E C O N O M I C S   B U L L E T I N

Homework and performance for time−constrained students

William Neilson
Texas AMUniversity

Abstract

Most studies of homework effectiveness relate time spent on homework to test performance, and find a nonmonotonic relationship. A theoretical model shows that this can occur even when additional homework helps all students because of the way in which variables are defined. However, some students are time−constrained, limiting the amount of homework they can complete. In the presence of time constraints, additional homework can increase the spread between the performance of the best and worst students, even when homework would reduce the spread in the unconstrained case.

Citation: Neilson, William, (2005) "Homework and performance for time−constrained students." *Economics Bulletin*, Vol. 9, No. 1 pp. 1−6
Submitted: June 2, 2004. **Accepted:** February 12, 2005.
URL: http://www.economicsbulletin.com/2005/volume9/EB−04I20008A.pdf

1. Introduction

The purpose of this paper is to construct a model that relates the amount of homework students do to their performance on exams. The model is purely theoretical, rather than empirical, so that it does not reflect any actual numerical relationships between homework and performance. Rather, it develops a framework for constructing empirical studies of the relationship, and it aids the interpretation of empirical results. More importantly, it also sheds light on how homework affects students differentially in the presence of constraints on students' time.

The model itself rests on four assumptions: (1) Students have differences in abilities. Some students are just better at certain subjects than other students are. (2) Homework is beneficial, at least in small amounts. This should be unobjectionable. After all, Betts (1997, p. 22) concludes that "a student who received no homework at all would have to spend approximately two extra years in school before learning as much as an identical student who had received half an hour of homework a night during Grades 7 through 11." (3) Different students require different amounts of time to complete the same homework assignment, and these differences are correlated with their performance in the class. In particular, the best students (as measured by performance) take less time than the worst students to complete the same assignment. The idea behind this assumption is that the best students in that particular subject are able to answer questions more quickly than the worst students and therefore require less time to complete the homework, and these same factors also lead them to perform better on tests in that subject. (4) Students are time-constrained, so that at some point it becomes impossible for them to do any more homework.

These four assumptions lead to an important result. Since the worst students require the most time to complete assignments, they are the first to hit their time constraints. Consequently, further assignments help the best students, who have not yet reached their time constraints, but do nothing for the worst students, thereby increasing the spread between the best and worst students. Because of this, increasing homework may do nothing to help the students who need it the most. The lesson for homework is clear: Some homework is beneficial, in that it helps everyone who does it, but too much homework makes the weak students look even worse in comparison with the strong ones.

The model also has implications for the interpretation of data that comes from surveys. The simplest possible survey would ask students to keep a diary of the time spent on homework and then record their scores on a test, and most studies of homework effectiveness use this technique.[1] The model implies a negative correlation between the amount of time spent on homework and test performance, at least among the students spending the most time on homework. The immediate deduction from this fact would be that some students should spend less time on homework, since students who spend less time on homework get better test scores. This turns out to be an artifact of using the wrong measure of homework, and it leads to the wrong conclusion. In fact, the negative correlation can arise even when it is explicitly assumed that more homework always helps performance.

[1] Examples include Aksoy and Link (2000) and the studies used by Cooper (1989).

2. Ability, Homework, and Performance

Let a_i be a measure of the student's raw ability. Let h_i be the number of units of homework a student completes, where a "unit" of homework is determined by how much homework is assigned, not be how long it takes to complete it. So, for example, a unit of homework could be an assignment that the teacher believes will take the average student thirty minutes to complete. Students take a standardized test, and their scores reflect both their innate ability and the homework they have done. Let s_i be student i's test score, and let S be the production function that transforms homework and ability into a test score: $s_i = S(a_i,h_i)$. It is assumed that higher ability leads to higher test scores, so that $\partial S/\partial a > 0$, which corresponds to assumption (1) in the introduction. More homework also leads to higher test scores, so that $\partial S/\partial h > 0$, corresponding to assumption (2) in the introduction.

