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Infantile mortality and fertility decisions in a stochastic environment

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Abstract

We analyze the effect of stochastic survival of children on fertility decision in a dynastic utility model where saving, so to speak, can only be made through having children, the number of which is an endogenous decision to the household. In our stochastic framework where the rate of population change undergoes a process of Brownian motion, the probability distribution of the steady state is well determined, and saving via the number of offsprings incorporates a precautionary component. Any health care assistance proposed to reduce the variance of the Brownian process, for example, to reduce the risk of premature infantile mortality, would have a negative effect on the fertility rate and a positive effect on the per capita consumption in the long run.

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1. Introduction

It is well know that over-population is a significant problem for the development of less developed countries (LDC). For this reason, international aid programs aiming to reduce infantile mortality could have adverse effects if this reduction leads to further population growth. Building on recent work in endogenous fertility (see Tamura (2000)), this paper presents a model of fertility decisions in a stochastic environment. Our conclusion is that a reduction of infantile mortality would be beneficial, as it would actually lead to lower population growth. In fact, in this model, children perform the role of precautionary savings. Reducing mortality would reduce the need for such savings, leading to lower population growth

This paper is organized as follows: the next section describes the model; the third section presents a solution to the model when the rate of time preference is zero, it also gives the main result of this paper which implies that a decrease in the variance of the population growth process will decrease fertility and enhance the per capita consumption at the steady state equilibrium; and the last section offers some concluding remarks.

2. The Model

The model considered here is the continuous time version of the Barro-Becker[1] dynastic utility framework suggested by Barro and Sala-i-Martin (1995). In this model, the dynasty head's utility function is

$$V_0 = \int_0^\infty e^{-\rho t} N_t^{1-\epsilon} \frac{c_t^{1-\sigma}}{1-\sigma} dt, \tag{2.1}$$

where N_t is the adult population at time t, c_t is the consumption at time t, ϵ is the degree of altruism which has a constant elasticity with respect to the number of offspring and $1/\sigma$ is the elasticity of inter-temporal substitution. Finally, ρ is the rate of time preference.

We suppose a simple production process which requires essentially the use of labour, its supply depends on agent's fertility decisions. The aggregate production in the economy can be written as $F(\overline{L}, N_t)$ where $\overline{L}$ represents a factor available in fixed quantity, say land, and N_t, the adult population being used as labour. At each period, production is allocated between consumption and the cost of having

[1]See Becker and Barro (1988) and Barro and Becker (1989).

n_t children, which includes parental time, education, food, etc. In order to simplify algebraic manipulations, we assume that

$$F(\overline{L}, N_t) = \overline{L}^{1-\gamma} N_t^{\gamma} = AN_t^{\gamma}. \tag{2.2}$$

If we consider the marginal cost of having one child, b, as constant, we have the following resources constraint at time t:

$$AN_t^{\gamma} = N_t c_t + bN_t n_t. \tag{2.3}$$

In a deterministic framework, since $\dot{N}_t = (n_t - 1)N_t$, equation (2.3) can be written in terms of the following diffusion process[2]:

$$dN_t = \left[\frac{1}{b} AN_t^{\gamma} - \frac{1}{b} N_t c_t - N_t \right] dt, N_0 \text{ given.} \tag{2.4}$$

Life and death, however, are not matters of certainty. A child in our model is faced with some probability of premature death, assumed to be $1 - p$, where, for simplicity, $0 < p < 1$ is a constant and represents the probability of survival. Let E denotes the expectation operator, then $E\left[dN_t\right] = \left(pn_t - 1\right) N_t dt = \left[\frac{1}{\beta} AN_t^{\gamma} - \frac{1}{\beta} N_t c_t - N_t\right] dt$ where $\beta = b/p$ corresponds to the expected cost of a surviving child. Because all adults die during the period, $1 - p$ will be identified as infantile mortality. Assume by now that population growth undergoes a stochastic process with a mean of $E\left[dN_t\right]$ and a variance which is proportional to the number of living people $Var\left[dN_t\right] = \xi N_t dt$. This functional form of the variance implies that the survival processes of children are all mutually independent. This assumption rules out the possibility of epidemiological factors. The corresponding process can be conveniently described by the familiar Ito's stochastic differential equation:

$$dN_t = \left[\frac{1}{\beta} AN^{\gamma} - \frac{1}{\beta} N_t c_t - N_t \right] dt + \left(\xi N_t\right)^{1/2} dZ, \tag{2.5}$$

where $dZ \sim N(0, dt)$ is a Brownian motion.

