* $c_0'(\cdot) = c'(\cdot)$, *social welfare in the mixed oligopoly is higher than that in the public monopoly.*

According to this proposition, when the marginal effect of the public firm's investment for each private firm is equal to that for the public firm, the emergence of private firms has a positive impact for social welfare.

The proof of Proposition 3 is as follows:

$$\begin{aligned} W^M(I^M) - W^P(I^P) &= -\{c_0(I^M)q_s^{**} + nc(I^M)q_p^{**}\} + c_0(I^P)q_s^*, \\ &> -\{c_0(I^M)q_s^{**} + nc_0(I^M)q_p^{**}\} + c_0(I^P)q_s^*, \\ &= -c_0(I^M)\{q_s^{**} + nq_p^{**}\} + c_0(I^P)q_s^*, \\ &= 0 \end{aligned}$$

where we use Proposition 2 and Assumption 1.

4 Concluding Remarks

A brief summary of the paper is presented.

(i) We incorporate R&D spillover into Nishimori and Ogawa's (2002) model.

(ii) It is shown that the investment in the mixed oligopoly is not higher than that in the public monopoly.

(iii) When the spillover effect is extremely strong ($c_0' = c'$), the existence of private firms is beneficial for social welfare.

Finally, other relevant research and directions of future work are mentioned. First, the number of private firms is exogenous in this paper. There exist several papers of the mixed oligopoly, which consider the free entry equilibrium.[1] In some countries, we often observe a mixed market under free entry of private firms. The comparison to the mixed oligopoly free entry is a reminded issue. Second, R&D investment by the state-owned public firm is considered in this study. A case of R&D investment by both the public firm and private firms would be an interesting topic for future research.

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[1] For example, see Matsumura and Kanda (2005).