The simplest possible study to perform on the impact of homework on performance would ask students to keep a diary of the time spent on homework and then determine the relationship between time spent and performance. Rather than providing a measure of the partial derivative $\partial S/\partial h$, however, it would find a measure of the derivative of a different function. Let t_i be the amount of time spent on homework by student i. Then $t_i = T(a_i,h_i)$, so that time spent depends on both ability and the amount of homework assigned. Obviously, if the student faces no time constraints and does all of his homework, $\partial T/\partial h > 0$. The third assumption, which is crucial and drives the results, is that $\partial T/\partial a < 0$, so that students who have higher ability complete their homework assignments more quickly.

To see why, again think of a "unit" of homework as an amount that would take an average student thirty minutes to complete. An above-average student, who has a higher-than-average level of ability, would require less than thirty minutes to complete the assignment. The same traits that make this student able to complete the homework more quickly, such as more advanced thought processes, greater understanding of the material, or a better grasp of the vocabulary, would also help the student perform better than average on the exam. In the same way, a below-average ability student would require more time to complete the assignment, and would also tend to do worse than average on the test.

Proposition 1. Suppose that all students are assigned the same amount of homework, and there are no constraints on their time. Then $ds/dt < 0$.

Proof. By assumption, $t_i = T(a_i,h_i)$ and $s_i = S(a_i,h_i)$. Since all students receive and complete the same amount of homework, $h_i = h$ for all i. Since $\partial T(a,h)/\partial a < 0$, and ignoring h as an argument of the functions, it is possible to define implicitly the function $a(t)$ with $da/dt < 0$. Now define the function $s(t) = S(a(t),h)$. Then $ds/dt = (\partial S/\partial a)\cdot(da/dt) < 0$. ❑

Proposition 1 states that if all students are assigned the same amount of homework, a survey of the type described above will find a negative relationship between time spent on homework and performance. The reasoning behind the result is that the same factors that help a student do better on an exam also enable them to complete their homework more quickly, which implies that the students with the highest performance spend the least time doing homework, and the students with the worst performance spend the most time.

Since it shows a decreasing relationship between time spent on homework and test performance, the data from such a survey would be easy to misinterpret. In fact, it leads to two false implications. The first is that some students could increase their performance by spending less time on homework, essentially doing only part of the assignment. After all, students who spend only 20 minutes on a 30-minute assignment get higher test scores than students who work twice that long. But, the assumption that $\partial S/\partial h > 0$ states clearly that students of all ability levels increase their performance by doing *more* homework, not less. So, even though the data would appear to support it, the conclusion that students would benefit from less homework is simply wrong.

The second unwarranted implication is that students are getting too much homework. After all, if the best scores are earned by students who work for 20 minutes on a 30-minute assignment, would it not follow that a 20-minute assignment would be more beneficial than a 30-minute one? Once again, the assumption that $\partial S/\partial h > 0$ implies that students benefit from more homework, not less, so reducing the size of the homework assignment hurts everyone, including the best students.

To get an accurate measure of how homework affects performance, a study must be designed to capture the derivative $\partial S/\partial h$ instead of ds/dt. Asking students to record the time spent on homework yields a false negative correlation. Instead, the researcher must identify homework "units" and then measure the number of "units" assigned, and correlate that with test performance. Betts (1997) uses this technique in his study showing how increases in homework can improve school quality.

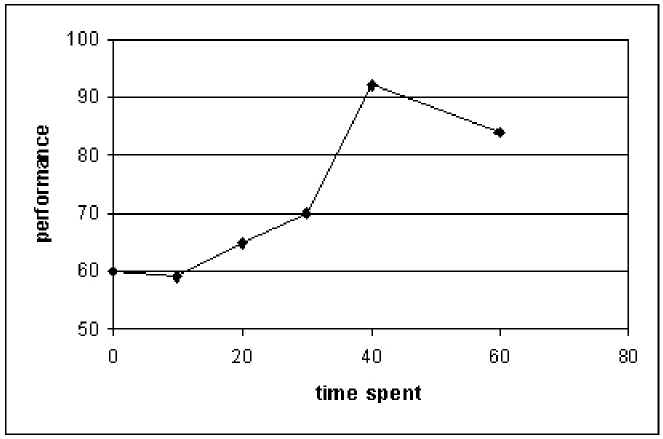
If one measures time spent on homework instead of the amount of homework assigned, and if different students have different amounts of homework assigned, then all sorts of patterns can arise. For example, suppose that there are six students who have the characteristics in Table 1. The students are ordered by ability from low to high, but they get different amounts of homework. Figure 1 graphs the relationship between time spent on homework and the test score. Looking at the graph, one could surmise that the optimal amount of homework is 40 minutes, since that leads to the highest test score.[2] However, such a conclusion is unfounded. All students would do better with more homework.