The problem for an individual living at time t is to choose his consumption, c_t, and his number of offspring, n_t, under the budget constraint (2.5). Since his utility

[2] We assume that, at time t, the agent chooses to have n_t children and that the mortality rate is equal to 1 so that each agent lives just two periods. For the first period they are children and during the second, they are adults offering one unit of labour. In this context, the population growth rate becomes $n_t - 1$.

depends not only on his own consumption but also on the well being of all of his descendants, this recurrent relationship allows us to subsume utilities of immediate descendants as well as all ensuing descendants in a single patriarch's utility function (2.1). Note that this function satisfies the Strotz-consistency requirement, therefore all individual decisions could be studied by examining the patriarch's decisions regarding the number of offspring and the consumption stream over the whole time horizon. This problem therefore consists of

$$\max_{\{c_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} N_t^{1-\epsilon} U(c_t) dt \tag{2.6}$$

$$dN_t = \left[\frac{1}{\beta} A N^{\gamma} - \frac{1}{\beta} N_t c_t - N_t\right] dt + (\xi N_t)^{1/2} dZ,$$

which is a stochastic planning problem.

3. Endogenous Population under Stochastic Growth

In this section, we will analyze the solution of our problem (2.6). For this purpose, we shall use the analytical apparatus provided by Bourguignon (1974) and Merton (1975, 1990). We first derive the probability density function of the population at the stochastic steady state, then use it to find the optimal stochastic consumption function.

For the stochastic process (2.5), let the transition probability be

$$G(N,t) = \Pr\left[N_t \leq N \mid N_0\right]. \tag{3.1}$$

where N is random. We make the assumption that $G(N,t,N_0)$ has a conditional probability density $g(N,t,N_0)$ which is well defined on the interval $[0,\infty)$.

Following closely Merton (1990, p 601-604), the variation of the probability density during dt is described by the following Kolmogorov-Fokker-Planck forward equation:

$$\begin{aligned} \frac{\partial g(N,t,N_0)}{\partial t} &= \frac{1}{2}\frac{\partial^2}{\partial N^2}\left[\xi N_t g(N,t,N_0)\right] \\ &\quad -\frac{\partial}{\partial N}\left[\left(\frac{1}{\beta} A N_t^{\gamma} - \frac{1}{\beta} N_t c_t - N_t\right) g(N,t,N_0)\right]. \end{aligned} \tag{3.2}$$

It is theoretically possible to obtain a full description of the process over the time horizon by integrating equation (3.2) subject to the following initial conditions:

$$g(N,0,N_0)=\begin{cases}1, & N=N_0\\ 0, & N\neq N_0\end{cases}. \qquad (3.3)$$

On the other hand, if the stochastic steady state exists, $g(N,t,N_0)$ becomes time independent. Let us denote the probability density function at the stochastic steady state by $g(N)$. We can then find the steady-state density, $g(N)$, by solving:

$$\frac{1}{2}\frac{d^2}{dN^2}\left[\xi N g(N)\right]-\frac{d}{dN}\left[\left(\frac{1}{\beta}AN^{\gamma}-\frac{1}{\beta}Nc(N)-N\right)g(N)\right]=0. \qquad (3.4)$$

The solution of this differential equation is

$$\begin{aligned} g(N) &= \frac{m_1}{\xi N}\exp\left\{2\int^{N}\frac{Ay^{\gamma}-yc(y)-\beta y}{\beta\xi y}dy\right\} \\ &\quad +\frac{m_2}{\xi N}\int^{N}\exp\left\{2\int_{y}^{N}\frac{As^{\gamma}-sc(s)-\beta s}{\beta\xi s}ds\right\}dy. \end{aligned} \qquad (3.5)$$

Mandl (1968)[3] states a theorem on the existence of the stochastic steady state. This theorem assumes that the boundaries of the distribution are inaccessible. This means that $\lim_{x\to 0}\Pr\left[N_t\leq x\right]=0$ and $\lim_{x\to\infty}\Pr\left[N_t\geq x\right]=0$. Intuitively, if $\lim_{x\to\infty}\Pr\left[N_t\geq x\right]\neq 0$, there will be no stochastic steady state because $N=\infty$ becomes an absorbing bound. On the other hand, if $\lim_{x\to 0}\Pr\left[N_t\leq x\right]\neq 0$, there will exist a $\widehat{t}$ such as $N_t=0$, $\forall t\geq\widehat{t}$, a degenerate case not considered in this paper[4]. These bounds are inaccessible if and only if:

$$\left.\begin{aligned} &\int_0^N\frac{1}{\xi N}\int^x\exp dx\left\{2\int_y^N\frac{As^{\gamma}-sc(s)-\beta s}{\beta\xi s}ds\right\}dydx=\infty\ , \\ &\int_N^{\infty}\frac{1}{\xi N}\int^x\exp\left\{2\int_y^N\frac{As^{\gamma}-sc(s)-\beta s}{\beta\xi s}ds\right\}dydx=\infty\ , \\ &\int_0^{\infty}\frac{1}{\xi N}\exp\left\{2\int^x\frac{Ay^{\gamma}-yc(y)-\beta y}{\beta\xi y}dy\right\}dx<\infty\ . \end{aligned}\right\} \qquad (3.6)$$

These three conditions stipulated as (3.6) imply that $m_2=0$ and $m_1\neq 0$. We now obtain:

[3]Quoted by Bourguignon (1974): Mandl, P. (1968), One Dimensional Markov Processes, *Die Grundlehren der Mathematischen Wissenschaften*, Band 151, Prague

[4]It is possible to have $N=0$ as an accessible bound and under some conditions for which a stochastic steady state still exists. Bourguignon (1974) gives a discussion on those conditions.

Lemma 3.1. *Under (3.6), the population at the stochastic steady state has the following probability density function*

$$g(N) = \frac{m}{\xi N} \exp\left\{ 2 \int^{N} \frac{Ay^{\gamma} - yc(N) - \beta y}{\beta \xi y} dy \right\}, \tag{3.7}$$

where m is such that $\int_0^{\infty} g(N)\, dN = 1$.

Given the probability density function of the population at the stochastic steady state, we can find an asymptotic approximation of the consumption function. For a problem in which the utility function is time independent, Merton (1975) has shown that solving the stochastic problem over the infinite time horizon is equivalent to maximizing $E\left[N^{1-\epsilon}\frac{c^{1-\sigma}}{1-\sigma}\right]$ [5]. By solving this problem, we obtain $\widehat{c}(N)$ which is the optimal consumption rule at the stochastic steady state for a time independent utility function.

If we assume that utility is time independent, we can provide an analytical solution to the problem. In this framework, Merton's result teaches us that the consumption rule at the stochastic steady state can be obtained by solving the following problem:

$$\max_{\{c_N\}_{N=0}^{\infty}} \int_0^{\infty} N^{1-\epsilon} \frac{c_N^{1-\sigma}}{1-\sigma} g(N)\, dN \tag{3.8}$$

subject to

$$\int_0^{\infty} g(N)\, dN = 1.$$

Defining $v(N) = \int^{N} \frac{Ay^{\gamma} - yc(y) - \beta y}{y} dy$, we have $g(N) = \frac{m}{\xi N} \exp\left\{\frac{2}{\beta\xi} v(N)\right\}$. Moreover, note that $v'(N) = AN^{\gamma-1} - c(N) - \beta$ and $v''(N) = (\gamma - 1) AN^{\gamma-2}$. We can then rewrite the problem as:

$$\max \int_0^{\infty} N^{1-\epsilon} \frac{\left[AN^{\gamma-1} - v'(N) - \beta\right]^{1-\sigma}}{1-\sigma} \frac{m}{\xi N} \exp\left\{\frac{2}{\beta\xi} v(N)\right\} dN \tag{3.9}$$
$$-\lambda \left[1 - \int_0^{\infty} \frac{m}{\xi N} \exp\left\{\frac{2}{\beta\xi} v(N)\right\} dN\right].$$

[5] See also Merton (1990, p 592-598) on a lucid account of the stochastic Ramsey problem.

Euler's conditions to this problem are:

$$\begin{aligned} 0 &= -\frac{d}{dN}\left[N^{-\epsilon}\left[AN^{\gamma-1}-v'(N)-\beta\right]^{-\sigma}\frac{1}{\xi}\exp\left\{\frac{2}{\xi\beta}v(N)\right\}\right] \\ &\quad +\frac{2}{\xi^2\beta N}\exp\left\{\frac{2}{\xi\beta}v(N)\right\}\left[N^{1-\epsilon}\frac{\left[AN^{\gamma-1}-v'(N)-\beta\right]^{1-\sigma}}{1-\sigma}-\lambda\right], \end{aligned} \tag{3.10}$$

$$0=\int_0^\infty N^{1-\epsilon}\frac{c^{1-\sigma}}{1-\sigma}g(N)\,dN-\lambda\int_0^\infty g(N)\,dN, \tag{3.11}$$

$$0=1-\int_0^\infty \frac{m}{\xi N}\exp\left\{\frac{2}{\xi\beta}v(N)\right\}dN. \tag{3.12}$$

We now establish the following:

Proposition 3.2. *The stochastic process of population growth brings about a precautionary component of the saving rate in terms of offsprings beyond its certainty-equivalent level.*