TABLE 1

Student	Time per unit of homework	Units of homework assigned	Time spent on homework	Test score
1	60	1/6	30	59
2	50	0	0	60
3	40	1/2	20	65
4	30	1	30	70
5	20	3	60	84
6	10	4	40	92

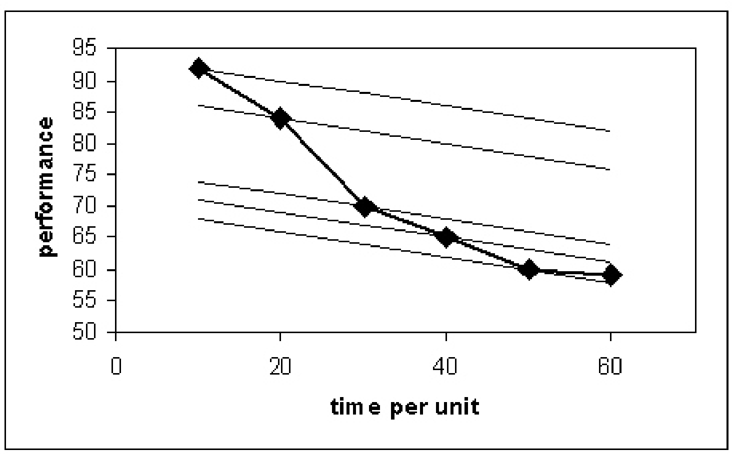
[2] In fact, Cooper and Valentine (2001) include a figure that is shaped very much like Figure 1, with the same conclusions about the optimal amount of homework.

Figure 1



A more useful graph that shows the true relationships is found in Figure 2. The horizontal axis is the time it takes to complete one unit of homework, so that movements to the right correspond to decreases in ability. The thin lines are the test scores holding the amounts of homework fixed, with the highest line corresponding to four units of homework and the lowest line corresponding to zero units. (Since increases in homework improve test scores, higher lines correspond to more homework.) The heavy line shows the performances of the six students in Table 1, and the graph shows the effects of both increased ability and increased homework.

FIGURE 2



3. Time Constraints

The fourth assumption of the paper, which has not yet been used, is that students are time-constrained. There are many reasons why students have constraints on the amount of time they can spend on homework. All students need sleep, some are involved in sports or other extracurricular activities, some have jobs, and so on. For simplicity the initial assumption is that all students have the same time constraint, followed by a brief discussion of the case of different time constraints.

Let M be the amount of time each student has available for doing homework. For ease of exposition, suppose that each unit of homework takes the same length of time for a given student, i.e. T is homogeneous of degree one with respect to homework. Then the most homework student i can do is $M/T(a_i,1)$ units, since each unit takes time $T(a_i,1)$. Order the students so that student 1 has the lowest ability and therefore takes the longest time to complete a homework assignment, and student n has the most ability and requires the least time to complete an assignment.

Proposition 2. Assume that all students have the same time constraint M and the same amount of assigned homework h. For any two students i and j with $a_i > a_j$, there exists a level of homework above which $s_i - s_j$ is nondecreasing in h.