Proof. From (3.11) and (3.12) we can deduce $\lambda=E\left[N^{1-\epsilon}\frac{\widehat{c}(N)^{1-\sigma}}{1-\sigma}\right]$. Computing the derivative in equation (3.10) and substituting for $v'(N)$, $v''(N)$ and λ, we obtain an equation that describes $\widehat{c}(N)$ implicitly:

$$\begin{aligned} 0 &= \widehat{c}(N)^{-\sigma}N^{-\epsilon}\left[-\frac{\epsilon}{N}+\frac{2}{\xi\beta}\left[AN^{\gamma-1}-\widehat{c}(N)-\beta\right]\right] \\ &\quad +\frac{2}{N\xi\beta}\left\{N^{1-\epsilon}\frac{\widehat{c}(N)^{1-\sigma}}{1-\sigma}-E\left[N^{1-\epsilon}\frac{\widehat{c}(N)^{1-\sigma}}{1-\sigma}\right]\right\}. \end{aligned} \tag{3.13}$$

Instead of solving (3.13) for $\widehat{c}(N)$, we get some useful information by closely examining this equation at the certainty equivalent population. Let us define the certainty equivalent population level as $\widetilde{N}$ which satisfies $\widetilde{N}^{1-\epsilon}\frac{c^{1-\sigma}}{1-\sigma}=E\left[N^{1-\epsilon}\frac{c^{1-\sigma}}{1-\sigma}\right]$. From (3.13), we find the certainty equivalent consumption equation

$$\widehat{c}\left(\widetilde{N}\right)=A\widetilde{N}^{\gamma-1}-\beta-\frac{\epsilon\xi\beta}{2\widetilde{N}}. \tag{3.14}$$

where, clearly, the consumption is reduced by the last term of the right hand side. Thus the certainty equivalent rate of saving should be increased by the same amount, which corresponds to the precautionary motive in a stochastic environment. ■

We note that $\lim_{\xi\to 0}\widehat{c}\left(\widetilde{N}\right)=A\widetilde{N}^{\gamma-1}-\beta$. This means that when the variance of the stochastic process vanishes and the probability of survival equals 1, we have exactly the same consumption rule obtained for the deterministic steady state[6]. From equation (3.14), one can easily see that, beside the increase of the children survival probability, any decrease in the variance of the stochastic process ξ will increase the per capita consumption at the stochastic steady state. Whenever an increase in health care is aimed at reducing the variance of the stochastic process, for example the international aid carried out specifically to diminish the risk of infantile mortality, it will induce a decrease in the endogenous fertility and an increase in the per capita consumption. Thus, the fear that international aids in health care provided to the LDC could exacerbate the population pressure which, often thought to impede the development process, is not always justified.

4. Concluding Remarks

When children constitute the sole asset in the kind of economy considered in this paper (where fertility decision is endogenous), saving in term of offspring will be lowered if precautionary motives would be reduced. With a Brownian process of population growth, this could be done through any effort that decreases the variance of this process. International aids specifically aimed at reducing the variance of infantile mortality rate in the LDC may be a good example for such cases, entirely justified on the ground they promote a lower population growth and a higher consumption level in the long run equilibrium state. In that perspective, they are compatible with the objective of poverty reduction in the Third World. Yet, as such, this latter objective requires much more vision and policy interventions with regard to the process of development and industrialisation which brings an economy from stage to stage as suggested in the recent work by Galor and Weil (2000). In the transition to modern world, the LDC should consider many issues, in particular the choices associated to Technology Transfers and the switch to Human and Knowledge Capital accumulation.

[6]From (2.4), we can easily see that if $\frac{dN_t}{dt}=0$, we have $c_\infty=AN_\infty^{\gamma-1}-b$. Recall also that $\beta=b/p$ which implies that if $p=1$, we have $\beta=b$.

References

[1] Barro, R.J. and G.S. Becker (1989), Fertility Choice in a Model of Economic Growth, *Econometrica*, 57, 481-501.

[2] Barro, R.J. and X. Sala-i-Martin (1995), *Economic Growth*, McGraw-Hill.

[3] Becker, G.S. and R.J. Barro (1988), A Reformulation of the Economic Theory of Fertility, *Quarterly Journal of Economics*, 103, 1-25.

[4] Bourguignon, F. (1974), A Particular Class of Continuous-Time Stochastic Growth Models, *Journal of Economic Theory*, 9, 141-158.

[5] Galor, O. and D. Weil (2000), Population, Technology, and Growth: from Malthusian Stagnation to the Demographic Transition and Beyond, *American Economic Review*, 90, 806-828.

[6] Merton, R.C. (1975), An Assymptotic Theory of Growth Under Uncertainty, *Review of Economic Studies*, 42, 375-393.

[7] Merton, R.C. (1990), Continuous-Time Finance, Blackwell.

[8] Tamura, R. (2000), Growth, Fertility and Human Capital: A Survey, *Spanish Economic Review*, 2, 183-229.