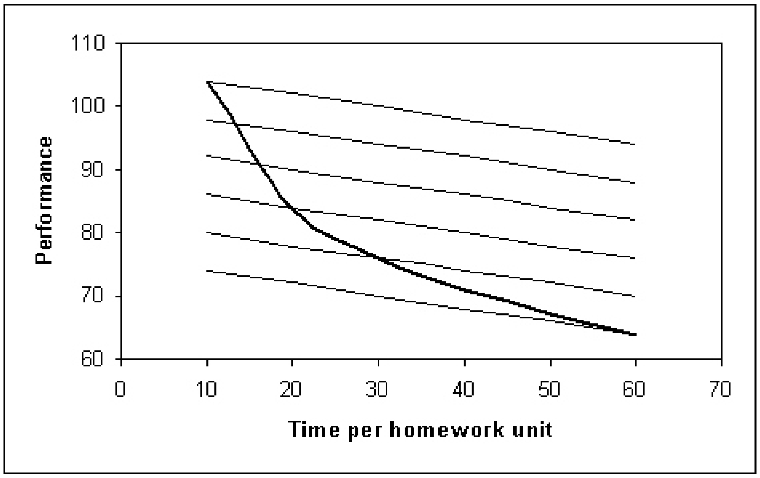
Proof. Define $h_j = M/T(a_j,1)$, so that h_j is the most homework student j can complete when his time is constrained to M. Since T is decreasing in a, $h_i > h_j$. For h in $[h_j, h_i)$, a small increase in h causes s_i to rise but causes no change in s_j since j's time constraint is binding , so that $s_i - s_j$ increases. For $h \geq h$, a small increase in h causes no change in either s_i or s_j since both time constraints are binding, and there is no change in $s_i - s_j$. For $h \geq h_j$, then, an increase in h leads to either an increase in $s_i - s_j$ or it leaves it unchanged. ❑

The reasoning behind Proposition 2 is straightforward. The students who hit their time constraints first are the ones who require the most time to complete a homework assignment. When the slowest students have reached the constraint but the fastest students have not, further homework only affects the fastest students. Furthermore, these are the best students, so additional homework helps only the best students, and does nothing for the worst students, increasing the performance gap between the best and worst students. So, even if homework would help the lower-performing students catch up, once time constraints are taken into consideration additional homework may help the higher-performing students pull away.

Figure 3 depicts the results of Proposition 3 in a different way. Assume that the time constraint is 60 minutes, and that the horizontal axis measures the length of time it takes to complete an assignment. The thin lines correspond to the performance profiles with different amounts of homework assuming that students are unconstrained, with the bottom line corresponding to $h = 1$ and the top line corresponding to $h = 6$. The thick line shows the performances when all students reach their time constraints. The thick line is steeper than the thin lines, and so homework increases the spread between students.

Both Proposition 2 and Figure 3 show that when students are time constrained, additional homework can help the better students but not the worse ones, thereby increasing any performance gap between the best and the worst students. Both figures were drawn with the assumption that all students face the same time constraint. This assumption is unrealistic, though. If some students have more time to devote to homework than others, the students with the least time will hit their constraints first. In these circumstances, additional homework helps the better students and the ones without outside activities, but not the worse students or the ones with extracurricular activities.

FIGURE 3



4. Conclusion

Using a model based on four simple assumptions, this paper produces some clear results. First, if there are no time constraints, or if there are time constraints but students have not yet reached them time constraints, homework can benefit everyone, although simple surveys might misrepresent this fact. Second, once time constraints come into play, additional homework can help the strongest students but not the weakest, thereby increasing any performance gap.

These results might be taken to suggest that there is an optimal amount of homework related to the time constraint. For example, one might consider as optimal the largest amount of homework that all students can fit inside their time constraints, which makes it the amount of homework that just meets either the lowest-ability student's time constraint or the most time-constrained student's time constraint, whichever is lower. Of course, finding this amount of homework is complicated by the facts that students take many courses, often with different teachers, and that different students excel at different subjects. Students who take a shorter-than-average length of time to complete an assignment in one subject might need a longer-than-average amount of time to complete one in a different subject, but a single time constraint governs all of the homework assignments combined.

One should be cautious about trying to find an optimum, though, because finding an optimum requires defining an objective function to maximize, and it must surely include such considerations as the mix of student abilities in the class, value judgments about which students to target, and, of course, the time it takes for teachers to create and grade homework assignments. All of these are well beyond the scope of this paper.

References

Aksoy, T. and C.R. Link (2000), "A Panel Analysis of Student Mathematics Achievement in the US in the 1990s: Does Increasing the Amount of Time in Learning Activities Affect Math Achievement?" *Economics of Education Review* **19**, 261-277.

Betts, J. (1997), "The Role of Homework in Improving School Quality," UC San Diego Discussion Paper.

Cooper, H. (1989), *Homework*, New York: Longman.

Cooper, H. And J.C. Valentine (2001), "Using Research to Answer Practical Questions About Homework," *Educational Psychologist* **36**, 142-153.